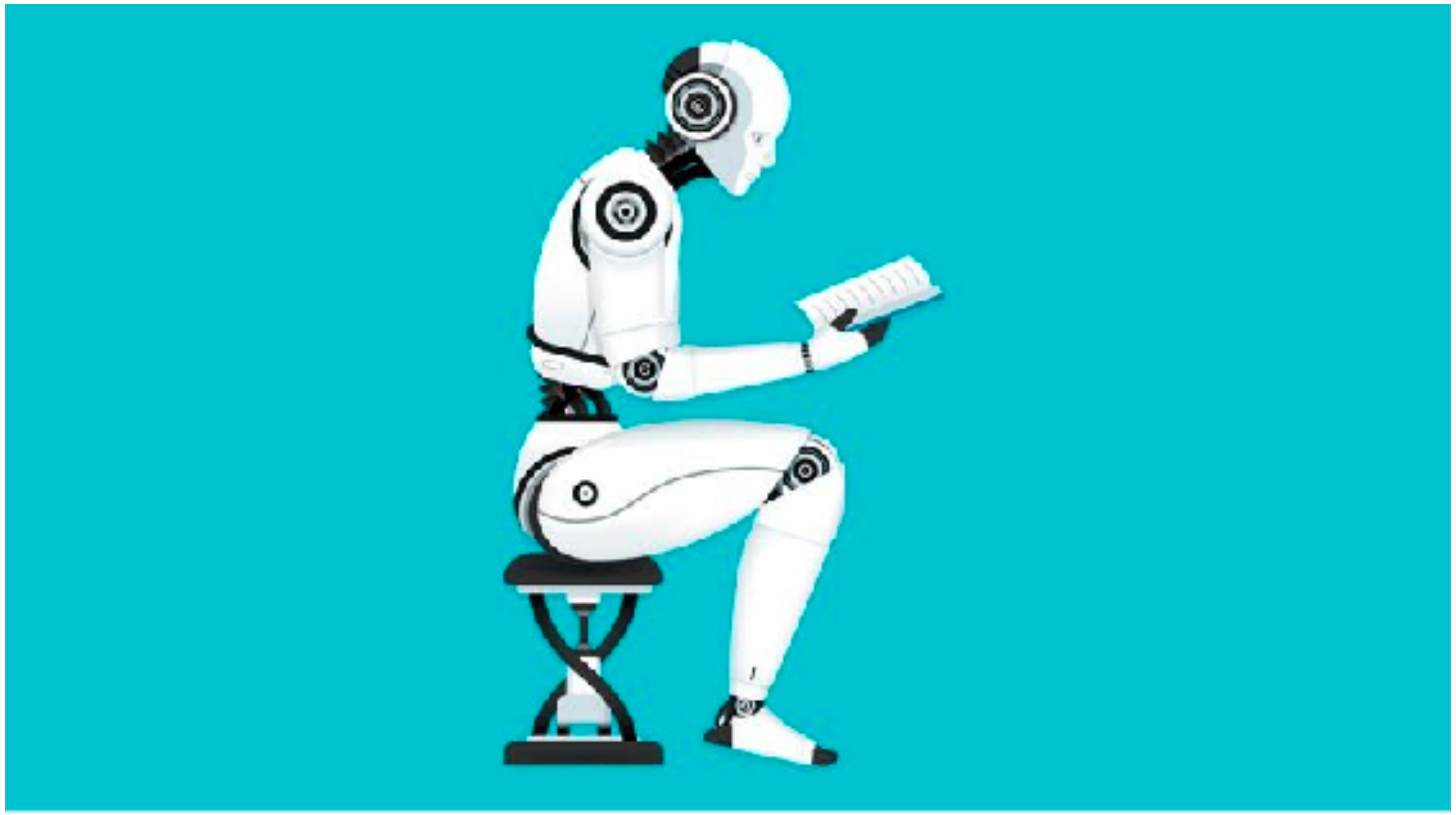
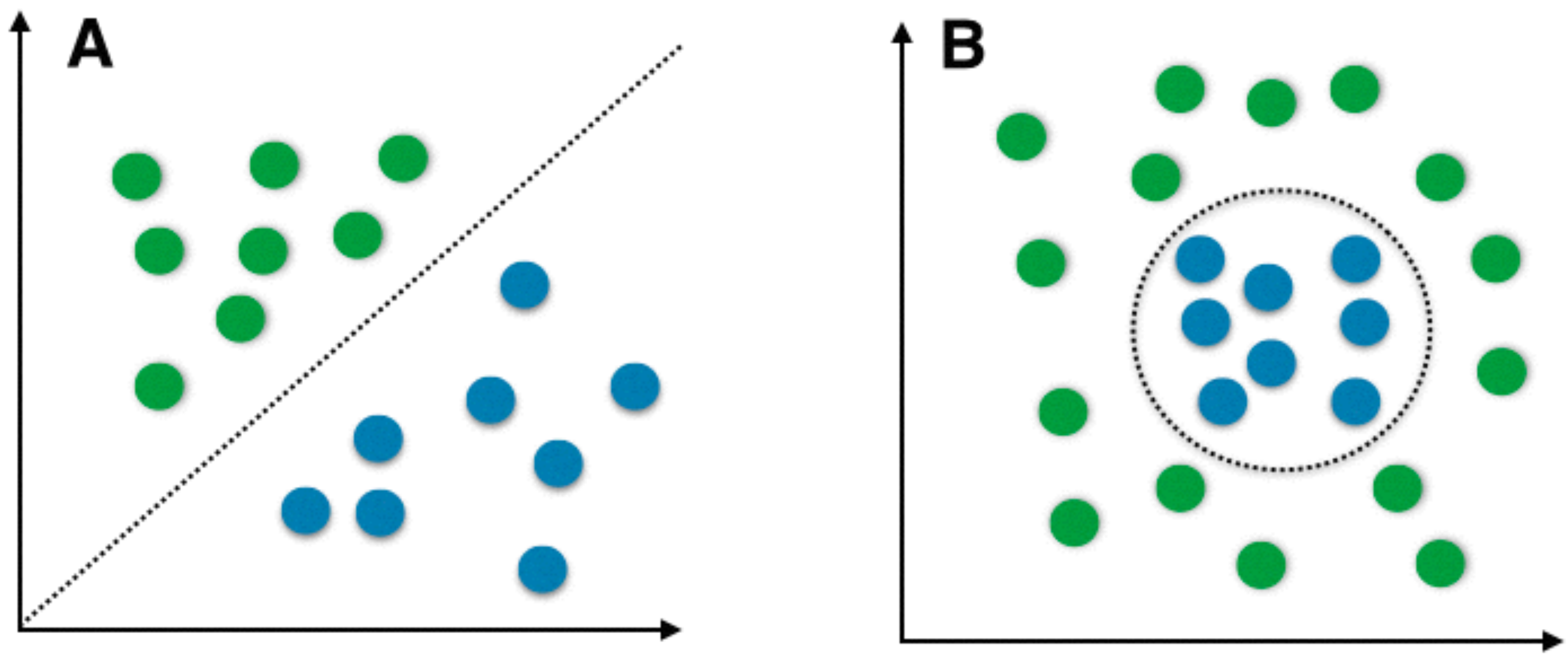


Classification

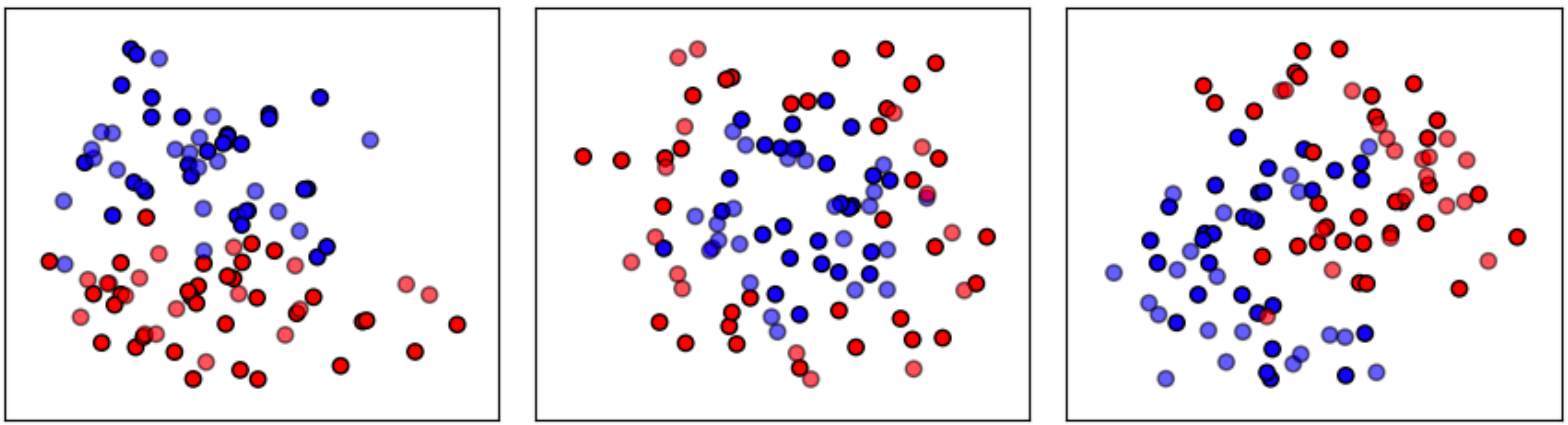
Santi Seguí | 2019-2020

Classification

What happens when the response is qualitative (also named categorical)?



Which is the optimal boundary for these datasets?



