GEN024 Final Exam 2018/9

Write your name and student ID number, and all your answers in the places provided on the separate answer sheets. (5pts$\times$ 20)

Part I.

1. Complete the truth tables of $(p \Rightarrow r) \wedge (q \Rightarrow r)$ and $(p \vee q) \Rightarrow r$, and determine whether they are logically equivalent.

2. Write $\boldsymbol{T}$ for true and $\boldsymbol{F}$ for false in the answer sheets.

$$\begin{cases} -3x - y + 7z & = & a \\ 2x + 2y - 2z & = & b \\ x - 3z & = & c \end{cases}, \quad A = \begin{bmatrix} -3 & -1 & 7 \\ 2 & 2 & -2 \\ 1 & 0 & -3 \end{bmatrix}.$$

   (a) The matrix A is the coefficient matrix of the system of linear equations above.
   (b) The matrix A is invertible.
   (c) The system is consistent for some values of a, b, c.
   (d) The system is always consistent for all values of a, b, c.
   (e) The system has a unique solution for some values of a, b, c.

Part II. Write the answers of the following in the places provided in answer sheets. Show work! If you apply a proposition, state the number or the statement clearly.

3. Write a polynomial $g(x)$ of degree at most three such that $g(-2) = 6$, $g(-1) = 0$, $g(0) = 2$ and $g(1) = -3$, and a polynomial $h(x)$ with degree exactly five such that $h(-2) = 6$, $h(-1) = 0$, $h(0) = 2$, $h(1) = -3$ and $g(2) = h(2)$.

4. Let $f(x) = x^4 - 2x^3 - 16x^2 + 56x - 45 = q(x)(x-2) + r = c_4(x-2)^4 + c_3(x-2)^3 + c_2(x-2)^2 + c_1(x-2) + c_0$. Find a polynomial $q(x)$, constants r and c_4, c_3, c_2, c_1, c_0.

5. Let $f(x)$ be a polynomial in Problem 4. (a) Show that there is a zero between 0 and 2, i.e., there is c with $0 < c < 2$ such that $f(c) = 0$. (b) Determine whether $c \geqq 1$ or $c < 1$.

6. Let $f(x)$ be a polynomial in Problem 4. Determine whether $f(x)$ is increasing, decreasing at $x = 2$ or $f(2)$ is a local maximum or a local minimum. Why?

7. Find the limit $\displaystyle\lim_{x \to 2} \frac{x^3 + x^2 - 16x + 20}{x^4 - 2x^3 - 16x^2 + 56x - 48}$.

8. Find the limit $\displaystyle\lim_{x \to 0} \frac{e^{-3x} - 1 + 3x}{x^2}$. First compute $(e^{-3x})'$.

9. Find the derivative of $\displaystyle\frac{1}{(2x-1)^9}$.

10. Find the derivative of $x^3 e^{-x^3}$.

11. Find the definite integral $\int_0^1 x^2(1-x^3)e^{-x^3}\,dx$.

12. Find the indefinite integral $\int \left(\frac{8}{x^3} + \frac{x}{2} + \sqrt{x}\right) dx$.

13. Find the indefinite integral $\int \frac{1}{(2x-1)^{10}} dx$.

14. Find the derivative of $F(x)$, where $F(x) = \int_{-1}^{x} t^3 e^{-t^3}\,dt$.

Part III. Write your answers on the answer sheets.

$$B = \begin{bmatrix} 1 & 0 & -2 & 0 & 0 & -2 & 4 \\ 2 & 0 & -4 & 0 & 1 & 5 & 7 \\ -3 & 3 & -3 & 3 & 0 & 3 & -15 \\ 2 & -1 & -1 & -2 & 0 & -2 & 7 \end{bmatrix} \to\to\to C = \begin{bmatrix} 1 & 0 & -2 & 0 & 0 & -2 & 4 \\ 2 & -1 & -1 & -2 & 0 & -2 & 7 \\ -1 & 1 & -1 & 1 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 & 1 & 9 & -1 \end{bmatrix}$$

15. The matrix C is obtained from the matrix B by performing elementary row operations three times. Write them in order using notation $[i, j; c]$ (add c times row j to row i), $[i, j]$ (interchange row i and row j), $[i; c]$ (multiply every entry in row i by c).

