MPC formulation description template

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ABSTRACT

This document describes the MPC formulation template as used within the frame of the IBPSA Project 1. The proposed template aims to uniformize all possible formulations for the different types of models and key performance indicators (KPIs) in consideration. It has to be general. To this end, we choose the control engineering notation as our starting point, since it considers the formulation in a general way from an abstract point of view. From here, we move to a formulation with physical meaning based on the specific characteristics of buildings.

1. CONTROL ENGINEERING NOTATION

We build up the control engineering notation based on a general building model structure presented in Fig. 1. The set of abstract domain variables represents the states of a system x, the heat flows to a system u, the disturbances d, the controlled variables y, the actuator variables y, the additional measured variables m, and the reference signals r.

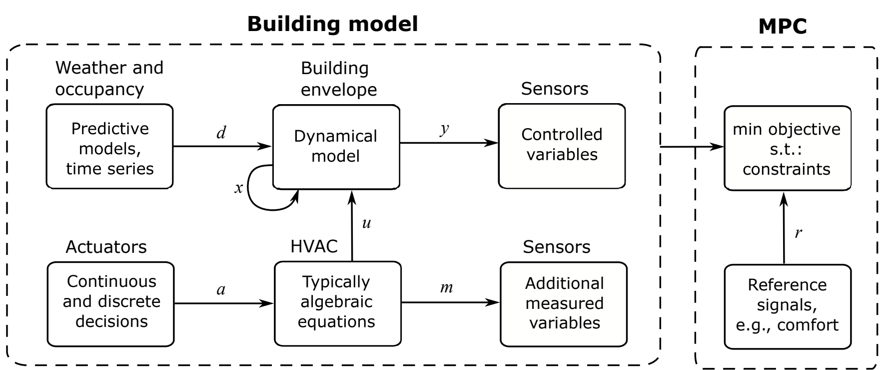

Figure 1: General structure of the building model with MPC.

The general MPC formulation can be cast as an optimal control problem (OCP) in discrete time:

$$\min_{u_0,\ldots,u_{N-1}} \ell_N(x_N) + \sum_{k=0}^{N-1} \ell_k(x_k, y_k, u_k, r_k, s_k) \tag{1a}$$

$$\text{s.t. } x_{k+1} = f(x_k, u_k, d_k), \quad k \in \mathbb{N}_0^{N-1} \tag{1b}$$

$$y_k = g(x_k, u_k, d_k), \quad k \in \mathbb{N}_0^{N-1} \tag{1c}$$

$$u_k = f_{\text{HVAC}}(a_k, m_k), \quad k \in \mathbb{N}_0^{N-1} \tag{1d}$$

$$s_k = h(x_k, y_k, u_k, r_k), \quad k \in \mathbb{N}_0^{N-1} \tag{1e}$$

$$x_k \in \mathcal{X}, \quad k \in \mathbb{N}_0^{N-1} \tag{1f}$$

$$u_k \in \mathcal{U}, \quad k \in \mathbb{N}_0^{N-1} \tag{1g}$$

$$a_k \in \mathcal{A}, \quad k \in \mathbb{N}_0^{N-1} \tag{1h}$$

$$s_k \in \mathcal{S}, \quad k \in \mathbb{N}_0^{N-1} \tag{1i}$$

$$d_k = d(t + kT_s), \quad k \in \mathbb{N}_0^{N-1} \tag{1j}$$

$$r_k = r(t + kT_s), \quad k \in \mathbb{N}_0^{N-1} \tag{1k}$$

$$x_0 = x(t), \tag{1l}$$

where $x_k \in \mathbb{R}^{n_x}$, $y_k \in \mathbb{R}^{n_y}$, $u_k \in \mathbb{R}^{n_u}$, $a_k \in \mathbb{R}^{n_a}$, $m_k \in \mathbb{R}^{n_m}$, $d_k \in \mathbb{R}^{n_d}$, $r_k \in \mathbb{R}^{n_r}$ and $s_k \in \mathbb{R}^{n_s}$ denote the values of states, outputs, envelope inputs, HVAC actuators, additional measured variables, disturbances, reference signals and slack variables respectively, at the k-th step of the prediction horizon N with a sampling time T_s. Where $n_\star$ denotes the dimensionality of associated variable $\star$.