Supervised Learning

Supervised Learning

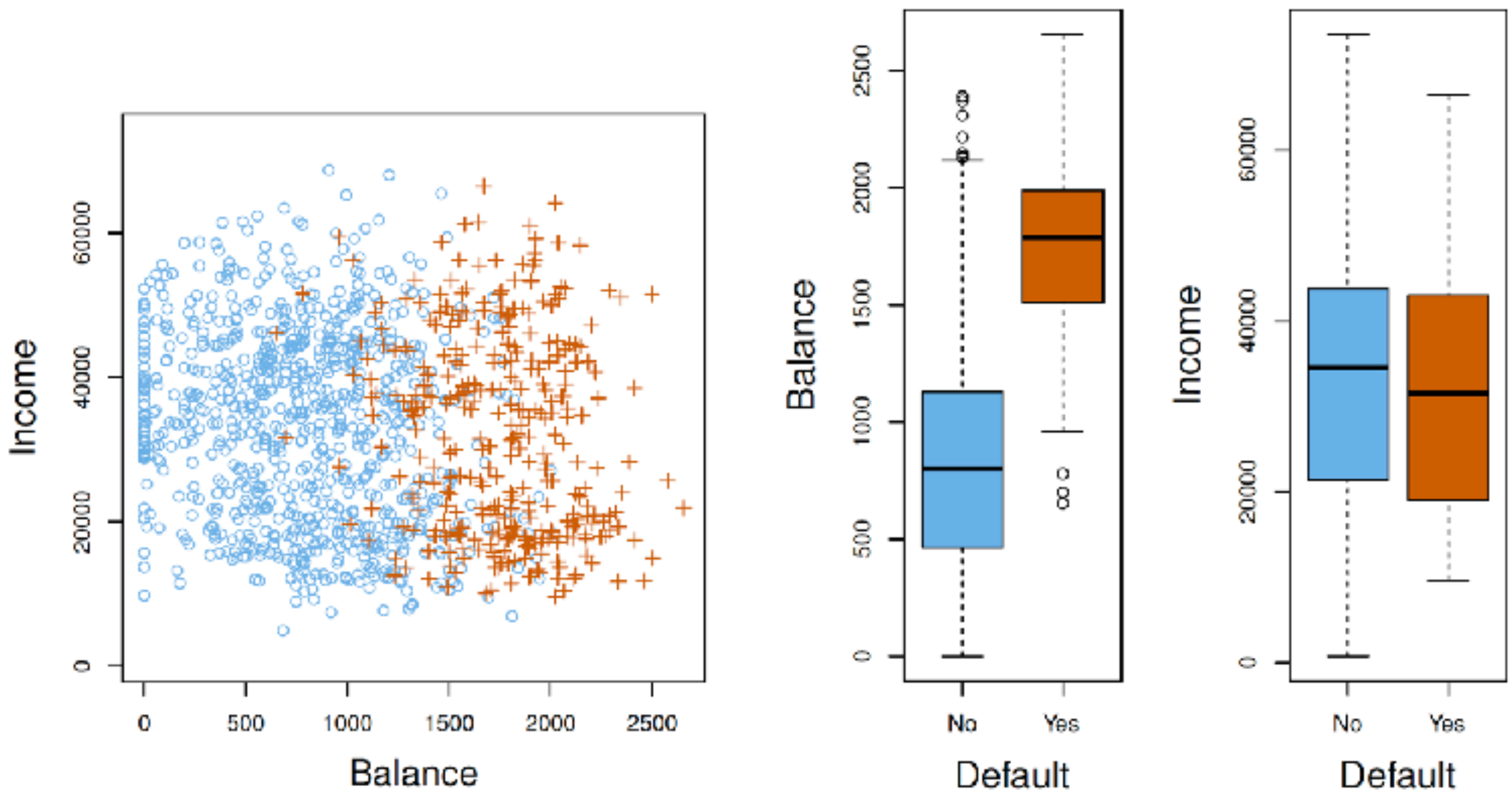
Regression →

Linear Regression	Bayesian Regression	Support Vector Regressor	Decision Trees
Lasso, Ridge & Elastic Net	Polynomial Regression	Random Forest	....
KNN Regression	Kernel Ridge Regression	Neural Networks	....

Classification →

KNN	Naive Bayes Classifier	Linear SVM’s	GBM
Logistic Regression	Decision Tree	Kernel SVM	...
Random Forest	Boosting	Neural Network	...

Linear Regression?



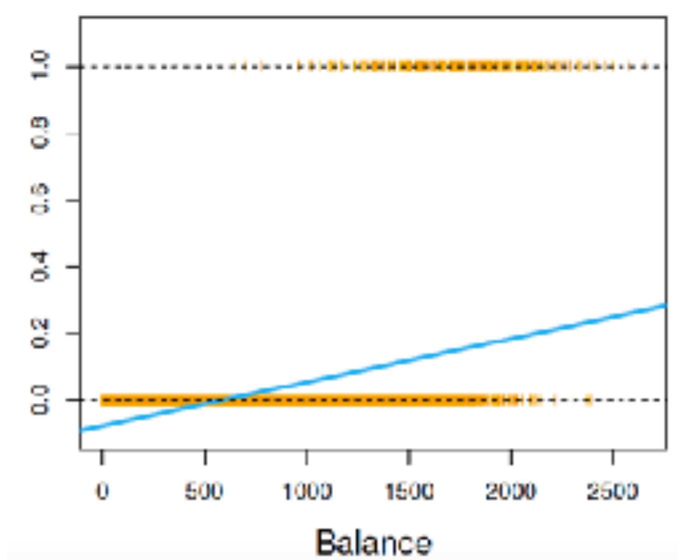
Can we use Linear Regression?

- Suppose for the Default classification task that we code as:

$$Y = \begin{cases} 0 \text{ if No} \\ 1 \text{ if Yes} \end{cases}$$

- Can we simply perform a linear regression of Y on X and classify as Yes if $\hat{Y} > 0.5$?

$$p(X) = \beta_0 + \beta_1 X$$



Can we use Linear Regression?

- Suppose for the Default classification task that we code as:

$$Y = \begin{cases} 0 \text{ if No} \\ 1 \text{ if Yes} \end{cases}$$

- Can we simply perform a linear regression of Y on X and classify as Yes if $\hat{Y} > 0.5$?
  - In this case of a binary outcome, linear regression does a good job as a classifier, and is equivalent to linear discriminant analysis which we discuss later.
  - Since in the population $E(Y|X = x) = Pr(Y = 1|X = x)$, we might think that regression is perfect for this task.
  - However, linear regression might produce probabilities less than zero or bigger than one. Logistic regression is more appropriate.

Can we use Linear Regression?

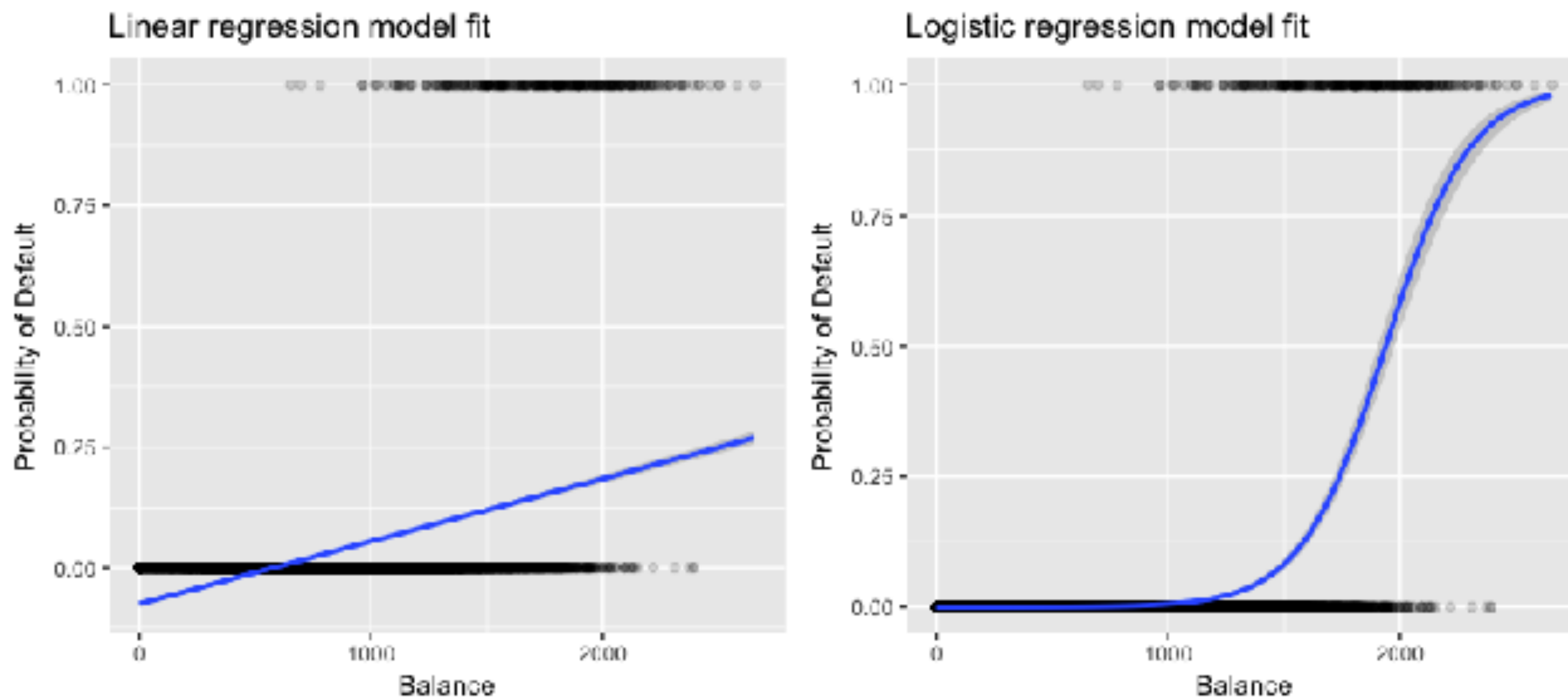
- Supose that we are trying to predict medical conditions of a patient in emergency room based on their symptoms. Imagine there is three possible diagnosis: **stroke**, **drug overdose** and **epileptic seizure**.

- We could consider encoding these values as a quantitative responses variable, Y, as follows:

$$Y = \begin{cases} 1 \text{ if stroke} \\ 2 \text{ if drug overdose} \\ 3 \text{ if epileptic seizure} \end{cases}$$

Any problem?

Logistic Regression



The orange marks indicate the response Y, either 0 or 1. Linear regression does not estimate $Pr(Y = 1 | X)$ well. Logistic regression models the probability the Y belongs to a particular category.

$$p(X) = \frac{e^{\beta_0 + \beta_1 X}}{1 + e^{\beta_0 + \beta_1 X}}$$

Logistic Regression

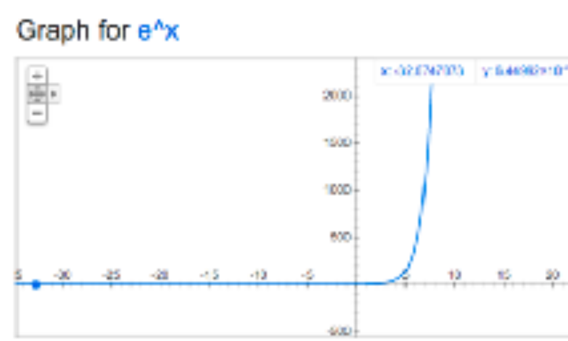
- How should we model the relationship between $p(X) = Pr(Y = 1 | X)$ and X using a function that gives outputs between 0 and 1 for all values of X?

- Logistic regression uses the form:

$$p(X) = \frac{e^{\beta_0 + \beta_1 X}}{1 + e^{\beta_0 + \beta_1 X}}$$

($e = 2.71828$ is a mathematical constant [Euler's number.])

- Remember the e_x function is:



- It is easy to see that no matter what values 0,1 or X take, $p(X)$ will have values between 0 and 1.

Logistic Regression

- How should we model the relationship between $p(X) = Pr(Y = 1 | X)$ and X using a function that gives outputs between 0 and 1 for all values of X?

- Logistic regression uses the form:

$$p(X) = \frac{e^{\beta_0 + \beta_1 X}}{1 + e^{\beta_0 + \beta_1 X}}$$

- A bit of rearrangement we find that:

$$\frac{p(X)}{1 - p(X)} = e^{\beta_0 + \beta_1 X}$$

- and taking the logarithm in both sides we arrive at:

$$log\left(\frac{p(X)}{1 - p(X)}\right) = \beta_0 + \beta_1 X$$

- which is called the log odds or logit transformation of p(X).

Maximum Likelihood

- As in linear regression, the coefficients β_0 and β_1 are unknown, and must be estimated using the training data.

- To fit the model we use a method called **maximum likelihood**. The basic intuition behind maximum likelihood to fit a logistic regression model as follows: we seek estimates β_0 and β_1 such that the predicted probabilities $\hat{p}(x_i)$ of default for each individual, corresponds as closely as possible to the individual's observed default status.

