Muslim Mathematicians: The Pioneer Archetypes of Mathematics Studies

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Abstract

Mathematics is the key factor behind the development of modern civilization. At the age of today's globalization, science and technology education is completely based on mathematics. But it is not well-known to most of us that when and by whom the different branches of mathematics are originated and enriched afterwards. Muslim mathematicians initiated the different branches of mathematics first. In this article we have made an attempt to focus on the Muslim scholars and mathematicians and their pioneer contributors to modern mathematics studies.

Introduction

In all early civilizations, the first expression of mathematical understanding appears in the form of counting systems[1]. From childhood we learn to count and gradually become familiar with the numbers 1, 2, 3,, etc. As we grow older, we come across other kinds of numbers and learn to add, subtract, multiply and divide numbers in our arithmetic lessons. We develop our ideas about numbers and manipulation with them in an intuitive and "common sense" manner, drawing from familiar examples in our daily life[12]. Thus we introduce ourselves with the foundations of mathematics. The handmaiden of the sciences, as mathematics is called, affects our early school years in such a way that few of us afterwards want to have any give and take with the subject. Yet, ironically, mathematics is all around us. One way or another, almost every aspect of our modern civilization is based on calculations. Be it in architecture, astronomy, medicine, technological development, Computing and the Internet, or mere payment in the local store, we make use of mathematical reckoning throughout our lives. And, are we lucky to live in this advanced age, taking for granted all historically accumulated knowledge in science, much of which is based on mathematics[3]?

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One of the greatest advances was the introduction of "Arabic" numerals. The "Arabic" numerals were influenced by India's mathematics. It is a system based on place values and a decimal system of tens. This system had a zero to hold a place. These numbers were much easier to use for calculation than the Roman system which used numbers, like I, V, X, L, C, M, etc. Addition, subtraction, multiplication and division now became easy. With Arabic numerals, simple fractions and decimal fractions were also possible. Fractions and decimal fractions were also described by Muslim mathematicians during the Middle Ages.



The Muslim mathematicians invented the symbol for zero, and they organized the numbers into the decimal system - base 10. The introduction of the zero was destined to revolutionize mathematics as it allowed for key innovations. It was proposed by **Muhammad Bin Ahmad** in 967 AD. Zero arrived in the West much later, in 13th century. Additionally, they invented the symbol to express an unknown quantity, i.e. variables like x.

Al Khwarismi reworked these numbers and gave us Arabic numerals. Algebra was first fully developed by Al Khwarism, the "father of algebra". In Khwarizmi's own words what he wanted to teach: "...what is easiest and most useful in arithmetic, such as men constantly require in cases of inheritance, legacies, partition, lawsuits, and trade, and in all their dealings with one another, or where the measuring of lands, the digging of canals, geometrical computations, and other objects of various sorts and kinds are concerned...". Al-Khwarizmi wrote about squares and square roots, first studied by the Greeks and Egyptians. Al-Khashi (from Persia, 15th century) invented decimal fractions: 5.25

The Muslims also excelled in geometry as reflected in their art. The brothers Banu Musa (the name means "sons of Musa" and refers to the three brothers, Muhammad, Ahmad, and Al-Hasan) who lived in the 9th century may be said to be the first outstanding Muslim geometers while their contemporary Thabit Ibn Qurrah used the method of exhaustion, giving a glimpse of what was to become integral calculus. They supervised the translation of Greek scientific works into Arabic and helped to found the Arabic school of mathematics. Muslims further defined Euclidian geometry, and pointed the way toward the discovery of independent, non-Euclidean geometry developed in the most recent centuries.

Trigonometry is also mostly a Muslim creation. Al-Tusi, a Muslim, is the "father of trigonometry". It is a branch of mathematics which studies plane and spherical triangles. It developed from the need of astronomers to map points in the sky on a heavenly sphere. Trigonometry's functions, involving ratios such as sine and cosine, tangent and cotangent, were greatly developed and refined in the Islamic lands. Muslim mathematicians invented spherical trigonometry, discovered the tangent and were first, "to introduce the sine of arc in Trigonometrical Calculations" Zero is an invaluable addition made to mathematical science by the Muslims. They have also shown remarkable progress in mathematical geography^[5].

