ICRA 2018 Workshop on:

Dynamic Legged Locomotion in Realistic Terrains

Simple yet powerful walking gait generation based on the Divergent Component of Motion (DCM)

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DLR

Our walking reference generation …

- is based on Divergent Component of Motion (DCM)
- yields „smooth“ (continuous) eCMP/VRP trajectories (~ „3D ZMP“)
- fulfills terminal constraint for DCM
- provides analytical solutions for VRP, DCM and CoM (matrix form)
- is (super!-) real-time capable

➢ here: focus on reference trajectories

➢ also applicable for e.g. step adjustment / push recovery

Presentation outline

- Basics:
  - ➢ Divergent Component of Motion
  - ➢ 3D force application points

- Building stone: single transition phase

- Complete trajectories in matrix form
  (built via concatenation of single transition phases)

= „ IROS 2017 ++ “

Divergent Component of Motion (DCM)

definition introduced by **Pratt et al.** (**2D**)

$$\xi = x + b\,\dot{x}$$

3D

CoM position

time constant

differentiation

CoM velocity

$$\dot{x} = -\frac{1}{b}\,(x - \xi)$$

stable

$$\dot{\xi} = \frac{1}{b}\,(\xi - v)$$

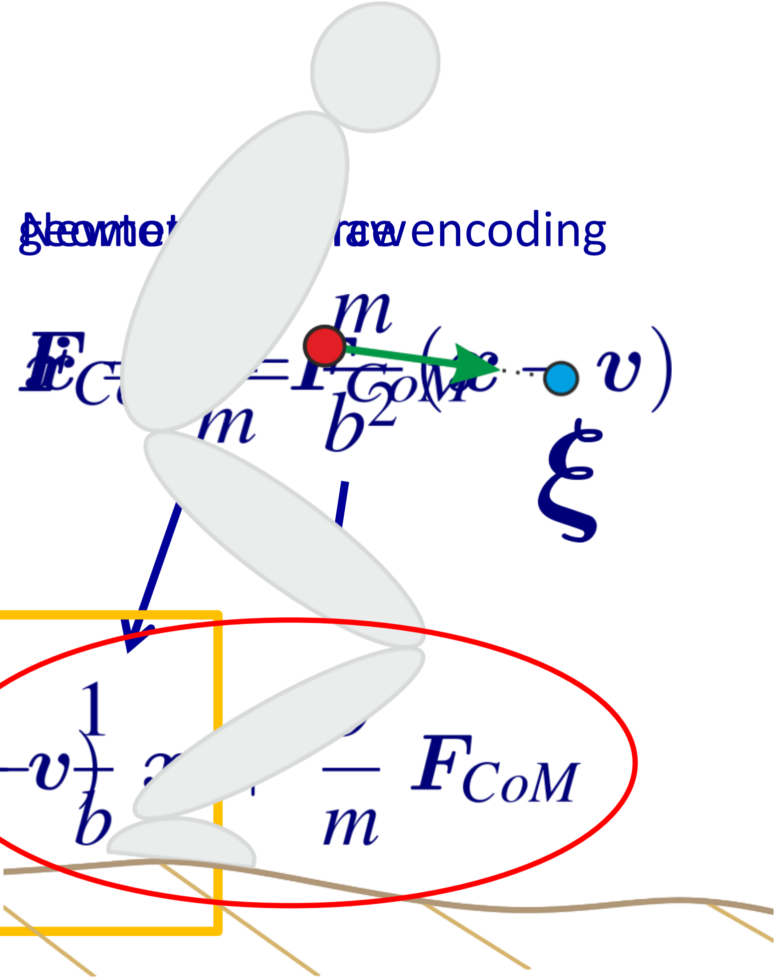
$$\frac{1}{b}\; x \quad \frac{}{m}\,F_{CoM}$$

