Dynamic Programming II

Coin Change?

How many ways to make **N** cents with pennies, nickels, dimes, and quarters?

Order matters

E.g. 6 = {(1,1,1,1,1,1), (1,5), (5,1)}

Coin Change?

What is the optimal subproblem?

- Make change for smaller **N**?
- Make change with fewer coins?

Coin Change?

```
ans[0] = 1
for (i = 1 to N)
  for(c in coins)
    if(i-c >= 0)
      ans[i] += ans[i-c];
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What is time complexity?

Coin Change II

How many ways to make **N** cents with pennies, nickels, dimes, and quarters?

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Coin Change II

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Hmm…

Coin Change II

Simpler case: only pennies and nickels

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Make 5**M** cents using only nickels, then
N-5**M** cents using only pennies

Coin Change II

What is the optimal subproblem?

- Make change for smaller **N** *and* with fewer coins
- ans[i,j] = ways to make **j** cents using first **i** coins

Coin Change II

simple test case: coins are {1¢, 2¢}

i \\ j	0	1	2	3	4	5
0						
1						
2						

ans[i,j] = ways to make **j** ¢ w/ first **i** coins

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2	1	1	**2**	2	3	

ans[i,j] = ways to make **j** ¢ w/ first **i** coins

Coin Change II

```
ans[0, 0] = 1
for (c = 0 to num_coins)
  for (i = 0 to N)
    if(c >= 1)
      ans[c, i] += ans[c-1, i]
    if(i – vals[c] >= 0)
      ans[c, i] += ans[c, i –
 vals[c]]
```

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```

