Control Challenges: Solutions

Iacopo Moles

Table of contents

1 Introduction **3**
1.1 What is this? 3

2 Block With Friction **4**
2.1 State Space representation 4
2.2 Pole Placement 6
2.3 Simulation 9

1 Introduction

1.1 What is this?

This is a collection of write ups on how to solve the various problems presented by Github user “Janismac”.

$$\dot{x} = \mathbb{A}x + \mathbb{B}u$$

2 Block With Friction

Position Control with friction. Using Pole Placement + PD.

2.1 State Space representation

We can convert the set of ODE into a state space representation. The final bode plot of the block position is:

```
using DiscretePIDs, ControlSystems, Plots, LinearAlgebra

# System parameters
Ts = 0.02 # sampling time
Tf = 2.5; #final simulation time
g = 9.81 #gravity
α = 0.0 # slope
μ = 1.0 # friction coefficient
x_0 = -2.0 # starting position
dx_0 = 0.0 # starting velocity
τ = 20.0 # torque constant

# State Space Matrix
A = [0 1 0
    0 -μ 1
    0 0 -τ
] * 1.0;
B = [0
    0
    τ] * 1.0;
C = [1 0 0
    0 1 0] * 1.0

sys = ss(A, B, C, 0)      # Continuous

bodeplot(tf(sys))
```

