

A minicourse of CAMB/CosmoMC

Gong-Bo Zhao

NAOC

gbzhao@nao.cas.cn

April, 2019

Why CAMB?

- Very efficient
- Well-structured (modulised) -- easy to tweak and hack
- Well-supported – cosmocoffee.info
- Popular – mainstream numeric tool in cosmology, used by WMAP, Planck, SDSS, etc

Purpose of this course

- Show you how to use CAMB
- Understand the basics of CAMB by linking CAMB equations to Ma & Bertschinger 1996 paper (astro-ph/9506072)
- Show one example of modifying CAMB – make it work for **modified gravity**
- Guide you how to modify CAMB for your own research purpose

Look into the code...

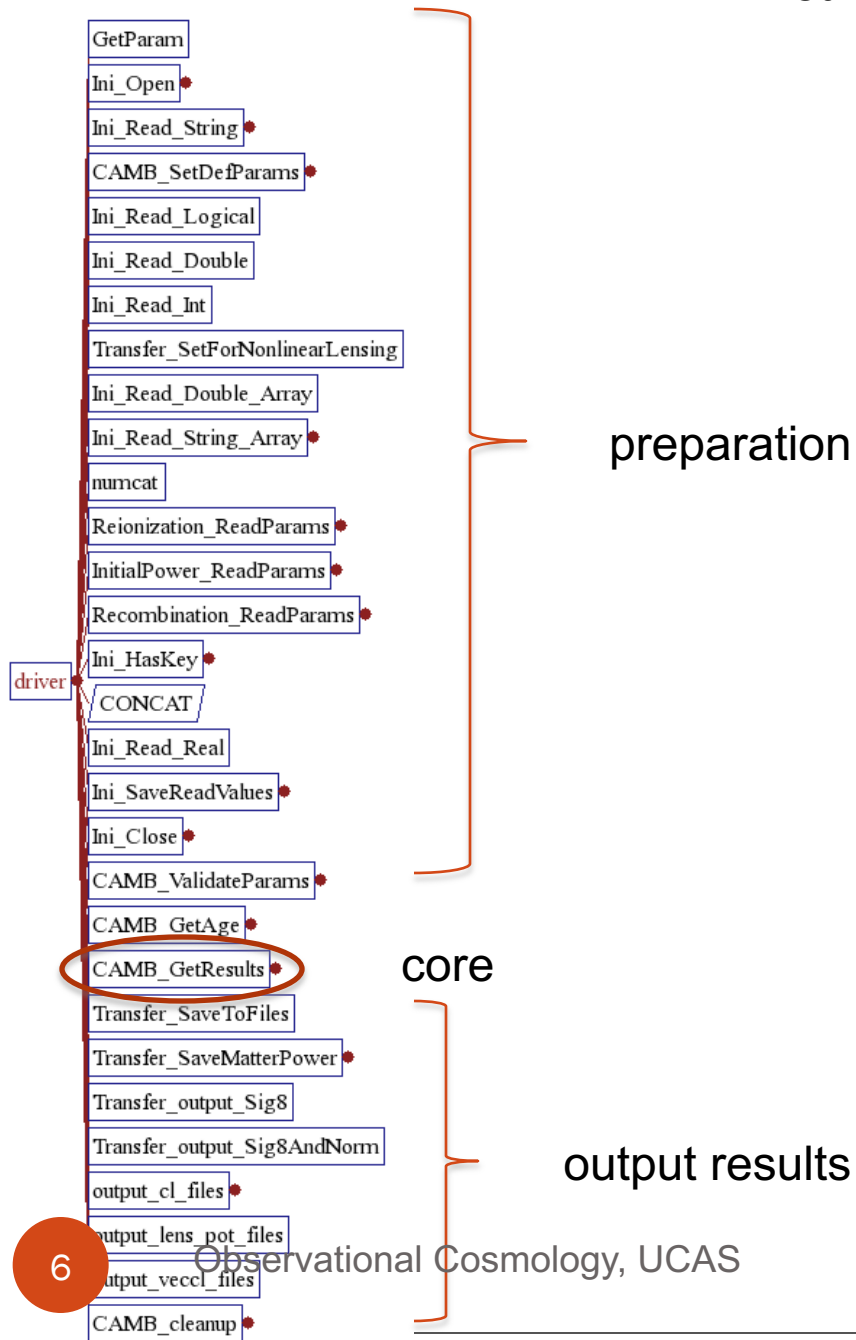
Analyze the code using
“understand for fortran”

