MPC formulation description template – An example

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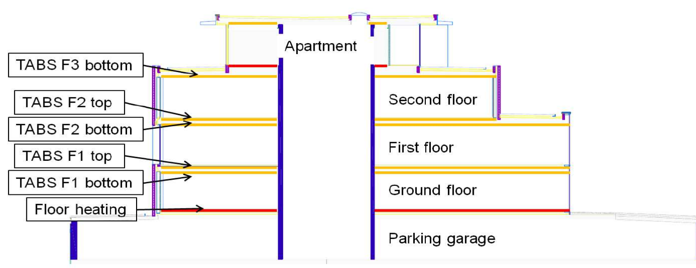
ABSTRACT

This document presents an example of the MPC formulation, using IBPSA Project 1 template. Twelve-zone office building is used for demonstation of the methodology. Equivalence of MPC formulations using physical notation and control engineering notation is demonstrated using translation template.

1. OFFICE BUILDING

This section provides a high-level overview of the building and its systems. This section is based on prior modeling work done by Picard [1]. The case study office building shown in Fig. 1a, called *Hollandsch Huys*, is located in Hasselt, Belgium. Its construction was finished in 2007 and it was designed to be a low-energy, innovative building. Fig. 1b shows the building's layout, which consists of five floors: underground parking, three floors, and a roof apartment. The following sections describe the building envelope, the HVAC system, the control oriented model, the occupancy, internal gains, comfort bounds assumed and RBC.

(a) Building from outside.



(b) Building's layout.

Figure 1: Hollandsch Huys office building [2].

1.1 Building Envelope

The general parameters of the building envelope are summarized in Tab. 1. The U-value is an average value for the whole building and ACH stands for *air changes per hour*. The building is divided into 12 thermal zones, 4 per floor. All transparent parts of the façade are equipped with triple glazing. The window surface lies $40\,\mathrm{cm}$ deeper than the façade. Each of them is equipped with an external slat shading device whose angle is adjusted automatically to the solar radiation intensity: the shading device is controlled by a hysteresis controller which closes the shading when the horizontal solar radiation exceeds $150\,\mathrm{W\,m^{-2}}$ and re-opens it when the solar radiation is lower than $80\,\mathrm{W\,m^{-2}}$.

1.2 Heating Ventilation and Air Conditioning System

Fig. 2 shows the hydraulic scheme of the building. Hollandsch Huys represents a so-called *hybrid GEOTABS* building with the emission system composed of a central air handling unit (AHU), floor heating (FH) on the ground floor, and

Table 1: General building parameters [1].

Floor area	[m^2]	3760	U-value	[$W\,m^{-2}\,K^{-1}$]	0.216
Conditioned volume	[m^3]	10526	Loss area	[m^2]	4438
Window-to-wall ratio	[-]	34%	ACH (n50)	[h^{-1}]	0.9

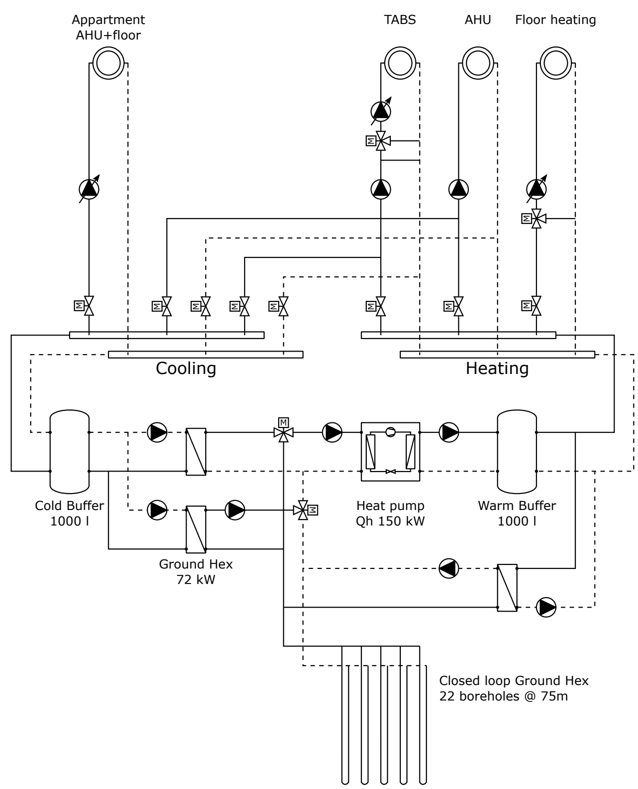


Figure 2: Hollandsch Huys hydraulic scheme. The components are: borefield, heat exchangers, buffers, heat pump, TABS, floor heating, and 12 circulation pumps.

