

Analyzing RCTs: A Cookbook

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Contents

1	Preface	5
1.1	Structure of the book	5
1.2	Acknowledgements	5
2	Introduction	7
2.1	Trial Flow	7
2.2	Simulated dataset	7
3	Continuous endpoints	9
3.1	Single follow-up	9
3.2	Repeated follow-up	9
4	Dichotomous endpoints	11
4.1	Single follow-up	11
4.2	Repeated follow-up	18
4.3	Treatment-time interaction model	19
5	Time to event analyses	29
6	Sample size calculations	31

Chapter 1

Preface

Placeholder

1.1 Structure of the book

1.2 Acknowledgements

Chapter 2

Introduction

Placeholder

2.1 Trial Flow

2.2 Simulated dataset

Chapter 3

Continuous endpoints

Placeholder

3.1 Single follow-up

3.1.1 Stata code

3.1.2 R code

3.1.3 Reporting

3.2 Repeated follow-up

3.2.1 Simple model

3.2.2 Model with treatment-time interaction

3.2.3 Model with treatment-time interaction and baseline information

Chapter 4

Dichotomous endpoints

4.1 Single follow-up

For a single follow-up assessment of a dichotomous endpoint, the main method I use is a standard logistic regression. Then we can adjust for stratification factors in the randomisation in addition to other pre-specified covariates, both categorical and continuous. In the simulated example, we define that the primary outcome is the dichotomous categorical outcome at time 3. Note that usually the baseline status of all patients are negative for the outcome, so adjusting for baseline is not necessary.

4.1.1 Stata code

```
use "stata/rct", clear
tabulate catout trt if time == 3, column
logistic catout i.trt i.site covar if time==3, coef
```

(all strata combined)

```
+-----+
| Key          |
+-----+
| frequency    |
| column percentage |
+-----+
```

Categorical	Treatment		
1 outcome	Placebo	Active	Total
-----+-----+-----			

Negative		9	22		31
		18.00	45.83		31.63
-----+					
Positive		41	26		67
		82.00	54.17		68.37
-----+					
Total		50	48		98
		100.00	100.00		100.00

Logistic regression

Number of obs = 98

LR chi2(5) = 48.59

Prob > chi2 = 0.0000

Log likelihood = -36.862204

Pseudo R2 = 0.3973

-----+-----						
catout		Coefficient	Std. err.	z	P> z	[95% conf. interval]
-----+-----						
trt						
Active		-2.890301	.7850252	-3.68	0.000	-4.428922 -1.351679
site						
2		.7783404	.8580245	0.91	0.364	-.9033566 2.460037
3		1.423791	.7786531	1.83	0.067	-.1023412 2.949923
4		.0253234	.8082887	0.03	0.975	-1.558893 1.60954
covar		1.001078	.2329461	4.30	0.000	.5445124 1.457644
_cons		-2.463577	.8925892	-2.76	0.006	-4.21302 -.7141344
-----+-----						

Note that the use the `coef` option to get the log odds ratio estimates.

4.1.2 R code

```
rct <- read_dta("stata/rct.dta") %>%
  modify_at(c("trt","catout"), haven::as_factor, levels = "labels") %>%
  modify_at(c("site","time"), haven::as_factor)
rct %>%
  filter(time==3) %>%
  glm(catout ~ trt + site + covar , data=., family = binomial) %>%
  summary
```

Call:

```
glm(formula = catout ~ trt + site + covar, family = binomial,
     data = .)
```

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)	
(Intercept)	-2.46358	0.89259	-2.760	0.005780	**
trtActive	-2.89030	0.78502	-3.682	0.000232	***
site2	0.77834	0.85802	0.907	0.364337	
site3	1.42379	0.77865	1.829	0.067470	.
site4	0.02532	0.80829	0.031	0.975007	
covar	1.00108	0.23295	4.297	1.73e-05	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 122.318 on 97 degrees of freedom
 Residual deviance: 73.724 on 92 degrees of freedom
 AIC: 85.724

Number of Fisher Scoring iterations: 6

Not surprisingly, the estimates are identical.

4.1.3 Reporting

Reporting for dichotomous endpoints is a bit tricky. The natural estimates from a logistic regression is odds and odds ratios, but these are less interpretable than risk differences or relative risk. As New England Journal of Medicine states in their Statistical Guidelines: “Odds ratios should be avoided, as they may overestimate the relative risks in many settings and be misinterpreted.” Fortunately, both Stata and R can estimate adjusted risk differences and relative risks from logistic regressions.