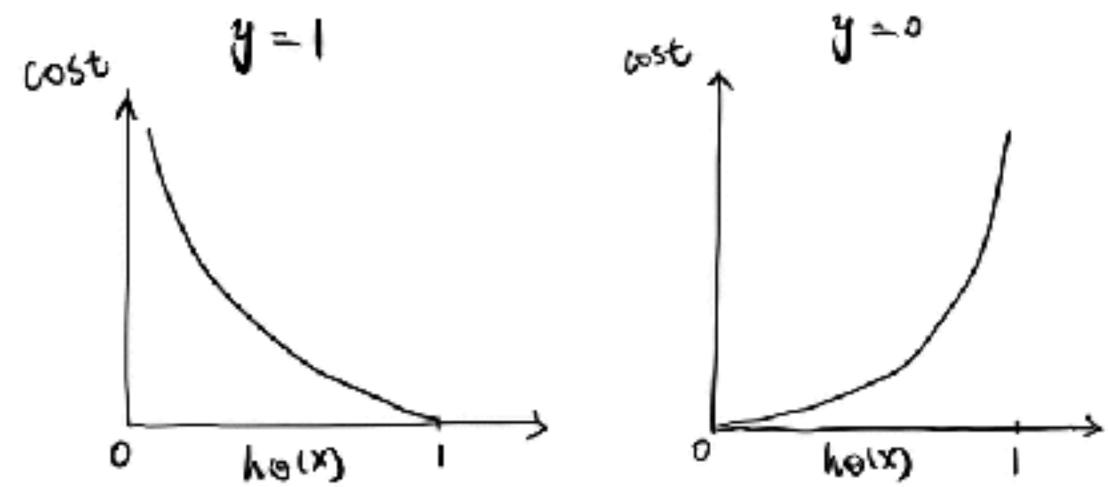
$$l(\beta_0, \beta_1) = \prod_{i:y_i=1} p(x_i) \prod_{i:y_i=0} (1 - p(x_i))$$

- This **likelihood** gives the probability of the observed zeros and ones in the data. We pick β_0 and β_1 to maximize the likelihood of the observed data.

Maximum Likelihood

$$l(\beta_0, \beta_1) = \prod_{i:y_i=1} p(x_i) \prod_{i:y_i=0} (1 - p(x_i))$$

$$Cost(h_\theta(x), y) = \begin{cases} -log(h_\theta(x)) & \text{if } y = 1 \\ -log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$



$$Cost(h_\theta(x), y) = -ylog(h_\theta(x)) - (1 - y)log(h_\theta(x))$$

$$J(\theta) = \frac{1}{m} \sum_{i=1}^{m} Cost(h_\theta(x^{(i)}), y^{(i)})$$

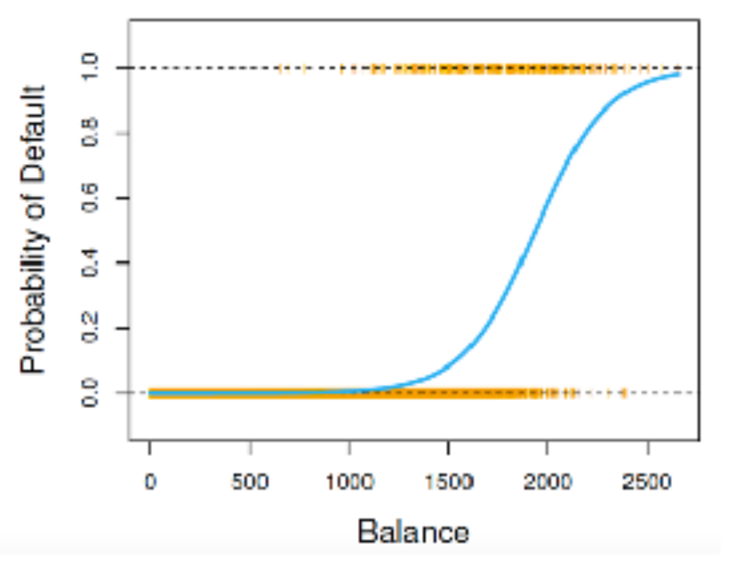
$$J(\theta) = \frac{1}{m}[\sum_{i=1}^{m} -y^{(i)} log(h_\theta(x^{(i)})) + (1 - y^{(i)}) log(1 - h_\theta(x^{(i)}))]$$

$$m = number\ of\ samples$$

Maximum Likelihood

- Most statistical packages can fit linear logistic regression models by maximum likelihood. In R we use the glm function.

	Coefficient	Std. Error	Z-statistic	p-value
Intercept	-10.6513	0.3612	-29.5	<0.0001
balance	0.0055	0.0002	24.9	< 0.0001



Maximum Likelihood

	Coefficient	Std. Error	Z-statistic	p-value
Intercept	-10.6513	0.3612	-29.5	<0.0001
balance	0.0055	0.0002	24.9	< 0.0001

- What is our estimated probability of default for someone with a balance of 1000$?

$$\widehat{p}(X) = \frac{e^{\beta_0+\beta_1 X}}{1 + e^{\beta_0+\beta_1 X}} = \frac{e^{-10.6513+0.0055x1000}}{1 + \epsilon^{-10.6513+0.0055x1000}} = 0.006$$

- What is our estimated probability of default for someone with a balance of 2000$?

$$\widehat{p}(X) = \frac{e^{\beta_0+\beta_1 X}}{1 + e^{\beta_0+\beta_1 X}} = \frac{e^{-10.6513+0.0055x2000}}{1 + \epsilon^{-10.6513+0.0055x2000}} = 0.586$$

- Let’s do it again, using student as the predictor.

	Coefficient	Std. Error	Z-statistic	p-value
Intercept	-3.5041	0.0707	-49.55	<0.0001
student[YES]	0.4049	0.1150	3.52	0.0004

$$\widehat{Pr}\,(default = Yes\,|\,student = Yes) = \frac{e^{-3.5041+0.4049x1}}{1+e^{-3.5041+0.4049x1}} = 0.0431$$

$$\widehat{Pr}\,(default = Yes\,|\,student = No) = \frac{e^{-3.5041+0.4049x0}}{1+e^{-3.5041+0.4049x0}} = 0.0292$$

Logistic regression with several variables

$$log(\frac{p(X)}{1-p(X)}) = \beta_0 + \beta_1 X_1 + \ldots + \beta_p X_p$$

$$p(X) = \frac{e^{\beta_0+\beta_1 X_1+\ldots+\beta_p X_p}}{1+e^{\beta_0+\beta_1 X_1+\ldots+\beta_p X_p}}$$

- Lets do it again, using **balance**, **income** and **student** as the predictors

	Coefficient	Std. Error	Z-statistic	p-value
Intercept	-10.8690	0.4923	-22.08	<0.0001
balance	0.0057	0.0002	24.74	< 0.0001
income	0.0030	0.0082	0.37	0.7115
student[Yes]	-0.6468	0.2362	-2.74	0.0062

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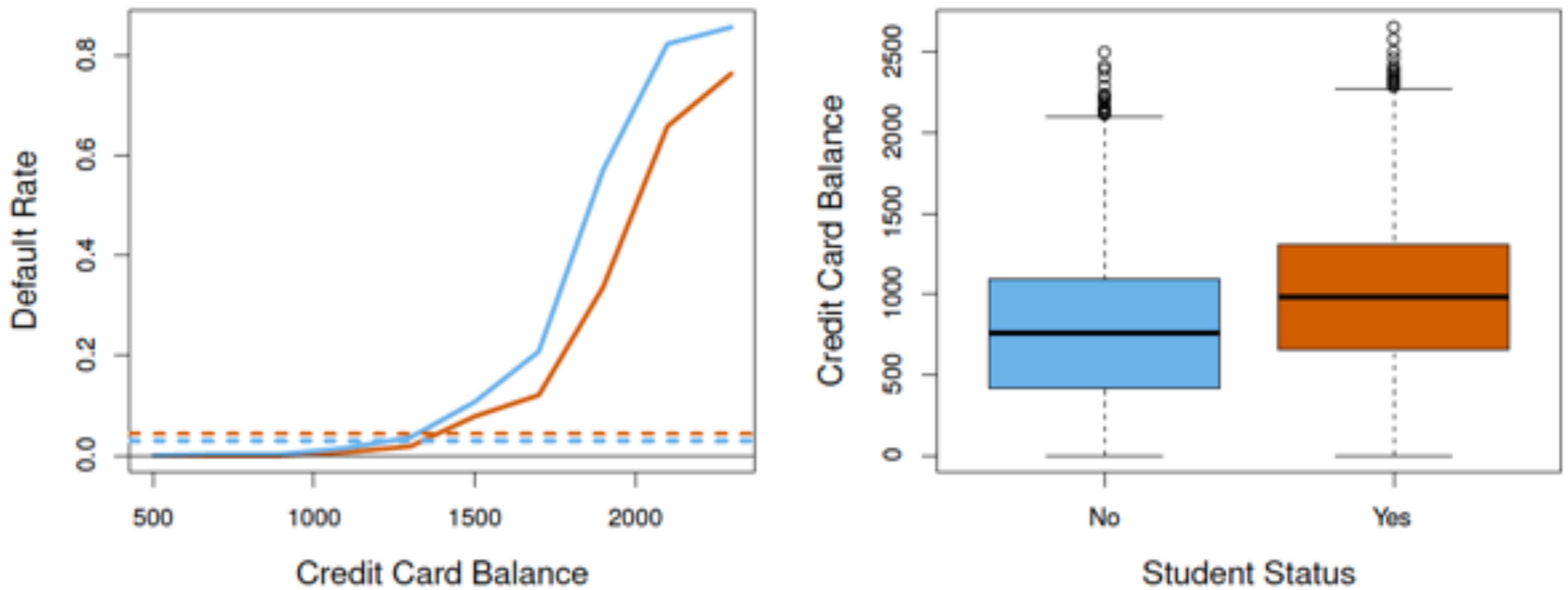
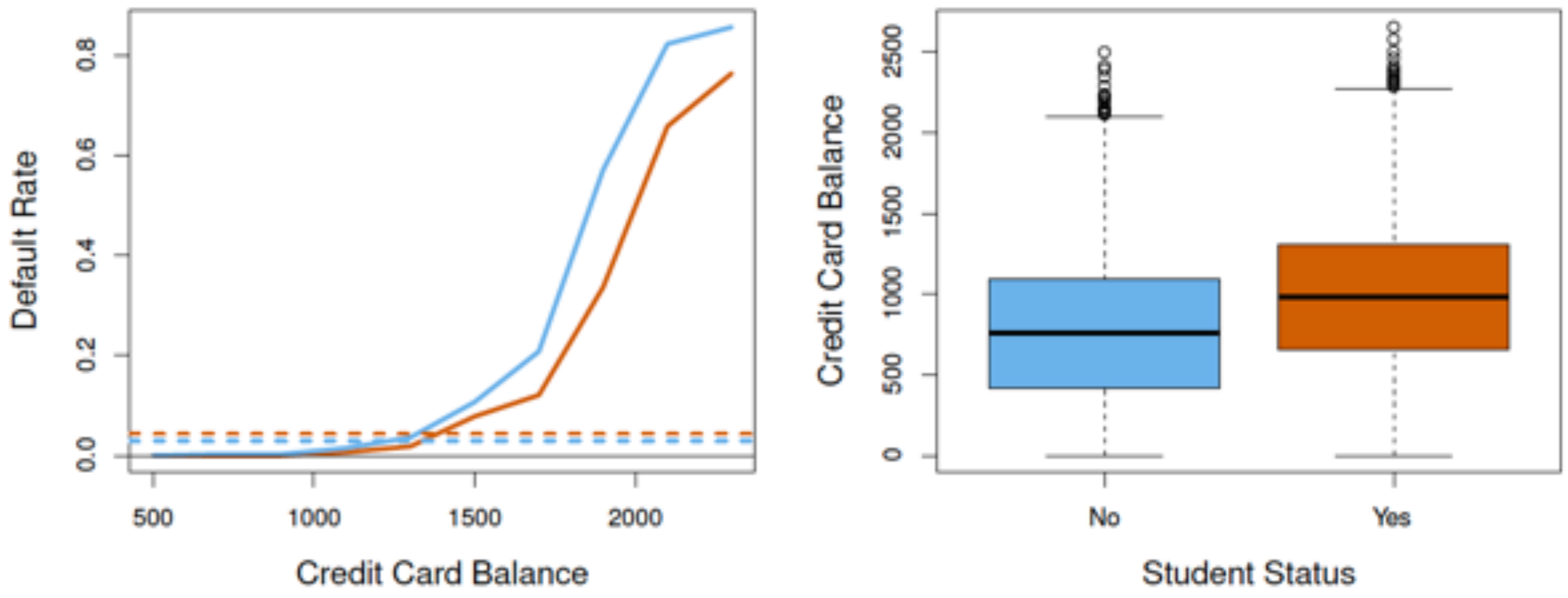
Why is coefficient for **student** negative, while it was positive before?

