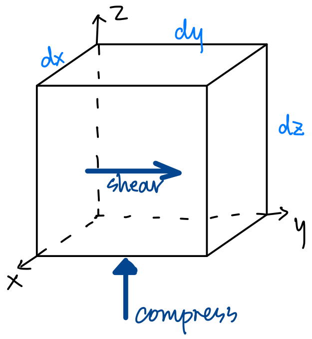
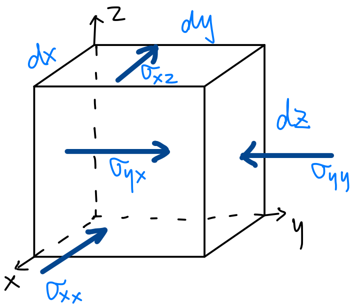
# Subsurface imaging and detection

2021.09.16. Seismic method: elastic medium




## SRESS




direction of vector — unit

$$\sigma_{ij} = \frac{Force}{Surface} \left[\frac{Nw}{m^2}\right] = Pressure\,[Pascal]$$

direction normal to surface — unit

# i, j = x, y, z

$\begin{cases} i = j \rightarrow \text{compress stress} \\ i \neq j \rightarrow \text{shear stress} \end{cases}$

# For equilibrium : $\sigma_{ij} = \sigma_{ji}$

⇓ Deformation

Displacement vector

$$\vec{u}(x,y,z) = u_x\vec{i} + u_y\vec{j} + u_z\vec{k}$$

## STRAIN

strains are described by the partial derivatives of the displacement vector with respect to the spatial coordinates.

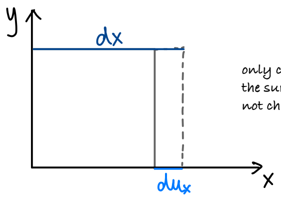
$$\varepsilon_{kl} = \frac{1}{2}\left(\frac{\partial u_k}{\partial l} + \frac{\partial u_l}{\partial k}\right) \quad \text{with } k,l = x,y,z$$

Adimensional: it measures the dilatation(膨胀度) in one direction when the other directions are compressed.

### Compressional Deformation

$$\varepsilon_{xx} = \frac{1}{2}\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_x}{\partial x}\right) = \frac{\partial u_x}{\partial x}$$

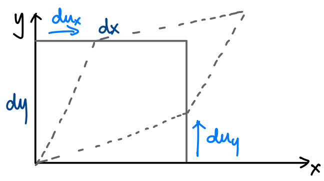
# same for $\varepsilon_{yy}$. $\varepsilon_{zz}$




only change the position of the surface, the angular is not change, so does the shape.

### Angular Deformation (Shear stress)

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right) \qquad \frac{\partial u_x}{\partial y} = \frac{\partial u_y}{\partial x} \neq 0$$



# Hook's Law

Linear relation between stress and strain (elastic medium)

stress, strain

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl}$$

elastic const, $3^4 = 81$ coeff

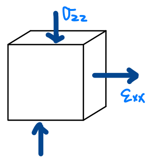
Due to reciprocity, number of independent coefficient reduce to 21

Reciprocity (相互作用)
$C_{klij} = C_{ijkl}$

For example:

$\sigma_{xx} = C_{xxxx}\,\varepsilon_{xx}$

$\sigma_{xy} = C_{xyxx}\,\varepsilon_{xx}$




$\sigma_{zz} = C_{zzxx}\,\varepsilon_{xx}$
$\sigma_{xx} = C_{xxzz}\,\varepsilon_{zz}$
Reciprocity

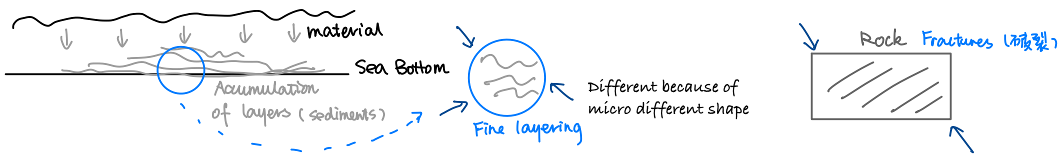
# Elastic Medium

**Isotropic Medium (各向同性介质)**
The elastic constants are all the same along the direction. (Elastic constants do not change with directions.)

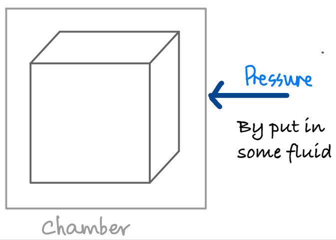
2 independent elastic constants
$C_{xxxx} = C_{yyyy} = C_{zzzz}$ (compressional stress)
$C_{xyxy} = C_{xzxz} = \cdots$ ( shear stress )

**Anisotropic medium**

例.

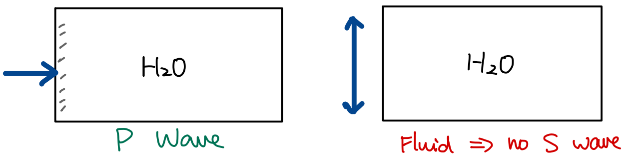



**How to measure?**

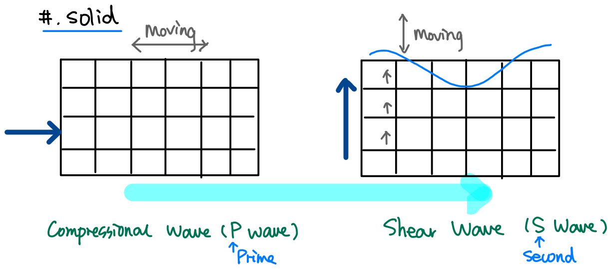


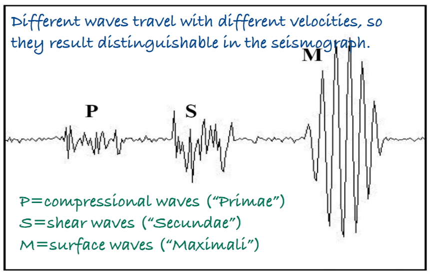

**What can be put into the container?**

\# Fluid { Liquid, Gas




\#. solid

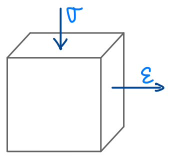


2021.09.21. Seismic method: wave equation

What kinds of medium do we consider?

Linear Elastic Medium




static scenerio

$\sigma = C\varepsilon$ (Hook's Law)

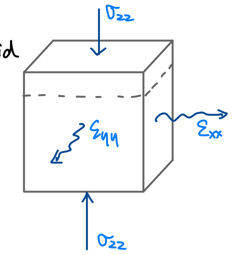
C: by medium

ISOTROPIC → 2 Elastic Consts → Compress, Shear

## Poisson Ratio (泊松比)

Poisson's ratio is a measure of the Poisson Effect, the deformation of a material in directions perpendicular to the specific direction of loading

Solid




Free Faces

$$\nu = -\frac{\varepsilon_{xx}}{\varepsilon_{zz}} = -\frac{\varepsilon_{yy}}{\varepsilon_{zz}}$$

$\varepsilon_{zz}$: direction of σ

In general, $-1 \le \nu \le \frac{1}{2}$ (strange!)

For most materials:

$0 \le \nu \le \frac{1}{2}$

0: Not conserved (cork); $\frac{1}{2}$: const volumn (clay)

## Young modulus (杨氏模量)

$$E = \frac{\sigma_x}{\varepsilon_x} = \frac{\sigma_y}{\varepsilon_y} = \frac{\sigma_z}{\varepsilon_z}$$

E: [Pa]; σ: [Pa]; ε: [Adim]

一般将杨氏模量习惯称为弹性模量，是材料力学中的名词。弹性材料承受正向应力时会产生正向应变，在形变量没有超过对应材料的一定弹性限度时，定义正向应力与正向应变的比值为这种材料的杨氏模量。

# Looking for E from table.

E.g. $E_{steel} = 2.1\times10^{11}$ Pa

$\sigma = C\varepsilon \rightarrow 1\,\mathrm{Pa} = C\varepsilon$

Measure rigidity, high C -> small ε

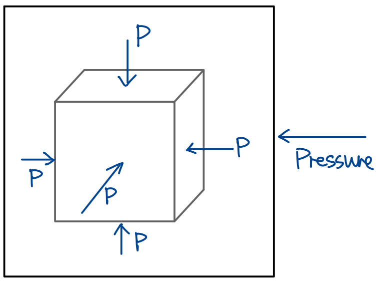
## Bulk modulus (体积弹性模量)

$$\boxed{k} = \frac{p}{\frac{\Delta V}{V}} = \frac{1}{3}\frac{E}{1-2\nu}$$

p: Pressure; V: volume; ν: Poisson Coefficient

E.g. $K_{steel} = 1.6\times10^{11}$ Pa

$K_{water} = 2.2\times10^{9}$ Pa




体积模量（K）也称为不可压缩量，是材料对于表面四周压强产生形变程度的度量。它被定义为产生单位相对体积收缩所需的压强。

# Lame parameter (拉梅常数)

$$\mu = \frac{\sigma_{ik}}{\varepsilon_{ik}} = \frac{E}{2(1+v)} \ [Pa] \ i \neq k$$

Shear Elastic Const (shear modulus) equals $c_{ikik}$

If $\mu$=0, medium is fluid (liquid+gases)

$$\lambda = \frac{vE}{(1+v)(1-2v)} = \text{Rigidity} = [Pa]$$

Note:
🌻 by now, we talk about the static scenarios (not changed by time)
🌻 Now we have equations between ν, E, k, μ, λ. If you know two of them, you can derive others.

(ν: Poisson; E: young; k: Bulk; μ, λ: lamé parameter)

# Wave Equation

Static Scenario : $\sigma = C\varepsilon$ (Hook's Law)
Dynamic Scenario : $\sigma(t)$

$\vec{F} = m\vec{a}$ (Newton Law)
mass = volume × density, $\rho$ = kg/m³
$\vec{F}$ ← $\sigma$; $\vec{a}$ ← $\frac{d^2\vec{u}}{dt^2}$ ← $\varepsilon$

2 Elastic Constants + Density = 3 Numbers
=> Needed by Dynamic Scenario

Elastic Const + Density

$$\boxed{V^2 \nabla^2 \Phi = \frac{d^2\Phi}{dt^2}}$$

Pressure, Displacement, Velocity

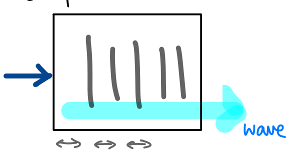
$$\Phi = \Phi(x, y, z, t) \Rightarrow \nabla^2\Phi = \frac{d^2\Phi}{dx^2} + \frac{d^2\Phi}{dy^2} + \frac{d^2\Phi}{dz^2}$$

Source / Forcing : $\Phi(x_0, y_0, z_0, t_0)$

Suppose: Volume Linear Elastic Isotropic Homogenous (do not change with position) Medium

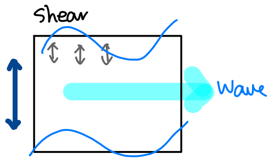
For one source => 2 solution for wave equation (2 waves with different velocity)

Compressional

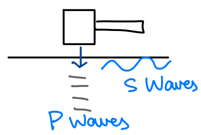



Particles oscillate // Direction of wave

Shear




particles oscillate ⊥ direction of wave




P Waves

Velocity $V_P = \alpha = \sqrt{\frac{\lambda+2\mu}{\rho}} = \sqrt{\frac{k+\frac{4}{3}\mu}{\rho}} = \sqrt{\frac{E(1+v)}{\rho(1+v)(1-2v)}}$

>

S Waves

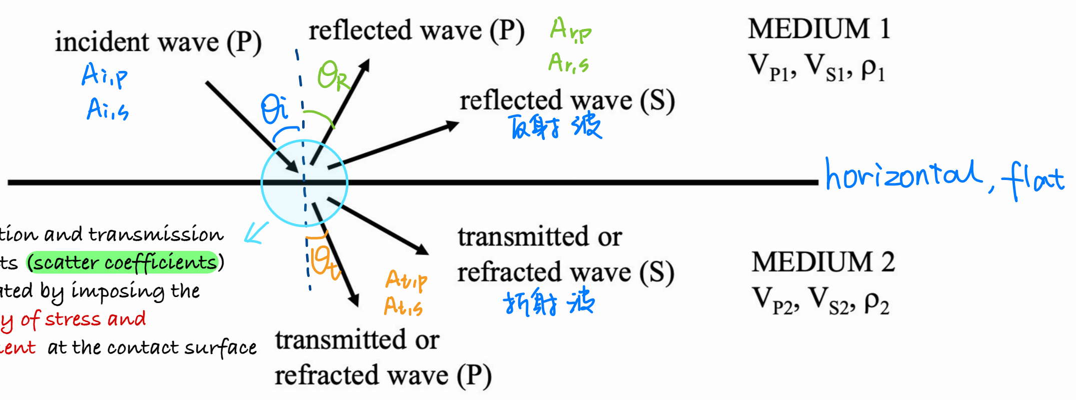
Velocity $V_S = \beta = \sqrt{\frac{\mu}{\rho}} = \sqrt{\frac{E(1+v)}{2\rho(1+v)}}$

Note : $0 \le v \le 0.5$

$$\sqrt{2} < \frac{V_P}{V_S} = \sqrt{\frac{2(1-v)}{1-2v}} < \infty$$

CASE: $\boxed{\mu = 0}$ [fluid] => Vs = 0 (no S Waves) => Only P waves (e.g. AIR)

2021.09.23. Seismic method: scatter coefficient, surface/guided waves

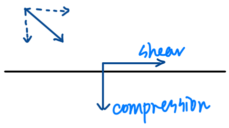



The reflection and transmission coefficients (scatter coefficients) are evaluated by imposing the continuity of stress and displacement at the contact surface

## Snell Law

$$\frac{\sin\vartheta_i}{V_i} = \frac{\sin\vartheta_r}{V_r} = \frac{\sin\vartheta_t}{V_t} = \text{const for all the waves (P, S)}$$

$\theta_{i,P} = \theta_{r,P} \neq \theta_{r,S}$ , $V_S < V_P$




In general the incident wave produces at the interface horizontal and vertical stresses, giving rise to reflected and transmitted waves of both P- and S-type (mode conversion).

## Scatter Coefficinets

The reflection (transmission) coefficients are the ratio between the amplitude of the reflected (transmitted) wave and the amplitude of the incident wave.

incident, reflected, Wave Amplitude

$$A_{r,P} = R_{P,P}A_{i,P} + R_{S,P}A_{i,S}$$

medium 2

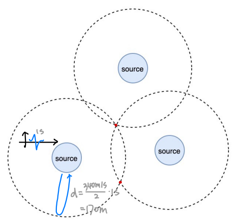
$$R = function(\boxed{V_{P1}, V_{S1}, P_1}, \boxed{V_{P2}, V_{S2}, P_2}, \theta)$$

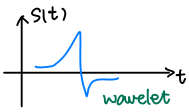
medium 1

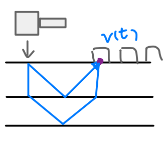
$$\left.\begin{array}{l}\begin{bmatrix} A_{r,P} \\ A_{r,S} \end{bmatrix} = \begin{bmatrix} R_{PP} & R_{SP} \\ R_{PS} & R_{SS} \end{bmatrix} \begin{bmatrix} A_{i,P} \\ A_{i,S} \end{bmatrix} \\ \begin{bmatrix} A_{t,P} \\ A_{t,S} \end{bmatrix} = \begin{bmatrix} T_{PP} & T_{SP} \\ T_{PS} & T_{SS} \end{bmatrix} \begin{bmatrix} A_{i,P} \\ A_{i,S} \end{bmatrix}\end{array}\right\} \text{SCATTER COEFFICIENT}$$

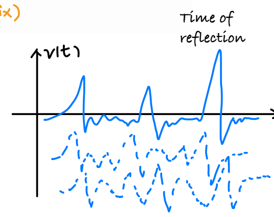
$\mathbb{R}$ (reflected matrix)

$\mathbb{T}$ (Transmission matrix)




$t \longrightarrow z$ (depth)

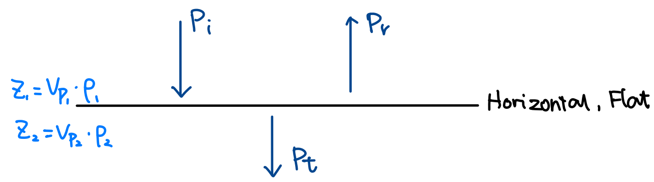
$\xrightarrow{\text{Processing}}$ Position of Reflectors

(velocity)

(seperate P and S)

↑ faster

## Scatter coefficients for normal incidence




For **normal incidence** (wave direction perpendicular to the interface):
* shear waves generate only shear stress at the interface,
* compressional wave generate only compressional stress at the interface
=> **no mode conversion**

$Z = V \cdot \rho$

$$R = \frac{Z_2 - Z_1}{Z_2 + Z_1} \quad \text{with} \quad \begin{cases} Z_n = V_{Pn}\rho_n & \text{for } R_{PP} \\ Z_n = V_{Sn}\rho_n & \text{for } R_{SVSV} \text{ and } R_{SHSH} \end{cases}$$

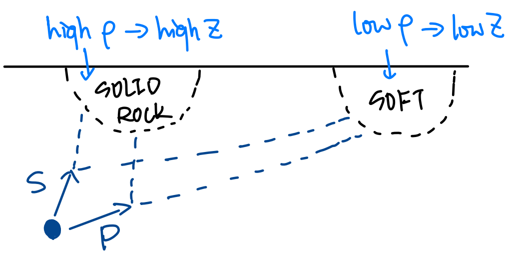
$$T = \frac{2Z_1}{Z_2 + Z_1} \qquad Z = \text{medium (P or S) impedance}$$

阻抗

例1. AIR / WALL: $Z_{AIR} = 340 \frac{m}{s} \cdot 1 \frac{kg}{m^3}$, $Z_{WALL} = 3000\ m/s \cdot 3000\ kg/m^3$

$Z_2 >> Z_1$: $R \simeq \frac{Z_2}{Z_2} \simeq 1$, $T \simeq 0$

WALL / AIR: $Z_1 >> Z_2$: $R \simeq -\frac{Z_1}{Z_1} \simeq -1$ (neg), $T \simeq \frac{2Z_1}{Z_1} \simeq 2$

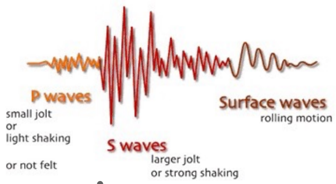



**Short Summary:**
* Z2 >> Z1: R–>1, T–> 0
* Z2 = Z1: R=0, T=1
* Z1 >> Z2: R–>–1, T–>2

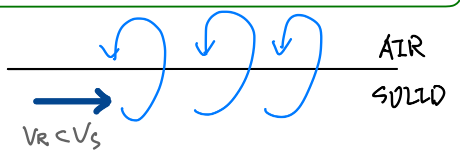
The amplitude of the reflected wave is smaller (at least equal) than the amplitude of the incident wave.
The amplitude of the transmitted wave can be higher than the amplitude of the incident wave

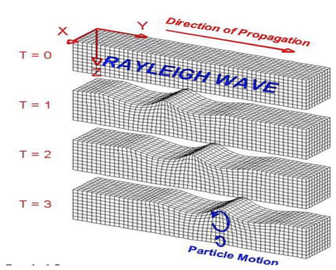
## Surface Waves

a wave that is guided along the interface between two different media.

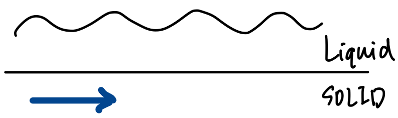



A **Rayleigh wave** rolls along the ground just like a wave rolls across a lake or an ocean. Because it rolls, it moves the ground up and down, and side-to-side in the same direction that the wave is moving. Most of the shaking felt from an earthquake is due to the Rayleigh wave, which can be much larger than the other waves.

