

LECTURE 11: THE MEASUREMENT OF POWER SPECTRUM MULTIPOLES

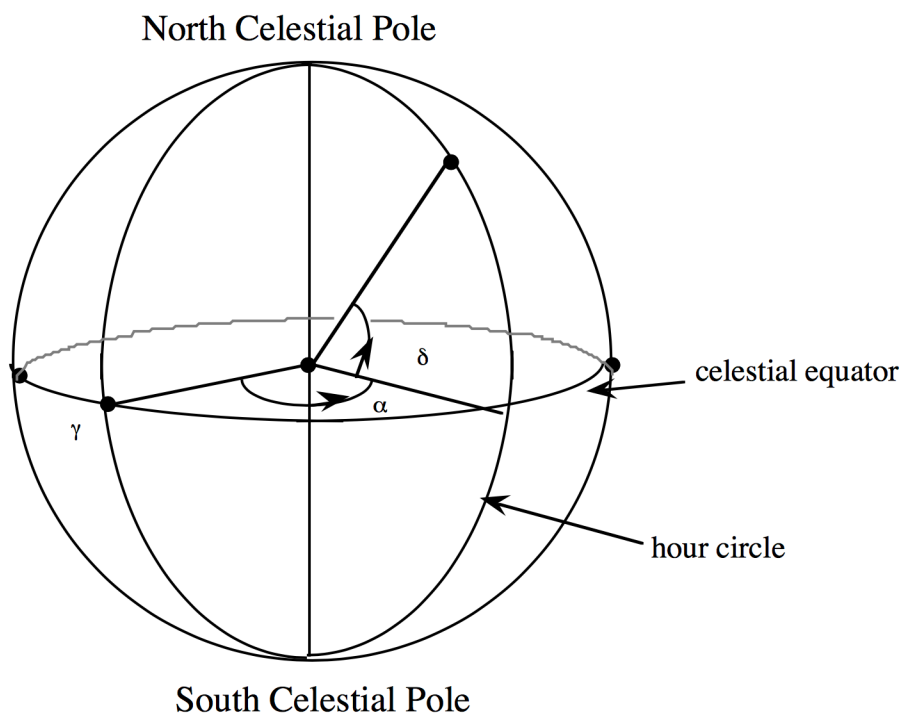
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1. MEASUREMENT OF POWER SPECTRUM MULTIPOLES

(1)

$$\begin{aligned}x &= \chi \cos \delta \cos \alpha \\y &= \chi \cos \delta \sin \alpha \\z &= \chi \sin \delta\end{aligned}$$

The Yamamoto estimator,



$$\hat{P}_\ell(k) = \frac{(2\ell+1)}{I} \int \frac{d\Omega_k}{4\pi} \left[\int d\mathbf{r}_1 \int d\mathbf{r}_2 F(\mathbf{r}_1) F(\mathbf{r}_2) \times e^{i\mathbf{k} \cdot (\mathbf{r}_1 - \mathbf{r}_2)} \mathcal{L}_\ell(\hat{\mathbf{k}} \cdot \hat{\mathbf{r}}_h) - P_\ell^{\text{noise}}(\mathbf{k}) \right],$$

$$\hat{P}_\ell^{\text{Yama}}(k) = \frac{(2\ell+1)}{I} \int \frac{d\Omega_k}{4\pi} \left[\int d\mathbf{r}_1 F(\mathbf{r}_1) e^{i\mathbf{k} \cdot \mathbf{r}_1} \times \int d\mathbf{r}_2 F(\mathbf{r}_2) e^{-i\mathbf{k} \cdot \mathbf{r}_2} \mathcal{L}_\ell(\hat{\mathbf{k}} \cdot \hat{\mathbf{r}}_2) - P_\ell^{\text{noise}}(\mathbf{k}) \right],$$

which can not be evaluated using FFTs.

One solution is the following [1],

$$(2) \quad A_n(\mathbf{k}) = \int d\mathbf{r} (\hat{\mathbf{k}} \cdot \hat{\mathbf{r}})^n F(\mathbf{r}) e^{i\mathbf{k} \cdot \mathbf{r}}$$

$$(3) \quad \begin{aligned} \hat{P}_0^{\text{Yama}}(k) &= \frac{1}{I} \int \frac{d\Omega_k}{4\pi} [A_0(\mathbf{k}) A_0^*(\mathbf{k})] - P_0^{\text{noise}} \\ \hat{P}_2^{\text{Yama}}(k) &= \frac{5}{2I} \int \frac{d\Omega_k}{4\pi} A_0(\mathbf{k}) [3A_2^*(\mathbf{k}) - A_0^*(\mathbf{k})] \\ \hat{P}_4^{\text{Yama}}(k) &= \frac{9}{8I} \int \frac{d\Omega_k}{4\pi} A_0(\mathbf{k}) [35A_4^*(\mathbf{k}) - 30A_2^*(\mathbf{k}) + 3A_0^*(\mathbf{k})] \end{aligned}$$

