Simulate the Orbit of Earth Around the Sun with the Moon

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April 5, 2023

1 Introduction

A problem that is common in the world of physics is modeling the orbit of earth around the sun. Though what starts making it more complex is when we add other bodies to the problem. In this case we are adding earth single moon to the problem. The earths moon is interesting in how small it is compared to the sun and the earth. We are just considering the case of the earth and the Moon orbiting the sun and the sun not moving.

2 Orbit Equations Used

For orbiting the earth around the sun the following equation is used:

$$\frac{M_{earth}v^2}{r} = F_g = \frac{GM_{sun}M_{earth}}{r^2} \tag{1}$$

G is the universal gravitational constant, r is the distance between the earth and the sun, and v is the velocity of the earth. If we astronomical units we find that $GM_{sun} = v^2r = 4\pi^2\frac{AU^3}{yr^2}$. Then we can simplify the acceleration of just the earth to be:

$$a_{earth} = \frac{-4\pi^2}{r^2} \tag{2}$$

This also applies for the moon. Though add the moon will add another term to this equation do that it is another gravitational force acting on the earth. This leads to the following equation:

$$a_{earth} = \frac{-4\pi^2}{r^2} + \frac{4\pi^2(r_{moon} - r_{earth})M_{moon}}{M_{sun}|r_{earth}^3|} \tag{3}$$

For modeling the system it is recommended that you break the acceleration of the moon and the earth into components in the x and y directions that leads to the following equations for the earth:

$$a_x = \frac{-4\pi^2}{r^2} + \frac{4\pi^2(r_{moonx} - r_{earthx})M_{moon}}{M_{sun}|r^3_{earth}|} \tag{4}$$

$$a_y = \frac{-4\pi^2}{r^2} + \frac{4\pi^2(r_{moony} - r_{earthy})M_{moon}}{M_{sun}|r^3_{earth}|} \tag{5}$$

To set up the equations for the Moon just swap the positions of the earth for the moon and vise versa.

3 Numerical Method

The numerical method chosen to solve the differential equation above is Fourth order Runge-Kutta

$$k_1 = dtf(x,t) \tag{6}$$

$$k_2 = dtf(x + \frac{1}{2}k_1, t + \frac{1}{2}dt) \tag{7}$$

$$k_3 = dtf(x + \frac{1}{2}k_2, t + \frac{1}{2}dt) \tag{8}$$

$$k_4 = dtf(x + k_3, t + dt) \tag{9}$$

$$x(t+h) = x(t) + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{10}$$

The solution to the equation look like the following:

$$\frac{dx}{dt} = v_x \tag{11}$$

$$\frac{dv_x}{dt} = \frac{-4\pi^2}{r^2} + \frac{4\pi^2(r_{moonx} - r_{earthx})M_{moon}}{M_{sun}|r^3_{earth}|} \tag{12}$$

$$\frac{dy}{dt} = v_y \tag{13}$$

$$\frac{dv_y}{dt} = \frac{-4\pi^2}{r^2} + \frac{4\pi^2(r_{moony} - r_{earthy})M_{moon}}{M_{sun}|r^3_{earth}|} \tag{14}$$

This solution works for both the moon and the earth. You could use other Numerical Methods like Euler Method of a Second order Runge-Kutta but to the sensitivity of the initial condition of the this method a Forth order Runge-Kutta is more stable.

4 Results

The result of the equation lead to the following graph in figure 1 and to see the obit of the moon look at figure 2:

