

# GEN024 Final Exam 2017/8

Write your name and student ID number, and all your answers in the places provided on the separate answer sheets. (5pts× 20)

## Part I.

1. Complete the truth tables of  $p \Rightarrow (q \vee r)$  and  $(\neg(p \wedge \neg q)) \vee r$ , and determine whether they are logically equivalent.
2. Write **T** for true and **F** for false in the answer sheets.

$$\begin{cases} x - y &= 3 \\ 2x - y + z &= 0 \\ 6x - 2y + 3z &= 1 \end{cases}, A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & -1 & 1 \\ 6 & -2 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 & -3 & 1 \\ 0 & -3 & 1 \\ -2 & 4 & -1 \end{bmatrix}.$$

- (a) The matrix  $A$  is the augmented matrix of the system of linear equations above.
- (b) The matrix  $B$  is the inverse matrix of  $A$ .
- (c) The rank of  $A$  is 3.
- (d) The rank of  $B$  is 3.
- (e) There are infinitely many solutions to the system of linear equations above.

**Part II.** Write the answers of the following in the places provided in answer sheets. Show work! If you apply a proposition, state the number or the statement clearly.

3. Let  $p(x)$  be a polynomial satisfying  $p(0) = 0$ ,  $p(5) = 3$ ,  $p(10) = 0$  and  $p(15) = 1$ . Write two such polynomials  $p(x)$ , one with degree at most three and the other with degree exactly four.
4. Let  $f(x) = 3x^3 - 26x^2 + 75x - 54 = q(x)(x - 3) + r = c_3(x - 3)^3 + c_2(x - 3)^2 + c_1(x - 3) + c_0$ . Find a polynomial  $q(x)$ , constants  $r$  and  $c_3, c_2, c_1, c_0$ .
5. Let  $f(x)$  be a polynomial in Problem 4. (a) Show that there is a zero between 0 and 3, i.e., there is  $c$  with  $0 < c < 3$  such that  $f(c) = 0$ . (b) Determine whether  $c \geq 2$  or  $c < 2$ .
6. Let  $f(x)$  be a polynomial in Problem 4. Determine whether  $f(x)$  is increasing, decreasing at  $x = 3$  or  $f(3)$  is a local maximum or a local minimum. Why?
7. Find the limit  $\lim_{x \rightarrow 3} \frac{2x^3 - 13x^2 + 24x - 9}{x^3 - 11x^2 + 39x - 45}$ .
8. Find the limit  $\lim_{x \rightarrow 0} \frac{\log(x + 1) - x}{x^2}$ . Note that  $\log 1 = 0$ .
9. Find the derivative of  $\frac{1}{(2x - 3)^7}$ .
10. Find the derivative of  $x^2 e^{x^3}$ .

11. Find the indefinite integral  $\int \left( \frac{1}{2x^2} + 1 - 3\sqrt{x} \right) dx$ .

12. Find the indefinite integral  $\int \frac{1}{(2x-3)^8} dx$ .

13. Find the definite integral  $\int_0^1 e^{-3x} dx$ .

14. Find the derivative of  $F(x)$ , where  $F(x) = \int_0^x e^{-t^4} dt$ .

**Part III.** Write your answers on the answer sheets.

$$B = \begin{bmatrix} 2 & 0 & -6 & 10 & -3 & 3 & -10 \\ 0 & 1 & 2 & 0 & -1 & -2 & -8 \\ 0 & -2 & -4 & 1 & 2 & 4 & 17 \\ 1 & 0 & -3 & 5 & -2 & 2 & -9 \end{bmatrix} \rightarrow \rightarrow \rightarrow C = \begin{bmatrix} 1 & 0 & -3 & 5 & -2 & 2 & -9 \\ 0 & 1 & 2 & 0 & -1 & -2 & -8 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 8 \end{bmatrix}$$

15. The matrix  $C$  is obtained from the matrix  $B$  by performing elementary row operations three times. Write them in order using notation  $[i, j; c]$  (add  $c$  times row  $j$  to row  $i$ ),  $[i, j]$  (interchange row  $i$  and row  $j$ ),  $[i; c]$  (multiply every entry in row  $i$  by  $c$ ).

16. Find a  $4 \times 4$  matrix  $T$  satisfying  $TB = C$ .

17. Find the inverse of the matrix  $T$  in Problem 16 above.

18. Suppose the matrix  $B$  above is an augmented matrix of a system of linear equations with unknowns  $x_1, x_2, x_3, x_4, x_5, x_6$ . Find (a) the reduced row echelon form of  $B$ , and (b) the solutions of the system.