$$(4) \quad \hat{\mathbf{k}} \cdot \hat{\mathbf{r}} = \frac{k_x r_x + k_y r_y + k_z r_z}{kr}$$

$$(5) \quad \begin{aligned} A_2(\mathbf{k}) &= \frac{1}{k^2} \{ k_x^2 B_{xx}(\mathbf{k}) + k_y^2 B_{yy}(\mathbf{k}) + k_z^2 B_{zz}(\mathbf{k}) \\ &\quad + 2[k_x k_y B_{xy}(\mathbf{k}) + k_x k_z B_{xz}(\mathbf{k}) + k_y k_z B_{yz}(\mathbf{k})] \} \end{aligned}$$

$$(6) \quad B_{ij}(\mathbf{k}) \equiv \int d\mathbf{r} \frac{r_i r_j}{r^2} F(\mathbf{r}) e^{i\mathbf{k} \cdot \mathbf{r}}$$

$$(7) \quad \begin{aligned} A_4(\mathbf{k}) &= \frac{1}{k^4} \{ k_x^4 C_{xxx} + k_y^4 C_{yyy} + k_z^4 C_{zzz} \\ &\quad + 4[k_x^3 k_y C_{xxy} + k_x^3 k_z C_{xxz} + k_y^3 k_x C_{yyx} \\ &\quad + 4_y^3 k_z C_{yyz} + k_z^3 k_x C_{zzx} + k_z^3 k_y C_{zzx}] \\ &\quad + 6[k_x^2 k_y^2 C_{xyy} + k_x^2 k_z^2 C_{xzz} + k_y^2 k_z^2 C_{yzz}] \\ &\quad + 12k_x k_y k_z [k_x C_{xyz} + k_y C_{yxz} + k_z C_{zxy}] \} \end{aligned}$$

$$(8) \quad C_{ijl}(\mathbf{k}) \equiv \int d\mathbf{r} \frac{r_i^2 r_j r_l}{r^4} F(\mathbf{r}) e^{i\mathbf{k} \cdot \mathbf{r}}$$

This works, but not efficient. A better way is using the addition theorem [2],

$$(9) \quad \mathcal{L}_\ell(\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{r}}_2) = \frac{4\pi}{2\ell+1} \sum_{m=-\ell}^{\ell} Y_{\ell m}(\hat{\mathbf{r}}_1) Y_{\ell m}^*(\hat{\mathbf{r}}_2)$$

which is a generalisation of the well-known formula below,

$$(10) \quad \begin{aligned} \sin(x+y) &= \sin(x)\cos(y) + \cos(x)\sin(y) \\ \cos(x+y) &= \cos(x)\cos(y) - \sin(x)\sin(y) \end{aligned}$$

Now,

$$(11) \quad \hat{P}_\ell(k) = \frac{2\ell+1}{I} \int \frac{d\Omega_k}{4\pi} F_0(\mathbf{k}) F_\ell(-\mathbf{k})$$

$$(12) \quad \begin{aligned} F_\ell(\mathbf{k}) &\equiv \int d\mathbf{r} F(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}} \mathcal{L}_\ell(\hat{\mathbf{k}} \cdot \hat{\mathbf{r}}) \\ &= \frac{4\pi}{2\ell+1} \sum_{m=-\ell}^{\ell} Y_{\ell m}(\hat{\mathbf{k}}) \int d\mathbf{r} F(\mathbf{r}) Y_{\ell m}^*(\hat{\mathbf{r}}) e^{i\mathbf{k}\cdot\mathbf{r}} \end{aligned}$$

where

$$(13) \quad Y_{\ell m}(\theta, \phi) \equiv \begin{cases} \sqrt{\frac{2\ell+1}{2\pi} \frac{(\ell-m)!}{(\ell+m)!}} \mathcal{L}_\ell^m(\cos \theta) \cos m\phi & m > 0 \\ \sqrt{\frac{2\ell+1}{4\pi}} \mathcal{L}_\ell^m(\cos \theta) & m = 0 \\ \sqrt{\frac{2\ell+1}{2\pi} \frac{(\ell-|m|)!}{(\ell+|m|)!}} \mathcal{L}_\ell^{|m|}(\cos \theta) \sin |m|\phi & m < 0 \end{cases}$$

REFERENCES

- [1] D. Bianchi, H. Gil-Marín, R. Ruggeri and W. J. Percival, “Measuring line-of-sight dependent Fourier-space clustering using FFTs,” *Mon. Not. Roy. Astron. Soc.* **453**, no. 1, L11 (2015) doi:10.1093/mnras/slv090 [arXiv:1505.05341 [astro-ph.CO]].
- [2] N. Hand, Y. Li, Z. Slepian and U. Seljak, “An optimal FFT-based anisotropic power spectrum estimator,” *JCAP* **1707**, no. 07, 002 (2017) doi:10.1088/1475-7516/2017/07/002 [arXiv:1704.02357 [astro-ph.CO]].