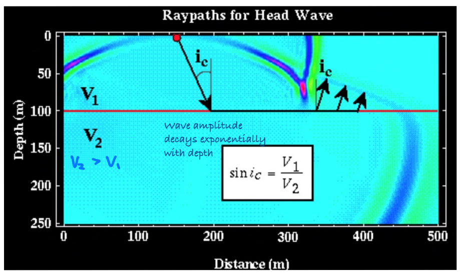



**Scholte-Stoneley waves**: rayleigh-like waves that propagate along a solid-fluid interface, such as along the walls of a fluid-filled borehole (Stoneley) or along the sea bottom (Scholte).




**Refracted waves** (head waves/critical wave)



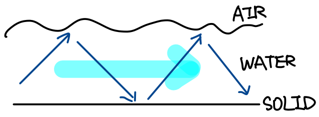

By SNELL LAW:

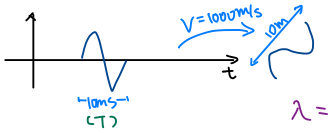
$$\frac{\sin\theta_i}{V_1} = \frac{\sin\theta_t}{V_2} \quad \text{and} \quad \theta_t = \frac{\pi}{2}$$

$$\Rightarrow \frac{\sin\theta_i}{V_1} = \frac{1}{V_2}$$

$$\theta_i = \operatorname{asin}\frac{V_1}{V_2} = \theta_{cr} = \text{Critical Angle}$$

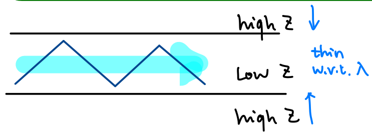
## Guided wave

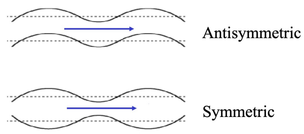


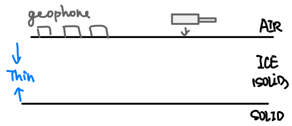




$\lambda = VT$ = wavelet length

**Lamb Waves:** For an object sufficiently thin to allow penetration to the opposite surface, e.g. a plate having a thickness of the order of a wavelength or so, Rayleigh waves degenerate to Lamb waves, which can propagate in a number of modes, either symmetrical or antisymmetrical.

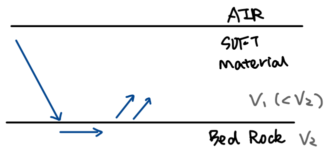


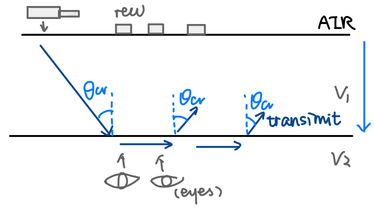



2021.09.28. Seismic method, refracted wave. Bandwidth of a signal/system, resolution

## Typical Situation



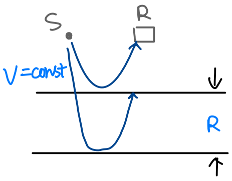

$\Rightarrow$

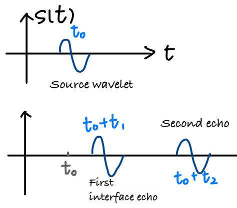



Refraction Seismic Method

## Resolution

Duration of wavelet $\Delta t$




$\Delta t = t_2 - t_1$
> Duration of wavelet

## Bandwidth

**Bandwidth of System:** range of frequencies that the system is able to record/process ("hear")

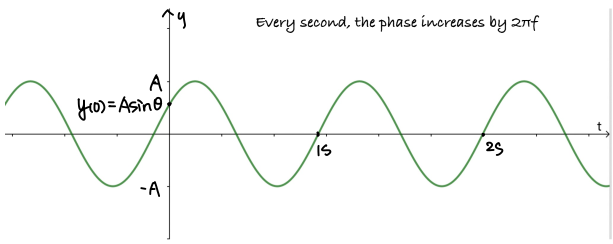
Number of Period in 1s (const)

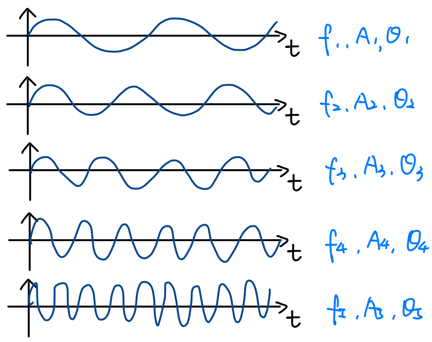
Phase (Angle)

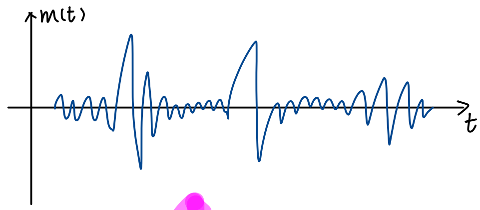
$$y(t) = A \sin(\boxed{2\pi f \cdot t + \theta})$$

Amplitude; time in second (const); Initial Phase (const)

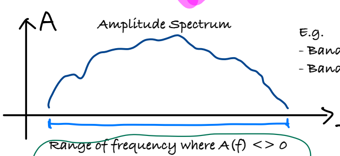
$f$ = frequency = $[\frac{1}{s}]$ = [Hz]   f越大，声音越尖锐



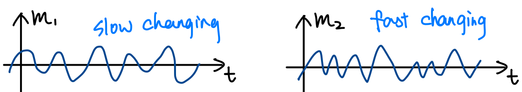




Fourier Transform



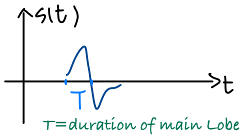

Bandwidth of Signal

E.g.
- Bandwidth of EAR: 20 Hz ~ 20000Hz
- Bandwidth of Voice



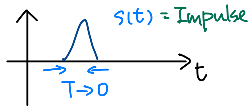

$B_1 < B_2$

fast (changing) signal -> high bandwidth
slow (changing) signal -> low bandwidth

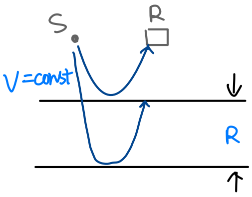



T = duration of main Lobe

$$\boxed{T \propto \frac{1}{B}}$$




E.g. Hitting Table : 100 ~ 1000 (range of f) → $B \simeq 1kHz$ → $T \simeq \frac{1}{1000}s = 1ms$



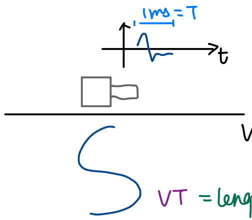

NB : $R \doteq \lambda \doteq VT \doteq \frac{V}{B}$

$\Delta t = t_2 - t_1 = \frac{R}{V} \cdot 2 > \frac{T}{2}$ ← Wave Overlap

$$\boxed{R \geq \frac{V \cdot T}{4} = \frac{\lambda}{4}}$$

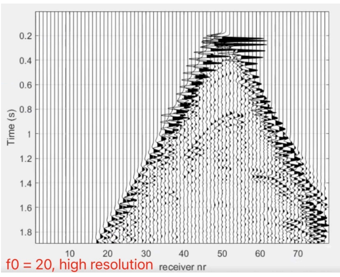
High resolution -> small R -> high B -> low T and low V

e.g.

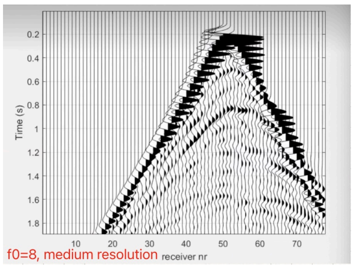



V = 1000m/s

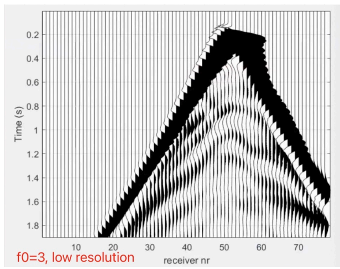
VT = length of wavelet = $\lambda$



f0 = 20, high resolution



f0=8, medium resolution

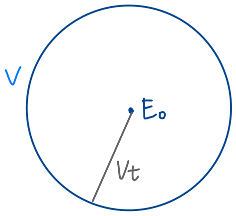


f0=3, low resolution

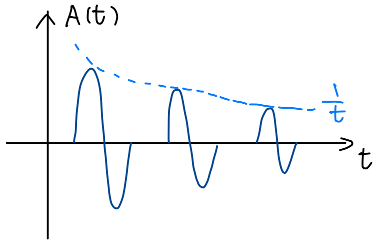
2021.09.30. Seismic method: attenuation

## Geometrical Divergence

### 3D Propagation (P, S)




SPERE = $(RADIUS)^2$
$\div (Vt)^2$




Wave energy decays as pow(R, -2)

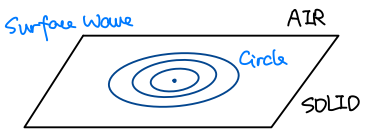
Energy ↓ $E_{Geosphere} \div$ Total Energy ↓ $\frac{E_0}{surface} = \frac{E_0}{(radius)^2} \Rightarrow \boxed{E \div \frac{1}{(radius)^2}}$

$E_x = \int_{-\infty}^{+\infty} |\underset{\uparrow \text{Pressure}}{x(t)}|^2 dt \Rightarrow E \div \underset{\text{Amplitude}}{A^2}$

Amplitude decays as pow(R,-1)

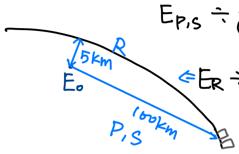
$\boxed{A \div \frac{1}{radius}}$

### 2D Propagation




$\boxed{A \div \frac{1}{\sqrt{radius}} \quad E \div \frac{1}{radius}}$

例1.



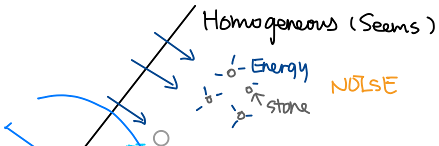

⇑

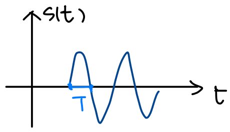
$E_{P,S} \div \frac{E_0}{(100km)^2} = \frac{1}{10000}$

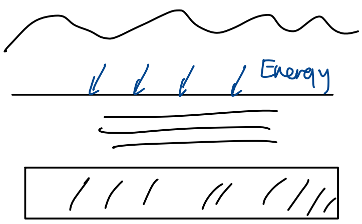
$\Leftarrow E_R \div \frac{E_0}{(5km)^2} \cdot \frac{1}{100km} = \frac{1}{2500}$

## Scattering of Small Heterogeneities (MIE scattering)

当微粒半径的大小接近于或者大于入射光线的波长λ的时候，大部分的入射光线会沿着前进的方向进行散射，这种现象被称为米氏散射。这种大微粒包括灰尘，水滴，来自污染物的颗粒物质，如烟雾等。即是形成所谓的廷得耳效应。



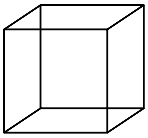



## Absorption

$\div$ Frequency

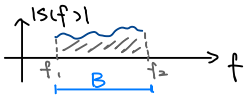
Elastic $\rightarrow$ Heat

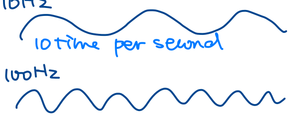
$\longrightarrow$ Frictions



$\Delta E$ = Energy lost on 1 compression

$S(t) \Longleftrightarrow$




| | Frequency | Energy |
|---|---|---|
| 10Hz, 10 time per second |  | $10\Delta E$ |
| 100Hz | | $100\Delta E$ |

Wave amplitude decays exponentially (absorbing media)

$$\text{Wave Amplitude} = e^{-\alpha x}, \quad \alpha = \frac{\pi}{\lambda Q}$$

$x$: Distance (radius)

$\alpha$: Absorption coeff

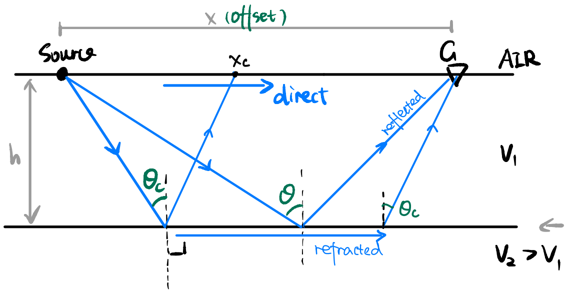
$Q$: Quality Factor

예. $Q_{WATER} = 10000$

$Q_{SOFT\text{-}SOIL} = 10$

**Dispersion**: the frequency components of the wave are attenuated differently. Higher frequencies are more attenuated

2021.10.05. Seismic method: refraction seismic




$x_c$ = Critical Distance. First appear of First Arrival.

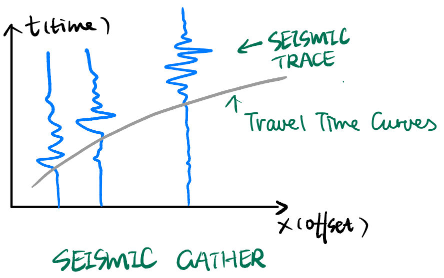
**For Direct**

$t_{direct}(x) = \frac{x}{V_1}$ ⇒ Linear

**For Reflected**

hyperbola (双曲线)

$t_{reflected} = \frac{1}{V_1} \cdot 2\sqrt{h^2 + (\frac{x}{2})^2} = \frac{1}{V_1}\sqrt{(2h)^2 + x^2} = \frac{2}{V_1} \cdot \frac{h}{\cos\theta}$




👈 one of the goal of the lesson

$\lim_{x \to \infty} t_{reflected}(x) \simeq \frac{x}{V_1}$

$t_{reflected}(x=0) = \frac{2h}{V_1}$

**For Refracted**

$t_{refracted}(x) = \underbrace{\frac{2}{V_1} \cdot \frac{h}{\cos\theta_c}}_{\text{Time in First Layer (const)}} + \underbrace{\frac{x - x_c}{V_2}}_{\text{Time in 2nd Layer}}$ (const)

# only for $x_c < x$

$x_c = 2h\tan\theta_c$ = Critical Distance

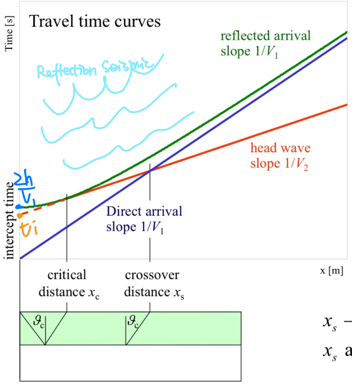
$\frac{\sin\theta_c}{V_1} = \frac{\sin\frac{\pi}{2}}{V_2} \Rightarrow V_2 = \frac{V_1}{\sin\theta_c}$

$\Rightarrow t_{refracted}(x) = \frac{2}{V_1} \cdot \frac{h}{\cos\theta_c} + \frac{x}{V_2} - \frac{x_c}{V_2}$

$= \frac{2}{V_1}\frac{h}{\cos\theta_c} - \frac{2h\tan\theta_c \sin\theta_c}{V_1} + \frac{x}{V_2}$

$= \frac{2h}{V_1} \cdot \frac{1}{\cos\theta_c}[1 - \sin^2\theta_c] + \frac{x}{V_2}$

$= \underbrace{\boxed{\frac{2h\cos\theta_c}{V_1}}}_{t_i} + \frac{x}{V_2}$ ⇒ linear




$x_s \rightarrow$

$x_s$ alv

$x_s$ = crossover distance (交叉距离)

$t_{direct}(x_s) = t_{refracted}(x_s)$

$\rightarrow x_s = 2h\sqrt{\frac{V_2+V_1}{V_2-V_1}}$ $(2h < x_s < \infty)$

$t_i$ = intercept time, measured by Ruler

$t_i = \frac{2h\cos\theta_c}{V_1}$

## How to obtain h, V1, V2?

1. COLLECT a seismic gather (1 source + many receivers)
2. on seismic gather, for each trace, MARK the "first arrival" (Picking)
3. Connect points (in step2) and Look for different slops
4. First Slope:

$$\frac{\Delta x_1}{\Delta t_1} = V_1$$

5. Second Slope:

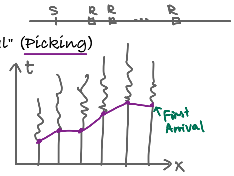
$$\frac{\Delta x_2}{\Delta t_2} = V_2$$

6. Critical Angle:

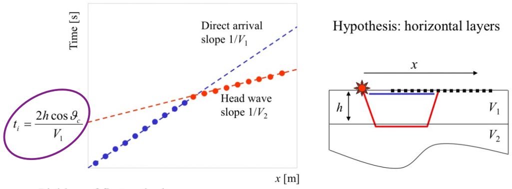
$$\theta_c = \sin^{-1}\frac{V_1}{V_2}$$

7. Time intercept method:

$$h = \frac{t_i}{2}\frac{V_1}{\cos\theta_c}$$



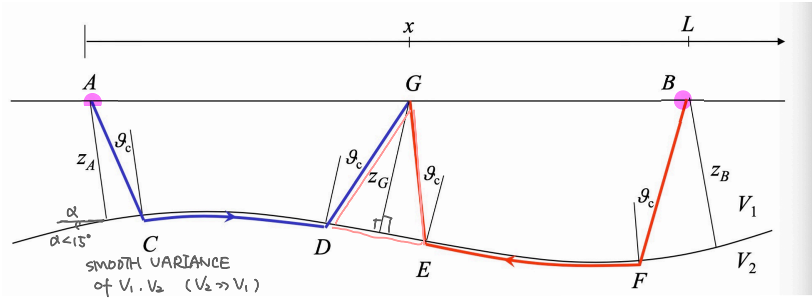

Refraction profiling: time intercept method




- Picking of first arrival
- $V_1$ from slope of direct arrival curve or on-situ measures
- $V_2$ from slope of head wave arrival curve
- $\vartheta_c$ from $V_1$ and $V_2$ $\Rightarrow \vartheta_c = \sin^{-1}[V_1/V_2]$
- $t_i$ by continuation of head wave arrival line

$$\Rightarrow h = \frac{t_i}{2}\frac{V_1}{\cos\vartheta_c}$$

## Reciprocal methods: PLUS-MINUS



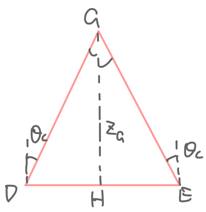

A, B = reciprocal shots

$Z_G(x)$
$V_1(x)$, $V_2(x)$
$\theta_c(x)$

UNKNOWN

// From A: A → C → D → G
// From B: B → F → E → G

$$\begin{cases} t_{AG} = \dfrac{AC}{V_1} + \dfrac{CD}{V_2} + \dfrac{DG}{V_1} \\ t_{BG} = \dfrac{BF}{V_1} + \dfrac{EF}{V_2} + \dfrac{EG}{V_1} \\ t_{AB} = t_{BA} \quad \text{(Reciprocity)} \end{cases}$$




$$T^+(x) = \frac{1}{2}(t_{AG} + t_{BG} - t_{AB})$$

time PLUS

$$= \frac{1}{2}\left[\left(\frac{AC}{V_1} + \frac{CD}{V_2} + \frac{DG}{V_1}\right) + \left(\frac{BF}{V_1} + \frac{EF}{V_2} + \frac{EG}{V_1}\right) - \left(\frac{AC}{V_1} + \frac{CD}{V_2} + \frac{DE}{V_2} + \frac{EF}{V_2} + \frac{BF}{V_1}\right)\right]$$

$$= \frac{1}{2}\left[\frac{DG}{V_1} + \frac{EG}{V_1} - \frac{DE}{V_2}\right]$$

$$= \frac{1}{2}\left[\frac{2EG}{V_1} - \frac{2DH}{V_2}\right]$$

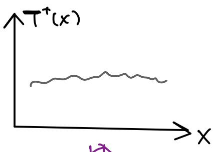
$$= \frac{EG}{V_1} - \frac{DH}{V_2}$$

$$= \frac{Z_G}{V_1\cos\theta_c} - \frac{Z_G\tan\theta_c}{V_2}$$

$$\frac{\sin\theta_c}{V_1} = \frac{1}{V_2} \Rightarrow V_2 = \frac{V_1}{\sin\theta_c}$$

$$= \frac{Z_G}{V_1\cos\theta_c} - \frac{Z_G}{V_1}\,\frac{\sin^2\theta_c}{\cos\theta_c}$$

$$= \frac{Z_G\cos\theta_c}{\boxed{V_1}} \leftarrow V_1(x)$$




time MINUS

$$T^-(x) = \frac{1}{2}\left[t_{AG} - t_{BG} + t_{AB}\right]$$

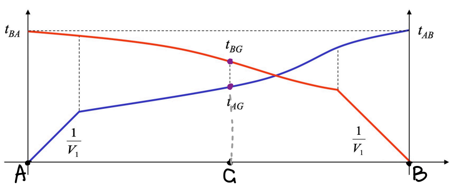
$$= \frac{1}{2}\left[\left(\frac{AC}{V_1} + \frac{CD}{V_2} - \frac{DG}{V_1}\right) - \left(\frac{BF}{V_1} + \frac{EF}{V_2} - \frac{EG}{V_1}\right) + \left(\frac{AC}{V_1} + \frac{CD}{V_2} + \frac{2DH}{V_2} + \frac{EF}{V_2} + \frac{BF}{V_1}\right)\right]$$

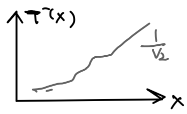
$$= \frac{1}{2}\left[\frac{2AC}{V_1} + \frac{2CD}{V_2} + \frac{2DH}{V_2}\right]$$

$$\overset{\text{Const}}{=} \boxed{\frac{AC}{V_1}} + \frac{CH}{V_2}$$

} Based on the assumption that α<15°

$$\cong \text{const} + \boxed{\frac{x}{V_2}}$$

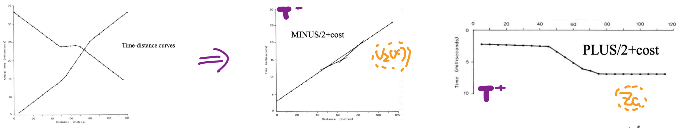


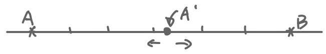
Step

1. COLLECT traces for at least 2 shots reciprocal
2. PICKING of first arrivals from A and from B
3. COMPUTE $T^+(x)$ and $T^-(x)$ ?

