# **Ecoadvisor model formulas**

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## Compilation command (bash)

groff -e -t -ms formulas.ms -T pdf -K utf-8

### 1. State Variables

### 1.1. Inputs

$$Q$$
 (ML/d)

$$S_{VFA}, S_{FBSO}, X_{BPO}, X_{UPO}, S_{USO}, X_{iSS}, S_{FSA}, S_{OP}, S_{NOx}$$
 (mg/L)

## 1.2. Total chemical oxygen demand (COD = $S_t$ )

$$S_t = S_{VFA} + S_{FBSO} + X_{BPO} + X_{UPO} + S_{USO}$$
 (mg/L)

also:

$$\begin{split} S_t &= S_b + S_u = S_s + S_p = S_{bs} + S_{bp} + S_{up} + S_{us} \\ S_b &= S_{bs} + S_{bp} \\ S_u &= S_{us} + S_{up} \\ S_s &= S_{bs} + S_{us} \\ S_p &= S_{bp} + S_{up} \\ S_{bs} &= S_{VFA} + S_{FBSO} \\ S_{bp} &= X_{BPO} \\ S_{up} &= X_{UPO} \\ S_{us} &= S_{USO} \end{split}$$

### 1.3. Mass ratios for COD, N, P and C

State Variable	$\mathbf{COD}(f_{cv})$	$\mathbf{N}(f_N)$	$\mathbf{P}(f_P)$	$\mathbf{C}(f_C)$
$S_{VFA}$	1.0667	0.0000	0.0000	0.400
$S_{FBSO}$	1.4200	0.0464	0.0118	0.471
$X_{BPO}$	1.5230	0.0323	0.0072	0.498
$X_{UPO}$	1.4810	0.1000	0.0250	0.518
$S_{USO}$	1.4930	0.0366	0.0000	0.498
OHO	1.4810	0.1000	0.0250	0.518
ANO	1.4810	0.1000	0.0250	0.518
PAO	1.4810	0.1000	0.0250	0.518

### **1.4.** Total Kjeldahl Nitrogen (TKN = $N_t$ )

$$N_t = N_o + S_{FSA} \tag{mg/L}$$

where:

$$\begin{split} N_{o} &= N_{obs} + N_{obp} + N_{ous} + N_{oup} \\ N_{obs} &= S_{VFA} \frac{f_{N_{VFA}}}{f_{cv_{VFA}}} + S_{FBSO} \frac{f_{N_{FBSO}}}{f_{cv_{FBSO}}} \\ N_{obp} &= X_{BPO} \frac{f_{N_{BPO}}}{f_{cv_{BPO}}} \\ N_{ous} &= S_{USO} \frac{f_{N_{USO}}}{f_{cv_{USO}}} \\ N_{oup} &= X_{UPO} \frac{f_{N_{USO}}}{f_{cv_{UPO}}} \end{split}$$

## **1.5.** Total Phosphorus (TP = $P_t$ )

$$P_t = P_o + S_{OP} \tag{mg/L}$$

where:

$$\begin{split} P_o &= P_{obs} + P_{obp} + P_{ous} + P_{oup} \\ P_{obs} &= S_{VFA} \frac{f_{P_{VFA}}}{f_{cv_{VFA}}} + S_{FBSO} \frac{f_{P_{FBSO}}}{f_{cv_{FBSO}}} \\ P_{obp} &= X_{BPO} \frac{f_{P_{BPO}}}{f_{cv_{BPO}}} \\ P_{ous} &= S_{USO} \frac{f_{P_{USO}}}{f_{cv_{USO}}} \\ P_{oup} &= X_{UPO} \frac{f_{P_{UPO}}}{f_{cv_{UPO}}} \end{split}$$

### **1.6.** Total Organic Carbon (TOC = $C_t$ )

$$C_t = C_{obs} + C_{obp} + C_{ous} + C_{oup}$$
 (mg/L)

where:

$$C_{obs} = S_{VFA} \frac{f_{C_{VFA}}}{f_{C_{VVFA}}} + S_{FBSO} \frac{f_{C_{FBSO}}}{f_{cv_{FBSO}}}$$

$$C_{obp} = X_{BPO} \frac{f_{C_{BPO}}}{f_{cv_{BPO}}}$$

$$C_{ous} = S_{USO} \frac{f_{C_{USO}}}{f_{cv_{USO}}}$$

$$C_{oup} = X_{UPO} \frac{f_{C_{UPO}}}{f_{cv_{UPO}}}$$

## **1.7.** Total suspended solids (TSS = $X_T$ )

$$X_T = X_V + X_{iSS}$$
 (mg/L)

where:

$$X_V = \frac{X_{BPO}}{f_{cv_{BPO}}} + \frac{X_{UPO}}{f_{cv_{UPO}}}$$

## 2. Activated sludge

### 2.1. Inputs

$$T, V_{p,}R_{s,}RAS, mass_{FeCl_3}$$

#### 2.2. Volatile suspended solids (VSS)

$$Y_H = 0.45$$
 (gVSS/gCOD)

$$b_H = 0.24$$
 (1/d)

$$b_{HT} = b_H (1.029)^{(T-20)} \tag{1/d}$$

$$f_{X_{BH}} = \frac{Y_H \cdot R_s}{1 + b_{HT} \cdot R_s}$$
 (gVSS·d/gCOD)

$$MX_{BH} = FS_{bi} \cdot f_{X_{BH}}$$
 (kgVSS)

$$MX_{EH} = f_H \cdot b_{HT} \cdot R_s \cdot MX_{BH}$$
 (kgVSS)

$$MX_I = FX_{ti} \cdot R_s \tag{kgVSS}$$

$$MX_V = MX_{BH} + MX_{EH} + MX_I$$
 (kgVSS)

$$X_V = \frac{MX_V}{V_p}$$
 (kgVSS/m<sup>3</sup>)

#### 2.3. Total inert solids (iSS)

$$f_{iOHO} = 0.15$$
 (giSS/gVSS)

$$MX_{IO} = FiSS \cdot R_s + f_{iOHO} \cdot MX_{BH} + F_{iSS\ precipitation} \cdot R_s$$
 (kgiSS)

### 2.4. Total suspended solids (TSS)

$$MX_T = MX_V + MX_{IO}$$
 (kgTSS)

$$X_T = \frac{MX_T}{V_p}$$
 (kgTSS/m<sup>3</sup>)

### 2.5. Nominal hydraulic retention time

$$HRT = \frac{V_p}{Q} \tag{d}$$

### 2.6. Wastage flowrate

$$Q_w = \frac{V_p}{R_c} \tag{ML/d}$$

#### 2.7. Effluent flowrate

$$Q_e = Q - Q_w \tag{ML/d}$$

## 2.8. Nitrogen and Phosphorus required for sludge production

$$N_s = \frac{f_{N_{OHO}}(MX_{BH} + MX_{EH}) + f_{N_{UPO}}MX_I}{R_s \cdot Q}$$
 (mg/L)

$$P_{s} = \frac{f_{P_{OHO}}(MX_{BH} + MX_{EH}) + f_{P_{UPO}}MX_{I}}{R_{s} \cdot Q}$$
 (mg/L)

#### 2.9. Effluent Ammonia concentration

$$N_{ae} = N_{ti} - N_s - N_{ouse}$$
 (mg/L)

### 2.10. Effluent Orthophosphate concentration

$$P_{se} = P_{ti} - P_s - P_{ouse} - P_{precipitation}$$
 (mg/L)

### 2.11. Wastage solids concentration

$$BPO_{was} = f_{cv_{RPO}}(1 - f_H)X_{BH}$$
 (mg/L)

$$UPO_{was} = f_{cv_{IPO}}(f_H X_{BH} + X_{EH} + X_I)$$
 (mg/L)

$$iSS_{was} = \frac{MX_{IO}}{V_n}$$
 (mg/L)

## 2.12. Oxygen demand

$$FO_c = Y_H (1 - f_{cv_{OHO}}) + f_{cv_{OHO}} (1 - f_H) \cdot b_{HT} \cdot f_{X_{RH}}$$
 (kgO/d)

$$FO_n = 4.57 \cdot Q \cdot N_{ae} \tag{kgO/d}$$

$$FO_t = FO_c + FO_n \tag{kgO/d}$$

$$OUR = \frac{FO_t}{V_p}$$
 (mgO/L·h)

## 3. Nitrification

### 3.1. Inputs

$$SF, f_{xt}, DO, pH$$

#### 3.2. Nitrifier kinetics

$$\mu_{Am} = 0.45$$
 (1/d)

$$\mu_{AmT} = \mu_{Am} (1.123)^{T-20} \tag{1/d}$$

$$K_O = 0.3 \tag{mgO/L}$$

$$\mu_{AmO} = \mu_{AmT} \frac{DO}{K_O + DO} \tag{1/d}$$

$$K_i = 1.13, K_{ii} = 0.3, K_{\text{max}} = 9.5$$

$$\mu_{AmpH} = \mu_{AmO}(2.35)^{pH-7.2} K_i \frac{K_{\text{max}} - pH}{K_{\text{max}} + K_{ii} - pH}$$
(1/d)