Confounding



Confounding



- Students tend to have higher balances than non-students, so their marginal default rate is higher than for non-students.

- But for each level of balance, students default less than non-students.

- **Student is less risky than a non-student with the same credit balance!**

Suposa que hem recol·lectat dades d'un grup d'estudiants d'aprenentatge automàtic i mineria de dades amb les següents variables:

X_1 : hores d'estudi.

X_2 : nota d'accés a la universitat

Y: Aprovat (1) / Suspès (0)

Hem après una regressió logística i els coeficients obtinguts han estat els següents: $B_0 = -1.3$, $\beta_1 = 0.1$, $\beta_2 = 1$.

Recordeu que el model de la regressió logística té la següent formulació:

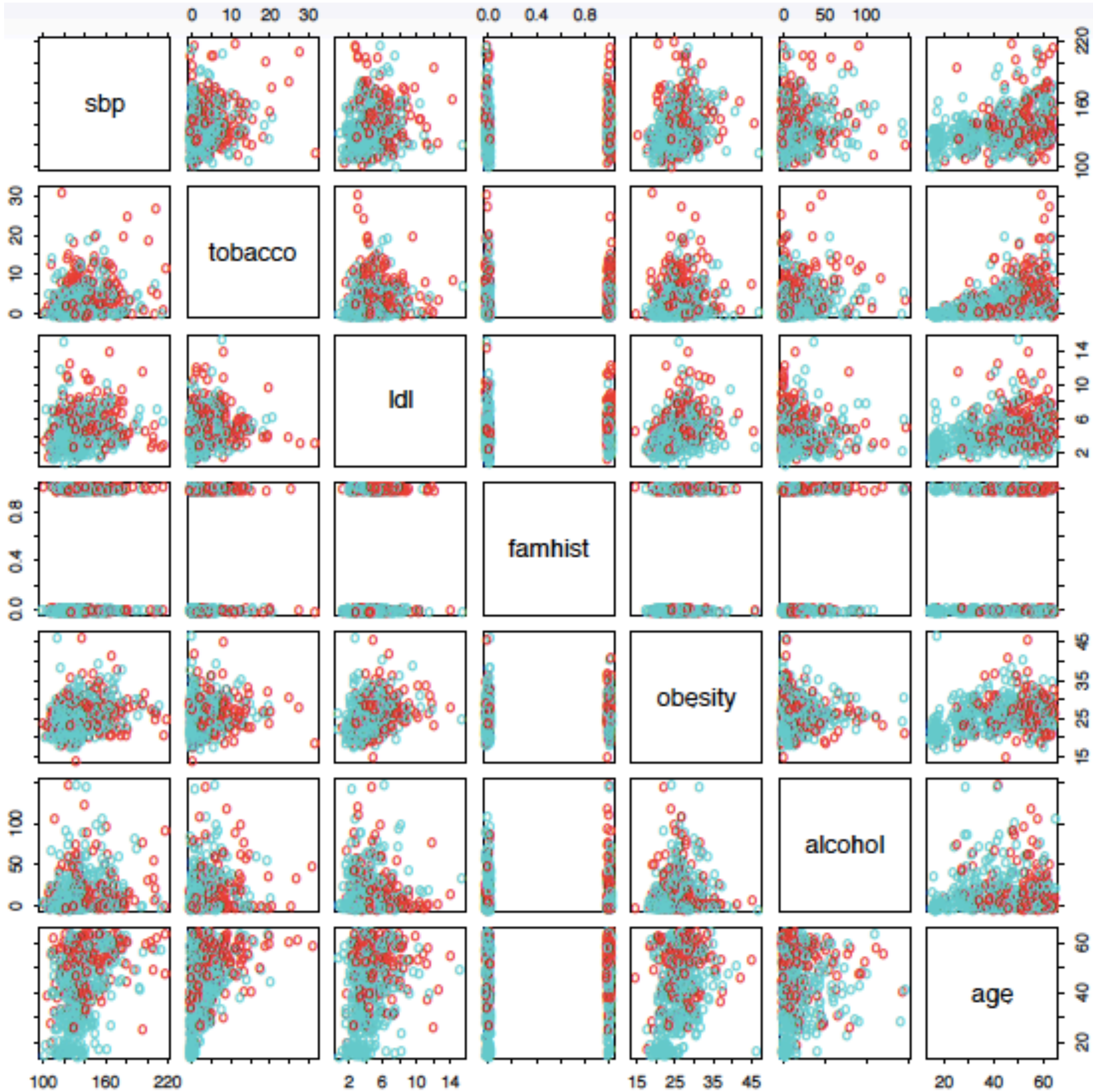
$$p(X) = \frac{e^{\beta_0+\beta_1 X}}{1 + e^{\beta_0+\beta_1 X}}$$

(a) [1 punt] Estima la probabilitat que té l'estudiant si ha dedicat 60 hores i la nota d'accés a la universitat ha estat un 10.

(b) [1 punt] Quantes hores hauria d'estudiar un estudiant per tenir més d'un 50% de possibilitats d'aprovar l'assignatura d'estadística?

Example: South African Heart Disease

- **160 cases of MI** (myocardial infarction) and **302 controls** (all male in age range 15-64), from Western Cape, South Africa in early 80s.
- Overall **prevalence** very high in this region: **5.1%**
- Measurements on seven predictors (risk factors), shown in scatterplot matrix.
- Goal is to identify relative strengths and directions of risk factors.



Scatterplot matrix of the South African Heart Disease data. The response is color coded - The cases (MI) are red, the controls turquoise. famhist is a binary variable, with 1 indicating family history of MI

```
> heartfit<-glm(chd~.,data=heart,family=binomial)
> summary(heartfit)

Call:
glm(formula = chd ~ ., family = binomial, data = heart)

Coefficients:
                Estimate Std. Error z value Pr(>|z|)
(Intercept)   -4.1295997  0.9641558  -4.283 1.84e-05 ***
sbp            0.0057607  0.0056326   1.023  0.30643
tobacco        0.0795256  0.0262150   3.034  0.00242 **
ldl            0.1847793  0.0574115   3.219  0.00129 **
famhistPresent 0.9391855  0.2248691   4.177 2.96e-05 ***
obesity       -0.0345434  0.0291053  -1.187  0.23529
alcohol        0.0006065  0.0044550   0.136  0.89171
age            0.0425412  0.0101749   4.181 2.90e-05 ***

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 596.11  on 461  degrees of freedom
Residual deviance: 483.17  on 454  degrees of freedom
AIC: 499.17
```

Which is the probability of having a heart attack?

Case-control sampling and logistic regression

- In South African data, there are **160 cases**, **302 controls** - $\tilde{\pi} = 0.35$ are cases. Yet the prevalence of MI in this region is $\pi = 0.05$.

- With case-control samples, we can estimate the regression parameters β_j accurately (if our model is correct); the constant term β_0 is incorrect.

- We can correct the estimated intercept by a simple transformation

$$\hat{\beta}_0^* = \hat{\beta}_0 + log\frac{\pi}{1-\pi} - log\frac{\tilde{\pi}}{1-\tilde{\pi}}$$

- Often cases are rare and we take them all; up to five times that number of controls is sufficient.

Linear Discriminant Analysis

Linear Discriminant Analysis

- Here the approach is to model the distribution of X in each of the classes separately, and then use Bayes theorem to flip things around and obtain $Pr(Y|X)$.

- When these data distribution are assumed to be normal, it turns out that the model is very similar to logistic regression.

- Although, this approach is quite general, and other distributions can be used as well, we will focus on normal distributions.

Linear Discriminant Analysis

- Thomas Bayes was a famous mathematician whose name represents a big subfield of statistical and probabilistic modeling. Here we focus on a simple result, known as Bayes theorem:

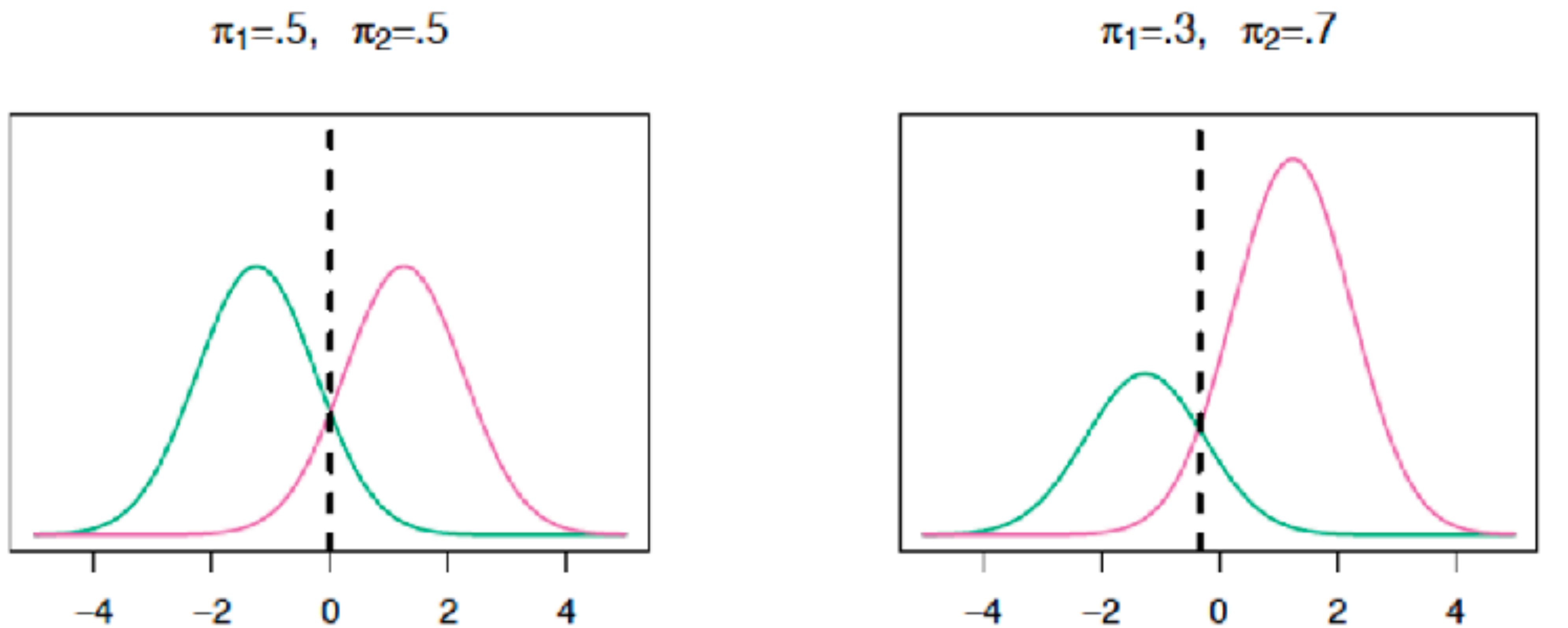
$$Pr(Y = k \mid X = x) = \frac{Pr(X = x \mid Y = k) \cdot Pr(Y = k)}{Pr(X = x)}$$

- One writes this slightly differently for discriminant analysis:

$$Pr(Y = k \mid X = x) = \frac{\pi_k f_k(x)}{\sum_{l=1}^{K} \pi_l f_l(x)}$$

- where,
  - $f_k(x) = Pr(X = x \mid Y = k)$ is **the density function** for X in class k . Here we will use normal densities for these, separately in each class
  - $\pi_k = Pr(Y = k)$ is the **marginal** or **prior** probability for class k.

