1. Confidence Intervals for Curved Gaussian Family

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(a)

1/1 point (graded)

Let $X_1, \ldots, X_n$ be i.i.d. random variables with distribution $\mathcal{N}(\theta, \theta)$, for some unknown parameter $\theta > 0$.

True or False: The sample average $\overline{X}_n$ follows a normal distribution for any integer $n \geq 1$.

- (selected) True
- False

Submit | You have used 1 of 1 attempt | Show Answer

(b)

2/2 points (graded)

What is the expectation and the variance of $\overline{X}_n$?

$\mathbb{E}[\overline{X}_n] =$ theta

θ

$\mathsf{Var}(\overline{X}_n) =$ theta/n

$\frac{\theta}{n}$

STANDARD NOTATION

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(c)

2/2 points (graded)

Find an interval $\mathcal{I}_\theta$ (that depends on θ) centered about $\overline{X}_n$ such that

$$\mathbf{P}(\mathcal{I}_\theta \ni \theta) = 0.9 \qquad \text{for all } n \text{(i.e, not only for large } n).$$

(Write `barX_n` for $\overline{X}_n$. Use the estimate $q_{0.05} \approx 1.6448$ for best results.)

$\mathcal{I}_\theta = [A_\theta, B_\theta]$ for

$A_\theta =$ barX_n-1.6448/sqrt(n/theta)

$$\overline{X}_n - \frac{1.6448}{\sqrt{\frac{n}{\theta}}}$$

$B_\theta =$ barX_n+1.6448/sqrt(n/theta)

$$\overline{X}_n + \frac{1.6448}{\sqrt{\frac{n}{\theta}}}$$

STANDARD NOTATION

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(d)

2/2 points (graded)

Again, use the estimate $q_{0.05} \approx 1.6448$ for best results.

Now, find a confidence interval $\mathcal{I}_{\text{plug-in}}$ with **asymptotic** confidence level 90% by plugging in $\overline{X}_n$ for all occurrences of θ in $\mathcal{I}_\theta$.

$\mathcal{I}_{\text{plug-in}} = [A_{\text{plug-in}}, B_{\text{plug-in}}]$ for

$A_{\text{plug-in}} =$ barX_n-1.6448/sqrt(n/barX_n)

$$\overline{X}_n - \frac{1.6448}{\sqrt{\frac{n}{\overline{X}_n}}}$$

$B_{\text{plug-in}} =$ barX_n+1.6448/sqrt(n/barX_n)

$$\overline{X}_n + \frac{1.6448}{\sqrt{\frac{n}{\overline{X}_n}}}$$

STANDARD NOTATION

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(e)

0/2 points (graded)

Finally, find a confidence interval $\mathcal{I}_{\text{solve}}$ for θ with **nonasymptotic** level 90% solving the bounds in $\mathcal{I}_\theta$ for θ.

$\mathcal{I}_{\text{solve}} = [A_{\text{solve}}, B_{\text{solve}}]$ for

$A_{\text{solve}} =$ barX_n+1.6448^2/(2*n)-1/2*sqrt(1.6448^4/(4*n^2)+barX_n*1.64

$$\overline{X}_n + \frac{1.6448^2}{2 \cdot n} - \frac{1}{2} \cdot \sqrt{\frac{1.6448^4}{4 \cdot n^2} + \overline{X}_n \cdot \frac{1.6448^2}{n}}$$

$B_{\text{solve}} =$ barX_n+1.6448^2/(2*n)+1/2*sqrt(1.6448^4/(4*n^2)+barX_n*1.6

$$\overline{X}_n + \frac{1.6448^2}{2 \cdot n} + \frac{1}{2} \cdot \sqrt{\frac{1.6448^4}{4 \cdot n^2} + \overline{X}_n \cdot \frac{1.6448^2}{n}}$$

