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## 논문 핵심 알고리즘

**Subroutine 3: Test Allocation Sub-Routine**

**Input:** Posterior distribution estimates  $\hat{\alpha}_k, \hat{\beta}_k$  for each type  $k \in K_t$ , arrivals  $A_{ke}(t)$  and budgets  $B_e(t)$  for each type  $k$  and point of entry  $e$ , number of pipeline tests  $Q_k$  for each type  $k$ , and tuning parameters  $\{c, \gamma\}$

**for**  $k = 1: K_t$

$\hat{\alpha}_k \leftarrow c \hat{\alpha}_k, \hat{\beta}_k \leftarrow c \hat{\beta}_k$  # prior widening

Compute  $\lambda_k$  from Eq. (6) using  $\hat{\alpha}_k, \hat{\beta}_k$

$\lambda_k \leftarrow$  pseudo-update index of type  $k$  (repeat  $Q_k$  times) # account for pipeline tests

$Y_k \leftarrow \sum_e A_{ke}(t)$  # type  $k$  passengers not yet allocated a test

**for**  $e = 1: \mathcal{E}$

$Y_{ke} \leftarrow A_{ke}(t)$  # type  $k$  passengers at point of entry  $e$  not yet allocated a test

$C_e \leftarrow B_e(t)$  # remaining (un-allocated) tests at point of entry  $e$

$N_{ke} \leftarrow 0$  # initialize allocations

**end**

**end**

**while**  $\max_e C_e > 0$  **and**  $\max_{k, e' \in \{e \mid C_e > 0\}} Y_{ke'} > 0$

$k^* = \operatorname{argmax}_k \{\lambda_k: Y_k > 0\}$

$e^* = \operatorname{argmax}_e \{C_e: Y_{k^*e} > 0\}$

$N_{k^*e^*} \leftarrow N_{k^*e^*} + 1$  # allocate a test to a type  $k$  passenger at point of entry  $e$

$C_{e^*} \leftarrow C_{e^*} - 1, Y_{k^*e^*} \leftarrow Y_{k^*e^*} - 1, Y_{k^*} \leftarrow Y_{k^*} - 1$

$\lambda_{k^*} \leftarrow$  pseudo-update index of type  $k^*$

**end**

**return**  $\{N_{ke}\}_{k \in K_t, e \in \{1, \dots, \mathcal{E}\}}$  # number of tests allocated to each type at each port of entry

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**Subroutine 1: Estimating Prevalence via Empirical Bayes**


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**Input:** # of positives  $P_k$ , negatives  $N_k$  and tests  $T_k$  for each type  $k$  in the past 14 days of test results.

**for** each color designation of countries  $l \in \{\mathcal{W}_t, \mathcal{G}_t, \mathcal{B}_t\}$

    Compute  $M_{1l}, M_{2l}$  from Eq. (2) and (3)

$$\hat{\alpha}_l \leftarrow \frac{M_{1l}^2(1 - M_{1l})}{M_{2l}^2 - M_{1l}^2} - M_{1l}, \hat{\beta}_l \leftarrow \hat{\alpha}_l \frac{(1 - M_{1l})}{M_{1l}}$$

**for** all types  $k \in \mathcal{L}_l$  whose origin country is in  $l$ :

$$\hat{\alpha}_k \leftarrow \hat{\alpha}_l + P_k \text{ and } \hat{\beta}_k \leftarrow \hat{\beta}_l + N_k$$

**end**

**end**

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Specifically, consider countries of color  $l \in \{\mathcal{W}_t, \mathcal{G}_t, \mathcal{B}_t\}$ , and the corresponding set of types  $\mathcal{L}_l = \{k \in \mathcal{K}_t : k \text{ is from a country in } l\}$ . Let  $T_k = P_k + N_k$  be the total number of tests (with results) allocated to type  $k$  over the last 16 days. Conditional on  $T_k$ ,  $P_k$  is approximately binomially distributed with  $T_k$

$$\begin{aligned} \hat{\alpha}_l &= \frac{M_{1l}^2(1 - M_{1l})}{M_{2l}^2 - M_{1l}^2} - M_{1l} \\ \hat{\beta}_l &= \hat{\alpha}_l \frac{(1 - M_{1l})}{M_{1l}} \\ M_{1l} &= \frac{1}{|\mathcal{L}_l|} \sum_{k \in \mathcal{L}_l} \frac{P_k}{T_k} \end{aligned} \quad (2)$$

$$M_{2l} = \frac{1}{|\mathcal{L}_l|} \sum_{k \in \mathcal{L}_l} \frac{P_k(P_k - 1)}{T_k(T_k - 1)} \quad (3)$$

$$\lambda_k = \frac{\alpha_k}{\alpha_k + \beta_k} \left( 1 - \gamma F_{\alpha_k+1, \beta_k}(\lambda_k) \right) + \gamma \lambda_k \left( 1 - F_{\alpha_k, \beta_k}(\lambda_k) \right).$$

