1. Statistical Models and Identifiability

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For each of the following examples, define a statistical model and check whether the parameter of interest is identifiable. Follow the definitions closely; it is helpful to consider the following: What is Θ and P_θ? What would it mean for the model to be identifiable?

(a)

3/4 points (graded)

1. One observes n i.i.d. Poisson random variables with unknown parameter λ.

- (•) λ identifiable
- ( ) λ not identifiable

✔

2. One observes n i.i.d. exponential random variables with parameter λ, which is unknown but a priori known to be no larger than 10.

- ( ) λ identifiable
- (•) λ not identifiable

✖

3. One observes n i.i.d. uniform random variables in the interval $[0, \theta]$, where θ is unknown.

- (•) θ identifiable
- ( ) θ not identifiable

✔

4. One observes n i.i.d. Gaussian random variables with unknown parameters μ, σ^2.

- (•) (μ, σ^2) identifiable
- ( ) (μ, σ^2) not identifiable

✔

Submit You have used 1 of 1 attempt

Show Answer

(b)

4/4 points (graded)

1. One observes the sign of n i.i.d. Gaussian random variables with unknown parameters μ, σ^2.

- ( ) (μ, σ^2) identifiable
- (•) (μ, σ^2) not identifiable

✔

2. *StatGen* is a statistical procedure to test the relevance of genes. When well calibrated, it outputs the (random) proportion of active genes in a (random) cell. We want to estimate the distribution of this proportion. To that end, we take n iid cells and submit them to *StatGen*. We model the output of *StatGen* as n random variables $X_1, \ldots, X_n$ that have uniform distribution on $[0, \theta]$ for some unknown theta.

- (•) θ identifiable
- ( ) θ not identifiable

✔

3. The US Census Bureau is interested in finding out the average commute time of Bostonians. To that end, it randomly selects n individuals, with replacement, among the people who work and live in the Boston area, and asks to each if their commute time is at least 20 minutes. The commute time of a random person is assumed to follow an exponential distribution with parameter λ.

- (•) λ identifiable
- ( ) λ not identifiable

✔

4. Willy Wonka's contains 67 identical machines. Each machine has a lifetime that is modeled as an exponential random variable with some unknown parameter λ. After a certain time $T = 500$ days, one has observed the lifetimes of all machines that have stopped working before T. The parameter of interest is λ.

- (•) λ identifiable
- ( ) λ not identifiable

✔

Submit You have used 1 of 1 attempt

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Discussion

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Topic: Unit 2 Foundation of Inference:Homework 1: Estimation, Confidence Interval, Modes of Convergence / 1. Statistical Models and Identifiability

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- [STAFF] question b.4 / lifetimes vs proportion of machines still running — 4
- B3 confusing — The Census Bureau is interested in **the average commutetime** which is **continuous**. However, it asks to each **if their commute time is at least 20 minutes** which i... — 5
- [STAFF] B4: please explain part of the solution — 1
- (b)4. Finite observations — 1
- (b) 4 other possible ways to resolve this? — 1
- screenshots — Lectures are currently missing, but in case you know their content already and don't need this course anyways, screenshots for this set of exercises are located here: https://d... — 11
- B3 and B4 - Infinite n, extreme cases? — I haven't submitted this yet and am hoping to talk through with others. So I don't know if any of the below reasoning is correct. In B3, I can't figure out if we should assume w... — 14
- Homework Extension request — hello, I hope this post finds everyone well and safe. normally I wouldn't posit such a question for a class at any level, let alone the masters level, however recent events have g... — 9
- b(1) Observing sign of Gaussian random variable with unknown (mu, sigma^2) — I approached this problem in this way. Domain of the random variable is (-infinity, +infinity) Observation that the Gaussian random variable is (+) or (-) can de defined by a ne... — 1
- b) 2 What is the input to StatGen? — I don't understand.... Are there multiple genes in one cell? That is, once a cell is selected at random, does that imply only one gene selected? What does it mean a "random pr... — 9
- Strange Questions — I love MIT courses, I only have this one left in MicroMasters, but I don't understand the point of some questions, in here the b(3)/b(4) in particular. These are very tricky questi... — 7
- STAFF: May I dispute an answer/solution? — Please let me know how to do so. — 4
- (μ,σ^2) interpretation — Hello, I have a doubt, so to be sure, when asked if (μ,σ^2) identifiable. Does it mean: - μ is identifiable and σ^2 is identifiable, or - the value of a function of (μ and σ^2) is identifiabl... — 5
- Explanation of answers to (b) — It would be useful to have an explanation of the answers, so I can understand why the answer I gave was incorrect. — 4