$\uparrow z_G$ $\uparrow V_2$


4. Obtain $V_1(x)$ with local measurements (extra shots)



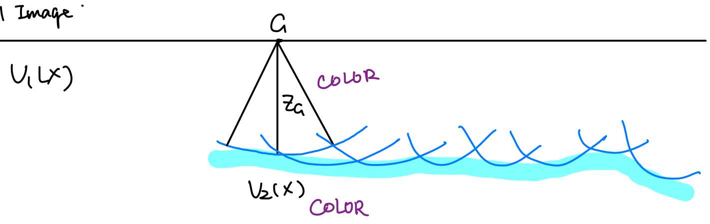

5. From slope of $T^-(x) \to V_2(x)$ (high slope $\to$ Low $V_2$)

6. $\theta_c(x) = \operatorname{asin} \dfrac{V_1(x)}{V_2(x)}$

7. From $T^+(x) \Rightarrow z_G(x) = \dfrac{T^+(x)\, V_1(x)}{\cos \theta_c(x)}$

Inaccuracies of PLUS-MINUS are due to ambiguity between (first layer) thickness and (second layer) velocity variations along the segment DE

Final Image:



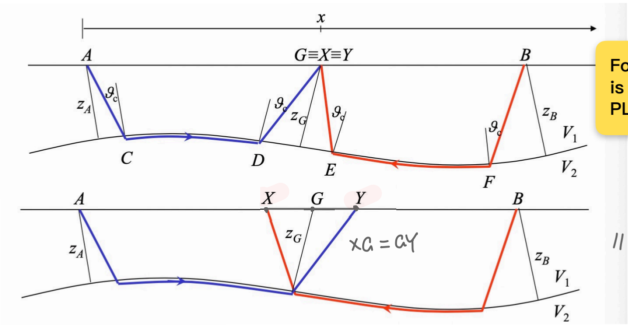

Envelope

Position OF Interface

In physics and engineering, the envelope of an oscillating signal is a smooth curve outlining its extremes.

## Generalized Reciprocal Method (GRM)

GRM looks for optimum XY, under the hypothesis that lateral variations of V2 are slower than the variations of the thickness of the first layer.

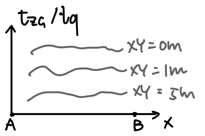



For XY=0, GRM is equivalent to PLUS–MINUS

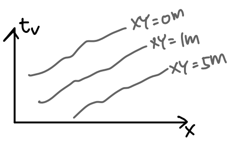
// $t_{AG} \to t_{AY}$

$t_{BG} \to t_{BX}$

Procedure:
- Build $t_g$ and $t_v$ curves for different values of $XY$
- Compute $V_1$ from slope of direct arrivals or other measurements
- Obtain $XY_{opt}$ from examination of $t_g$ and $t_v$ curves: $XY$ value with minimum lateral variation of $V_2$ (that is the "straigthest" $t_v$ curve)
- Compute $V_2$ from the curve $t_v(XY_{opt})$
- Obtain $\vartheta_c$ from $V_1$ e $V_2$
- Obtain $z_g$ from $t_g$ curve




$\Leftarrow$




$\Leftarrow$

Case:

If $V_2(x)$ is sharp, choose sharp one from $x$-$t_v$, then choose $T^+$ accordingly

$T^+ \to t_{zG}(XY)$

$\uparrow$ many $t_{zG}$

$t_{zG} = \frac{1}{2}[t_{AY} + t_{BX} - t_{AB}]$

$\Rightarrow$ Time Depth Function $\Rightarrow z_g$

$t_v = \frac{1}{2}[t_{AY} - t_{BX} + t_{AB}]$

$\Rightarrow$ velocity analysis function

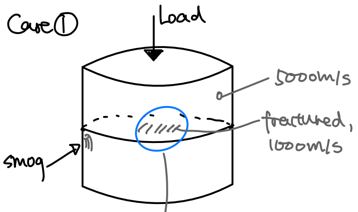
$\Rightarrow V_2$

// For more details, refer to "Slide 04"

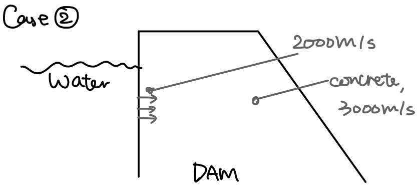
## 2021.10.14. Travel time tomography (X线断层摄影术)

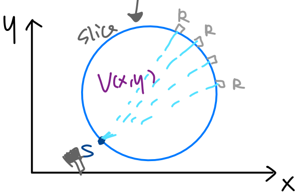
# See experiments in 2021.11.23

Case ①




Case ②

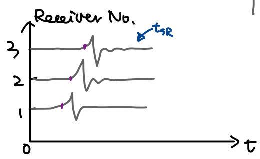



// One Source A Time

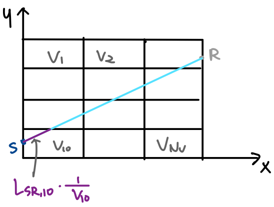
### Time Tomography

Measurement: $t_{SR}$ $\quad \begin{cases} S = \text{source} = 1 \ldots N_S \\ R = \text{receiver} = 1 \ldots N_R \end{cases}$




# of Measurement $= N_S \times N_R$

Gridde $V(x,y)$




$$\underset{\text{measure}}{t_{SR}} = \sum_{n=1}^{N_V} \underset{\text{geometry}}{L_{SR,n}} \cdot \underbrace{\frac{1}{V_n}}_{\text{unknown}}$$

(Cell $N_R$; linearize)

$$= \sum_{n=1}^{N_V} L_{SR,n} \cdot Q_n$$

$$\boxed{Q = \frac{1}{V} = \text{slowness} = [\tfrac{s}{m}]}$$

$$\underset{(N_S \cdot N_R,\,1)}{\begin{bmatrix} t_{11} \\ t_{12} \\ \vdots \end{bmatrix}} = \underset{(N_S \cdot N_R,\,N_V)}{\begin{bmatrix} L_{11,1} & L_{11,2} & \cdots & L_{11,N_V} \\ L_{12,1} & L_{12,2} & \cdots & L_{12,N_V} \\ & \text{History of Ray } S \to R & & \end{bmatrix}} \underset{(N_V,\,1)}{\begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_{N_V} \end{bmatrix}}$$

$$T = A \quad Q$$

$\xrightarrow{\text{Inversion}}$ $Q = A^{-1}T$ ← solution of the inverse problem (ill conditioned) $\Rightarrow V = \frac{1}{Q}$

$\xrightarrow{\text{Least Squares}}$ $Q = (A^TA)^{-1}A^TT$

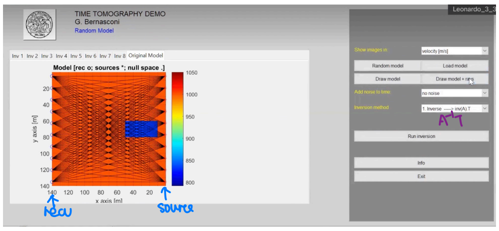
### Regularization of Inverse Problem

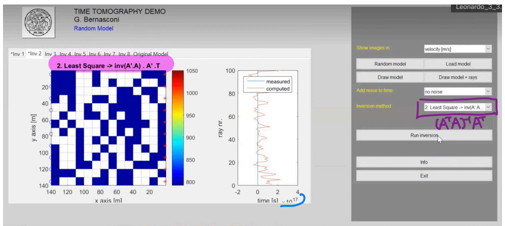
The standard remedy is to impose an additional **minimum-energy** requirement which, in effect, regularizes the solution.

$T = AQ + \underbrace{\cdots}_{\text{constraints.}}$ e.g. minimal derivatives

$\Rightarrow Q = (A^TA + \lambda \cdot I)^{-1}A^TT$

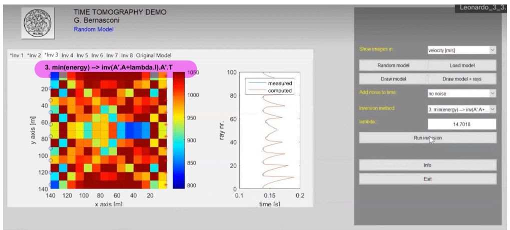
Note: The direction w.r.t the exploration limits the possibility to see one direction with the other direction. So the better solution is to look around, that is, to have the receivers in the positions around the body.

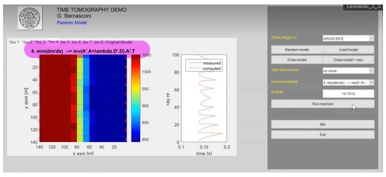


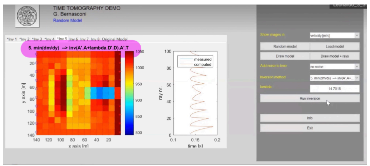


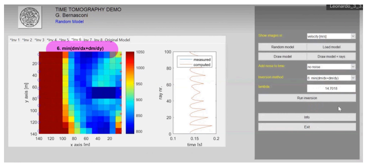
CASE: one direction, from right (source) to left (rec)

* inversion method => Matrix not square, cannot compute inverse
* least square method => do not work because some of the cells are unreliable, they produce noise. This solution is unstable (see the time up to 10+e17) in this case
* min(energy) => The result is not bad.

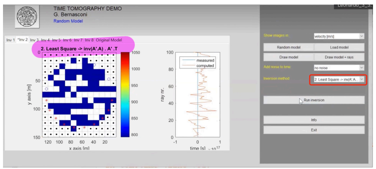


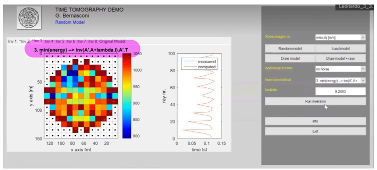






CASE: the model is a circle and close to the border





2021.10.19. 1st homework

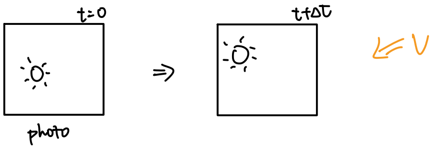
$$V^2 \nabla^2 \Phi = \frac{d^2\Phi}{dt^2}$$

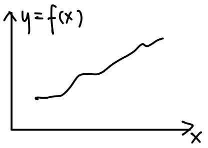
velocity

$$\frac{d^2\Phi}{dx^2} + \frac{d^2\Phi}{dy^2} + \frac{d^2\Phi}{dz^2}$$

$\Phi$ = pressure = $\Phi(x,y,z,t)$

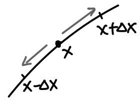
$V = V(x,y,z)$




$$y' = f'(x) = \frac{f(x+dx)-f(x)}{dx}$$ for $dx \to 0$ # slope

$$\xrightarrow{dx \simeq \Delta x} \quad y' \simeq \frac{f(x+\Delta x)-f(x)}{\Delta x}$$ ⇐ Finite Difference Approximation of Derivative




$$y''(x) = \frac{y'(x+\Delta x) - y'(x)}{\Delta x} = \frac{f(x+\Delta x) - 2f(x) + f(x-\Delta x)}{(\Delta x)^2}$$ ⇐ Centered Formula

$$\longrightarrow \quad V^2 \nabla^2 \Phi = \frac{d^2\Phi}{dt^2}$$

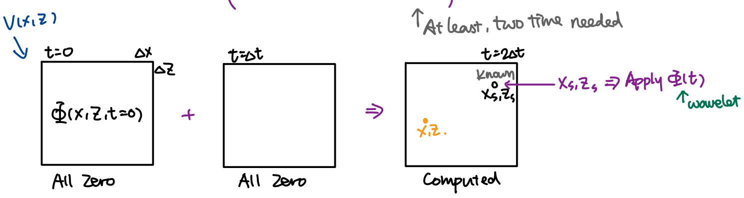
$$V^2(x,y,z)\left[\begin{array}{c} \frac{\Phi(x+\Delta x,y,z,t) - 2\Phi(x,y,z,t) + \Phi(x-\Delta x,y,z,t)}{\Delta x^2} \\ \frac{\Phi(x,y+\Delta y,z,t) - 2\Phi(x,y,z,t) + \Phi(x,y-\Delta y,z,t)}{\Delta y^2} \\ \frac{\Phi(x,y,z+\Delta z,t) - 2\Phi(x,y,z,t) + \Phi(x,y,z-\Delta z,t)}{\Delta z^2} \end{array}\right] = \frac{\Phi(x,y,z,t+\Delta t) - 2\Phi(x,y,z,t) + \Phi(x,y,z,t-\Delta t)}{\Delta t^2}$$

2D version → x, z, t

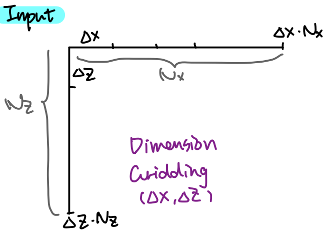
$$\Phi(x,y,z,t+\Delta t) = 2\Phi(x,y,z,t) - \Phi(x,y,z,t-\Delta t) + \Delta t^2 \underline{V^2(x,y,z)} \left[\begin{array}{c} \frac{\Phi(x+\Delta x,y,z,t) - 2\Phi(x,y,z,t) + \Phi(x-\Delta x,y,z,t)}{\Delta x^2} \\ \frac{\Phi(x,y+\Delta y,z,t) - 2\Phi(x,y,z,t) + \Phi(x,y-\Delta y,z,t)}{\Delta y^2} \\ \frac{\Phi(x,y,z+\Delta z,t) - 2\Phi(x,y,z,t) + \Phi(x,y,z-\Delta z,t)}{\Delta z^2} \end{array}\right]$$

$$= f\left[\Phi\begin{pmatrix} x+\Delta x & z+\Delta z & \\ x & & t \\ x-\Delta x & z-\Delta z & t-\Delta t \end{pmatrix}\right]$$

↑ At least, two time needed




**Input**




↙ matrix $(N_x, N_z)$

$V(x,z)$

$\underbrace{x_s,\ z_s}$ Source Position ⇓ $\Phi_s(t)$

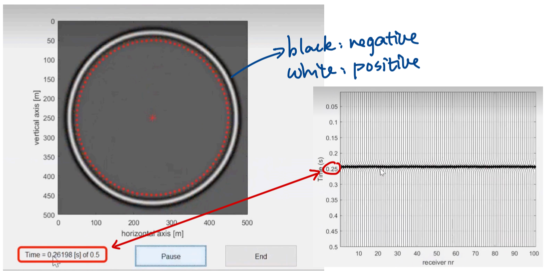
Time Step ↓ $[\ \Delta t\ ]$ computed

$t_{max}$ ↑ total time

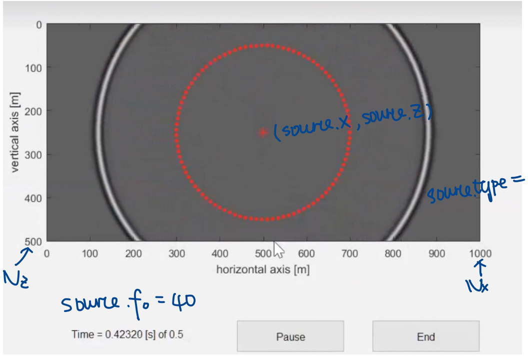
Receivers

$\overbrace{x_R,\ z_R}$ ⇓ $\Phi(x_R, z_R, t)$

Experiment part




black: negative
white: positive




```
% 1. Model parameters in structure 'model'
%
% Compulsory parameters
% model.x: vector of x grid coordinates [m], Nx elements
% model.z: vector of z grid coordinates [m], Nz elements
% model.vel: matrix of velocity values [m/s], (Nz,Nx) elements   V(x,z)
%
% Optional parameters
% model.recx: vector of x coordinates of receivers [m], Nr elements   XR
% model.recz: vector of z coordinates of receivers [m], Nr elements   ZR
% model.dtrec: max time sampling interval for seismic traces [s]   Δt
%
```

```
% 2. Source parameters in structure 'source'
% source coordinates are rounded to nearest grid point
% all of them can be **vectors**, in order to simulate multiple sources
%
% Compulsory parameters
% source.x:  x coordinate of source [m]   Xs
% source.z:  z coordinate of source [m]   Zs
% source.f0: central frequency of source Ricker wavelet [Hz]
% source.t0: time of source emission (referred to max peak of Ricker)
% source.type: 1 is Ricker, 2 is sinusoid at frequency source.f0
% source.amp: multiplier of source amplitude   A
%
```

```
% 3. Simulation and graphic parameters in structure 'simul'
%
% simul.timeMax:     max simulation time [s]   tmax
% simul.borderAlg:   Absorbing boundaries (Yes:1, No:0)
% simul.printRatio:  pressure map shown every printRatio comput. time steps
% simul.higVal:  colormap between -highVal and + highVal (from 0 to 1)
% simul.lowVal:  values between -lowVal and +lowVal zeroed (from 0 to 1)
% simul.bkgVel:  velocity matrix as  a "shadow" in the images (1:yes, 0:no)
%
% Optional parameters
% simul.cmap:    colormap (default 'gray')
%
```