The objective function is given by (1a) where $\ell_N(x_N)$ represents the terminal penalty, while $\ell_k(x_k, y_k, u_k, r_k, s_k)$ is called a stage cost and its purpose is to assign a cost to a particular choice of x_k, y_k, u_k, r_k and s_k. For most of the building control applications the terminal penalty is omitted. The predictions of the state values are obtained from the state update equation (1b), while values of the predicted outputs are given by the output equation (1c). The building envelope inputs are given by the HVAC model (1d). Slack variables are defined as additional constraints (1e), they ususally represent algebraic relations of state, output, or input variables. States, envelope inputs, actuators, and slack variables are subject to constraints (1f), (1g), (1h), and (1i), where sets $\mathcal{X}$, $\mathcal{U}$, $\mathcal{A}$, and $\mathcal{S}$ represent feasible subset of $\mathbb{R}^n$. The initial conditions for the state variables are given by (1l), which are either measured or estimated. Eq. (1j) and (1k) represent initialization of the disturbances and references for the whole prediction horizon, obtained as a forecast. For the sake of generality we denote by ξ the vector that encapsulates all initial conditions of (1), i.e. the current states $x(t)$, current and future disturbance variables $d(t), \ldots, d(t + NT_s)$, and possibly parameters such as comfort boundaries or reference signals which depend on the specific formulation. The parameter T_s here represents the sampling time. Compression of all initial conditions into single vector ξ is convenient for compact representation of MPC feedback law in the form $U = f_{MPC}(\xi)$, where $U = \begin{bmatrix} u_0 & u_1 & \cdots & u_{N-1} \end{bmatrix}$ is the vector of computed optimal control actions obtained as a solution of the problem (1).

2. MPC FORMULATION IN BUILDINGS: GRANTING A PHYSICAL MEANING

In Section 1 the abstract MPC formulation has been defined, now physical meaning and ranges of variables and parameters have to be specified. For the sake of clarity, we make a distinction between the meaning of variable and parameter in this document. Variable represents a value which is computed by or is dynamically influencing the model equations (1b) and (1c). For this reason, we will denote the measured disturbances such as weather forecast a variable. Parameter on the other hand is a value which is not involved in model equations in a dynamical vay (1b) and (1c). Parameters can be either set up by the engineer (e.g., prediction horizon N, objective weights $Q_\star$), or can be known a priory during the design process (e.g., specific heat capacity c_p), or even continuously measured as time-varying signals (e.g., coefficient of performance COP, or price factor p). The slack variables s, are labeled as variables because they are computed within MPC and usually depend on model dynamics.

Table 1 summarizes the most common variables used in building control and maps them with the abstract notation from the control engineering notation presented in previous section. Because these variables are usually vectors, we indicate the i-th element of the vector variable via supercsript i, for instance the operative temperature in the i-th zone is denoted as T_z^i. Please note that presented mapping of variables from physical domain to abstract domain generalizes most common cases found in the literature. However, less common but technically feasible mapping can be applied in some specific cases, e.g. treating envelope temperatures as disturbance signals.

An overview of the most common modifying and bounding parameters in building control together with their associated variables is presented in Table 2 and Table 3, respectively. These parameters can be treated either as constant numerical or time-varying values. For example, in the latter case the parameters can represent reference r within the MPC formulation.

Table 4 recaps the most common objectives used in the MPC formulation. These objectives can be combined in a multi-objective function. Since the main objective of building control is the maximization of occupants comfort while using as little resources as possible, usually two conflicting terms are found: an energy minimization term and a comfort maximization term. Eq. (2) is an example of such multi-objective function, where $||Q_\mathrm{s}s_k||_2^2$ represents an arbitrary discomfort term in the form of the weighted squared 2-norm of the slack variables, and $Q_\mathrm{u}u_k$ stands for the weighted linear energy term. The matrices Q_s and Q_u here stand for the weighting factors.

$$\min_{u_0,\dots,u_{N-1}} \sum_{k=0}^{N-1} (||Q_\mathrm{s}s_k||_2^2 + Q_\mathrm{u}u_k) \tag{2}$$

