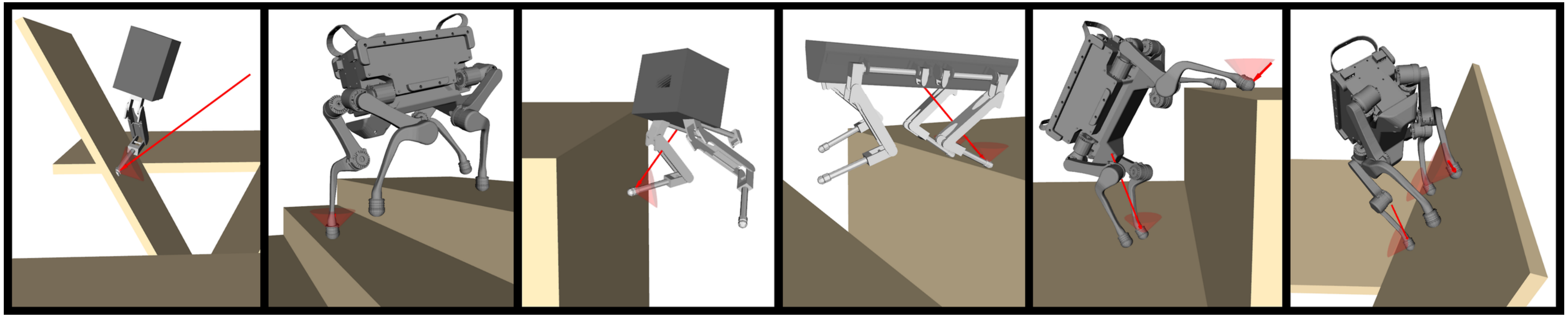


Robotic and Automation Letters (RA-L) · 2018

Gait and Trajectory Optimization for Legged Systems through Phase-based End-Effector Parameterization

Alexander W. Winkler, Dario Bellicoso, Marco Hutter, Jonas Buchli

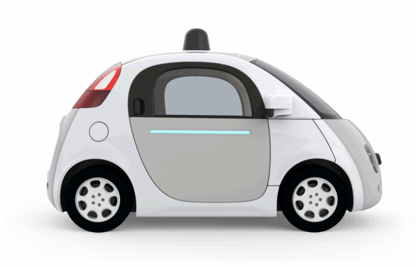
A D R L

RSL Robotic Systems Lab

www.awinkler.me

Why legged machines?


VS




Source:
ANYbotics, Anymal bear, "Image: https://www.anybotics.com/anymal", 2018; Boston Dynamics, Atlas, "Image: https://www.bostondynamics.com/atlas", 2016; Italian Institute of Technology, HyQ2Max "Image: https://dls.iit.it/robots/hyq2max, 2018; Alphabet Waymo, Firefly car, "Image: https://waymo.com", 2016, DJI, Phantom 2 drone, "Image: https://www.dji.com/phantom-2", 2016

Agility ...vs rolling

• traverse rubble in earthquake • reach trapped humans • climb stairs •...

Source: https://www.youtube.com/watch?v=NX7QNWEGcNIa

Strength ...vs flying

• carry heavy payload • open heavy doors • rescue humans • ...

Source: https://www.youtube.com/watch?v=arCOVKxGy9E

Optimization-based Motion Planning

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Task

Robot · kinematic · dynamic
Goal · position · velocity · duration ·
Environment · terrain · friction · ...

Direct Method
Collocation

Mathematical Optimization Problem

$$\min_{\mathbf{w}} a(\mathbf{w}) \quad \text{subject to} \quad \mathbf{b}(\mathbf{w}) = \mathbf{0}, \quad \mathbf{c}(\mathbf{w}) \geq \mathbf{0}$$