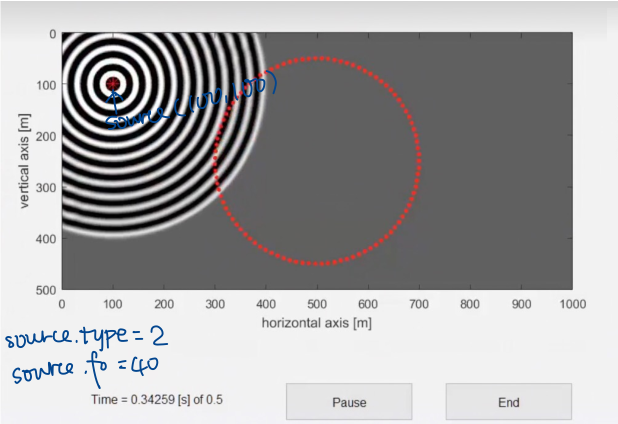
```
% 4. Acoustic simulator program call
%
% recfield=acu2Dpro(model,source,simul);
%
%  recfield.time: time axis of recorded signal [s], Nt elements
%  recfield.data: matrix of pressure at the receivers, (Nt,Nr1)
%  recfield.recx: vector of x grid coordinates of receivers [m], Nr1 elements
%  recfield.recz: vector of z grid coordinates of receivers [m], Nr1 elements
%
```

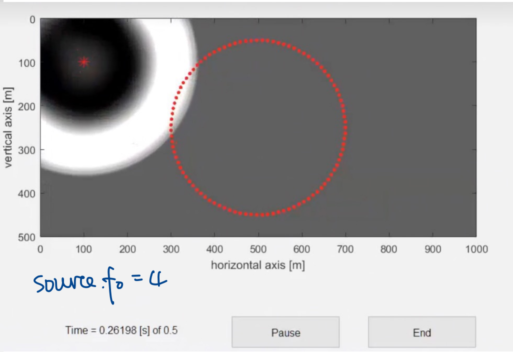
```
% 5. Plotting seismic trace program call
%
% seisplot2(recfield.data,recfield.time)
%
%   [fact]=seisplot2(datain,t,tr,scal,pltflg,scfact,colour,clip)
%
%   function for plotting seismic traces
%
%   INPUT
%   datain  - input matrix of seismic traces
%   t       - time axis
%   tr      - trace axis
%   scal    - 1 for global max, 0 for global ave, 2 for trace max
%   pltflg  - 1 plot only filled peaks, 0 plot wiggle traces and filled peaks,
%             2 plots wiggle traces only, 3 imagesc gray, 4 pcolor gray
%   scfact  - scaling factor
%   colour  - trace colour, default is black
%   clip    - clipping of amplitudes (if <1); default no clipping
%
%   OUPTPUT
%   fact    - factor that matrix was scaled by for plotting
%   if you want to plot several matrices using the same scaling factor,
%   capture 'fact' and pass it as input variable 'scal' to future matrices
%   with 'scfact' set to 1
%
```

```
% 6. Essential theory
%
%  Sampling interval for stability (computed automatically)
%  dt=0.95*sqrt(1/2)*min(dx,dz)/vMax
%
%  Courant-Friedrick-Levy (CFL) stability condition (display warning)
%  dx<(vMin/(f0*2*8)))
%  dz<(vMin/(f0*2*8)))
%
%  ----------
%  Ricker wavelet
%
%  positive (white)  *                       !    ***
%                   * *                      !   * ! *
%    ___________*___*__________ T            !  *  !   *
%    ********   *     *   *******            ! *   !     *
%  negative  ***        ***                  !*    !      * * * *
%  (black)     >   TD    <                   ------+---------------- F
%                                                  F0
%
%    s(t) = (1-Y2*T*T)*exp(-Y2*T*T/2)   S(f) = 2*F^2/(f0^3*sqrt(pi))*
%    Y2 = 2*pi^2*f0^2                          *exp(-F*F/(f0^2))
%    TD = sqrt(6)/(pi*f0)
%    f0 ↑ , TD ↓
%  ----------
```

Note:

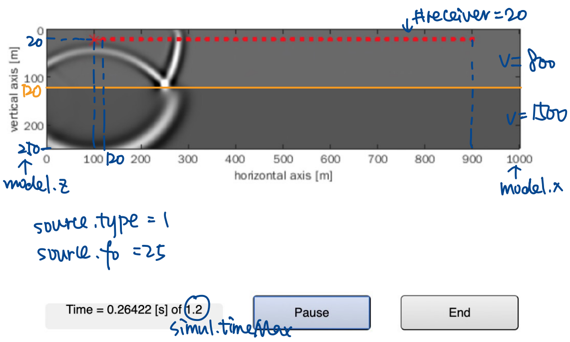
1. higher source.f0, shorter wavelet
2. slower velocity, more compact wavelet

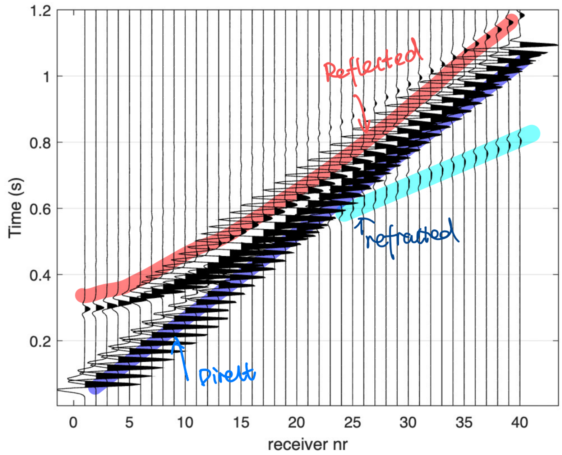


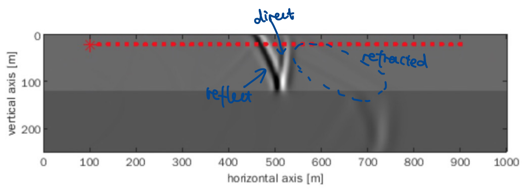



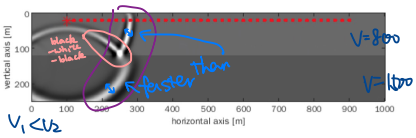

A01_example

## A02_example $(V_2 > V_1)$

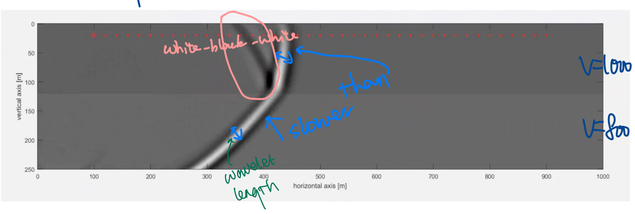



More (if $V_1 > V_2$)




NB: the wave changes sign because of reflected coefficients

From high speed to low speed:
- reflected wave: change sign
- refracted wave: sign does not change

From low speed to high speed:
- reflected wave: not change sign
- refracted wave: not change sign

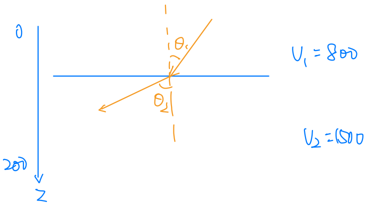
How to produce a plane wave? See A03_example

# Tips for B03_exercise

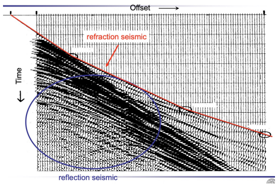

Design an example with 2 layers, and build a plane wave by using a line of sources. Now change the incident angle of the plane wave by activating the sources in the line delayed in sequence. Check the Snell law.

$$\frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{V_2}$$

$V_1 < V_2 \Rightarrow \theta_1 < \theta_2$



# 2021.10.21. Reflection seismic

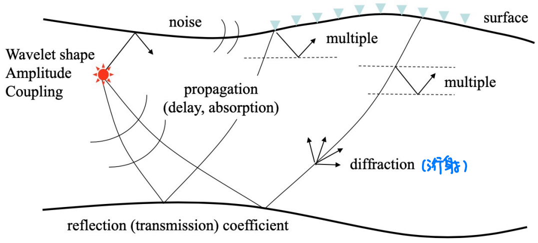



Seismic
- refraction
- time tomography

Need to detect first arrivals

- reflection: generation, recording and processing of seismic waves reflected by elastic discontinuities (弹性不连续面) to obtain a map (position) of the reflectors and of the velocity model.

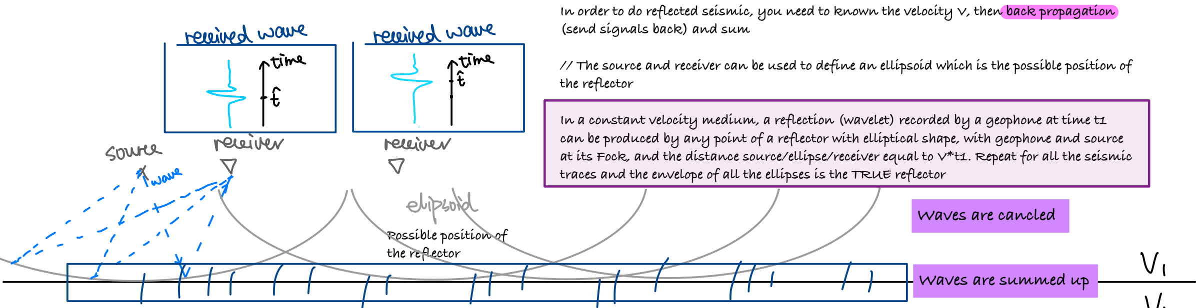
Factors influencing shape and amplitude of the reflected seismic signal:




In order to do reflected seismic, you need to known the velocity v, then back propagation (send signals back) and sum

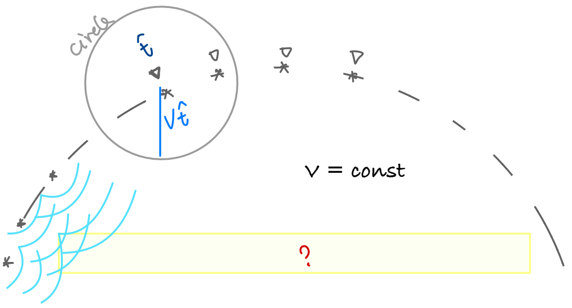
// The source and receiver can be used to define an ellipsoid which is the possible position of the reflector

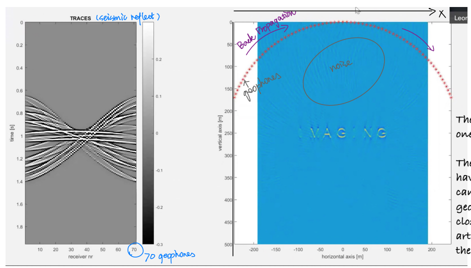
In a constant velocity medium, a reflection (wavelet) recorded by a geophone at time t1 can be produced by any point of a reflector with elliptical shape, with geophone and source at its Fock, and the distance source/ellipse/receiver equal to V*t1. Repeat for all the seismic traces and the envelope of all the ellipses is the TRUE reflector




Constructive interference (相长干涉)

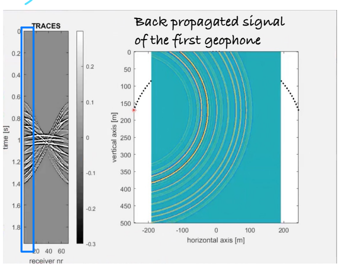
若两波的波峰（或波谷）同时抵达同一地点，称两波在该点同相，干涉波会产生最大的振幅，称为相长干涉（建设性干涉）

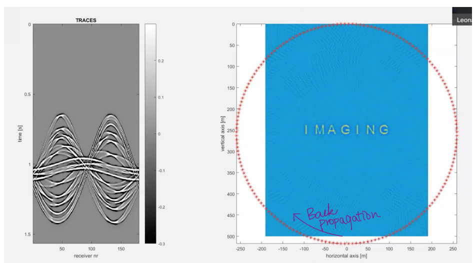


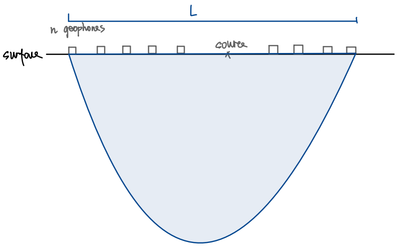

The geophones play back one by one.

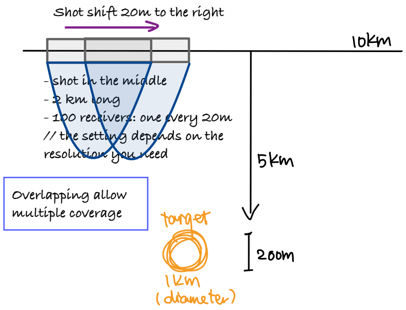
The noise is because we do not have enough geophones to cancle. If we have enough geophones around the body, one close to another, this circular artifact should disappear and the image would be more clear.



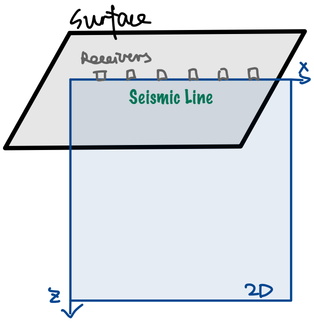



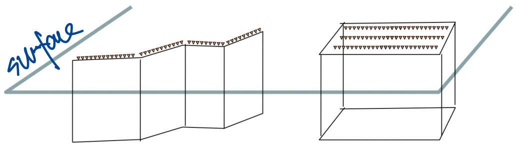
Case: the data is collected on the surface, not around the body
- setting: one source and n geophones




Acquisition geometry: 2D and 3D

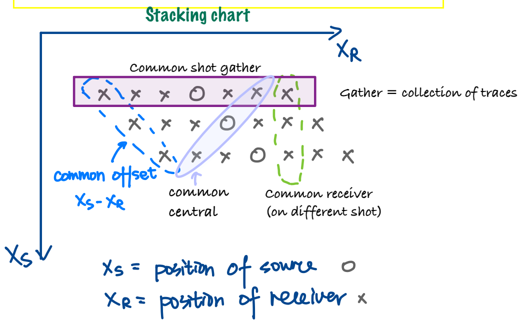


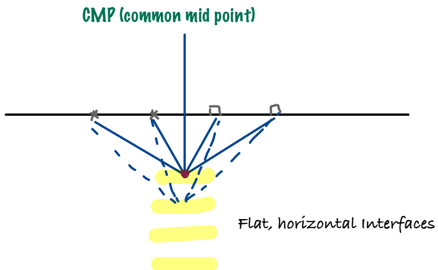




2D acquisition: sources and receivers along a line to obtain a 2D vertical map in the subsurface

3D acquisition: sources and receivers on a surface to obtain a 3D volume map in the subsurface

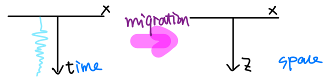
How to restore the information above in some charts?



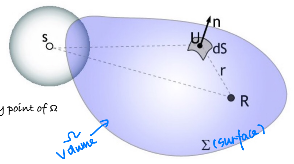
# 2021.10.26. Reflection seismic (ctd)

From time to space domain: migration (possible if v is known)




**Mathematical support: Kirkhhoff integral**

Pressure on Σ
+ known velocity
+ wave equation
=> pressure in any point of Ω




S source out of an arbitrary surface surrounding the receiver R

The free-field pressure emitted by the source

The Green function corresponding to the propagation from U to R

$$P(R) = P_0(R) - \int_\Sigma \left( P(U)\nabla G(U,R) - \boxed{G(U,R)}\nabla P(U) \right) \cdot n\,dS$$

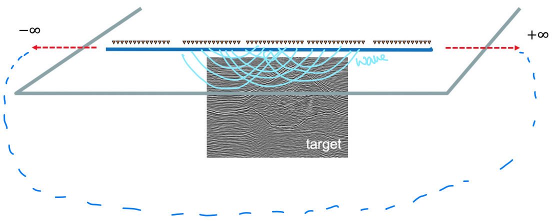
↑ pressure at R

$n\,dS$ (n unit vector)

The diffracted field

If you know (possibly by measure) the field on a closed surface and how the wave field propagates (velocity) is also known, you can obtain the field in any point and the time in the volume closed by the surface. Therefore, you can obtain the image at the time the reflectors were scattering.

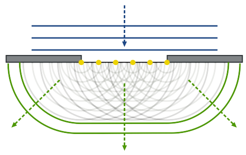
**How Kirkhhoff integral applies to seismic acquisition?**



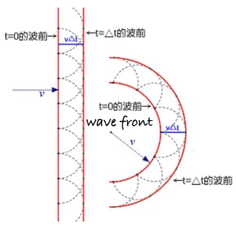

2D version: the extension of the acquisition line is higher than that of the target. We can think that the line closes at infinite and that the contribution from the target to the missing geophone s is negligible

**Huygens principle**

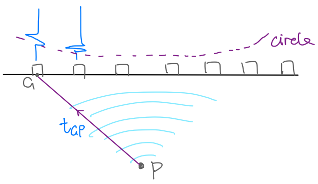
惠更斯－菲涅耳原理（英语：Huygens-Fresnel principle）是研究波传播问题的一种分析方法。惠更斯－菲涅耳原理指出，波前的每一点发出次波，这些次波互相干涉，叠加形成新的波前。这个原理同时适用于远场极限和近场衍射。



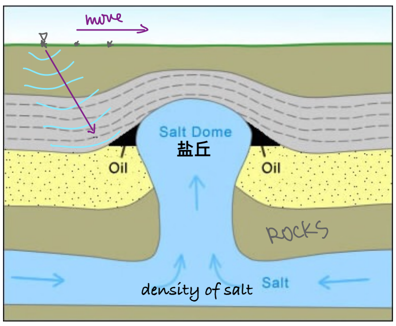
根据惠更斯–菲涅耳原理，波遇到障碍物时会产生衍射




Every point of a wave front is a source of spherical wave

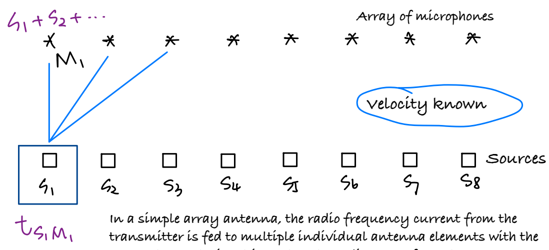



CASE: A salt dome is a type of structural dome formed when salt (or other evaporite minerals) intrudes into overlying rocks in a process known as diapirism. Salt domes can have unique surface and subsurface structures, and they can be discovered using techniques such as seismic reflection. They are important in petroleum geology as they can function as petroleum traps.



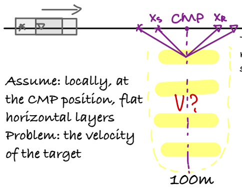

这里有两个Matlab实验，要试试！！

CASE: 波束赋形（Beamforming）又叫波束成型、空域滤波，是一种使用传感器阵列定向发送和接收信号的信号处理技术。波束赋形技术通过调整相位阵列的基本单元的参数，使得某些角度的信号获得相长干涉，而另一些角度的信号获得相消干涉。波束赋形既可以用于信号发射端，又可以用于信号接收端。




In a simple array antenna, the radio frequency current from the transmitter is fed to multiple individual antenna elements with the proper phase relationship so that the radio waves from the separate elements combine (superpose) to form beams, to increase power radiated in desired directions and suppress radiation in undesired directions.

How to obtain the velocity model?