16. Find a 4×4 matrix T satisfying $TB = C$.

17. Express the inverse T^{-1} of the matrix T in Problem 16 as a product of elementary matrices using the notation $E(i, j; c)$, $E(i, j)$ and $E(i; c)$.

18. Suppose the matrix B above is an augmented matrix of a system of linear equations with unknowns $x_1, x_2, x_3, x_4, x_5, x_6$. Find (a) the reduced row echelon form of B, and (b) the solutions of the system.

19. Let $f(x) = e^{-x} + 2$. Find the equation of the tangent line to $y = f(x)$ at $x = 0$.

20. Apply Proposition 7.4, and solve the differential equation below. (a) identify $h(x)$, $g(y)$, and (b) find $G(y)$, $H(x)$ and write the equation $G(y) = H(x) + C$. Finally (c) solve it with an initial condition below for $y = f(x)$.

$$y \cdot y' = y \cdot \frac{dy}{dx} = \frac{3}{2}x^2, \quad y(0) = f(0) = 1.$$

$$\boxed{y' = \frac{h(x)}{g(y)}, H'(x) = h(x), G'(y) = g(y) \Rightarrow G(y) = H(x) + C.}$$

GEN024 FINAL 2018/9 Answer Sheets

ID#: **Name:**

Part I-1.

p	q	r	$(p \Rightarrow r) \wedge (q \Rightarrow r)$	$(p \vee q) \Rightarrow r$
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

Are these logically equivalent?

2.

(a)	(b)	(c)	(d)	(e)

No.	PTS.
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
11.	
12.	
13.	
14.	
15.	
16.	
17.	
18.	
19.	
20.	
Total	

Message: Did you enjoy mathematics, or did you suffer a lot? I appreciate your feedbacks on the following. 数学少しは楽しめましたか。苦しんだ人もいるかな。以下のことについて書いて下さい。(If you don't want your message to be posted, write "Do Not Post."「HP 掲載不可」は明記の事。)

(A) About this class, especially on improvements. この授業について。改善点など何でもどうぞ。

(B) About the education at ICU, especially on improvements. Any comments concerning ICU are welcome. ICU の教育一般について。改善点など、ICU に関すること何でもどうぞ。

Part II.

3. $g(x) =$

   $h(x) =$

4. Let $f(x) = x^4 - 2x^3 - 16x^2 + 56x - 45 = q(x)(x-2) + r = c_4(x-2)^4 + c_3(x-2)^3 + c_2(x-2)^2 + c_1(x-2) + c_0$. Find a polynomial $q(x)$, constants r and c_4, c_3, c_2, c_1, c_0.

5. Let $f(x)$ be a polynomial in Problem 4.
   (a) Show that there is a zero between 0 and 2, i.e., there is c with $0 < c < 2$ such that $f(c) = 0$.

   (b) Determine whether $c \geqq 1$ or $c < 1$.

6. Let $f(x)$ be a polynomial in Problem 4. Determine whether $f(x)$ is increasing, decreasing at $x = 2$ or $f(2)$ is a local maximum or a local minimum. Why?

7. Find the limit $\lim_{x \to 2} \frac{x^3 + x^2 - 16x + 20}{x^4 - 2x^3 - 16x^2 + 56x - 48}$.

8. Find the limit $\lim_{x \to 0} \frac{e^{-3x} - 1 + 3x}{x^2}$. First compute $(e^{-3x})'$.

9. Find the derivative of $\frac{1}{(2x-1)^9}$.

10. Find the derivative of $x^3 e^{-x^3}$.

11. Find the definite integral $\int_0^1 x^2(1 - x^3)e^{-x^3}\, dx$.

12. Find the indefinite integral $\int \left(\frac{8}{x^3} + \frac{x}{2} + \sqrt{x} \right) dx$.

13. Find the indefinite integral $\displaystyle\int \frac{1}{(2x-1)^{10}}dx$.

14. Find the derivative of $F(x)$, where $F(x) = \displaystyle\int_{-1}^{x} t^3 e^{-t^3}\,dt$.

Part III.