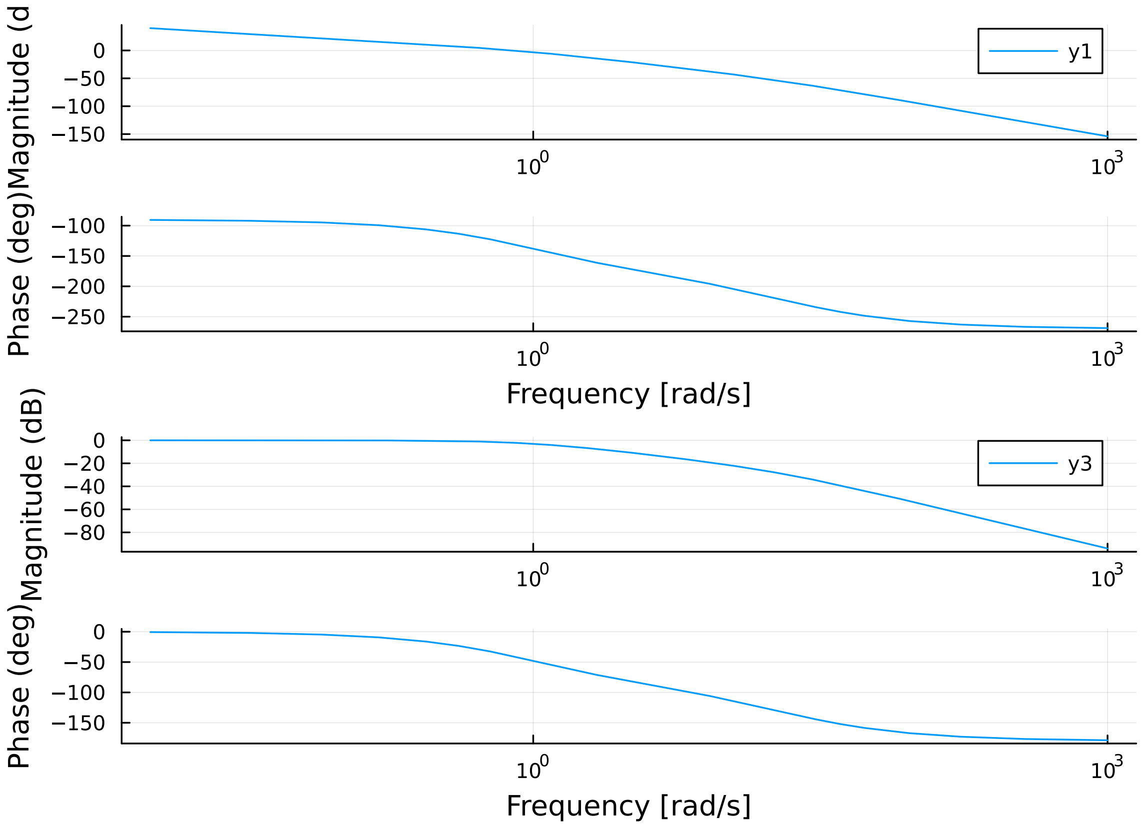


Figure 2.1: Starting Bode Plot

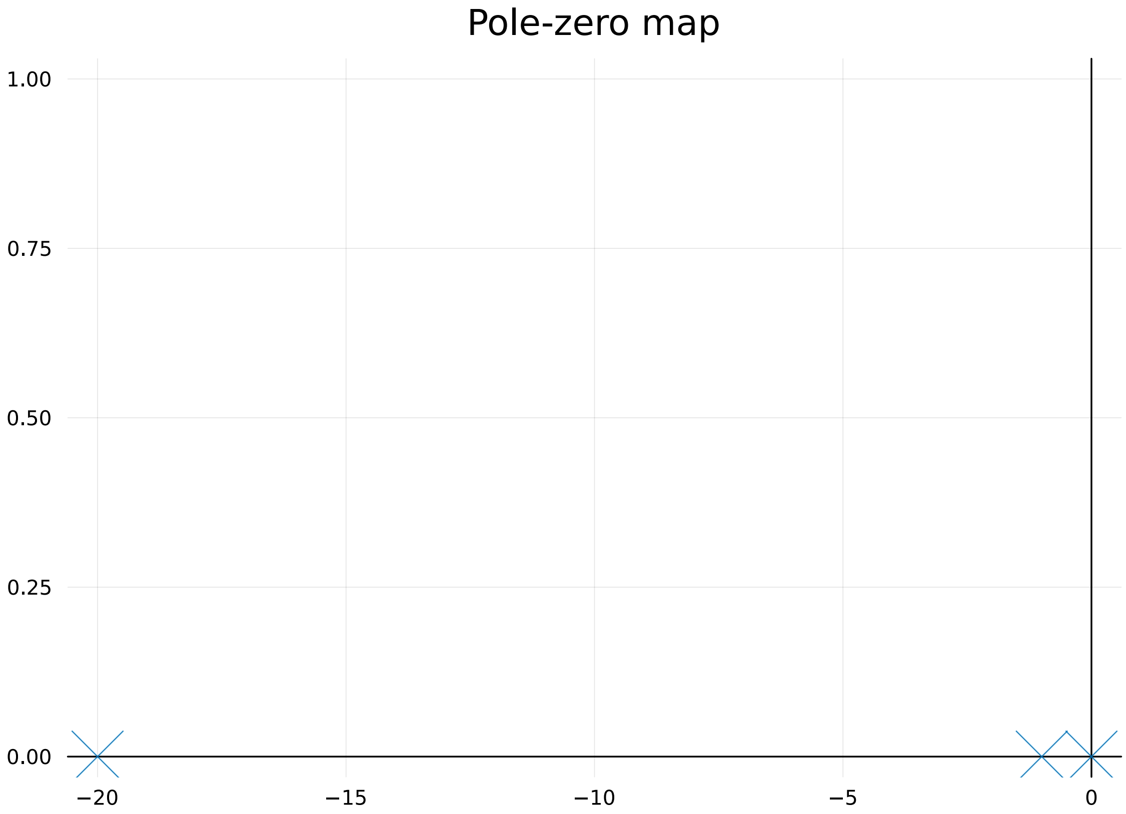
It has the shape we expect from a motor + friction. Slow pole for the mass + friction and a faster pole for the current & inductance.

Numerically they are:

```
display(eigvals(A)) # -20 , -1, 0
display(pzmap(tf(sys)))
```

```
3-element Vector{Float64}:
 -20.0
  -1.0
   0.0
```

(a) Starting PZ map



(b)

Figure 2.2

In Figure 2.2 we see that we start with all the pole in the left-half plane, which is good.

2.2 Pole Placement

We can design a controller with pole placement.