19. Let  $f(x) = (x+1)e^x$ . Find the equation of the tangent line to  $y = f(x)$  at  $x = 0$ .

20. Apply Proposition 7.4, and solve the differential equation below. (a) identify  $h(x)$ ,  $g(y)$ , and (b) find  $G(y)$ ,  $H(x)$  and write the equation  $G(y) = H(x) + C$ . Finally (c) solve it with an initial condition below for  $y = f(x)$ .

$$y' = \frac{dy}{dx} = 8x^3\sqrt{y}, \quad y(0) = f(0) = 1.$$

$$y' = \frac{h(x)}{g(y)}, H'(x) = h(x), G'(y) = g(y) \Rightarrow G(y) = H(x) + C.$$

# GEN024 FINAL 2017/8 Answer Sheets

ID#:

Name:

## Part I-1.

$p$	$q$	$r$	$p \Rightarrow (q \vee r)$	$(\neg (p \wedge \neg q)) \vee r$
$T$	$T$	$T$		
$T$	$T$	$F$		
$T$	$F$	$T$		
$T$	$F$	$F$		
$F$	$T$	$T$		
$F$	$T$	$F$		
$F$	$F$	$T$		
$F$	$F$	$F$		

Are these logically equivalent?

2.

(a)	(b)	(c)	(d)	(e)

**Message:** Did you enjoy mathematics, or did you suffer a lot? I appreciate your feedbacks on the following. 数学少しは楽しめましたか。苦しんだ人もいるかな。以下のことについて書いて下さい。( If you don't want your message to be posted, write "Do Not Post." 「HP 掲載不可」は明記の事。)

- (A) About this class, especially on improvements. この授業について。改善点など何でもどうぞ。
- (B) About the education at ICU, especially on improvements. Any comments concerning ICU are welcome. ICU の教育一般について。改善点など、ICU に関すること何でもどうぞ。

No.	PTS.
1.	
2.	
3.	
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20.	
<b>Total</b>	

## Part II.

3. degree at most three:

degree four:

4. Let  $f(x) = 3x^3 - 26x^2 + 75x - 54 = q(x)(x - 3) + r = c_3(x - 3)^3 + c_2(x - 3)^2 + c_1(x - 3) + c_0$ . Find a polynomial  $q(x)$ , constants  $r$  and  $c_3, c_2, c_1, c_0$ .

5. Let  $f(x)$  be a polynomial in Problem 4.

(a) Show that there is a zero between 0 and 3, i.e., there is  $c$  with  $0 < c < 3$  such that  $f(c) = 0$ .

(b) Determine whether  $c \geq 2$  or  $c < 2$ .

6. Let  $f(x)$  be a polynomial in Problem 4. Determine whether  $f(x)$  is increasing, decreasing at  $x = 3$  or  $f(3)$  is a local maximum or a local minimum. Why?

7. Find the limit  $\lim_{x \rightarrow 3} \frac{2x^3 - 13x^2 + 24x - 9}{x^3 - 11x^2 + 39x - 45}$ .
8. Find the limit  $\lim_{x \rightarrow 0} \frac{\log(x+1) - x}{x^2}$ . Note that  $\log 1 = 0$ .
9. Find the derivative of  $\frac{1}{(2x-3)^7}$ .
10. Find the derivative of  $x^2 e^{x^3}$ .
11. Find the indefinite integral  $\int \left( \frac{1}{2x^2} + 1 - 3\sqrt{x} \right) dx$ .

12. Find the indefinite integral  $\int \frac{1}{(2x-3)^8} dx$ .

13. Find the definite integral  $\int_0^1 e^{-3x} dx$ .

14. Find the derivative of  $F(x) = \int_0^x e^{-t^4} dt$ .

### Part III.

15. The matrix  $C$  is obtained from the matrix  $B$  by performing elementary row operations three times. Write them in order using notation  $[i, j; c]$  (add  $c$  times row  $j$  to row  $i$ ),  $[i, j]$  (interchange row  $i$  and row  $j$ ),  $[i; c]$  (multiply every entry in row  $i$  by  $c$ ).

16. Find a  $4 \times 4$  matrix  $T$  satisfying  $TB = C$ .

17. Find the inverse of the matrix  $T$  in Problem 16 above.

18. Suppose the matrix  $B$  above is an augmented matrix of a system of linear equations with unknowns  $x_1, x_2, x_3, x_4, x_5, x_6$ . Find (a) the reduced row echelon form of  $B$ , and (b) the solutions of the system.