The energy used by the building stands for the sum of the energy used by each HVAC component Q^i_{HVAC}. Generally, the power P^i_{HVAC} of i-th HVAC component can be calculated as the time integral of the ratio between the thermal power that it delivers Q^i_{HVAC} and its efficiency COP^i, EER^i or η^i. These efficiencies are parameters that can be considered either constant or variable as a function of the inputs, states and disturbances. The energy use minimization function can be further transformed to monetary cost using a price factor p^i, which can be added to the formulation as a time-varying (in electric-based components), constant (in fuel-based components) or even negative (renewable-based components that inject energy into the grid) parameter. The emission factor e^i converts the energy used into the associated CO_2 emissions for the i-th component which can be variable for electric-based components, constant for fuel-based components and zero for renewable-based components. The renewable energy sources (RES) share RES^i represents the fraction of renewable energy coming from the i-th component and depends on the RES share of the grid for electric-based components. The share is zero for fuel-based ones and for renewable-based components the share would be 100%. The balance between the different objectives is typically adjusted by means of weighting factors $Q_\star$ to give more or less priority to the associated objective term $\ell(\star)$ for a particular variable $\star$. For example the penalization on control inputs u can be by denoted by a weighting term Q_u.

In general, there are two types of constraints: inequality (control inputs range, comfort zones, etc.) and equality (building model dynamics, rate limits, etc.) constraints. A recap of the most common constraints in building control can be found in Table 5. The constraints which satisfaction is mandatory are called *hard constraints*. For example, hard control action bounds define the minumum $\underline{u}_k$ and maximum $\overline{u}_k$ allowed values for the contol variables. On the other hand, the constraints which can be violated are known as *soft constraints*. They are usually relaxed by slack variables s_k that are added to and penalized in the objective function (see comfort objective in Table 4 and soft inequality constraints in Table 5), trying to keep them within the bounds set. For example the thermal discomfort is typically evaluated via a slack variable s_k which denotes the violation of a thermal comfort zone associated with zone operative temperature T_z. All constraints can be time-varying.

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Table 1: MPC variables notation and translation between physical and abstract domain.

Physical domain			Abstract domain						
Variables	**Symbol**	**Description**	x	y	m	a	u	d	s
Temperatures [°C]	T	Envelope temperatures (wall, concrete, glazing...)	•	-	-	-	-	-	-
	T_{z}	Zone operative temperatures	-	•	-	-	-	-	-
	$T_{\mathrm{sup}}^{\mathrm{HVAC}}$	Supply temperatures of HVAC	-	-	•	-	-	-	-
	$T_{\mathrm{ret}}^{\mathrm{HVAC}}$	Return temperatures of HVAC	-	-	•	-	-	-	-
	$T_{\mathrm{sp}}^{\mathrm{HVAC}}$	Set-points of HVAC supply temperatures	-	-	-	•	-	-	-
	T_{e}	Ambient temperature	-	-	-	-	-	•	-
	T_{b}	Borefield wall temperature	•	-	-	-	-	-	-
	T_{g}	Borefield ground temperatures	•	-	-	-	-	-	-
Thermal energy $[J]$	Q_{HVAC}	Thermal energy of the HVAC components	-	-	•	-	-	-	-
	Q_{b}	Thermal energy injected/extracted into/from the borefield	-	-	•	-	-	-	-
Thermal power $[W]$	$\dot{Q}_{\mathrm{HVAC}}$	Thermal power of HVAC components	-	-	-	-	•	-	-
	$\dot{Q}_{\mathrm{rad}}$	Solar radiation	-	-	-	-	-	•	-
	$\dot{Q}_{\mathrm{occ}}$	Occupancy internal gains	-	-	-	-	-	•	-
	$\dot{Q}_{\mathrm{lig}}$	Lighting internal gains	-	-	-	-	•	•	-
	$\dot{Q}_{\mathrm{app}}$	Appliances internal gains	-	-	-	-	•	•	-
Electrical energy $[J]$	E_{HVAC}	Electrical energy used by HVAC components	-	-	•	-	-	-	-
	E_{lig}	Electrical energy used by lighting	-	-	•	-	-	-	-
	E_{app}	Electrical energy used by appliances	-	-	•	-	-	-	-
Electrical power $[W]$	P_{HVAC}	Power of HVAC components	-	-	•	-	-	-	-
	P_{lig}	Lighting power	-	-	•	-	-	-	-
	P_{app}	Appliances power	-	-	•	-	-	-	-
Mass flows $[kg/s]$	$\dot{m}_{\mathrm{HVAC}}$	Water/air mass flow rates of HVAC	-	-	•	•	-	-	-
Component signals	x_{HVAC}	ON/OFF/Modulated signal of HVAC components $[\{0,1\},\%]$	-	-	-	•	-	-	-
	Δp	Pump/fan difference pressure $[kPa]$	-	-	-	•	-	-	-
	x_{mov}	Pump/fan speed signal $[\%, kPa]$	-	-	-	•	-	-	-
	x_{val}	Valve positions [%]	-	-	-	•	-	-	-
	x_{dam}	Damper positions [%]	-	-	-	•	-	-	-
Indoor Environmental Quality (IEQ)	PMV	Predicted mean vote $[-]$	-	•	-	-	-	-	-
	CO_2	CO_2 concentration in air $[ppm]$	-	•	-	-	-	-	-
	ϕ	Relative humidity [%]	-	•	-	-	-	-	-
	E_{v}	Illuminance $[lx]$	-	•	-	-	-	-	-
Constraints violations	$s^{\star}$	Violations of constraints on variable $\star$ [-]	-	-	-	-	-	-	•
	$s^{T_{\mathrm{z}}}$	Violations of thermal comfort zones [°C]	-	-	-	-	-	-	•
	s^{PMV}	Violations of PMV zones [-]	-	-	-	-	-	-	•
	s^{CO_2}	Violations of CO_2 concentration limits $[ppm]$	-	-	-	-	-	-	•
	s^{ϕ}	Violations of relative humidity limits [%]	-	-	-	-	-	-	•
	$s^{E_{\mathrm{v}}}$	Violations of illuminance limits $[lx]$	-	-	-	-	-	-	•