Linear Discriminant Analysis



- We classify a new point according to which density is highest. When the priors are different, we take them into account as well, and compare $\pi_k f_k(x)$. On the right, we favor the pink class (the decision boundary has shifted to the left).

Why discriminant Analysis?

- When the classes are well-separated, the parameter estimates for the logistic regression model are surprisingly **unstable**. Linear discriminant analysis does not suffer from this problem.

- If n is small and the distribution of the predictors X is approximately normal in each of the classes, the linear discriminant model is again more stable than the logistic regression model.

- Linear discriminant analysis is popular when we have more than two response classes, because it also provides low-dimensional views of the data.

Linear Discriminant Analysis when p = 1

- The Gaussian density has the form

$$f_k(x) = \frac{1}{\sigma_k\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu_k}{\sigma_k}\right)^2}$$

- Here μ_k is the mean, and σ_k^2 the variance (in class k). We will assume that all the $\sigma_k = \sigma$ are the same.

- Plugging this into Bayes formula, we get a rather complex expression for $p_k(x) = Pr(Y = k | X = x)$:

$$p_x(k) = \frac{\pi_k \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{1}{2}\left(\frac{x-\mu_k}{\sigma}\right)^2}}{\sum_{l=1}^{K} \pi_l \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{1}{2}\left(\frac{x-\mu_l}{\sigma}\right)^2}}$$

- Happily, there are simplifications and cancellations.

Discriminate functions

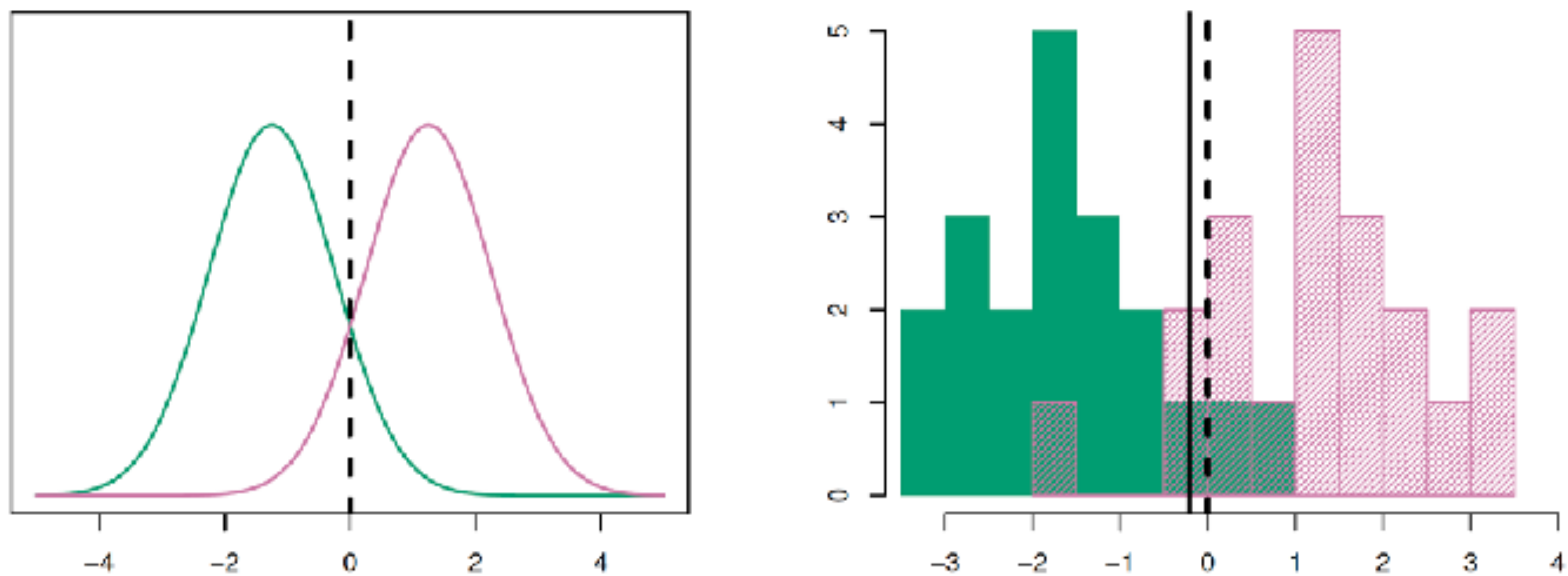
- To classify at the value $X = x$, we need to see which of the $p_k(x)$ is largest. Taking logs, and discarding terms that do not depend on k , we see that this is equivalent to assigning x to the class with the largest discriminant score:

$$\lambda_k(x) = x \cdot \frac{\mu_k}{\sigma^2} - \frac{\mu_k^2}{2\sigma^2} + log(\pi_k)$$

- Note that $\lambda_k(x)$ is a linear function of x.

- If there are K = 2 classes and $\pi_1 = \pi_2 = 0.5$, then one can see that the decision boundary is at

$$x = \frac{\mu_1 + \mu_2}{2}$$



Example with $\mu_1 = -1.5$, $\mu_2 = 1.5$,

$$\pi_1 = \pi_2 = 0.5 \text{ and } \sigma^2 = 1.$$

Typically we don't know these parameters; we just have the training data. In that case we simply estimate the parameters and plug them into the rule.

Estimating the parameters

$$\hat{\pi}_k = \frac{n_k}{n}$$

$$\hat{\mu}_k = \frac{1}{n_k} \sum_{i:y_i=k} x_i$$

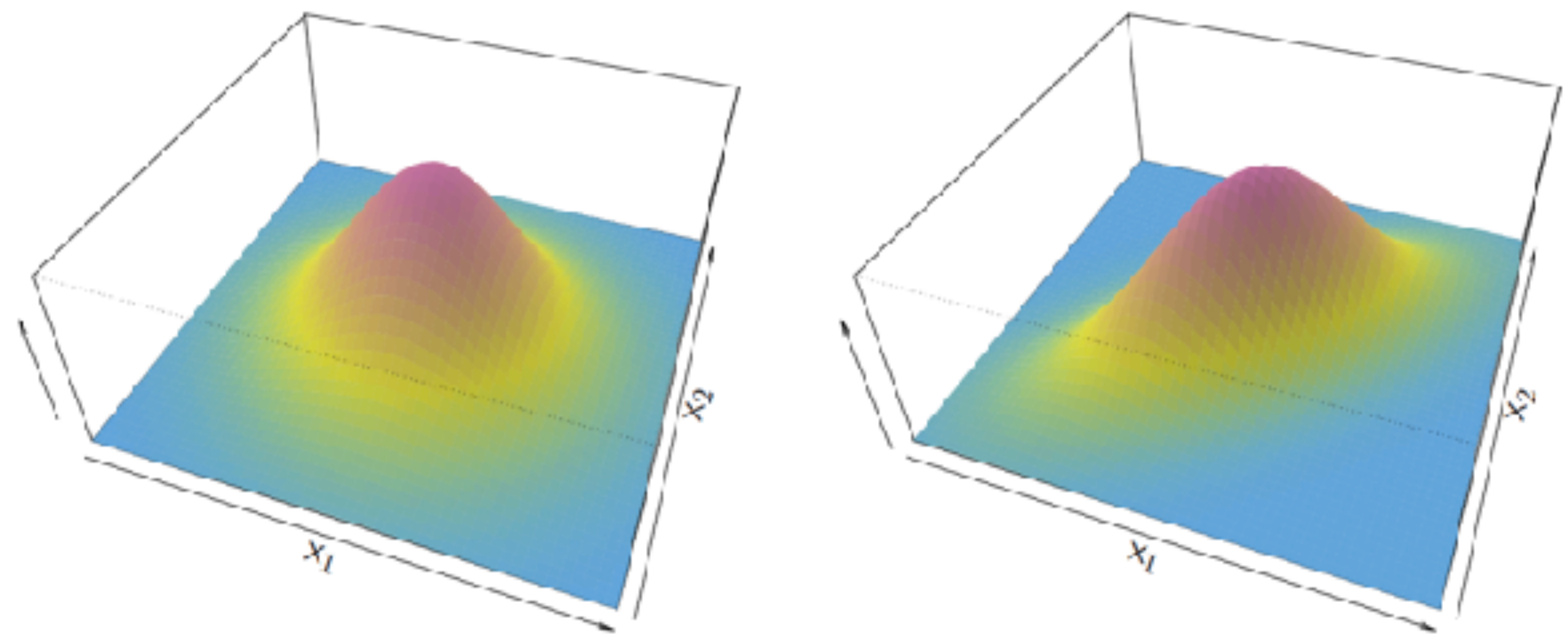
$$\hat{\sigma}^2 = \frac{1}{n-K} \sum_{k=1}^{K} \sum_{i:y_i=k} (x_i - \hat{\mu}_k)^2 = \sum_{k=1}^{K} \frac{n_k - 1}{n-K} \cdot \hat{\sigma}_k^2$$

Where

$$\hat{\sigma}_k^2 = \frac{1}{n_k - 1} \sum_{:y_i=k} (x_i - \hat{\mu}_k)^2$$

is the usual formula for the estimated variance in the kth class

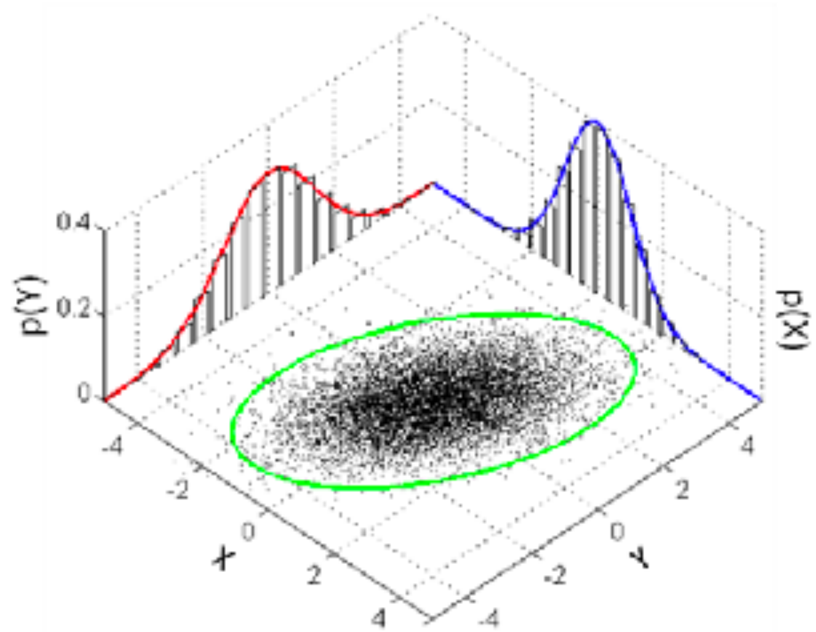
Linear Discriminant Analysis when p>1



To extend the LDA classifier to the case of multiple predictors we will assume that $X = (X_1, X_2, \ldots, X_p)$ is drawn from a multivariate Gaussian distribution, with a class-specific mean vector and a common covariance matrix Σ.