off-the-shelf
NLP Solver

Desired Motion-Plan

$\mathbf{x}(t), \mathbf{u}(t)$

Tracking
Controller

Actuator Commands

force · torque

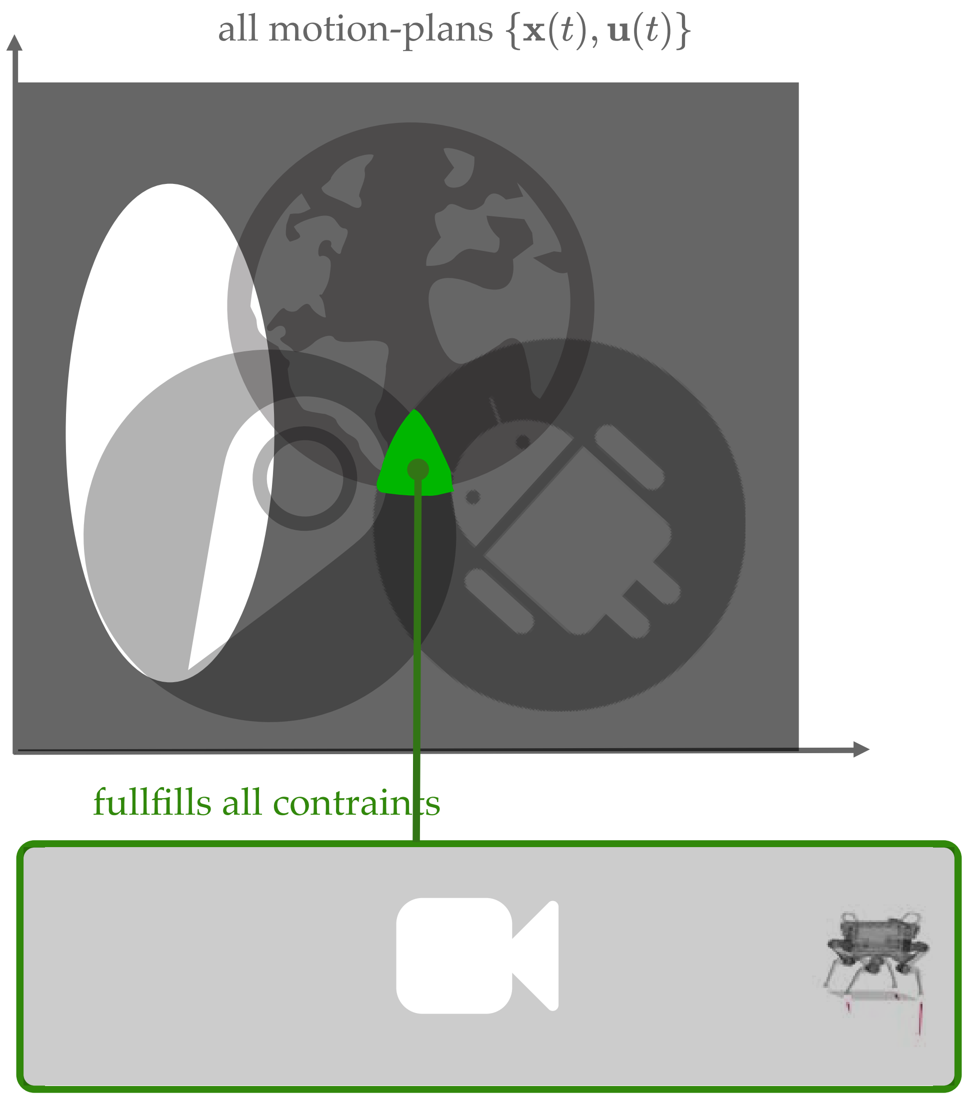
Why integrated motion-planning?

Mathematical Optimization Problem

predefined / "factorized":

- Contact schedule
- CoM height (no jumps)
- Body orientation (horizontal)
- Foothold height (flat ground)

restrict search space



Towards integrated motion-planning

keeping search-space as open as possible

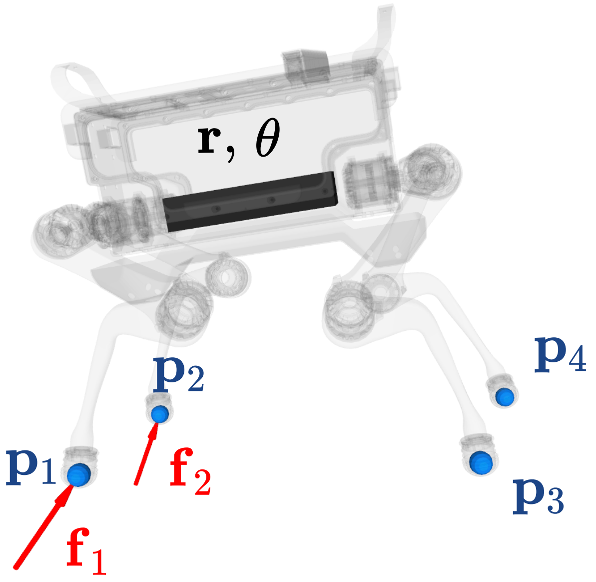
find $\mathbf{r}(t) \in \mathbb{R}^3$ (CoM)

$\theta(t) \in \mathbb{R}^3$ (Base orientation)

for every foot $i \in \{1, \ldots, n_{ee}\}$:

$\mathbf{p}_i(t) \in \mathbb{R}^3$ (Foot position)