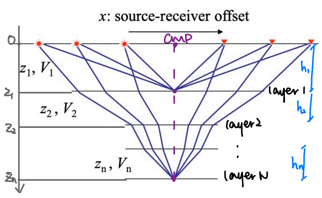

The sources and the receivers have the same central point

$X_S - X_R = X_0$ offset

Assume: locally, at the CMP position, flat horizontal layers

Problem: the velocity of the target

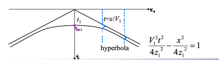
CMP Gather: consider traces with same CMP




Time to first interface normally

$$t_1(x_0) = \frac{2}{V_1}\sqrt{h_1^2 + \left(\frac{x_0}{2}\right)^2} = \sqrt{\boxed{\left(\frac{2h_1}{V_1}\right)^2}_{(t_{0,1})^2} + \left(\frac{x_0}{V_1}\right)^2} = \sqrt{(t_{0,1})^2 + \left(\frac{x_0}{V_1}\right)^2}$$

Reflection hyperbola

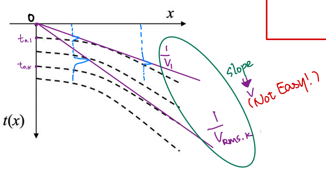


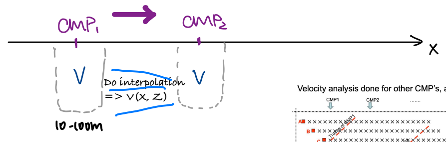

$$t_k(x_0) = \sqrt{(t_{0,k})^2 + \left(\frac{x_0}{V_{RMS,k}}\right)^2}, \quad V_{RMS,k} = \sqrt{\frac{\sum_{m=1}^{k} V_m^2 \cdot t_m}{\sum_{m=1}^{k} t_m}}$$

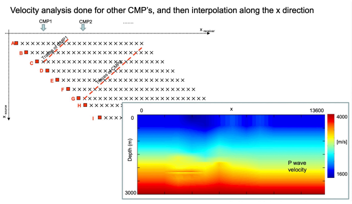
RMS = Root Mean Square

Total wait time in layer m

$$V_{RMS}(t) \Rightarrow V(Z)$$



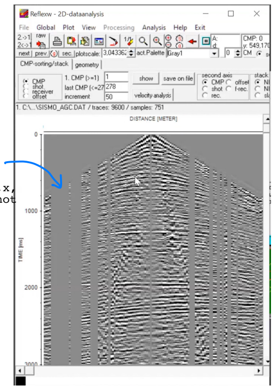




CASE in Reflexw

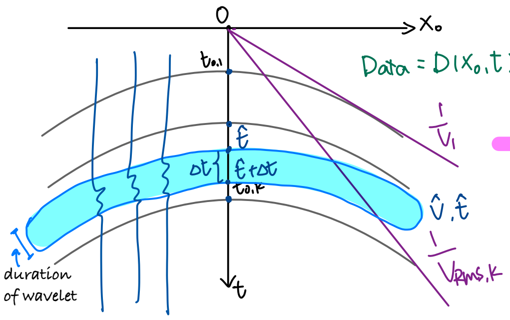
Why there are holes (missing data)?

These are places (e.x, water) where we cannot put geophones to collect data

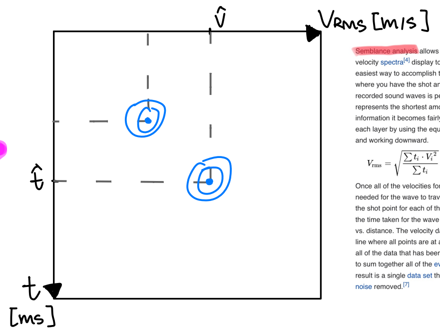



The data is used two time:
1. To get the velocity (slopes)
2. To make an array. Put the wavelets over the reflector to get the position

# 2021.10.28. Reflection seismic: velocity analysis on CMP gather




CMP Gather




Semblance analysis allows for the refinement of seismic data. This is done by developing a velocity spectra[4] display to determine the velocity through different layers at depth.[5] The easiest way to accomplish this is by recording the normal incidence path (NIP). The NIP is where you have the shot and the geophone in the same location and the path taken by the recorded sound waves is perpendicular to the boundaries between the layers.[6] This path represents the shortest amount of time that can be taken to reach a layer and return. With this information it becomes fairly easy to calculate the velocity of the waves as they travel through each layer by using the equation for the root mean square velocity starting with the top layer and working downward.

$$V_{\text{rms}} = \sqrt{\frac{\sum t_i \cdot V_i^2}{\sum t_i}}$$

Once all of the velocities for the layers are known then it is possible to calculate the time needed for the wave to travel the distance down to the midpoint between each geophone and the shot point for each of the layers. As the geophones are farther away from the shot, the more the time taken for the wave to travel there increases, this forms a hyperbola in a graph of time vs. distance. The velocity data is used to correct the curves of the hyperbolas and create a flat line where all points are at an equal depth. The final step for the semblance analysis is to sum all of the data that has been corrected for velocity. This is done with the use of a computer filter to sum together all of the events that the traces share, then remove the ones they don't. The result is a single data set that has all of the primary peaks strongly displayed with most of the noise removed.[7]

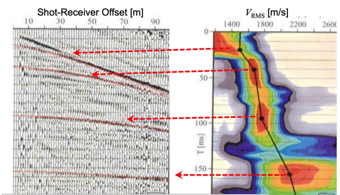
Semblance: each point in the semblance gather is the sum along an hyperbolic window in the CMP gather

Given $\hat v, \hat t$, $\quad t(x_0) = \sqrt{\hat t^2 + \frac{x_0^2}{\hat v^2}}$

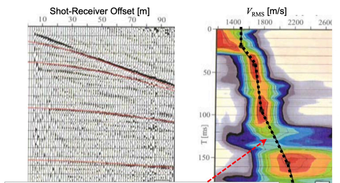
$$S(\hat v, \hat t) = \frac{\sum_{t=t(x_0)}^{t(x_0)+\delta t} \left[\sum_{x_0} D(x_0, t)\right]^2}{\sum_{t=t(x_0)}^{t(x_0)+\delta t} \sum_{x_0} \left[D(x_0, t)\right]^2}$$

Normalization
$0 <= S <= 1$

Semblance -> velocity picking

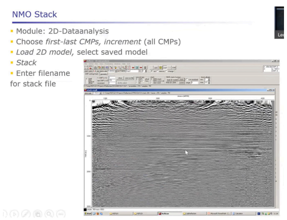


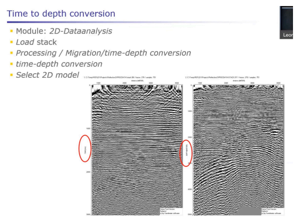

A high semblance value means the traces along the hyperbolic window are laterally "coherent": probable hyperbolic reflection event, with slope asymptotic to 1/*Vrms*



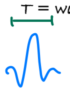

Velocity picking is done by connecting high semblance values along time (dotted black line): smaller velocity events at the same time are generally multiples and/or converted waves (*Vs*<*Vp*). The velocity picking produces $V_{RMS}(t)$

Example

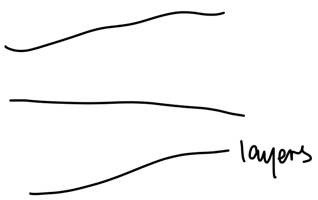



T = wavelet duration



Representation of Geology (True one)




Reflection Seismic

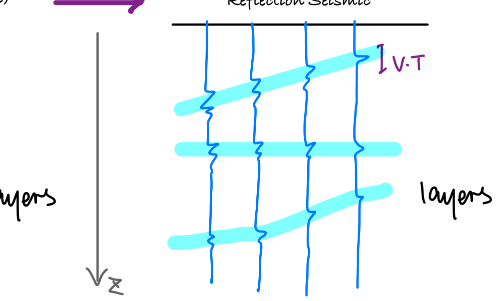
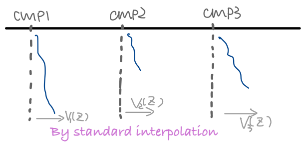



Image of Seismic reflection
-> position of the reflectors with resolution V*T
-> velocity

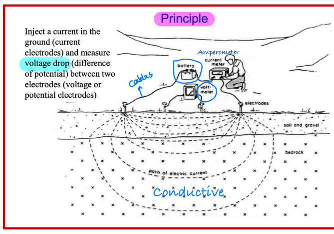
2D velocity model

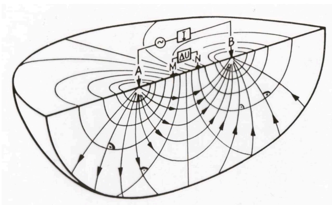


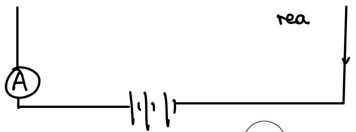
2021.11.11 Electrical method (slide 7)

**Objective:** measure electrical properties (resistivity) of the ground.

Principle





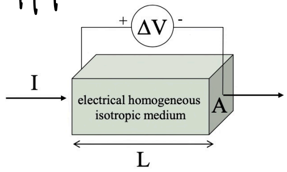

## OHM Law

$$\Delta V = R\,I$$

$\Delta V$: Voltage drop [volts]; $R$: [volts/amps] = [ohm] = [Ω]; $I$: Current intensity [amps]

$$R = \rho\frac{L}{A} \iff \rho = R\frac{Area}{L} = [\Omega\cdot m]$$

$L$: Length of the rod; $Area$: Cross-sectional area of the rod; $\rho$: Resistivity [Ω*m]



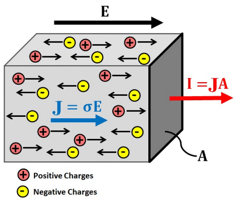

### Conduction

$$\frac{1}{\rho} = \sigma = \text{Conductivity} = \left[\frac{1}{\Omega m}\right] = \left[\frac{\text{siemens}}{m}\right] = \left[\frac{\text{MHO}}{m}\right]$$

**Electronic conduction (电子传导)**
- rapid movement of electrons which are free to move through the molecular matrix (metals).
- Resistivity increases with temperature, due to increase in particle collisions

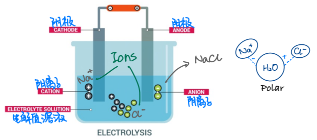
Metals: $10^{-10} < \rho < 10^{-6}\ \Omega\cdot m$

Conductivity in Materials




**Electrolytic Conduction (电解传导)**
- slow movement of ions within an electrolyte. Depends upon type of ion, ionic concentration and mobility.
- Resistivity decreases with temperature, as more ions are released

$10^{-3} < \rho < 10\ \Omega\cdot m$

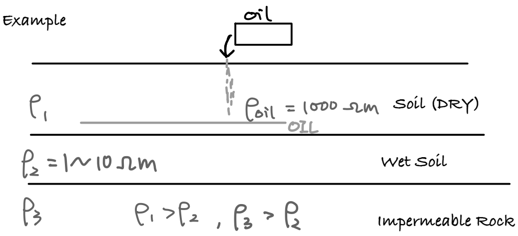



**Electronic vs Ionic Conduction**

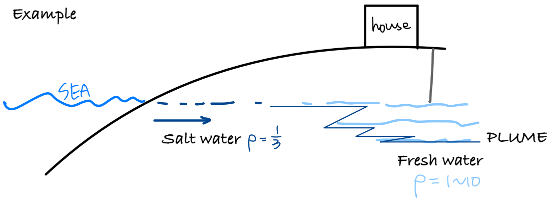
More Information Online WWW.DIFFERENCEBETWEEN.COM

| | Electronic Conduction | Ionic Conduction |
| --- | --- | --- |
| DEFINITION | Electronic conduction is the process of transferring energy in the form of an electric current | Ionic conduction is the process of transferring energy via the movement of ionic species |
| CHEMICAL SPECIES | Electrons | Ions |
| CHARGE OF TRANSFERRING MATERIAL | Negative charge | Either negative or positive charges |
| METHOD | Free electrons transfer from the orbital of one atom to an orbital of an adjacent atom | Ions move from one defect to another in a crystal lattice |

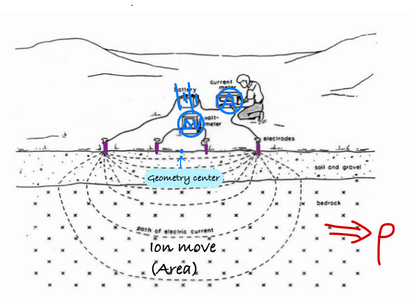
Example

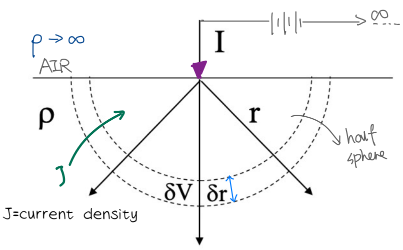



Example




### Current Flow in a homogenous medium




Single current source, air behaves as an insulator (绝缘体).

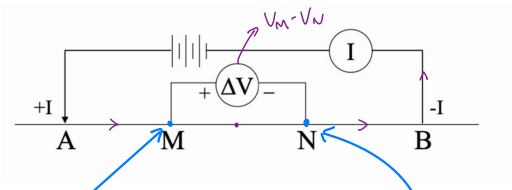
The current density (J) is the current (I) divided by the area over which the current is distributed (a hemisphere).

$$J = \text{Current Density} = \left[\frac{I}{Area}\right] = \frac{I}{2\pi r^2}$$

The voltage drop between any two points can be described by the potential gradient, which is negative because the potential decreases in the direction of current flow (potential zero at infinite):

$$\frac{dV}{dr} = -\rho J = -\rho\cdot\frac{I}{2\pi r^2} \rightarrow \boxed{V(r) = \int dV = -\int \rho\cdot\frac{I}{2\pi r^2}\,dr = \rho\frac{I}{2\pi r}}$$

## Resistivity Measurements




A, B: current electrodes (电极), used to inject the current into the ground
M, N: potential electrodes that measure the resulting potential

$$V_M = \rho \cdot \frac{I}{2\pi \cdot AM} - \rho \cdot \frac{I}{2\pi \cdot MB}$$
$$V_N = \rho \cdot \frac{I}{2\pi \cdot AN} - \rho \cdot \frac{I}{2\pi \cdot NB}$$

$$\Delta V = V_M - V_N = \rho \cdot \frac{I}{2\pi} \left[ \frac{1}{AM} - \frac{1}{MB} - \frac{1}{AN} + \frac{1}{NB} \right]$$

$$\rightarrow \rho = \frac{\Delta V}{I} \cdot \boxed{2\pi \cdot \left[ \frac{1}{AM} - \frac{1}{MB} - \frac{1}{AN} + \frac{1}{NB} \right]^{-1}} = \frac{\Delta V}{I} \cdot K$$

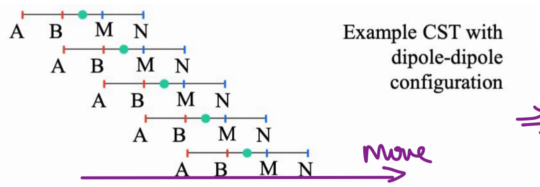
ΔV and I from readings

Apparent resistivity.
The resistivity value is associated to the center of the geometry. $\rho_a$

K: geometrical factor (m), related to acquisition geometry

## Constant Separation Traverse (CST)

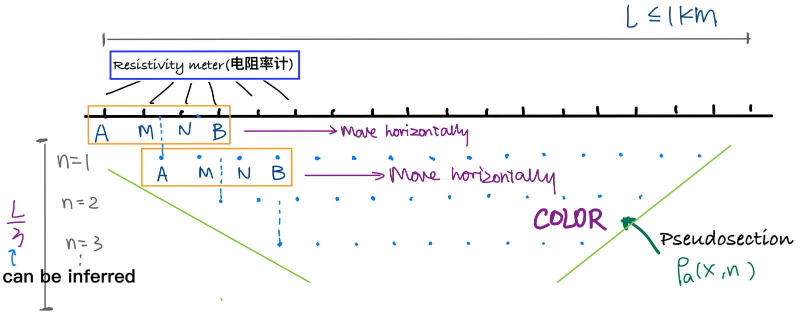
Obtains apparent resistivity along the horizontal axis by moving all the electrodes




$\Rightarrow \rho_a(x, ?)$

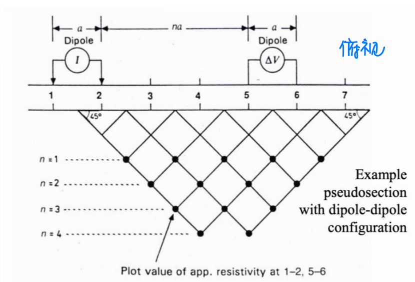
## Pseudosection

obtains apparent resistivity along the horizontal axis by moving all the electrodes and along the vertical axis by increasing n.
Apparent resistivity displayed as contour lines and/or colormap.




$\rightarrow \rho_a(x, z)$
(Processing by computer)

Data measure
(collect)




### Current flow with depth

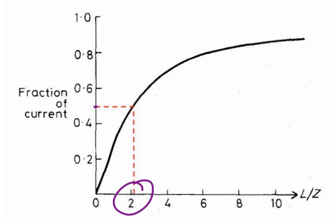



**Fig. 8.5** The fraction of current penetrating below a depth Z for a current electrode separation L. (After Telford *et al.* 1976.)

In order for at least 50% of the current to flow through an interface at a depth of Z meters into a second medium, the current electrode separation L needs to be at least twice (preferable more than three times) the depth. Practical implications: electrical sounding is limited to few hundred of meters in depth

## True and apparent resistivity (真电阻率和视电阻率)

In reality, the subsurface ground does not conform to a homogenous medium, and thus the resistivity obtained is no longer the true resistivity but the apparent resistivity, which can even be negative.

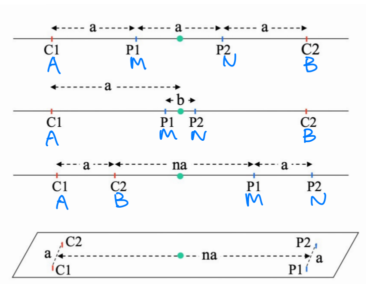
The apparent resistivity is not a physical property of the subsurface media, unlike the true resistivity.

Consequently, all field resistivity data are apparent resistivity while those obtained by interpretation techniques are "true" resistivities.

## Electrode configurations

More: https://www.agiusa.com/blog/comparison-11-classical-electrode-arrays

$$\rho = \frac{\Delta V}{I} \cdot K, \quad K = \left(\frac{1}{AM} - \frac{1}{MB} - \frac{1}{AN} + \frac{1}{NB}\right)^{-1} \cdot 2\pi$$




Wenner Geometry
$K=2\pi a$

Schlumberger Array
$K=\pi a/b\,[1-b^2/(4a^2)]$ // a>>b

(longitudinal) dipole-dipole
$K=\pi n(n+1)(n+2)a$

transverse dipole-dipole

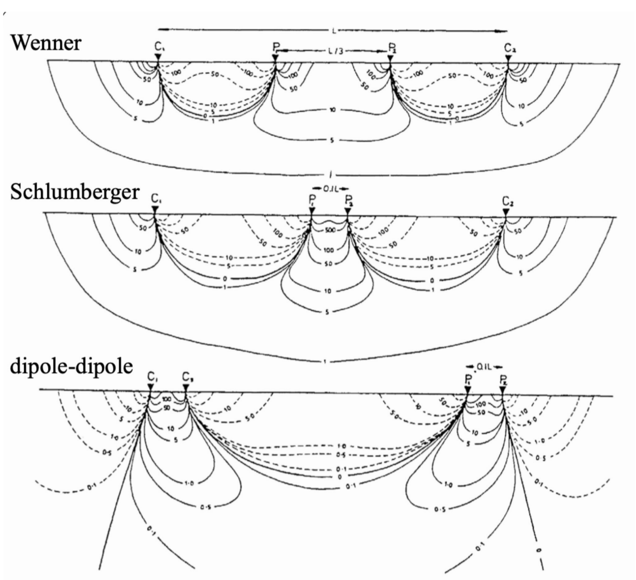
Wenner: there is an equal spacing "a" between the four electrodes AMNB.

Schlumberger: four electrodes are placed in a line, centered around a midpoint.
- pros:
  * faster: only move 2 electrodes for most readings, not 4
  * more accurate: because MN and AB can be changed independently, lateral variations between MN electrodes can be detected. These near-surface lateral variations could be is interpreted in terms of depth variations in resistivity.
  * deeper data: it has slightly greater depth of investigation

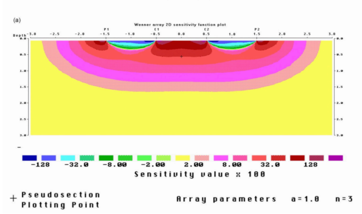
Question: how to choose the configuration?

