

Control Challenges: Solutions

Iacopo Moles

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1 Introduction

1.1 Intro

This is a collection of write ups on how to solve the various problems presented by [Github user "Janismac"](#).

1.2 What do I need?

1.2.1 Software

- A real OS like Linux or Windows.
- The [Julia Programming Language](#)
 - Clone the [repo](#)
 - Activate the package by running in your terminal:

```
julia --project -e 'using Pkg; Pkg.instantiate()'
```

- (Nice to have) [OpenModelica Editor](#)

1.2.2 Theory

- Basic Julia knowledge
- Basic JS knowledge
- Control Theory knowledge
 - Frequency Based Control
 - State Space Based control
- Misc knowledge:
 - Linear Algebra
 - Differential Equations

Part I

The Damped Mass problem

Modeling

To better understand the problem let's take a [peek](#) at how the simulated model works.

Listing 1.1 BlockOnSlope.js

```
Models.BlockOnSlope.prototype.vars =  
{  
    g: 9.81,  
    x: 0,           // distance from objective s  
    dx: 0,          // velocity v  
    slope: 1,        // slope coefficient alpha = dy/dx in the  
    ↪ cartesian plane  
    F: 0,           // Requested u  
    F_cmd: 0,        // Saturated u  
    friction: 0,     // Coulomb friction coefficient mu  
    T: 0,           // Simulation Time  
};  
  
Models.BlockOnSlope.prototype.simulate = function (dt,  
    ↪ controlFunc)  
{  
    this.F_cmd = controlFunc({x:this.x,dx:this.dx,T:this.T});  
    if(typeof this.F_cmd !== 'number' || isNaN(this.F_cmd)) throw  
    ↪ "Error: The controlFunction must return a number.";  
    this.F_cmd = Math.max(-20,Math.min(20,this.F_cmd));  
    integrationStep(this, ['x', 'dx', 'F'], dt);  
}  
  
Models.BlockOnSlope.prototype.ode = function (x)  
{  
    return [  
        x[1],  
        (x[2]) - (Math.sin(this.slope) * this.g) -  
    ↪ (this.friction * x[1]),  
        20.0 * (this.F_cmd - x[2])  
    ];  
}
```

-
- ① The model has obviously some default values for the parameters that can be modified for the different scenarios.
 - ② The control command u is generated by the `controlFunction(block)` function provided by us. There are some checks to see if it's a number. If it's acceptable then it passes through a saturation between ± 20 .

③ The model is a simple ODE with equations:

$$\begin{cases} \dot{s} = v \\ \dot{v} = F - \sin(\alpha) \cdot g - \mu \cdot v \\ \dot{F} = -20 \cdot F + 20 \cdot u_{sat} \end{cases}$$

Converting it in state-space representation:

$$\begin{bmatrix} \dot{s} \\ \dot{v} \\ \dot{F} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\mu & 1 \\ 0 & 0 & -20 \end{bmatrix} \begin{bmatrix} s \\ v \\ F \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ u_{sat} \end{bmatrix} + \begin{bmatrix} 0 \\ -\sin(\alpha) \cdot g \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} s \\ v \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} s \\ v \\ F \end{bmatrix}$$

Obviously the gravitational term acts as a disturbance.

2 Block without Friction

Position Control with friction. Using Pole Placement + PD.

2.1 State Space representation

We can convert the set of ODE into a state space representation. The final bode plot of the block position is:

```
using DiscretePIDs, ControlSystems, Plots, LinearAlgebra

# System parameters
Ts = 0.02 # sampling time
Tf = 2.5; #final simulation time
g = 9.81 #gravity
α = 0.0 # slope
μ = 1.0 # friction coefficient
x_0 = -2.0 # starting position
dx_0 = 0.0 # starting velocity
τ = 20.0 # torque constant

# State Space Matrix
A = [0 1 0
     0 -μ 1
     0 0 -τ];
B = [0
     0
     τ];
C = [1 0 0
     0 1 0];

sys = ss(A, B, C, 0.0) # Continuous

plot!(bodeplot(tf(sys)), pzmap(tf(sys)))
```

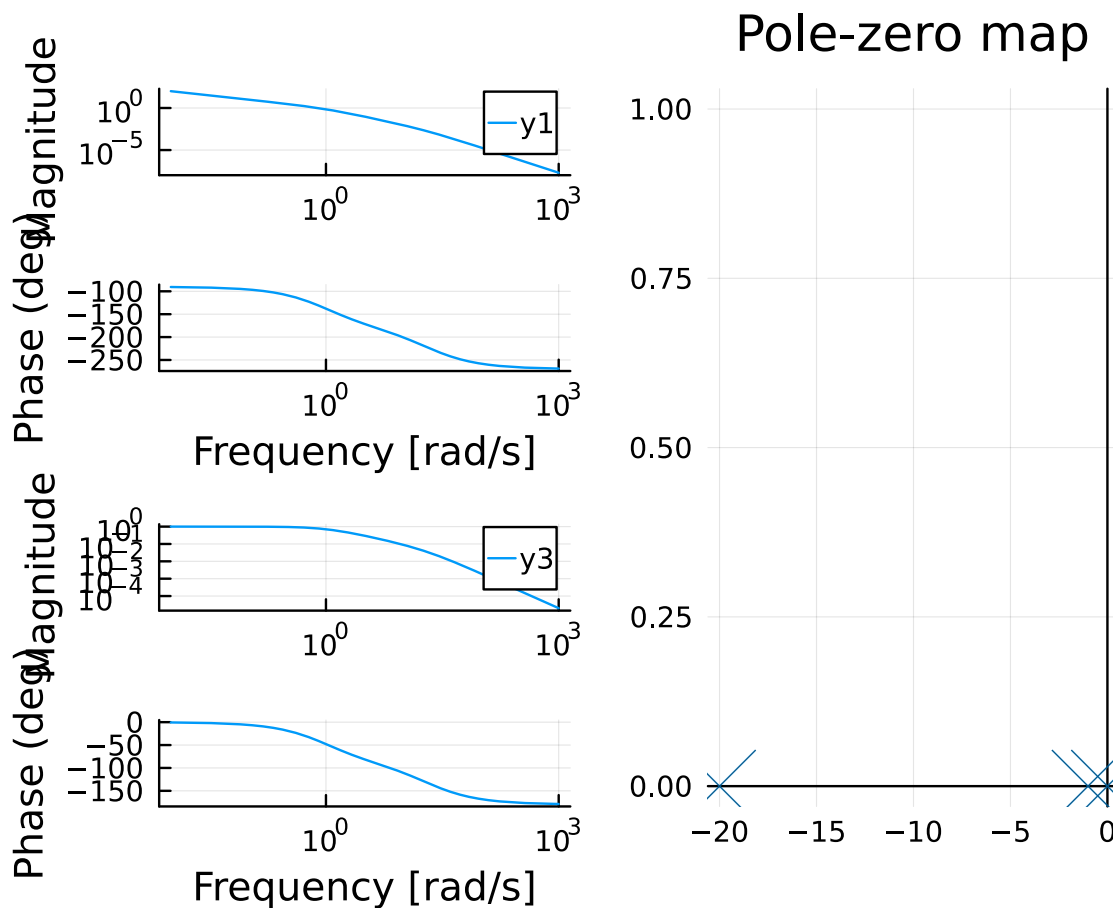


Figure 2.1: Starting Bode Plot and PZ Map

It has the shape we expect from a motor + friction. Slow pole for the mass + friction and a faster pole for the current & inductance.

Numerically they are:

```
display(eigvals(A))
```

3-element Vector{Float64}:

```
-20.0
-1.0
0.0
```

We see that we start with all the pole in the left-half plane, which is good.