4.1.3.1 Stata code

First we compute the average predicted marginal probabilities. Basically this is done by calculating the predicted probability of a positive outcome for each patient, under both treatments, and then averaging. The standard errors are computed by the delta method.

```
use "stata/rct", clear
quietly logistic catout i.trt i.site covar if time==3, coef
margins trt
```

(all strata combined)

Predictive margins
Model VCE: OIM

Number of obs = 98

Expression: Pr(catout), predict()

		Delta-method					
		Margin	std. err.	z	P> z	[95% conf. interval]	
	trt						
	Placebo	.8499905	.0387218	21.95	0.000	.7740972	.9258839
	Active	.5111833	.0533918	9.57	0.000	.4065374	.6158293

The adjusted risk difference is calculated similarly.

```
use "stata/rct", clear
quietly logistic catout i.trt i.site covar if time==3, coef
margins, dydx(trt)
```

(all strata combined)

Average marginal effects
Model VCE: OIM

Number of obs = 98

Expression: Pr(catout), predict()
dy/dx wrt: 1.trt

		Delta-method					
		dy/dx	std. err.	z	P> z	[95% conf. interval]	
	trt						
	Active	-.3388072	.0661086	-5.13	0.000	-.4683777	-.2092367

Note: dy/dx for factor levels is the discrete change from the base level.

We see that the risk difference is the difference of the estimated marginal probabilities we computed previously.

The relative risk is a bit more difficult to calculate, but not much. It uses the nlcom method to compute non-linear combinations of estimates.

```
use "stata/rct", clear
quietly logistic catout i.trt i.site covar if time==3, coef
quietly margins trt, post
```

```
margins, coeflegend
nlcom (ratio1: (_b[1.trt]/_b[0bn.trt]))
```

```
(all strata combined)
```

```
Predictive margins
Model VCE: OIM
```

```
Number of obs = 98
```

```
Expression: Pr(catout), predict()
```

	Margin	Legend
trt		
Placebo	.8499905	_b[0bn.trt]
Active	.5111833	_b[1.trt]

```
ratio1: (_b[1.trt]/_b[0bn.trt])
```

	Coefficient	Std. err.	z	P> z	[95% conf. interval]
ratio1	.6013989	.0686524	8.76	0.000	.4668426 .7359552

The trick is to know what goes into the `_b[]`-brackets, which will be revealed using the ‘coeflegend’-option. Note that I do not know the properties of this estimator, and it might be clever to check the estimates using bootstrap.

Some journals require calculation of the number needed to treat (NNT), at least if the confidence interval of the adjusted risk difference does not include zero (for which the NNT is undefined). This is simply done by inverting the adjusted risk difference estimate (both point estimate and the confidence limits).

4.1.3.2 R code

The average predicted marginal probabilities was previously not easily computed in R, but with the emergence of the very nice `marginalEffects`-package, this is now much easier:

```
mod <- rct %>%
  filter(time == 3) %>%
  glm(catout ~ trt + site + covar, data=., family = binomial)
```

```
mod %>%
  avg_predictions(variables = list(trt = c("Active", "Placebo")), type = "response")
```

	trt	Estimate	Std. Error	z	Pr(> z)	S	2.5 %	97.5 %
	Active	0.511	0.0534	9.57	<0.001	69.7	0.407	0.616
	Placebo	0.850	0.0387	21.95	<0.001	352.4	0.774	0.926

Columns: trt, estimate, std.error, statistic, p.value, s.value, conf.low, conf.high
Type: response

We notice that the estimates are equal to the Stata output..

Another option is to bootstrap the predicted marginal predictions:

```
library(boot)

fpred <- function(formula, data, indices){
  d <- data[indices,]
  fit <- glm(formula, data = d, family = binomial)
  pred <- prediction(fit, data = d, at = list(trt = c("Active", "Placebo"))) %>%
    as_tibble %>%
    group_by(trt) %>%
    summarise(mean = mean(fitted)) %>%
    ungroup() %>%
    mutate(name = paste0(trt)) %>%
    select(name, mean) %>%
    spread(name, mean) %>%
    as_vector
  return(pred)
}

data <- filter(rct, time == 3)
result <- boot(data = data,
               statistic = fpred,
               R = 10000,
               formula = catout ~ trt + site + covar,
               parallel = "multicore",
               ncpus = 4) %>%
  tidy(conf.int = TRUE)
```

Error in prediction(fit, data = d, at = list(trt = c("Active", "Placebo"))): could not

```
result %>%
  select(-bias) %>%
  knitr::kable(digits = 3)
```


Error in eval(expr, envir, enclos): object 'result' not found

We see that the estimates are identical to the Stata estimates, although the standard errors and confidence limits are a bit different. But I actually think the bootstrap estimates are better.