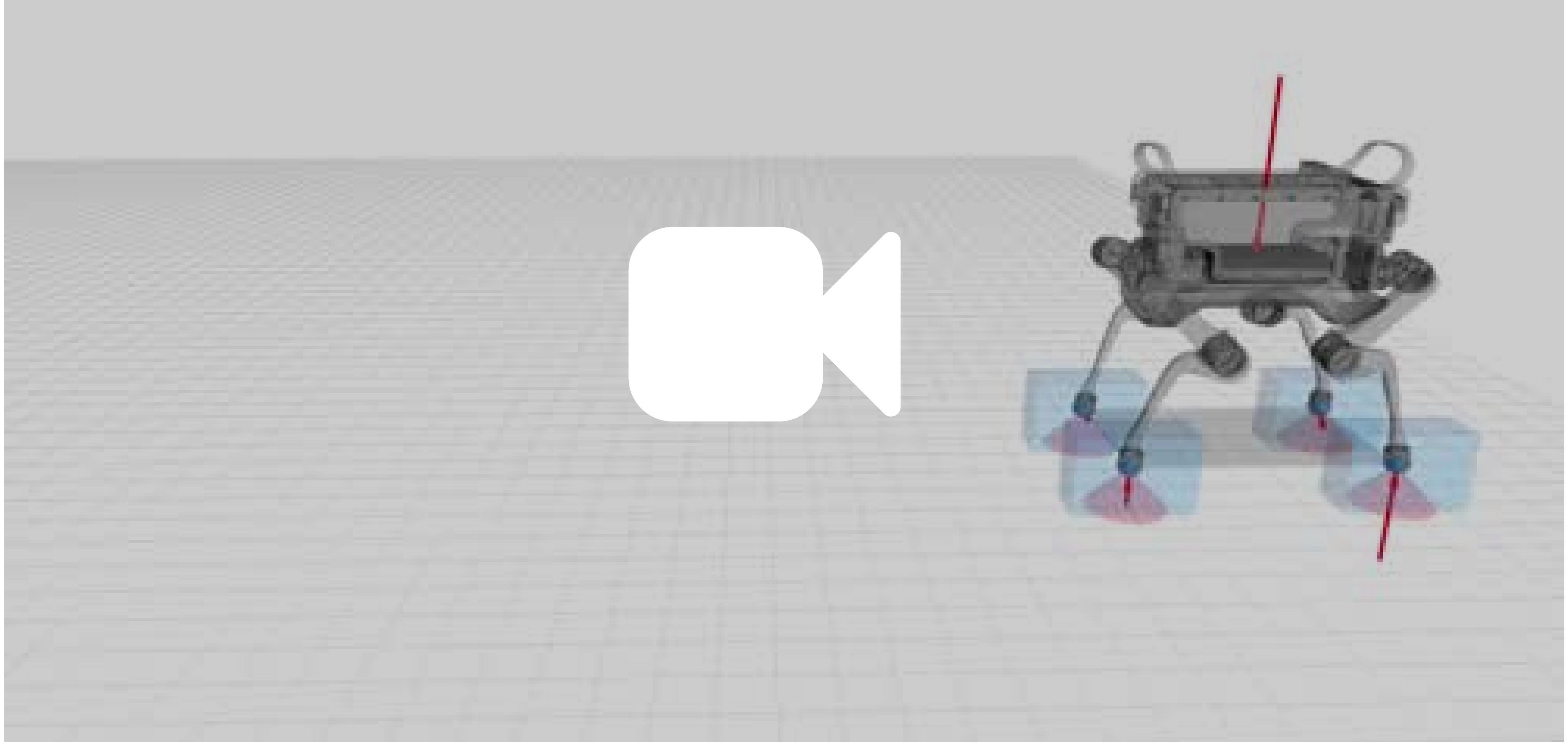
$\mathbf{f}_i(t) \in \mathbb{R}^3$ (Foot force)