2.2 Pole Placement

We can design a controller with pole placement.

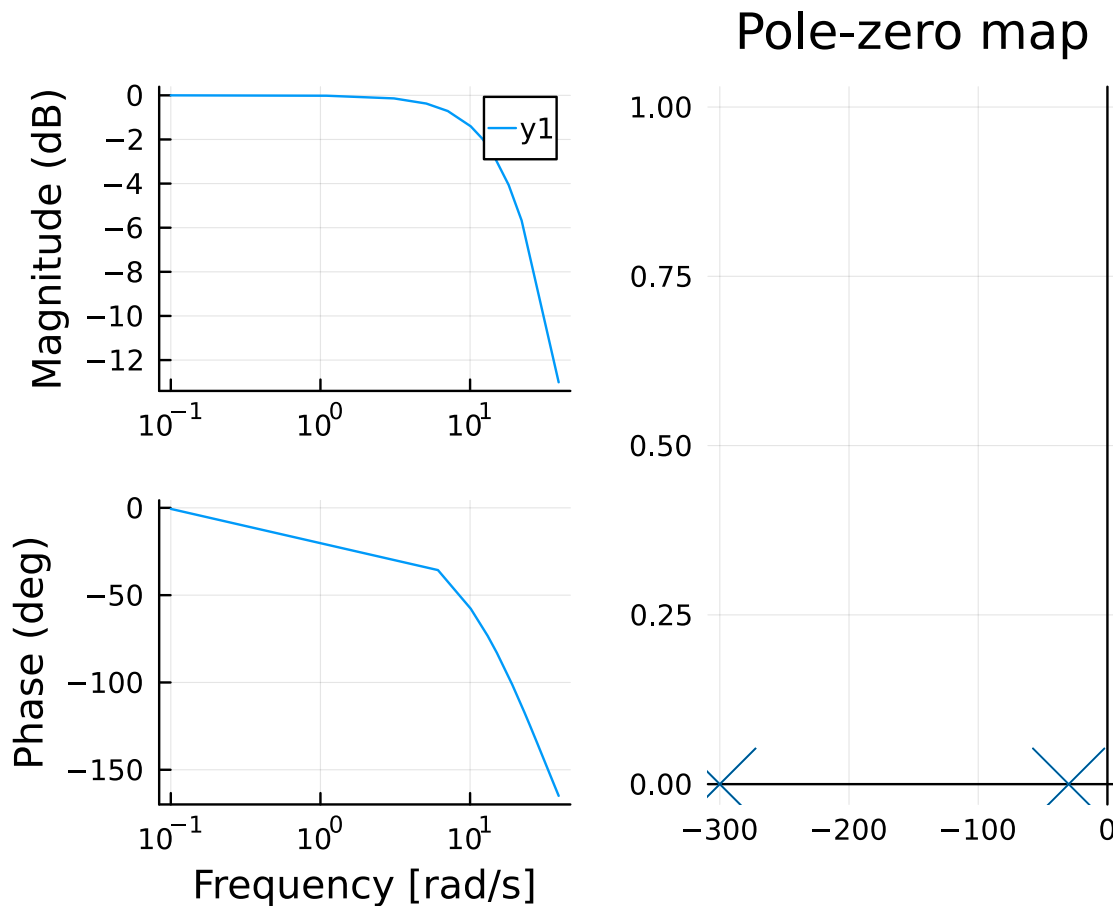
For some reason pole placement doesn't work for the observer, I use a Kalman Filter with random fast values.

```
observability(A, C).isobservable &  
controllability(A, B).iscontrollable; #OK  
  
 $\varepsilon$  = 0.01;  
pp = 15.0;  
poles_cont = -2.0 * [pp +  $\varepsilon$ , pp -  $\varepsilon$ , pp];  
L = real(place(sys, poles_cont, :c));  
  
poles_obs = poles_cont * 10.0;  
K = place(1.0 * A', 1.0 * C', poles_obs)'  
cont = observer_controller(sys, L, K; direct=false);
```

We can check the effect of the new controller on the loop

```
closedLoop = feedback(sys * cont)  
print(poles(closedLoop));  
setPlotScale("dB")  
plot!(bodeplot(closedLoop[1, 1], 0.1:40), pzmap(closedLoop))
```

```
ComplexF64[-29.979998924597755 + 0.0im, -30.000002152199972 +  
0.0im, -30.019998923202337 + 0.0im, -300.00000000000004 + 0.0im,  
-300.19999999999752 + 0.0im, -299.800000000004117 + 0.0im]
```



We can compare this to the open-loop response in @start-bode. We can see that we achieve unitary gain throughout the whole low-frequency range.

We can convert the pole placement controller into the standard PD gain form.

```
K = L[1];
Ti = 0;
Td = L[2] / L[1];
pid = DiscretePID(; K, Ts, Ti, Td);
```

2.3 Simulation

We can simulate this with a motor that only outputs the position:

```
sysreal = ss(A, B, [1 0 0], 0.0)
ctrl = function (x, t)
    y = (sysreal.C*x)[] # measurement
```