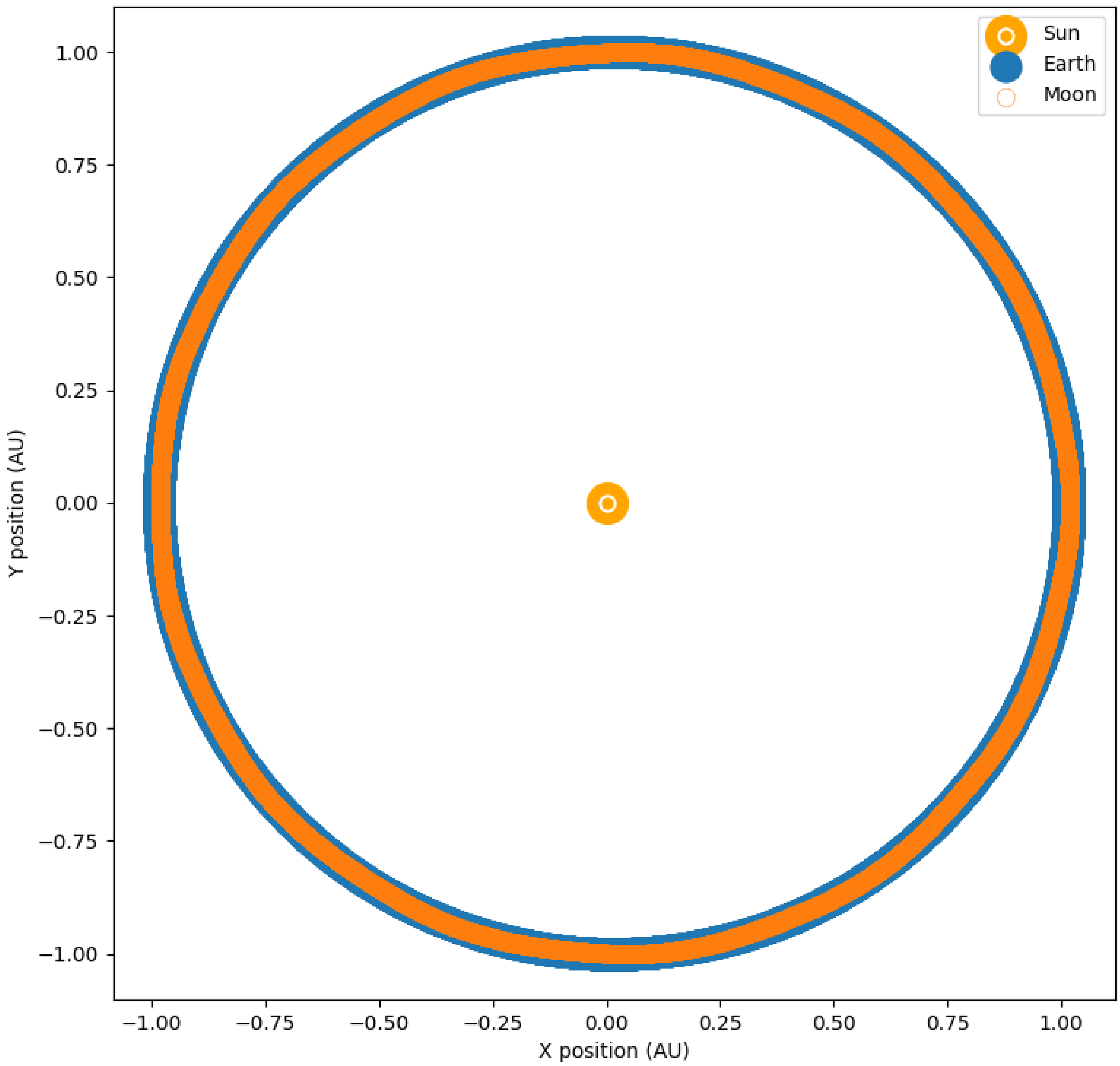

Figure 1: This shows the full path of the Earth and the Moon. Figure 2 is a better look of the path of the moon.