The estimated marginal risk difference in R is computed using the `marginaleffects`-package again.

```
rlogistic <- rct %>%
  filter(time==3) %>%
  glm(catout ~ trt + site + covar , data=., family = binomial)

rlogistic %>%
  avg_comparisons(variables = list(trt = c("Active", "Placebo")), type = "response")
```

Term	Contrast	Estimate	Std. Error	z	Pr(> z)	S	2.5 %	97.5 %
trt	Placebo - Active	0.339	0.0661	5.13	<0.001	21.7	0.209	0.468

Columns: term, contrast, estimate, std.error, statistic, p.value, s.value, conf.low, conf.high
Type: response

We see that the estimates are identical to the Stata estimates.

The relative risk is very easily computed in R using the `marginaleffects`-package:

```
rlogistic <- rct %>%
  filter(time==3) %>%
  glm(catout ~ trt + site + covar , data=., family = binomial)

rlogistic %>%
  avg_comparisons(variables = list(trt = c("Active", "Placebo")), type = "response", comparison = "riskratio")
```

Term	Contrast	Estimate	Std. Error	z	Pr(> z)	S	2.5 %	97.5 %
trt	mean(Placebo) / mean(Active)	1.66	0.19	8.76	<0.001	58.8	1.29	2.03

Columns: term, contrast, estimate, std.error, statistic, p.value, s.value, conf.low, conf.high, p
Type: response

The estimate is the inverse of the Stata estimate, and the confidence limits are very similar. There is probably a slight difference in how these are computed.

This is possible to do also by bootstrapping, but it is a bit more complicated:

```

library(boot)
library(prediction)

fpred <- function(formula, data, indices){
  d <- data[indices,]
  fit <- glm(formula, data = d, family = binomial)
  pred <- prediction(fit, data = d, at = list(trt = c("Active", "Placebo"))) %>%
    as_tibble %>%
    group_by(trt) %>%
    summarise(mean = mean(fitted)) %>%
    ungroup() %>%
    mutate(name = paste0(trt)) %>%
    select(name, mean) %>%
    spread(name, mean) %>%
    as_vector
  return(pred["Active"]/pred["Placebo"])
}

data <- filter(rct, time == 3)
result <- boot(data = data,
               statistic = fpred,
               R = 10000,
               formula = catout ~ trt + site + covar,
               parallel = "multicore",
               ncpus = 4) %>%
  tidy(conf.int = TRUE)

result %>%
  select(-bias) %>%
  knitr::kable(digits = 3)

```

term	statistic	std.error	conf.low	conf.high
Active	0.601	0.081	0.446	0.763

4.2 Repeated follow-up

When there are repeated dichotomous endpoints, there are usually two methods available, either the generalized estimating equations method or the generalized mixed model method. I prefer the mixed model approach because it has better missing data properties, and I like that the parameter estimates are interpretable conditional on the subject. In my mind it is more aligned to a causal interpretation. I will show how to do the mixed logistic regression model. We skip the simple model and go straight to a model with treatment-time interaction. Note that usually a dichotomous endpoint all have the same value at baseline (all

subjects are in the same state), thus we rarely include the baseline. The model is a simple random intercept model, but it could of course also be expanded to a random intercept and slope model.

4.3 Treatment-time interaction model

In Stata, the model is coded as:

```
use "stata/rct", clear
bysort time: tabulate catout trt, column
melogit catout i.trt i.site covar i.time i.trt#i.time if time != 0 || pid:
```

(all strata combined)

-> time = 0

```
+-----+
| Key          |
|-----|
|      frequency      |
| column percentage |
+-----+
```

Categorical 1 outcome	Treatment		Total
	Placebo	Active	
Negative	50	48	98
	100.00	100.00	100.00
Total	50	48	98
	100.00	100.00	100.00

-> time = 1

```
+-----+
| Key          |
|-----|
|      frequency      |
| column percentage |
+-----+
```

Categorical 1 outcome	Treatment		Total
	Placebo	Active	

Negative	24	32	56
	48.00	66.67	57.14
Positive	26	16	42
	52.00	33.33	42.86
Total	50	48	98
	100.00	100.00	100.00