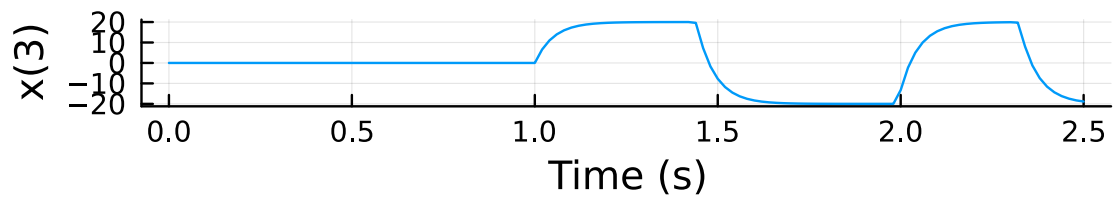
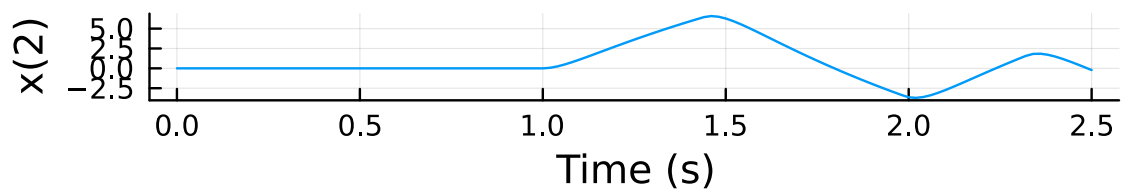
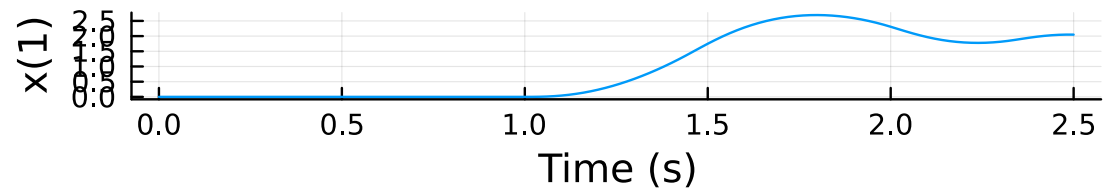
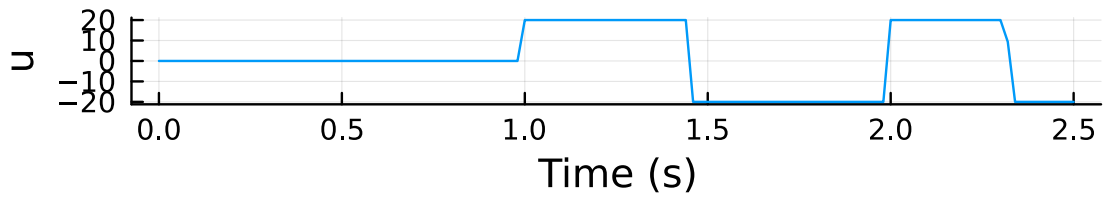
```

d = 0 * [1.0]          # disturbance
r = 2.0 * (t ≥ 1) # reference
# u = pid(r, y) # control signal
# u + d # Plant input is control signal + disturbance
# u =1
e = x - [r; 0.0; 0.0]
e[3] = 0.0 # torque not observable, just ignore it in the
↳ final feedback
u = -L * e + d
u = [maximum([-20.0 minimum([20.0 u]))]]
end
t = 0:Ts:Tf

res = lsim(sysreal, ctrl, t)

display(plot(res,
    plotu=true,
    plotx=true,
    ploty=false
))
ylabel!("u", sp=1);
ylabel!("x", sp=2);
ylabel!("v", sp=3);
ylabel!("T", sp=4);

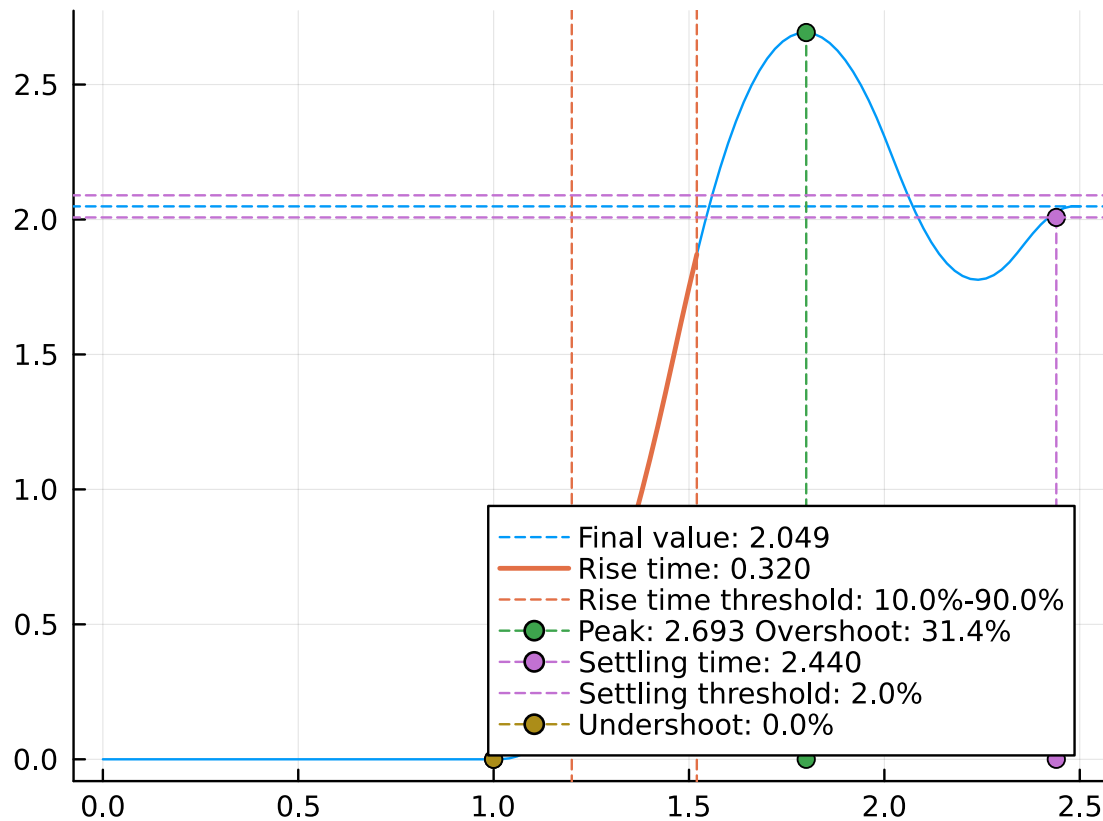
```



For more stats:

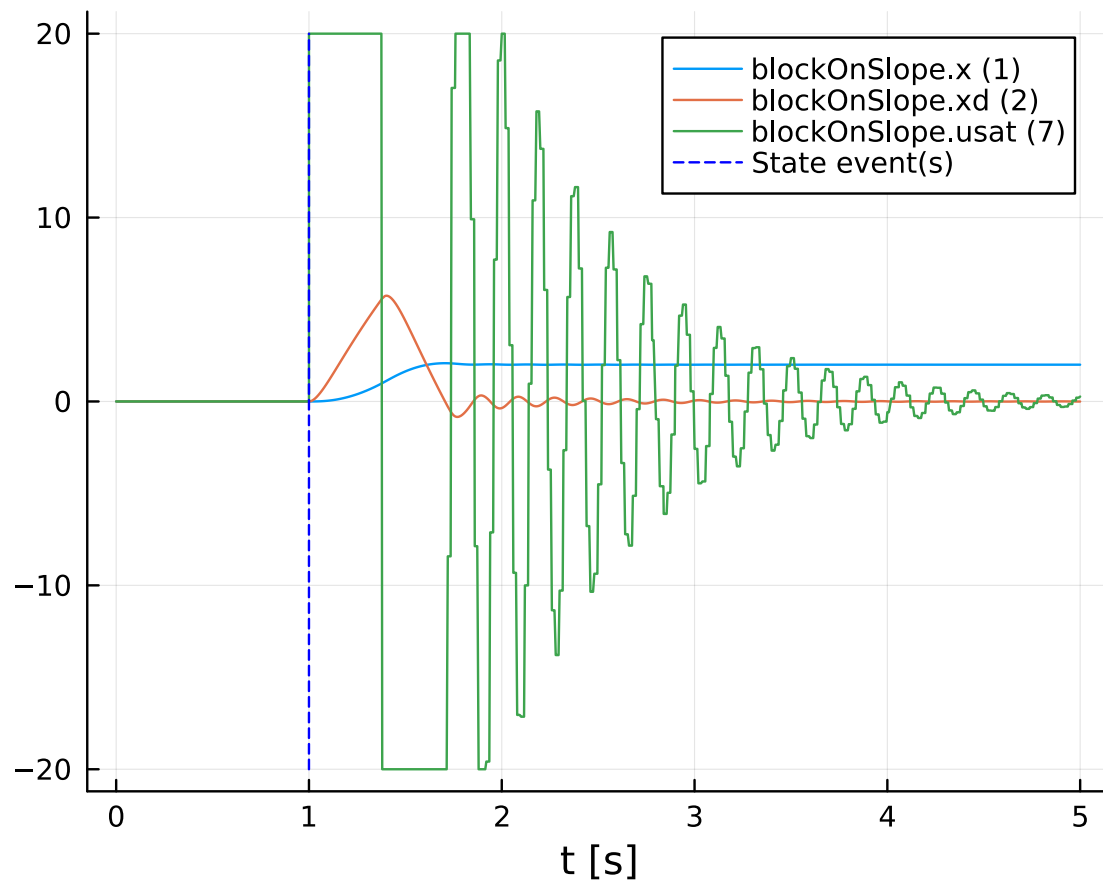
```
si = stepinfo(res);
plot(si);title("Step Response")
```

Step Response



We can also simulate it in a SIMULINK-like environment:

```
using FMI, DifferentialEquations
fmuPath = abspath(joinpath(@__DIR__,
    "..", "..",
    "modelica",
    "ControlChallenges",
    "ControlChallenges.BlockOnSlope_Challenges.Examples.WithFriction.fmu"))
fmu = loadFMU(fmuPath);
simData = simulateME(
    fmu,
    (0.0, 5.0);
    recordValues=["blockOnSlope.x",
        "blockOnSlope.xd",
        "blockOnSlope.usat"],
    showProgress=false);
unloadFMU(fmu);
plot(simData, states=false, timeEvents=false)
```



There is a slight difference between the `lsim` simulation and the FMU simulation. I need to recheck some stuff.