Table 2: MPC formulation parameters - modifiers and tuning parameters.

Energy efficiencies	**Symbol**	**Description**	**Associated variables**
Coefficient of performance	COP^i	Coefficient of performance of heat pump i	$T_{\text{sup}}^{\text{HVAC}}, \dot{Q}_{\text{HVAC}}^i, P_{\text{HVAC}}^i$
Energy efficiency ratio	EER^i	Cooling efficiency of air conditioning system i	$T_{\text{sup}}^{\text{HVAC}}, \dot{Q}_{\text{HVAC}}^i, P_{\text{HVAC}}^i$
Efficiency	η^i	Efficiency of other i-th system	$T_{\text{sup}}^{\text{HVAC}}, \dot{Q}_{\text{HVAC}}^i, P_{\text{HVAC}}^i$
Seasonal coefficient of performance	$SCOP^i$	Seasonal coefficient of performance of heat pump i	$T_{\text{sup}}^{\text{HVAC}}, Q_{\text{HVAC}}^i, E_{\text{HVAC}}^i$
Seasonal energy efficiency ratio	$SEER^i$	Seasonal cooling efficiency of air conditioning system i	$T_{\text{sup}}^{\text{HVAC}}, Q_{\text{HVAC}}^i, E_{\text{HVAC}}^i$
Other energy factors	**Symbol**	**Description**	**Associated variables**
Price factor	p^i	Conversion factor from energy to monetary cost of component i	Q_{HVAC}^i
Emission factor	e^i	Conversion factor from energy to CO_2 emissions of component i	Q_{HVAC}^i
RES factor	RES^i	Share of renewable energy of the considered component i	Q_{HVAC}^i
Heat flow parameters	**Symbol**	**Description**	**Associated variables**
Specific heat capacity	c_p^i	Specific heat capacity of i-th medium	$T_{\text{sup}}^{\text{HVAC}}, T_{\text{ret}}^{\text{HVAC}}, \dot{m}_{\text{wat}}, \dot{m}_{\text{air}}, \dot{Q}_{\text{HVAC}}^i$
Tuning parameters	**Symbol**	**Description**	**Associated variables**
Weighting factor	$Q_\star$	Weighting for the particular term in the objective function	all
Sampling time	T_s	Time-step used in the optimization problem	all
Dimensionality quantifier	$n_\star$	Cardinality of the vector elements	all