(a)

(b)

19. Let  $f(x) = (x + 1)e^x$ . Find the equation of the tangent line to  $y = f(x)$  at  $x = 0$ .

20. Apply Proposition 7.4, and solve the differential equation below. (a) identify  $h(x)$ ,  $g(y)$ , and (b) find  $G(y)$ ,  $H(x)$  and write the equation  $G(y) = H(x) + C$ . Finally (c) solve it with an initial condition below for  $y = f(x)$ .

$$y' = \frac{dy}{dx} = 8x^3\sqrt{y}, \quad y(0) = f(0) = 1.$$

(a)

(b)

(c)

# Solutions to GEN024 FINAL 2017/8

## Part I.

1.

$p$	$q$	$r$	$p \Rightarrow (q \vee r)$	$(\neg(p \wedge \neg q)) \vee r$
$T$	$T$	$T$	$T$	$T$
$T$	$T$	$F$	$T$	$T$
$T$	$F$	$T$	$T$	$T$
$T$	$F$	$F$	$F$	$F$
$F$	$T$	$T$	$T$	$T$
$F$	$T$	$F$	$T$	$T$
$F$	$F$	$T$	$T$	$T$
$F$	$F$	$F$	$T$	$T$

Are these logically equivalent? : **YES**

2.

(a)	(b)	(c)	(d)	(e)
<b>F</b>	<b>T</b>	<b>T</b>	<b>T</b>	<b>F</b>

(a)  $A$  is the coefficient matrix. (b)  $AB = BA = I$ . (c), (d) By Invertible Matrix Theorem (IMT) and (b),  $\text{rank } A = \text{rank } B = 3$ . (e) By IMT, the system has a unique solution.

## Part II.

3. degree at most three:

$$p(x) = 3 \frac{x(x-10)(x-15)}{(5)(5-10)(5-15)} + \frac{x(x-5)(x-10)}{15(15-5)(15-10)} = \frac{x(x-10)(2x-25)}{375}.$$

degree four: Let  $p(x)$  be the one with degree at most three above. Then the following is a polynomial of degree four satisfying the conditions. See Proposition 4.2.

$$p(x) + x(x-5)(x-10)(x-15).$$

4. Let  $f(x) = 3x^3 - 26x^2 + 75x - 54 = q(x)(x-3) + r = c_3(x-3)^3 + c_2(x-3)^2 + c_1(x-3) + c_0$ . Find a polynomial  $q(x)$ , constants  $r$  and  $c_4, c_3, c_2, c_1, c_0$ .

**Soln.**  $f(x) = 3x^3 - 26x^2 + 75x - 54 = (3x^2 - 17x + 24)(x-3) + 18 = 3(x-3)^3 + (x-3)^2 + 18$ . Hence  $q(x) = 3x^3 - 17x + 24$ ,  $r = c_0 = 18$ ,  $c_1 = 0$ ,  $c_2 = 1$ ,  $c_3 = 3$ . Use synthetic division.

5. Let  $f(x)$  be a polynomial in Problem 4. (a) Show that there is a zero between 0 and 3, i.e., there is  $c$  with  $0 < c < 3$  such that  $f(c) = 0$ . (b) Determine whether  $c \geq 2$  or  $c < 2$ .

**Soln.** Since  $f(x)$  is a polynomial, it is continuous in a closed interval  $[0, 3]$ . Since  $f(0) = -49 < 0$  and  $f(3) = 18 > 0$ , by Intermediate Value Theorem, there is  $c \in [0, 3]$  such that  $f(c) = 0$ . Since  $f(2) = 16 > 0$ , there is a zero between 0 and 2. Hence  $c < 2$ .



6. Let  $f(x)$  be a polynomial in Problem 4. Determine whether  $f(x)$  is increasing, decreasing at  $x = 3$  or  $f(3)$  is a local maximum or a local minimum. Why?

**Soln.**  $f'(x) = 9(x-3)^2 + 2(x-3)$ ,  $f''(x) = 18(x-3) + 2$ . Hence,  $f'(3) = 0$ ,  $f''(3) = 2$ . Thus  $f(3)$  is a local minimum by Second Derivative Test.