2. Biased and unbiased estimation for variance of Bernoulli variables

Bookmark this page

(a)

2/2 points (graded)

Let $X_1, \ldots, X_n$ be i.i.d. Bernoulli random variables, with unknown parameter $p \in (0, 1)$. The aim of this exercise is to estimate the common variance of the X_i.

First, recall what $\mathsf{Var}(X_i)$ is for Bernoulli random variables.

$\mathsf{Var}(X_i) =$ p*(1-p)

$p \cdot (1 - p)$

Let $\overline{X}_n$ be the sample average of the X_i,

$$\overline{X}_n = \frac{1}{n} \sum_{i=1}^{n} X_i.$$

We are interested in finding an estimator for $\mathsf{Var}(X_i)$, and propose to use

$$\hat{V} = \overline{X}_n (1 - \overline{X}_n).$$

Check the correct statement that applies to $\hat{V}$:

- $\hat{V}$ is not consistent because $\mathsf{Var}(X_i)$ is not linear in p
- **$\hat{V}$ is consistent because of the Law of Large Numbers and Continuous Mapping Theorem**
- $\hat{V}$ is consistent because of the Central Limit Theorem

STANDARD NOTATION

Submit You have used 1 of 4 attempts

Show Answer

(b)

1/2 points (graded)

Now, we are interested in the bias of $\hat{V}$. Compute:

$\mathbb{E}[\hat{V}] - \mathsf{Var}(X_i) =$ 1/n*p*(p-1)

$\frac{1}{n} \cdot p \cdot (p - 1)$

Using this, find an unbiased estimator $\hat{V}'$ for $p(1 - p)$ if $n \geq 2$.

Write barX_n for $\overline{X}_n$.

$\hat{V}' =$ n*barX_n

$n \cdot \overline{X}_n$

STANDARD NOTATION

Submit You have used 3 of 4 attempts

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Discussion

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Topic: Unit 2 Foundation of Inference:Homework 1: Estimation, Confidence Interval, Modes of Convergence / 2. Biased and unbiased estimation for variance of Bernoulli variables

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- Continuous Mapping Theorem in the lecture — 3
  We did not covered this is the lecture and it was not contemplated in the probability course either. Can some one give me some references? Thank you
- [STAFF] Question 2 (b): Alternative answer possible? — 9
  Hi, Why is it that $\hat{V}' = (n/n-1) * X_n(1-X_n)$ and not $V'' = (1+1/n) * X_n(1-X_n)$. I'm confused, I thought the bias should be 0 and that E[V] - bias = V would give us the unbiased estimat...
- [STAFF] Any explanation why the index n in the second part of 2b is the why it is? — 4
  Performing the calculation I would get the same formula with the index incremented by one, which derives from adding the bias. The correct answer has a different index, bu...
- Second part of B — 1
  Would it be posibble to elaborate a bit more about the second part of B? I have not seen this in the course and it seems that it will be useful in the future.
- Hint for part 2 — 23
  If you are stuck in part 2 and cannot understand how to get the result, watch Lec.20 Video 12 from the Probability - The science of Uncertainty and Data course. I guess this wi...
- Hint B part 2? — 6
  I have calculated B part 1, and gotten it correct. However, I'm unsure how to convert this into an unbiased estimator. My attempts to do so have things like Var(Xi) or (E[Xn])^2...
- Variance Estimator Expectation — 4
- Kindly request — 1
- Why do we compute bias of V' that way — 9
  According to the definition, bias is E[barX_n] - var(barX_n), however, when computing bias we are asked to calculate E[barX_n] - var(X_i), which is wrong from my understandin...
- [Staff] Show Answer in HW1 Questions 2-a & 2-b — 7
  Would it be possible to activate the "show answer" option, please?
  Community TA
- [STAFF] Getting error: "Could not parse '...' as a formula" while answering 2(b) — 3

3. Consistency

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Quantifying Consistency (optional)

0 points possible (ungraded)

Let $X_1, \ldots, X_n \overset{i.i.d.}{\sim} \text{Ber(p)}$ and let $\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$ be an estimator p.