<http://www.scitools.com/download/>
(choose the free 15-day trial version)

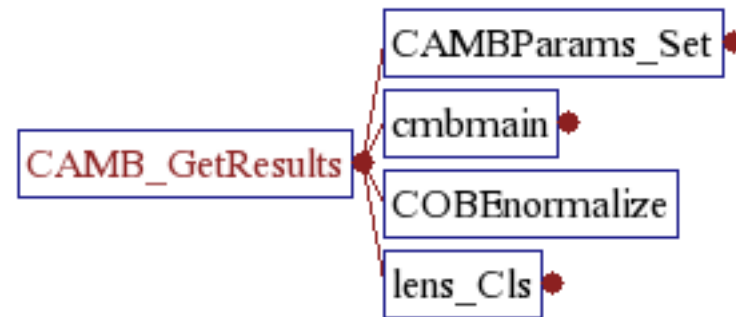
Structure of CAMB



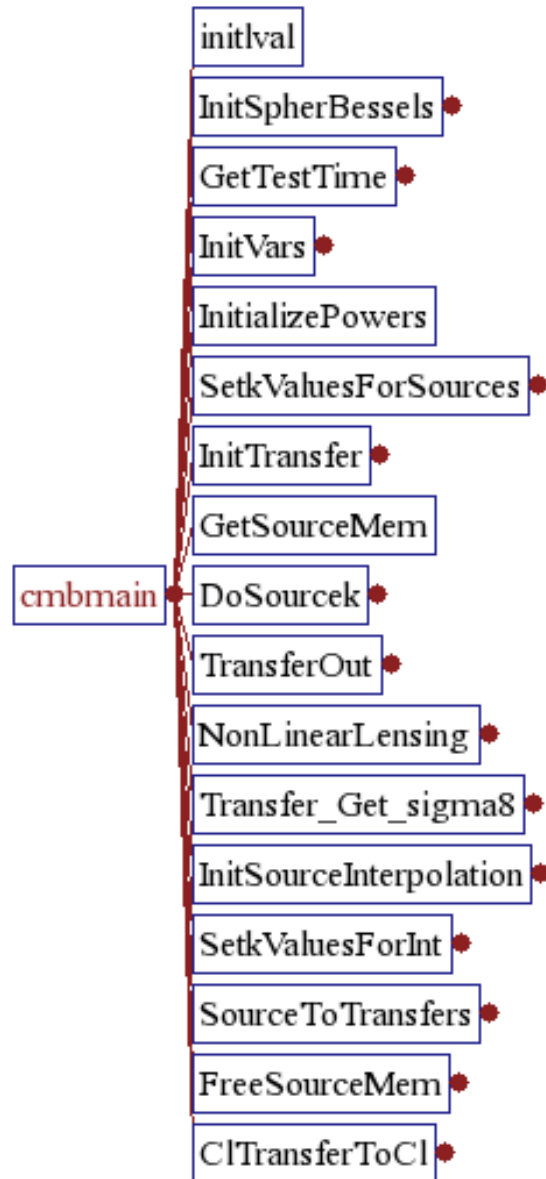
Structure of CAMB



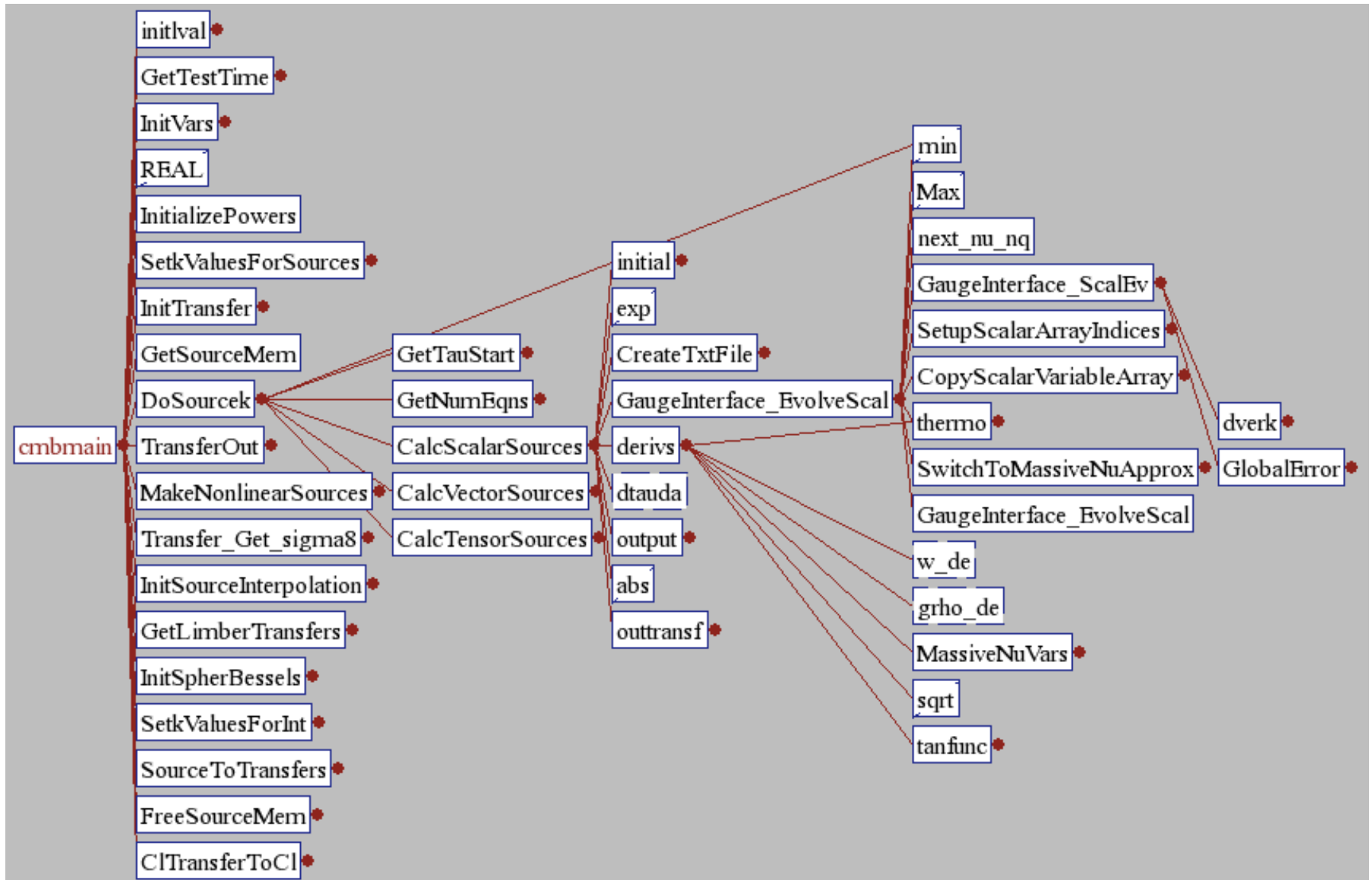
Structure of CAMB_GetResults



Structure of CMBmain



Structure of CMBmain



Initial Conditions

```
graph TD; A[Initial Conditions] --> B[Solve for the coupled differential equations]; B --> C[Do and output statistics (Cls, P(k))];
```

Solve for the coupled differential equations

Do and output statistics
(Cls , $P(k)$)