STANDARD NOTATION

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- [STAFF] Please check my answer for 1.e) (slight variation in notation) — 3
  I have the same answer, but a slightly varying notation. In the end both results are the same, so could you please check and if you agree, award this one point. And if not, prov...
- [Staff]Kindly consider my answer for 1.e — 1
  I have got the same answer but I have written the wrong value for constant q_(alpha/2). There is a typo at 10^-4 decimal place. Could you please consider my answer and chec...
- [question for staff and students] Best online solver for (e) — 1
  Yesterday I used an online equation solver and it provided different roots. Today, seeing the results and the equation to be solved in the solution, again I used the same equa...
- Wrong computation for Delta in quadratic equation. — 1
  A = 1 B = (2x+t^2/2) C = 4x Delta = B^-4*A*C = (2x+t^2/n)^2-4x = 4x^2 + t^4/n^2 + 4x*t^2/n -4x^2 = 4x*t^2/n + t^4/n^2 4 is coefficient in numerator not in denominator
- ? [STAFF] last try for d — 1
  Staff, I spend my last chance for (d) because I put the answer related to (e) by mistake in (d). But I think I have the correct one in (d). Could you please reconsider it and give m...
- This class lacks so much context — 8
  I must be the only totally lost, but there is such vagueness in the questions that the TA's or professors think would be obvious, but I mean, it isn't... There is almost no context ...
- ? General confusion about how to apply CIs to different distributions — 1
  When the examples were given with exponential distributions or uniform distributions, I got them, but I don't understand the background information enough to be able to a...
- ? A bit of help, overthinking 1.b — 5
  I overthought 1.b, however I think the following questions are based upon it. Could someone provide a hint to this please? Thank you
- Hints part e — 9
  Did it twice with wolfram, same expression: 1-first I plugged the number for q in the expression before solving ; gives birth to a monster, got it wrong 2-same expression, left ...
- Tip for 1e — 9
- [STAFF] One more try for (e) please ? — 1
  Im am so frustrated, I think i got the correct calculation but I inverted upper and lower bound as I wrote them (upper bound first)... Can someone on the staff please look at m...
- ? [Staff] Grader issue? — 1
  Dear staff: I guess there´s an issue with grader on question e). Would you please check my answer? Kind regards Rodolfo
- ? [STAFF] confidence interval 1c — 6
- ? Part e: How do we know which root to use? — 3

2. Delta method and asymptotic variances

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(a) (Optional)

0 points possible (ungraded)

In this problem, you are going to compute the **asymptotic variance** of some estimators. Recall that the asymptotic variance of an estimator $\hat{\theta}$ for a parameter θ is defined as $V(\hat{\theta})$, if

$$\sqrt{n}(\hat{\theta}-\theta) \xrightarrow[n\to\infty]{(D)} \mathcal{N}(0, V(\hat{\theta})).$$

The arguments that we use to establish asymptotic normality are often the same in our setups, namely the Law of Large Numbers, the Central Limit Theorem, and the Delta Method. First, we review the assumptions and statements of those theorems:

Let $X_1, X_2, \ldots,$ be random variables. The (weak) Law of Large Numbers says that under suitable assumptions, with

$$\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i,$$

we have

$$\overline{X}_n \xrightarrow{\mathbf{P}} \mathbb{E}[X_1].$$

What are the assumptions we need for the weak Law of Large Numbers? (Choose all that apply.)

- [x] $\mathbb{E}[|X_i|] < \infty$ for all i
- [x] $\mathsf{Var}(X_i) < \infty$ for all i
- [x] $X_1, X_2, \ldots$ independent
- [ ] There exists $M > 0$ such that $|X_i| \leq M$ for all i
- [ ] $|X_i| \geq |X_{i+1}|$ almost surely for all i
- [x] $X_1, X_2, \ldots$ identically distributed

✖

The Central Limit Theorem states that under some assumptions, there is a V such that

$$\sqrt{n}(\overline{X}_n - \mathbb{E}[X_1]) \xrightarrow{(D)} \mathcal{N}(0, V).$$

What are the assumptions we need for the Central Limit Theorem? Pick all that apply.

- [ ] $\mathbb{E}[|X_i|] < \infty$ for all i
- [ ] $\mathsf{Var}(X_i) < \infty$ for all i
- [x] $X_1, X_2, \ldots$ independent
- [ ] There exists $M > 0$ such that $|X_i| \leq M$ for all i
- [ ] $|X_i| \geq |X_{i+1}|$ almost surely for all i
- [x] $X_1, X_2, \ldots$ identically distributed

✖

The Delta Method gives us a way to control the asymptotic variance of a transformation of a random variable. Let $\theta \in \mathbb{R}$ be a parameter and $Z_n \in \mathbb{R}$ be a sequence of random variables that satisfies

$$\sqrt{n}(Z_n - \theta) \xrightarrow[n\to\infty]{(D)} \mathcal{N}(0, V)$$

for some $V > 0$.

Given a function $g : \Omega \subseteq \mathbb{R} \to \mathbb{R}$,

$$\sqrt{n}(g(Z_n) - g(\theta)) \xrightarrow[n\to\infty]{(D)} \mathcal{N}(0, W).$$

for some $W > 0$.