thermally activated building structures (TABS) with a floor and a ceiling circuits on individual floors, see Fig. 1b. The nominal mass flow rates are listed in Tab.2. The main production system is a $150\,\mathrm{kW}$ Daikin EWWP145 KAW1M heat pump (HP) coupled to ground heat exchangers (22 with $75\,\mathrm{m}$ depth), two buffer tanks of $1\,\mathrm{m}^3$ each, three heat exchangers, and circulation pumps. An additional gas boiler (GB) is installed in the building to back up the heating of the ventilation air but it is not included in the model as it is not needed when proper control is used.

Table 2: Nominal mass flow rates for TABS and floor heating.

Emission	Nominal mass flow rate			
TABS-ceiling	7	$[\mathrm{l\,h^{-1}\,m^{-2}}]$	0.0019	$[\mathrm{kg\,s^{-1}\,m^{-2}}]$
TABS-floor	6	$[\mathrm{l\,h^{-1}\,m^{-2}}]$	0.0017	$[\mathrm{kg\,s^{-1}\,m^{-2}}]$
Floor heating	4	$[\mathrm{l\,h^{-1}\,m^{-2}}]$	0.0011	$[\mathrm{kg\,s^{-1}\,m^{-2}}]$
Entire building	47600	$[\mathrm{l\,h^{-1}}]$	13.22	$[\mathrm{kg\,s^{-1}}]$

2. VARIABLES AND PARAMETERS

The abstract domain variables represent states x, controlled outputs y, other measured variables m, actuator variables a, inputs to the building envelope u, disturbances d, and slack variables s, respectively. The MPC formulation parameters for modifiers and bounds are given in Tab. 4 and Tab. 5, respectively. Please note that the following tables list only a problem-relevant subset of variables from complete IBPSA Project 1 template.

Table 3: Notation of MPC variables and translation between physical and abstract domain for this case.

Physical domain			**Abstract domain**						
Variables	**Symbol**	**Description**	x	y	m	a	u	d	s
Temperatures [°C]	T	Envelope temperatures	•	-	-	-	-	-	-
	T_{z}	Zone operative temperatures	-	•	-	-	-	-	-
	$T_{\mathrm{ret}}^{\mathrm{TABS}}$	Water return temperatures TABS	-	-	•	-	-	-	-
	$T_{\mathrm{sup}}^{\mathrm{HP}}$	Water supply temperature heat pump	-	-	•	-	-	-	-
	$T_{\mathrm{sp}}^{\mathrm{HP}}$	Set-point supply temperature heat pump	-	-	-	•	-	-	-
	$T_{\mathrm{sp}}^{\mathrm{TABS}}$	Set-point supply temperature TABS	-	-	-	•	-	-	-
	$T_{\mathrm{sup}}^{\mathrm{AHU}}$	Air supply temperatures AHU	-	-	-	-	-	•	-
	T_{e}	Ambient temperature	-	-	-	-	-	•	-
Thermal power [W]	$\dot{Q}_{\mathrm{HVAC}}$	Thermal power of TABS and FH	-	-	-	-	•	-	-
	$\dot{Q}_{\mathrm{rad}}$	Solar radiation	-	-	-	-	-	•	-
	$\dot{Q}_{\mathrm{occ}}$	Occupancy internal gains	-	-	-	-	-	•	-
Mass flows $[\mathrm{kg\,s^{-1}}]$	$\dot{m}_{\mathrm{HVAC}}$	Water mass flow rates of TABS and FH	-	-	-	•	-	-	-
Component signals	x_{TABS}	ON/OFF signal of TABS and FH pump $[\{0,1\}]$	-	-	-	•	-	-	-
	x_{heat}	ON/OFF heating circuit valve $[\{0,1\}]$	-	-	-	•	-	-	-
	x_{cool}	ON/OFF cooling circuit valve $[\{0,1\}]$	-	-	-	•	-	-	-
Comfort violations [°C]	s^{T_z}	Violations of thermal comfort zones	-	-	-	-	-	-	•

Table 4: MPC formulation parameters - modifiers.