What is the smallest exponent c such that $n^c\left(\overline{X}_n - p\right)$ does **not** converge to 0 almost surely as $n \to \infty$?

$c =$ 1

STANDARD NOTATION

Submit You have used 2 of 4 attempts

Show Answer

Discussion

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Topic: Unit 2 Foundation of Inference:Homework 1: Estimation, Confidence Interval, Modes of Convergence / 3. Consistency

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? Not sure I completely follow the approximation step in the second equation of the solution
From CLT I follow the pivoted statistic converges in distribution to N(0,1). How does this imply the approximation in the second equation where we swap out the pivoted rand... 1

Tip for this question
if a random sequence converges with probability 1, it will converge to the mean and all other higher moments (Variance, skewness, kurtosis) will converge to 0, that is to say, i... 3

? Reading Material
Can you suggest some reading material for the topic of "Consistency"? Also how important is this topic for applications of Statistics? 3

? Is this definitely asking about convergence almost surely (rather than in probability)
Almost the exact question came up in Lecture 4. Have tried to answer as stated (convergence almost surely) and got the answer wrong. Wanted to double check before using ... 4

Answer not shown after failing
Hi, This exercise is not showing the answer after using up all attempts. I would like to know how the correct answer is reached. 3

[STAFF] 0 point questions number of tries
I understand the question was made to have 0 weight (i.e. no grade awarded). Is it possible to increase the number of attempts for such 0-point questions? If it's not worth an... 1

4. Estimation of an exponential parameter

Bookmark this page

(a)

0/1 point (graded)

Let $X_1, \ldots, X_n$ be i.i.d. Exp(λ) random variables, where λ is unknown.

What is the distribution of $\min_{1 \le i \le n} (X_i)$? Enter the pdf $f_{\min}(x)$ of $\min_i (X_i)$ in terms of x.

$f_{\min}(x)$ e^(-n*lambda*x)

$e^{-n \cdot \lambda \cdot x}$

STANDARD NOTATION

Submit You have used 3 of 4 attempts

Show Answer

(b)

1/1 point (graded)

Use the previous question to give an **unbiased** estimator $\hat{\theta}$ for $1/\lambda$.
(Enter `min`, with no subscripts, for the expression $\min_i (X_i)$.

$\hat{\theta} =$ n*min

$n \cdot min$

STANDARD NOTATION

Submit You have used 1 of 4 attempts

Show Answer

(c)

2/2 points (graded)

What is the variance and quadratic risk of the unbiased estimator $\hat{\theta}$ in the previous part?

Var $(\hat{\theta}) =$ 1/lambda^2

$\frac{1}{\lambda^2}$

Quadratic risk of $\hat{\theta}$: 1/lambda^2

$\frac{1}{\lambda^2}$

STANDARD NOTATION

Submit You have used 2 of 4 attempts

Show Answer

(d)

3/3 points (graded)

Compute $\mathbf{P}\left(\frac{1}{\lambda} \ge \frac{n \min_i X_i}{\ln(5)}\right)$.

$\mathbf{P}\left(\frac{1}{\lambda} \ge \frac{n \min_i X_i}{\ln(5)}\right) =$ 0.8

This computation allows us to compute a confidence interval. The interpretation is as follows:
Let α be a value such that $1 - \alpha = \mathbf{P}\left(\frac{1}{\lambda} \le \frac{n \min_i (X_i)}{\ln(5)}\right)$. (This value depends on the answer you just computed.)