unstable

Newton / geometric encoding

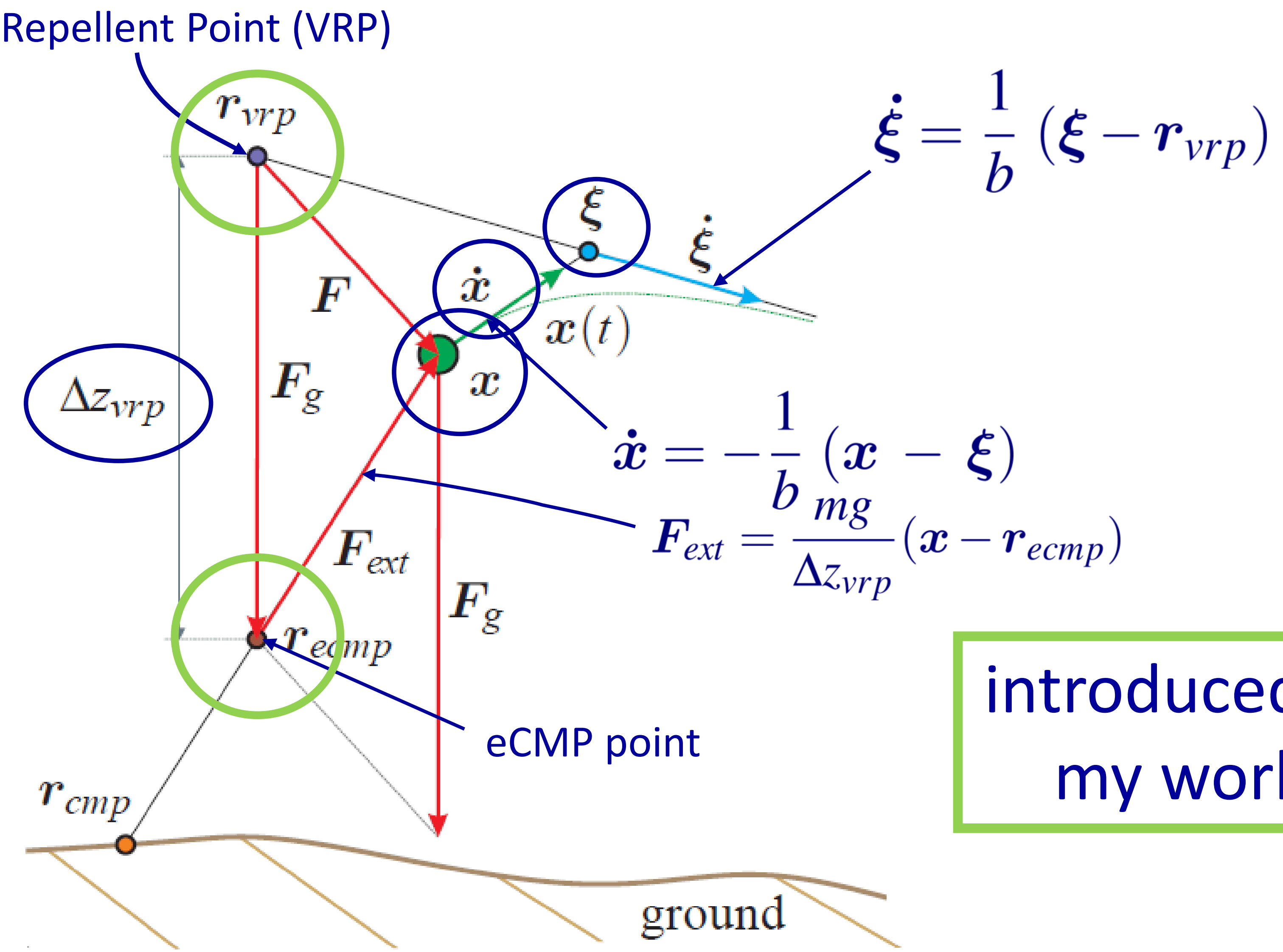
$$F_{CoM} = \frac{m}{b^2}\,(x - v)$$

$$\xi$$



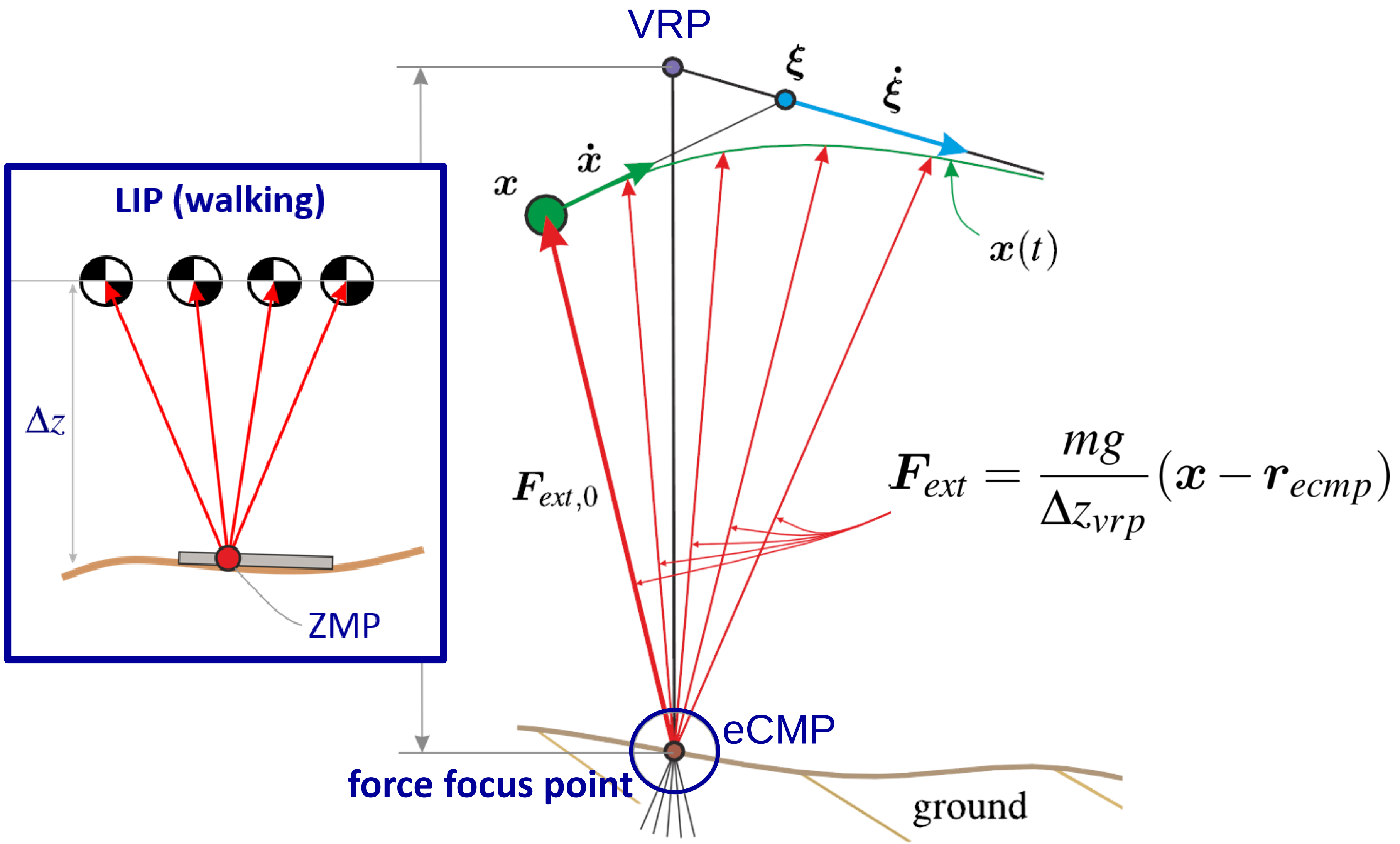
Geometrical force encoding (3D)

IROS 2013

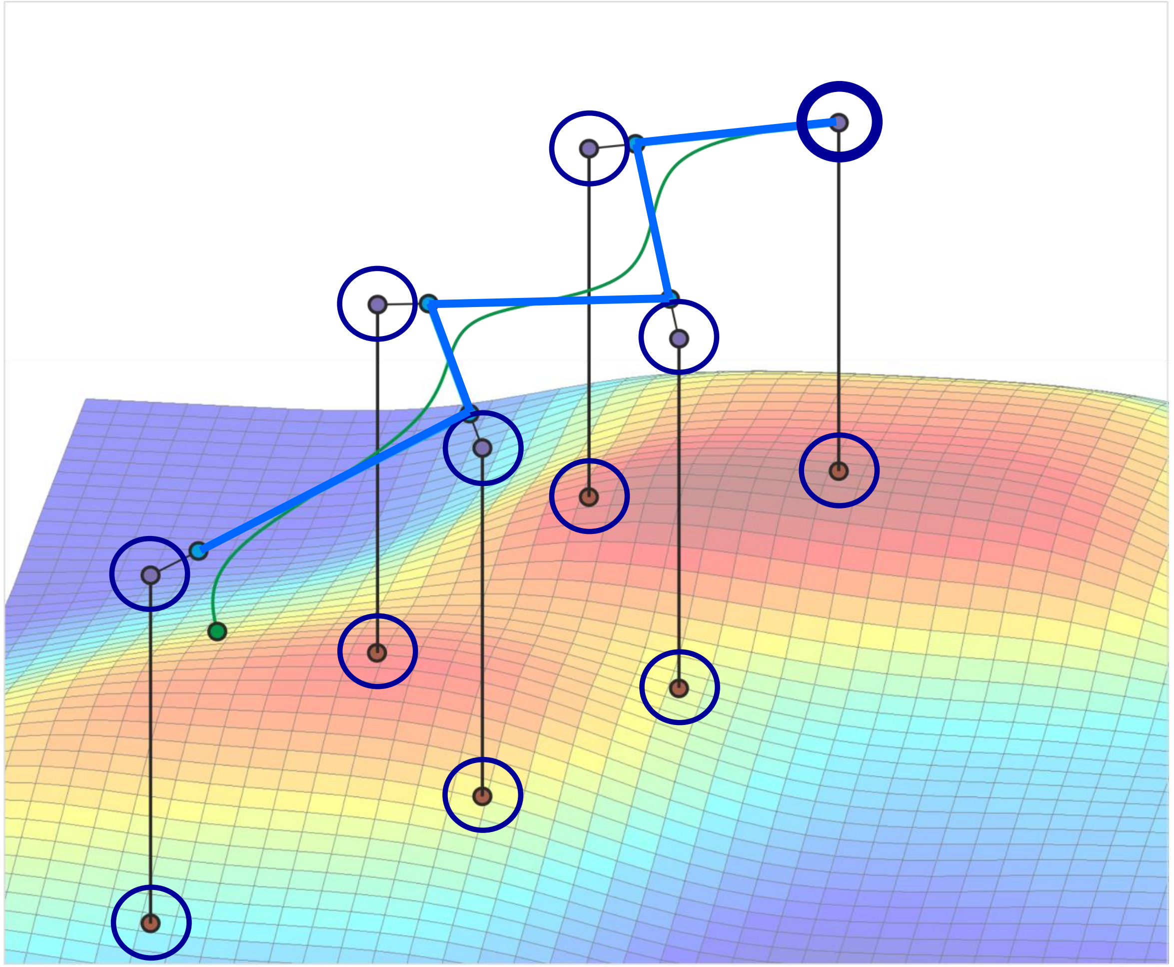


introduced in my work

Force ray focusing (point foot example)



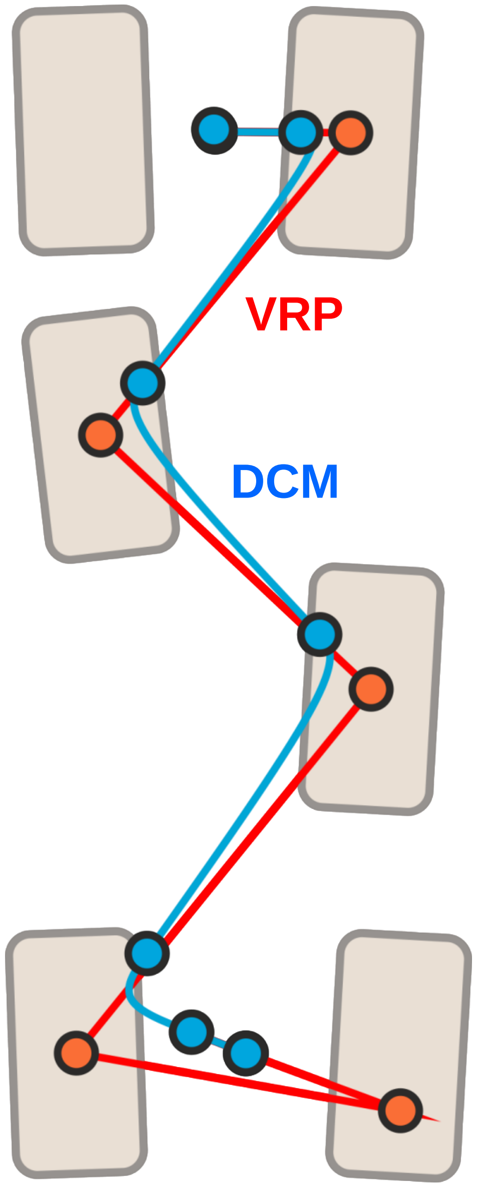
DCM trajectory generation (point foot example)