Pick the following assumptions and conditions that apply to the Delta Method as stated in class:

- [ ] g is monotonically increasing
- [x] g is continuously differentiable at θ
- [x] $W = g'(\theta)^2 V$
- [ ] $W = g(\theta)^2 V$
- [ ] $W = |g'(\theta)| V$

✔

Submit You have used 1 of 3 attempts Show Answer

Instructions:
Now, in each of the following questions, argue that both proposed estimators are consistent and asymptotically normal. Then, give their asymptotic variances and decide if one of them is always bigger than the other.

(b)

2/2 points (graded)

Argue that the proposed estimators $\hat{\lambda}$ and $\tilde{\lambda}$ below are both consistent and asymptotically normal. Then, give their **asymptotic variances** $V(\hat{\lambda})$ and $V(\tilde{\lambda})$, and decide if one of them is always bigger than the other.

Let $X_1, \ldots, X_n \overset{i.i.d.}{\sim} \mathsf{Poiss}(\lambda)$, for some $\lambda > 0$. Let $\hat{\lambda} = \overline{X}_n$ and $\tilde{\lambda} = -\ln(\overline{Y}_n)$, where $Y_i = \mathbf{1}\{X_i = 0\}, i = 1, \ldots, n$.

$V(\hat{\lambda}) =$ lambda ✔
λ

$V(\tilde{\lambda}) =$ exp(lambda)-1 ✔
$\exp(\lambda) - 1$

STANDARD NOTATION

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(c)

3/3 points (graded)

As above, argue that both proposed estimators $\hat{\lambda}$ and $\tilde{\lambda}$ are consistent and asymptotically normal. Then, give their **asymptotic** variances $V(\hat{\lambda})$ and $V(\tilde{\lambda})$, and decide if one of them is always bigger than the other.

Let $X_1, \ldots, X_n \overset{i.i.d.}{\sim} \mathsf{Exp}(\lambda)$, for some $\lambda > 0$. Let $\hat{\lambda} = \frac{1}{\overline{X}_n}$ and $\tilde{\lambda} = -\ln(\overline{Y}_n)$, where $Y_i = \mathbf{1}\{X_i > 1\}, i = 1, \ldots, n$.

$V(\hat{\lambda}) =$ lambda^2 ✔
λ^2

$V(\tilde{\lambda}) =$ exp(lambda)-1 ✔
$\exp(\lambda) - 1$

- ( ) $V(\hat{\lambda}) > V(\tilde{\lambda})$ for all λ.
- (•) $V(\hat{\lambda}) < V(\tilde{\lambda})$ for all λ.
- ( ) $V(\hat{\lambda}) = V(\tilde{\lambda})$ for all λ.
- ( ) There exists λ_1 such that $V(\hat{\lambda}) > V(\tilde{\lambda})$ and λ_2 such that $V(\hat{\lambda}) < V(\tilde{\lambda})$

✔

STANDARD NOTATION

Submit You have used 3 of 3 attempts Show Answer

(d) (Optional)

0 points possible (ungraded)

Ungrading Note: *This question needs techniques that you will only learn in a later unit, and we will revisit this problem in a later homework. For now, this is ungraded. We are sorry for the oversight on our part, but hope that the time you have spent on this undoable question (for now) will still be worthwhile in the long run.*

As above, argue that both proposed estimators $\widehat{\sigma^2}$ and $\widetilde{\sigma^2}$ are consistent and asymptotically normal. Then, give their asymptotic variances $V(\widehat{\sigma^2})$ and $V(\widetilde{\sigma^2})$ and decide if one of them is always bigger than the other.

Let $X_1, \ldots, X_n \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma^2)$, for some $\sigma^2 > 0$. Let

$$\widehat{\sigma^2} = \frac{1}{n}\sum_{i=1}^{n} X_i^2, \quad \text{and} \quad \widetilde{\sigma^2} = \frac{1}{n}\sum_{i=1}^{n} (X_i - \overline{X}_n)^2.$$

$V(\widehat{\sigma^2}) =$

$V(\widetilde{\sigma^2}) =$

STANDARD NOTATION

Submit You have used 0 of 2 attempts

(e)

3/3 points (graded)

As above, argue that both proposed estimators $\hat{p}, \tilde{p},$ are consistent and asymptotically normal. Then, give their **asymptotic** variances $V(\hat{p})$ and $V(\tilde{p})$ and decide if one of them is always bigger than the other.

