**UNESCO-IHE INSTITUTE FOR WATER EDUCATION**
**Department of Urban Water and Sanitation**

**Conventional Wastewater treatment module**

**Question set for video conference 26 February to 2 March, 2018**

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Note: **(1) Equations you require for doing the some of questions are given in a fact sheet on pages 5 and 6 of this question paper.**
(2) When doing these questions, you must write down you calculations and working in the necessary detail so that the examiners can see HOW you obtained your answers. This is for your own protection in case you make a numerical error. Simply writing down the formula and the answer, even if correct, will not attract marks. The DEMONSTRATION of your understanding of the problem and its solution will attract marks.

**QUESTION 1 - Primary settling and wastewater characterization** **Total 42 marks**

A wastewater treatment plant (WWTP) is to be designed for the flow weighted average concentrations measured in the raw and settled wastewater (WW) as follows:

***Table 1:*** *Tests on wastewater*

| **Tests on wastewater** | **COD** | **VFA** | **TOC** | **TKN** | **FSA** | **TP** | **OP** | **TSS** | **ISS** |
|---|---|---|---|---|---|---|---|---|---|
| Unfiltered (Raw WW) | 750 | -- | 250.8 | 60.0 | -- | 12.0 | - | 416 | 60.0 |
| Supernatant (Settled WW) | 475 | -- | 158.8 | 50.0 | -- | 9.60 | - | 189 | 15.0 |
| 0.45 μm membrane filtered | 210 | 50 | 71.9 | 44.0 | 39.1 | 8.23 | 7.28 | - | - |

(COD= Chemical Oxygen Demand in mgCOD/ℓ; VFA =Volatile Fatty acids in mgCOD/ℓ; TOC=Total Organic Carbon in mgC/ℓ; FSA=Free and saline ammonia in mgFSA-N/ℓ; TP= total Phosphorus in mgTP-P/ℓ; OP=Ortho-phosphate in mg$PO_4^{3-}$-P/ℓ; TSS= total suspended solids in mgTSS/ℓ, ISS= inorganic suspended solids in mgISS/ℓ).

The raw and settled wastewaters (WWs) above were each treated in a long sludge age activated sludge (AS) system and the following filtered effluent concentrations were measured -

| ***Table 2:*** *Tests on effluents* | COD | TOC | TKN | FSA | $NO_3$-N | TP | OP | TSS | ISS |
|---|---|---|---|---|---|---|---|---|---|
| Raw WW AS system | 45 | 15.0 | 1.48 | 0.38 | 40.8 | 7.54 | 7.54 | 0.0 | 0.0 |
| Settled WW AS system | 45 | 15.0 | 1.95 | 0.85 | 37.4 | 6.94 | 6.94 | 0.0 | 0.0 |

If the (i) raw WW flow is 25.0 Mℓ/d (125 000 person equivalent),
(ii) primary settling tank (PST) under flow is 0.5% of the raw WW influent flow,
(iii) unbiodegradable particulate organic (UPO) COD fraction ($f_{S'up}$) of the raw and settled WWs are 0.133 and 0.021 respectively,
(iv) the COD/VSS ($f_{cv}$), TOC/VSS ($f_c$), N/VSS ($f_n$) and P/VSS ($f_p$) ratios of the influent UPO are 1.481 mgCOD/mgVSS, 0.518 mgC/mgVSS, 0.100 mgN/mgVSS and 0.025 mgP/mgVSS, respectively....

(1.1) Calculate by mass balance the raw WW, settled WW and primary sludge (PS) COD, TKN, TP, TOC, TSS, VSS and ISS concentrations and fluxes. Write your results in a Table as shown in Table 3 below:

Table 3: Mass balance over primary settling tank (PST).

| | Raw WW | | Settled WW | | Primary Sludge | |
|---|---|---|---|---|---|---|
| Flow - Mℓ/d | | | | | | |
| | Conc | Flux | Conc | Flux | Conc | Flux |
| Units | mg/ℓ | kg/d | mg/ℓ | kg/d | mg/ℓ | kg/d |

(14)

(1.2) Characterize the raw WW, settled WW and PS COD, TOC, TKN and TP concentrations into their biodegradable and unbiodegradable, soluble and particulate (BPO, UPO, BSO, USO) concentrations in the block diagram format; (21)

(1.3) What is the unbiodegradable particulate COD fraction of the PS? (1)

(1.4) Given the UPO $f_{cv}$, $f_C$ and $f_N$ mass ratios in (iv) above, calculate these ratios for the BPO in the PS by difference between the particulate organics (PO=BPO+UPO) and the UPO. Write your results in a Table as shown below.