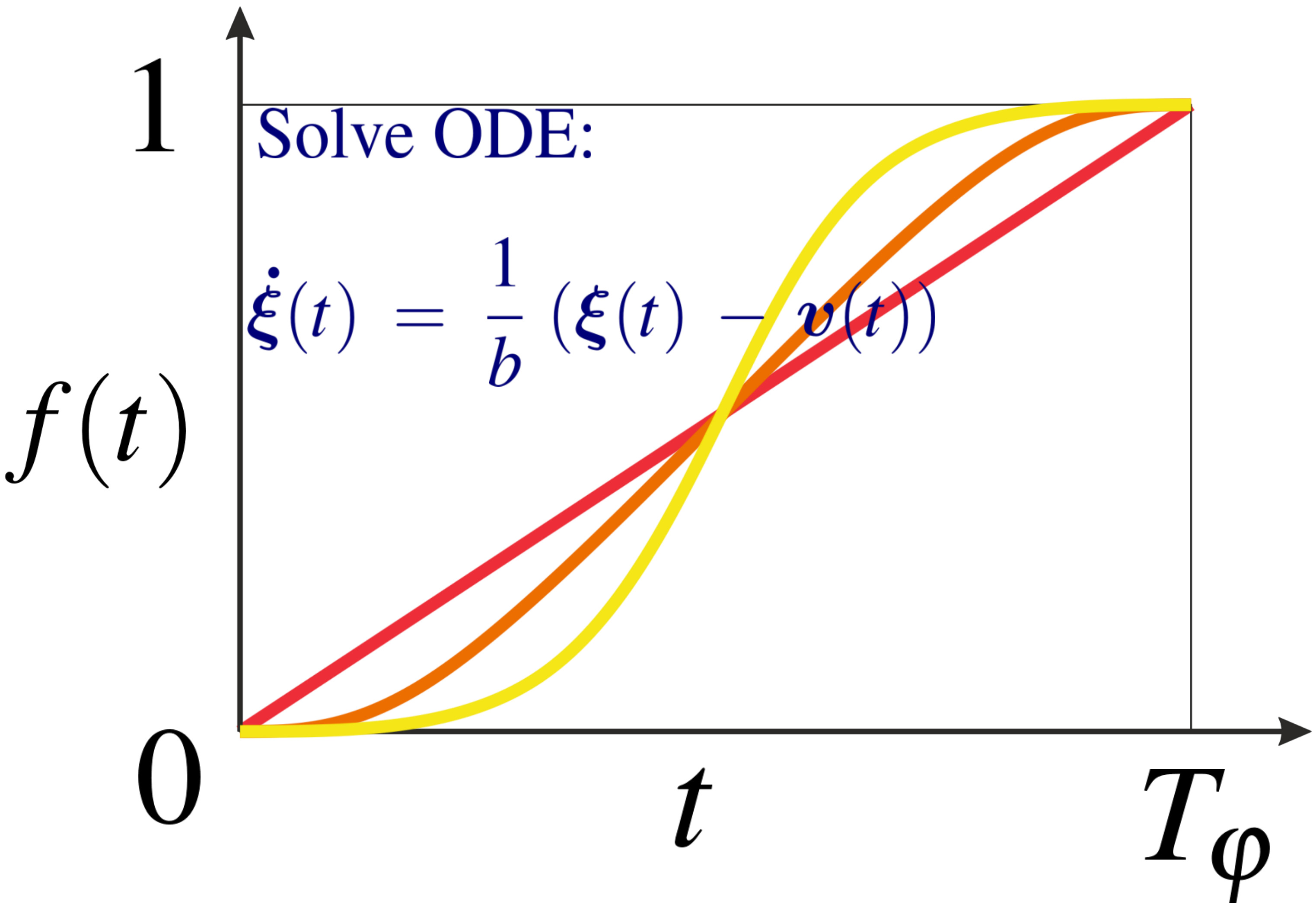
DCM reference trajectory (analytical)

single transition phase

VRP



$$v(t) = (1 - f(t))\, v_0 + f(t)\, v_T$$



Solve ODE:

$$\dot{\xi}(t) = \frac{1}{b}\left(\xi(t) - v(t)\right)$$

single transition phase

VRP

$$v(t) = (1 - f(t))\, v_0 + f(t)\, v_T$$

n_{poly}	$f(t)$	boundary conditions
0^{th}	0	$v(t) = v_0 = const.$
1^{st}	$\frac{t}{T}$	$v(0) = v_0$ $v(T) = v_T$
3^{rd}	$(\frac{t}{T})^2 \left(3 - \frac{2\,t}{T}\right)$	$v(0) = v_0$ $\dot{v}(0) = 0$ $v(T) = v_T$ $\dot{v}(T) = 0$
5^{th}	$(\frac{t}{T})^3 \left(10 - \frac{15\,t}{T} + \frac{6\,t^2}{T^2}\right)$	$v(0) = v_0$ $\dot{v}(0) = 0$ $\ddot{v}(0) = 0$ $v(T) = v_T$ $\dot{v}(T) = 0$ $\ddot{v}(T) = 0$

Johannes Englsberger, DLR

$$\dot{\xi}(t) = \frac{1}{b}\left(\xi(t) - v(t)\right)$$

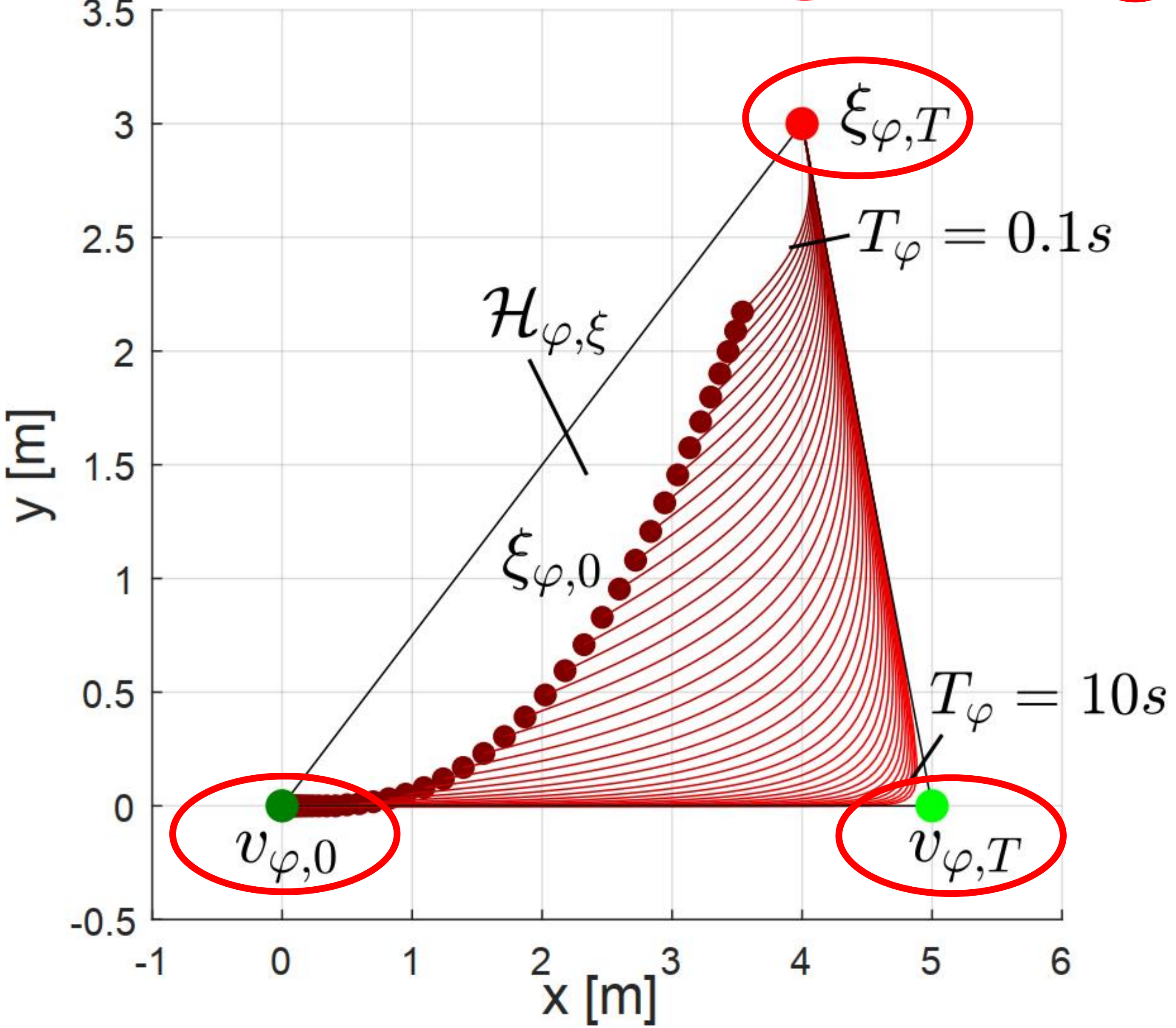
$$\xi(t) = \underbrace{\left(1 - e^{\frac{t-T}{b}}\right)}_{\alpha_\xi(t)} v_0 + \underbrace{\beta_\xi(t)}_{\beta_\xi(t)} v_T + \underbrace{e^{\frac{t-T}{b}}}_{\gamma_\xi(t)} \xi_T$$