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**Gittins Pseudo-Update for type  $k$** 


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**Input:**  $\hat{\alpha}_k, \hat{\beta}_k$  for type  $k$

$$\hat{r}_k^{EB} \leftarrow \frac{\hat{\alpha}_k}{\hat{\alpha}_k + \hat{\beta}_k}$$

$$\hat{\alpha}_k \leftarrow \hat{\alpha}_k + \hat{r}_k^{EB} \text{ and } \hat{\beta}_k \leftarrow \hat{\beta}_k + 1 - \hat{r}_k^{EB}$$

Compute  $\lambda_k$  from Eq. (6) using  $\hat{\alpha}_k, \hat{\beta}_k$

**return**  $\lambda_k$

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세팅

- L(W, G, B) → 20개의 그룹
  - 컬럼 40개 이용하여 KMeans
    - 40개 컬럼: app+기초 변수(ip, device, channel, os), 기초 변수 + app LDA topic 5개씩
    - Kmeans에 사용되지 않는 나머지 4개의 컬럼: ip, app, time\_dh, is\_attributed
  - 20개의 그룹에서 ip가 독립적으로 존재하지 않고 흩어져 있는 문제
    - 각각의 ip에 대해 평균적으로 20개의 그룹 중 8개의 그룹에 존재하고 있음

```

0 120571
1 101588
2 87611
3 88804
4 77213
5 74833
6 101787
7 21624
8 99143
9 100748
10 91925
11 89932
12 56543
13 57005
14 99180
15 104129
16 67226
17 78827
18 76366
19 58881
8.01403236747747

```

- 총 ip 유저 수 : 206,380
- k(L그룹 안의 나라 목록) → ??
  - L그룹을 더 세분화해야 될까?
  - L: 대그룹, K:소그룹
- 개개인의 승객 → ip

## 규칙

[시간 단위로 반복  $T_i$ ]

[L그룹 별로 반복  $L_i$ ]

Case1.  $L_i$  가 처음 접하는 그룹인 경우

- inspection rate 무작위로 ip 추천해서 검사

Case2  $L_i$  가 이미 접한 적 있는 그룹인 경우

- Optimistic Gittins Indices(람다식) 계산

Case2에 해당하는 그룹 중  $\text{argmax}$ 해서  $k$  소그룹에 있는 ip rate의 비율로 선택 후 검사

검사 대상 ip가 fraud\_ip면 해당 ip 끝까지 블락

검사 결과를 바탕으로  $A_i, B_i, A_k, B_k$  파라미터 업데이트

- 시간 단위 : 1시간
- 답지를 깔 때 걸리는 딜레이 시간: ? 시간

- 딜레이 반영 알고리즘

관찰 결과 P/N 확률을 먼저 정해두고 반영

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### Gittins Pseudo-Update for type $k$

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$$\hat{r}_k^{EB} \leftarrow \frac{\hat{\alpha}_k}{\hat{\alpha}_k + \hat{\beta}_k}$$

$$\hat{\alpha}_k \leftarrow \hat{\alpha}_k + \hat{r}_k^{EB} \text{ and } \hat{\beta}_k \leftarrow \hat{\beta}_k + 1 - \hat{r}_k^{EB}$$

Compute  $\lambda_k$  from Eq. (6) using  $\hat{\alpha}_k, \hat{\beta}_k$

**return**  $\lambda_k$

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- 무작위: inspection rate
- detect/undetected

→ 질문: 분모 분자가 지워져서 결국 양성 판별 횟수 아닌가, 즉, 정확도에 무관하게 검사를 많이 하면 할수록 좋은 값이 되는거 아닌가??

## 알고리즘 평가

### Objective

Note that, conditional on  $T_{xe}(t)$ , the number of positive tests observed at entry  $e$  is binomially distributed with  $T_{xe}(t)$  trials and (unknown) success probability  $R_x(t)$ . Our goal is to maximize the expected total number of infections caught at the border, i.e.,

$$\mathbb{E} \left[ \sum_{t=1}^T \sum_{x \in \mathcal{X}} \sum_{e=1}^{\varepsilon} T_{xe}(t) R_x(t) \right].$$