15. The matrix C is obtained from the matrix B by performing elementary row operations three times. Write them in order using notation $[i,j;c]$ (add c times row j to row i), $[i,j]$ (interchange row i and row j), $[i;c]$ (multiply every entry in row i by c).

16. Find a 4×4 matrix T satisfying $TB = C$.

17. Express the inverse T^{-1} of the matrix T in Problem 16 as a product of elementary matrices using the notation $E(i,j;c)$, $E(i,j)$ and $E(i;c)$.

18. Suppose the matrix B above is an augmented matrix of a system of linear equations with unknowns $x_1, x_2, x_3, x_4, x_5, x_6$. Find (a) the reduced row echelon form of B, and (b) the solutions of the system.

(a)

(b)

19. Let $f(x) = e^{-x} + 2$. Find the equation of the tangent line to $y = f(x)$ at $x = 0$.

20. Apply Proposition 7.4, and solve the differential equation below. (a) identify $h(x)$, $g(y)$, and (b) find $G(y)$, $H(x)$ and write the equation $G(y) = H(x) + C$. Finally (c) solve it with an initial condition below for $y = f(x)$.

$$y \cdot y' = y \cdot \frac{dy}{dx} = \frac{3}{2}x^2, \quad y(0) = f(0) = 1.$$

(a)

(b)

(c)

Solutions to GEN024 FINAL 2018/9

Part I.

1.

p	q	r	$(p$	$\Rightarrow$	$r)$	$\wedge$	$(q$	$\Rightarrow$	$r)$	$(p$	$\vee$	$q)$	$\Rightarrow$	r
T	T	T				$\boldsymbol{T}$							$\boldsymbol{T}$	
T	T	F		F		$\boldsymbol{F}$		F					$\boldsymbol{F}$	F
T	F	T				$\boldsymbol{T}$							$\boldsymbol{T}$	
T	F	F		F		$\boldsymbol{F}$							$\boldsymbol{F}$	F
F	T	T				$\boldsymbol{T}$							$\boldsymbol{T}$	
F	T	F				$\boldsymbol{F}$		F					$\boldsymbol{F}$	F
F	F	T				$\boldsymbol{T}$					F		$\boldsymbol{T}$	
F	F	F				$\boldsymbol{T}$					F		$\boldsymbol{T}$	F

Are these logically equivalent? : **YES**

2.

(a)	(b)	(c)	(d)	(e)
$\boldsymbol{T}$	$\boldsymbol{F}$	$\boldsymbol{T}$	$\boldsymbol{F}$	$\boldsymbol{F}$

(a) A is the coefficient matrix. (b) The reduced row echelon form of A is $\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$. Hence A is not invertible. (c) If $a = b = c = 0$, the system is consistent as $x = y = z = 0$ is a solution. (d) A does not have a leading one in every row, the system is inconsistent for some a, b, c. (e) Since there is a column which does not have a leading one, if it is consistent, the general solution always contains a free parameter.

Part II.

3. $g(x)$: $g(-2) = 6$, $g(-1) = 0$, $g(0) = 2$ and $g(1) = -3$, $\deg g(x) \leqq 3$.

$$\begin{aligned} g(x) &= 6\frac{(x+1)x(x-1)}{(-2+1)(-2)(-2-1)} + 2\frac{(x+2)(x+1)(x-1)}{(0+2)(0+1)(0-1)} - 3\frac{(x+2)(x+1)x}{(1+2)(1+1)1} \\ &= -(x+1)x(x-1) - (x+2)(x+1)(x-1) - \frac{1}{2}(x+2)(x+1)x \\ &= -\frac{5}{2}x^3 - \frac{7}{2}x^2 + x + 2. \end{aligned}$$

$h(x)$: $h(-2) = 6$, $h(-1) = 0$, $h(0) = 2$, $h(1) = -3$ and $g(2) = h(2)$, $\deg h(x) = 5$.

$$h(x) = g(x) + c(x+2)(x+1)x(x-1)(x-2), \text{ where } c \text{ is a nonzero constant.}$$

4. Let $f(x) = x^4 - 2x^3 - 16x^2 + 56x - 45 = q(x)(x-2) + r = c_4(x-2)^4 + c_3(x-2)^3 + c_2(x-2)^2 + c_1(x-2) + c_0$. Find a polynomial $q(x)$, constants r and c_4, c_3, c_2, c_1, c_0.