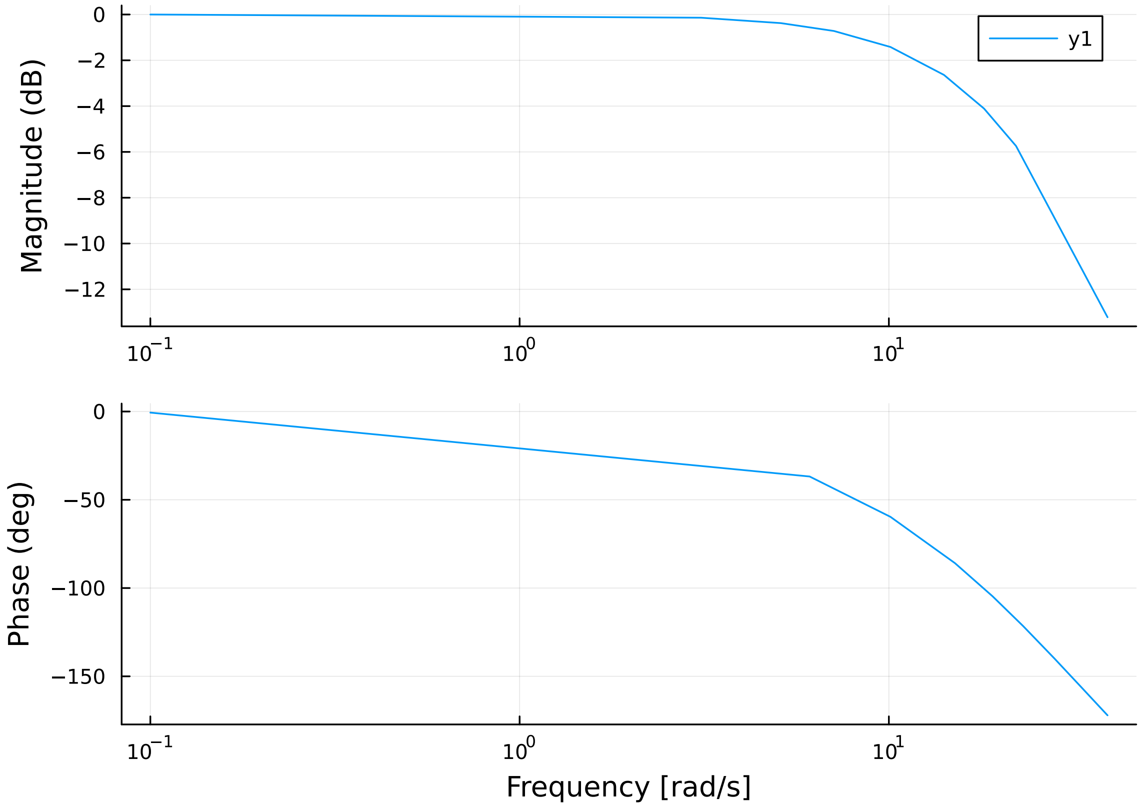
For some reason pole placement doesn't work for the observer, I use a Kalman Filter with random fast values.

```
observability(A, C).isobservable & controllability(A, B).iscontrollable; #OK

ε = 0.01;
pp = 15;
poles_cont = -2.0 * [pp + ε, pp - ε, pp];
L = real(place(sys, poles_cont, :c));

poles_obs = poles_cont * 5.0;
K = place(1.0 * A', 1.0 * C', poles_obs)'
cont = observer_controller(sys, L, K; direct=false);
```

