Lecture Note on Terwilliger Algebra

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2022-11-11

Contents

About this lecturenote

Setting

This note is created by `bookdown` package on RStudio.

1. Log-in to my GitHub Account
2. Go to RStudio/bookdown-demo repository: https://github.com/rstudio/bookdown-demo
3. Use This Template
4. Input Repository Name
5. Select Public - default
6. Create repository from template
7. From Code download ZIP
8. Move the extracted folder into a favorite directory
9. Open RStudio Project in the folder
10. Use Terminal in the buttom left pane

    - confirm that the current directory is the home directry of the project by pwd

11. (failed to proceed by ssh)
12. Use Console

    1. library(usethis)
    2. use_git()
    3. use_github() — Error
    4. gh_token_help()
    5. create_github_token(): create a token in the github page. Copy the token
    6. gitcreds::gitcreds_set(): paste the token, the token is to be expired in 30 days

13. Use Terminal

    1. git remote add origin https://github.com/icu-hsuzuki/t-alagebra.git
    2. git push -u origin main

    3. type in the password of the computer

14. Use GIT in R Studio

Access from Another Host

1. library(usethis)
2. use_git()
3. create_github_token()
4. gitcreds::gitcreds_set(): Replace these credentials

Chapter 1

Lecture 1

[Wednesday, January 20, 1993]style="float:right"

A graph (undirected, without loops or multiple edges) is a pair $\Gamma = (X, E)$, where

$$X = \text{finite set (of vertices)} \tag{1.1}$$
$$E = \text{set of (distinct) 2-element subsets of } X \text{ (= edges of) } \Gamma. \tag{1.2}$$

vertices x and $y \in X$ are adjacent if and only if $xy \in E$.

Example 1.1. Let Γ be a graph. $X = \{a, b, c, d\}$, $E = \{ab, ac, bc, bd\}$.

Set $n = |X|$, the order of Γ.

Pick a field K (= $\mathbb{R}$ or $\mathbb{C}$). Then $\mathrm{Mat}_X(K)$ denotes the K algebra of all $n \times n$ matrices with entries in K. (rows and columns are indexed by X)

Adjacency matrix $A \in \mathrm{Mat}_X(K)$ is defined by

$$A_{xy} = \begin{cases} 1 & \text{if } \ xy \in E \\ 0 & \text{else } . \end{cases} \tag{1.3}$$

Example 1.2. Let a, b, c, d be labels of rows and columns. Then

$$A = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

The subalgebra M of $\mathrm{Mat}_X(K)$ generated by A is called the *Bose-Mesner algebra* of Γ.

Set $V = K^n$, the set of n-dimensional column vectors, the coorinates are indexed by X.

Let $\langle\ ,\ \rangle$ denote the Hermitean inner product:

$$\langle u, v\rangle = u^\top \cdot v \quad (u, v \in V)$$

V with $\langle\ ,\ \rangle$ is the *standard module* of Γ.

M acts on V: For every $x \in X$, write

$$\hat{x} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

where 1 is at the x position.

Then

$$A\hat{x} = \sum_{y \in X, xy \in E} \hat{y}.$$

Since A is a real symmetrix matrix,

$$V = V_0 + V_1 + \cdots + V_r \qquad \text{some } r \in \mathbb{Z}^{\geq 0},$$

the orthogonal direct sum of maximal A-eigenspaces.

Let $E_i\mathrm{Mat}_X(K)$ denote the orthogonal projection,

$$E_i : V \longrightarrow V_i.$$

Then $E_0, \dots, E_r$ are the primitive idempotents of M.

Chapter 2

Lecture 2

Here is a review of existing methods.

Chapter 3

Lecture 3

We describe our methods in this chapter.

Math can be added in body using usual syntax like this

3.1 math example

p is unknown but expected to be around 1/3. Standard error will be approximated

$$SE = \sqrt(\frac{p(1-p)}{n}) \approx \sqrt{\frac{1/3(1-1/3)}{300}} = 0.027$$

You can also use math in footnotes like this[1].

We will approximate standard error to 0.027[2]

[1] where we mention $p = \frac{a}{b}$

[2] p is unknown but expected to be around 1/3. Standard error will be approximated

$$SE = \sqrt(\frac{p(1-p)}{n}) \approx \sqrt{\frac{1/3(1-1/3)}{300}} = 0.027$$

Chapter 4

Applications

Some *significant* applications are demonstrated in this chapter.

4.1 Example one

4.2 Example two

Chapter 5

Final Words

We have finished a nice book.