Based on this setup, the corresponding, non-asymptotic, one-sided confidence interval at level $1 - \alpha$ for $1/\lambda$ is:
(Type `min` for $\min(X_i)$.)
(Note the confidence interval is finite.)
Note: The value of α is unusually large ($\alpha > 0.5$) in this problem. Please do not worry and proceed with the question as written.

[0 , (n*min)/ln(5)]

0

$\frac{n \cdot min}{\ln(5)}$

STANDARD NOTATION

Submit You have used 1 of 4 attempts

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Topic: Unit 2 Foundation of Inference:Homework 1: Estimation, Confidence Interval, Modes of Convergence / 4. Estimation of an exponential parameter

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- 4 a) what does t mean — hi, in the given solution. what does t in p(min(x)<t) mean? — 1
- Very confused about part (c) — 3
- Value of α — 1
- philosophical question deriving from part a) — Here we compute everything deriving from: P(min < t) = 1 - P(min>t) =,...= exp(-lambda*n*t) and I agree with this. But what if I compute directly: P(min < t) = ... = (1 - exp(-lamb... — 1
- [Staff] Important; regarding incorrect grading — The answer to question 4 part a) n*lambda*e^(-n*lambda*x). The answer that I entered as of today is (-n*lambda*exp(-x*n*lambda)) Now, I don't know why this happened o... — 2
- any hint for a & b? — I still don't get this part, any materials that i can learn further? thank you very much. — 30
- On my last Attempt 4(d) upper interval — I have one last attempt and I wanted to make it count. I got the PDF computation right as well as the lower limit. I think I am close, I know that the one sided confidence interv... — 3
- part d - need some hint — I am not able to understand why the answer to confidence interval not dependent on alpha. Till now in lecture it was always dependent on value at alpha. I am not sure what ... — 20
- some last minute tipp — good teaching material: https://www2.stat.duke.edu/courses/Spring12/sta104.1/Lectures/Lec15.pdf — 2
- Part a — 3
- Hint for solving part d — I struggled quite a bit on solving the upper bound of the confidence interval and finally found out that the question is much simpler than what I expected. There are no compl... — 5
- What does this question ask y f min(X)?part a — Hi, I have problem in understanding question part a. I am not familiar with this question type and I don't remember it from the lectures. Can you please elaborate? — 1
- part d - help!! — I am unable to get the right answer on the probability and the upper bound. I got the lower bound correct. To solve this problem, I converted to standard normal to arrive up... — 1
- Question b — I think the expectation of the x is 1/lambda, and of course, the expectation of the minimum of X should also be 1/lambda, so I think the answer of the question b should be mi... — 5

Previous Next

5. A confidence interval for Poisson variables

Bookmark this page

(a)

2/2 points (graded)

Let $X_1, \ldots, X_n$ be i.i.d. Poisson random variables with parameter $\lambda > 0$ and denote by $\overline{X}_n$ their empirical average,

$$\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i.$$

Find two sequences $(a_n)_{n\geq 1}$ and $(b_n)_{n\geq 1}$ such that $a_n(\overline{X}_n - b_n)$ converges in distribution to a standard Gaussian random variable $Z \sim N(0,1)$.

$a_n =$ sqrt(n/lambda) ✔

$\sqrt{\frac{n}{\lambda}}$

$b_n =$ lambda ✔

λ

STANDARD NOTATION

Submit You have used 3 of 4 attempts

Show Answer

(b)

1.0/1 point (graded)

Secondly, express $\mathbf{P}(|Z| \leq t)$ in terms of $\Phi(r) = \mathbf{P}(Z \leq r)$ for $t > 0$.

Write `Phi(t)` (with capital `P`) for $\Phi(t)$.

$\mathbf{P}(|Z| \leq t) =$ 2*Phi(t)-1 ✔

$2 \cdot \Phi(t) - 1$

STANDARD NOTATION

Submit You have used 1 of 4 attempts

Show Answer

(c)

2/2 points (graded)

Using the previous questions, find an interval I_λ that **depends on** λ and that is centered around $\overline{X}_n$ such that

$$\mathbf{P}[I_\lambda \ni \lambda] \to .95, \quad n \to \infty.$$

(In other words, the interval before applying any of the 3 methods.)