-> time = 2

Key
frequency
column percentage

Categorical outcome	Treatment		Total
	Placebo	Active	
Negative	12	29	41
	24.00	60.42	41.84
Positive	38	19	57
	76.00	39.58	58.16
Total	50	48	98
	100.00	100.00	100.00

-> time = 3

Key
frequency
column percentage

Categorical outcome	Treatment		Total
	Placebo	Active	
Negative	9	22	31

		18.00	45.83		31.63
-----+-----					
Positive		41	26		67
		82.00	54.17		68.37
-----+-----					
Total		50	48		98
		100.00	100.00		100.00

Fitting fixed-effects model:

Iteration 0: Log likelihood = -133.59899
 Iteration 1: Log likelihood = -129.32428
 Iteration 2: Log likelihood = -129.32244
 Iteration 3: Log likelihood = -129.32244

Refining starting values:

Grid node 0: Log likelihood = -130.87679

Fitting full model:

Iteration 0: Log likelihood = -130.87679
 Iteration 1: Log likelihood = -129.7867
 Iteration 2: Log likelihood = -129.37214
 Iteration 3: Log likelihood = -129.31054
 Iteration 4: Log likelihood = -129.31042
 Iteration 5: Log likelihood = -129.31042

Mixed-effects logistic regression
 Group variable: pid

Number of obs = 294
 Number of groups = 98

Obs per group:

min = 3
 avg = 3.0
 max = 3

Integration method: mvaghermite

Integration pts. = 7

Log likelihood = -129.31042

Wald chi2(9) = 57.65
 Prob > chi2 = 0.0000

catout		Coefficient	Std. err.	z	P> z	[95% conf. interval]
-----+-----						
trt						

Active		-1.445692	.5336445	-2.71	0.007	-2.491616	-.3997679
site							
2		-.0293421	.4666747	-0.06	0.950	-.9440077	.8853234
3		.72617	.4145006	1.75	0.080	-.0862363	1.538576
4		.5185491	.4721013	1.10	0.272	-.4067524	1.443851
covar		.9174909	.1319527	6.95	0.000	.6588684	1.176113
time							
2		1.787348	.5856394	3.05	0.002	.6395156	2.93518
3		2.365878	.6353582	3.72	0.000	1.120599	3.611157
trt#time							
Active#2		-1.406754	.7657322	-1.84	0.066	-2.907561	.0940536
Active#3		-1.127376	.785537	-1.44	0.151	-2.667	.4122484
_cons		-4.424402	.7595232	-5.83	0.000	-5.91304	-2.935764
-----+-----							
pid							
var(_cons)		.0684106	.4545936			1.51e-07	31004.17
-----+-----							

LR test vs. logistic model: $\text{chibar2}(01) = 0.02$ Prob >= $\text{chibar2} = 0.4384$

In R, this model is coded as:

```
library(lme4)
rct %>%
  filter(time != 0) %>%
  glmer(catout ~ trt + time + trt*time + site + covar + (1|pid),
        data = .,
        family = binomial,
        nAGQ = 7) %>%
  summary()
```

Generalized linear mixed model fit by maximum likelihood (Adaptive Gauss-Hermite Quadrature, nAGQ = 7) [glmerMod]
 Family: binomial (logit)
 Formula: catout ~ trt + time + trt * time + site + covar + (1 | pid)
 Data: .

AIC	BIC	logLik	deviance	df.resid
280.6	321.1	-129.3	258.6	283

Scaled residuals:

Min	1Q	Median	3Q	Max
-2.8902	-0.5490	0.1407	0.5132	5.6565

Random effects:

```
Groups Name      Variance Std.Dev.
pid  (Intercept) 0.06842  0.2616
Number of obs: 294, groups: pid, 98
```

Fixed effects:

```
Estimate Std. Error z value Pr(>|z|)
(Intercept) -4.42440    0.75952  -5.825 5.70e-09 ***
trtActive    -1.44556    0.53364  -2.709 0.006751 **
time2        1.78740    0.58564   3.052 0.002273 **
time3        2.36568    0.63534   3.724 0.000196 ***
site2       -0.02946    0.46667  -0.063 0.949661
site3        0.72598    0.41449   1.751 0.079861 .
site4        0.51846    0.47210   1.098 0.272110
covar        0.91750    0.13195   6.953 3.57e-12 ***
trtActive:time2 -1.40690    0.76574  -1.837 0.066163 .
trtActive:time3 -1.12718    0.78553  -1.435 0.151305
---
```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:

```
(Intr) trtAct time2 time3 site2 site3 site4 covar trtA:2
trtActive -0.117
time2      -0.555  0.349
time3      -0.604  0.295  0.475
site2      -0.188 -0.031  0.029  0.042
site3      -0.292 -0.115  0.059  0.077  0.477
site4      -0.306 -0.061  0.052  0.078  0.398  0.462
covar      -0.818 -0.226  0.293  0.373 -0.074  0.032  0.086
trtActv:tm2  0.385 -0.601 -0.750 -0.343 -0.020 -0.031 -0.037 -0.181
trtActv:tm3  0.363 -0.594 -0.336 -0.745 -0.029 -0.015 -0.048 -0.161  0.470
```