Heat flow parameters	**Symbol**	**Description**	**Associated variables**
Specific heat capacity	c_p	Specific heat capacity of water [$\mathrm{J\,kg^{-1}\,K^{-1}}$]	$T_{sup}^{\text{TABS}}, T_{ret}^{\text{TABS}}, \dot{Q}_{\text{HVAC}}^{i}$
Auxiliary parameters	**Symbol**	**Description**	**Associated variables**
Weighting factor	$Q_{\star}$	Weighting for the particular term in the objective function	s, u
Sampling time	T_s	Time-step used in the optimization problem	all
Dimensionality quantifier	$n_{\star}$	Cardinality of the vector elements	all

Table 5: MPC formulation parameters - bounds.

Bounds	**Symbol**	**Description**	**Associated variables**	**Abstract domain**
Comfort bounds [°C]	$\underline{T}_{\text{z}}, \overline{T}_{\text{z}}$	Lower/upper bounds of thermal comfort zones	T_{z}	$r = \{\underline{y}, \overline{y}\}$
Thermal power limits [W]	$\underline{\dot{Q}}_{\text{HVAC}}, \overline{\dot{Q}}_{\text{HVAC}}$	Min/max thermal powers	Q_{HVAC}	$\underline{u}, \overline{u}$
Supply temp. bounds [°C]	$\underline{T}_{\text{sup}}^{\text{TABS}}, \overline{T}_{\text{sup}}^{\text{TABS}}$	Min/max temperatures	$T_{\text{sup}}^{\text{TABS}}$	$a^{T_{\text{sup}}}$

3. MPC FORMULATION: PHYSICAL NOTATION

The MPC minimizing the energy consumption and thermal discomfort for Hollandsch Huys building is given as the following quadratic optimization problem using physical notation:

$$\min_{\substack{s_0^{T_{\text{z}}},\dots,s_{N-1}^{T_{\text{z}}},\\ \dot{m}_{\text{HVAC},0},\dots,\dot{m}_{\text{HVAC},N_{\text{c}}-1},\\ T_{\text{sup},0}^{\text{TABS}}\dots,T_{\text{sup},N_{\text{c}}-1}^{\text{TABS}}}} \sum_{k=0}^{N-1} ||s_k^{T_{\text{z}}}||_{Q_{\text{s}}}^2 + \sum_{k=0}^{N_{\text{c}}-1} COP_k p_k ||\dot{Q}_{\text{HVAC},k}||_{Q_{\text{u}}}^2 \tag{1a}$$

$$\text{s.t. } T_{k+1} = AT_k + B\dot{Q}_{\text{HVAC},k} + E[\dot{Q}_{\text{rad},k}, \dot{Q}_{\text{occ},k}, T_{\text{e},k}]^T, \quad k \in \mathbb{N}_0^{N-1} \tag{1b}$$

$$T_{\text{z},k} = CT_k, \quad k \in \mathbb{N}_0^{N-1} \tag{1c}$$

$$\underline{T}_{\text{z},k} - s_k^{T_{\text{z}}} \leq T_{\text{z},k} \leq \overline{T}_{\text{z},k} + s_k^{T_{\text{z}}}, \quad k \in \mathbb{N}_0^{N-1} \tag{1d}$$

$$\mathbf{0} \leq s_k^{T_{\text{z}}}, \quad k \in \mathbb{N}_0^{N-1} \tag{1e}$$

$$\underline{\dot{Q}}_{\text{HVAC},k} \leq \dot{Q}_{\text{HVAC},k} \leq \overline{\dot{Q}}_{\text{HVAC},k}, \quad k \in \mathbb{N}_0^{N-1} \tag{1f}$$

$$\dot{Q}_{\text{HVAC},k} = \dot{Q}_{\text{HVAC},N_{\text{c}}}, \quad k \in \mathbb{N}_{N_{\text{c}}}^{N-1} \tag{1g}$$

$$\dot{Q}_{\text{HVAC},k} = \dot{m}_{\text{HVAC}}^{\text{nom}}\, c_p\, (T_{\text{sup},k}^{\text{TABS}} - T_{\text{ret},k}^{\text{TABS}}), \quad k \in \mathbb{N}_0^{N_{\text{c}}-1} \tag{1h}$$