(Write barX_n for $\overline{X}_n$.)

(*Hint*: The 97.5%-quantile of the standard Gaussian distribution is 1.96.)

$I_\lambda = [A, B]$ for

$A =$ barX_n-1.96*sqrt(lambda/r ✔

$\overline{X}_n - 1.96 \cdot \sqrt{\frac{\lambda}{n}}$

$B =$ barX_n+1.96*sqrt(lambda/ ✔

$\overline{X}_n + 1.96 \cdot \sqrt{\frac{\lambda}{n}}$

STANDARD NOTATION

Submit You have used 1 of 4 attempts

Show Answer

(d)

1/1 point (graded)

Which of the following is a confidence interval $\mathcal{J}$ that fulfills

$$\mathbf{P}[\mathcal{J} \ni \lambda] \to .95, \quad n \to \infty.$$

(Choose all that apply.)

- [ ] $\mathcal{J} = [\overline{X}_n - 1.96\sqrt{\lambda/n},\ \overline{X}_n + 1.96\sqrt{\lambda/n}]$
- [ ] $\mathcal{J} = [\overline{X}_n - 1.96\sqrt{\overline{X}_n/n^2},\ \overline{X}_n + 1.96\sqrt{\overline{X}_n/n^2}]$
- [x] $\mathcal{J} = [\overline{X}_n - 1.96\sqrt{\overline{X}_n/n},\ \overline{X}_n + 1.96\sqrt{\overline{X}_n/n}]$
- [ ] $\mathcal{J} = [\overline{X}_n - 1.96\sqrt{100/n},\ \overline{X}_n + 1.96\sqrt{100/n}]$

✔

Submit You have used 2 of 2 attempts

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Topic: Unit 2 Foundation of Inference:Homework 1: Estimation, Confidence Interval, Modes of Convergence / 5. A confidence interval for Poisson variables

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- Low Standard compared to probability course — The standard just dropped from a '10' to '3'. I miss 'The Professor' already. — 5
- Part d- feedback requested — Hi Can someone advise how is part (d) above different from part(c)? I got part(d) wrong - didn't notice it had only 2 tries! Despite this, I think this was an interesting problem. C... — 5
- Deadline for submitting the Homework — I got email saying that the deadline for submitting the Homework was extended by 24 hrs but i cannot submit my homework now. Please help. — 1
- Perhaps it is time for a need for evaluation, of some sort lol. Seriously. — Perhaps it is time for a need for evaluation, of some sort lol. Seriously. — 1
- Can anyone give me a hint on letter (b)? — 3
- 5.c doesn't seem to make sense — 7
- 5.d) Is the grader correct? — What I think is the right answer is marked incorrect. My second best guess is also incorrect. I would love to read the answer because I am clueless. — Community TA — 2 new — 9
- This was a vert good review exercise — Big thank you for the guys who framed it. Cheers. — 4
- [STAFF] Notation and format for part (a) — I'm baffled by the notation for part (a). It would seem that a sequence is meant to indicate that it's formulated in terms of the *i* subscript, but then I'm being asked for the v... — 12
- Part d question — In part d, is J also an interval before applying any of the 3 methods, or a proper CI? — 5
- I am stuck at part a, Any hint.. — I am stuck at part a, Any hint.. I have read all discussion, nothing helps me here.. I know Lim(n->inf) shoud converge to zero.. — 12
- [Staff] 5(b) disappeared — Could not format HTML for problem. Contact course staff in the discussion forum for assistance. — 3
- Error in Part b — The question says to express in terms of [mathjaxinline}\Phi(r)[/mathjaxinline] but it tells us to use [mathjaxinline}\Phi(t)[/mathjaxinline]. I ended up getting it accepted using [... — 2

6. A confidence interval for uniform distributions

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(a)

2.0/2 points (graded)

Let $X_1, \ldots, X_n$ be i.i.d. uniform random variables in $[0, \theta]$, for some $\theta > 0$. Denote by

$$M_n = \max_{i=1,\ldots,n} X_i.$$

Compute the following probabilities:

$\mathbf{P}(M_n \geq \theta) =$ 0 ✔

0

For all $0 \leq t \leq \theta$:

$\mathbf{P}(M_n \leq \theta - t) =$ (1-t/theta)^n ✔

$\left(1 - \frac{t}{\theta}\right)^n$

(*Food for thought:* What can you conclude?)