4.3.1 Reporting

Plotting i Stata

```
use stata/rct, clear
```

```
quietly melogit catout i.trt i.site covar i.time i.trt#i.time if time != 0 || pid:
```

```
*Compute the predictive margins by time and treatment
```

```
margins time#trt
```

```
*Plot the predictive margins. Note that the arguments after the comma is just to prettify the plot
```

```
marginsplot, graphregion(color(white)) graphregion(color(white)) plotregion(color(white))
graph export stata/figures/cat_fig1.png, replace
```

(all strata combined)

Predictive margins
Model VCE: OIM

Number of obs = 294

Expression: Marginal predicted mean, predict()

		Delta-method				[95% conf. interval]	
		Margin	std. err.	z	P> z		
time#trt							
1#Placebo		.5476187	.0577701	9.48	0.000	.4343913	.660846
1#Active		.3187847	.0540835	5.89	0.000	.2127831	.4247863
2#Placebo		.7897137	.0463852	17.03	0.000	.6988004	.880627
2#Active		.3770772	.0557815	6.76	0.000	.2677475	.4864068
3#Placebo		.8467085	.0410465	20.63	0.000	.7662588	.9271581
3#Active		.5147246	.0571038	9.01	0.000	.4028032	.626646

Variables that uniquely identify margins: time trt

```
(file stata/figures/cat_fig1.png not found)
file stata/figures/cat_fig1.png written in PNG format
knitr::include_graphics("stata/figures/cat_fig1.png")
```

The same in R using the `marginaleffects`-package:

```
mod <- rct %>%
  filter(time != 0) %>%
  glmer(catout ~ trt + time + trt*time + site + covar + (1|pid),
        data = .,
        family = binomial,
        nAGQ = 7)

pred <- mod %>%
  avg_predictions(variables = list(trt = c("Placebo", "Active"), time = c("1", "2", "3"))
```

Warning: For this model type, ``marginaleffects`` only takes into account the uncertainty in fixed-effect parameters. You can use the ``re.form=NA``