$$\underline{T}_{\text{sup},k}^{\text{TABS}} \leq T_{\text{sup},k}^{\text{TABS}} \leq \overline{T}_{\text{sup},k}^{\text{TABS}}, \quad k \in \mathbb{N}_0^{N_{\text{c}}-1} \tag{1i}$$

$$\dot{Q}_{\text{rad},k} = \dot{Q}_{\text{rad}}(t + kT_s), \quad k \in \mathbb{N}_0^{N-1} \tag{1j}$$

$$\dot{Q}_{\text{occ},k} = \dot{Q}_{\text{occ}}(t + kT_s), \quad k \in \mathbb{N}_0^{N-1} \tag{1k}$$

$$T_{\text{e},k} = T_{\text{e}}(t + kT_s), \quad k \in \mathbb{N}_0^{N-1} \tag{1l}$$

$$\underline{T}_{\text{z},k} = \underline{T}_{\text{z}}(t + kT_s), \quad k \in \mathbb{N}_0^{N-1} \tag{1m}$$

$$\overline{T}_{\text{z},k} = \overline{T}_{\text{z}}(t + kT_s), \quad k \in \mathbb{N}_0^{N-1} \tag{1n}$$

$$T_0 = \hat{T}(t) \tag{1o}$$

4. MPC FORMULATION: CONTROL ENGINEERING NOTATION

The same MPC formulation using control engineering notation:

$$\min_{\substack{s_0^{T_z},\ldots,s_{N-1}^{T_z},\\ a_0^{T_{\text{sup}}},\ldots,a_{N_c-1}^{T_{\text{sup}}},\\ a_0^{\dot{m}}\ldots,a_{N_c-1}^{\dot{m}}}} \sum_{k=0}^{N-1} ||s_k^{T_y}||_{Q_s}^2 + \sum_{k=0}^{N_c-1} COP_k p_k ||u_k||_{Q_u}^2 \tag{2a}$$

$$\text{s.t. } x_{k+1} = Ax_k + Bu_k + Ed_k, \qquad k \in \mathbb{N}_0^{N-1} \tag{2b}$$

$$y_k = Cx_k, \qquad k \in \mathbb{N}_0^{N-1} \tag{2c}$$

$$\underline{y}_k - s_k^y \leq y_k \leq \overline{y}_k + s_k^{T_y}, \qquad k \in \mathbb{N}_0^{N-1} \tag{2d}$$

$$\mathbf{0} \leq s_k^y, \qquad k \in \mathbb{N}_0^{N-1} \tag{2e}$$

$$\underline{u_k} \leq u_k \leq \overline{u_k}, \qquad k \in \mathbb{N}_0^{N-1} \tag{2f}$$

$$u_k = u_{N_c}, \qquad k \in \mathbb{N}_{N_c}^{N-1} \tag{2g}$$

$$u_k = a_k^{\dot{m}}\, c_p\, (a_k^{T_{\text{sup}}} - m_k^{T_{\text{ret}}}), \qquad k \in \mathbb{N}_0^{N_c-1} \tag{2h}$$

$$\underline{a}_k^{T_{\text{sup}}} \leq a_k^{T_{\text{sup}}} \leq \overline{a}_k^{T_{\text{sup}}}, \qquad k \in \mathbb{N}_0^{N_c-1} \tag{2i}$$

$$d_k = d_(t + kT_s), \qquad k \in \mathbb{N}_0^{N-1} \tag{2j}$$

$$\underline{y}_k = \underline{y}(t + kT_s), \qquad k \in \mathbb{N}_0^{N-1} \tag{2k}$$

$$\overline{y}_k = \overline{y}(t + kT_s), \qquad k \in \mathbb{N}_0^{N-1} \tag{2l}$$

$$x_0 = \hat{x}(t) \tag{2m}$$

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REFERENCES

D. Picard. Modeling, Optimal Control and HVAC Design of Large Buildings using Ground Source Heat Pump Systems, PhD Thesis, KU Leuven, Belgium. 2017.

E. Zácecková, Z. Vána, J. Hoogmartens, C. Verhelst, M. Sourbron, L. Ferkl, and L. Helsen. Identification for model based predictive control applied to an office building with a thermally activated building systems. pages 2430–2439. Teloy, Tanja, VDI-Society für Civil Engineering and Building Services; Düsseldorf, 2013.