7. Find the limit  $\lim_{x \rightarrow 3} \frac{2x^3 - 13x^2 + 24x - 9}{x^3 - 11x^2 + 39x - 45}$ .

**Soln.** Since  $2x^3 - 13x^2 + 24x - 9 = (x-3)^2(2x-1)$  and  $x^3 - 11x^2 + 39x - 45 = (x-3)^2(x-5)$  by synthetic division,

$$= \lim_{x \rightarrow 3} \frac{(x-3)^2(2x-1)}{(x-3)^2(x-5)} = \lim_{x \rightarrow 3} \frac{2x-1}{x-5} = \frac{5}{-2} = -\frac{5}{2}.$$

8. Find the limit  $\lim_{x \rightarrow 0} \frac{\log(x+1) - x}{x^2}$ . Note that  $\log 1 = 0$ .

**Soln.** Use l'Hôpital's rule.

$$\lim_{x \rightarrow 0} \frac{\log(x+1) - x}{x^2} = \lim_{x \rightarrow 0} \frac{\frac{1}{x+1} - 1}{2x} = \lim_{x \rightarrow 0} \frac{-(x+1)^{-2}}{2} = -\frac{1}{2}.$$

9. Find the derivative of  $\frac{1}{(2x-3)^7}$ .

**Soln.**

$$\left( \frac{1}{(2x-3)^7} \right)' = ((2x-3)^{-7})' = -7(2x-3)^{-8}(2x-3)' = -14(2x-3)^{-8} = -\frac{14}{(2x-3)^8}.$$

10. Find the derivative of  $x^2 e^{x^3}$ .

**Soln.** Since  $(e^{x^3})' = e^{x^3}(x^3)' = 3x^2 e^{x^3}$  by the Chain Rule, using the Product Rule we have

$$(x^2 e^{x^3})' = (x^2)' e^{x^3} + x^2 (e^{x^3})' = 2x e^{x^3} + 3x^4 e^{x^3} = x(2 + 3x^3) e^{x^3}.$$

11. Find the indefinite integral  $\int \left( \frac{1}{2x^2} + 1 - 3\sqrt{x} \right) dx$ .

**Soln.** Since  $\sqrt{x} = x^{\frac{1}{2}}$ ,

$$\int \left( \frac{1}{2x^2} + 1 - 3\sqrt{x} \right) dx = \int \left( \frac{1}{2} x^{-2} + 1 - 3x^{\frac{1}{2}} \right) dx = -\frac{1}{2x} + x - 2x^{\frac{3}{2}} + C = -\frac{1}{2x} + x - 2\sqrt{x^3} + C.$$

12. Find the indefinite integral  $\int \frac{1}{(2x-3)^8} dx$ .

**Soln.** By Problem 9,  $-\frac{1}{14(2x-3)^7}$  is an antiderivative of  $\frac{1}{(2x-3)^8}$ . Hence

$$\int \frac{1}{(2x-3)^8} dx = -\frac{1}{14(2x-3)^7} + C.$$

13. Find the definite integral  $\int_0^1 e^{-3x} dx$ .

**Soln.**

$$\int_0^1 e^{-3x} dx = \left[ -\frac{1}{3} e^{-3x} \right]_0^1 = \frac{1}{3} (1 - e^{-3}) = \frac{1}{3} \left( 1 - \frac{1}{e^3} \right).$$

14. Find the derivative of  $F(x)$ , where  $F(x) = \int_0^x e^{-t^4} dt$ .

**Soln.** Since  $F(x)$  is an antiderivative of  $e^{-x^4}$ ,  $F'(x) = e^{-x^4}$  by the Fundamental Theorem of Calculus.

### Part III.

15. The matrix  $C$  is obtained from the matrix  $B$  by performing elementary row operations three times. Write these operations in order using notation  $[i, j; c]$ ,  $[i, j]$ , and  $[i; c]$ .

**Soln.**

$$B = \begin{bmatrix} 2 & 0 & -6 & 10 & -3 & 3 & -10 \\ 0 & 1 & 2 & 0 & -1 & -2 & -8 \\ 0 & -2 & -4 & 1 & 2 & 4 & 17 \\ 1 & 0 & -3 & 5 & -2 & 2 & -9 \end{bmatrix} \xrightarrow{[1,4]} \begin{bmatrix} 1 & 0 & -3 & 5 & -2 & 2 & -9 \\ 0 & 1 & 2 & 0 & -1 & -2 & -8 \\ 0 & -2 & -4 & 1 & 2 & 4 & 17 \\ 2 & 0 & -6 & 10 & -3 & 3 & -10 \end{bmatrix} \xrightarrow{[4,1;-2]} \begin{bmatrix} 1 & 0 & -3 & 5 & -2 & 2 & -9 \\ 0 & 1 & 2 & 0 & -1 & -2 & -8 \\ 0 & -2 & -4 & 1 & 2 & 4 & 17 \\ 0 & 0 & 0 & 0 & 1 & -1 & 8 \end{bmatrix} \xrightarrow{[3,2;2]} C = \begin{bmatrix} 1 & 0 & -3 & 5 & -2 & 2 & -9 \\ 0 & 1 & 2 & 0 & -1 & -2 & -8 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 8 \end{bmatrix}$$

Hence the operations are  $[1, 4]$ ,  $[4, 1; -2]$ ,  $[3, 2; 2]$  in this order. There are other solutions.