3 Block With Friction

Position Control with friction. Using Pole Placement + PD.

3.1 Response Analysis

```
using CCS: blockModel
using ControlSystems, Plots, LinearAlgebra,
    ↳ RobustAndOptimalControl

contSys = blockModel.csys(;g = 0, a = 0, μ = 1, τ = 20)

plot!(bodeplot(contSys[1,1]), pzmap(contSys))
```

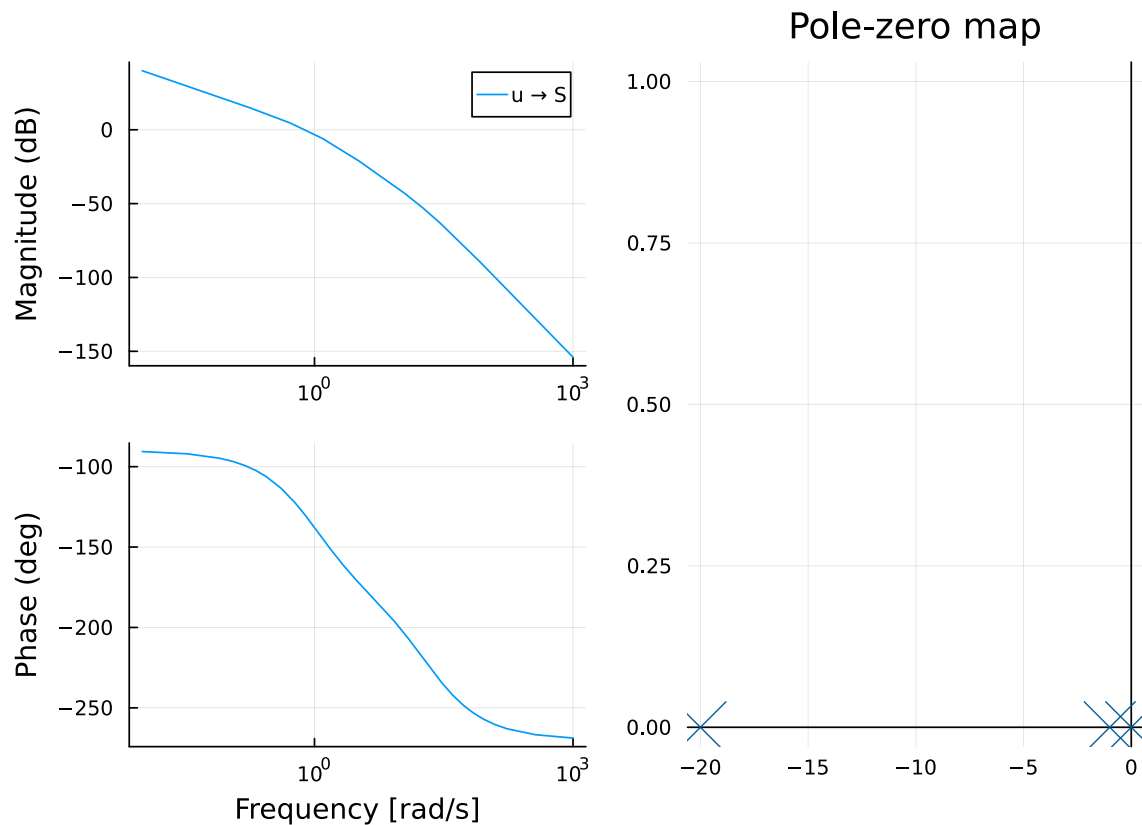


Figure 3.1: Starting Bode Plot and PZ Map

It has the shape we expect from a motor + friction. Slow pole for the mass + friction and a faster pole for the current & inductance.

Numerically they are:

```
display(eigvals(contSys.A))
```

3-element Vector{Float64}:

```
-20.0
-1.0
0.0
```

We see that we start with all the poles in the left-half plane, which is good.

3.2 Pole Placement

We can design a controller with pole placement.

For some reason pole placement doesn't work for the observer, I use a Kalman Filter with random fast values.

```
observability(contSys.A,contSys.C).isobservable || error("System
↳ is not observable")
controllability(contSys.A,contSys.B).iscontrollable ||
↳ error("System is not controllable")

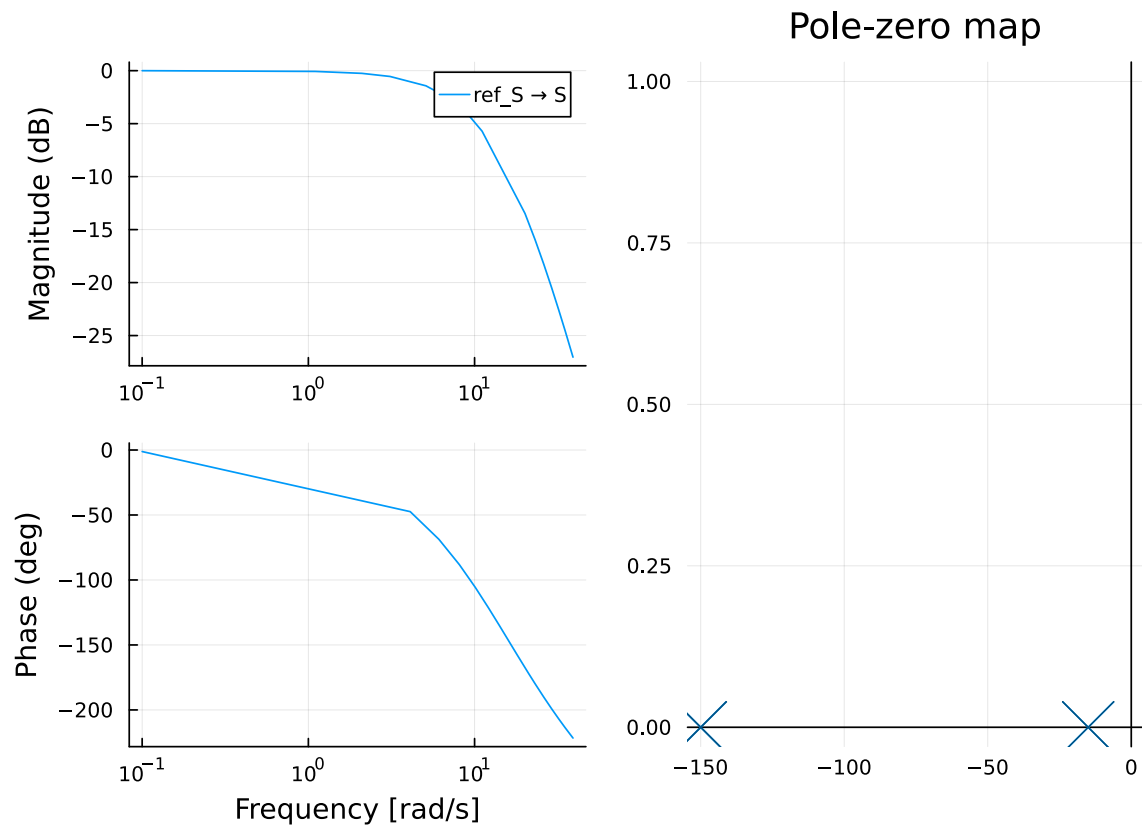
ε = 0.01;
pp = 15.0;
poles_cont = - [pp + ε, pp - ε, pp];
L = real(place(contSys, poles_cont, :c));

poles_obs = poles_cont * 10.0;
K = place(contSys, poles_obs, :o)
obs_controller = observer_controller(contSys, L, K;
↳ direct=false);
fsf_controller = named_ss(obs_controller, u = [:ref_S, :ref_V],
↳ y = [:u]);
```

We can check the effect of the new controller on the loop

```
closedLoop = feedback( contSys * fsf_controller);
print(poles(closedLoop));
setPlotScale("dB")
plot!(bodeplot(closedLoop[1,1], 0.1:40), pzmap(closedLoop))
```

```
ComplexF64[-14.990000366343788 + 0.0im, -14.999999266673722 +
0.0im, -15.010000366982666 + 0.0im, -149.9999999999986 + 0.0im,
-150.100000000002432 + 0.0im, -149.89999999999607 + 0.0im]
```



We can compare this to the open-loop response in @start-bode. We can see that we achieve unitary gain throughout the whole low-frequency range.