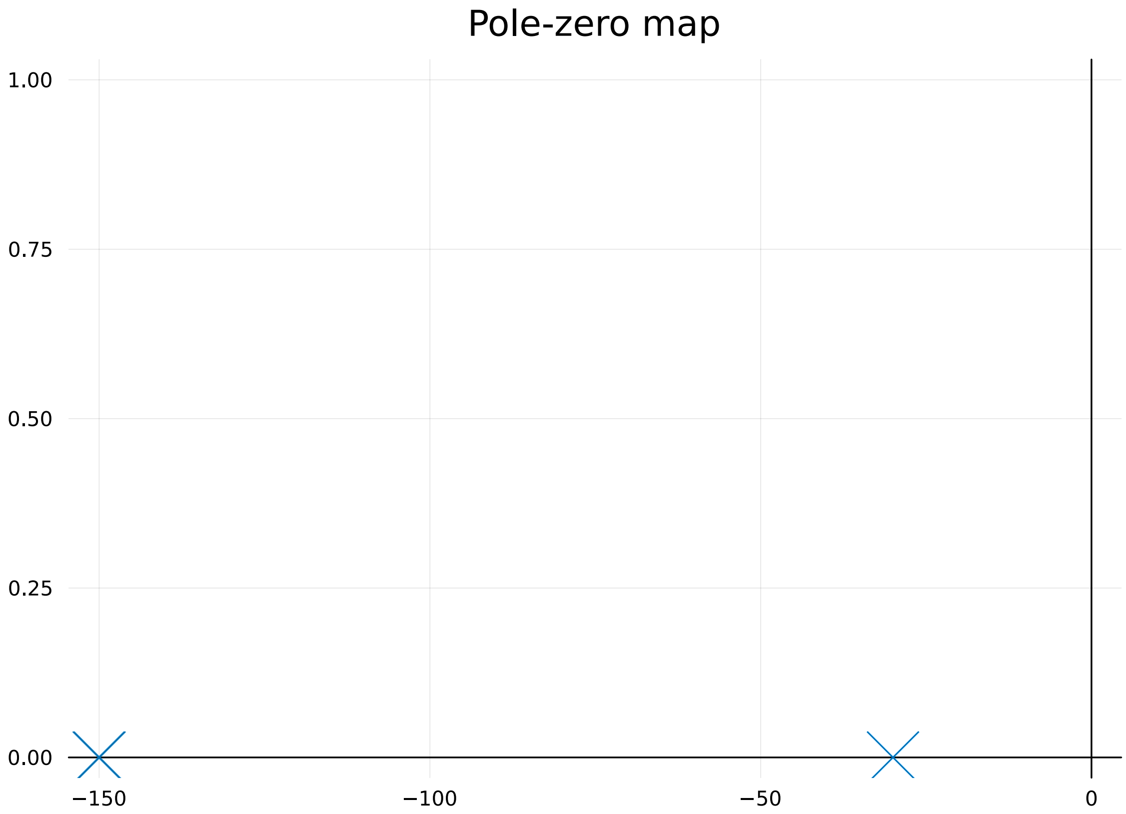
We can check the effect of the new controller on the loop

```
closedLoop = feedback(sys * cont)
print(poles(closedLoop));
setPlotScale("dB")
display(bodeplot(closedLoop[1, 1], 0.1:40))
display(pzmap(closedLoop))
```

```
ComplexF64[-29.980000105166976 + 0.0im, -29.999999789554334 + 0.0im, -
30.020000105278555 + 0.0im, -150.00000000000009 + 0.0im, -150.09999999988327 + 0.0i
149.90000000010167 + 0.0im]
```



Magnitude (dB)
0
−2
−4
−6
−8
−10
−12
y1
10^{-1}
10^{0}
10^{1}
Phase (deg)
0
−50
−100
−150
10^{-1}
10^{0}
10^{1}
Frequency [rad/s]



We can compare this to the open-loop response in Figure 2.1. We can see that we achieve unitary gain throughout the whole low-frequency range.

We can convert the pole placement controller into the standard PD gain form.

```
K = L[1];
Ti = 0;
Td = L[2] / L[1];
pid = DiscretePID(; K, Ts, Ti, Td);
```

2.3 Simulation

We can simulate this with a motor that only outputs the position:

```
sysreal = ss(A, B, [1 0 0], 0)
ctrl = function (x, t)
    y = (sysreal.C*x)[] # measurement
    d = 0 * [1.0]        # disturbance
    r = 2 * (t ≥ 1) # reference
    # u = pid(r, y) # control signal
```