Soln. $f(x) = x^4 - 2x^3 - 16x^2 + 56x - 45 = (x^3 - 16x + 24)(x - 2) + 3 = (x - 2)^4 + 6(x - 2)^3 - 4(x - 2)^2 + 3$. Hence $q(x) = x^3 - 16x + 24$, $r = c_0 = 3$, $c_1 = 0$, $c_2 = -4$, $c_3 = 6$, $c_4 = 1$. Use synthetic division.

5. Let $f(x)$ be a polynomial in Problem 4. (a) Show that there is a zero between 0 and 2, i.e., there is c with $0 < c < 2$ such that $f(c) = 0$. (b) Determine whether $c \geqq 1$ or $c < 1$.

   Soln. Since $f(x)$ is a polynomial, it is continuous in a closed interval $[0, 2]$. Since $f(0) = -45 < 0$ and $f(2) = 3 > 0$, by Intermediate Value Theorem, there is c with $0 < c < 2$ such that $f(c) = 0$. Since $f(1) = -6 < 0$, there is a zero between 1 and 2. Hence $c \geqq 1$.

6. Let $f(x)$ be a polynomial in Problem 4. Determine whether $f(x)$ is increasing, decreasing at $x = 2$ or $f(2)$ is a local maximum or a local minimum. Why?

   Soln. $f'(x) = 4(x-2)^3 + 18(x-2)^2 - 8(x-2)$, $f''(x) = 12(x-2)^2 + 36(x-2) - 8$. Hence, $f'(2) = 0$, $f''(2) = -8 < 0$. Thus $f(2)$ is a local maximum by Second Derivative Test.

7. Find the limit $\displaystyle\lim_{x\to 2} \frac{x^3 + x^2 - 16x + 20}{x^4 - 2x^3 - 16x^2 + 56x - 48}$.

   Soln. Since $x^3 + x^2 - 16x + 20 = (x - 2)^2(x + 5)$ by synthetic division, and $x^4 - 2x^3 - 16x^2 + 56x - 48 = (x - 2)^2((x - 2)^2 + 6(x - 2) - 4)$ by Problem 4 above,

$$= \lim_{x\to 2} \frac{(x-2)^2(x+5)}{(x-2)^2((x-2)^2 + 6(x-2) - 4)} = \lim_{x\to 2} \frac{x+5}{(x-2)^2 + 6(x-2) - 4} = \frac{7}{-4} = -\frac{7}{4}.$$

8. Find the limit $\displaystyle\lim_{x\to 0} \frac{e^{-3x} - 1 + 3x}{x^2}$. First compute $(e^{-3x})'$.

   Soln. By Chain Rule, $(e^{-3x})' = -3e^{-3x}$. Now use l'Hôpital's rule.

$$\lim_{x\to 0} \frac{e^{-3x} - 1 + 3x}{x^2} = \lim_{x\to 0} \frac{-3e^{-3x} + 3}{2x} = \lim_{x\to 0} \frac{9e^{-3x}}{2} = \frac{9}{2}.$$

9. Find the derivative of $\dfrac{1}{(2x-1)^9}$.

   Soln.

$$\left(\frac{1}{(2x-1)^9}\right)' = ((2x-1)^{-9})' = -9(2x-1)^{-10}(2x-1)' = -18(2x-1)^{-10} = -\frac{18}{(2x-1)^{10}}.$$

10. Find the derivative of $x^3 e^{-x^3}$.

    Soln. Since $(e^{-x^3})' = e^{-x^3}(-x^3)' = -3x^2 e^{-x^3}$ by the Chain Rule, using the Product Rule we have

$$(x^3 e^{-x^3})' = (x^3)' e^{-x^3} + x^3 (e^{-x^3})' = 3x^2 e^{-x^3} - 3x^5 e^{-x^3} = 3x^2(1 - x^3)e^{-x^3}.$$