***Table 4:*** *Composition of primary sludge organics*

| Primary Sludge | Units | PO | UPO | BPO |
|---|---|---|---|---|
| COD | mgCOD/ℓ | | | |
| VSS | mgVSS/ℓ | | | |
| TOC | mgC/ℓ | | | |
| OrgN | mgN/ℓ | | | |
| OrgP | mgP/ℓ | | | |
| $f_{cv}$ | mgCOD/mgVSS | | | |
| $f_c$ | mgC/mgVSS | | | |
| $f_n$ | mgN/mgVSS | | | |
| $f_p$ | mgP/mgVSS | | | |

(6)

**QUESTION 2 - Activated sludge system design** **Total 21 marks**

A 15 day sludge age activated sludge system is to be designed for treating the ***settled*** wastewater with influent COD, TKN and FSA concentrations given in Q1 above. Accepting the following design conditions:

(a) influent unbiodegradable soluble ($f_{S'us}$) and particulate ($f_{S'up}$) COD fractions are 0.095 and 0.021 respectively,

(b) effluent biodegradable COD concentration = zero and temperature = 16°C,

(c) average influent flow rate ($Q_i$) = 24.875 Mℓ/d from 125 000 people,

(d) reactor TSS concentration ($X_t$) = 4 500 mgTSS/ℓ.

Calculate for the system -

2.1 the influent fluxes in kg/d of total COD, biodegradable COD, unbiodegradable particulate VSS and ISS. (2)

2.2 masses of volatile (VSS) and total suspended solids (TSS) in the reactor in kg. (2)

2.3 volume of the reactor in $m^3$ and the nominal hydraulic retention time in h. (2)

2.4 waste flow rate in $m^3$/d and the mass of sludge (TSS) wasted per day (kg/d). (2)

2.5 active fraction with respect to VSS and TSS and the VSS/TSS ratio ($f_i$) of the AS. (3)

2.6 effluent TKN, FSA and nitrate concentrations for (a) no and (b) complete nitrification. (3)

2.7 mass of oxygen utilized per day in kgO/d and the oxygen utilization rate in the reactor in mgO/(ℓ.h) (a) excluding and (b) including nitrification. (2)

2.8 effluent TP concentration in mgP/ℓ (1)

2.9 Check you answers by doing a COD and N balance over the designed plant. (4)

**QUESTION 3 - Nitrification and Denitrification** **Total 20 marks**

Biological N removal needs to be included in the settled wastewater activated sludge system you designed for 16ºC in Q2 above. Accepting the following for nitrification and denitrification:

(i) Maximum specific growth rate of nitrifiers at 20ºC ($\mu_{Am20}$) = 0.45/d
(ii) Factor of safety on nitrification ($S_f$) = 1.25
(ii) Half saturation constant for nitrifiers ($K_{n20}$) = 1.0 mgN/ℓ
(iii) Endogenous respiration rate for nitrifiers ($b_{A20}$) = 0.04/d
(iv) $K_2$ denitrification rate at 20ºC in MLE ($K_{220}$) = 0.101 mg$NO_3$-N/(mgOHOVSS.d)
(v) Dissolved oxygen concentrations in the a and s recycles $O_a$ = 2 and $O_s$=1 mgO/ℓ.
(vi) Underflow s recycle ratio (s) = 1.0:1

3.1 For the factor of safety for nitrification ($S_f$) of 1.25, what is the maximum unaerated sludge mass fraction ($f_{xm}$), effluent ammonia concentration ($N_{ae}$), nitrification capacity ($N_c$) and oxygen demand for nitrification ($FO_n$)? (4)

3.2 From the COD block diagram for the settled WW from Q1, what is the readily biodegradable COD fraction with respect to the biodegradable COD ($f_{Sb's}=S_{bsi}/S_{bi}$)? (1)

3.3 Accepting a MLE N removal system with an anoxic mass fraction ($f_{x1}$) equal to 0.39, calculate the nitrification capacity ($N_c$) and denitrification potential of the primary anoxic reactor ($D_{p1}$) *NOTE: use the $D_{p1}$ equation on the fact sheet on Page 5, not Eq 5.49 in the text book on page 116 because the text book equation has typing errors.* (4)