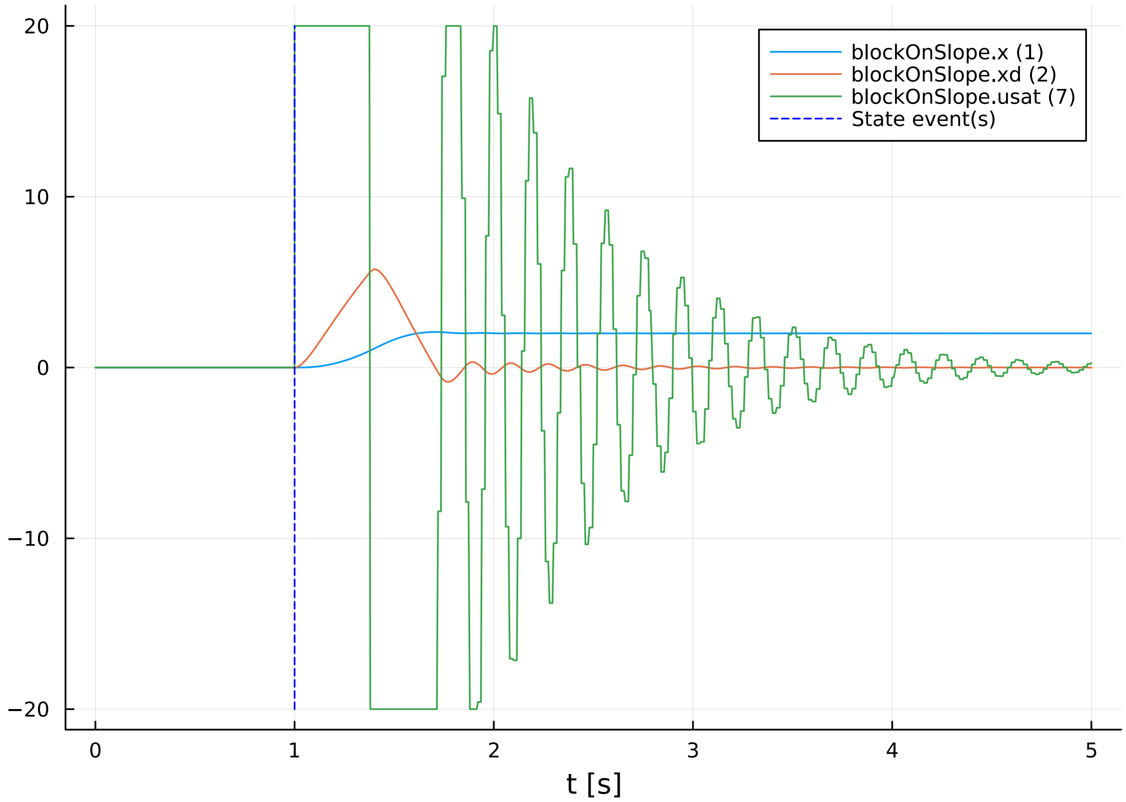
```
    # u + d # Plant input is control signal + disturbance
    # u =1
    e = x - [r; 0; 0]
    e[3] = 0 # torque not observable, just ignore it in the final feedback
    u = -L * e + d
    u = [maximum([-20 minimum([20 u])])]
end
t = 0:Ts:Tf


res = lsim(sysreal, ctrl, t)

plot(res, plotu=true, plotx=true, ploty=false);
ylabel!("u", sp=1);
ylabel!("x", sp=2);
ylabel!("v", sp=3);
ylabel!("T", sp=4);
```

For more stats:

```
si = stepinfo(res);
plot(si);
title!("Step Response");
```

We can also simulate it in a SIMULINK-like environment:

```
using FMI, DifferentialEquations
fmu = loadFMU(abspath("../modelica/ControlChallenges/ControlChallenges.BlockOnSlop
simData = simulateME(
    fmu,
    (0.0, 5.0);
    recordValues=["blockOnSlope.x", "blockOnSlope.xd", "blockOnSlope.usat"],
    showProgress = false);
unloadFMU(fmu);
plot(simData, states=false, timeEvents=false)
```



There is a slight difference between the `lsim` simulation and the FMU simulation. I need to recheck some stuff.