$$\sigma(t) = \sum_{k=0}^{\infty} \left(b^k v^{(k)}(t)\right) \beta_\xi(t) + \gamma_\xi(t)\, \sigma_T = \sigma(T)$$

single transition phase

DCM

$$\boldsymbol{\xi}(t) = \alpha_\xi(t)\, \boldsymbol{v}_0 + \beta_\xi(t)\, \boldsymbol{v}_T + \gamma_\xi(t)\, \boldsymbol{\xi}_T$$



single transition phase

CoM

$$\boldsymbol{x}(t) = \underbrace{\left(1 - \rho(t) - e^{-\frac{t}{b}}(1 - \rho_0) - \frac{1}{2}\left(e^{\frac{t-T}{b}} - e^{\frac{-t-T}{b}}\right)(1 - \sigma_T)\right)}_{\alpha_x(t)} \boldsymbol{v}_0$$

$$+ \underbrace{\rho(t) - e^{-\frac{t}{b}}\rho_0 - \frac{1}{2}\left(e^{\frac{t-T}{b}} - e^{\frac{-t-T}{b}}\right)\sigma_T}_{\beta_x(t)} \boldsymbol{v}_T$$

$$+ \underbrace{\frac{1}{2}\left(e^{\frac{t-T}{b}} - e^{\frac{-t-T}{b}}\right)}_{\gamma_x(t)} \boldsymbol{\xi}_T$$

$$+ \underbrace{e^{-\frac{t}{b}}}_{\delta_x(t)} \boldsymbol{x}_0$$

$$\dot{\boldsymbol{x}}(t) = -\frac{1}{b}\left(\boldsymbol{x}(t) - \boldsymbol{\xi}(t)\right)$$

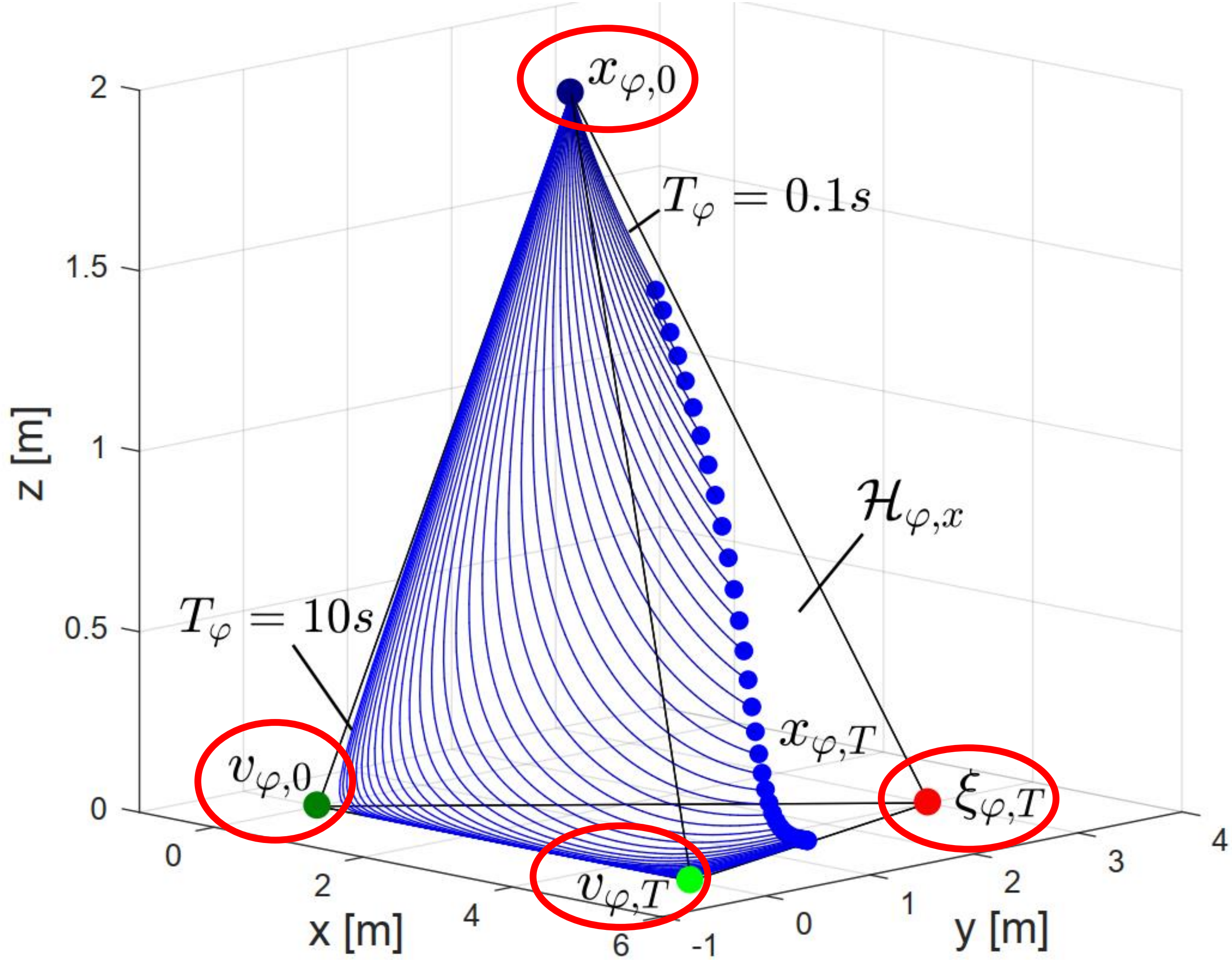
$$\boldsymbol{x}(t) = \alpha_x(t)\,\boldsymbol{v}_0 + \beta_x(t)\,\boldsymbol{v}_T + \gamma_x(t)\,\boldsymbol{\xi}_T + \delta_x(t)\,\boldsymbol{x}_0$$

$$\alpha_x(t) + \beta_x(t) + \gamma_x(t) + \delta_x(t) = 1$$

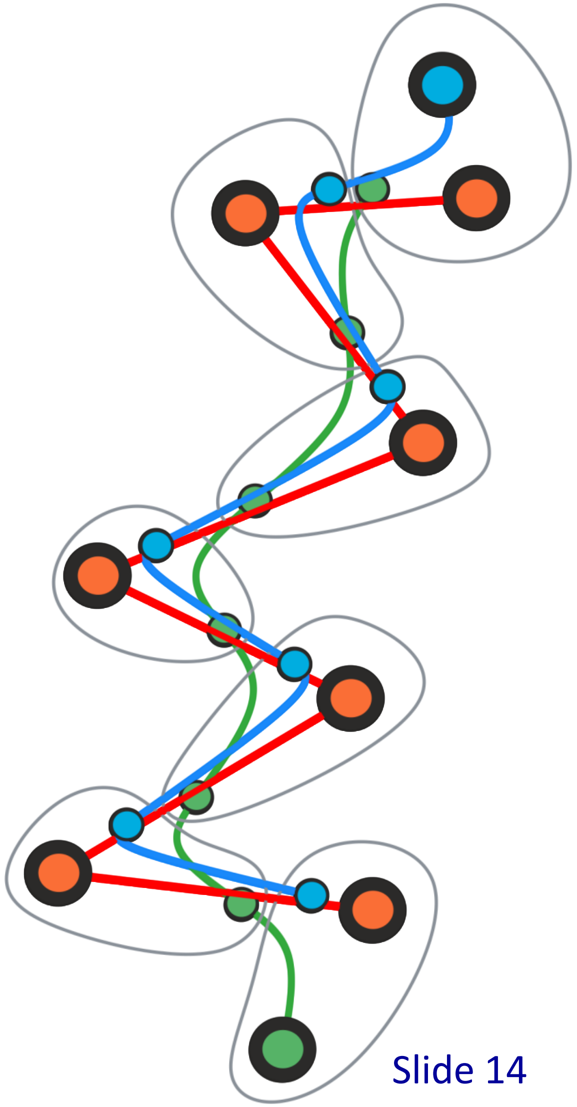
single transition phase

CoM

$$x(t) = \alpha_x(t)\, v_0 + \beta_x(t)\, v_T + \gamma_x(t)\, \xi_T + \delta_x(t)\, x_0$$