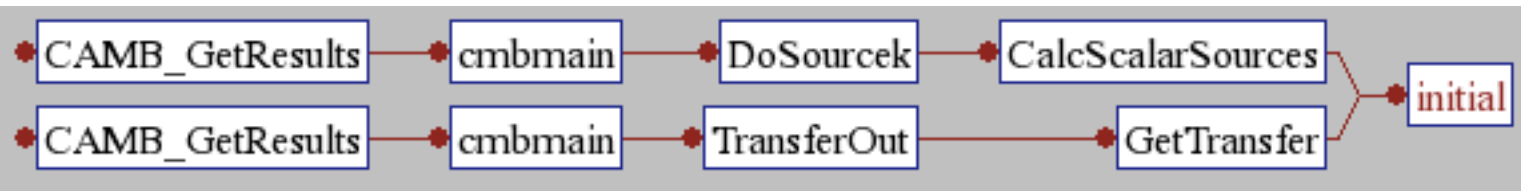
Initial conditions

CAMB notes:

$$\Delta_\gamma = \Delta_\nu = \frac{\beta_2}{3}(k\tau)^2 - \frac{\beta_2}{15}\omega k^2\tau^3$$
$$\Delta_c = \Delta_b = \frac{\beta_2}{4}(k\tau)^2 - \frac{\beta_2}{20}\omega k^2\tau^3$$

CAMB code:

```
x=k*tau
x2=x*x
EV%Kf(1:EV%MaxINeeded)=1._dl (if flat)
chi=1 (if flat)
initv(1,i_clxg)=-chi*EV%Kf(1)/3*x2*(1-omtau/5)
initv(1,i_clxr)= initv(1,i_clxg)
initv(1,i_clxb)=0.75_dl*initv(1,i_clxg)
initv(1,i_clxc)=initv(1,i_clxb)
```



Coupled differential equations

Background

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G a^2 \bar{\rho} - \kappa ,$$
$$\frac{d}{d\tau} \left(\frac{\dot{a}}{a}\right) = -\frac{4\pi}{3} G a^2 (\bar{\rho} + 3\bar{P})$$

Linear perturbation in FRW universe

$$ds^2 = -a^2(\eta)[(1 + 2\Psi(\vec{x}, \eta))d\eta^2 - (1 - 2\Phi(\vec{x}, \eta))d\vec{x}^2]$$

$$\nabla_\mu T^{\mu\nu} = 0 \quad \Longrightarrow \quad \begin{aligned} \Phi' &= \frac{1}{3}(\delta' + \frac{k}{aH}v) \\ \Psi &= \frac{aH}{k}(v' + v) \end{aligned}$$

General Relativity



$$\begin{aligned} k^2 \Phi &= -4\pi G a^2 \rho \delta \\ \frac{\Phi}{\Psi} &= 1 \end{aligned}$$

Coupled differential equations

Perturbations

(Perturbed Einstein's equations in synchronous gauge)

$$k^2\eta - \frac{1}{2} \frac{\dot{a}}{a} \dot{\eta} = 4\pi G a^2 \delta T^0_0(\text{Syn}) ,$$

$$k^2\dot{\eta} = 4\pi G a^2 (\bar{\rho} + \bar{P})\theta(\text{Syn}) ,$$

$$\ddot{\eta} + 2 \frac{\dot{a}}{a} \dot{\eta} - 2k^2\eta = -8\pi G a^2 \delta T^i_i(\text{Syn}) ,$$

$$\ddot{\eta} + 6\ddot{\eta} + 2 \frac{\dot{a}}{a} (\dot{\eta} + 6\dot{\eta}) - 2k^2\eta = -24\pi G a^2 (\bar{\rho} + \bar{P})\sigma(\text{Syn}) .$$

Coupled differential equations

Energy conservation

$$T^{\mu\nu}{}_{;\mu} = \partial_{\mu} T^{\mu\nu} + \Gamma^{\nu}{}_{\alpha\beta} T^{\alpha\beta} + \Gamma^{\alpha}{}_{\alpha\beta} T^{\nu\beta} = 0$$

