8-Puzzle Game, Part I: Solvability decision

Input: a state
Output: a Boolean value indicating whether there is a solution for the given state or not

[Hint] Solvability of N-puzzle problems

Note that the board of an N-puzzle game is an $n \times n$ square where $N = n \times n - 1$.

If n is odd, and the number of disorder digits (in the state presentation) is even, the problem is solvable. Number of disorder digits is measured by counting pairs of digits (i, j) where $1 \le i < j \le N$ but position(i) > position(j). For example,

3	1	2
4	5	7
6		8

is unsolvable, because its state representation 31245768 (discarding 0) has 3 disordered pairs: (1,3) (2,3) (6,7).

If n is even, and the index of the row containing the empty tile plus the number of disorder digits is even, the problem is solvable. For example,

1	2	7	4
3		6	10
8	5	13	11
9	12	14	15

is unsolvable,
because its state representation 127436A85DB9CEF (discarding 0) has 14 disordered pairs: (3,7) (4,7) (5,7) (6,7) (3,4) (5,6) (5,A) (8,A) (9,A) (5,8) (9,D) (B,D) (C,D) (9,B),
and the empty tile is at Row #1 (index starts from 0),
so 14 + 1 = 15 is odd.
(Note that A ~ F stand for 10 ~ 15.)

Testing Input (for 8-puzzle)

Input	Meaning
10 312457680 724506831 438126507 167352480 104782563 817365204 320685741 426031785 102345678 041235678	Number of test data State of test data #1 State of test data #2 . . .
Output	Meaning

NO YES NO NO YES YES NO YES YES NO	Output of test data #1 Output of test data #2 . . .