multiple transition phases



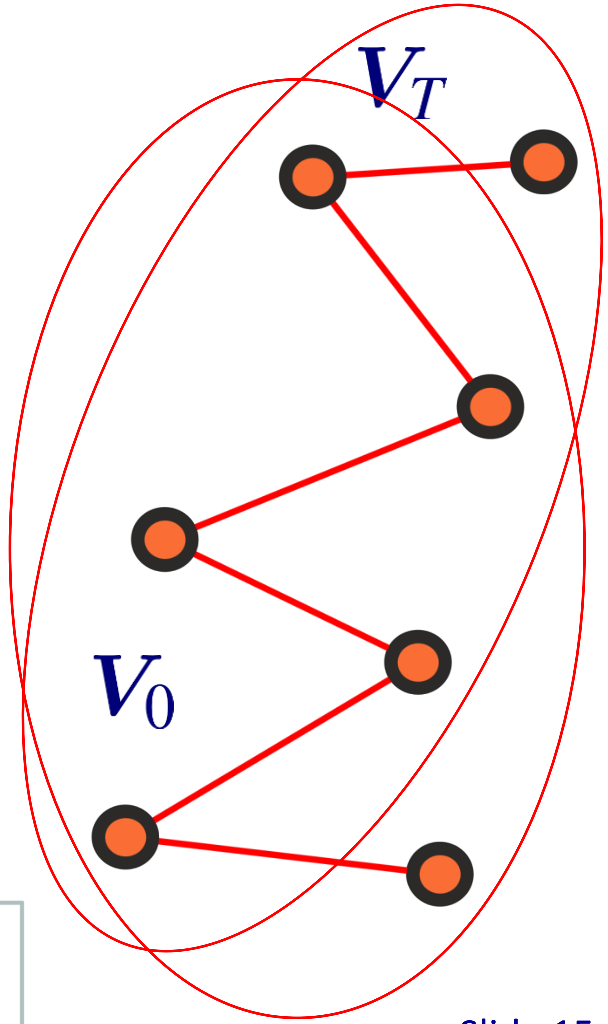
each transition phase $\varphi \in \{1,..,n_\varphi\}, \quad n_\varphi = n_{wp} - 1$

$$\boldsymbol{v}_\varphi(t_\varphi) = (1 - f(t_\varphi))\, \boldsymbol{v}_{\varphi,0} + f(t_\varphi)\, \boldsymbol{v}_{\varphi,T}$$

$$\underbrace{\begin{bmatrix} \boldsymbol{v}_{1,0}^T \\ \vdots \\ \vdots \\ \boldsymbol{v}_{n_\varphi,0}^T \end{bmatrix}}_{\boldsymbol{V}_0 \in \mathbb{R}^{n_\varphi \times 3}} = \underbrace{\begin{bmatrix} \boldsymbol{I}_{n_\varphi \times n_\varphi} & \boldsymbol{0}_{n_\varphi \times 1} \end{bmatrix}}_{\boldsymbol{A}_0 \in \mathbb{R}^{n_\varphi \times n_{wp}}} \underbrace{\begin{bmatrix} \boldsymbol{v}_{wp,1}^T \\ \vdots \\ \vdots \\ \boldsymbol{v}_{wp,n_{wp}}^T \end{bmatrix}}_{\boldsymbol{V}_{wp} \in \mathbb{R}^{n_{wp} \times 3}}$$

$$\underbrace{\begin{bmatrix} \boldsymbol{v}_{1,T}^T \\ \vdots \\ \vdots \\ \boldsymbol{v}_{n_\varphi,T}^T \end{bmatrix}}_{\boldsymbol{V}_T \in \mathbb{R}^{n_\varphi \times 3}} = \underbrace{\begin{bmatrix} \boldsymbol{0}_{n_\varphi \times 1} & \boldsymbol{I}_{n_\varphi \times n_\varphi} \end{bmatrix}}_{\boldsymbol{A}_T \in \mathbb{R}^{n_\varphi \times n_{wp}}} \underbrace{\begin{bmatrix} \boldsymbol{v}_{wp,1}^T \\ \vdots \\ \vdots \\ \boldsymbol{v}_{wp,n_{wp}}^T \end{bmatrix}}_{\boldsymbol{V}_{wp} \in \mathbb{R}^{n_{wp} \times 3}}$$

stacked VRP waypoint matrix



stacked DCM waypoint matrix:

$$\boldsymbol{\Xi}_{wp} = \begin{bmatrix} \boldsymbol{\xi}_{wp,1}^T \\ \vdots \\ \vdots \\ \boldsymbol{\xi}_{wp,n_{wp}}^T \end{bmatrix} \in \mathbb{R}^{n_{wp} \times 3}$$

$$\begin{aligned}
\boldsymbol{\xi}_{\varphi,0} &= \underbrace{\left(1-\boldsymbol{\sigma}_{\varphi,0}-e^{-\frac{T\varphi}{b}}\left(1-\boldsymbol{\sigma}_{\varphi,T}\right)\right)}_{\alpha_{\xi,\varphi,0}} \boldsymbol{v}_{\varphi,0} \\
&+ \underbrace{\left(\boldsymbol{\sigma}_{\varphi,0}-e^{-\frac{T\varphi}{b}}\boldsymbol{\sigma}_{\varphi,T}\right)}_{\beta_{\xi,\varphi,0}} \boldsymbol{v}_{\varphi,T} \\
&+ \underbrace{e^{-\frac{T\varphi}{b}}}_{\gamma_{\xi,\varphi,0}} \boldsymbol{\xi}_{\varphi,T}
\end{aligned}$$

$$\boldsymbol{\xi}_{\varphi,0} = \alpha_{\xi,\varphi,0}\,\boldsymbol{v}_{\varphi,0} + \beta_{\xi,\varphi,0}\,\boldsymbol{v}_{\varphi,T} + \gamma_{\xi,\varphi,0}\,\boldsymbol{\xi}_{\varphi,T}$$

multiple transition phases

DCM

$$\boldsymbol{\xi}_{\varphi,0} = \alpha_{\xi,\varphi,0}\, \boldsymbol{v}_{\varphi,0} + \beta_{\xi,\varphi,0}\, \boldsymbol{v}_{\varphi,T} + \gamma_{\xi,\varphi,0}\, \boldsymbol{\xi}_{\varphi,T}$$

continuity

$$\underbrace{\boldsymbol{A}_0\, \boldsymbol{\Xi}_{wp}}_{\boldsymbol{\Xi}_0 \in \mathbb{R}^{n\varphi \times 3}} = \underbrace{\begin{bmatrix} \alpha_{\xi,1,0} & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \alpha_{\xi,n_\varphi,0} \end{bmatrix}}_{\boldsymbol{A}_{\alpha_\xi} \in \mathbb{R}^{n\varphi \times n\varphi}} \underbrace{\boldsymbol{A}_0\, \boldsymbol{V}_{wp}}_{\boldsymbol{V}_0 \in \mathbb{R}^{n\varphi \times 3}} + \underbrace{\begin{bmatrix} \beta_{\xi,1,0} & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \beta_{\xi,n_\varphi,0} \end{bmatrix}}_{\boldsymbol{A}_{\beta_\xi} \in \mathbb{R}^{n\varphi \times n\varphi}} \underbrace{\boldsymbol{A}_T\, \boldsymbol{V}_{wp}}_{\boldsymbol{V}_T \in \mathbb{R}^{n\varphi \times 3}}$$

$$+ \underbrace{\begin{bmatrix} \gamma_{\xi,1,0} & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \gamma_{\xi,n_\varphi,0} \end{bmatrix}}_{\boldsymbol{A}_{\gamma_\xi} \in \mathbb{R}^{n\varphi \times n\varphi}} \underbrace{\boldsymbol{A}_T\, \boldsymbol{\Xi}_{wp}}_{\boldsymbol{\Xi}_T \in \mathbb{R}^{n\varphi \times 3}}$$