Many Muslim mathematicians such as Omar Khayyam and Al-Tusi also dealt with the fifth postulate of Euclid and the problems that follow if one tries to prove this postulate within the confines of Euclidian geometry. Another branch of mathematics developed by Muslims is trigonometry which was established as a distinct branch of mathematics by Al-Biruni. The Muslim mathematicians, especially Al-Battani, Abu'l-Wafa', Ibn Yunus and Ibn al-Haytham, also developed spherical astronomy and applied it to the solution of astronomical problems.

The love for the study of magic squares and amicable numbers led Muslims to develop the theory of numbers. Abu Jafar Al-Khazan and Abu Kamil of the 10^{th} century were involved in the investigation of the equations of Diophantine. They were also working to prove a special case of what later became popular as Fermat's last theorem. It maintains that rational solution to the equation $x^3 + y^3 = z^3$ does not exist. That is, Al-Khujandi discovered a particular case of Fermat's theorem that "the sum of two cubes cannot be another cube", while Al Karaji analyzed arithmetic and geometric progressions such as: $1^3+2^3+3^3+...+n^3=(1+2+3+...+n)^2$.

In order to understand the history of mathematics studies, it is essential to review its development in the Muslim world in the period from the 9th to the 15th centuries AD. It is really a tremendous matter to have so many Muslim scholars in the 11th century. So many contributions to mathematics studies were not found anywhere of the world in that century except Muslim world. Whatever was done in the western world were the primary levels, the childhood stage of mathematics.

In India only two men who studied on mathematics were found in the 11th century^[4]. One of them was Shatananda Vaskar and another was Sridhar Acharja. In 1020 C, Sridhar Acharja wrote his famous book on mathematics 'Ganit Saar Sangraha' where he described the currently used method of calculating the Least Common Multiple (LCM) of given numbers^[1]. Shatananda Vaskar wrote a book on Astronomy 'Bhasyati' in the 1099 C.

In China only three men who studied a bit on mathematics were found in the 11th century^[4]. One of them was **Chou Tsung** and another was **Su sung** and the third one **Shen Cua** discussed the Astronomy. Everything of them was the mathematical studies of initial stage and was not so remarkable. Dropping the unremarkable things it can be noted that mathematics studies was not discussed anywhere in the world except the Muslim world in that century. That century was enriched by the

contributions of mathematics of the Muslim world. According to George Sarton, "It is almost like passing from the shade to the open sun and from a sleepy world into one tremendously active."

FamousMuslim Mathematicians of the Middle Ages and their Advancements

Muhammad Ibn Musa Al-Khwarizmi (770 - 840 C.E.) was the first great Muslim mathematician, the 9th century scientist who ever lived and is called the "Father of Algebra". He settled in Baghdad which was under the caliphates of al-Ma'mun and al-Mu'tasim in what is considered to be the first Golden Age of Muslim Science. He wrote a treatise on arithmetic whose Latin translation brought what is known as Arabic numerals to the West. To this day *guarismo*, derived from his name, means figure or digit in Spanish while algorithm is still used in English. Khwarizmi influenced the growth of science and mathematics. His approach was systematic and logical.



অহ-কযধিৎরুসর

Al-Khwarazmi is also the author of the first book on algebra and thus famous as the father of Algebra. He produced his most important work in about 830 AD, and called it *Kitab al-jabr wa'l-muqabalah*, or the *Book of Restoring and Balancing*. It dealt with "what is easiest and most useful." From the term "al-jabr," translating as "restoring," in the title, we get "algebra" which was the way the term was translated into Latin in the 12th century. Restoring, in this case, referred to the method of taking a subtracted quantity from one side and placing it to the other side of an equation.

