

# LECTURE 12: THE MEASUREMENT OF POWER SPECTRUM AND CORRELATION FUNCTION

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## 1. THE FKP WEIGHT

The FKP weight is the optimal weight for measurement of power spectrum multipoles [1].

$$(1) \quad P(k) = P(\mathbf{k}) \equiv \int d^3r \xi(\mathbf{r}) e^{i\mathbf{k} \cdot \mathbf{r}}$$

$$(2) \quad F(\mathbf{r}) \equiv \frac{w(\mathbf{r}) [n_g(\mathbf{r}) - \alpha n_s(\mathbf{r})]}{[\int d^3r \bar{n}^2(\mathbf{r}) w^2(\mathbf{r})]^{1/2}}$$

$$\langle |F(\mathbf{k})|^2 \rangle = \frac{\int d^3r \int d^3r' w(\mathbf{r}) w(\mathbf{r}') \langle [n_g(\mathbf{r}) - \alpha n_s(\mathbf{r})] [n_g(\mathbf{r}') - \alpha n_s(\mathbf{r}')] \rangle e^{i\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}')}}{\int d^3r \bar{n}^2(\mathbf{r}) w^2(\mathbf{r})}$$

$$(3) \quad \begin{aligned} \langle n_g(\mathbf{r}) n_g(\mathbf{r}') \rangle &= \bar{n}(\mathbf{r}) \bar{n}(\mathbf{r}') [1 + \xi(\mathbf{r} - \mathbf{r}')] + \bar{n}(\mathbf{r}) \delta(\mathbf{r} - \mathbf{r}') \\ \langle n_s(\mathbf{r}) n_s(\mathbf{r}') \rangle &= \alpha^{-2} \bar{n}(\mathbf{r}) \bar{n}(\mathbf{r}') + \alpha^{-1} \bar{n}(\mathbf{r}) \delta(\mathbf{r} - \mathbf{r}') \\ \langle n_g(\mathbf{r}) n_s(\mathbf{r}') \rangle &= \alpha^{-1} \bar{n}(\mathbf{r}) \bar{n}(\mathbf{r}') \end{aligned}$$

$$(4) \quad \langle |F(\mathbf{k})|^2 \rangle = \int \frac{d^3k'}{(2\pi)^3} P(\mathbf{k}') |G(\mathbf{k} - \mathbf{k}')|^2 + (1 + \alpha) \frac{\int d^3r \bar{n}(\mathbf{r}) w^2(\mathbf{r})}{\int d^3r \bar{n}^2(\mathbf{r}) w^2(\mathbf{r})}$$

$$(5) \quad G(\mathbf{k}) \equiv \frac{\int d^3r \bar{n}(\mathbf{r}) w(\mathbf{r}) e^{i\mathbf{k} \cdot \mathbf{r}}}{[\int d^3r \bar{n}^2(\mathbf{r}) w^2(\mathbf{r})]^{1/2}}$$

$$(6) \quad \langle |F(\mathbf{k})|^2 \rangle \simeq P(\mathbf{k}) + P_{\text{shot}}$$

$$(7) \quad P_{\text{shot}} \equiv \frac{(1 + \alpha) \int d^3r \bar{n}(\mathbf{r}) w^2(\mathbf{r})}{\int d^3r \bar{n}^2(\mathbf{r}) w^2(\mathbf{r})}$$

$$(8) \quad \hat{P}(\mathbf{k}) = |F(\mathbf{k})|^2 - P_{\text{shot}}$$

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$$(9) \quad \hat{P}(k) \equiv \frac{1}{V_k} \int_{V_k} d^3k' \hat{P}(\mathbf{k}')$$

$$(10) \quad \sigma_P^2(k) \simeq \frac{1}{V_k} \int d^3k' |P(k)Q(\mathbf{k}') + S(\mathbf{k}')|^2$$

where

$$(11) \quad Q(\mathbf{k}) \equiv \frac{\int d^3r \bar{n}^2(\mathbf{r}) w^2(\mathbf{r}) e^{i\mathbf{k} \cdot \mathbf{r}}}{\int d^3r \bar{n}^2(\mathbf{r}) w^2(\mathbf{r})}$$

and

$$(12) \quad S(\mathbf{k}) \equiv \frac{(1 + \alpha) \int d^3r \bar{n}(\mathbf{r}) w^2(\mathbf{r}) e^{i\mathbf{k} \cdot \mathbf{r}}}{\int d^3r \bar{n}^2(\mathbf{r}) w^2(\mathbf{r})}$$

$$(13) \quad \frac{\sigma_P^2(k)}{P^2(k)} = \frac{(2\pi)^3 \int d^3r \bar{n}^4 w^4 [1 + 1/\bar{n}P(k)]^2}{V_k [\int d^3r \bar{n}^2 w^2]^2}$$