Table 3: **MPC formulation parameters - bounds and references r.**

Comfort bounds r	**Symbol**	**Description**	**Associated variables**
Temperature	$\underline{T_\text{z}}, \overline{T_\text{z}}$	Zone operative temperature comfort bounds	T_z
CO_2 concentration	$\underline{CO_2}, \overline{CO_2}$	CO_2 concentration comfort bounds	CO_2
Predicted mean vote	$\underline{PMV}, \overline{PMV}$	Predicted mean vote comfort bounds	PMV
Humidity	$\underline{\phi}, \overline{\phi}$	Humidity comfort bounds	ϕ
Illuminance	$\underline{E_\text{v}}, \overline{E_\text{v}}$	Illuminance comfort bounds	E_v
Component limitations	**Symbol**	**Description**	**Associated variables**
Thermal power limit	$\underline{\dot{Q}^i_\text{HVAC}}, \overline{\dot{Q}^i_\text{HVAC}}$	Min/max thermal power of the i-th HVAC component	Q^i_HVAC
Rate of change of thermal power rate	$\underline{\Delta\dot{Q}^i_\text{HVAC}}, \overline{\Delta\dot{Q}^i_\text{HVAC}}$	Min/max rate of change of thermal power of the i-th HVAC component	Q^i_HVAC
Pump difference pressure limit	$\underline{\Delta p^i}, \overline{\Delta p^i}$	Min/max diff. pressure of the i-th fan/pump	Δp^i
Rate of change of pump difference pressure	$\underline{\Delta\Delta p^i}, \overline{\Delta\Delta p^i}$	Min/max rate of change of diff. pressure of the i-th fan/pump	Δp^i
Pump speed limit	$\underline{x^i_\text{mov}}, \overline{x^i_\text{mov}}$	Min/max speed of the i-th fan/pump	x^i_mov
Rate of change of pump speed	$\underline{\Delta x^i_\text{mov}}, \overline{\Delta x^i_\text{mov}}$	Min/max rate of change of speed of the i-th fan/pump	x^i_mov
Valve position limits	$\underline{x^i_\text{val}}, \overline{x^i_\text{val}}$	Min/max position of the i-th valve	x^i_val
Rate of change of valve position	$\underline{\Delta x^i_\text{val}}, \overline{\Delta x^i_\text{val}}$	Min/max change of position of the i-th valve	x^i_val
Damper position limits	$\underline{x^i_\text{dam}}, \overline{x^i_\text{dam}}$	Min/max position of the i-th damper	x^i_dam
Rate of change of damper position	$\underline{\Delta x^i_\text{dam}}, \overline{\Delta x^i_\text{dam}}$	Min/max rate of change of position of the i-th damper	x^i_dam

Table 4: Additive terms $\star$ of MPC objective $\min \sum_{k=0}^{N-1} \star$**.**

Linguistic objective	**MPC formulation**		
	Class	Control notation	Physical notation
Minimize energy use	linear	$\sum_{i=1}^{n_u} u_k^i$	$\sum_{i=1}^{n_{\dot{Q}_{\mathrm{HVAC}}}} \dot{Q}_{\mathrm{HVAC},k}^i$
Minimize power use	linear	$\sum_{i=1}^{n_u} \frac{u_k^i}{\eta_k^i}$	$\sum_{i=1}^{n_{\dot{Q}_{\mathrm{HVAC}}}} \frac{\dot{Q}_{\mathrm{HVAC},k}^i}{\eta_k^i}$
Minimize cost	linear	$\sum_{i=1}^{n_u} p_k^i \frac{u_k^i}{\eta_k^i}$	$\sum_{i=1}^{n_{\dot{Q}_{\mathrm{HVAC}}}} p_k^i \frac{\dot{Q}_{\mathrm{HVAC},k}^i}{\eta_k^i}$
Minimize CO_2 emissions	linear	$\sum_{i=1}^{n_u} e_k^i \frac{u_k^i}{\eta_k^i}$	$\sum_{i=1}^{n_{\dot{Q}_{\mathrm{HVAC}}}} e_k^i \frac{\dot{Q}_{\mathrm{HVAC},k}^i}{\eta_k^i}$
Maximise RES	linear	$\sum_{i=1}^{n_u} (1 - RES_k^i) \frac{u_k^i}{\eta_k^i}$	$\sum_{i=1}^{n_{\dot{Q}_{\mathrm{HVAC}}}} (1 - RES_k^i) \frac{\dot{Q}_{\mathrm{HVAC},k}^i}{\eta_k^i}$
Minimize discomfort	quadratic	$\|\|s_k\|\|_2^2$	$\|\|s_k^{T_z}\|\|_2^2$
Flexibility	TBD	TBD	TBD