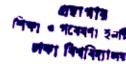
Al-Khwarizmi's book *Kitab al-jabr wa'l-muqabala* proposes a set of rules for arithmetical solutions of linear and quadratic equations, and for elementary geometry. It also resolves inheritance problems regarding the division of money according to proportions, which was in line with the complex requirements of Muslim religious law. The whole work built upon an extended tradition beginning with Babylonian mathematics of the 2nd century BC, going through stages of Greek, Hebrew, and Hindu development. The book served as the prime example for later scientists such as the Egyptian Abu Kamil. Even millennia later, *Kitab al-jabr wa'l-muqabala* was still used as the standard mathematics text at universities in Europe until the 16th century.

Working as an instructor in the academic institution known as the "House of Wisdom", Al-Khwarizmi explicitly presented Indian influence in his works and produced a book on Hindu arithmetic. It was entitled "The Book of Addition and Subtraction According to the Hindu Calculation". This volume was translated in Latin as Algoritmi de Numero Indorum, which means "Al-Khwarizmi Concerning the Hindu Art of Reckoning." The "Algoritmi" in this translation of the title made popular the term "Algorithm". The word "algorithm" is derived from his name. In the book, Al-Khwarizmi tackled and found solutions for specific algebraic equations

called "quadratic equations", which are widely used in science today. As a sign of service to the Muslim faith, al-Khwarizmi's developed a method to calculate the time of visibility of the new moon, indicating the beginning of the Muslim month calculation.

Al-Khwarizmi first reduces an equation (linear or quadratic) to one of six standard forms:

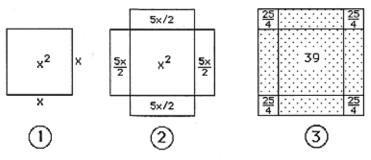
- 1. Squares equal to roots; e.g. $x^2 = \sqrt{2}$.
- Squares equal to numbers; e.g. x² = 2.
- Roots equal to numbers; e.g. x = 2.
- Squares and roots equal to numbers; e.g. x² + 10x = 39.
- Squares and numbers equal to roots; e.g. x² + 21 = 10x.
- Roots and numbers equal to squares; e.g. 3x + 4 = x².



Al-Khwarizmi then shows how to solve these six standard types of equations. He uses both algebraic methods of solution and geometric methods. For example to solve the equation $x^2 + 10x = 39$ he wrote:

... a square and 10 roots are equal to 39 units. The question therefore in this type of equation is about as follows: what is the square which combined with ten of its roots will give a sum total of 39? The manner of solving this type of equation is to take one-half of the roots just mentioned. Now the roots in the problem before us are 10. Therefore take 5, which multiplied by itself gives 25, an amount which you add to 39 giving 64. Having taken then the square root of this which is 8, subtract from it half the roots 5, leaving 3. The number 3 therefore represents one root of this square, which itself, of course is 9. Therefore 9 gives the square.

The geometric proof by completing the square follows. Al-Khwarizmi starts with a square of side x, which therefore represents x^2 (Fig.1). To the square we must add 10x and this is done by adding four rectangles each of breadth 10/4 and length x to the square (Fig. 2). Fig. 2 has area $x^2 + 10x$ which is equal to 39. We now complete the square by adding the four little squares each of area $\frac{5}{2} \times \frac{5}{2} = \frac{25}{4}$. Hence the outside square in Fig 3 has area $4 \times \frac{25}{4} + 39 = 25 + 39 = 64$. The side of the square is therefore 8. But the side is of length $\frac{5}{2} + x + \frac{5}{2}$ so x + 5 = 8, gives x = 3.



al-Khwarizmi completes the square

Abu Al-Wafa Al-Buzajani (940-998 C.E.), the Arab mathematician translated and improved on the works of the Greek mathematicians Euclid and Diophantus and of forerunner Al-Khwarizmi. The exceptional achievements of Al-Wafa include the invention of a field in geometry, which deals with problems leading to equations in algebra of a higher degree than the second. His labor included work on the polyhedral theory and on the development of trigonometry, much of which he put to use in astronomy. Al-Wafa's success in astronomy was marked by his creation of the first wall quadrant for studying the stars. Al-Wafa formulated the tangent and cotangent trigonometric functions, the secant and cosecant functions, showed the generality of the sine theorem for spherical triangles, and thought of a technique of using sine tables.