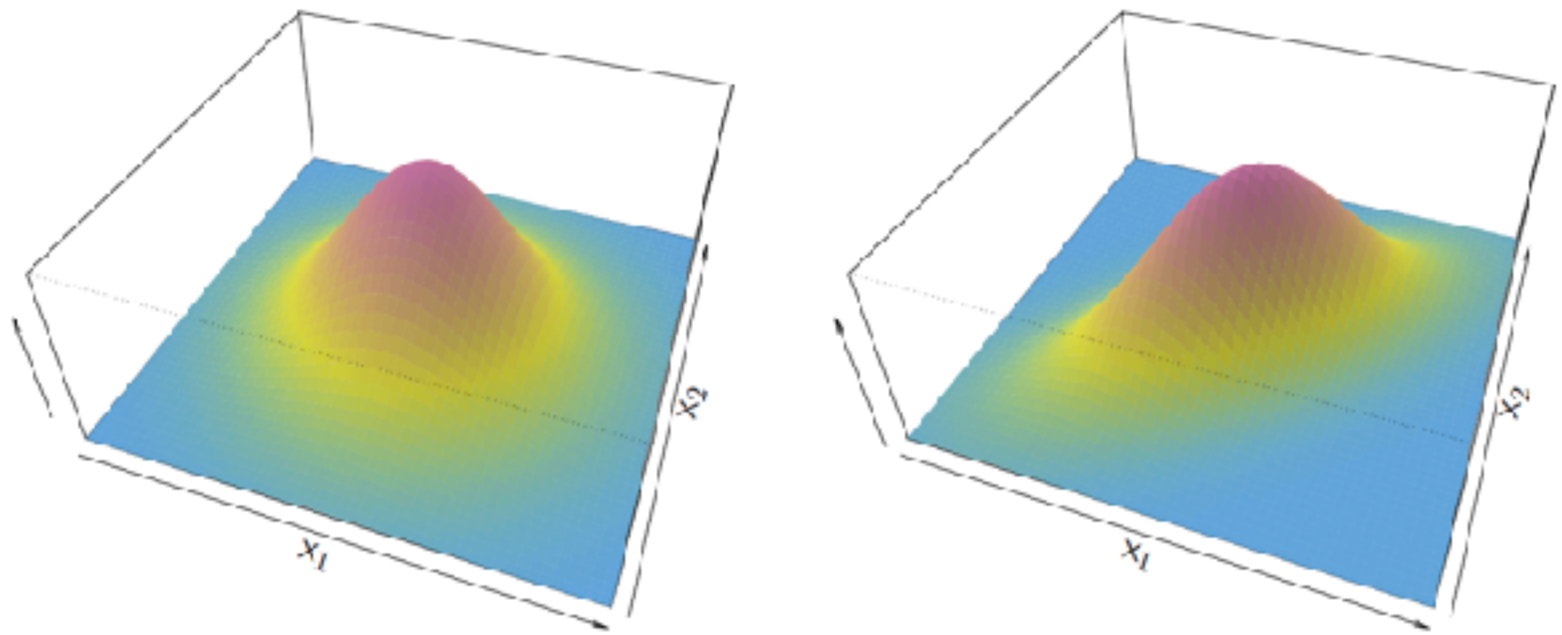
Linear Discriminant Analysis when p>1

It is defined as $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$.



Many sample points from a multivariate normal distribution with $\boldsymbol{\mu} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $\boldsymbol{\Sigma} = \begin{bmatrix} 1 & 3/5 \\ 3/5 & 2 \end{bmatrix}$, shown along with the 3-sigma ellipse, the two marginal distributions, and the two 1-d histograms.

Linear Discriminant Analysis when p>1

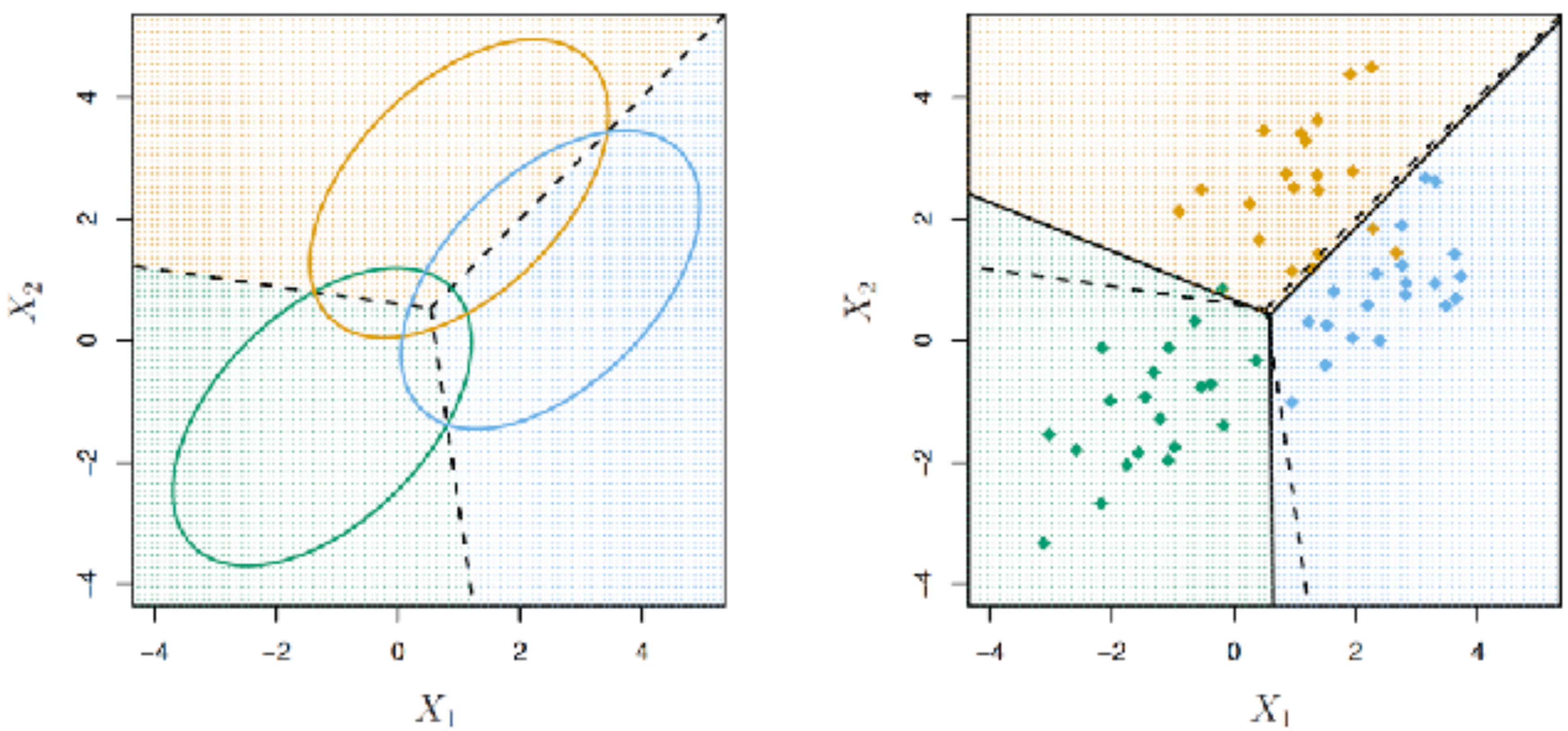


Density function: $f(x) = \frac{1}{(2\pi)^{p/2}|\Sigma|^{1/2}} e^{\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)}$

Discriminant function: $\delta_k(x) = x^T\Sigma^{-1}\mu_k - \frac{1}{2}\mu_k^T\Sigma^{-1}\mu_k + log\pi_k$

Despite its complex form, $\delta_k(x) = c_{k0} + c_{k1}x_1 + c_{k2}x_2 + \ldots c_{kp}x_p$ **is a linear combinations**

Illustration: p=2 and K = 3 classes



Here $\pi_1 = \pi_2 = \pi_3 = 1/3$ the dashed lines are known as the **Bayes decision** boundaries. Where they known, they would yield the fewest misclassification errors, among all possible classifiers.

Classifier Evaluation

How can we evaluate a classifier?

Binary Classification

Classification on Credit Data

		True Default Status		
		No	Yes	Total
Predicted	No	9644	252	9896
Default Status	Yes	23	81	104
	Total	9667	333	10000

- (23 + 252) / 10000 errors - 2.75% misclassification rate! Some points to consider:
- This is **training error**, and we may be overfitting. Not a big concern here since n = 10000 and p = 4!
- If we classified to the prior - **always to class No in this case** - we would make 333/10000 errors, or only 3.33 %.
- Of the true No's, we make 23/9667 = 0.2% errors; of the true Yes's, we make 252/333 = 75.7% errors!

Types of errors

- **False positive rate**: The fraction of negative examples that are classified as positive - 0.2% in example.

- **False negative rate**: The fraction of positive examples that are classified as negative - 75.7% in example.

- We produced this table by classifying to class Yes if

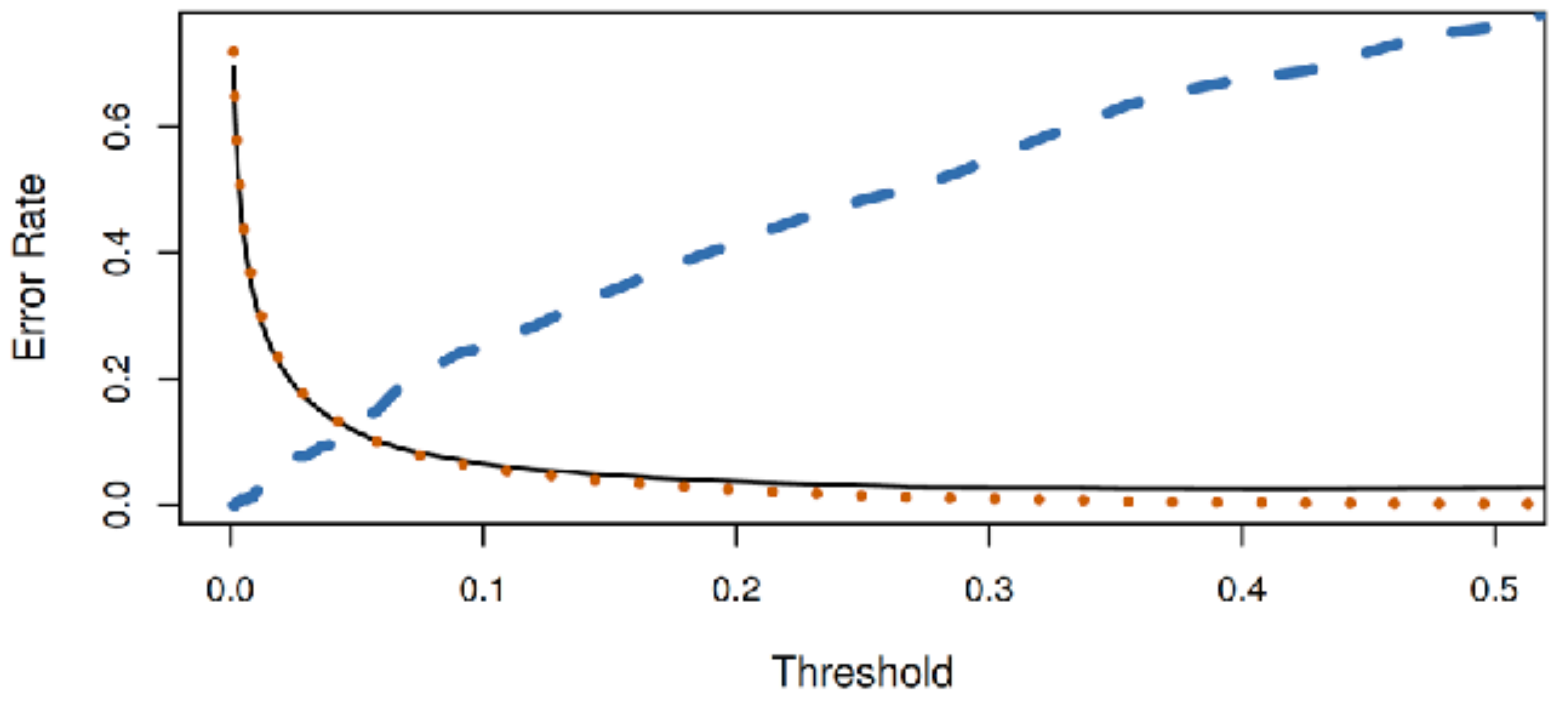
$$Pr(Default = Yes \mid Balance, Student) \geq 0{,}5$$