We can convert the pole placement controller into the standard PD gain form.

```
using DiscretePIDs
Ts = 0.02 # sampling time
Tf = 2.5; #final simulation time

K = L[1];
Ti = 0;
Td = L[2] / L[1];

pid = DiscretePID(; K, Ts, Ti, Td);
```

3.3 Simulation

We can simulate this with a motor that only outputs the position:

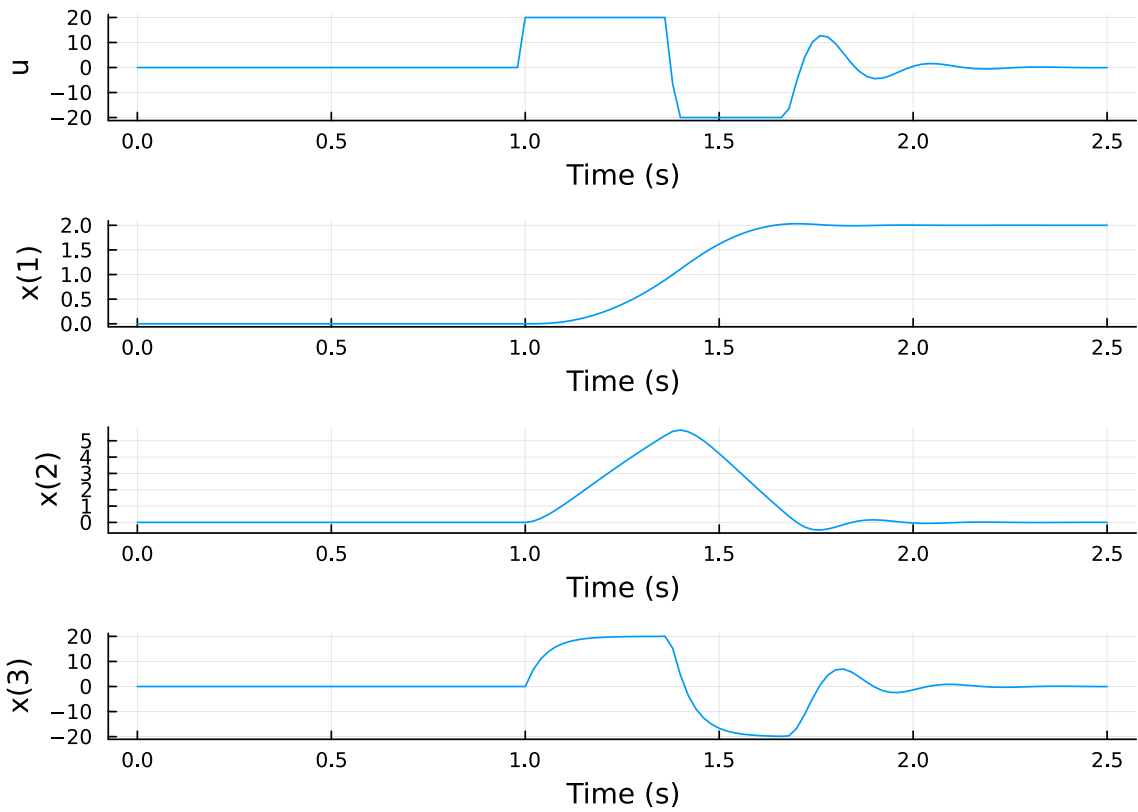
```

sysreal = ss(contSys.A, contSys.B, [1 0 0], 0.0)
ctrl = function (x, t)
    y = (sysreal.C*x)[] # measurement
    d = 0 * [1.0] # disturbance
    r = 2.0 * (t ≥ 1) # reference
    # u = pid(r, y) # control signal
    # u + d # Plant input is control signal + disturbance
    # u = 1
    e = x - [r; 0.0; 0.0]
    e[3] = 0.0 # torque not observable, just ignore it in the
    ↪ final feedback
    u = -L * e + d
    u = [maximum([-20.0 minimum([20.0 u]))]]
end
t = 0:Ts:Tf

res = lsim(sysreal, ctrl, t)

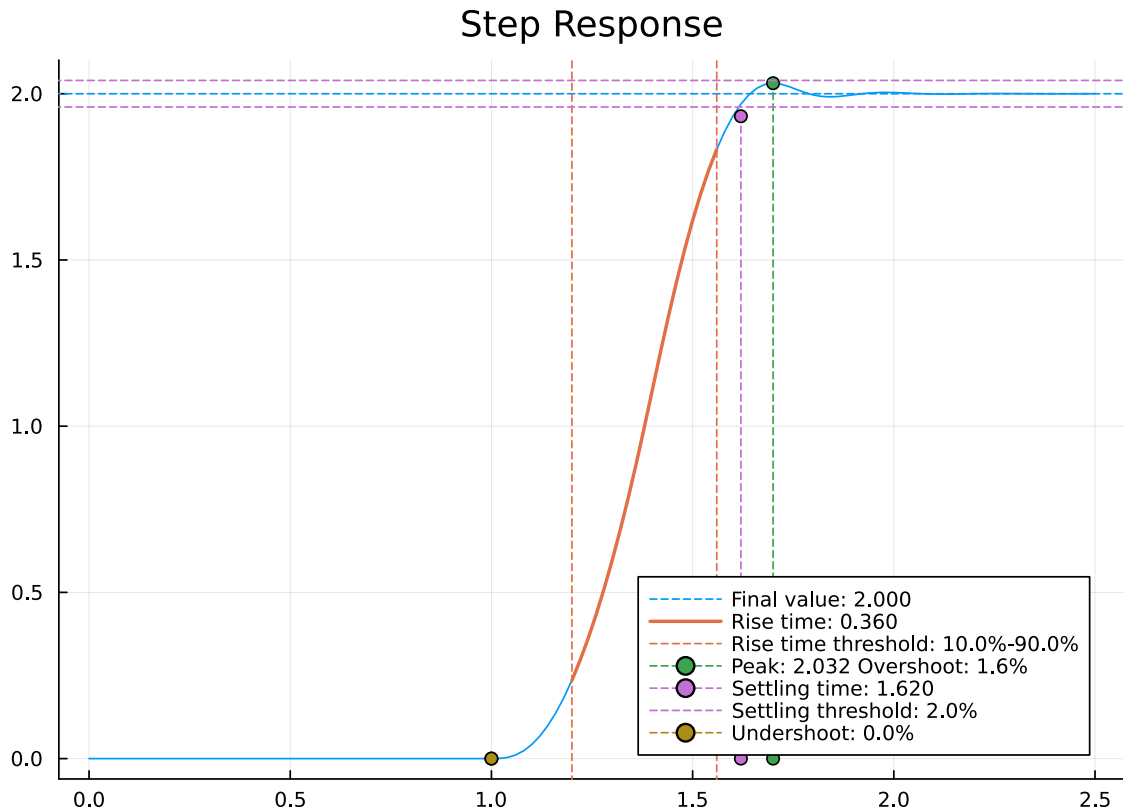
display(plot(res,
    plotu=true,
    plotx=true,
    ploty=false
))
ylabel!("u", sp=1);
ylabel!("x", sp=2);
ylabel!("v", sp=3);
ylabel!("T", sp=4);