To begin with, there was a relationship between early Muslim mathematics and the mathematics of Hellenistic and Sanskritic schools, and it looks like the Arabs found useful those earlier Greek and Hindu pre-algebra efforts. This science was developed by Muslims on the basis of earlier Greek and Indian works of a rudimentary nature. Learned Muslim men found interest in the mathematical questions. As a point in case, Thabit Ibn Qurrah (836-901) translated the works of key Greek mathematicians for the time. As a typical scenario, Ibn Qurrah studied and improved those Hellenistic compositions. One such work was Nicomachus of Gerasa's Arithmetic, which he revised and which led him to finding the rule for amicable numbers. These are a pair of numbers such that each number of the pair is the sum of the set of proper divisors of the other number. The search for amicable numbers since has established a long-term fascination with them in the Muslim centers of study. Later in history, Kamal Ad-Din Al-Farisi in the 14th century found the pair 17,926 and 18,416 as an illustration of Ibn Qurrah's rule, and Muhammad Baqir Yazdi in the 17th century produced the pair 9,363,584 and 9,437,056.

Abu Raihan Al-Biruni (973 - 1048 C.E.) was a philosopher, astronomer, pharmacologist (one who studies drugs and herbs used for health), botanist (one who studies plants), geologist and mathematician. Certainly by the age of seventeen Al-Biruni was engaged in serious scientific work for it was in 990 that he computed the latitude of Kath by observing the maximum altitude of the sun. He wrote 14 books amongst which 'Kitab-ul-Hind' gained the most popularity. He was the first Muslim to write about the Hindu society. His book 'Kitab-ul-Hind' is considered an important source of knowledge concerning the history and society of South Asia during the early eleventh century. In the book he has given a detailed account of the geography, literature, philosophy, customs, laws and religious believes of Hindus. His research convinced him that there is a marked difference between Hindus and Muslims and that they are two different nations that have almost nothing in common.

He translated Euclid's work into Sanskrit (an Indian language), and calculated the earth's circumference (distance around the earth) and radius (distance to the center) with an accuracy that is close to today's measurements. He also dealt with progressions while Ghiyath al-Din Jamshid al-Kashani brought the study of number theory among Muslims to its peak. Al-Biruni wrote a mathematical treatise on shadows that helped calibrate sundials accurately. Al-Biruni studied Indian literature in the original, translating several Sanskrit texts into Arabic. He also wrote several

treatises devoted to certain aspects of Indian astronomy and mathematics which were of particular interest to him. Al-Biruni was amazingly well read, having knowledge of Sanskrit literature on topics such as astrology, astronomy, chronology, geography, grammar, mathematics, medicine, philosophy, religion, and weights and measures.

Ibn Sina (980-1037 C.E.) was not only the greatest medical scholars of mediaeval times but also made a lasting contribution to the development of mathematical Science. His full name is Abu Ali Al-Hussain Ibn Abdallah Ibn Sina and in somewhere he is known Avicenna. He was born in 980 C.E. at Afshana near Bukhara. The young Bu Ali received his early education in Bukhara, and by the age of ten had become well versed in the study of the Qur'an and various sciences. While still young, he attained such a degree of expertise in medicine that his renown spread far and wide. At the age of 17, he was fortunate in curing Nooh Ibn Mansoor, the King of Bukhhara, of an illness in which all the well-known physicians had given up hope. On his recovery, the King wished to reward him, but the young physician only desired permission to use his uniquely stocked library.