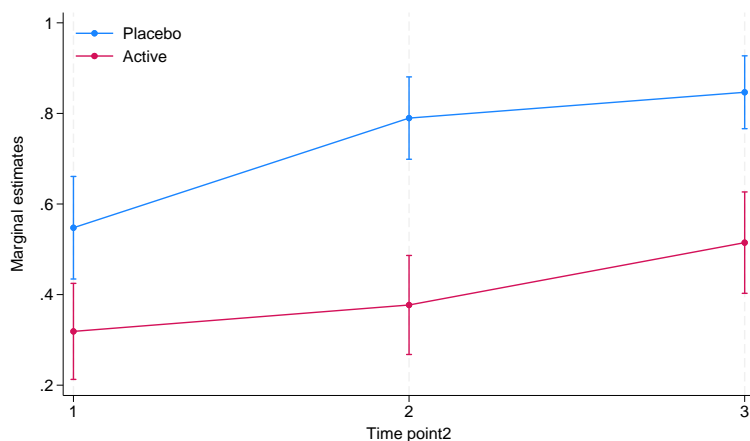


Figure 4.1: Margins plot by Stata

argument to acknowledge this explicitly and silence this warning.

```
pred
```

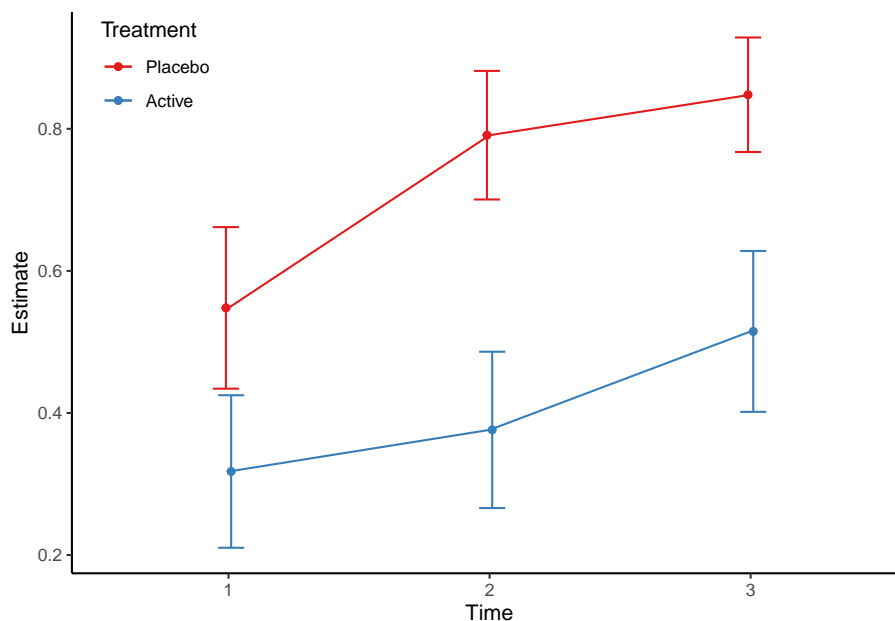
time	trt	Estimate	Std. Error	z	Pr(> z)	S	2.5 %	97.5 %
1	Placebo	0.548	0.0580	9.45	<0.001	67.9	0.434	0.662
1	Active	0.318	0.0548	5.80	<0.001	27.2	0.210	0.425
2	Placebo	0.791	0.0462	17.13	<0.001	216.1	0.701	0.882
2	Active	0.376	0.0561	6.70	<0.001	35.5	0.266	0.486
3	Placebo	0.848	0.0411	20.62	<0.001	311.3	0.767	0.929
3	Active	0.515	0.0578	8.91	<0.001	60.8	0.402	0.628

Columns: trt, time, estimate, std.error, statistic, p.value, s.value, conf.low, conf.high
Type: response

The estimated marginal plot is then given by:

```
pred %>%
  ggplot(aes(time, estimate, color=trt, group=trt)) +
  geom_point(position = position_dodge(0.04)) +
  geom_line() +
  geom_errorbar(aes(ymin = conf.low, ymax = conf.high),
                width=.2,
                position = position_dodge(0.04)) +
  ylab("Estimate") +
  xlab("Time") +
  theme_classic() +
  theme(legend.position=c(0.1,0.9)) +
```

```
scale_colour_brewer(palette = "Set1", name = "Treatment")
```



The treatment differences at different timepoints are then calculated with:

```
use "stata/rct", clear
quietly melogit catout i.trt i.site covar i.time i.trt#i.time if time != 0 || pid:
margins time, dydx(trt)
```

(all strata combined)

Average marginal effects
Model VCE: OIM

Number of obs = 294

Expression: Marginal predicted mean, predict()
dy/dx wrt: 1.trt

		Delta-method				
		dy/dx	std. err.	z	P> z	[95% conf. interval]

0.trt		(base outcome)				

1.trt						

time							
1		-.228834	.079152	-2.89	0.004	-.383969	-.0736989
2		-.4126365	.0726801	-5.68	0.000	-.5550869	-.2701861
3		-.3319839	.070613	-4.70	0.000	-.4703829	-.1935849

Note: dy/dx for factor levels is the discrete change from the base level.

In R this is done with the `marginalEffects`-package:

```
mod <- rct %>%
  filter(time != 0) %>%
  glmer(catout ~ trt + time + trt*time + site + covar + (1|pid),
        data = .,
        family = binomial,
        nAGQ = 7)

mod %>%
  avg_comparisons(variables = list(trt = c("Active", "Placebo")), by = "time", type = "response",
```

Term	Contrast	time	Estimate	Std. Error	z	Pr(> z)	S
trt mean(Placebo) - mean(Active)		1	0.230	0.0797	2.89	0.00386	8.0
trt mean(Placebo) - mean(Active)		2	0.414	0.0728	5.69	< 0.001	26.2
trt mean(Placebo) - mean(Active)		3	0.333	0.0704	4.73	< 0.001	18.7
2.5 % 97.5 %							
0.0741 0.387							
0.2717 0.557							
0.1947 0.471							

Columns: term, contrast, time, estimate, std.error, statistic, p.value, s.value, conf.low, conf.h
 Type: response

We see the results are slightly different to the Stata results, but the differences are small.

Chapter 5

Time to event analyses

To be done

Chapter 6

Sample size calculations

To be done