Dynamic Model

Single Rigid Body · Newton-Euler Equations

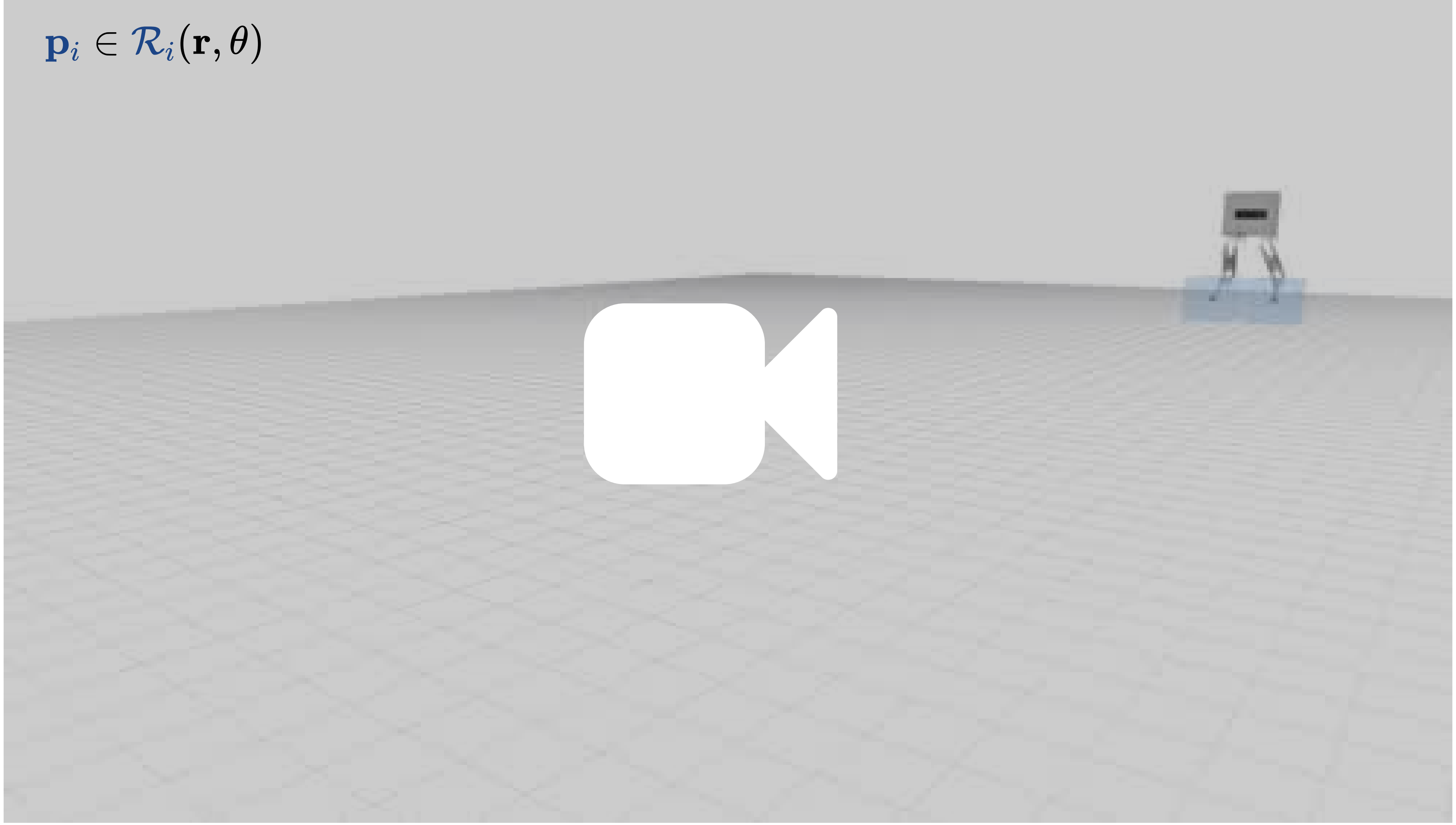
$$
\begin{aligned}
m\,\ddot{\mathbf{r}} &= \textstyle\sum_{i=1}^{4} \mathbf{f}_i - m\mathbf{g} \\
\mathbf{I}(\theta)\,\dot{\omega} + \omega \times \mathbf{I}(\theta)\omega &= \textstyle\sum_{i=1}^{4} \mathbf{f}_i \times (\mathbf{r} - \mathbf{p}_i)
\end{aligned}
$$