Synchronous gauge:

$$\dot{\delta} = -(1+w)\left(\theta + \frac{\dot{h}}{2}\right) - 3 \frac{\dot{a}}{a} \left(\frac{\delta P}{\delta \rho} - w\right) \delta ,$$

$$\dot{\theta} = -\frac{\dot{a}}{a} (1-3w)\theta - \frac{\dot{w}}{1+w} \theta + \frac{\delta P/\delta \rho}{1+w} k^2 \delta - k^2 \sigma .$$

Coupled differential equations

CAMB language A,B,C

Background: $\text{grho} = 8\pi G \rho a^2$, $\text{adotoa} = \frac{a'}{a} = \frac{da/d\tau}{a}$, tau =conformal time

$$\delta T^{\mu\nu}: \quad \text{dgrho} = 8\pi G a^2 \sum_i \rho_i \delta_i, \quad \text{dgq} = 8\pi G a^2 \sum_i (\rho_i + p_i) v_i$$

$$\text{clxc} = \delta_c, \text{clxb} = \delta_b, \text{clxq} = \delta_{\text{DE}}$$

$$\delta G^{\mu\nu}: \quad \text{etak} = \eta k, \text{z} = h'/(2k), \quad \text{sigma} = \frac{h' + 6\eta'}{2k}$$

Coupled differential equations

CAMB code

M+B '96

astro-ph/9506072

$$\eta' k = dgq/2$$

$$\eta' k^2 = 4\pi G a^2 (\bar{\rho} + \bar{P}) \theta$$

Differential equations to evolve in CAMB

$$clxcdot = -kz$$

$$\delta'_c = -\frac{1}{2} h'$$

$$z = (0.5dgrho/k + \eta k)/adotoa$$

$$k^2 \eta - \frac{1}{2} \frac{a'}{a} h' = 4\pi G a^2 \delta T_0^0$$

Constraint equations (algebraic)

$$\sigma = z + 1.5dgq/k^2$$

$$\sigma = \frac{h' + 6\eta'}{2k}$$



Baryons

$$\dot{\delta}_b = -\theta_b - \frac{1}{2} \dot{h} ,$$

! Baryon equation of motion.
 $\dot{\kappa} = -k(z + v_b)$
 $a y''(4) = \dot{\kappa}$

$$\dot{\theta}_b = -\frac{\dot{a}}{a} \theta_b + c_s^2 k^2 \delta_b + \frac{4\bar{\rho}_\gamma}{3\bar{\rho}_b} a n_e \sigma_T (\theta_\gamma - \theta_b) ,$$

$$\dot{v}_b = -\frac{\dot{a}}{a} v_b + c_s^2 k \kappa - \frac{4}{3} \frac{\bar{\rho}_\gamma}{\bar{\rho}_b} a n_e \sigma_T (v_\gamma - v_b)$$

Can be simplified in the tightly-coupling limit (homework)

Photons

$$\dot{\delta}_\gamma = -\frac{4}{3} \theta_\gamma - \frac{2}{3} \dot{h} ,$$

! Photon equation of motion
 $\text{clxgdot} = -k \cdot (4 \cdot \text{_dl} / 3 \cdot \text{_dl} \cdot z + qg)$

$$\dot{\theta}_\gamma = k^2 \left(\frac{1}{4} \delta_\gamma - \sigma_\gamma \right) + a n_e \sigma_T (\theta_b - \theta_\gamma) ,$$

!Once know slip, recompute qgdot, pig, pigdot
 $qgdot = k \cdot (\text{clxg} / 4 \cdot \text{_dl} - \text{pig} / 2 \cdot \text{_dl}) + \text{opacity} \cdot \text{slip}$