DCM terminal constraint

$$\boldsymbol{\xi}_{n_\varphi,T} = \boldsymbol{\xi}_{wp,n_{wp}} = \boldsymbol{\xi}_f$$

$$\underbrace{\begin{bmatrix} \mathbf{0}_{n_\varphi \times n_\varphi} & \mathbf{0}_{n_\varphi \times 1} \\ \mathbf{0}_{1 \times n_\varphi} & 1 \end{bmatrix}}_{\boldsymbol{S}_{n_{wp}} \in \mathbb{R}^{n_{wp} \times n_{wp}}} \boldsymbol{\Xi}_{wp} = \underbrace{\begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}}_{\boldsymbol{A}_{TC} \in \mathbb{R}^{n_{wp} \times 1}} \boldsymbol{\xi}_f^T$$

continuity + DCM terminal constraint

$$\boldsymbol{\Xi}_{wp} = \underbrace{(\boldsymbol{I}_{n_{wp}\times n_{wp}} - \boldsymbol{A}_0^T\,\boldsymbol{A}_{\gamma_\xi}\,\boldsymbol{A}_T)^{-1}(\boldsymbol{A}_0^T\,\boldsymbol{A}_{\alpha_\xi}\,\boldsymbol{A}_0 + \boldsymbol{A}_0^T\,\boldsymbol{A}_{\beta_\xi}\,\boldsymbol{A}_T)}_{\boldsymbol{A}_{V_{wp}\rightarrow\Xi_{wp}} \in \mathbb{R}^{n_{wp}\times n_{wp}}}\,\boldsymbol{V}_{wp}$$

$$+ \underbrace{(\boldsymbol{I}_{n_{wp}\times n_{wp}} - \boldsymbol{A}_0^T\,\boldsymbol{A}_{\gamma_\xi}\,\boldsymbol{A}_T)^{-1}\boldsymbol{A}_{TC}}_{\boldsymbol{A}_{\xi_f\rightarrow\Xi_{wp}} \in \mathbb{R}^{n_{wp}\times 1}}\,\boldsymbol{\xi}_f^T$$

$$\boldsymbol{\Xi}_{wp} = \boldsymbol{A}_{V_{wp}\rightarrow\Xi_{wp}}\,\boldsymbol{V}_{wp} + \boldsymbol{A}_{\xi_f\rightarrow\Xi_{wp}}\,\boldsymbol{\xi}_f^T$$

multiple transition phases

DCM

$$\boldsymbol{\xi}_{\varphi}(t_{\varphi}) = \boldsymbol{A}_{V_{wp} \rightarrow \xi_{\varphi}}(t_{\varphi}) \boldsymbol{V}_{wp} + \boldsymbol{A}_{\xi_f \rightarrow \xi_{\varphi}}(t_{\varphi}) \boldsymbol{\xi}_f^T$$

$$\begin{aligned} \boldsymbol{A}_{V_{wp} \rightarrow \xi_{\varphi}}(t_{\varphi}) = \; & \alpha_{\xi,\varphi}(t_{\varphi}) \, row_{\varphi}(\boldsymbol{A}_0) \\ + \; & \beta_{\xi,\varphi}(t_{\varphi}) \, row_{\varphi}(\boldsymbol{A}_T) \\ + \; & \gamma_{\xi,\varphi}(t_{\varphi}) \, row_{\varphi}(\boldsymbol{A}_T \boldsymbol{A}_{V_{wp} \rightarrow \Xi_{wp}}) \end{aligned}$$

$$\boldsymbol{A}_{\xi_f \rightarrow \xi_{\varphi}}(t_{\varphi}) = \gamma_{\xi,\varphi}(t_{\varphi}) \, row_{\varphi}(\boldsymbol{A}_T \boldsymbol{A}_{\xi_f \rightarrow \Xi_{wp}})$$

stacked CoM waypoint matrix:

$$\boldsymbol{X}_{wp} = \begin{bmatrix} \boldsymbol{x}_{wp,1}^T \\ \vdots \\ \vdots \\ \boldsymbol{x}_{wp,n_{wp}}^T \end{bmatrix} \in \mathbb{R}^{n_{wp} \times 3}$$

multiple transition phases

CoM

$$
\boldsymbol{x}_{\varphi,T} = \underbrace{\left(1-\rho_{\varphi,T}-e^{-\frac{T\varphi}{b\varphi}}(1-\rho_{\varphi,0})-\frac{1}{2}(1-e^{-\frac{2T\varphi}{b\varphi}})(1-\sigma_{\varphi,T}\right)}_{\alpha_{x,\varphi,T}}\boldsymbol{v}_{\varphi,0}
$$

$$
+\underbrace{\left(\rho_{\varphi,T}-e^{-\frac{T\varphi}{b\varphi}}\rho_{\varphi,0}-\frac{1}{2}(1-e^{-\frac{2T\varphi}{b\varphi}})\,\sigma_{\varphi,T}\right)}_{\beta_{x,\varphi,T}}\boldsymbol{v}_{\varphi,T}
$$

$$
+\underbrace{\frac{1}{2}(1-e^{-\frac{2T\varphi}{b\varphi}})}_{\gamma_{x,\varphi,T}}\boldsymbol{\xi}_{\varphi,T}
$$

$$
+\underbrace{e^{-\frac{T\varphi}{b\varphi}}}_{\delta_{x,\varphi,T}}\boldsymbol{x}_{\varphi,0}
$$

multiple transition phases

CoM

$$\boldsymbol{x}_{\varphi,T} = \alpha_{x,\varphi,T}\, \boldsymbol{v}_{\varphi,0} + \beta_{x,\varphi,T}\, \boldsymbol{v}_{\varphi,T} + \gamma_{x,\varphi,T}\, \boldsymbol{\xi}_{\varphi,T} + \delta_{x,\varphi,T}\, \boldsymbol{x}_{\varphi,0}$$

continuity

$$\underbrace{\boldsymbol{A}_T\, \boldsymbol{X}_{wp}}_{\boldsymbol{X}_T \in \mathbb{R}^{n\varphi \times 3}} = \underbrace{\begin{bmatrix} \alpha_{x,1,T} & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \alpha_{x,n_\varphi,T} \end{bmatrix}}_{\boldsymbol{A}_{\alpha_x} \in \mathbb{R}^{n\varphi \times n\varphi}} \underbrace{\boldsymbol{A}_0\, \boldsymbol{V}_{wp}}_{\boldsymbol{V}_0 \in \mathbb{R}^{n\varphi \times 3}} + \underbrace{\begin{bmatrix} \beta_{x,1,T} & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \beta_{x,n_\varphi,T} \end{bmatrix}}_{\boldsymbol{A}_{\beta_x} \in \mathbb{R}^{n\varphi \times n\varphi}} \underbrace{\boldsymbol{A}_T\, \boldsymbol{V}_{wp}}_{\boldsymbol{V}_T \in \mathbb{R}^{n\varphi \times 3}}$$

$$+ \underbrace{\begin{bmatrix} \gamma_{x,1,T} & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \gamma_{x,n_\varphi,T} \end{bmatrix}}_{\boldsymbol{A}_{\gamma_x} \in \mathbb{R}^{n\varphi \times n\varphi}} \underbrace{\boldsymbol{A}_T\, \boldsymbol{\Xi}_{wp}}_{\boldsymbol{\Xi}_T \in \mathbb{R}^{n\varphi \times 3}} + \underbrace{\begin{bmatrix} \delta_{x,1,T} & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \delta_{x,n_\varphi,T} \end{bmatrix}}_{\boldsymbol{A}_{\delta_x} \in \mathbb{R}^{n\varphi \times n\varphi}} \underbrace{\boldsymbol{A}_0\, \boldsymbol{X}_{wp}}_{\boldsymbol{X}_0 \in \mathbb{R}^{n\varphi \times 3}}$$

The CoM initial constraint ...