Kinematic Model

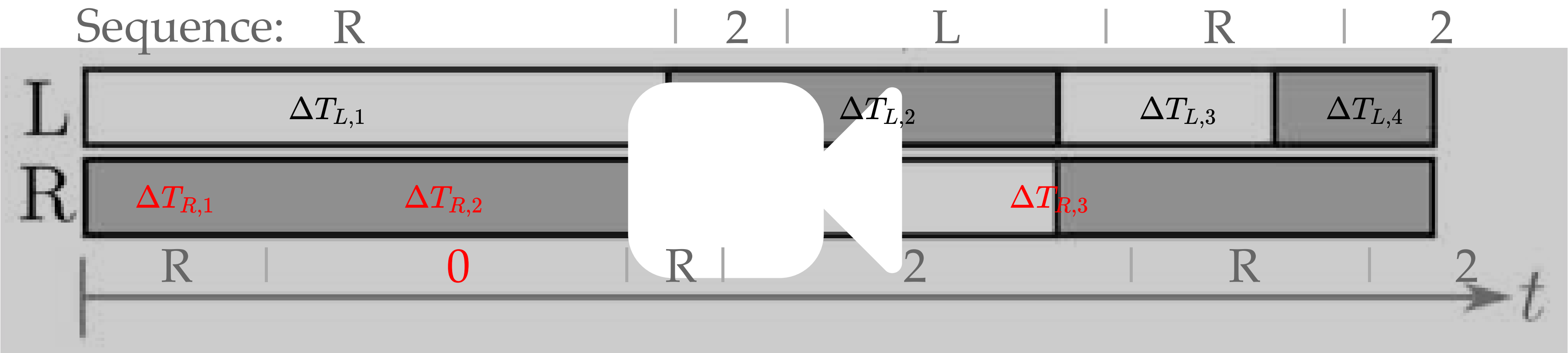
Range-of-Motion Box ≈ Joint limits

$$\mathbf{p}_i \in \mathcal{R}_i(\mathbf{r}, \theta)$$



Gait Optimization

without Integer Programming

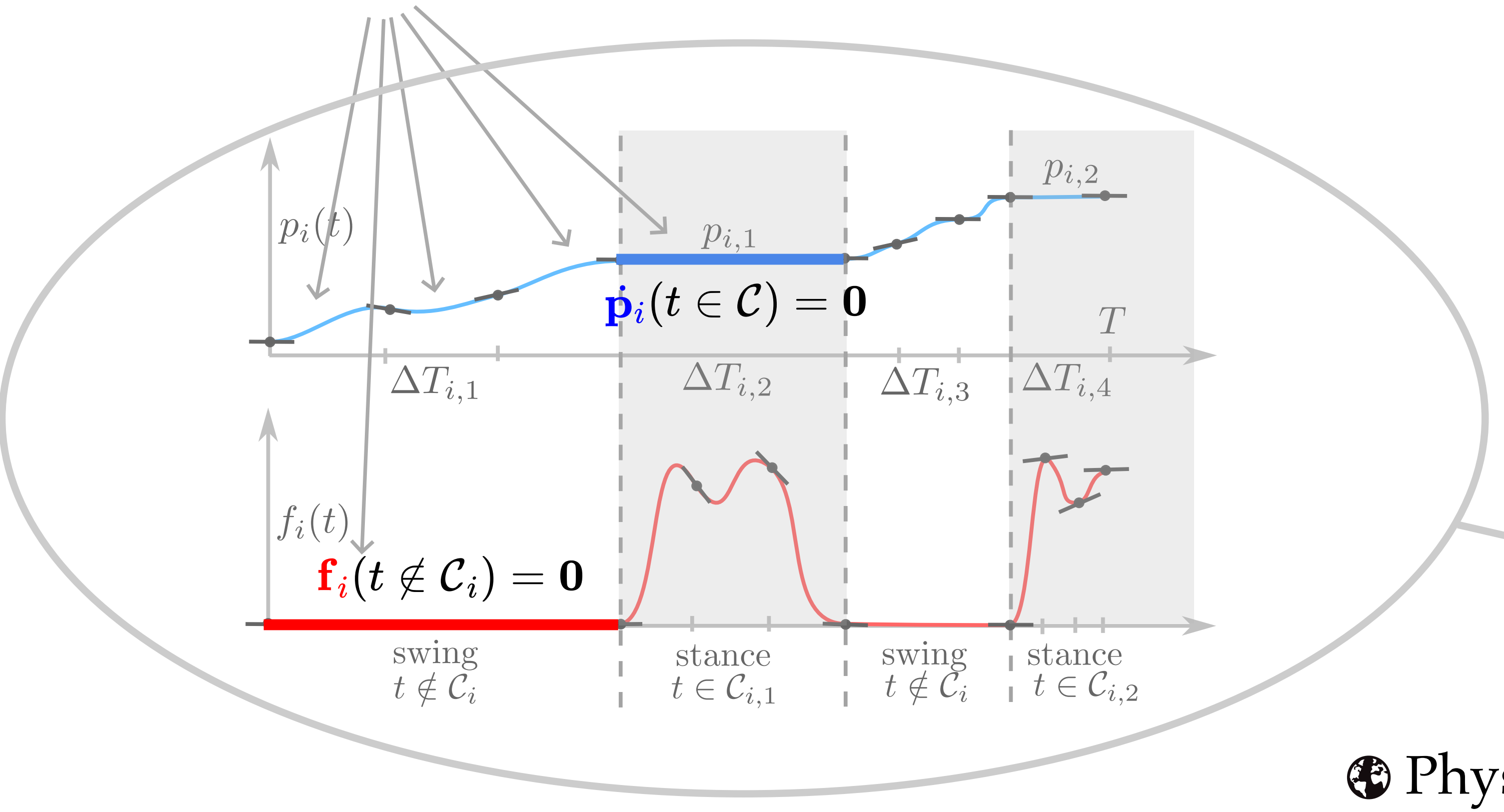


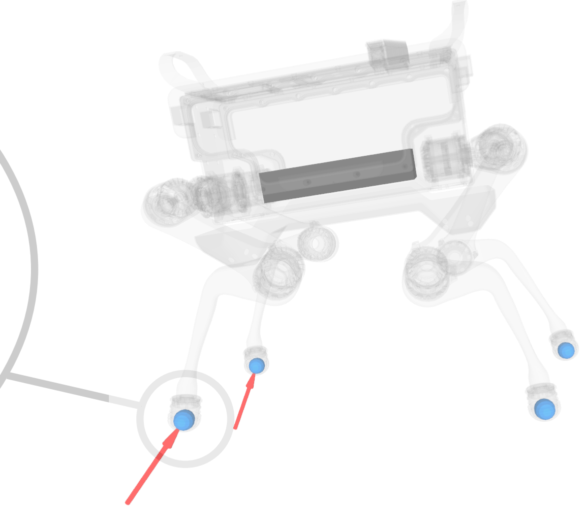
individual foot *always* alternates between swing and stance

