

## **Ecoadvisor model formulas**

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### **Compilation command (bash)**

```
groff -e -t -ms formulas.ms -T pdf -K utf-8
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## 1. State Variables

### 1.1. Inputs

$$Q \quad (\text{ML/d})$$

$$S_{VFA}, S_{FBSO}, X_{BPO}, X_{UPO}, S_{USO}, X_{iSS}, S_{FSA}, S_{OP}, S_{NOx}, X_{OHO} \quad (\text{mg/L})$$

### 1.2. Total chemical oxygen demand (COD = $S_t$ )

$$S_t = S_{VFA} + S_{FBSO} + X_{BPO} + X_{UPO} + S_{USO} + X_{OHO} \quad (\text{mgCOD/L})$$

and:

$$\begin{aligned} S_{bs} &= S_{VFA} + S_{FBSO} \\ S_{bp} &= X_{BPO} \\ S_{up} &= X_{UPO} \\ S_{us} &= S_{USO} \\ S_{active} &= X_{OHO} \\ S_t &= S_{bs} + S_{bp} + S_{up} + S_{us} + S_{active} \end{aligned}$$

The components can be grouped as follows:

	$S_s$	$S_p$
$S_b$	$S_{bs}$	$S_{bp}$
$S_u$	$S_{us}$	$S_{up}$

So,

$$S_t = S_b + S_u + S_{active}$$

$$S_t = S_s + S_p + S_{active}$$

### 1.3. Mass ratios for COD, N, P and C

State Variable	COD ( $f_{cv}$ )	N ( $f_N$ )	P ( $f_P$ )	C ( $f_C$ )
$S_{VFA}$	1.0667	0.0000	0.0000	0.400
$S_{FBSO}$	1.4200	0.0464	0.0118	0.471
$X_{BPO}$	1.5230	0.0323	0.0072	0.498
$X_{UPO}$	1.4810	0.1000	0.0250	0.518
$S_{USO}$	1.4930	0.0366	0.0000	0.498
$X_{OHO}$	1.4810	0.1000	0.0250	0.518
$ANO$	1.4810	0.1000	0.0250	0.518
$PAO$	1.4810	0.1000	0.0250	0.518

### 1.4. Total Kjeldahl Nitrogen (TKN = $N_t$ )

$$N_t = N_o + S_{FSA} + N_{active} \quad (\text{mgN/L})$$

where:

$$\begin{aligned}
 N_o &= N_{obs} + N_{obp} + N_{ous} + N_{oup} \\
 N_{obs} &= S_{VFA} \frac{f_{N_{VFA}}}{f_{cv_{VFA}}} + S_{FBSO} \frac{f_{N_{FBSO}}}{f_{cv_{FBSO}}} \\
 N_{obp} &= X_{BPO} \frac{f_{N_{BPO}}}{f_{cv_{BPO}}} \\
 N_{ous} &= S_{USO} \frac{f_{N_{USO}}}{f_{cv_{USO}}} \\
 N_{oup} &= X_{UPO} \frac{f_{N_{UPO}}}{f_{cv_{UPO}}} \\
 N_{active} &= X_{OHO} \frac{f_{N_{OHO}}}{f_{cv_{OHO}}}
 \end{aligned}$$

### 1.5. Total Phosphorus (TP = $P_t$ )

$$P_t = P_o + S_{OP} + P_{active} \quad (\text{mgP/L})$$

where:

$$\begin{aligned}
 P_o &= P_{obs} + P_{obp} + P_{ous} + P_{oup} \\
 P_{obs} &= S_{VFA} \frac{f_{P_{VFA}}}{f_{cv_{VFA}}} + S_{FBSO} \frac{f_{P_{FBSO}}}{f_{cv_{FBSO}}} \\
 P_{obp} &= X_{BPO} \frac{f_{P_{BPO}}}{f_{cv_{BPO}}} \\
 P_{ous} &= S_{USO} \frac{f_{P_{USO}}}{f_{cv_{USO}}} \\
 P_{oup} &= X_{UPO} \frac{f_{P_{UPO}}}{f_{cv_{UPO}}} \\
 P_{active} &= X_{OHO} \frac{f_{P_{OHO}}}{f_{cv_{OHO}}}
 \end{aligned}$$

### 1.6. Total Organic Carbon (TOC = $C_t$ )

$$C_t = C_{obs} + C_{obp} + C_{ous} + C_{oup} + C_{active} \quad (\text{mgC/L})$$

where:

$$\begin{aligned}