Table 5: MPC formulation constraints.

Linguistic constraint	**MPC formulation**		
	Class	Control notation	Physical notation
Envelope dynamics	hard equality	$x_{k+1} = f(x_k, u_k, d_k)$	$T_{k+1} = f(T_k, \dot{Q}_{\mathrm{HVAC},k}, T_{\mathrm{e},k}, \dot{Q}_{\mathrm{rad},k}, \dot{Q}_{\mathrm{occ},k}, \dot{Q}_{\mathrm{lig},k})$
HVAC model	hard equality	$u_k = f_{\mathrm{HVAC}}(a_k, m_k)$	$\dot{Q}_{\mathrm{HVAC},k} = f_{\mathrm{HVAC}}(\dot{m}_{\mathrm{HVAC}}, T_{\mathrm{sup}}^{\mathrm{HVAC}}, T_{\mathrm{ret}}^{\mathrm{HVAC}}, P_{\mathrm{HVAC}})$
Component limitations	hard inequality	$\underline{u}_k \leq u_k \leq \overline{u}_k$	$\underline{\dot{Q}_{\mathrm{HVAC}}}_k \leq \dot{Q}_{\mathrm{HVAC},k} \leq \overline{\dot{Q}_{\mathrm{HVAC}}}_k$
Component difference	hard equality	$\Delta u_k = u_{k-1} - u_k$	$\Delta\dot{Q}_{\mathrm{HVAC},k} = \dot{Q}_{\mathrm{HVAC},k-1} - \dot{Q}_{\mathrm{HVAC},k}$
Component rate of change	hard inequality	$\underline{\Delta u}_k \leq \Delta u_k \leq \overline{\Delta u}_k$	$\underline{\Delta\dot{Q}_{\mathrm{HVAC}}}_k \leq \Delta\dot{Q}_{\mathrm{HVAC},k} \leq \overline{\Delta\dot{Q}_{\mathrm{HVAC}}}_k$
Temperature comfort zone	soft inequality	$\underline{y}_k - s_k \leq y_k \leq \overline{y}_k + s_k$	$\underline{T_{\mathrm{z}}}_k - s_k^{T_{\mathrm{z}}} \leq T_{\mathrm{z}} \leq \overline{T_{\mathrm{z}}}_k + s_k^{T_{\mathrm{z}}}$
CO_2 concentration	soft inequality	$\underline{y}_k - s_k \leq y_k \leq \overline{y}_k + s_k$	$\underline{CO_2}_k - s_k^{CO_2} \leq CO_2 \leq \overline{CO_2}_k + s_k^{CO_2}$
Predicted mean vote	soft inequality	$\underline{y}_k - s_k \leq y_k \leq \overline{y}_k + s_k$	$\underline{PMV}_k - s_k^{PMV} \leq PMV \leq \overline{PMV}_k + s_k^{PMV}$
Humidity	soft inequality	$\underline{y}_k - s_k \leq y_k \leq \overline{y}_k + s_k$	$\underline{\phi}_k - s_k^{\phi} \leq \phi \leq \overline{\phi}_k + s_k^{\phi}$
Illuminance	soft inequality	$\underline{y}_k - s_k \leq y_k \leq \overline{y}_k + s_k$	$\underline{E_{\mathrm{v}}}_k - s_k^{E_{\mathrm{v}}} \leq E_{\mathrm{v}} \leq \overline{E_{\mathrm{v}}}_k + s_k^{E_{\mathrm{v}}}$