$$\boldsymbol{x}_{1,0} = \boldsymbol{x}_{wp,1} = \boldsymbol{x}_s$$

$$\underbrace{\begin{bmatrix} 1 & \mathbf{0}_{1\times n_\varphi} \\ \mathbf{0}_{n_\varphi \times 1} & \mathbf{0}_{n_\varphi \times n_\varphi} \end{bmatrix}}_{\boldsymbol{S}_1 \in \mathbb{R}^{n_{wp} \times n_{wp}}} \boldsymbol{X}_{wp} = \underbrace{\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{\boldsymbol{A}_{IC} \in \mathbb{R}^{n_{wp} \times 1}} \boldsymbol{x}_s^T$$

multiple transition phases

CoM

... together with continuity

$$
\begin{aligned}
\boldsymbol{X}_{wp} = &\underbrace{\left(\boldsymbol{I}_{n_{wp}\times n_{wp}} - \boldsymbol{A}_T^T \boldsymbol{A}_{\delta_x} \boldsymbol{A}_0\right)^{-1} \left(\boldsymbol{A}_T^T \boldsymbol{A}_{\gamma_x} \boldsymbol{A}_T \boldsymbol{A}_{V_{wp}\rightarrow\Xi_{wp}}\right)}_{\boldsymbol{A}_{V_{wp}\rightarrow X_{wp}} \in \mathbb{R}^{n_{wp}\times n_{wp}}} \boldsymbol{V}_{wp} \\
&+ \underbrace{\left(\boldsymbol{I}_{n_{wp}\times n_{wp}} - \boldsymbol{A}_T^T \boldsymbol{A}_{\delta_x} \boldsymbol{A}_0\right)^{-1} \boldsymbol{A}_T^T \boldsymbol{A}_{\gamma_x} \boldsymbol{A}_T \boldsymbol{A}_{\xi_f\rightarrow\Xi_{wp}}}_{\boldsymbol{A}_{\xi_f\rightarrow X_{wp}} \in \mathbb{R}^{n_{wp}\times 1}} \boldsymbol{\xi}_f^T \\
&+ \underbrace{\left(\boldsymbol{I}_{n_{wp}\times n_{wp}} - \boldsymbol{A}_T^T \boldsymbol{A}_{\delta_x} \boldsymbol{A}_0\right)^{-1} \boldsymbol{A}_{IC}}_{\boldsymbol{A}_{x_s\rightarrow X_{wp}} \in \mathbb{R}^{n_{wp}\times 1}} \boldsymbol{x}_s^T
\end{aligned}
$$

multiple transition phases

CoM

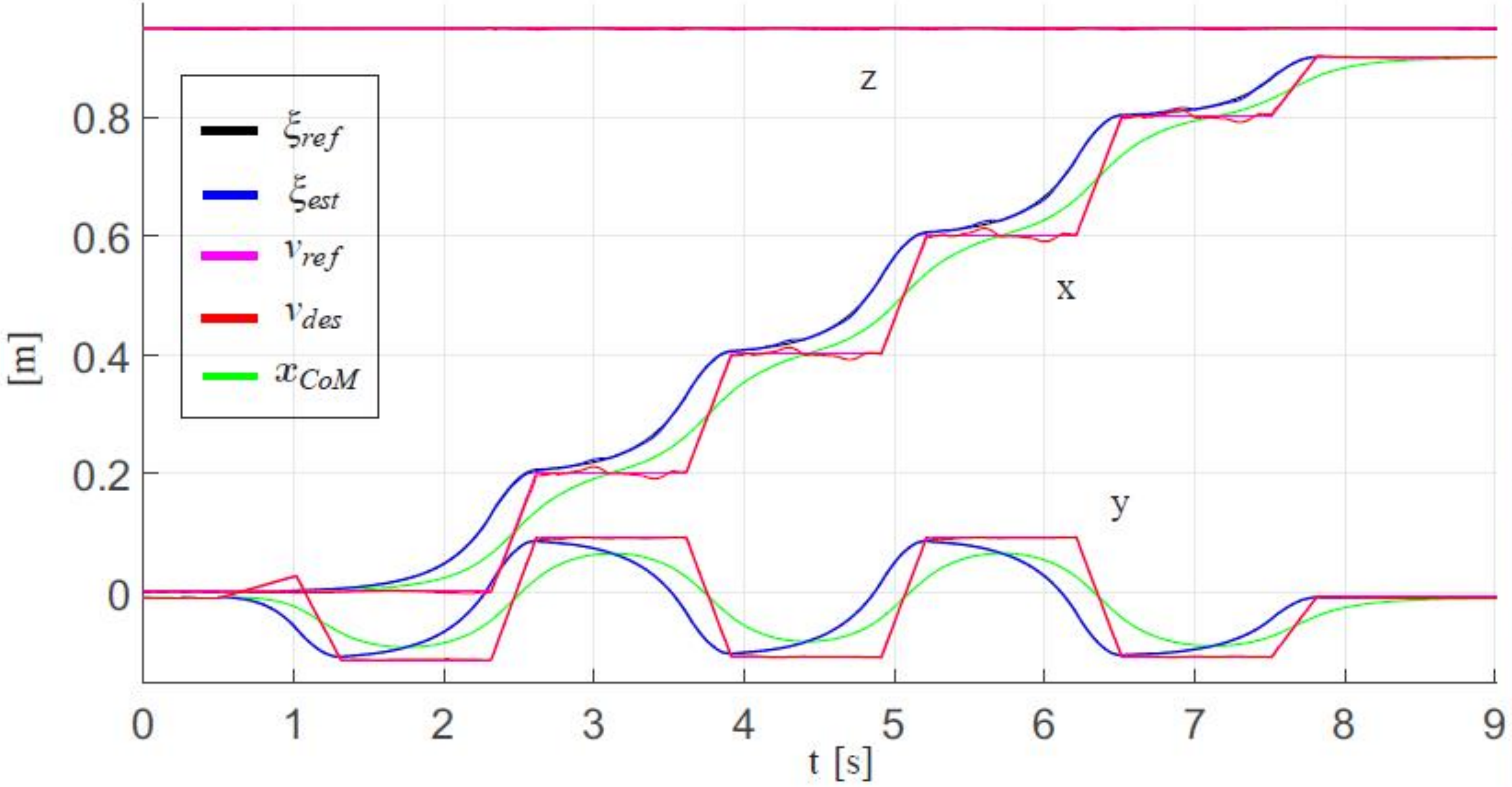
$$\boldsymbol{x}_\varphi(t_\varphi) = \boldsymbol{A}_{V_{wp}\rightarrow x_\varphi}(t_\varphi)\,\boldsymbol{V}_{wp} + \boldsymbol{A}_{\xi_f\rightarrow x_\varphi}(t_\varphi)\,\boldsymbol{\xi}_f^T + \boldsymbol{A}_{x_s\rightarrow x_\varphi}(t_\varphi)\,\boldsymbol{x}_s^T$$

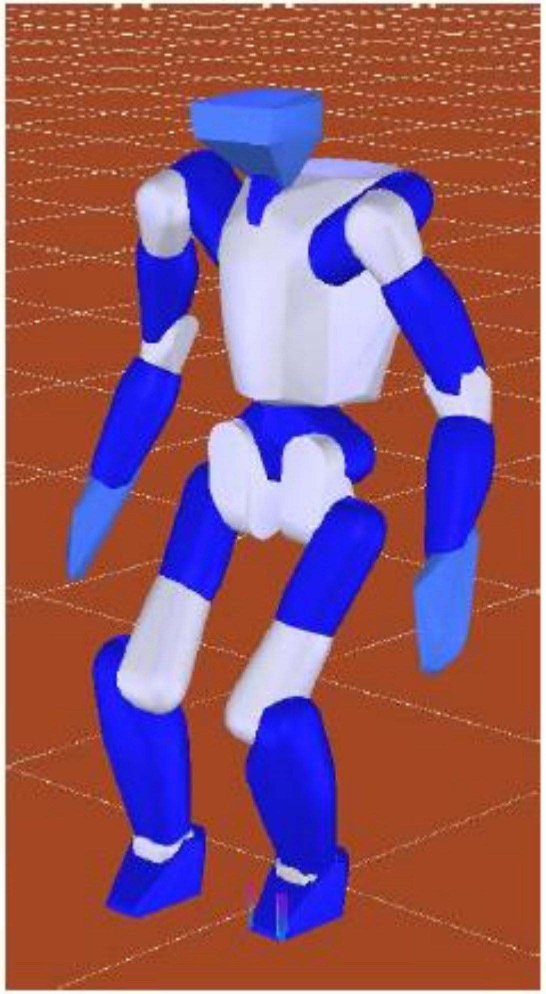
$$\begin{aligned}
\boldsymbol{A}_{V_{wp}\rightarrow x_\varphi}(t_\varphi) &= \alpha_{x,\varphi}(t_\varphi)\, row_\varphi(\boldsymbol{A}_0) \\
&+ \beta_{x,\varphi}(t_\varphi)\, row_\varphi(\boldsymbol{A}_T) \\
&+ \gamma_{x,\varphi}(t_\varphi)\, row_\varphi(\boldsymbol{A}_T\,\boldsymbol{A}_{V_{wp}\rightarrow \Xi_{wp}}) \\
&+ \delta_{x,\varphi}(t_\varphi)\, row_\varphi(\boldsymbol{A}_0\,\boldsymbol{A}_{V_{wp}\rightarrow X_{wp}}) \\
\boldsymbol{A}_{\xi_f\rightarrow x_\varphi}(t_\varphi) &= \gamma_{x,\varphi}(t_\varphi)\, row_\varphi(\boldsymbol{A}_T\,\boldsymbol{A}_{\xi_f\rightarrow \Xi_{wp}}) \\
&+ \delta_{x,\varphi}(t_\varphi)\, row_\varphi(\boldsymbol{A}_0\,\boldsymbol{A}_{\xi_f\rightarrow X_{wp}}) \\
\boldsymbol{A}_{x_s\rightarrow x_\varphi}(t_\varphi) &= \delta_{x,\varphi}(t_\varphi)\, row_\varphi(\boldsymbol{A}_0\,\boldsymbol{A}_{x_s\rightarrow X_{wp}})
\end{aligned}$$