 C_{obs} &= S_{VFA} \frac{f_{C_{VFA}}}{f_{cv_{VFA}}} + S_{FBSO} \frac{f_{C_{FBSO}}}{f_{cv_{FBSO}}} \\
 C_{obp} &= X_{BPO} \frac{f_{C_{BPO}}}{f_{cv_{BPO}}} \\
 C_{ous} &= S_{USO} \frac{f_{C_{USO}}}{f_{cv_{USO}}} \\
 C_{oup} &= X_{UPO} \frac{f_{C_{UPO}}}{f_{cv_{UPO}}} \\
 C_{active} &= X_{OHO} \frac{f_{C_{OHO}}}{f_{cv_{OHO}}}
 \end{aligned}$$

### 1.7. Total suspended solids (TSS = $X_T$ )

$$X_T = X_V + X_{iSS} \quad (\text{mgTSS/L})$$

where:

$$X_V = \frac{X_{BPO}}{f_{cv_{BPO}}} + \frac{X_{UPO}}{f_{cv_{UPO}}} + \frac{X_{OHO}}{f_{cv_{OHO}}} \quad (\text{mgVSS/L})$$

## 2. Activated sludge

### 2.1. Inputs

$$T, V_p, R_s, RAS, mass_{FeCl_3}$$

### 2.2. Volatile suspended solids (VSS)

$$Y_H = 0.45 \quad (\text{gVSS/gCOD})$$

$$b_H = 0.24 \quad (1/\text{d})$$

$$b_{HT} = b_H(1.029)^{T-20} \quad (1/\text{d})$$

$$f_{X_{BH}} = \frac{Y_H \cdot R_s}{1 + b_{HT} \cdot R_s} \quad (\text{gVSS} \cdot \text{d/gCOD})$$

$$k_{vT} = 0.007 \cdot (1.035)^{T-20} \quad (\text{L/gVSS} \cdot \text{d})$$

$$S_{nd} = \frac{1}{f_{X_{BH}} k_{vT}} \quad (\text{mgCOD/L})$$

$$FdS_{bi} = FS_{bi} - Q \cdot S_{nd} \quad (\text{kgCOD/d})$$

$$MX_{BH} = FdS_{bi} \cdot f_{X_{BH}} \quad (\text{kgVSS})$$

$$MX_{EH} = f_H \cdot b_{HT} \cdot R_s \cdot MX_{BH} \quad (\text{kgVSS})$$

$$MX_I = FX_{Ti} \cdot R_s \quad (\text{kgVSS})$$

$$MX_V = MX_{BH} + MX_{EH} + MX_I \quad (\text{kgVSS})$$

$$X_V = \frac{MX_V}{V_p} \quad (\text{kgVSS/m}^3)$$

### 2.3. Total inert solids (iSS)

$$f_{iOHO} = 0.15 \quad (\text{giSS/gVSS})$$

$$MX_{IO} = FiSS \cdot R_s + f_{iOHO} \cdot MX_{BH} + F_{iSS\_precipitation} \cdot R_s \quad (\text{kgiSS})$$

### 2.4. Total suspended solids (TSS)

$$MX_T = MX_V + MX_{IO} \quad (\text{kgTSS})$$

$$X_T = \frac{MX_T}{V_p} \quad (\text{kgTSS/m}^3)$$

## 2.5. Nominal hydraulic retention time

$$HRT = \frac{V_p}{Q} \quad (d) \quad (d)$$

## 2.6. Wastage flowrate

$$Q_w = \frac{V_p}{R_s} \quad (ML/d)$$

## 2.7. Effluent flowrate

$$Q_e = Q - Q_w \quad (ML/d)$$

## 2.8. Nitrogen and Phosphorus required for sludge production

$$N_s = \frac{f_{N_{OHO}}(MX_{BH} + MX_{EH}) + f_{N_{UPO}}MX_I}{R_s \cdot Q} \quad (mgN/L)$$

$$P_s = \frac{f_{P_{OHO}}(MX_{BH} + MX_{EH}) + f_{P_{UPO}}MX_I}{R_s \cdot Q} \quad (mgP/L)$$

## 2.9. Effluent Ammonia ( $N_{ae}$ ) and effluent Orthophosphate ( $P_{se}$ ) concentration

$$N_{ae} = N_{ti} - N_s - N_{ouse} \quad (mgN/L)$$

$$P_{se} = P_{ti} - P_s - P_{ouse} - P_{precipitation} \quad (mgP/L)$$

## 2.10. Wastage solids concentration

$$BPO_{was} = 0 \quad (mgCOD/L)$$