## Signal Contribution Sections

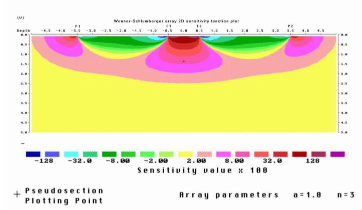
Contours indicate the relative contributions made by discrete volume elements of the subsurface to the total potential difference measured between the two potential electrodes



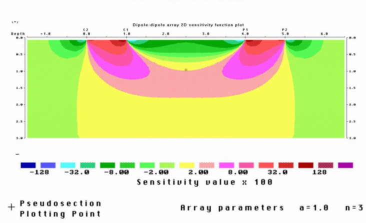

WENNER




SCHLUMBERGER




DIPOLE - DIPOLE

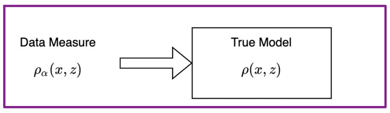


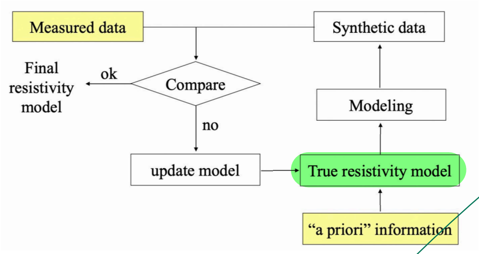

Ref: https://youtu.be/MvOR4xYIMb8

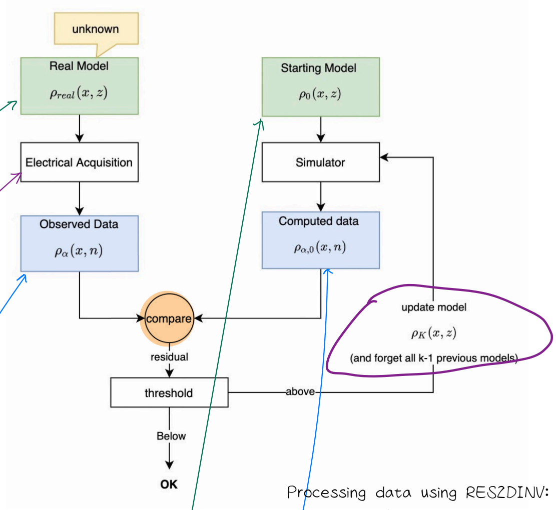
2021.11.16. 2nd homework

## Interpretation techniques

From data (apparent resistivity) to true resistivity model.
Computer aided iterative inversion

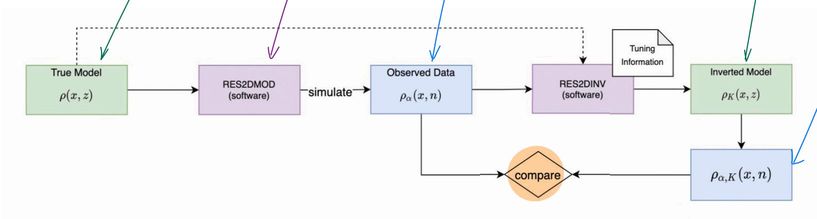



Synthetic method 合成法

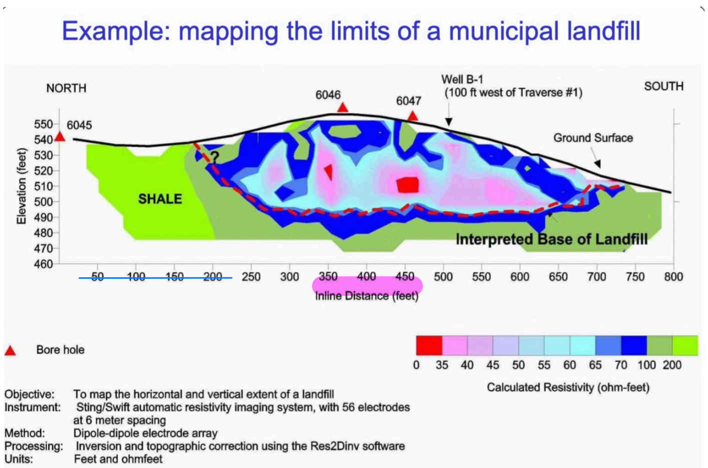
// ill conditioned

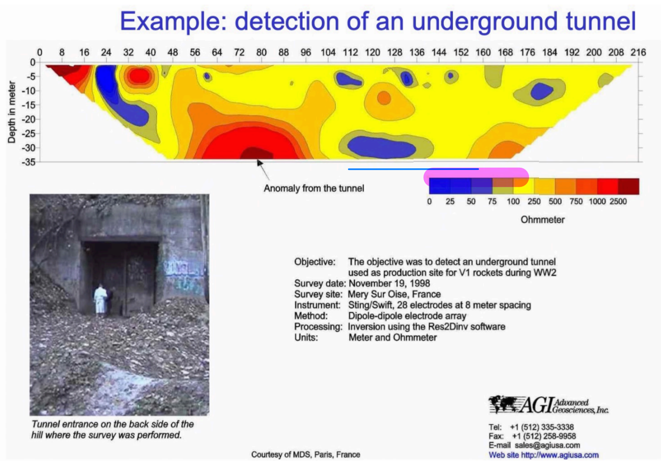



Processing data using RES2DINV:
1. Open RES2DINV program
2. Check system requirement
3. Read data file (…dat): File -> Read data file
4. Edit -> exterminate bad data points: After pick the bad data, exit display windows then save the dat.file as new file
5. Change settings -> inversion damping parameters -> damping factors
6. Change settings ->forward modeling method setting
   - horizontal mesh size: choose 4 nodes
   - vertical mesh size: Finest mesh
   - type of forward modeling: finite-difference method.
7. Inversion -> inversion methods and settings -> select robust inversion
   - change from absolute error to RMS error
8. Inversion -> model discretization -> use model refinement
   - use model cells with width of half of the unit spacing
9. Inversion -> model discretization -> use extended model -> yes
10 inversion -> carry out inversion: wait until the inversion complete
11 display:
   - Display -> show inversion results
   - Display sections -> model display -> display data and model sections: Insert number of iteration; select type of contour interval
   - Edit data -> RMS error statistics: reduce noise to reduce RMS error
   - save the trimmed data into a new file name, and exit->quit display window
12.

// https://youtu.be/5JYsGKMaO6E

## Example



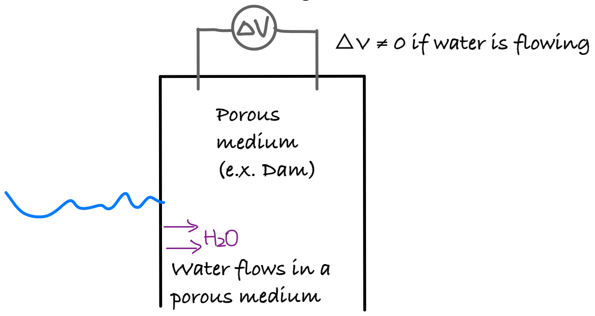

Lab/homework2 FROM 00:43:30 (see software/Homework2.pdf)

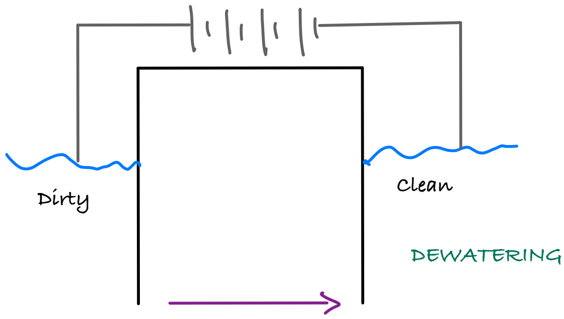
2021.11.18. Electrical method

Self potential Method

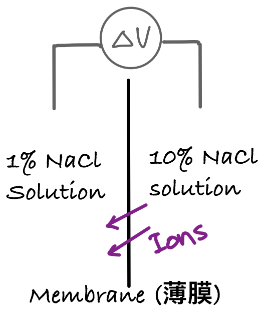
Used to detect the presence of massive ore bodies. Passive method, i.e. differences of potentials are generated by natural sources and they are measured on the surface.
Self potential means Natural "Battery".

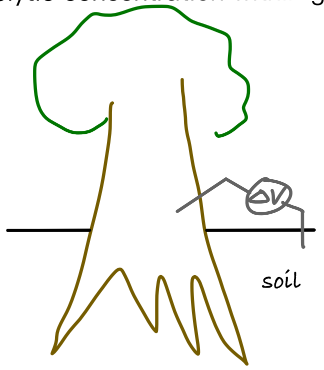
Electrokinetic Effect: the potentials are generated by the flow of an electrolyte (water) through a capillary porous media (electrofiltration or streaming potentials)



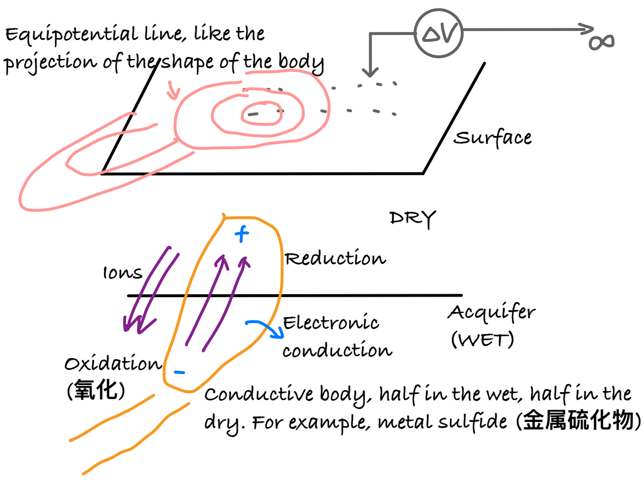




Electrochemical Effect the potentials are generated by imbalance in the electrolytic concentration within groundwater




Mineral potentials: ore bodies intersecting the water table. Reduction/oxidation processes occur at the body boundaries, and the body permits the flow of electrons.



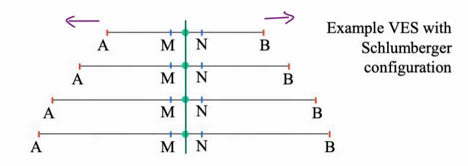

ORE Exploration

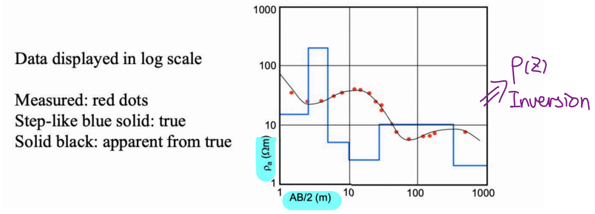
## Vertical Electrical Sounding (VES)

Obtains apparent resistivity data for increasing depth, by increasing electrodes separation with respect to the same central point. ρ(z)
Used in the study of near-horizontal interfaces. The electrode spread is progressively expanded about a central point

depth

z

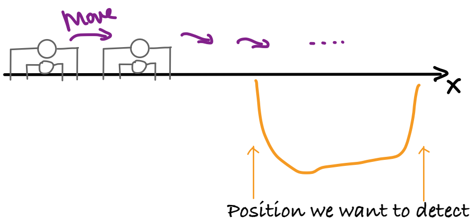


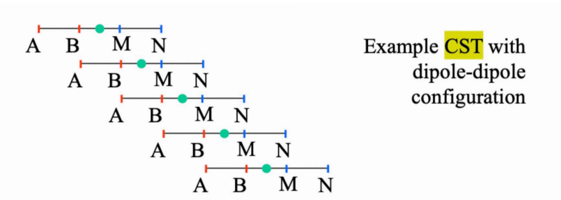




## Constant Separation Traverse (CST)

x (horizontal)

obtains apparent resistivity along the horizontal axis by moving all the electrodes. Used to determine lateral variations of resistivity.



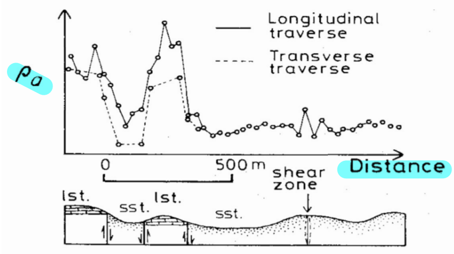




**Fig. 8.17** Longitudinal and transverse traverses across a series of faulted strata in Illinois, USA. (After Hubbert 1934.)

Data displayed in linear scale

lst: limestone
sst: sandstone

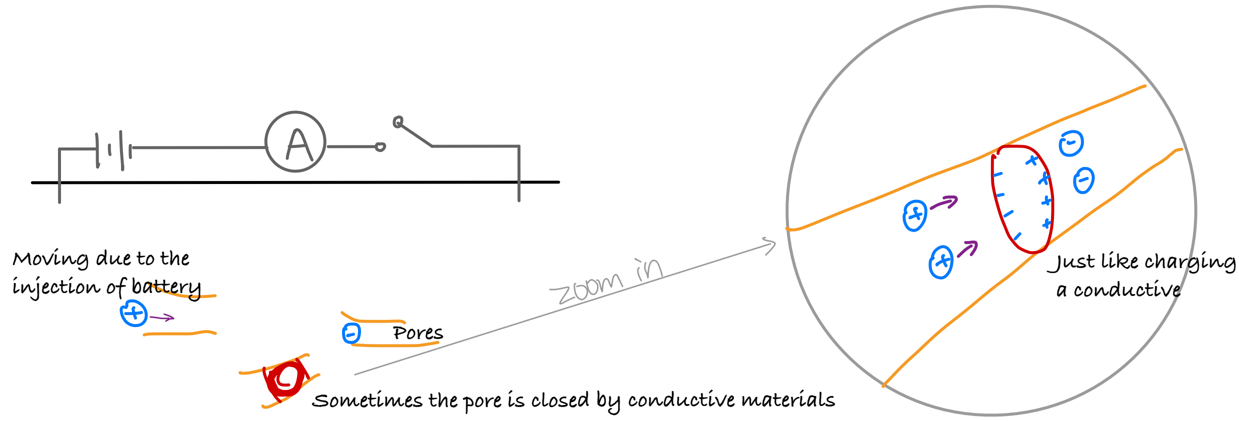
## Electrical method (Galvanic method / ERT = Electrical Resistivity Tomography / DC (Direct Currency) method)

(From WIKI) Electrical resistivity tomography (ERT) or electrical resistivity imaging (ERI) is a geophysical technique for imaging sub-surface structures from electrical resistivity measurements made at the surface, or by electrodes in one or more boreholes. If the electrodes are suspended in the boreholes, deeper sections can be investigated. It is closely related to the medical imaging technique electrical impedance tomography (EIT), and mathematically is the same inverse problem. In contrast to medical EIT, however, ERT is essentially a direct current method. A related geophysical method, induced polarization (or spectral induced polarization), measures the transient response and aims to determine the subsurface chargeability properties.

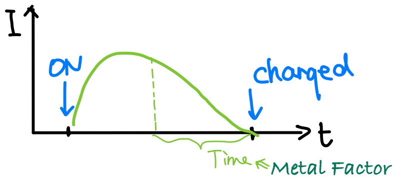
## Induced Polarization (IP)

Used to check the capacity of the metal.

(FROM WIKI) The survey method is similar to electrical resistivity tomography (ERT), in that an electric current is transmitted into the subsurface through two electrodes, and voltage is monitored through two other electrodes.

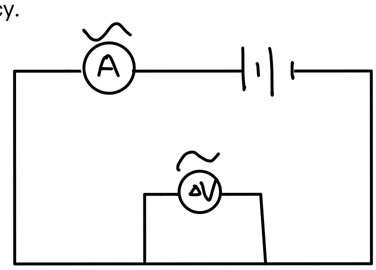


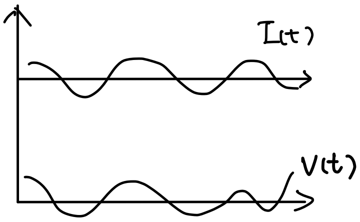
Time-domain induced polarization: (From WIKI) the voltage response is observed as a function of time after the injected current is switched off or on.




Frequency-domain induced polarization: (From WIKI) an alternating current is injected into the ground with variable frequencies. Voltage phase-shifts are measured to evaluate the impedance ~~spectrum at different~~ injection frequencies, which is commonly referred to as spectral IP.

The apparent resistivity is measured at two frequencies less than 10Hz. The apparent resistivity at low frequency is greater than that at higher frequency.




$$\left.\begin{array}{l} f_0 = 0.1\,\mathrm{Hz} \rightarrow \rho_0 = \dfrac{\Delta V}{I} K \\ f_1 = 1\,\mathrm{Hz} \rightarrow \rho_1 = \dfrac{\Delta V}{I} K \end{array}\right\} \rho_1 - \rho_0$$

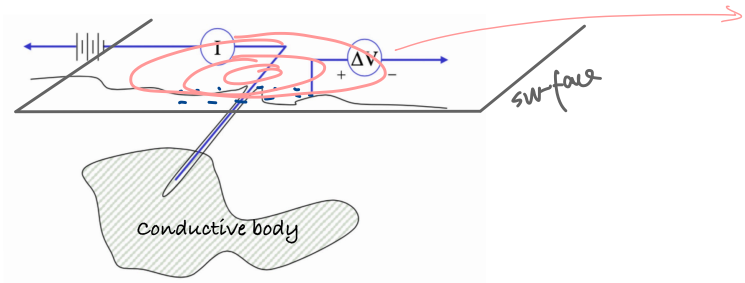
Mine-a-la-masse Method

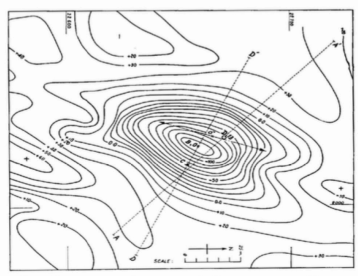
also called "charged body potential". used for the delineation of a conductive body in the surface.

One current electrode is placed within the conductive body and other current electrode at a semi-infinite distance away on the surface. The voltage between a pair of potential electrodes (one fixed) is measured with appropriate corrections for any self potentials.

For an isolated conductor in a homogeneous medium, the lines of equipotential should be concentric around the conductor. This principle is used to delimit the spatial extent of the body.

Mise-a-la-masse method is particularly useful in checking whether a particular conductive mineral-show forms an isolated mass or is part of a larger electrically connected ore body.



Surface equipotential contour lines

Note that it is different from the "mineral potentials"

2021.11.23. Time tomography lab

暂时跳过

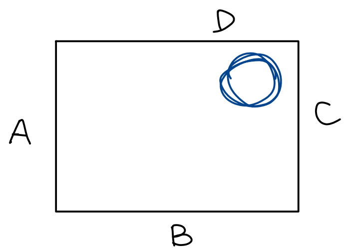
Box

without wax only water

$\frac{28\,cm}{174\,\mu s}$

with wax

$\frac{\quad}{162\,\mu s}$



## 2021.11.25 . EM (ElectroMagnetic) methods

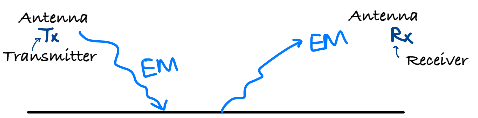
Slide: #8

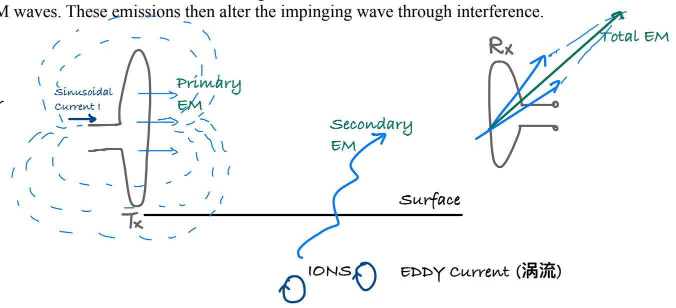
Electromagnetic (EM) radiation is produced by oscillating charges. It is a self- propagating wave in space with electric and magnetic components. These components oscillate at right angles to each other and to the direction of propagation, and are in phase with each other.