```



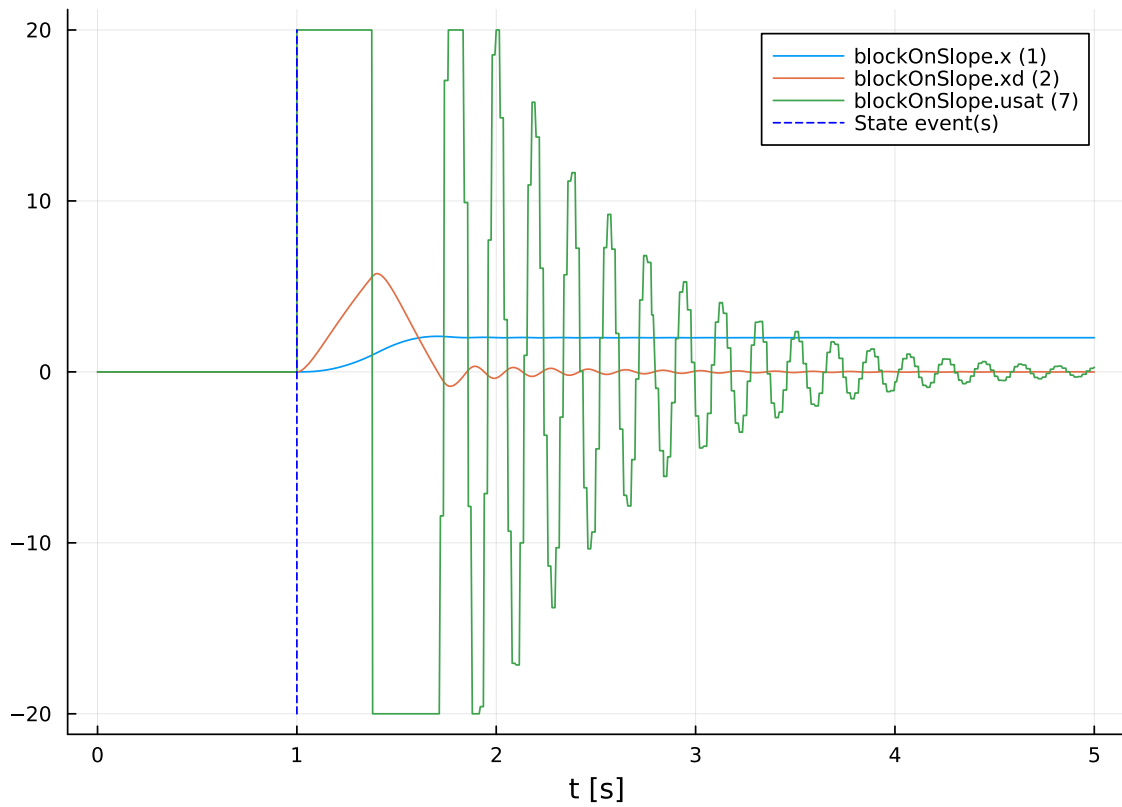
For more stats:

```
si = stepinfo(res);  
plot(si);title!("Step Response")
```



We can also simulate it in a SIMULINK-like environment:

```
using FMI, DifferentialEquations
fmuPath = abspath(joinpath(@__DIR__,
    "..", "..",
    "modelica",
    "ControlChallenges",
    "ControlChallenges.BlockOnSlope_Challenges.Examples.WithFriction",
    "ion.fmu"))
fmu = loadFMU(fmuPath);
simData = simulateME(
    fmu,
    (0.0, 5.0);
    recordValues=["blockOnSlope.x",
        "blockOnSlope.xd",
        "blockOnSlope.usat"],
    showProgress=false);
unloadFMU(fmu);
plot(simData, states=false, timeEvents=false)
```



There is a slight difference between the `lsim` simulation and the FMU simulation. I need to recheck some stuff.

4 Block on a slope

Position Control with friction. Using Pole Placement + PD.

4.1 Response Analysis

```
using CCS: blockModel
using ControlSystems, Plots, LinearAlgebra,
    ↳ RobustAndOptimalControl

contSys = blockModel.csys(;g = 0, a = 0, μ = 1, τ = 20)

plot!(bodeplot(contSys[1,1]), pzmap(contSys))
```

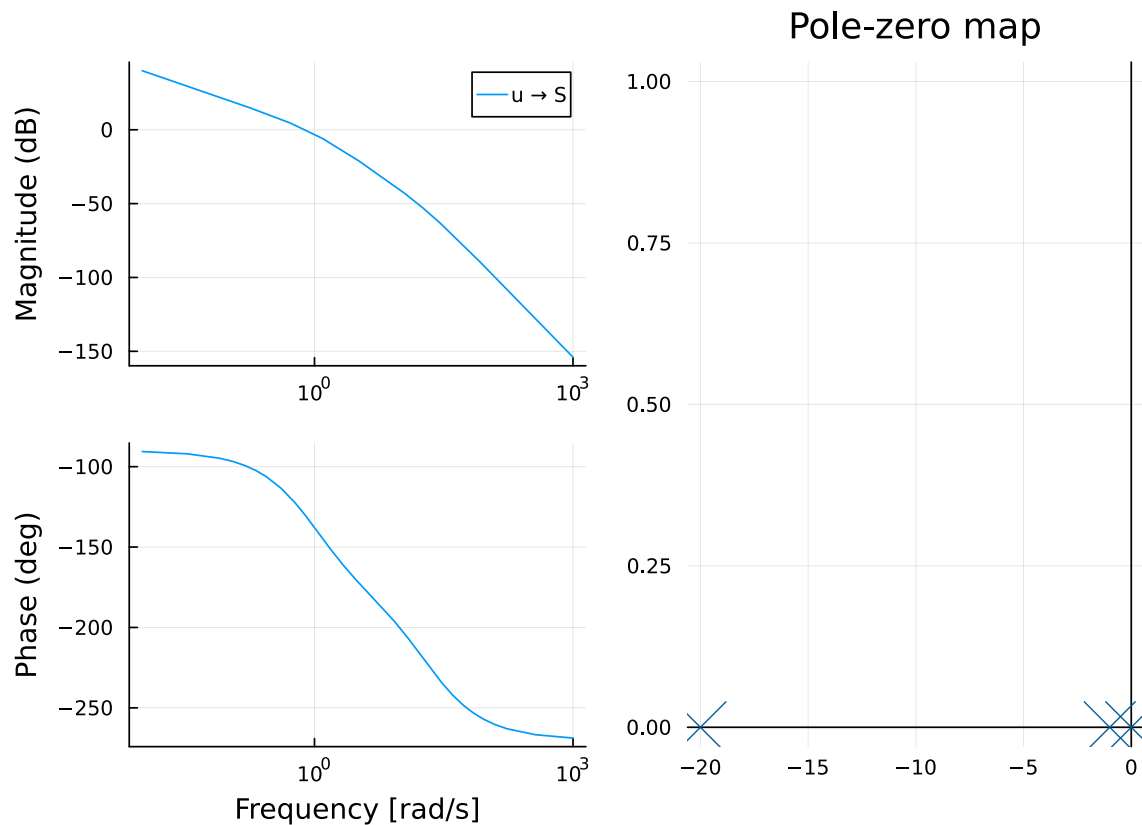


Figure 4.1: Starting Bode Plot and PZ Map

It has the shape we expect from a motor + friction. Slow pole for the mass + friction and a faster pole for the current & inductance.