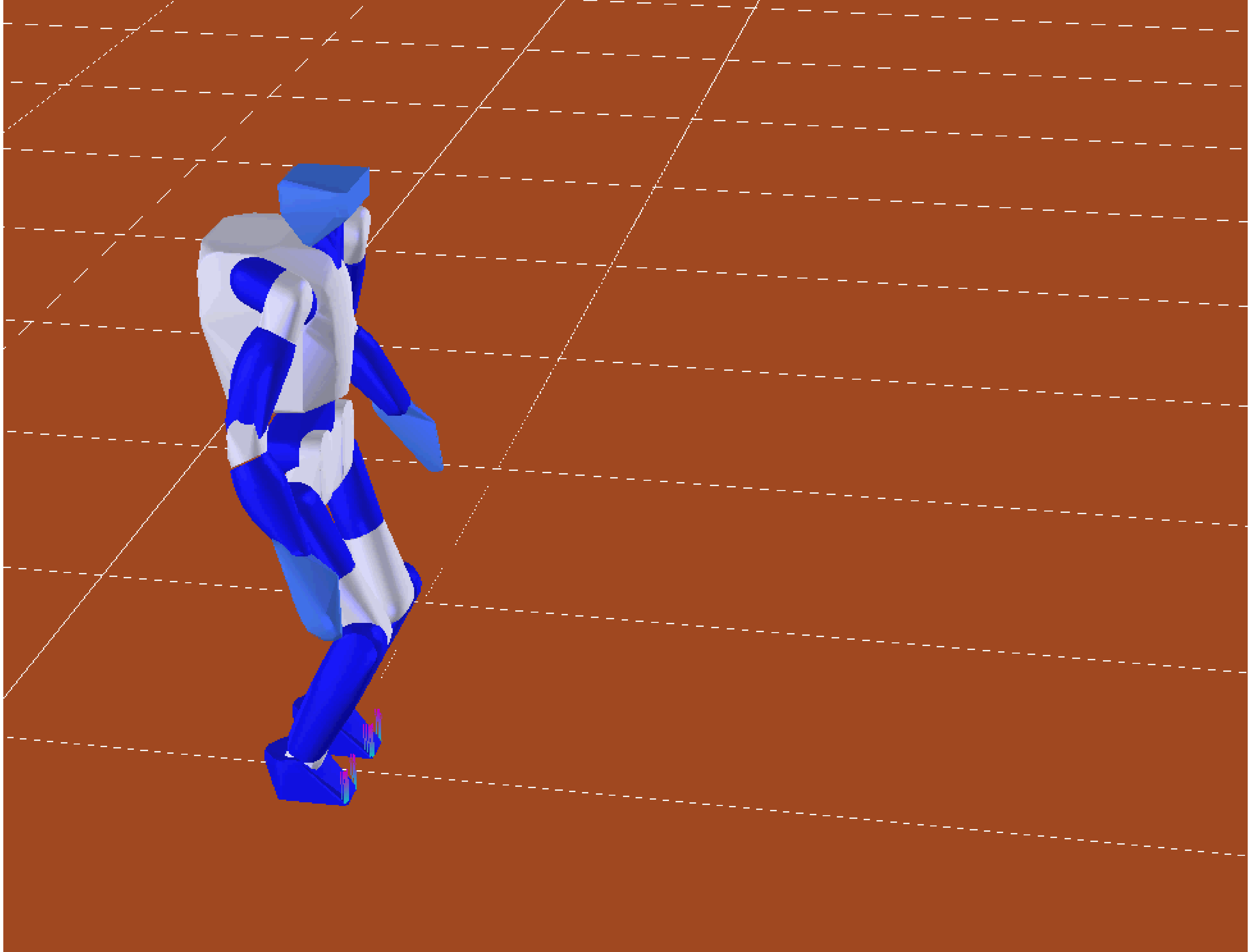
Smooth DCM trajectory generation (online computation)

$\Rightarrow$ **super-realtime capable:**
about **0.1 ms** computation time for **14 transition phases** in **Matlab**

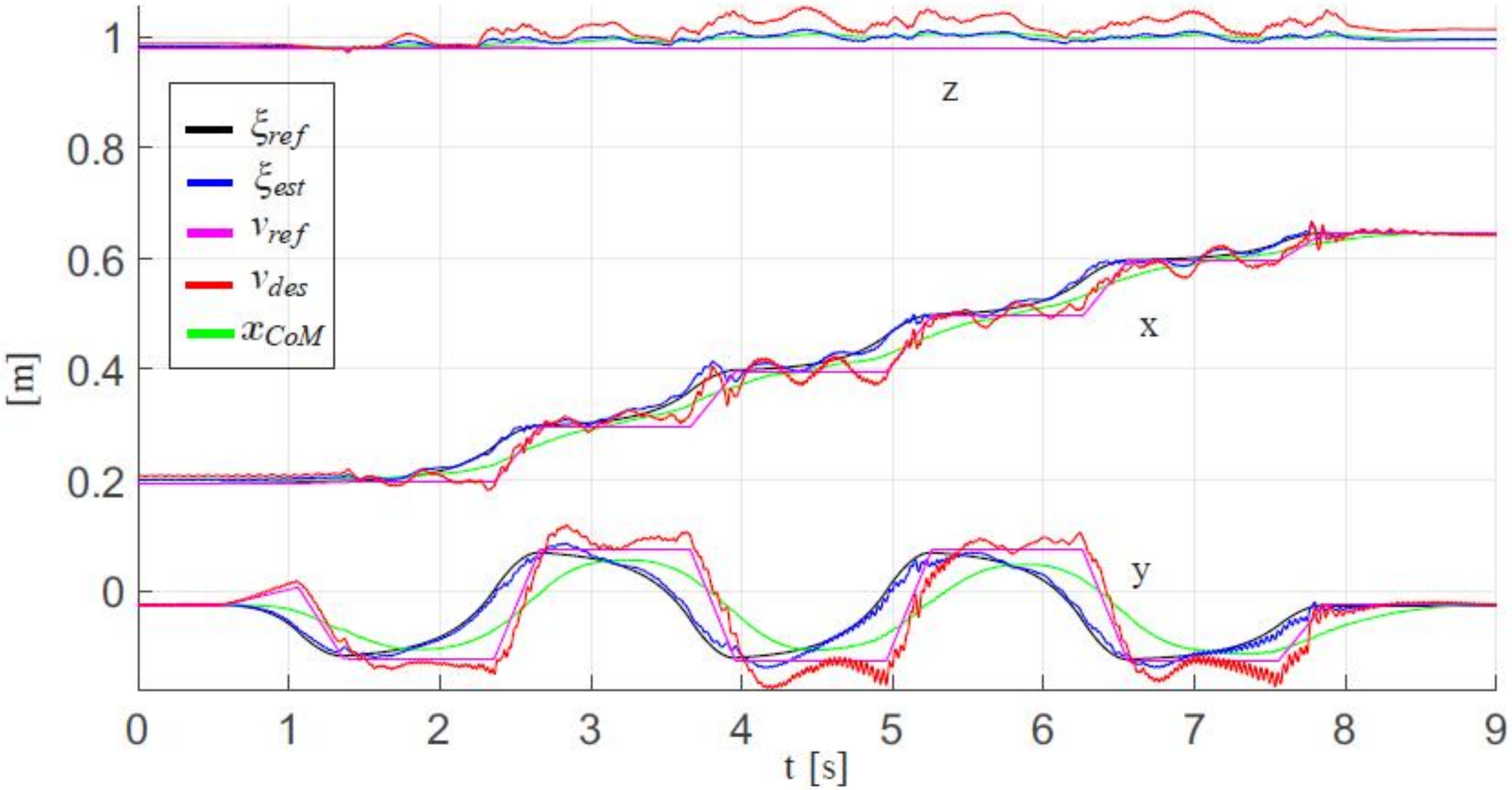
Simulation in OpenHRP => close to perfect tracking!







Torque-based walking with Toro



Some more walking videos ….

Future developments / progress (hopefully)

- improving performance in experiment (longer/faster steps)
- including toe-off motion for more natural gait
- including angular momentum in gait planning stage (open)
- development of unified framework for walking gaits of arbitrary legged robots (2-/4-/6-/n-legged)
  - ➢ stances to CoM trajectories (unified, arbitrary, optimal)

Thanks for your attention!