3.4 What is the optimum a recycle ratio ($a_{opt}$)? If $a_{opt}$>6, set a to the maximum practical ratio of 6:1. With a set to ≤6, what is the effluent nitrate ($N_{ne}$) and TKN ($N_{te}$) concentrations and the % N removal from the settled wastewater?. (6)

3.5 If the primary anoxic sludge mass fraction ($f_{x1}$) were set at the maximum unaerated sludge mass fraction ($f_{xm}$), would the effluent nitrate concentration ($N_{ne}$) be lower? With $f_{x1}$ set to 0.39, what are the volumes of the anoxic and aerobic reactors? (2)

3.6 Calculate the oxygen recovered by denitrification, the total oxygen demand in the aerobic reactor and the oxygen utilization rate (OUR) in the aerobic reactor in mgO/(l.h). (3)

3.7 Check your calculations with a COD, N and TOD balance over the system. (15)

**QUESTION 4 - Biological excess P removal with denitrification in a University of cape Town (UCT) system. Total 47 marks**

**Table 4: Wastewater characteristics, system design specifications, UPO and AS characteristics and kinetic constants for the UCT NDEBPR system.**

| 1. Wastewater characteristics | Value | 2. System Design specifications | Value |
|---|---|---|---|
| Average dry aether flow (ADWF, Mℓ/d) | 24.875 | Sludge age (d) | 15 |
| Minimum wastewater temperature (ºC) | 16 | Reactor TSS concentration ($X_t$, gTSS/ℓ) | 4.5 |
| Unbiodeg soluble COD fraction ($f_{S'us}$) | 0.095 | DO concentration in a recycle ($O_a$, mgO/ℓ) | 2 |
| Unbiodeg particulate COD fraction ($f_{S'up}$) | 0.021 | DO concentration in s recycle ($O_s$, mgO/ℓ) | 1 |
| Frac bio COD that is Bio soluble ($f_{Sb's}$) | 0.393 | Nitrate conc in r or s recycle to anaerobic ($N_{nr}$, mgN/ℓ) | 0 |
| Influent ISS concentration ($X_{IOi}$, mg/ℓ) | 15.0 | r recycle ratio (r) relative to ADWF | 1:1 |
| DO concentration influent ($O_i$, mgO/ℓ) | 0.0 | s recycle ratio (s) relative to ADWF | 1:1 |
| Nitrate concentration influent ($N_{ni}$, mgN/ℓ) | 0.0 | Total anoxic sludge mass fraction ($f_{xdt}$) | 0.390 |
| | | Anaerobic mass fraction ($f_{xa}$) | 0.15 |
| | | Anaerobic reactor is divided into | 3 |
| | | Maximum a recycle ratio ($a_{prac}$) | 6 |
| 3. UPO and AS characteristics | Value | 4. Kinetic constants | Value |
| COD/VSS ratio ($f_{cv}$, mgCOD/mgVSS) | 1.481 | Max spec growth rate of ANOs at 20ºC ($\mu_{Am20}$ /d) | 0.45 |
| TKN/VSS ratio ($f_n$, mgN/mgVSS) | 0.100 | Safety factor for nitrification ($S_f$) | 1.25 |
| P/VSS ratio ($f_p$, mgP/mgVSS) | 0.025 | Half saturation concentration for ANO ($K_{n20}$) | 1.0 |
| Max PolyP content of PAOs ($f_{XBGPP}$, mgPolyP/mgPAOVSS) | 0.355 | | |

4.1 What are the fluxes of influent biodegradable COD obtained by the OHOs and PAOs? (6)

4.2 Calculate the OHO and PAO biomass and endogenous masses, the unbiodegradable particulate VSS and VSS ($X_v$) masses in the reactor. (5)

4.3 What is the biological excess P removal and the expected effluent P concentration? (2)

4.4 What is the oxygen demand ($FO_c$) by the OHOs and PAOs? (2)

4.5 Check you calculations with a COD balance. (3)

4.6 Calculate the ISS and TSS masses in the reactor taking due account of the ISS contribution by the OHO and PAOs. What is the VSS/TSS ratio of the reactor sludge. (4)

4.7 What are the anaerobic, anoxic and aerobic reactor volumes in $m^3$? (3)

4.8 What is the waste flow rate and flux of TSS wasted per day? (2)

4.9 Calculate the effluent FSA and TKN concentrations for the UCT system. What is the nitrification oxygen demand ($FO_n$)? (3)