Higher moments, compare to CAMB equations in derivs.f90 (homework)

$$\dot{\delta}_\gamma = -\frac{4}{3} \theta_\gamma - \frac{2}{3} \dot{h},$$

$$\dot{\theta}_\gamma = k^2 \left(\frac{1}{4} \delta_\gamma - \sigma_\gamma \right) + an_e \sigma_T (\theta_b - \theta_\gamma),$$

$$\dot{F}_{\gamma 2} = 2\dot{\sigma}_\gamma = \frac{8}{15} \theta_\gamma - \frac{3}{5} k F_{\gamma 3} + \frac{4}{15} \dot{h} + \frac{8}{5} \dot{\eta}$$

$$- \frac{9}{5} an_e \sigma_T \sigma_\gamma + \frac{1}{10} an_e \sigma_T (G_{\gamma 0} + G_{\gamma 2}),$$

$$\dot{F}_{\gamma l} = \frac{k}{2l+1} [l F_{\gamma(l-1)} - (l+1) F_{\gamma(l+1)}] - an_e \sigma_T F_{\gamma l}, \quad l \geq 3,$$

$$\dot{G}_{\gamma l} = \frac{k}{2l+1} [l G_{\gamma(l-1)} - (l+1) G_{\gamma(l+1)}]$$

$$+ an_e \sigma_T \left[-G_{\gamma l} + \frac{1}{2} (F_{\gamma 2} + G_{\gamma 0} + G_{\gamma 2}) \left(\delta_{l0} + \frac{\delta_{l2}}{5} \right) \right],$$

CMB

$$C_l^{XY} \propto \int \frac{dk}{k} \Delta_{\mathcal{R}}^2 I_l^X(k) I_l^Y(k)$$
$$I_l^{X(Y)}(k) = \int \mathcal{S}^{X(Y)}(z) j_l[kr(z)] dz$$

The source term for CMB is in subroutine 'output' in equations.f90

$$\text{ISW} = (4.D0/3.D0*k*EV\%Kf(1)*sigma+(-2.D0/3.D0*sigma-2.D0/3.D0*etak/adotoa)*k \& \\ -diff_rho/k**2-1.D0/adotoa*dgrho/3.D0+(3.D0*gpres+5.D0*grho)*sigma/k/3.D0 \& \\ -2.D0/k*adotoa/EV\%Kf(1)*etak)*expmmu(j)$$

!e.g. to get only late-time ISW

! if $(1/a-1 < 30)$ ISW=0

!The rest, note $y(9) \rightarrow \text{octg}$, $yprime(9) \rightarrow \text{octgprime}$ (octopoles)

$$\text{sources}(1) = \text{ISW} + ((-9.D0/160.D0*pig-27.D0/80.D0*ypol(2))/k**2*opac(j)+ \& \\ (11.D0/10.D0*sigma- 3.D0/8.D0*EV\%Kf(2)*ypol(3)+vb- \\ 9.D0/80.D0*EV\%Kf(2)*octg+3.D0/40.D0*qg)/k- \& \\ (-180.D0*ypolprime(2)-30.D0*pigdot)/k**2/160.D0)*dvis(j) + \& \\ (-9.D0*pigdot+ \\ 54.D0*ypolprime(2))/k**2*opac(j)/160.D0+pig/16.D0+clxg/4.D0+3.D0/8.D0*ypol(2) + \& \\ (-21.D0/5.D0*adotoa*sigma-3.D0/8.D0*EV\%Kf(2)*ypolprime(3) + \& \\ vbdot+3.D0/40.D0*qgdot- 9.D0/80.D0*EV\%Kf(2)*octgprime)/k + \& \\ (-9.D0/160.D0*dopac(j)*pig-21.D0/10.D0*dgpi-27.D0/80.D0*dopac(j)*ypol(2))/k**2)*vis(j) \\ + \&$$