Electromagnetic radiation is classified into types according to the frequency of the wave: these types include, in order of increasing frequency, radio waves, microwaves, terahertz radiation, infrared radiation, visible light, ultraviolet radiation, X- rays and gamma rays.

According to Maxwell's equations, a time-varying electric field generates a magnetic field and vice versa. Therefore, as an oscillating electric field generates an oscillating magnetic field, the magnetic field in turn generates an oscillating electric field, and so on. These oscillating fields together form an electromagnetic wave.

Electric and magnetic fields obey the properties of superposition（叠加）, so fields due to particular particles or time-varying electric or magnetic fields contribute to the fields due to other causes. (As these fields are vector fields, all magnetic and electric field vectors add together according to vector addition.) These properties cause various phenomena including refraction and diffraction. For instance, a travelling EM wave incident on an atomic structure induces oscillation in the atoms, thereby causing them to emit their own EM waves. These emissions then alter the impinging wave through interference.




### Electrical / EM Parameters of a medium

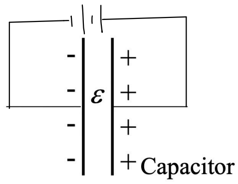
$\boxed{\text{Conductivity} = \sigma} = \frac{1}{\rho}$ [siemens/m]   $(10^{-1} < \rho\,[\Omega m] < 10^{3} \rightarrow 10^{-3} < \sigma < 10)$

Resistivity

$\boxed{\text{Dielectric constant (介电常数；电容率)} = \varepsilon}$: measures the "ability" of a material in storing electrical energy. It is related to the displacement currentt (polarization conduction)

$\varepsilon = \varepsilon_0 \cdot \varepsilon_r$  Vacuum; [Faraday/m]; A dimensional (relative dielectric constant)

$$\begin{cases} \text{Air} \Leftrightarrow \text{Vacuum} : \varepsilon_{r_{AIR}} = 1 \\ \text{Sandstone} = 4\,\text{Vacuum} : \varepsilon_{r_{sandstone}} = 4 \\ \varepsilon_{r_{Rock\_dry}} \simeq 4-10 \;;\; \varepsilon_{r_{water}} = 80 \end{cases}$$




capacity = C [Faraday] $= \varepsilon \cdot \frac{\text{Area}}{\text{distance}}$

$\boxed{\text{Magnetic permeability (导磁率)} = \mu} = \mu_0 \cdot \mu_r$  Vacuum; Relative

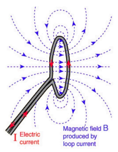
$$\begin{cases} \mu_{r_{rock}} = 1 \\ \mu_{r_{water}} = 1 \\ \mu_{r_{AIR}} = 1 \end{cases} \approx \mu_0 \cdot 1$$

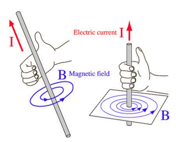
Magnetic induction (磁感应) = B [Tesla]  $= \mu \cdot H$

Magnetizing force (磁化力)/ magnetic field strength [amperes/m]

### Biot-Savart Law

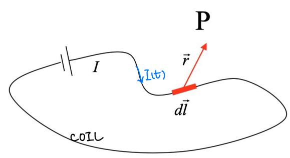
Relate the magnetic filed to the currents which are its sources A current in a conductor (antenna) producing EM field.




Magnetic filed B measured in P and produced by the "coil" where the current I is flowing

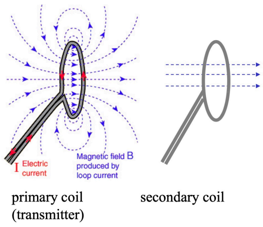
$$B(t) = I(t) \int_{COIL} \frac{\mu}{4\pi} \cdot \frac{dl \cdot r}{r^3}$$



## Faraday Law

Any change in the magnetic environment of a coil of wire will cause a voltage (electromotive force, EMF) to be "induced" in the coil. The induced EMF (generated voltage) can be calculated from Faraday's Law.

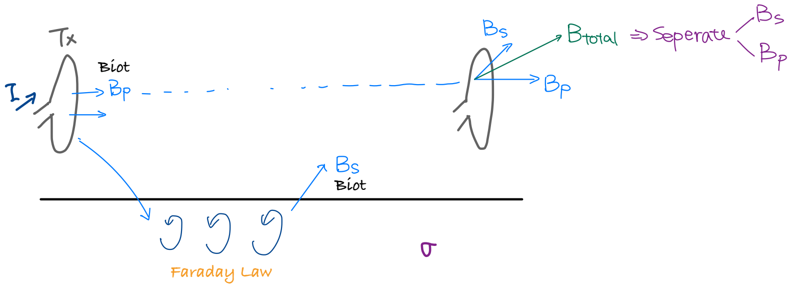
If the circuit is closed, an induced current circulates in the coil. This current generates a magnetic field to oppose the change in magnetic field which produced it.



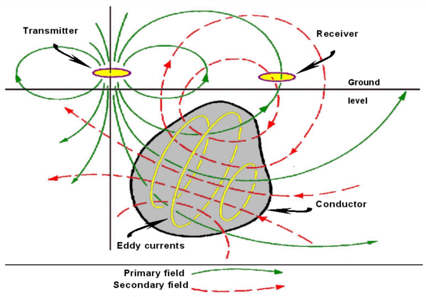

### Faraday-Neumann-Lenz Law

$$\Phi_B = B \cdot \text{Area} \quad , \quad EMF = -\frac{d\Phi_B}{dt}$$

Flux of B through the coil
[Weber]




**Principle of EM method:** measure the modifications produced by the conductivity of the target on an artificially / naturally generated EM Field



# 2021.11.30 . EM methods

## EM wave in a medium

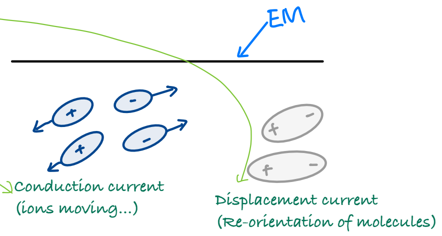
an electric field E applied to a (homogeneous Isotropic) material produces a movement of the charges in the medium.

The associated current flow J is given by:

Current Density = J [amps/m2]

$$J = \sigma E + \varepsilon \cdot \frac{dE}{dt}$$

Conductivity ($\sigma$); Dielectric constant ($\varepsilon$)




For a sinusoidal wave: $E = E_0 \cos(2\pi f_0 t)$

$$J = \sigma E_0 \cos(2\pi f_0 t) + \boxed{\varepsilon \cdot 2\pi f_0} \cdot E_0 \sin(2\pi f_0 t)$$

Depending on the frequency, the first and/or the second term become predominant (low f -> first term; high f -> second term), and so the associated variable σ or ε

For low f0 (fo << 10 MHz), the second term can be neglected -> get Ground Conductivity
For high f0 (fo > 10 MHz), the first term can be neglected -> Ground Penetrating radar

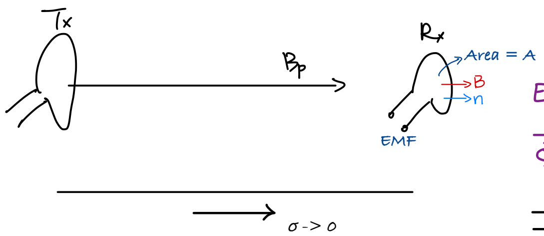
$f \ll \frac{\sigma}{2\pi\varepsilon}$ Conductivity Meters and Metal Detector

$f \gg \frac{\sigma}{2\pi\varepsilon}$ Ground penetrating radar

## Tilt Angle Method

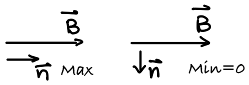
Primary (TX) and secondary (eddy currents) magnetic fields have different directions. The total field is the vectorial sum of the two magnetic fields.

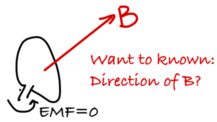
By keeping the TX in a fixed position and by moving the receiver coil, it is possible to measure the direction (polarization) angle θ of the total field in different positions along the surface.

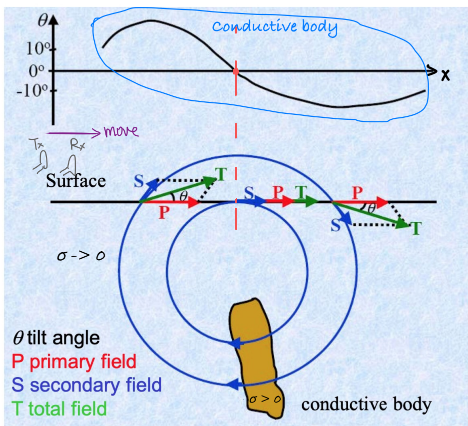



$$EMF = \frac{d\Phi_B}{dt}$$

$$\vec{\Phi} = \vec{B} \cdot \vec{n} A$$



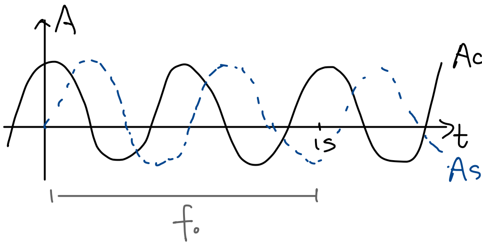




Used for the location of sub surface conductive bodies.

**PROCEDURE:**

- TX coil in a fixe position.
- RX coil moved along the surface. By rotating the RX coil, it is possible to measure the direction of the total field (generally by searching the angular position of the RX by which the induced EMF is zero, that is the RX coil plane is parallel to the total field).
- The position of the sign change of θ along the surface gives the horizontal location of the conductive body.
- asymmetry of the θ curve is associated to the inclination of the conductive body

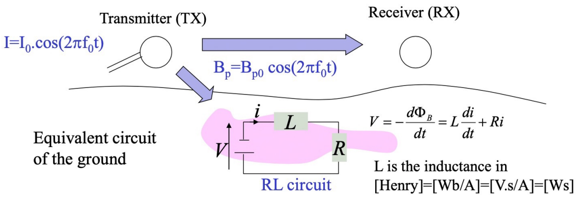
## Polarization Angle Method (Phase Measurement)




$A\cos(2\pi f_0 t)$

$A\sin(2\pi f_0 t) = A\cos(2\pi f_0 t - \frac{\pi}{2})$

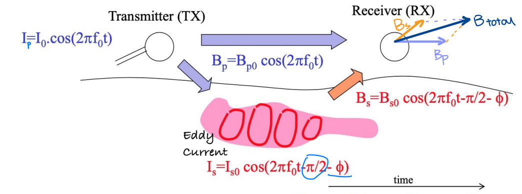
Angular Delay




A sinusoidal current I (fixed frequency) in the primary coil (TX) generates a sinusoidal primary EM wave in phase with the current (direct link between current and magnetic field given by the Biot-Savart law).

The resistance (R) accounts for the "difficult" of flowing a current in the medium.
The inductance (L) accounts for the "inertia" of the medium, that is the "difficulty" of starting/stopping the current flow. The inductance produces a "delay" in the induced current with respect to the inducing field. For increasing frequency, the inductance behaves as an open switch (no current flow).

This model is valid as long as conduction currents are predominant (electrical/EM methods)



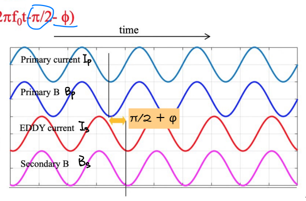

EM induced in the ground and the associated eddy currents) is linked to the derivative of the inducing magnetic field (Faraday law).

EMF has a phase difference with respect to the inducing field -> $-\pi/2$

A further phase difference is linked to the inductance of the ground (RL circuit) -> $\phi$

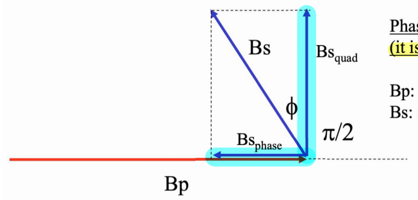
$\phi=\tan^{-1}(2\pi f_0 L/R)$

high conductivity material (low R) $\phi \rightarrow \pi/2$
low conductivity materila (high R) $\phi \rightarrow 0$




$B_{total}$ — $B_p$ $\rightarrow |B_p|$ Initial angle = 0
$B_{total}$ — $B_s$ Amplitude ?
Angle (between Bp, Bs sinusoids)

### Phase measurements




Phase representation
(it is not the field polarization!)

Bp: primary field
Bs: secondary field

Polarization plane

$Bs_{quad}$ = component of the secondary field in phase with the primary field
$Bs_{phase}$ = component of the secondary field in quadrature with the primaryfield

$\frac{Bs_{phase}}{Bs_{quad}} \rightarrow \infty$ : High conductivity material ($\varphi$ -> $\pi/2$)

### Ground conductivity meter

If $K_B$(induction number, $s/\delta$) << 1: skin length

Apparent ground conductivity [siemens/m]

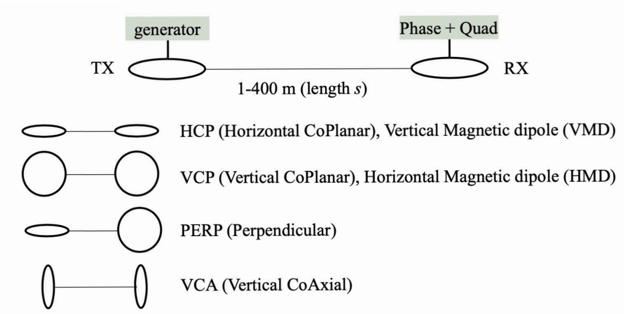
$$\sigma_a = \frac{4}{2\pi f_0 \mu s^2}\left[\frac{Bs_{quadrature}}{Bp}\right]$$

$\mu_0$ Distance btw TX and RX

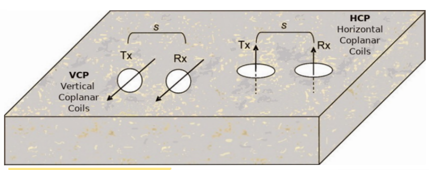
**Procedure**: with s fixed, $\delta$ (skin length) depending on frequency and on conductivity
- the frequency is chosen in order to have k_B << 1 (with an estimated maximum value of $\sigma$)
- depth of penetration can be increased by decreasing the frequency or by increasing s

$f_0, f_1, f_2 < 20kHz$

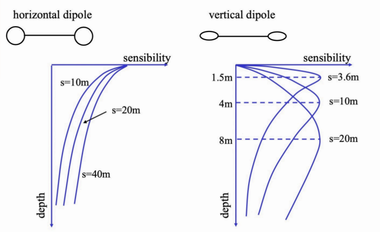
### Configuration




The cable sends the information about the phase of the primary field.
TX-RX distance constant. Measurement of ground conductivity associated to the center.

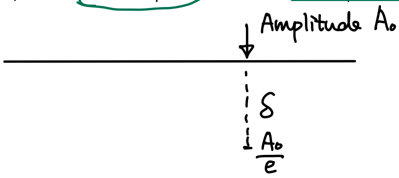



What change? Sensibility!



## Skin Depth (δ)

The amplitude of the EM field attenuates (减弱) exponentially during propagation (absorption). When the EM field enters a material, it is defined a skin depth δ, which is the depth where the amplitude is 1/e times the amplitude at the surface




$$A_\delta = A_0 \cdot e^{-1}$$

$$\underset{[m]}{\delta} = \sqrt{\frac{1}{2\pi f \sigma \mu}} = 503.8\sqrt{\frac{1}{f\sigma}}$$

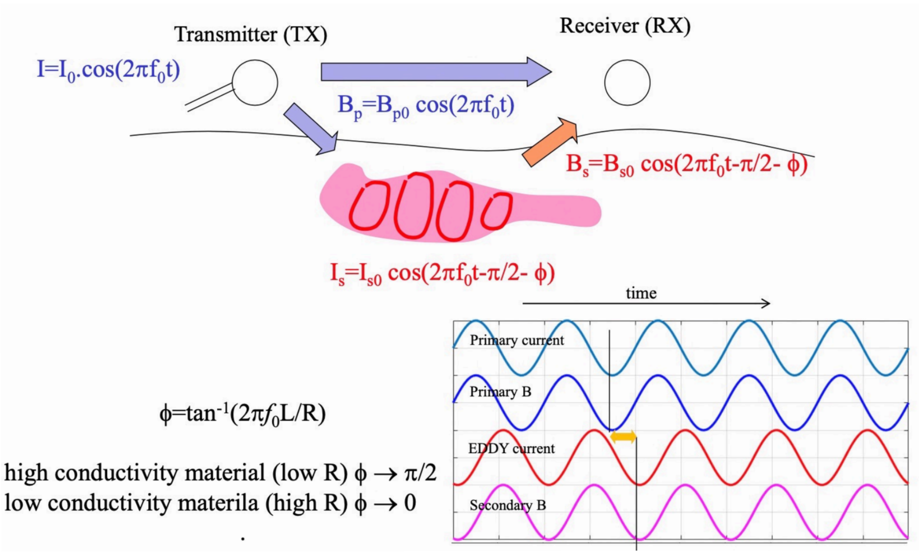
$\mu \simeq const = \mu_0$

Frequency.
Higher frequencies are attenated more than lower frequencies

Material conductivity

In practice, assume δ/5 as the maximum depth at which a material can produce identifiable EM effects.
Moreover, it is difficult to produce artificially low frequencies (<1000Hz)-> maxi depth of investigation (with artificial sources) around 500m

## Frequency Domain EM (FDEM)

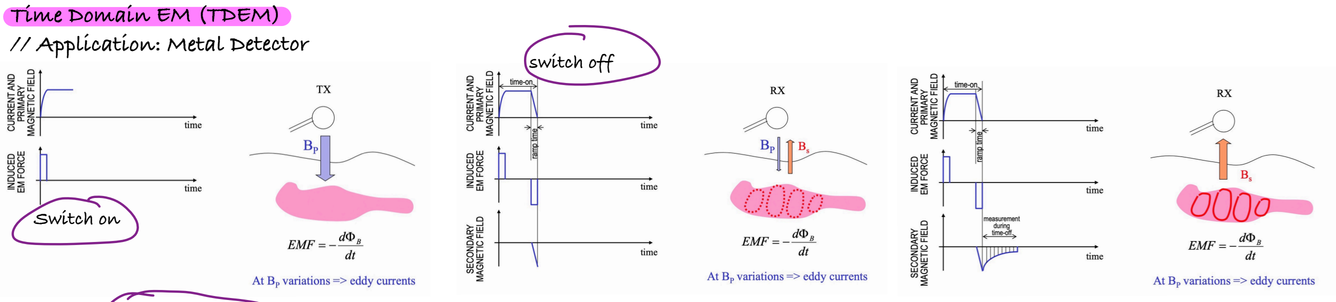


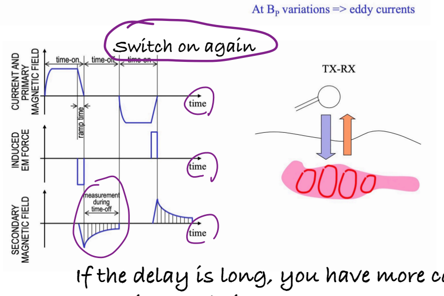
2021.12.02 GPR

# slide 8

## Time Domain EM (TDEM)

// Application: Metal Detector

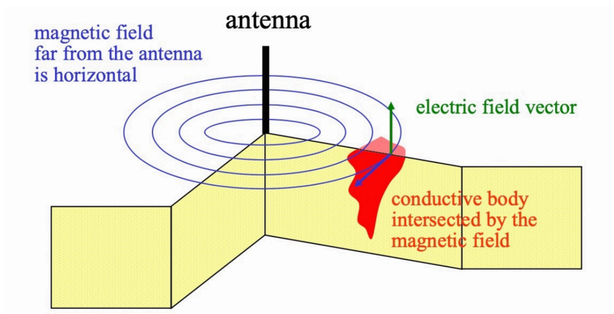



If the delay is long, you have more conductive material, and vice versa

Procedure:
– the time domain waveform of the secondary field is measured at fixed time intervals (usually regular in log scale) during the time–off period.
– the measurements are made with alternating sign in order to remove locally generated EM noise

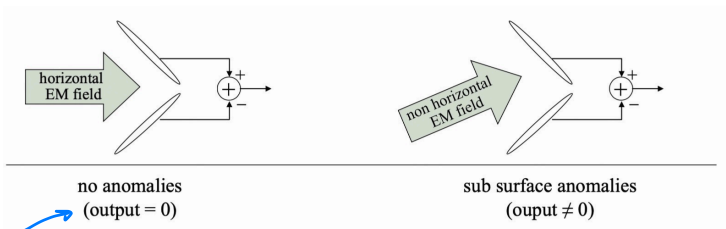
## Very Low Frequency (VLF) method




Sources are the EM fields generated by the VLF stations (radio navigation system, 5kHz ~ 25 kHz).
Medium anomalies layout has to intersect magnetic field.