4.10 When the primary anoxic reactor has an anoxic mass fraction ($f_{x1}$) equal to 0.39, calculate the nitrification capacity ($N_c$) and denitrification potential ($D_{p1}$). For the same $O_a$, $O_s$ and s as in Q3 above, calculate the optimum a-recycle ratio. If $a_{opt}$>6, set a to the maximum practical ratio of 6:1. With a set to ≤6, what is the effluent nitrate ($N_{ne}$) and TKN ($N_{te}$) concentrations and the % N removal from the settled wastewater? (9)

4.11 How much oxygen is recovered by denitrification? What is the total oxygen demand and OUR in the aerobic reactor?. (3)

4.12 Check your calculations with a TOD balance over the system. (10)

4.13 Compare the effluent COD, TKN, $NO_3$ and TP concentrations, reactor volume, oxygen demand, OUR, waste flow rate, and flux of TSS wasted per day in the MLE (Q3) and UCT (Q4) systems. (5)

**QUESTION 5 - Biological excess P removal with denitrification (Johannesburg, JHB system)**
**Total 47 marks**

For the settled WW, design a JHB NDEBPR system accepting that the SRT ($R_s$) and anaerobic mass fraction ($f_{xa}$) and other design specifications of the JHB system are the same as the UCT system as given in Table 4 above.

Because WW characteristics and SRT ($R_s$) and anaerobic mass fraction ($f_{xa}$) of JHB system are the same as the UCT system, the EBPR calculations are the same as for the UCT system. So the answers from Q4.1 to 4.6 apply to th JHB system. However, the denitrification changes.......

5.1 If the total anoxic mass fraction is set at the value given in Table 4, is the total unaerated mass fraction less than the maximum allowed to ensure nitrification? Calculate the effluent FSA, TKN and nitrification capacity ($N_c$) for the JHB system. What is the nitrification oxygen demand ($FO_n$)? (8)

5.2 What is the denitrification potential ($D_{pp}$). For the same $O_a$, $O_s$ and s as in Table 4, calculate the optimum a-recycle ratio. If $a_{opt}$>$a_{prac}$, set a to the maximum practical ratio of $a_{prac}$. With a set to ≤ $a_{prac}$, what are the primary and secondary anoxic sludge mass fractions, the effluent nitrate ($N_{ne}$) and TKN ($N_{te}$) concentrations and the % N removal from the settled wastewater? (23)

5.3 What are the anaerobic, primary and secondary anoxic and aerobic reactor volumes in $m^3$?(9)

5.4 What is the waste flow rate and flux of TSS wasted per day? (2)

5.5 How much oxygen is recovered by denitrification? What is the total oxygen demand and OUR in the aerobic reactor? What is the alkalinity and expected pH of the effluent? (6)

5.6 Compare in a table the effluent COD, TKN, $NO_3$, and TP concentrations, reactor volume, oxygen demand, OUR, waste flow rate and flux TSS wasted for the UCT (Q4) and JHB (Q5) systems. (10)

5.7 At the same SRT, unaerated and anaerobic mass fractions as the JHB system, will a UCT system perform better than the JHB system in (i) P removal and (ii) N removal? Explain the differences. (10)

5.8 How does the volume of the JBH system compare with that of UCT system at the same SRT? Do you expect the balanced SRT of the JHB system to be longer or shorter than the UCT system? How does this difference in balanced SRT of the JHB and UCT systems affect the volumes of the two systems?

............................................End of Question Set........................................................................

GAEkama - 16 Jan., 2018.

**FACT SHEETS**

$X_{Ii} = f_{S'up} S_{ti}/f_{cv}$ (mgVSS/ℓ); $N_{oupi} = f_n X_{Ii}$ (mgN/ℓ); $S_{bi} = (1 - f_{S'up} - f_{S'us}) S_{ti}$ (mgCOD/ℓ)

$FS_{ti} = f_{S'us} FS_{ti} + FO_c + f_{cv} MX_v / R_s$ mgCOD/d ; $S_b = \dfrac{(1 + b_H R_s)}{Y_{Hv} R_s K_v}$ mgCOD/ℓ;