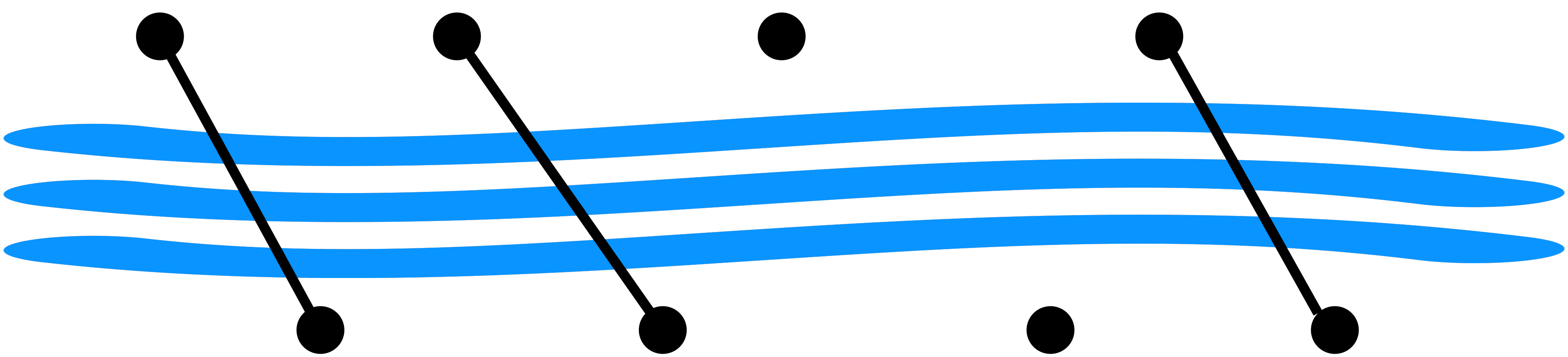
What is time complexity?

Bridge Building

Given points ($\mathbf{p}_i$,-1) and ($\mathbf{q}_i$,1) on opposite banks of river, build some bridges

Rules:

- no bridges can cross
- every ($\mathbf{p}_i$,-1) must have a bridge



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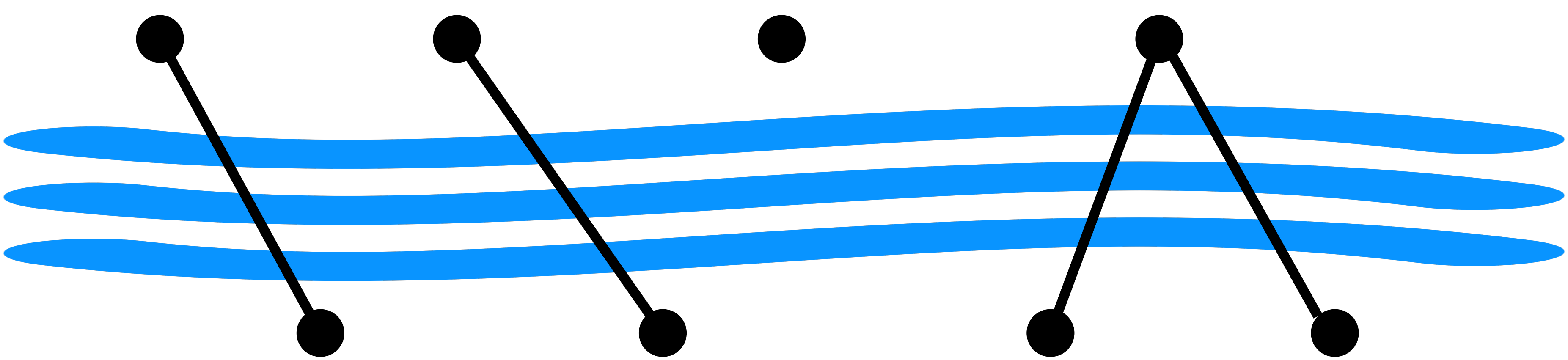
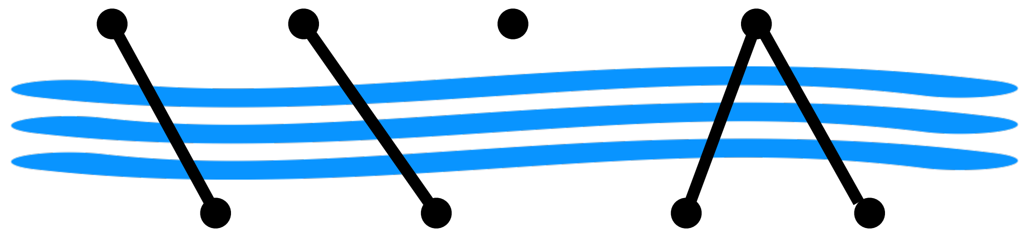


Bridge Building



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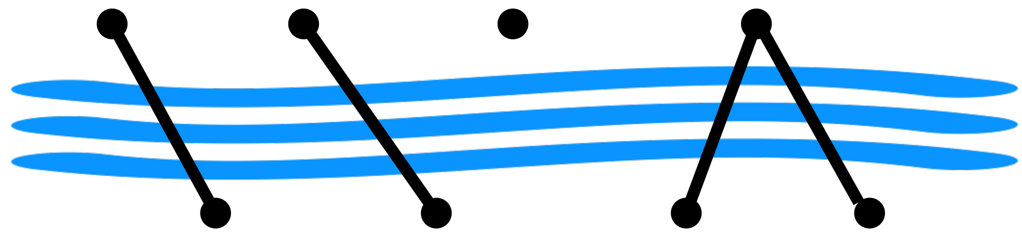
What is valid bridge network with shortest total length?

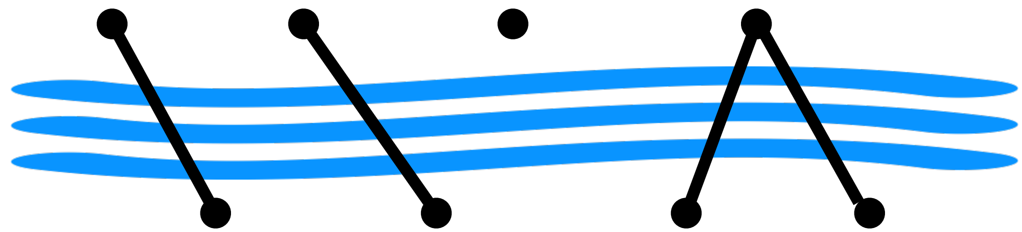
Bridge Building



Greedy algorithm

For each bottom point, connect it to nearest point on top

Bridge Building



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Rules:

- no bridges can cross
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- must be **maximally-connected**

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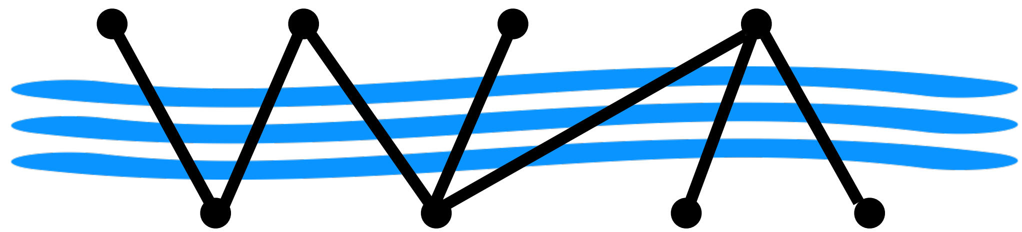

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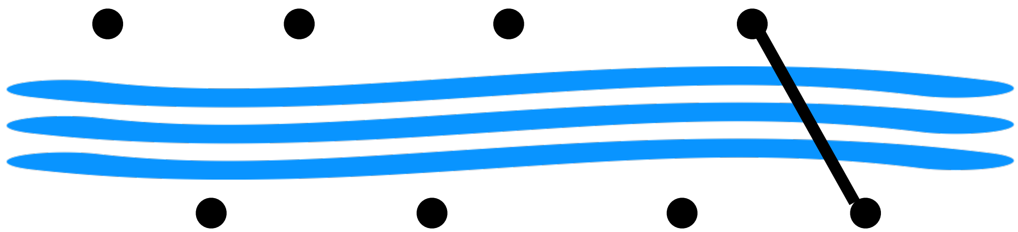
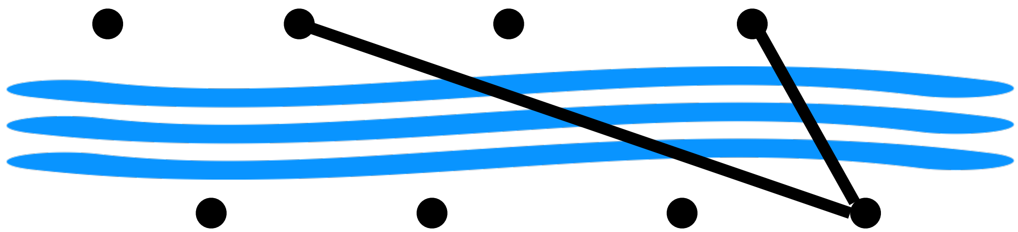


Some observations:

- rightmost bottom point **must** connect rightmost top point

Bridge Building



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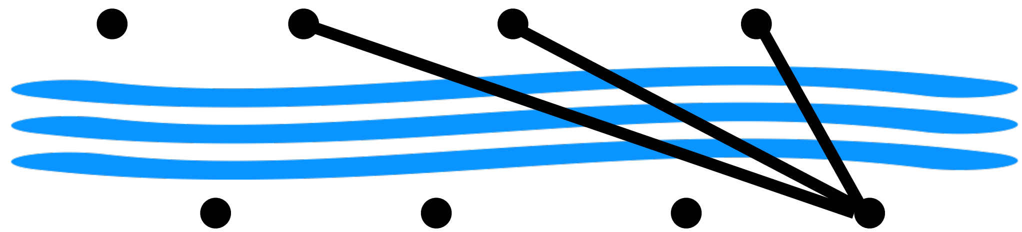
- could also connect to other top points

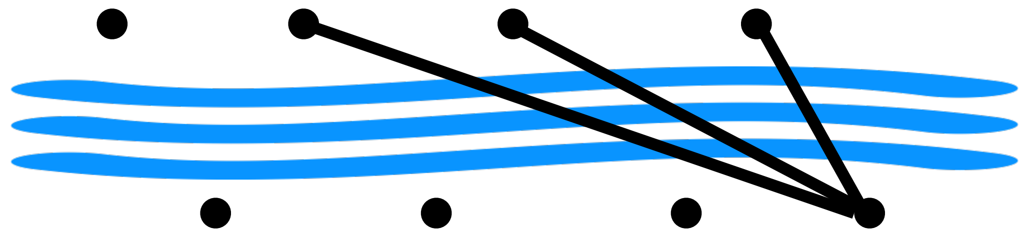
Bridge Building



Some observations:

- rightmost bottom point **must** connect rightmost top point
- could also connect to other top points