- We can change the two error rates by changing the threshold from 0.5 to some other value in [0, 1]:

$$Pr(Default = Yes \mid Balance, Student) \geq threshold$$

and vary threshold

Varying the threshold

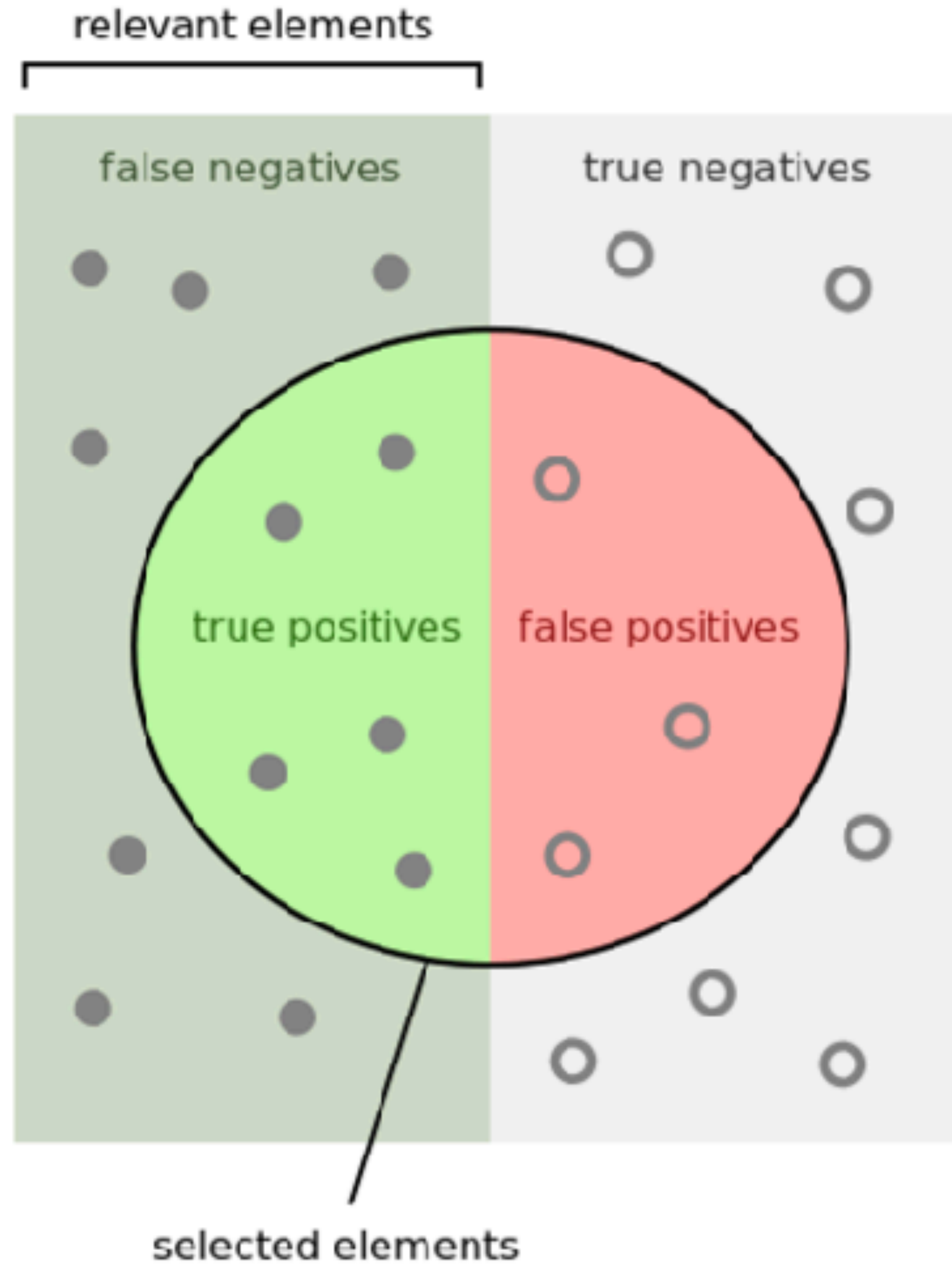


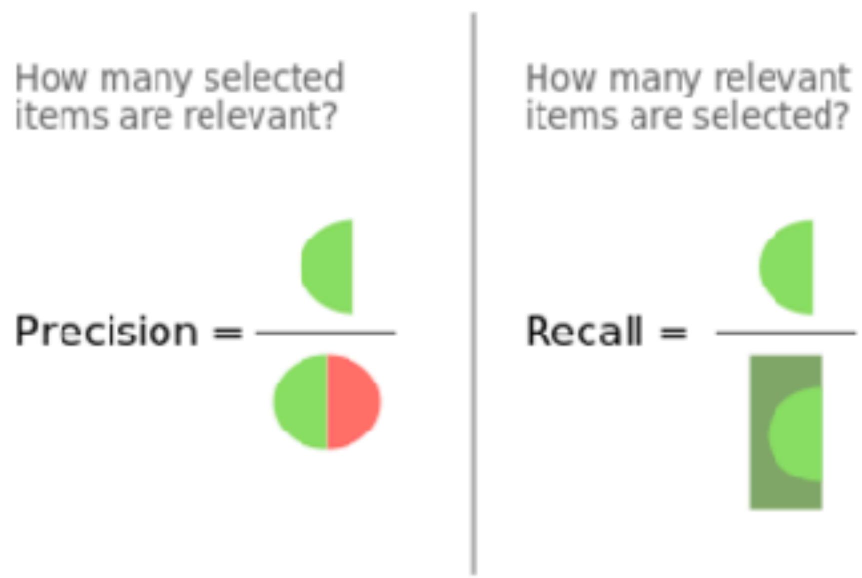
In order to reduce the false negative rate, we may want to reduce the threshold to 0.1 or less.

Classification Error

- **FP** = Negative considered Positive, also considered Type I Error
- **FN** = Positive considered Negative, also know as Type II error

- **What is best, FP or FN?**
  - **It depends on the problem.**
    - A FP may lead a subject to undergo an unnecessary treatment
    - A FN may not receive an intervention when one would have been beneficial


relevant elements
false negatives
true negatives
true positives
false positives
selected elements


How many selected items are relevant?
Precision =
How many relevant items are selected?
Recall =

Example: We want to train a model that for the diagnosis of Colon Cancer

Training:

Positive Cases: 10

Negative Cases: 90

Test:

Positive Cases: 10

Negative Cases: 90

Example: We want to train a model that for the diagnosis of Colon Cancer

Training:

Positive Cases: 10

Negative Cases: 90

Test:

Positive Cases: 10

Negative Cases: 90

Train Model

Evaluate Model

— — — — — —Result— — — — — —

Positive Cases:
- **7 classified as Positive**
- 3 classified as Negative

Negative Cases
- **81 classified as Negative**
- 9 classified as Positive

Evaluation Measures

Accuracy or Classification Error

Confusion Matrix

Precision

Recall/Sensitivity

Specificity

F1-Score

ROC Curve

AUC

Accuracy = (TP + TN) / N

(7 + 81) / 100 = 0.88

Evaluation Measures

Accuracy or Classification Error

Confusion Matrix

Precision

Recall/Sensitivity

Specificity

F1-Score

ROC Curve

AUC

		Predicted	
		Positive	Negative
Real	Positive	TP	FN
	Negative	FP	TN

		Predicted	
		Positive	Negative
Real	Positive	7	3
	Negative	9	81

Evaluation Measures

Accuracy or Classification Error

Confusion Matrix

Precision

Recall/Sensitivity

Specificity

F1-Score

ROC Curve

AUC

Precision = TP / (TP + FP)