$$UPO_{was} = f_{cv_{UPO}} \cdot X_I \quad (mgCOD/L)$$

$$OHO_{was} = f_{cv_{OHO}}(X_{BH} + X_{EH}) \quad (mgCOD/L)$$

$$iSS_{was} = \frac{MX_{IO}}{V_p} \quad (mgiSS/L)$$

## 2.11. Oxygen demand

$$FO_c = Y_H(1 - f_{cv_{OHO}}) + f_{cv_{OHO}}(1 - f_H) \cdot b_{HT} \cdot f_{X_{BH}} \quad (kgO/d)$$

$$FO_n = 4.57 \cdot Q \cdot N_{ae} \quad (kgO/d)$$

$$FO_t = FO_c + FO_n \quad (kgO/d)$$

$$OUR = \frac{FO_t}{V_p} \quad (\text{mgO/L}\cdot\text{h})$$

### 3. Chemical P removal

Sludge component	weight (g/mol)
$M_{Fe}$	55.845
$M_P$	30.974
$M_{FeCl_3}$	162.195
$M_{Fe(OH)_3}$	106.866
$M_{Fe_{1.6}H_2PO_4OH_{3.8}}$	250.9646

#### 3.1. Moles of Fe added

$$moles_{Fe} = \frac{mass_{FeCl_3}}{M_{FeCl_3}} \frac{1 mol_{Fe}}{1 mol_{FeCl_3}} \quad (\text{molFe/d})$$

#### 3.2. Moles of P available

$$P_{available} = P_{ti} - P_s - P_{ouse} - P_{obse} \quad (\text{mgP/L})$$

$$moles_P = \frac{Q \cdot P_{available}}{M_P} \quad (\text{molFe/d})$$

#### 3.3. Fe/P mole ratio

$$Fe/P_{ratio} = \frac{moles_{Fe}}{moles_P} \quad (\text{molFe/molP})$$

#### 3.4. Figure 6-13 Metcalf and Eddy 5th ed, page 484

Fe/P ratio	$P_{se}$ (mgP/L)
8.00	0.01
4.90	0.02
4.50	0.03
4.20	0.04
3.90	0.05
3.80	0.06
3.70	0.07
3.50	0.08
3.35	0.09
3.30	0.10
2.60	0.20
2.10	0.30
2.00	0.40
1.70	0.50
1.50	0.60
1.20	0.70
1.10	0.80
1.00	0.90



Fe/P ratio	$P_{se}$ (mgP/L)
1.00	1.00
0.20	2.00
0.10	3.00
0.10	4.00
0.01	5.00
0.01	6.00
0.005	7.00
0.001	8.00
0.001	9.00
0.0001	10.00

### 3.5. P removed by precipitation

$$P_{precipitation} = Q \cdot (P_{available} - P_{se}) \quad (\text{kgP/d})$$

### 3.6. Extra iSS production by precipitation

$$F_{iSS\_precipitation} = \frac{P_{precipitation}}{M_P} (M_{Fe_{1.6}H_2PO_4OH_{3.8}} + M_{Fe(OH)_3} \cdot (Fe/P_{ratio} - 1.6)) \quad (\text{kgiSS/d})$$

## 4. Nitrification

### 4.1. Inputs

$$SF, f_{xt}, DO, pH$$

### 4.2. Nitrifier kinetics

$$\mu_{Am} = 0.45 \quad (1/d)$$

$$\mu_{AmT} = \mu_{Am}(1.123)^{T-20} \quad (1/d)$$

$$K_O = 0.3 \quad (mgO/L)$$

$$\mu_{AmO} = \mu_{AmT} \frac{DO}{K_O + DO} \quad (1/d)$$

$$K_i = 1.13, K_{ii} = 0.3, K_{\max} = 9.5$$

$$\mu_{AmpH} = \mu_{AmO}(2.35)^{pH-7.2} K_i \frac{K_{\max} - pH}{K_{\max} + K_{ii} - pH} \quad (1/d)$$

$$Y_A = 0.1 \quad (gVSS/gN)$$

$$Y_{AT} = Y_A^{T-20} \quad (gVSS/gNH_4)$$

$$K_n = 1.0 \quad (mgN/L)$$

$$K_{nT} = K_n(1.123)^{T-20} \quad (mgN/L)$$

$$b_A = 0.04 \quad (1/d)$$

$$b_{AT} = b_A(1.029)^{T-20} \quad (1/d)$$

### 4.3. Maximum design unaerated sludge mass fraction

$$f_{xm} = 1 - SF \frac{b_{AT} + \frac{1}{R_s}}{\mu_{AmpH}} \quad (\emptyset)$$