He was the most famous physician, philosopher, encyclopaedist, mathematician and astronomer of his time. His major contribution to medical science was his famous book al-Qanun, known as the "Canon" in the West. The Qanun fi al-Tibb is an immense encyclo- paedia of medicine extending over a million words. It surveyed the entire medical knowledge available from ancient and Muslim sources. His important original contribution includes such advances as recognition of the contagious nature of phthisis and tuberculosis; distribution of diseases by water and soil, and interaction between psychology and health. He was also the first to describe meningitis and made rich contributions to anatomy, gynaecology and child health.

Ibn Sina also contributed to mathematics, physics, music and other fields. In physical mathematics, his contribution comprised the study of different forms of energy, heat, light and mechanical, and such concepts as force, vacuum and infinity. He made the important observation that if the perception of light is due to the emission of some sort of particles by the luminous source, the speed of light must be finite. He propounded an interconnection between time and motion, and also made investigations on specific gravity and used an air thermo-meter. Ibn Sina observed that in the series of consonances represented by (n + 1)/n, the ear is unable to distinguish them when n = 45. His treatise on minerals was one of the "main" sources of geology of the Christian encyclopaedists of the thirteenth century. Besides *Shifa* his well-known treatises in philosophy are *al-Najat* and *Isharat*.

Omar Khayyam (1044 - 1123 C.E.): Another great Muslim mathematician was Omar Khayyam. He further developed the most notably Al-Khawarizmi's work algebra. He is best known today for his poetry, but his contribution to mathematics was great. He showed how to express roots of cubic equations by line segments obtained by intersecting conic sections. Khayyam was an outstanding poet, mathematician, and astronomer. His astronomical observations also had practical applications including not only finding the



Omar Khayyam

direction of Makkah for prayers, but also devising almanacs (the word itself being of Arabic origin). His work on algebra was known throughout Europe in the Middle Ages, and he also applied his astronomical knowledge to questions of time keeping and the calendar. The most exact solar calendar existing to this day is the Jalali calendar devised under the direction of 'Omar Khayyam in the 12th century and still in use in Persia and Afghanistan. Khayyam refers in his algebra book to Pascal's triangle. The algebra of Khayyam is geometrical, solving linear and quadratic equations by methods appearing in Euclid's Elements. Khayyam also gave important results on ratios giving a new definition and extending Euclid's work to include the multiplication of ratios. He poses the question of whether a ratio can be regarded as a number but leaves the question unanswered.

Nasir al-Din al-Tusi (1201–1274 C.E.) was a Persian scientist, of Shi'a islamic belief, born in Tus, Khorasan (then Persia, present time Iran). He is known to be a philosopher, mathematicians, astronomer, theologian, physician, and a prolific writer. He is one of the founders of spherical Trigonometry that includes six fundamental formulas for the solution of spherical right-angled triangles. As the armies of Genghis Khan swept his homeland, he fled to join the Ismailis and made his most important contributions in science during this time, while he was moving from one stronghold to another. He finally joined Hulagu's (Ghenghis Khans' son) ranks, after the invasion of the Alamut castle of the Hashshashin (Assassins) by Mongol forces.

Tusi made very accurate tables of planetary movements as depicted in his book Zij-i ilkhani (the Ilkhanic Tables). This book contains tables for calculating the positions of the planets and the names of the stars. His planetary system was the most advanced of his period and was used extensively until the development of the heliocentric model in the time of Copernicus. Between Ptolemy and Copernicus, he is considered as the most eminent scientist on this field.

He is also known by Tusi-couple, which resolves linear motion into the sum of two circular motions. He also calculated the value of 51' for the precession of the equinoxes and contributed to construction and usage of some astronomical instruments including astrolabe. He gave the first extant exposition of the whole system of plane and spherical trigonometry. A 60-km diameter lunar crater located on the southern hemisphere of the moon is named after him as "Nasireddin".