11. Find the definite integral $\displaystyle\int_0^1 x^2(1 - x^3)e^{-x^3}\,dx$.

    Soln. By Problem 10,

$$\int_0^1 x^2(1 - x^3)e^{-x^3}\,dx = \left[\frac{1}{3}x^3 e^{-x^3}\right]_0^1 = \frac{1}{3}e^{-1} = \frac{1}{3e}.$$

12. Find the indefinite integral $\int \left(\frac{8}{x^3} + \frac{x}{2} + \sqrt{x} \right) dx$.

Soln. Since $\sqrt{x} = x^{\frac{1}{2}}$,

$$\begin{aligned}
&\int \left(\frac{8}{x^3} + \frac{x}{2} + \sqrt{x} \right) dx \\
= \quad & \int \left(8x^{-3} + \frac{1}{2}x^1 + x^{\frac{1}{2}} \right) dx = \frac{8}{-3+1}x^{-3+1} + \frac{1}{2} \cdot \frac{1}{1+1}x^{1+1} + \frac{1}{\frac{1}{2}+1}x^{\frac{1}{2}+1} + C \\
= \quad & -4x^{-2} + \frac{1}{4}x^2 + \frac{2}{3}x^{\frac{3}{2}} + C = -\frac{4}{x^2} + \frac{1}{4}x^2 + \frac{2}{3}x\sqrt{x} + C.
\end{aligned}$$

13. Find the indefinite integral $\int \frac{1}{(2x-1)^{10}} dx$.

Soln. By Problem 9, $-\frac{1}{18(2x-1)^9}$ is an antiderivative of $\frac{1}{(2x-1)^{10}}$. Hence

$$\int \frac{1}{(2x-1)^{10}} dx = -\frac{1}{18(2x-3)^9} + C.$$

14. Find the derivative of $F(x)$, where $F(x) = \int_{-1}^{x} t^3 e^{-t^3} dt$.

Soln. Since $F(x)$ is an antiderivative of $x^3 e^{-x^3}$, $F'(x) = x^3 e^{-x^3}$ by Fundamental Theorem of Calculus.

Part III.

15. The matrix C is obtained from the matrix B by performing elementary row operations three times. Write these operations in order using notation $[i, j; c]$, $[i, j]$, and $[i; c]$.

Soln.

$$B = \begin{bmatrix} 1 & 0 & -2 & 0 & 0 & -2 & 4 \\ 2 & 0 & -4 & 0 & 1 & 5 & 7 \\ -3 & 3 & -3 & 3 & 0 & 3 & -15 \\ 2 & -1 & -1 & -2 & 0 & -2 & 7 \end{bmatrix} \overset{[2,1;-2]}{\to} \begin{bmatrix} 1 & 0 & -2 & 0 & 0 & -2 & 4 \\ 0 & 0 & 0 & 0 & 1 & 9 & -1 \\ -3 & 3 & -3 & 3 & 0 & 3 & -15 \\ 2 & -1 & -1 & -2 & 0 & -2 & 7 \end{bmatrix} \overset{[3;1/3]}{\to}$$

$$\begin{bmatrix} 1 & 0 & -2 & 0 & 0 & -2 & 4 \\ 0 & 0 & 0 & 0 & 1 & 9 & -1 \\ -1 & 1 & -1 & 1 & 0 & 1 & -5 \\ 2 & -1 & -1 & -2 & 0 & -2 & 7 \end{bmatrix} \overset{[2,4]}{\to} C = \begin{bmatrix} 1 & 0 & -2 & 0 & 0 & -2 & 4 \\ 2 & -1 & -1 & -2 & 0 & -2 & 7 \\ -1 & 1 & -1 & 1 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 & 1 & 9 & -1 \end{bmatrix}$$

Hence the operations are $[2, 1; -2], [3; 1/3], [2, 4]$ in this order. There are other solutions.

16. Find a 4×4 matrix T satisfying $TB = C$.

Soln. We obtain the matrix T by applying $[2, 1; -2], [3; 1/3], [2, 4]$ to I in this order.