Pros:
– portable and easy to use equipment
– measurements possible also at thousands of km from the TX (also from airplane).

## Audio Frequency Magnetic Field (AFMAG) method

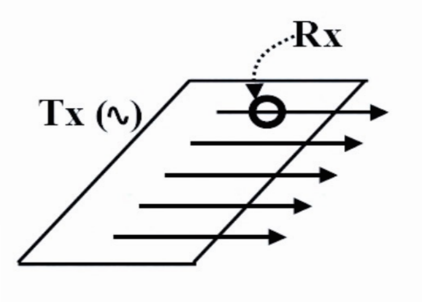



Source of EM field: thunderstorms

The EM field remains trapped between the earth surface and the ionosphere and propagates horizontally (in absence of sub–surface conductive anomalies). The direction of arrival is variable. Anomalies are detected as deviations of the horizontal propagation.

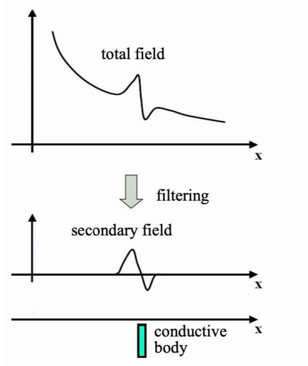
Frequency range: 100~1000Hz
Depth penetration: hundreds of meters (difficult to obtain with artificial sources)
Procedure: use two coils at 45 deg with respect to the horizontal (as the direction of arrival is unknown) and subtract the outputs (zero output for horizontal field)

## Fixed sources EM method: Sundberg method

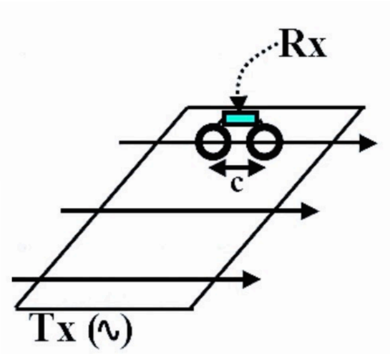



The RX measures the total field: by removing the primary field (filtering of the primaryfield trend), the secondary field is obtained.

Typical loop dimensions 1200x400m

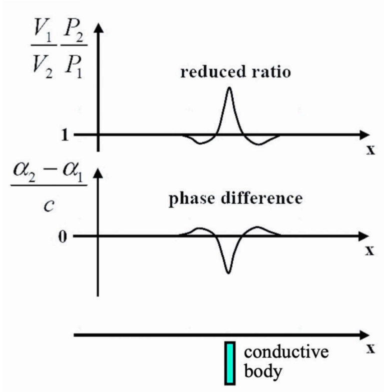



## Fixed sources EM method: Turam method




TX–RX distance: 10–20m
With two receiers, it is possible to measure field "gradients"

V: measured field
P: primary field (trend)
α: phase




## Airborne EM (AEM)

Use artificial/natural (AFMAG, VLF) EM fields. Both TDEM and FDEM methods

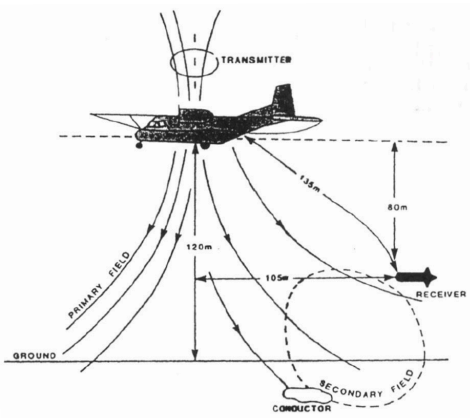



**Figure 10.16** Principle of airborne electromagnetic surveying. The system shown deployed is of the towed-bird type. (Reynolds).

// see examples in the slide #8

## Borehole EM measurements

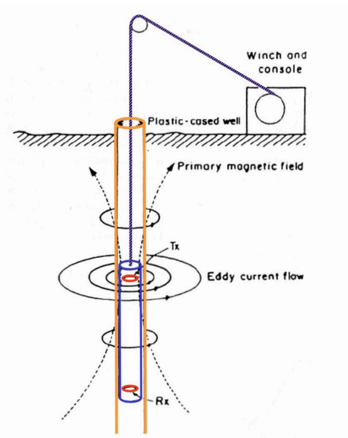



**Figure 10.22** The basic principle behind an EM induction logger for use in boreholes (Reynolds).

// see examples in the slide #8

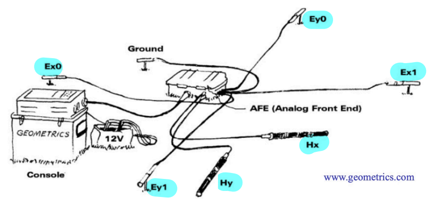
## Magneto-telluric (MT) Method

temporal variations in the Earth's magnetosphere and ionosphere, caused by factors such as the solar wind and diurnal variation of the Earth's magnetic field, result in natural low-frequency magneto-telluric fields across the globe with induce alternating telluric currents within the ground. Higher frequency signals resulting from worldwide electrical storms are superimposed on these low-frequency fields (up to 10kHz).

Conventional magneto-telluric survey techniques, such as natural source MT and audio frequency MT, utilize the magnetic and electric components of the MT fields and currents in order to map variations in subsurface resistivity to depths of up to several hundred kilometers.

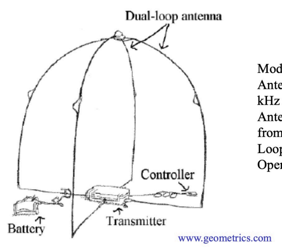
CSAMT (Controlled Source Audio Magneto Telluric) is a specific derivation of conventional natural source and audio frequency magneto-telluric methods, which utilizes an artificial source (typical in the range 0.1Hz to 10kHz) in order to speed up data acquisition and provide a more reliable and strong signal.
- The source normally comprises either a loop or long grounded dipole of up to several kilometers length. The dipole may be combined with a second orthogonal transmitters in order to provide two source polarizations.
- simultaneous measurements of five separate parameters are taken at each location: the two components of the electric field and the three components of the magnetic field.
    * electric field measurements are acquired using orthogonal dipoles
    * the magnetic field vectors are measured using multi-turn high permeability coils.
- measurement of the change in the electric and magnetic fields over a range of frequencies enables an apparent resistivity sounding curve to be constructed. Apparent resistivity is combined with a measure of the phase difference between the electric and magnetic components. Over isotropic homogeneous ground, the magnetic component will lag behind the electric component by $\pi/4$. However, if the resistivity varies with depth, the measured phase difference will be different. Join inversion of the data using both phase and apparent resistivity provides a more robust interpretation. The data are normally displayed as apparent resistivity versus frequency and phase difference versus frequency plots.

MT equipment: receiver




Ex, Ey: sensors (SSE Stainless-Steel Electrodes connected to voltmeters) for measuring the x and y components of the electrical field.
Hx, Hy: sensors (magnetometers) for measuring the x and y components of the magnetic field.

CSAM transmitter

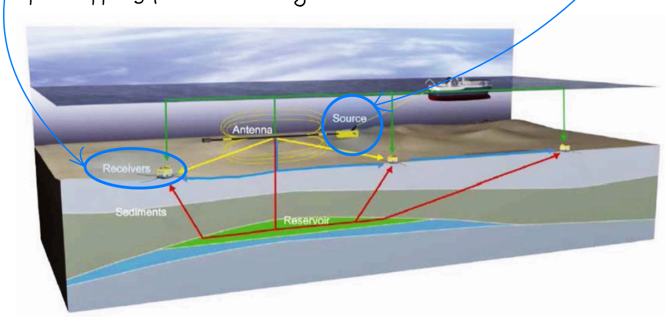


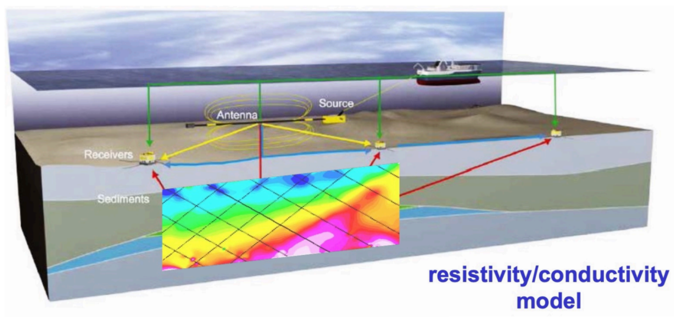

Model TxIM2 with Vertical Loop Antenna with Frequency Range of 1 kHz to 70 kHz.
Antenna Moment of 400 Amp-m2 from Two Perpendicular Vertical Loop Antennae Each 4 m3.
Operates on 12 V DC.

Note: Low frequency -> go deep

## Marine Controlled Source EM (MCSEM)

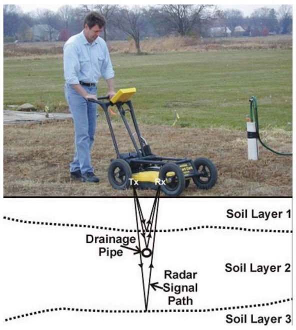
- Source: long antenna (cable, up to 1km) in the water, pulled by a vessel at 10-30 from the sea bottom. Frequency 0.1Hz ~ 3Hz
- Receivers: on the sea bottom, recovered at end of acquisition
- output image: conductivity of subsea media up to 1~3km from seabed.
- usage: Very interesting for hydrocarbon exploration, to distinguish oil and gas (resistive) from brine (conductive)
- pros: upping field shielded by the salt water




// more examples, see the slide #8

2021.12.09. GPR (Ground Penetrating Radar)
// slide #9

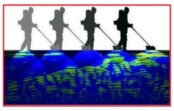


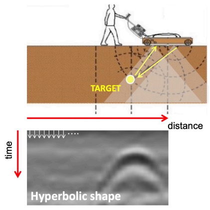

How it works?

– an odometer on a wheel sends a trigger signal every few cm of motion
– a high frequency EM impulse is transmitted by the TX antenna
– the receiver antenna RX records the reflections, produced by variations of the electrical impedance in the subsurface

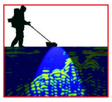
Examples:

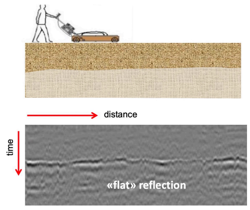
➢ point target



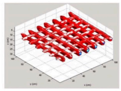


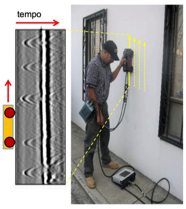

➢ flat interface






• Structural monitoring






GPR application
– ICE exploration
– location of subsurface cavities/structures
– detection of bedrock
– detection of water table
– bridge and road inspections

GPR Formulas

Maxwell formulas for EM wave propagation

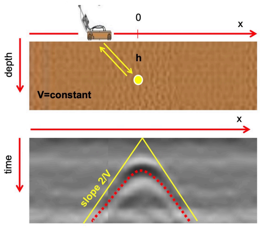
Ground Penetrating Radar

| | | Conduction current | Displacement current | |
|---|---|---|---|---|
| velocity | $V = \sqrt{\dfrac{2\omega}{\mu + \sigma}}$ | $f << \dfrac{\sigma}{2\pi\varepsilon}$ | $f >> \dfrac{\sigma}{2\pi\varepsilon}$ | $V = \dfrac{1}{\sqrt{\varepsilon\mu}}$ |
| Absorption coeff | $\alpha = \sqrt{\dfrac{\omega\mu\sigma}{2}}$ | The "transition" frequency is around 10MHz | | $\alpha = \sqrt{\dfrac{\mu}{\varepsilon}}\dfrac{\sigma}{2}$ |
| EM impedance | $Z = (1+i)\sqrt{\dfrac{\omega\mu}{2\sigma}}$ | $\varepsilon_0 = 8.85\ 10^{-12}$ F/m<br>$\mu_0 = 1{,}25\ 10^{-6}$ Wb/(A m) | | $Z = \sqrt{\dfrac{\mu}{\varepsilon}}$ |

GPR: hyperbolic traveltime…

$$t(x) = \frac{2}{V}\sqrt{h^2 + x^2}$$

Hyperbola with asymptotic slope 2/V

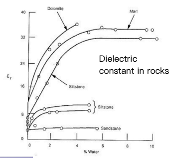



GPR: EM wave velocity

$$V = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{1}{\sqrt{\varepsilon_r\varepsilon_o \cdot \mu_r\mu_o}} = \frac{c}{\sqrt{\varepsilon_r\mu_r}} = \frac{c}{\sqrt{\varepsilon_r}}$$

c= velocity in vacuum
$\varepsilon = \varepsilon_r \varepsilon_o$
$\mu = \mu_r \mu_o$

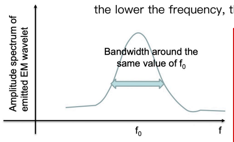
| Medium | Relative dielectric constant $\varepsilon_r$ | EM waves velocity $V$ [m/ns] |
|---|---|---|
| air | 1 | 0.30 |
| water | 81 | 0.033 |
| clay | 5 ÷ 40 | 0.06 |
| Limestone (calcare) | 4 ÷ 8 | 0.12 |
| Dry concrete | 6 | 0.12 |
| Wet concrete | 10.9 | 0.09 |
| Fractured dry concrete | 4.5 | 0.14 |
| Fractured wet concrete | 13.7 | 0.08 |
| granite | 4 ÷ 6 | 0.13 |
| Wet sand | 20 ÷ 30 | 0.06 |
| Dry sand | 3 ÷ 5 | 0.15 |
| Brick's wall | 6 | 0.12 |
| Asphalt | 3 | 0.17 |



Dielectric constant in rocks

## GPR with antenna at $f_0$

the lower the frequency, the bigger the antenna



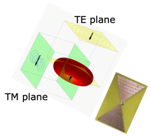

Commercial GPRs
10MHz < $f_0$ < some GHz

The greater $f_0$, the greater the bandwidth,
the shorter the duration of the emitted wavelet which is around 1/bandwidth, the higher the resolution

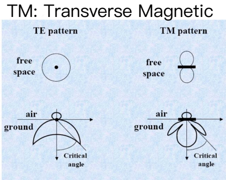
but....

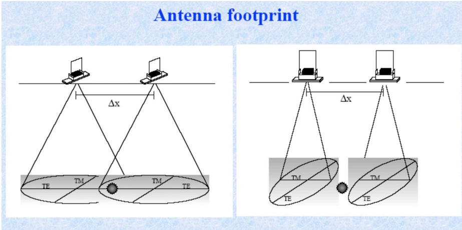
the smaller the skin depth
(proportional to 1/sqrt(sigma . f)

## Antenna directivity patterns

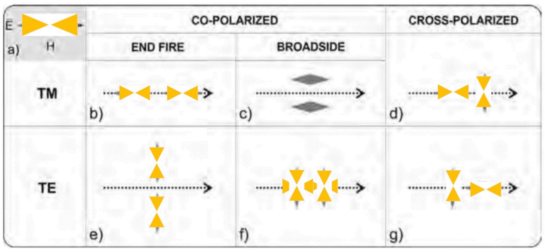



TE: Transverse Electric
TM: Transverse Magnetic

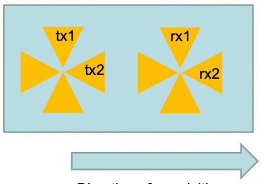



## TX-RX layout




## Multiple polarization GPR




Images:

tx1 on rx1
tx2 on rx2
tx1 on rx2
tx2 on rx1

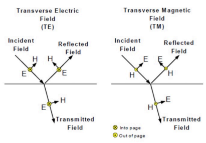
## Scatter coefficients

$$R_{TE} = \frac{Y_1 \cdot \cos\theta_1 - Y_2 \cdot \cos\theta_2}{Y_1 \cdot \cos\theta_1 + Y_2 \cdot \cos\theta_2}$$

$$R_{TM} = \frac{Z_1 \cdot \cos\theta_1 - Z_2 \cdot \cos\theta_2}{Z_1 \cdot \cos\theta_1 + Z_2 \cdot \cos\theta_2}$$

$$T_{TE} = 1 + R_{TE} = \frac{2 \cdot Y_1 \cdot \cos\theta_1}{Y_1 \cdot \cos\theta_1 + Y_2 \cdot \cos\theta_2}$$

$$T_{TM} = 1 + R_{TM} = \frac{2 \cdot Z_1 \cdot \cos\theta_1}{Z_1 \cdot \cos\theta_1 + Z_2 \cdot \cos\theta_2}$$

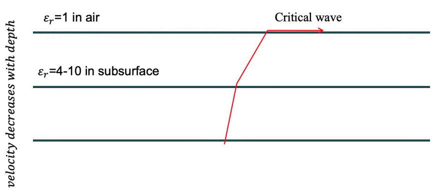



Y=1/Z is the admittance

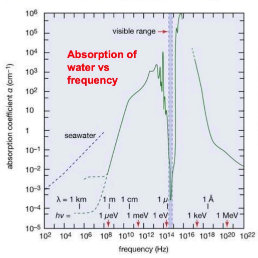
The expressions apply to the electric field in the TM case and the magnetic field in the TE case.

## Snell law

Sin(scatter angle)/velocity = const

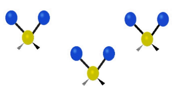



## GPR with water....




Water increase conductivity => decrease of skin depth => less penetration

Microwave oven works with EM waves of the order of GHz where there is an absorption peak => water becomes hot...



Fundamental vibration modes of water molecules
https://youtu.be/jhGhSXAqXmU

GPR data processing:
– Objective: map reflectors
– remove/reduce noise
  * background removal
  * band pass filter on emitted signal bandwidth
– compensate amplitude decay
  * gain recovery
– move reflections from their time to their position
  * velocity analysis
  * migration

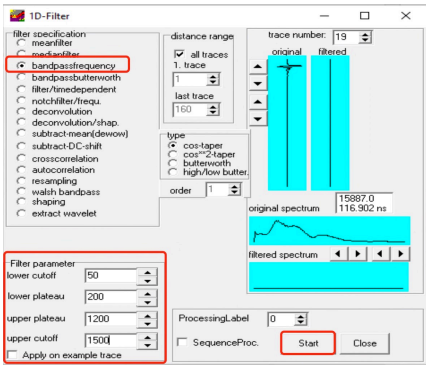
// for more detail, see the slide #09

Note that in the last 10 minutes of this course, report is described!!

2021.12.16. GPR field lab: data processing

Lzz1: antennas at 700MHz
Lzz2: antennas at 250MHz

Processing steps
1. look the raw data
2. band pass filter
   - processing -> 1D-Filter -> “bandpassfrequency” -> filter parameters -> start
3. background removal
   - processing -> 2D filter processing -> background removal ->
4. Move start time to 0
5. gain equalization
6. Velocity analysis
7. Migration



2021.12.23 . Homework 3 - Notes on time tomography lab