.... gait defined by *continuous* phase-durations ΔT_i

Phase-Based End-Effector *Parameterization*

Know if polynomial belongs to swing or stance phase





Physical Restrictions

- Foot $\mathbf{p}_i(t)$ cannot move while standing
- Forces $\mathbf{f}_i(t)$ cannot exist while swinging

Terrain constraints

Given:

- height map $h(x, y) \in \mathbb{R}$
- normals $\mathbf{n}(x, y) \in \mathbb{R}^3$
- tangents $\mathbf{t}(x, y) \in \mathbb{R}^3$

$t \in \mathcal{C}$

Foot can only stand *on* terrain

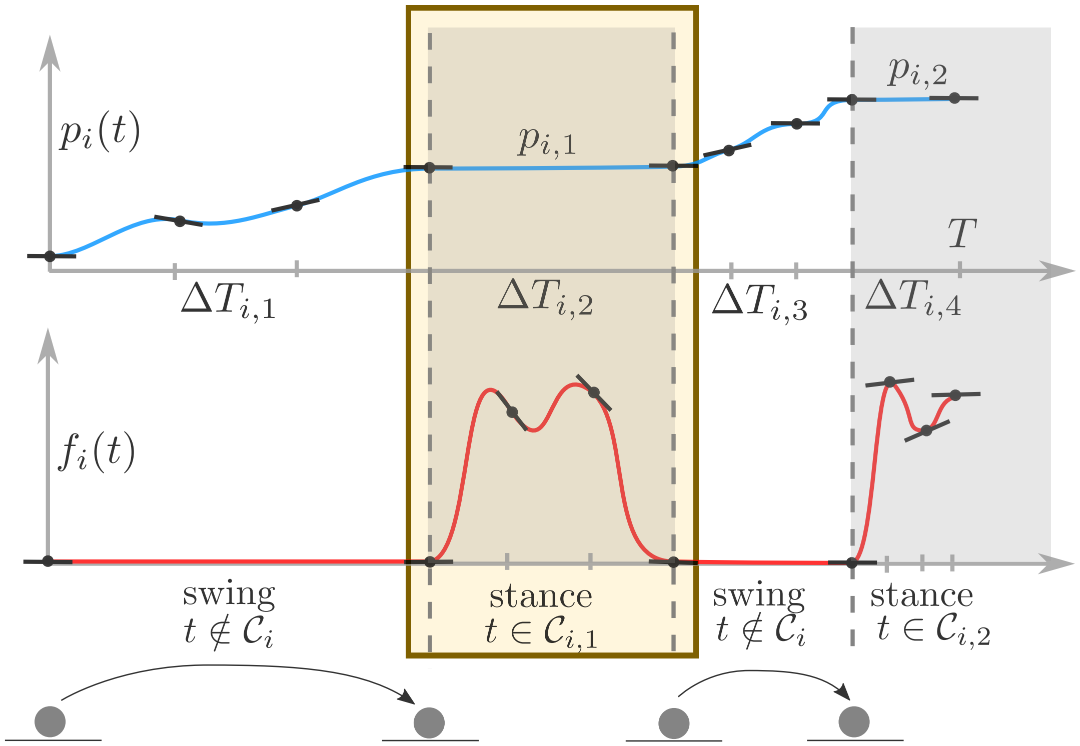
$p^z_{i,s} = h(p^x_{i,s}, p^y_{i,s})$

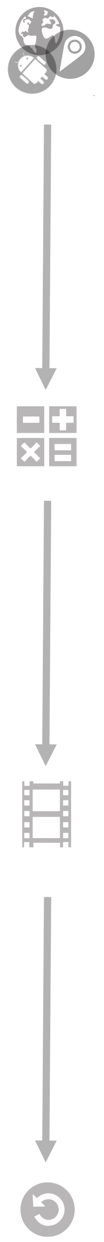
Forces can only push

$\mathbf{f}_i(t) \cdot \mathbf{n}(p^x_{i,s}, p^y_{i,s}) \geq 0$

Forces inside friction pyramid

$|\mathbf{f}_i(t) \cdot \mathbf{t}(p^x_{i,s}, p^y_{i,s})| < \mu \mathbf{f}_i(t) \cdot \mathbf{n}(p^x_{i,s}, p^y_{i,s})$





Summary

open-sourced software

towr

Dynamic model (accuracy)	SRBD
Optimized components	
Base motion	6D
Footholds	3D
Step sequence	✓
Step timing	✓
Contact force	✓
Swingleg motion	✓
Number of steps	✗
Difficulty of shown task	
Enforce friction cone	✓
Non-flat terrain	✓
Inclined terrain	✓
Flight-phases	✓
Sliding contacts	✗
Impulses	✗
Number of end-effectors	1, 2, 4
Computation Time 1s-horizon, 4-footstep motion for a quadruped	100 ms