Let $X_1, \ldots, X_n \overset{i.i.d.}{\sim} \mathsf{Geom}(p)$, for some $p \in (0, 1)$. That means that

$$\mathbf{P}(X_1 = k) = p(1-p)^{k-1}, \quad \text{for } k = 1, 2, \ldots.$$

Let

$$\hat{p} = \frac{1}{\overline{X}_n},$$

and $\tilde{p}$ be the **number of ones in the sample divided by** n.

$V(\hat{p}) =$ (1-p)*p^2 ✔
$(1-p) \cdot p^2$

$V(\tilde{p}) =$ p*(1-p) ✔
$p \cdot (1-p)$

- ( ) $V(\hat{p}) > V(\tilde{p})$ for all p.
- (•) $V(\hat{p}) < V(\tilde{p})$ for all p.
- ( ) $V(\hat{p}) = V(\tilde{p})$ for all p.
- ( ) There exists p_1 such that $V(\hat{p}) > V(\tilde{p})$ and p_2 such that $V(\hat{p}) < V(\tilde{p})$

✔

STANDARD NOTATION

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- Potential Bug in Provided Answer for B? — 2
  I understand most part of the answer but just the step of applying delta method. The g'(t) is calculated as -1/t and then we plug in t=e^(-lambda). My confusion is that in the le...
- [a] CLT assumptions — 1
  Can someone please explain the first two assumptions (according to solutions) for CLT? CLT has the Randomization Condition: The data must be sampled randomly. Are thes...
- help on (b) please. Strugling on finding function g(x)... Is there a rule for that? — 3
  I have been struggling with this whole day. I think the distribution is poisson. However, I can not convert this poission distribution to a form like sqrt(n)(g(x)-g(lambda))...includi...
- (b) confusion asymptotic variance Yn — 18
  Hi all! I am having difficulties with computing the asymptotic variance of the second sub-question, which I calculated with: g'(λ)^2 * var(Y_i) I assumed that the asymptotic vari...
- Homework2 Progress — 2
  When you complete homework2, the progress chart gives credit to homework3 instead of 2
- The display of the homework is messed up - This what I see — 2
  (a) (Optional) 0 points possible (ungraded) In this problem, you are going to compute the asymptotic variance of some estimators. Recall that the asymptotic variance of an es...
- Stuck at asymptotic variance of indicator function (b and c) — 32
- (b) & (c) unable to solve for asymptotic variance for λ̃ = -ln(Y_n) — 7
  [Please do not post answers/partial answers until the due date] For (b) I calculated the [*removed]*. Then used it in the variance formula for [*removed*] Then used the de...
- (e) - p_hat, any hints? — 9
  I tried to treat Geom as Exp and do all the magic professor did, but it did not work, any hints? for p_tilda i got my answer correct.
- Why the variance doesn't decrease inversely proportional to N in Questions B and C? — 3
  If we trying to find the mean of an IID sample, why we are not rescaling the variance to Var(X)/n?
- [Staff] d ungraded but still counts in progress — 2
  Is it possible that question d is still graded? It's afecting the total hw score in Progress. Thanks
- Request for extension — 3
  Dear team, Due to unforseen circumstances, I am unable to finish this week's lecture and homework. I humbly request you to extend it for a week if possible! Thanks!
- (e) p_tilde -> g(x) ? — 7
- Totally frustrated — 8

3. Application of Delta Method on Gamma Variables

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The **Gamma distribution** $\mathrm{Gamma}(\alpha, \beta)$ with paramters $\alpha > 0$, and $\beta > 0$ is defined by the density

$$f_{\alpha,\beta}(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, \quad \text{for all } x \geq 0.$$

The Γ function is defined by

$$\Gamma(s) = \int_0^\infty x^{s-1} e^{-x} dx.$$

As usual, the constant $\frac{\beta^\alpha}{\Gamma(\alpha)}$ is a normalization constant that gives $\int_0^\infty f_{\alpha,\beta}(x)\,dx = 1$.

In this problem, let $X_1, \ldots, X_n$ be i.i.d. Gamma variables with

$$\beta = \frac{1}{\alpha} \text{ for some } \alpha > 0.$$

That is, $X_1, \ldots, X_n \sim \mathrm{Gamma}\left(\alpha, \frac{1}{\alpha}\right)$ random variables for some $\alpha > 0$. The pdf for X_i is therefore

$$f_\alpha(x) = \frac{1}{\Gamma(\alpha)\,\alpha^\alpha} x^{\alpha-1} e^{-x/\alpha}, \quad \text{for all } x \geq 0.$$

(a)

1/1 point (graded)

What is the limit, in probability, of the sample average $\overline{X}_n$ of the sample in terms of α?