STANDARD NOTATION

Submit You have used 3 of 4 attempts

Show Answer

(b)

1.0/2 points (graded)

Compute the cumulative distribution function $F_n(t)$ of $n(1 - M_n/\theta)$ for fixed $t \in [0, n]$ and any positive integer n.

$F_n(t) =$ 1-(1-t/n)^n ✔

$1 - \left(1 - \frac{t}{n}\right)^n$

Compute the following limit.

$\lim_{n\to\infty} F_n(t) =$ 1 ✘

1

(*Food for thought:* Again, What can you conclude?)

STANDARD NOTATION

Submit You have used 2 of 4 attempts

Show Answer

(c)

2/2 points (graded)

Next, we will use the previous question to find an interval $\mathcal{I}$ of the form $\mathcal{I} = [M_n, M_n + c]$, that does not depend on θ and such that

$$\mathbf{P}[\mathcal{I} \ni \theta] \to .95, \text{ as } n \to \infty.$$

The strategy now is to use a plug-in estimator for θ to replace it in the expression for c. Parts (a) and (b) suggest that we use c of the form $\left(\frac{t}{n}\right) M_n$, where t ought to equal a certain value in order for $\mathbf{P}[\mathcal{I} \ni \theta] \to .95$. What is the appropriate numerical value of t?

$t =$ ln(20) ✔

Why can we use a plugin-estimator for the asymptotic confidence interval?

- By the Delta Method, the asymptotic variance scales with the square of the first derivative of the plugin function.
- By Slutsky's Theorem, we can combine convergence in distribution of Y_n and in probability of Z_n if Z_n converges to a constant. ✔
- By the Central Limit Theorem, the plugin variable will again be normally distributed.

✔

STANDARD NOTATION

Submit You have used 1 of 2 attempts

Show Answer

(d)

1/1 point (graded)

Compute the bias of M_n as an estimator of θ.

$\mathbb{E}[M_n] - \theta =$ -theta/(n+1) ✔

$-\frac{\theta}{n+1}$

STANDARD NOTATION

Submit You have used 3 of 4 attempts

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Topic: Unit 2 Foundation of Inference:Homework 1: Estimation, Confidence Interval, Modes of Convergence / 6. A confidence interval for uniform distributions

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- Part c - still uncertain how we can conclude that 1 - e^(-t) = .95 — Part c - still uncertain how we can conclude that 1 - e^(-t) = .95, which seems to be the basis for deriving the solution to this problem. Does anyone have any insight you could ... — 1
- So confused by the log() in part C's answer — I'm just wondering, why didn't we use **ln** instead of **log**? So confusing since we learned that log by default is based on 2. Also, any one who worked out part C in a bet... — 2
- Part b explanation of terminology used ... "...atom..." — Hello, Could somebody explain to me what the below statement, quoted from the provided solution of part b, means? "Together with the fact that the cdf of Mn does not hav... — 1
- Part (c) - what *can* I conclude? — 28
- [STAFF] Part b - grader correct? — Dear staff, For part (b) I think I had the correct answer, but entered e^ instead of exp. The grader then marked it wrong, and so I removed the "1-" on my last attempt and end... — 1
- delta method not covered in unit 4-5 — Can you explain why this matter is anticipated in this exam? — 1
- Part b: Like others, I don't understand the setup, and not because I don't understand what a CDF is — From the probability class I just finished, I understand that the CDF is the probability that n*(1-(M_n/theta)) is less than or equal to t. I don't understand how to calculate this w... — 6
- Part A can't be seen — Hi, I can see all of the problem except part A, which says "Could not format HTML for problem. Contact course staff in the discussion forum for assistance". What can I do to fi... — 5
- What I am missing here? — I've read all my notes from lectures 3 and 4, did the recitation exercises, studied the relevant subjects from the textbook all of statistics and still can't get why I'm doing the ite... — 4
- PArt b: Why is t in [0,n] instead of [0,theta]? — And with t no longer in [0,theta], as in part a, what notation do we use for the little m upper bound on the Mn value in the CDF of Mn? — 1
- Part d) — I think the grader isn't fed with the right answer (in part d)), or am I wrong? — 12
- Infinite Notation — Hi, what i should type for infinite symbol? Thanks. — 3
- Bias of M_n — Does the answer depend only on n? or also θ? — 7
- Use the Recitation