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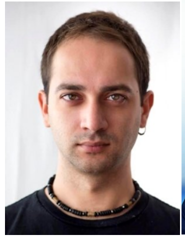
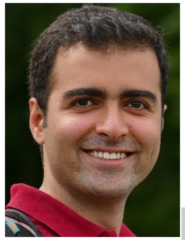

D. Bellicoso


M. Hutter


J. Buchli


F. Farshidian


D. Pardo


M. Neunert

Software

≡ ethz-adrl/**towr**

Trajectory Optimizer for Walking Robots using IPOPT

C++ ★ 11 7

≡ ethz-adrl/**ifopt**

Eigen-based Interface to Nonlinear Programming Solvers IPOPT and SNOPT

C++ ★ 14 6

≡ leggedrobotics/**xpp**

Visualization of Motions for Legged Robots in ros-rviz

C++ ★ 12 7

```
sudo apt-get install ros-kinetic-xpp
```

ADRL

RSL Robotic Systems Lab

robotics Swiss National Centre of Competence in Research

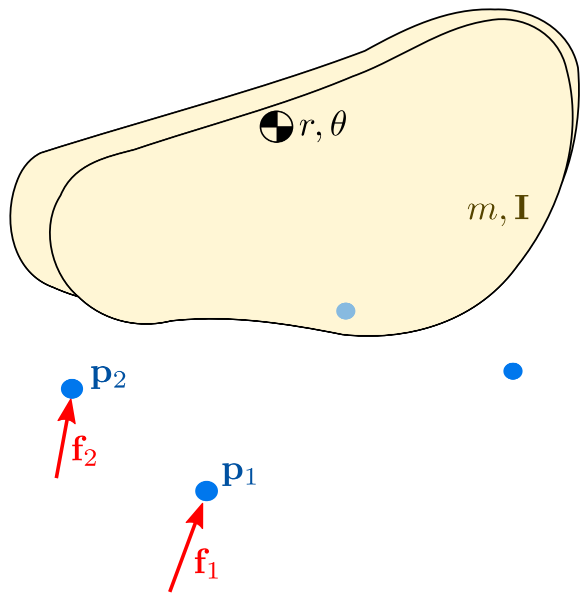
These slides and more at www.awinkler.me

Additional Material:

Centroidal Dynamics ⇒ Single Rigid Body Dynamics

Newton-Euler Equations

$$\mathbf{A}(\mathbf{q})\ddot{\mathbf{q}} + \dot{\mathbf{A}}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} = \begin{bmatrix} \sum_{i=1}^{4} \mathbf{f}_i - m\mathbf{g} \\ \sum_{i=1}^{n_i} \mathbf{f}_i \times (\mathbf{r}(\mathbf{q}) - \mathbf{p}_i(\mathbf{q})) \end{bmatrix}$$

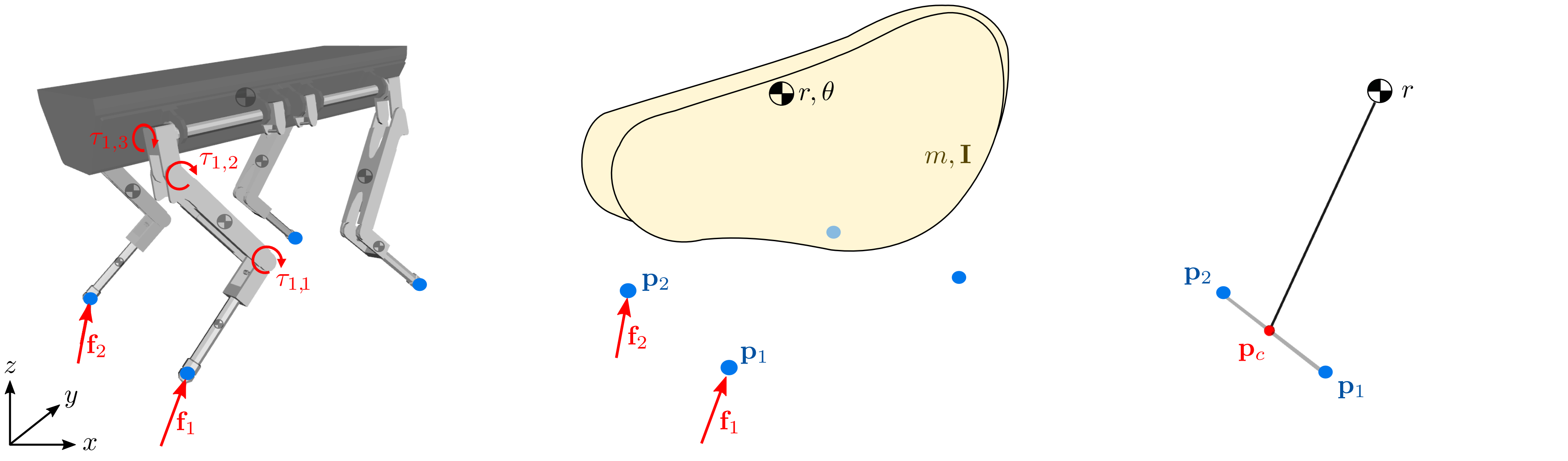


+ **Assumption A2**: *Momentum produced by the joint velocities is negligible.*
+ **Assumption A3**: *Full-body inertia remains similar to the one in nominal configuration.*