16. Find a  $4 \times 4$  matrix  $T$  satisfying  $TB = C$ .

**Soln.** We obtain the matrix  $T$  by applying  $[1, 4]$ ,  $[4, 1; -2]$ ,  $[3, 2; 2]$  to  $I$  in this order.

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{[1,4]} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{[4,1;-2]} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -2 \end{bmatrix} \xrightarrow{[3,2;2]} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 1 & 0 & 0 & -2 \end{bmatrix} = T,$$

or

$$T = E(3, 2; 2)E(4, 1; -2)E(1, 4).$$

17. Find the inverse of the matrix  $T$  in 16 above.

**Soln.**

$$[T, I] = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -2 & 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{[3,2;-2]} \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -2 & 1 & 0 \\ 1 & 0 & 0 & -2 & 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{[4,1;-2]} \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -2 & 1 & 0 \\ 1 & 0 & 0 & 0 & 2 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{[1,4]} \begin{bmatrix} 1 & 0 & 0 & 0 & 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}, T^{-1} = \begin{bmatrix} 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

18. Suppose the matrix  $B$  above is an augmented matrix of a system of linear equations with unknowns  $x_1, x_2, x_3, x_4, x_5, x_6$ . Find (a) the reduced row echelon form of  $B$ , and (b) the solutions of the system.

**Soln.** (a) Using  $B \rightarrow \rightarrow \rightarrow C$ , we start from  $C$ .

$$C = \begin{bmatrix} 1 & 0 & -3 & 5 & -2 & 2 & -9 \\ 0 & 1 & 2 & 0 & -1 & -2 & -8 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 8 \end{bmatrix} \xrightarrow{[1,3;-5]} \begin{bmatrix} 1 & 0 & -3 & 0 & -2 & 2 & -14 \\ 0 & 1 & 2 & 0 & -1 & -2 & -8 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 8 \end{bmatrix} \xrightarrow{[1,4;2]}$$

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 2 \\ 0 & 1 & 2 & 0 & -1 & -2 & -8 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 8 \end{bmatrix} \xrightarrow{[2,4;1]} \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 2 \\ 0 & 1 & 2 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 8 \end{bmatrix}, \quad \begin{cases} x_1 = 3s + 2, \\ x_2 = -2s + 3t, \\ x_3 = s: \text{ free}, \\ x_4 = 1, \\ x_5 = t + 8, \\ x_6 = t: \text{ free}. \end{cases}$$

19. Let  $f(x) = (x+1)e^x$ . Find the equation of the tangent line to  $y = f(x)$  at  $x = 0$ .

**Soln.** Since  $f'(x) = e^x + (x+1)e^x = (x+2)e^x$ ,  $f(0) = 1$ ,

$$y = f(0) + f'(0)(x-0) = 2x + 1.$$

20. Apply Proposition 7.4 and solve the differential equation below. (a) identify  $h(x)$ ,  $g(y)$ , and (b) find  $G(y)$ ,  $H(x)$  and write the equation  $G(y) = H(x) + C$ . Finally (c) solve it with an initial condition below for  $y = f(x)$ .

$$y' = \frac{dy}{dx} = 8x^3\sqrt{y}, \quad y(0) = f(0) = 1.$$

**Soln.**

$$\frac{dy}{dx} = y' = 8x^3\sqrt{y} = \frac{8x^3}{y^{-1/2}}.$$

So one of the choices is  $h(x) = 8x^3$  and  $g(y) = y^{-1/2}$ . Hence  $H(x) = 2x^4$  and  $G(y) = 2y^{1/2}$ .

$$2y^{1/2} = 2x^4 + C, \quad C = 2y(0)^{1/2} = 2.$$

Therefore,  $y = (x^4 + 1)^2$ .

$$y' = 2(x^4 + 1)(4x^3) = 8x^3(x^4 + 1) = 8x^3\sqrt{y}, \quad \text{and } y(0) = 1.$$