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{[2,1;-2]} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{[3;1/3]} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{[2,4]} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1/3 & 0 \\ -2 & 1 & 0 & 0 \end{bmatrix} = T,$$

or

$$T = E(2, 4)E(3; 1/3)E(2, 1; -2) = E(2, 4)E(2, 1; -2)E(3; 1/3) = E(3; 1/3)E(2, 4)E(2, 1; -2).$$

17. Express the inverse T^{-1} of the matrix T in Problem 16 as a product of elementary matrices using the notation $E(i,j;c)$, $E(i,j)$ and $E(i;c)$.

Soln. Since $T = E(2,4)E(3;1/3)E(2,1;-2)$,

$$\begin{aligned} T^{-1} &= (E(2,4)E(3;1/3)E(2,1;-2))^{-1} = E(2,1;-2)^{-1}E(3;1/3)^{-1}E(2,4)^{-1} \\ &= E(2,1;2)E(3;3)E(2,4). \end{aligned}$$

18. Suppose the matrix B above is an augmented matrix of a system of linear equations with unknowns $x_1, x_2, x_3, x_4, x_5, x_6$. Find (a) the reduced row echelon form of B, and (b) the solutions of the system.

Soln. (a) Using $B \to\to\to C$, we start from C.

$$C = \begin{bmatrix} 1 & 0 & -2 & 0 & 0 & -2 & 4 \\ 2 & -1 & -1 & -2 & 0 & -2 & 7 \\ -1 & 1 & -1 & 1 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 & 1 & 9 & -1 \end{bmatrix} \xrightarrow{[2,3;2]} \begin{bmatrix} 1 & 0 & -2 & 0 & 0 & -2 & 4 \\ 0 & 1 & -3 & 0 & 0 & 0 & -3 \\ -1 & 1 & -1 & 1 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 & 1 & 9 & -1 \end{bmatrix} \xrightarrow{[3,1;1]}$$

$$\begin{bmatrix} 1 & 0 & -2 & 0 & 0 & -2 & 4 \\ 0 & 1 & -3 & 0 & 0 & 0 & -3 \\ 0 & 1 & -3 & 1 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 9 & -1 \end{bmatrix} \xrightarrow{[3,2;-1]} \begin{bmatrix} 1 & 0 & -2 & 0 & 0 & -2 & 4 \\ 0 & 1 & -3 & 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 0 & 1 & 9 & -1 \end{bmatrix}, \quad \begin{cases} x_1 &= 2s + 2t + 4, \\ x_2 &= 3s - 3, \\ x_3 &= s\text{: free}, \\ x_4 &= t + 2, \\ x_5 &= -9t - 1, \\ x_6 &= t\text{: free}. \end{cases}$$

19. Let $f(x) = e^{-x} + 2$. Find the equation of the tangent line to $y = f(x)$ at $x = 0$.

Soln. Since $f'(x) = e^{-x}(-x)' = -e^{-x}$, $f(0) = e^0 + 2 = 3$,

$$y = f(0) + f'(0)(x - 0) = 3 + (-1)(x - 0) = 3 - x.$$

20. Apply Proposition 7.4 and solve the differential equation below. (a) identify $h(x)$, $g(y)$, and (b) find $G(y)$, $H(x)$ and write the equation $G(y) = H(x) + C$. Finally (c) solve it with an initial condition below for $y = f(x)$.

$$y \cdot y' = y \cdot \frac{dy}{dx} = \frac{3}{2}x^2, \quad y(0) = f(0) = 1.$$

Soln.

$$\frac{dy}{dx} = y' = \frac{3x^2}{2y}.$$

So one of the choices is $h(x) = 3x^2$ and $g(y) = 2y$. Hence $H(x) = x^3$ and $G(y) = y^2$.

$$y^2 = x^3 + C, \ 0 + C = y(0)^2 = 1.$$

Therefore, $y^2 = x^3 + 1$. $y = \sqrt{x^3 + 1}$, as $y(0) > 0$.

$$y' = \frac{1}{2}(x^3 + 1)^{-\frac{1}{2}}(x^3 + 1)' = \frac{3}{2}\frac{x^2}{\sqrt{x^3 + 1}} = \frac{3}{2}\frac{x^2}{y}, \text{ and } y(0) = 1.$$