$$Y_A = 0.1 (gVSS/gNH4)$$

$$Y_{AT} = Y_A^{T-20}$$
 (gVSS/gNH4)

$$K_n = 1.0 (mg/L)$$

$$K_{nT} = K_n (1.123)^{T-20}$$
 (mg/L)

$$b_A = 0.04$$
 (1/d)

$$b_{AT} = b_A (1.029)^{T-20} (1/d)$$

#### 3.3. Maximum design unaerated sludge mass fraction

$$f_{xm} = 1 - SF \frac{b_{AT} + \frac{1}{R_s}}{\mu_{AmpH}} \tag{\emptyset}$$

## 3.4. Minimum sludge age for nitrification

$$R_{sm} = \frac{1}{\mu_{AmpH}(1 - f_{xt}) - b_{AT}}$$
 (d)

### 3.5. Effluent Ammonia concentration

$$N_{ae} = \frac{K_{nT}(b_{AT} + \frac{1}{R_s})}{\mu_{AmpH}(1 - f_{xt}) - b_{AT} - \frac{1}{R_s}}$$
 (mg/L)

## 3.6. Nitrification capacity

$$N_c = N_{ti} - N_s - (N_{ae} + N_{ouse})$$
 (mg/L)

## 3.7. ANO biomass

$$f_{X_{BA}} = \frac{Y_{AT}R_s}{1 + b_{AT} \cdot R_s}$$
 (gVSS·d/gNH4)

$$MX_{BA} = Q \cdot N_c \cdot f_{X_{BA}}$$
 (kgVSS)

$$X_{BA} = \frac{MX_{BA}}{V_p}$$
 (kgVSS/m<sup>3</sup>)

## 4. Denitrification

### **4.1. Inputs**

$$IR, DO_{RAS}$$
,  $alk_i$ 

#### 4.2. Denitrification kinetics

$$K_1^{20} = 0.72, K_2^{20} = 0.10, K_3^{20} = 0.10, K_4^{20} = 0.00$$
 (mgN/mgVSS·d)

$$K_{1T} = K_1^{20} (1.200)^{T-20}$$
 (mgN/mgVSS·d)

$$K_{2T} = K_2^{20} (1.080)^{T-20}$$
 (mgN/mgVSS·d)

$$K_{3T} = K_3^{20} (1.029)^{T-20}$$
 (mgN/mgVSS·d)

$$K_{4T} = K_4^{20} (1.029)^{T-20}$$
 (mgN/mgVSS·d)

### 4.3. Denitrification potential

$$D_{p1RBSO} = \frac{S_{bsi}(1 - f_{cv}Y_H)}{2.86}$$
 (mgN/L)

$$D_{n1BPO} = K_{2T} f_{xt} S_{hi} f_{X_{pu}}$$
 (mgN/L)

$$D_{p1} = D_{p1RBSO} + D_{p1BPO}$$
 (mgN/L)

### 4.4. Optimum internal recirculation

$$a_{opt} = \frac{-B + \sqrt{B^2 + 4AC}}{2A} \tag{\emptyset}$$

where:

$$\begin{split} A &= \frac{DO}{2.86} \\ B &= N_c - D_{p1} + \frac{(1 + RAS) \cdot DO + RAS \cdot DO_{RAS}}{2.86} \\ C &= (1 + RAS) \cdot (D_{p1} - \frac{RAS \cdot DO_{RAS}}{2.86}) - RAS \cdot N_c \end{split}$$

## 4.5. Minimum effluent NOx concentration

$$N_{ne\_opt} = \frac{Nc}{a_{opt} + RAS + 1}$$
 (mgN/L)

# 4.6. Effluent nitrate

 $if(a < a_{opt})$ 

$$N_{ne} = \frac{N_c}{a + RAS + 1}$$
 (mgN/L)

 $if(a > a_{opt})$ 

$$N_{ne} = N_c - D_{p1} + \frac{a * DO + RAS * DO_{RAS}}{2.86}$$
 (mgN/L)

## 4.7. Nitrogen gas

$$FN_2 = Q(N_c - N_{ne}) (kgN/d)$$

## 4.8. Oxygen recovered by denitrification

$$FO_d = 2.86 \cdot Q \cdot (N_c - N_{ne}) \tag{kgO/d}$$

$$FO_t = FO_c + FO_n - FO_d (kgO/d)$$

## 4.9. Effluent alkalinity

$$alk_e = alk_i + 3.57 \cdot (N_{obi} - (N_s - N_{oupi})) - 7.14 \cdot N_c + 2.86 \cdot (N_c - N_{ne})$$
 (mg/L)