Numerically they are:

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display(eigvals(contSys.A))
```

3-element Vector{Float64}:

```
-20.0
-1.0
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```

We see that we start with all the poles in the left-half plane, which is good.

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↳ is not observable")
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ε = 0.01;
pp = 15.0;
poles_cont = - [pp + ε, pp - ε, pp];
L = real(place(contSys, poles_cont, :c));

poles_obs = poles_cont * 10.0;
K = place(contSys, poles_obs, :o)
obs_controller = observer_controller(contSys, L, K;
↳ direct=false);
fsf_controller = named_ss(obs_controller, u = [:ref_S, :ref_V],
↳ y = [:u])
```

NamedStateSpace{Continuous, Float64}

```
A =
  -150.0              8.033684828490095e-12    0.0
    1.0915557686859107e-5   -279.999999999851    1.0
  -3374.9970813429577   -17530.98989999806   -44.0
B =
  150.0              0.9999999999919663
  -1.0915557686859107e-5   278.999999999851
  -0.0014186570429435138  16899.989999998063
C =
  168.749925000000002  31.5499950000000003  1.20000000000000002
D =
  0.0  0.0
```

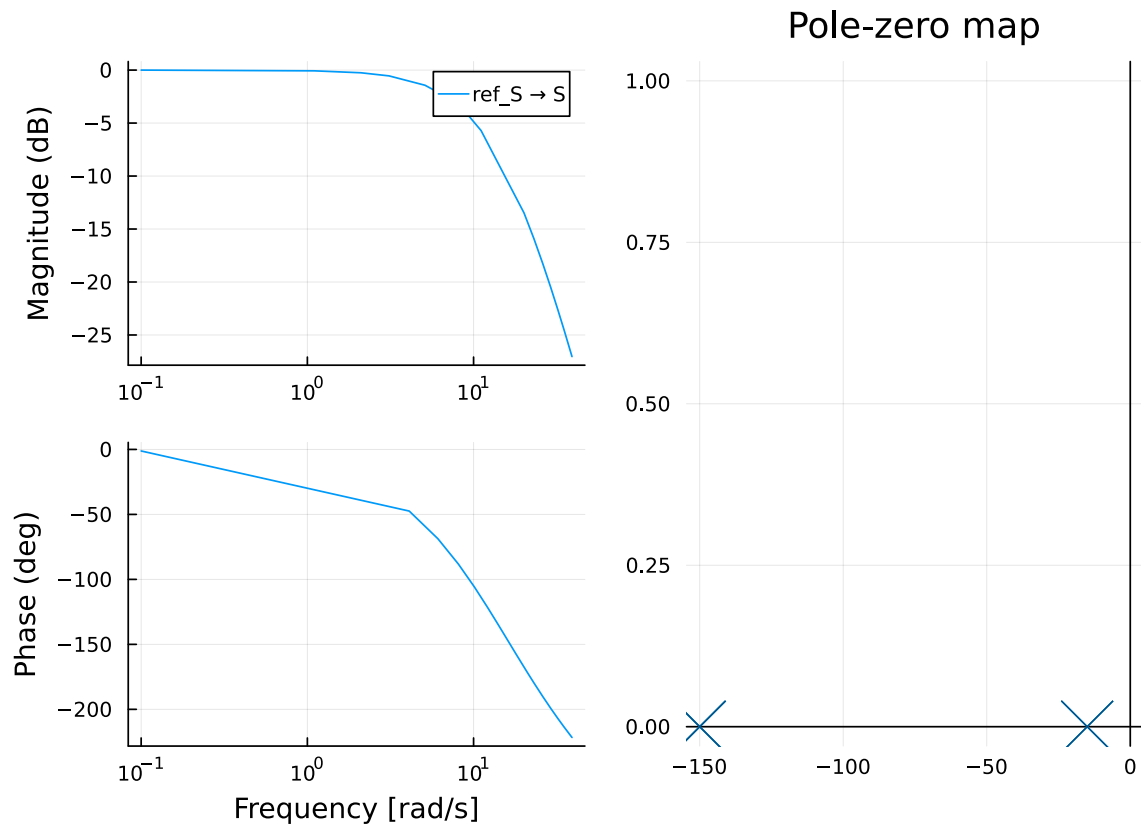
Continuous-time state-space model

```
With state names: x1 x2 x3
    input names: ref_S ref_V
    output names: u
```

We can check the effect of the new controller on the loop

```
closedLoop = feedback( contSys * fsf_controller);
print(poles(closedLoop));
setPlotScale("dB")
plot!(bodeplot(closedLoop[1,1], 0.1:40), pzmap(closedLoop))
```

```
ComplexF64[-14.990000366343788 + 0.0im, -14.999999266673722 + 0.0im, -15.010000366982666 + 0.0im, -149.9999999999986 + 0.0im, -150.10000000002432 + 0.0im, -149.8999999999607 + 0.0im]
```



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```
using DiscretePIDs
Ts = 0.02 # sampling time
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Td = L[2] / L[1];

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```

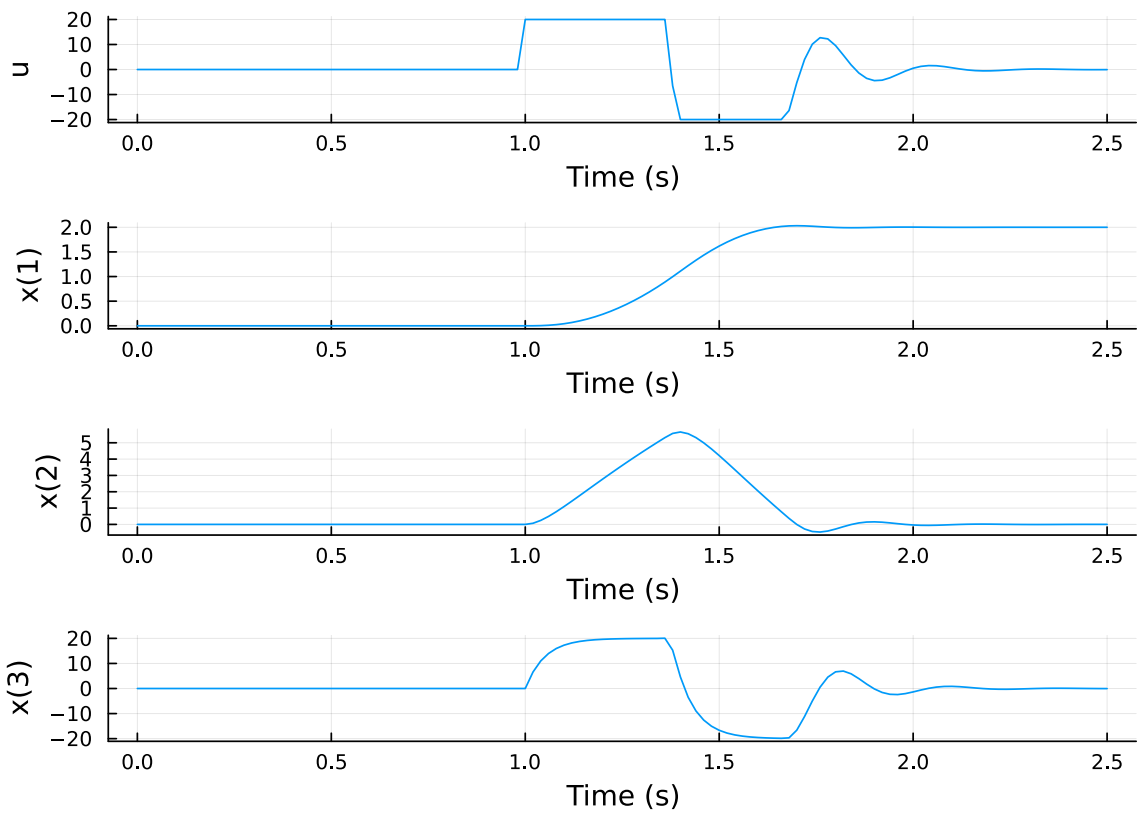
4.3 Simulation

We can simulate this with a motor that only outputs the position:

```
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ctrl = function (x, t)
    y = (sysreal.C*x)[] # measurement
    d = 0 * [1.0] # disturbance
    r = 2.0 * (t ≥ 1) # reference
    # u = pid(r, y) # control signal
    # u + d # Plant input is control signal + disturbance
    # u = 1
    e = x - [r; 0.0; 0.0]
    e[3] = 0.0 # torque not observable, just ignore it in the
    ↪ final feedback
    u = -L * e + d
    u = [maximum([-20.0 minimum([20.0 u]))]]
end
t = 0:Ts:Tf

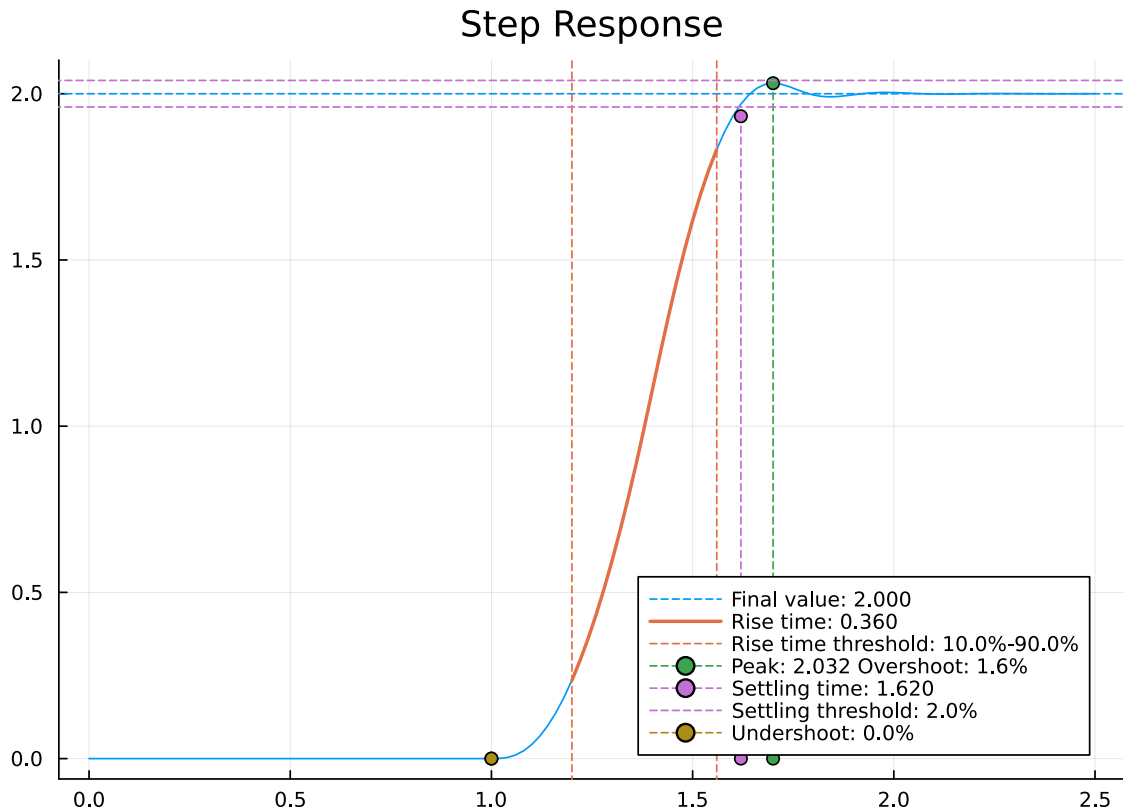
res = lsim(sysreal, ctrl, t)

display(plot(res,
    plotu=true,
    plotx=true,
    ploty=false
))
ylabel!("u", sp=1);
ylabel!("x", sp=2);
ylabel!("v", sp=3);
ylabel!("T", sp=4);
```



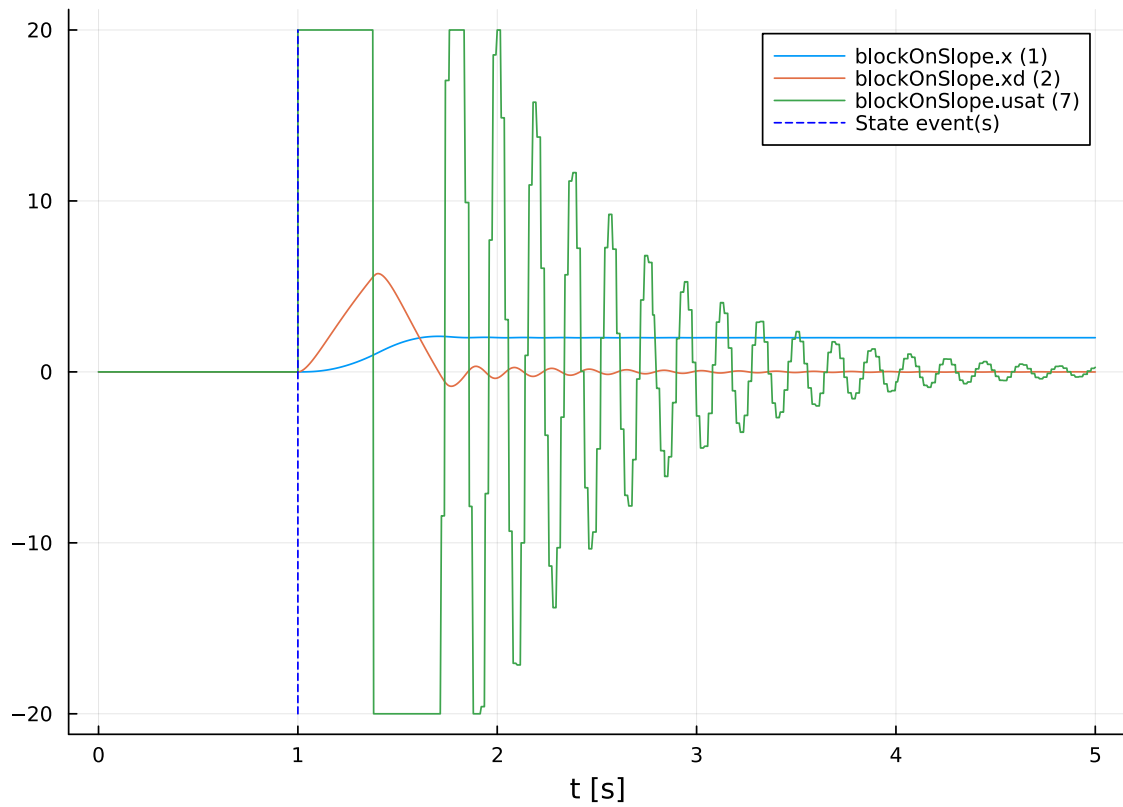
For more stats:

```
si = stepinfo(res);  
plot(si);title!("Step Response")
```



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fmu = loadFMU(fmuPath);
simData = simulateME(
    fmu,
    (0.0, 5.0);
    recordValues=["blockOnSlope.x",
        "blockOnSlope.xd",
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plot(simData, states=false, timeEvents=false)
```



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