$$\frac{7}{7+9} = \frac{7}{16} = 0.4375$$

Evaluation Measures

Accuracy or Classification Error

Confusion Matrix

Precision

Recall/Sensitivity

Specificity

F1-Score

ROC Curve

AUC

$$Recall = \frac{TP}{TP + FN} = \frac{TP}{P}$$

$$\frac{7}{7+3} = \frac{7}{10} = 0.7$$

Evaluation Measures

Accuracy or Classification Error

Confusion Matrix

Precision

Recall/Sensitivity

Specificity

F1-Score

ROC Curve

AUC

$$Specificity = \frac{TN}{TN + FP} = \frac{TN}{N}$$

$$\frac{81}{81 + 9} = \frac{81}{90} = 0.9$$

Evaluation Measures

Accuracy or Classification Error

Confusion Matrix

Precision

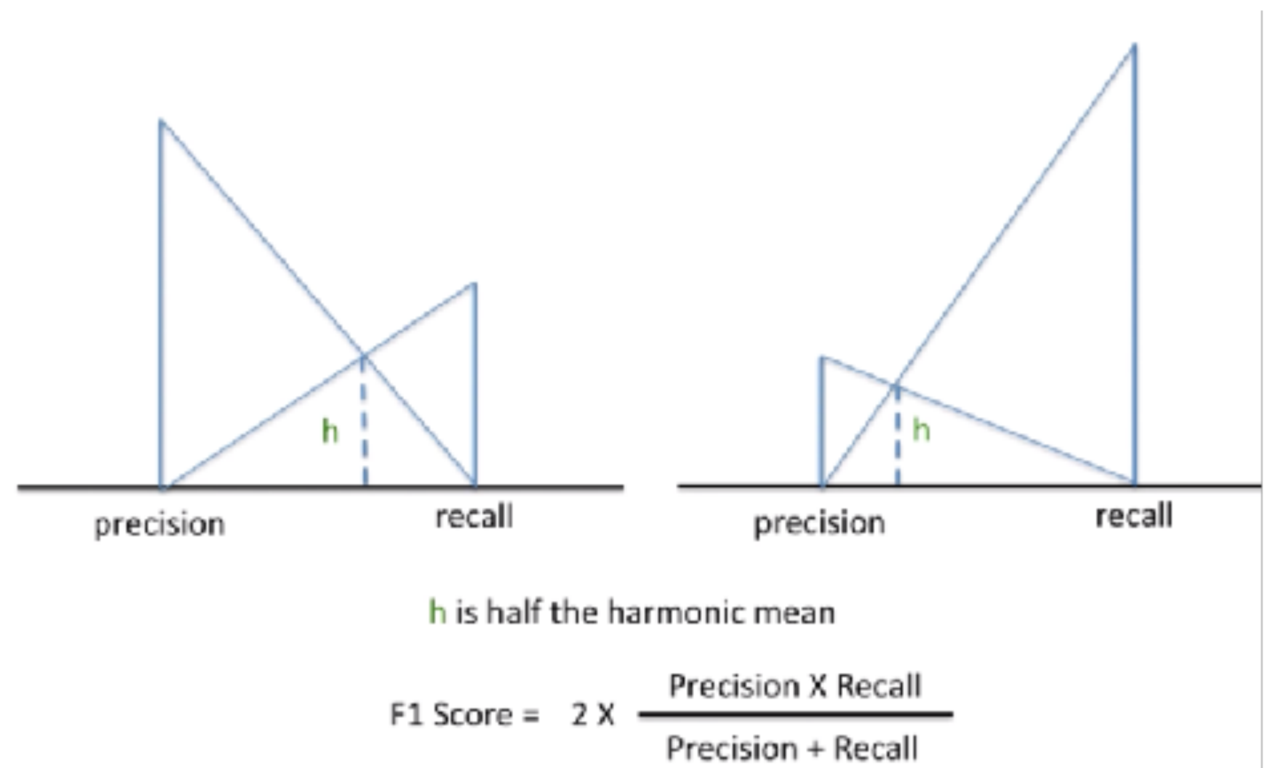
Recall/Sensitivity

Specificity

F1-Score

ROC Curve

AUC



Evaluation Measures

Accuracy or Classification Error

Confusion Matrix

Precision

Recall/Sensitivity

Specificity

F1-Score

ROC Curve

AUC

$$F_1 = 2 \cdot \frac{precision \cdot recall}{precision + recall}$$

$$2 \cdot \frac{0.4375 \cdot 0.7}{0.4375 + 0.7} = 0.538$$

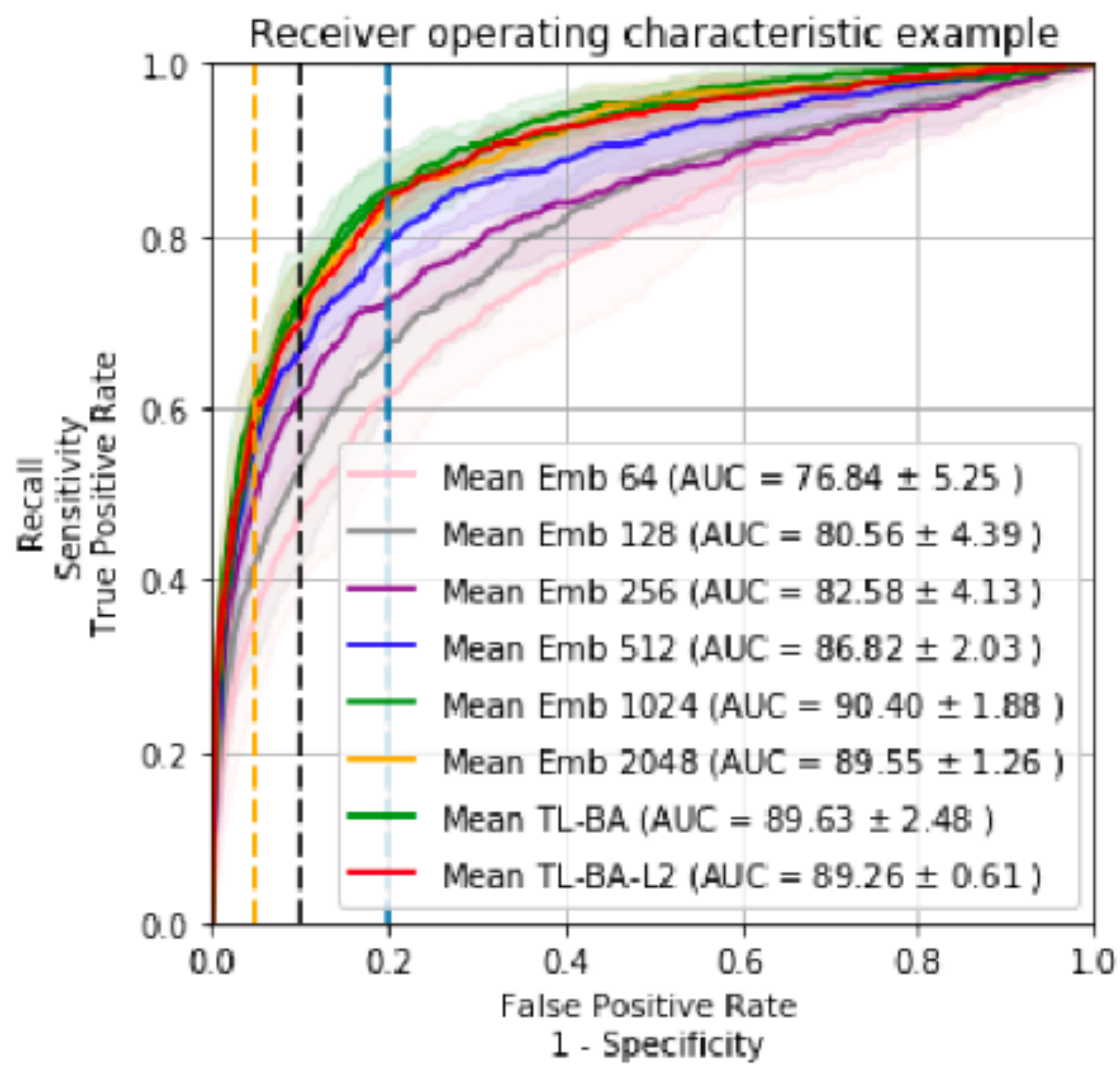
ROC-Curve & AUC

- There is, usually, a trade-off between *sensitivity* and *specificity*: this is controlled by the classification threshold θ, and can be displayed graphically through the receiver operating curve.

- The *area under the ROC* curve (AUC) provides a measure of the performance of the classification model over all values θ.

- The AUC is independent on the amount of positive and negative cases.

ROC-Curve & AUC



Which is the best?

MODEL A

Result

Positive Cases:
- **7 classified as Positive**
- 3 classified as Negative

Negative Cases
- **81 classified as Negative**
- 9 classified as Positive

MODEL B

Result

Positive Cases:
- **9 classified as Positive**
- 1 classified as Negative

Negative Cases
- **78 classified as Negative**
- 12 classified as Positive

Which is the best?

MODEL A

MODEL B

	Model A	Model B
Accuracy	**0.88**	0.87
Precision	**0.43**	0.42
Recall/Sensitivity	0.70	**0.9**
Specificity	**0.90**	0.86
F1-Score	0.53	**0.58**

MultiClass Classification

Definitions

- **Multi-Class:** classification task with more than two classes such that the input is to be classified into one, and only one of these classes. Example: classify a set of images of fruits into any one of these categories – apples, bananas, and oranges.

- **Multi-labels:** classifying a sample into a set of target labels. Example: tagging a blog into one or more topics like technology, religion, politics etc. Labels are isolated and their relations are not considered important.

- **Hierarchical:** each category can be grouped together with similar categories, creating meta-classes, which in turn can be grouped again until we reach the root level (set containing all data). Examples include text classification & species classification.

Example: We want to train a model that predict the type of Colon Cancer (A;B;C and D):

MODEL A

	Predicted			
	Type A	Type B	Type C	Type D
Type A	10	0	0	0
Type B	0	5	3	2
Type C	0	1	8	1
Type D	0	1	0	9

MODEL A

	Predicted			
	Type A	Type B	Type C	Type D
Type A	8	2	0	0
Type B	1	7	3	2
Type C	0	1	9	1
Type D	2	3	0	5

Example: We want to train a model that predict the type of Colon Cancer (A;B;C and D):

MODEL A

	Predicted			
	Type A	Type B	Type C	Type D
Type A	10	0	0	0
Type B	0	5	3	2
Type C	0	1	8	1
Type D	0	1	0	9

Accuracy = (10+5+8+9)/40 = 0.8

MODEL A

	Predicted			
	Type A	Type B	Type C	Type D
Type A	8	2	0	0
Type B	1	7	3	2
Type C	0	1	9	1
Type D	2	3	0	5

Accuracy = (8+7+9+5)/40 = 0.725

Example: We want to train a model that predict the type of Colon Cancer (A;B;C and D):
with imabalanced dataset

MODEL A

	Predicted			
	Type A	Type B	Type C	Type D
Type A	100	80	10	10
Type B	0	9	0	1
Type C	0	1	8	1
Type D	0	1	0	9
Precision	100/100	9/91	8/18	9/21

Average Accuracy = (0.5+0.9+0.8+0.9)/4 = 0.775

Average Precisions = 0.492

MODEL A

	Predicted			
	Type A	Type B	Type C	Type D
Type A	198	2	0	0
Type B	7	1	0	2
Type C	0	8	1	1
Type D	2	3	4	1
Precision	198/207	1/14	1/5	1/4

Average Accuracy = (0.99+0.1+0.1+0.1)/4 = 0.3225

Table 3
Measures for multi-class classification based on a generalization of the measures of Table 1 for many classes C_i: tp_i are true positive for C_i, and fp_i – false positive, fn_i – false negative, and tn_i – true negative counts respectively. μ and M indices represent micro- and macro-averaging.

Measure	Formula	Evaluation focus
Average Accuracy	$\frac{\sum_{i=1}^{l} \frac{tp_i+tn_i}{tp_i+fn_i+fp_i+tn_i}}{l}$	The average per-class effectiveness of a classifier
Error Rate	$\frac{\sum_{i=1}^{l} \frac{fp_i+fn_i}{tp_i+fn_i+fp_i+tn_i}}{l}$	The average per-class classification error
Precision$_\mu$	$\frac{\sum_{i=1}^{l} tp_i}{\sum_{i=1}^{l} (tp_i+fp_i)}$	Agreement of the data class labels with those of a classifiers if calculated from sums of per-text decisions
Recall$_\mu$	$\frac{\sum_{i=1}^{l} tp_i}{\sum_{i=1}^{l} (tp_i+fn_i)}$	Effectiveness of a classifier to identify class labels if calculated from sums of per-text decisions
Fscore$_\mu$	$\frac{(\beta^2+1) Precision_\mu Recall_\mu}{\beta^2 Precision_\mu + Recall_\mu}$	Relations between data's positive labels and those given by a classifier based on sums of per-text decisions
Precision$_M$	$\frac{\sum_{i=1}^{l} \frac{tp_i}{tp_i+fp_i}}{l}$	An average per-class agreement of the data class labels with those of a classifiers
Recall$_M$	$\frac{\sum_{i=1}^{l} \frac{tp_i}{tp_i+fn_i}}{l}$	An average per-class effectiveness of a classifier to identify class labels
Fscore$_M$	$\frac{(\beta^2+1) Precision_M Recall_M}{\beta^2 Precision_M + Recall_M}$	Relations between data's positive labels and those given by a classifier based on a per-class average

https://www.sciencedirect.com/science/article/pii/S0306457309000259

- In Micro-average method, you sum up the individual true positives, false positives, and false negatives of the system for different sets and then apply them to get the statistics.

- In Macro-average, you take the average of the precision and recall of the system on different sets

- **Micro-average** is preferable if there is a class imbalance problem.