$$m\ddot{\mathbf{r}} = \sum_{i=1}^{4} \mathbf{f}_i - m\mathbf{g}$$

$$\mathbf{I}(\theta)\dot{\omega} + \omega \times \mathbf{I}(\theta)\omega = \sum_{i=1}^{4} \mathbf{f}_i \times (\mathbf{r} - \mathbf{p}_i)$$

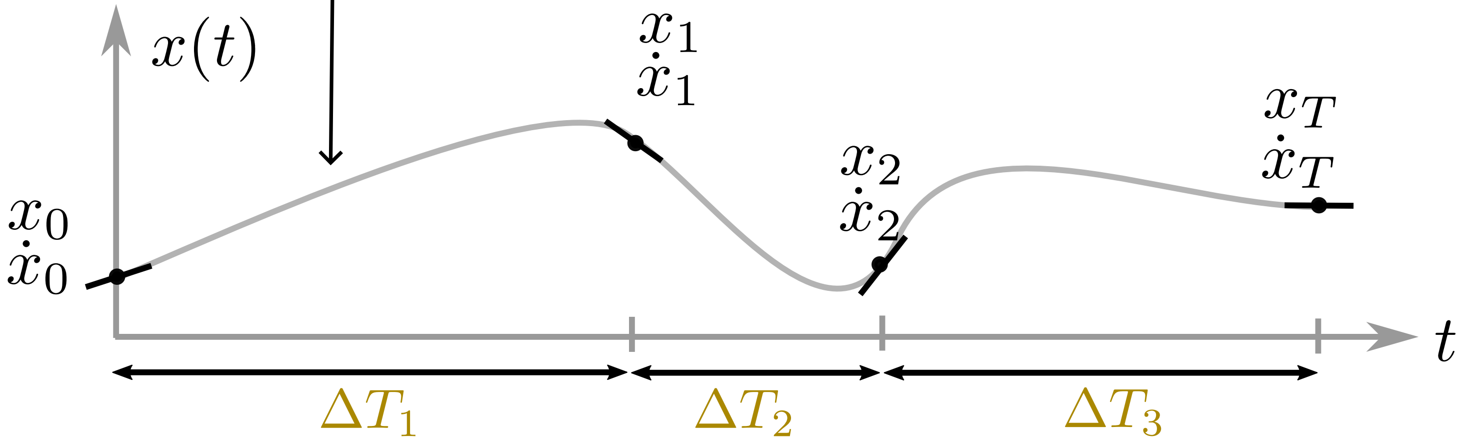
Rigid Body Dynamics (RBD) Single Rigid Body Dynamics (SRBD) Linear Inverted Pendulum (LIPM)



$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}(t), \mathbf{u}(t))$	$\mathbf{x}$ *(pos)*	$\mathbf{u}$	**Assumptions**
Rigid Body Dynamics (RBD)	$\mathbf{q}_b, \mathbf{q}_j$	$\tau, \mathbf{f}_i$	A1
Centroidal Dynamics (CD)	$\mathbf{q}_b, \mathbf{q}_j$	$\mathbf{f}_i$	A1
Single Rigid Body Dynamics (SRBD)	$\mathbf{r}, \theta, \mathbf{p}_i$ r_x, r_y	$\mathbf{f}_i$ $\mathbf{p}_c$	A1, A2, A3
Linear Inverted Pendulum (LIP)			A1, A2, A3, A4, A5, A6

Cubic-Hermite Spline for $f_{\{x,y,z\}}(t)$, $p_{\{x,y,z\}}(t)$

$$x(t) = \underbrace{a_0}_{x_0} + \underbrace{a_1}_{\dot{x}_0} t + \underbrace{a_2}_{-\Delta T_1{}^{-2}[3(x_0 - x_1) + \Delta T_1(2\dot{x}_0 + \dot{x}_1)]} t^2 + \underbrace{a_3}_{\Delta T_1{}^{-3}[2(x_0 - x_1) + \Delta T_1(\dot{x}_0 + \dot{x}_1)]} t^3$$



$$\mathbf{w}_j = \{x_0, \dot{x}_0, \Delta T_1, x_1, \dot{x}_1, \Delta T_2, x_2, \dot{x}_2, \Delta T_3, x_T, \dot{x}_T\}$$

Icons from: https://www.behance.net/gallery/22478593/ICONS-Pack-2