### 4.4. Minimum sludge age for nitrification

$$R_{sm} = \frac{1}{\frac{\mu_{AmpH}}{SF} (1 - f_{xt}) - b_{AT}} \quad (d)$$

#### 4.5. Effluent Ammonia concentration

$$N_{ae} = \frac{K_{nT}(b_{AT} + \frac{1}{R_s})}{\mu_{AmpH}(1 - f_{xt}) - b_{AT} - \frac{1}{R_s}} \quad (\text{mgN/L})$$

#### 4.6. Nitrification capacity

$$N_c = N_{ti} - N_s - (N_{ae} + N_{ouse}) \quad (\text{mgN/L})$$

#### 4.7. ANO biomass

$$f_{X_{BA}} = \frac{Y_{AT}R_s}{1 + b_{AT} \cdot R_s} \quad (\text{gVSS} \cdot \text{d/gNH}_4)$$

$$MX_{BA} = Q \cdot N_c \cdot f_{X_{BA}} \quad (\text{kgVSS})$$

$$X_{BA} = \frac{MX_{BA}}{V_p} \quad (\text{kgVSS/m}^3)$$

## 5. Denitrification

### 5.1. Inputs

$$a, DO_{RAS}, alk_i$$

### 5.2. Denitrification kinetics

$$K_1^{20} = 0.72, K_2^{20} = 0.10, K_3^{20} = 0.10, K_4^{20} = 0.00 \quad (\text{mgN/mgVSS}\cdot\text{d})$$

$$K_{1T} = K_1^{20}(1.200)^{T-20} \quad (\text{mgN/mgVSS}\cdot\text{d})$$

$$K_{2T} = K_2^{20}(1.080)^{T-20} \quad (\text{mgN/mgVSS}\cdot\text{d})$$

$$K_{3T} = K_3^{20}(1.029)^{T-20} \quad (\text{mgN/mgVSS}\cdot\text{d})$$

$$K_{4T} = K_4^{20}(1.029)^{T-20} \quad (\text{mgN/mgVSS}\cdot\text{d})$$

### 5.3. Denitrification potential

$$D_{p1RBSO} = \frac{S_{bsi}(1 - f_{cv}Y_H)}{2.86} \quad (\text{mgN/L})$$

$$D_{p1BPO} = K_{2T}f_{xi}(S_{bi} - S_{nd})f_{X_{BH}} \quad (\text{mgN/L})$$

$$D_{p1} = D_{p1RBSO} + D_{p1BPO} \quad (\text{mgN/L})$$

### 5.4. Optimum internal recirculation (a)

$$a_{opt} = \frac{-B + \sqrt{B^2 + 4AC}}{2A} \quad (\emptyset)$$

where:

$$\begin{aligned} A &= \frac{DO}{2.86} \\ B &= N_c - D_{p1} + \frac{(1 + RAS) \cdot DO + RAS \cdot DO_{RAS}}{2.86} \\ C &= (1 + RAS) \cdot (D_{p1} - \frac{RAS \cdot DO_{RAS}}{2.86}) - RAS \cdot N_c \end{aligned}$$

### 5.5. Minimum (optimum) effluent nitrate concentration

$$N_{ne\_opt} = \frac{N_c}{a_{opt} + RAS + 1} \quad (\text{mgN/L})$$

**5.6. Effluent nitrate ( $N_{ne}$ )**

*if* ( $a < a_{opt}$ )

$$N_{ne} = \frac{N_c}{a + RAS + 1} \quad (\text{mgN/L})$$

*if* ( $a > a_{opt}$ )

$$N_{ne} = N_c - D_{p1} + \frac{a \cdot DO + RAS \cdot DO_{RAS}}{2.86} \quad (\text{mgN/L})$$

**5.7. Nitrogen gas production ( $FN_2$ )**

$$FN_2 = Q(N_c - N_{ne}) \quad (\text{kgN/d})$$

**5.8. Oxygen recovered by denitrification and total oxygen demand corrected**

$$FO_d = 2.86 \cdot Q \cdot (N_c - N_{ne}) \quad (\text{kgO/d})$$

$$FO_t = FO_c + FO_n - FO_d \quad (\text{kgO/d})$$

**5.9. Effluent alkalinity**

$$alk_e = alk_i + 3.57 \cdot (N_{obi} - (N_s - N_{oupi})) - 7.14 \cdot N_c + 2.86 \cdot (N_c - N_{ne}) \quad (\text{mgCaCO}_3/\text{L})$$