$X_{BH} = \dfrac{Y_{Hv}(S_{bi} - S_b) R_s}{(1 + b_H R_s) R_{hn}}$ mgVSS/ℓ; $X_{EH} = f_H b_H R_s X_{BH}$ mgVSS/ℓ; $X_I = X_{Ii} \dfrac{R_s}{R_{hn}}$ mgVSS/ℓ

$X_{Io} = X_{Ioi} \dfrac{R_s}{R_{hn}} + f_{iOHO} X_{BH}$ mgISS/ℓ; $X_t = \dfrac{X_v}{f_i}$ mgTSS/ℓ;

$O_c = (1 - f_{cv} Y_{Hv}) \dfrac{(S_{bi} - S_b)}{R_{hn}} + f_{cv}(1 - f_H) b_H X_{BH}$ mgO/(ℓ.d)

$f_{av} = \dfrac{X_{BH}}{X_v}$ $f_{at} = \dfrac{X_{BH}}{X_t}$ $OUR_c = \dfrac{dO_c}{dt} = (1 - f_{cv} Y_h) \dfrac{K_m S_b}{(K_s + S_b)} X_a + f_{cv}(1 - f) b_h X_a$

$$FO_c = FS_{bi} \left[ (1 - f_{cv} Y_{Hv}) + f_{cv}(1 - f_H) b_H \frac{Y_{Hv} R_s}{(1 + b_H R_s)} \right]$$ (mgO/d)

Note that in these $FO_c$ and $MX_v$ equations, $S_b$=0.

$$MX_v = FS_{ti} \left[ (1 - f_{S'up} - f_{S'us}) \frac{Y_{Hv} R_s}{(1 + b_H R_s)} (1 + f_H b_H R_s) + \frac{f_{S'up} R_s}{f_{cv}} \right]$$ (mgVSS)

$$MX_t = FS_{ti} \left[ (1 - f_{S'up} - f_{S'us}) \frac{Y_{Hv} R_s}{(1 + b_H R_s)} (1 + f_H b_H R_s + f_{iOHO}) + \left( \frac{f_{S'up}}{f_{cv}} + \frac{X_{Ioi}}{S_{ti}} \right) R_s \right]$$ (mgTSS)

$N_s = f_n \dfrac{MX_v}{R_s Q_i}$ (mgOrgN-N/ℓ); $N_{ae} = N_{te} - N_{ouse}$ (mgFSA-N/ℓ);

$N_{te} = N_{ti} - N_s$ (mgTKN-N/ℓ) and $N_{ne} = 0$ (mg$NO_3$-N/ℓ) for no nitrification.

$N_{te} = N_{ouse} + N_{ae}$ (mgTKN-N/ℓ) and $N_{ne} = N_{ti} - N_s - N_{te}$ (mg$NO_3$-N/ℓ) for nitrification.

Oxygen required for nitrification = 4.57 mgO/mgFSA-N nitrified.

$P_s = f_p \dfrac{MX_v}{R_s Q_i}$ mgP/ℓ; $P_{te} = P_{ti} - P_s$ (mgP/ℓ)

where $f_{cv}$ = 1.48 mgCOD/mgVSS; $f_{EH}$ = 0.20; $b_H$ = 0.24/d at 20°C; $f_n$ = 0.10 mgN/mgVSS
$Y_{Hv}$ = 0.45 mgVSS/mgCOD; $K_v$ = 0.07 ℓ/(mgVSS.d); $f_p$ = 0.025 mgP/mgVSS;
$f_{iOHO}$=0.15 mgISS/mgOHOVSS;

$N_{ae} = K_{nT}/(S_f - 1)$ mgFSA-N/ℓ; $N_{ae} = \dfrac{K_{nT}(b_{AT} + 1/R_s)}{\mu_{AmT}(1 - f_{xm}) - (b_{AT} + 1/R_s)}$ mgFSA-N/

$f_{xm} = 1 - S_f(b_{AT} + 1/R_s)/\mu_{AmT}$ ; $N_c = N_{ti} - N_s - N_{te}$ (mg$NO_3$-N/ℓ);

$$D_{p1} = S_{bi}\left[\frac{f_{Sb's}(1 - f_{cv}Y_{Hv})}{2.86} + K_2 f_{x1}\frac{Y_{Hv}R_s}{(1 + b_H R_s)}\right]$$

$a_{opt} = \left[-B + \sqrt{B^2 + 4AC}\right]/(2A)$ mg$NO_3$-N/ℓ;