Al-Khashi (1390-1450 C.E.): born in Kashan, Iran and died in Samarkand (now Uzbek). He devised a method of computing the decimal expansion of pi (the ratio between a circle's circumference and its diameter)^4, obtaining a result accurate to 16 places which was the best until about 1700. He considered himself the inventor of decimal fractions. Al-Khashi also devised the first mechanical computing devices which could be used to predict the occurrence of important celestial phenomena such as solar and lunar eclipses. He wrote *The Reckoners' Key* which summarizes arithmetic and contains work on algebra and geometry.

Muslim scholars: At a glance

A brief history of Muslim mathematicians and their contributions to mathematical world in their timeline^[6,7] is summarized below(see Table-1). This chart depicts the lives of key Islamic Scientists, specially the Muslim Mathematicians from the 8th to the end of the 14th century. By placing each writer in a historical context, this will help us understand the influences and borrowing of ideas.

Muslim Mathematicians	Contributions	Ages
Al-Khwarizmi (Algorizm)	Mathematics, Astronomy, Geography. (Algorithm, Algebra, calculus) (Father of Algebra)	770 - 840
Ibn Ishaq Al-Kindi (Alkindus)	Physics, Optics, Mathematics.	800 - 873
Thabit Ibn Qurrah (Thebit)	Astronomy, Mechanics, Geometry, Anatomy.	836 - 901
'Abbas Ibn Firnas	Mechanics of Flight, Planetarium, Artificial Crystals.	Died 888
Ali Ibn Rabban Al-Tabari	Mathematics.	838 - 870
Al-Battani (Albategnius)	Astronomy, mathematics, Trigonometry.	858 - 929
Al-Farghani (Al-Fraganus)	Astronomy, Civil Engineering.	C. 860
Abul Hasan Ali Al-Masu'di	Geography.	Died 957
Al-Sufi (Azophi)	Astronomy	903 - 986
Muhammad Al-Buzjani	Mathematics, Astronomy, Geometry, Trigonometry.	940 - 997
Ibn Al-Haitham (Alhazen)	Physics, Optics, Mathematics.	965 - 1040
Abu Raihan Al-Biruni	Astronomy, Mathematics. (Determined Earth's circumference)	973-1048
Ibn Sina (Avicenna)	Mathematics, Astronomy.	981 - 1037
Al-Zarqali (Arzachel)	Astronomy (Invented Astrolabe).	1028 - 1087
Omar Al-Khayyam	Mathematics.	1044 - 1123
Al-Idrisi (Dreses)	Geography (World Map, First Globe).	1099 - 1166
Ibn Rushd (Averroes)	Astronomy.	1128 - 1198
Al-Bitruji (Alpetragius)	Astronomy	Died 1204
Nasir Al-Din Al-Tusi	Astronomy, Non-Euclidean Geometry.	1201 - 1274
Al-Fida (Abdulfeda)	Astronomy, Geography,	1273 - 1331
Ulugh Beg	Astronomy	1393 – 1449

Conclusion

From the above discussion of the context of situation we find that almost every branches of mathematics were invented and enriched by the touch of the Muslim. They were at the top of mathematics studies in their ages. So, the Muslim may be regarded as the pioneer archetype of mathematics studies. Most of the inventions and developments of mathematics were originated by the Muslim mathematicians from the timeline 700-1500 centuries.

Muslims distinguished themselves not only as theoretical mathematician and scientific thinkers, but contributed through innumerable inventions to the growth of the modern mathematics studies. Though the mediaeval Muslim mathematicians had very meager resources at their command as compared to those of the present age, they achieved a great deal. They replaced the old speculative method of the Greeks with an experimental method, which in later periods formed the basis of all scientific investigations.

The Muslim Mathematicians made additional contributions of their own, and through their study and written work, they preserved the knowledge of mathematics that otherwise might have been lost to the world. So from the beginning, "Arabic math" was a mixing of international knowledge.

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