$\overline{X}_n \xrightarrow[n\to\infty]{\mathbf{P}}$ alpha^2 ✓

α^2

STANDARD NOTATION

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Show Answer

(b)

1/1 point (graded)

Use the result from the previous problem to give a consistent estimator $\hat{\alpha}$ of α in terms of $\overline{X}_n$.

(Enter barX_n for $\overline{X}_n$)

$\hat{\alpha} =$ barX_n^(1/2) ✓

$\overline{X}_n^{\frac{1}{2}}$

Submit You have used 1 of 3 attempts

Show Answer

(c)

3/3 points (graded)

For the Delta method to apply, at what value of x does g need to be continuously differentiable? (Your answer should be in terms of α.)

$x =$ alpha^2 ✓

α^2

What distribution does $\sqrt{n}(\hat{\alpha} - \alpha)$ converge to as $n \to \infty$?

- ○ Gamma distribution
- ● Normal distribution
- ○ None of the above

✓

What is its **asymptotic** variance of $\hat{\alpha}$?

$V(\hat{\alpha}) =$ alpha/4 ✓

$\frac{\alpha}{4}$

STANDARD NOTATION

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(d)

3.0/4.0 points (graded)

Using the previous part, find confidence intervals for α with asymptotic level 90% using both the "solving" and the "plug-in" methods. Use $n = 25$, and $\overline{X}_n = 4.5$.
(Enter your answers accurate to 2 decimal places. Use the Gaussian estimate $q_{0.05} \approx 1.6448$ for best results.)

$\mathcal{I}_{\text{solve}} =$ [1.89 ✓ , 2.36 ✗]

$\mathcal{I}_{\text{plug-in}} =$ [1.88 ✓ , 2.36 ✓]

STANDARD NOTATION

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Starting to doubt if I should even continue with this course..... Don't get me wrong, I have been loving the course, and I've even completed the Probability course before this and enjoyed it thoroughly. It's just that for someone like me, I h...	54
Extension of one day Hello I planned to finish the homework this morning but I lost my grandmother early this morning. So i spent the day between the hospital and the cemetery. I wanted to kno...	7
Feelings: This Homeworks grades follow some sort of geometric process for how many points you can get in each section, each questions depends on previous one. if you get one wrong then, its simply impossible to score next one. Id be much happy to work 100 hours a week if I ...	2
this is so frustrating i kept trying to answer the d question and thought that i did something wrong only to discover that i got it wrong because of one decimal place that is really not cool and so fr...	1
Don't lose heart! Hi all, I see a lot of despairing students. I just want to encourage you all. I could only complete 35% of last week's homework. This week I did much better (even though I relied...	3
It's more like a comedy lol. Viewers will simply audit the course. It's more like a comedy lol. Viewers will simply audit the course. "What is the square root of the flintstones dino the dinosaur david bowie hitch-hikers guide red left angle in p...	1
Hints for all I just completed the Homework and I wanted to help anybody who is trying to meet the deadline today. Here are some hints for all *(a)* [mathjaxinline] X_1,\ldots ,X_ n\sim \t...	3
NO Consistency Between Exercise # 5 and Homework #3 NO!!!!! Consistency Between Exercise # 5 and Homework #3 (#2) My answers to (3d) are right Yet they are marked wrong. Are we truncating? Are we rounding? Are we using 4...	1
Grader does not work, hopeless, won't accept input,hopeless Grader does not work, hopeless, won't accept input, hopeless. This was before expiration time	1
(c): g differentiability Isn't **g** has to be continuously differentiable everywhere? What this question about?	13
HELP needed - part d- numerical values Hi TA's and everyone else! I'm in a fix- after solving the trickier pieces in this problem set, I'm making mistakes with the numerical answers! 3 out of 4 of my numerical answer...	3
Part c - asymptotic variance I thought that all I had to do for this part is just take the Gamma distribution variance, plug it in with the modified beta variable, and submit that but it's wrong. Any help?	5
STAFF:Problem with the grader on (d) Hi, i only use 2 attempts on question (d), and actually i got it partially right in the Iplug-in interval, some how in my third intent it roll back to my first answer and now states wr...	1