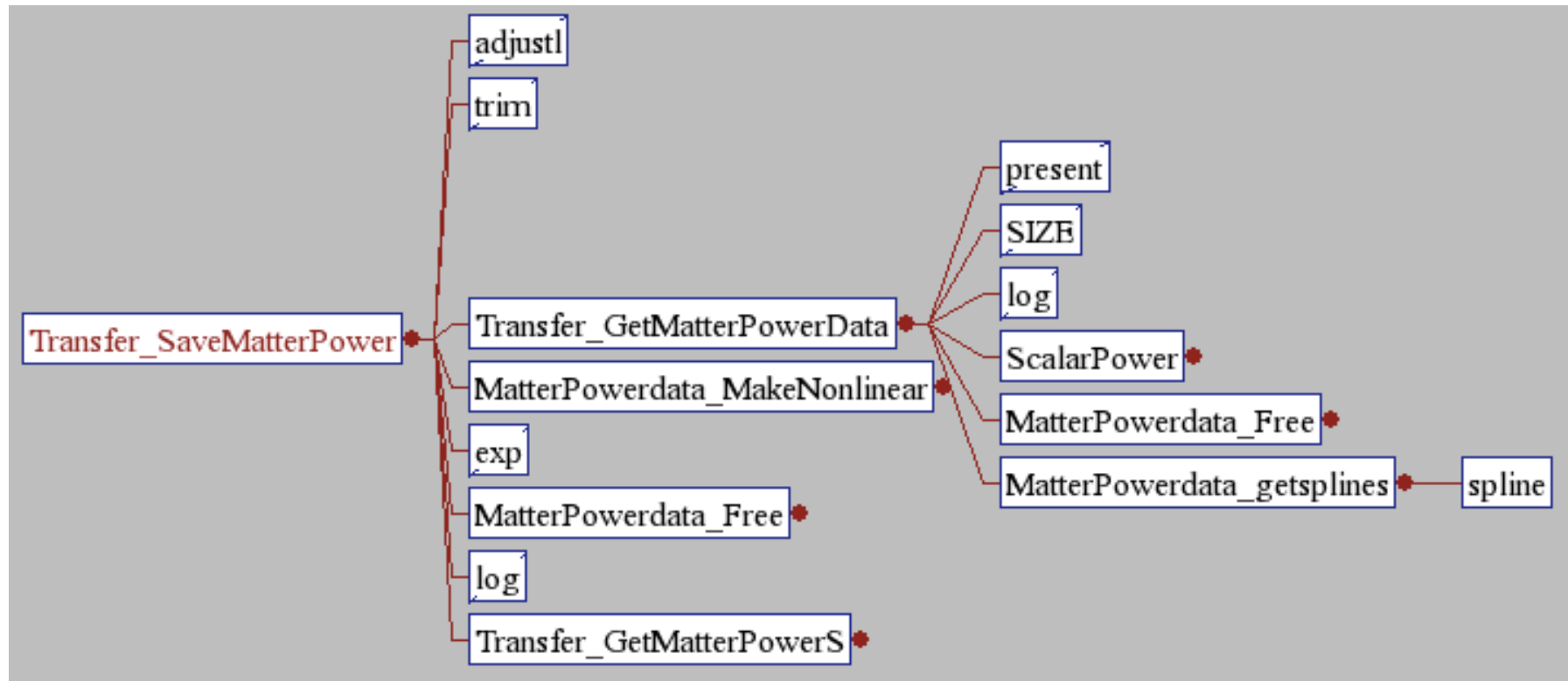
$$(3.D0/16.D0*ddvis(j)*pig+9.D0/8.D0*ddvis(j)*ypol(2))/k**2+21.D0/10.D0/k/EV\%Kf(1)*vis(j)* \\ etak$$

But it is understandable!!

Check <http://camb.info/theory.html>
for the maple files of this source calculated using
the famous line-of-sight integral! (homework)

3D Matter power spectrum

$$P_k = \frac{2\pi^2 \mathcal{P}_\chi}{k^3} T_\Delta^2.$$



Useful references

- Camb.info; cosmologist.info/cosmomc/
- Cosmocoffee.info
- M+B: astro-ph/9506072
- Jussi's ICG lectures: www.icg.port.ac.uk/~valiviiij/
- Wayne Hu's tutorials: <http://background.uchicago.edu/~whu/>
- Numerical Recipe: www.nr.com
- Plotting software: OriginPro, Matlab, IDL, gnuplot
- Icosmology.info