The KFP weight taking the following form minimises Eq (13):

$$(14) \quad w_0(\mathbf{r}) = \frac{1}{1 + \bar{n}(\mathbf{r})P(k)}$$

## 2. THE MEASUREMENT OF THE ANISOTROPIC CORRELATION FUNCTION

Pair counts in ‘data’ are compared to pair counts in ‘random’ samples that follow the geometry of the survey. We assume a catalogue of  $n_d$  objects in the data sample and  $n_r$  in the random sample and then calculate three sets of numbers of pairs as a function of the binned comoving separation  $s$ ,

- Data-data pairs  $dd(s)$  that normalized by the total number of data pairs:

$$(15) \quad DD(s) = \frac{dd(s)}{n_d(n_d - 1)/2};$$

- Random-Random pairs  $rr(s)$  that normalized by the total number of random pairs:

$$(16) \quad RR(s) = \frac{rr(s)}{n_r(n_r - 1)/2};$$

- Data-Random pairs  $dr(s)$  that normalized by the total number of cross pairs:

$$(17) \quad DR(s) = \frac{dr(s)}{n_r n_d}.$$

The most commonly used Landy & Szalay estimator [4]:

$$(18) \quad \hat{\xi}(s) = \frac{DD - 2DR + RR}{RR}.$$

The task now is to compute the pair counts quickly for a large sample, which is challenging. For example, for BOSS DR12 sample, there are  $\sim 1\text{M}$  data samples, and  $\sim 100\text{M}$

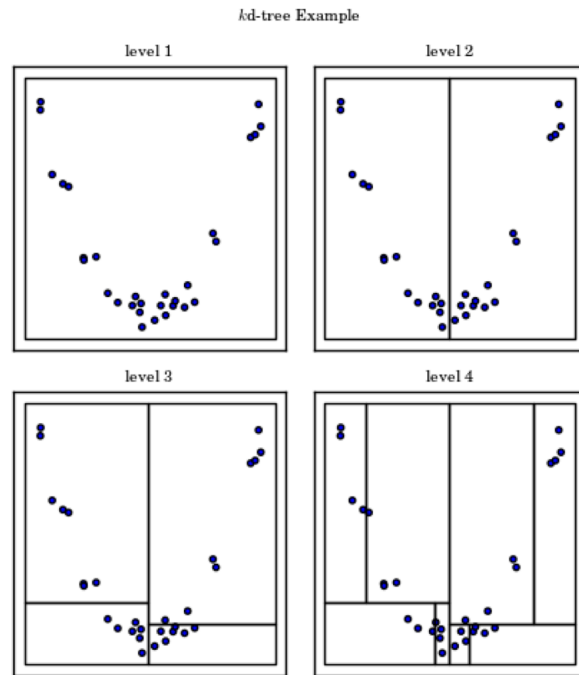


FIGURE 1. An example of kdtree.

random samples! So this is mission impossible for a brutal force algorithm with  $O(N^2)$  complexity.

The way out is to use the kd-tree algorithm <sup>1</sup>.

The kdtree code for the measurement: <https://github.com/w11745881210/KDTPCF>

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<sup>1</sup>[http://www.linuxclustersinstitute.org/conferences/archive/2008/PDF/Dolence\\_98279.pdf](http://www.linuxclustersinstitute.org/conferences/archive/2008/PDF/Dolence_98279.pdf)

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function dualtree(node1, node2) {
    If node1 and node2 have already been compared in reverse
        return
    Let dmin = minimum distance between node1 and node2
    If dmin is greater than maximum distance of interest
        return
    Let dmax = maximum distance between node1 and node2
    Let binmin = distance bin for distance dmin
    Let binmax = distance bin for distance dmax
    If binmin = binmax
        Add node1.cnt x node2.cnt to bin count
    Else if node1 and node2 are leaf nodes
        For all points i owned by node1
            Let smin = minimum distance between point i and node2
            Let smax = maximum distance between point i and node2

            Let binmin = distance bin for smin
            Let binmax = distance bin for distance smax
            If binmin = binmax
                Add node2.cnt to bin count
            Else
                Find distances to all points in node2 from point i
                Find distance bins and update the bin counts
    Else
        If node1.cnt > node2.cnt
            dualtree(node1.leftChild, node2)
            dualtree(node1.rightChild, node2)
        Else
            dualtree(node1, node2.leftChild)
            dualtree(node1, node2.rightChild)
}

```

FIGURE 2. The pseudo-code of kd-tree.

## REFERENCES

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