- if it connects to a top point, must connect to all other points to the right

Bridge Building



best(i,j) = shortest network using only up to bottom point **i** bottom and top point **j**

```
best(i,j):
  ans = infinity
  for (k = 0 to j)
    len = best(i-1,k)
    for (l = k to j)
      len += sqrt( (pi – ql)*(pi – ql) + 4 )
    ans = min(ans, len)
  return ans
```

Traveling Salesman

Given 16 cities and distances $\mathbf{d}_{ij}$ between them, what is cheapest itinerary that starts at city 0 and visits all cities exactly once?

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Traveling Salesman

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Find best paths visiting fewer cities?

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Find best paths visiting fewer cities?

- what if we **knew** that best path ended in city **c**?
- now can recurse

Traveling Salesman

best[**S**, **c**] = cheapest path visiting all cities in **S** and ending at city **c**

```
ans = infinity
for (x in S – {c, 0})
  len = best[S – {c}, x]
  len += d_xc
  ans = min(ans, len)
return ans
```

Traveling Salesman

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What is time complexity?

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- 2^{15} subsets containing city 0
- 15 possible ending cities
- 2^{15}*15 ~= 500,000

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What is time complexity?

- 2^{15} subsets containing city 0
- 15 possible ending cities
- 2^{15}*15 ~= 500,000
- NP-hard problem… no free lunch