$A = O_a/2.86 \quad B = N_c - D_{p1} + [(s+1)O_a + sO_s]/2.86 \quad C = (s+1)[D_{p1} - sO_s/2.86] - sN_c$ ;

$N_{ne} = N_c/(a+s+1)$ for $a \leq a_{opt}$; $N_{ne} = N_c - D_{p1} + (aO_a + sO_s)/2.86$ for $a \geq a_{opt}$

$FO_t = FO_c + FO_n - FO_d$ mgO/d ; $S'_{bsi} = S_{bsi} - 8.6\, r\, N_{nr} - 3O_i$

$$S_{bsfN} = \frac{\dfrac{S'_{bsi}}{(1+r)}}{\left[1 + \dfrac{K_{ferm}\dfrac{f_{xa}}{N}\dfrac{MX_{BH}}{Q_i}}{(1+r)}\right]^N} \qquad \frac{MX_{BH}}{Q_i} = [S_{bi} - (S'_{bsi} - (1+r)S_{bsfN})]\frac{Y_{Hv}R_s}{1 + b_H R_s}$$

$K_{ferm}$ = 0.06 ℓ/(mgOHOVSS.d)

$$MX_{BH} = FS_{biOHO}\left[\frac{Y_{Hv}R_s}{(1 + b_H R_s)}\right] \qquad MX_{EH} = f_H b_H R_s MX_{BH}$$

; $f_H$=0.20; $b_H$ = 0.24/d

$$MX_{BG} = FS_{biPAO}\left[\frac{Y_{Hv}R_s}{(1 + b_G R_s)}\right] \qquad MX_{EG} = f_G b_G R_s MX_{BG}$$

; $f_G$=0.25; $b_G$ = 0.04/d

$$MX_{Io} = FX_{Ioi}R_s + f_{iOHO}MX_{BH} + (f_{iPAO} + 3.286 f_{XBGPP})MX_{BG}$$

$f_{iOHO} = f_{iPAO}$ = 0.15 mgISS/mgOHOVSS or mgISS/mgPAOVSS ;
$f_{XBGPP}$ = 0.355 mgPolyphosphate-P/mgPAOVSS

**For denitrification in NDEBPR systems:**

$$D_{p1} = \frac{(1 - f_{cv}Y_{Hv})}{2.86}(1+r)S_{bsfN} + \frac{FS_{biOHO}}{Q_i}K'_{2T} f_{x1}\frac{Y_{Hv}R_s}{(1 + b_{HT}R_s)}$$

$K'_{220}$ = 0.255 mg$NO_3$-N/(mgOHOVSS.d) ($\theta_{K'2}$ =1.080);
$K'_{320}$ = 0.114 mg$NO_3$-N/(mgOHOVSS.d) ($\theta_{K'3}$ =1.029);

$a_{opt}$ in UCT system as for MLE system.
$D_{pp}$ in JHB system as per $D_{p1}$ equation above for UCT system.
$N_{ne}$ for UCT and JHB as for MLE system.

...........PTO for JHB system $a_{opt}$, $f_{x1}$ and $f_{x3}$ equations

$a_{opt}$, $f_{x1}$ and $f_{x3}$ in JHB system........

$$a_{opt} = \frac{-B + \sqrt{(B^2 + 4AC)}}{2A}$$

$$A = \frac{O_a}{2.86}$$

$$B = N_c - D_{pp} + (1+s)\frac{O_a}{2.86} + s\frac{K'_{2T}}{K'_{3T}}\frac{O_s}{2.86}$$

$$C = (1+s)\left[D_{pp} - s\frac{K'_{2T}}{K'_{3T}}\frac{O_s}{2.86}\right] - s\frac{K'_{2T}}{K'_{3T}}N_c$$

$$f_{x1} = \frac{\left[\frac{N_c}{(a+s+1)} + \frac{O_a}{2.86}\right]a - (1+s)\,S_{bsfN}\frac{(1-f_{cv}Y_H)}{2.86}}{K'_{2T}\frac{Y_H R_s}{(1+b_{HT}R_S)}\frac{FS_{bOHO}}{Q_i}}$$

$$f_{x3} = \frac{\left[\frac{N_c}{(a+s+1)} + \frac{O_s}{2.86}\right]s}{K'_{3T}\frac{Y_H R_s}{(1+b_{HT}R_S)}\frac{FS_{bOHO}}{Q_i}}$$