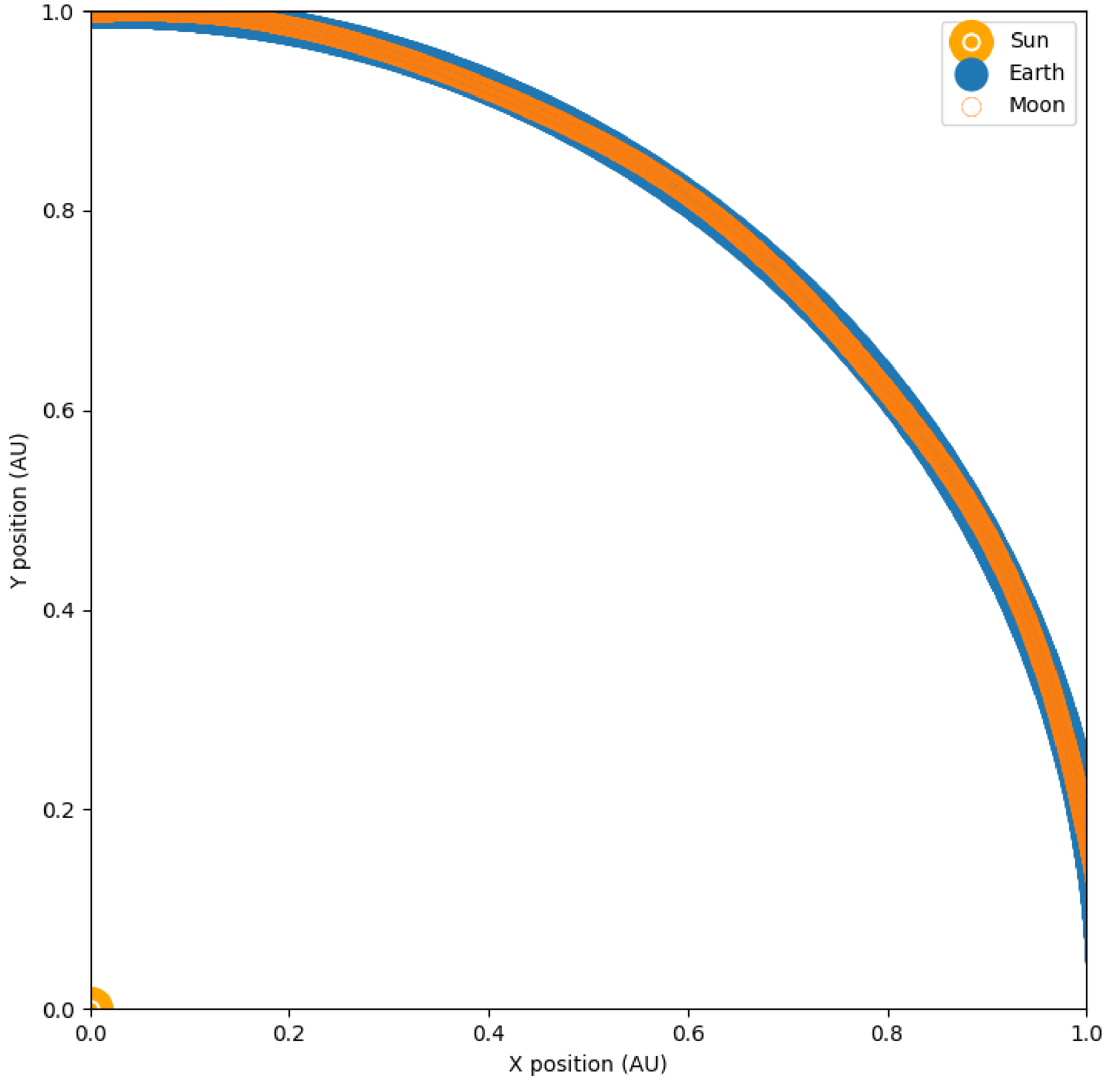

Figure 2: Zoom in version of Figure 1. Look at the deviation of the orange moon path vs the blue earth path.

Using the initial condition of earth starting at a initial x position of 1.017 AU a y position of 0. Earth mass being $6.0 * 10^2 4kg$, Earth initial velocity in the x direction being 0 and the y position being $\sqrt{\frac{4\pi^2(1-0.017}{(1+0.017}}$ with a time step of $\frac{1}{10000}$ of a year. Then for the moon the initial x position being $earthposition + 0.025 * (1 + 0.0549)$ and a y position of 0. The moon mass was $7.3 * 10^2 2kg$ with initial velocity of 0 in the x and y velocity being $\sqrt{\frac{4\pi^2(M_{earth}/M_{sun})(1-0.0549)}{0.025(1+0.0549)+v_{earthy}}}$ with the same time step of earth. The mass of the sun is $2.0 * 10^3 0kg$

5 Conclusion

You can see how the orbit of the moon is weaving round the earth and how the earth is orbiting the sun.