7. Modes of convergence

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Convergence in distribution

4.0/4 points (graded)

Let T_n be a sequence of random variables that converges to $\mathcal{N}(0,1)$ in distribution. What family of distribution does the limit of $2T_n + 1$ belong to?

- χ^2 distribution
- Normal distribution ✔

Call this limit Y. Compute:

$\mathbb{E}[Y] =$ 1 ✔

$\mathrm{Var}[Y] =$ 4 ✔

Let Φ be the cumulative distribution function (cdf) of the standard Gaussian distribution. In terms of Φ, what is the limit, as $n \to \infty$, of $\mathbf{P}(|T_n + 2| \leq 8)$?

(Write `Phi`, with capital `P`, for Φ).

Phi(6)-Phi(-10) ✔

$\Phi(6) - \Phi(-10)$

STANDARD NOTATION

Submit You have used 3 of 4 attempts

Show Answer

Convergence in probability and variance

3/3 points (graded)

For $n \geq 2$, let X_n be a random variable such that $\mathbf{P}\left(X_n = \frac{1}{n}\right) = 1 - \frac{1}{n^2}$ and $\mathbf{P}(X_n = n) = \frac{1}{n^2}$.

Does X_n converge in probability? If yes, enter the value of the limit; if no, enter `DNE`.

$X_n \xrightarrow[n\to\infty]{\mathbf{P}}$ 0 ✔

0

Compute $\lim_{n\to\infty} \mathbb{E}[X_n]$ and $\lim_{n\to\infty} \mathrm{Var}(X_n)$. Enter `DNE` if the limit diverges or does not exist.

$\lim_{n\to\infty} \mathbb{E}[X_n] =$ 0 ✔

0

$\lim_{n\to\infty} \mathrm{Var}(X_n) =$ 1 ✔

1

STANDARD NOTATION

Submit You have used 3 of 4 attempts

Show Answer

Modes of convergence

3/3 points (graded)

Let X_n and Y_n be two sequences of random variables. For each of the following statement, say whether it is true or false. When your answer is "false", try to think of a counter example.

1. If $X_n \xrightarrow[n\to\infty]{\text{a.s.}} X$ and $Y_n \xrightarrow[n\to\infty]{\text{a.s.}} Y$, then $X_n + Y_n \xrightarrow[n\to\infty]{\text{a.s.}} X + Y$.
   - True ✔
   - False
2. If $X_n \xrightarrow[n\to\infty]{\mathbf{P}} X$ and $Y_n \xrightarrow[n\to\infty]{\mathbf{P}} Y$, then $X_n + Y_n \xrightarrow[n\to\infty]{\mathbf{P}} X + Y$.
   - True ✔
   - False
3. If $X_n \xrightarrow[n\to\infty]{(\mathrm{d})} X$ and $Y_n \xrightarrow[n\to\infty]{(\mathrm{d})} Y$, then $X_n + Y_n \xrightarrow[n\to\infty]{(\mathrm{d})} X + Y$.
   - True
   - False ✔

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Topic: Unit 2 Foundation of Inference:Homework 1: Estimation, Confidence Interval, Modes of Convergence / 7. Modes of convergence

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- Convergence in Probability and Variance is Discrete!!! — 7
- Be careful with comparing last question with recitation 2! — I guess I didn't understand one of the videos of recitation 2 correctly so I changed my mind from correct answer I chose intuitively to wrong ones. Pay attention, rewatch vide... — 1
- The first part (worth 4 points) in Q7 is not rendering — I entered the solution but get this message "Could not format HTML for problem. Contact course staff in the discussion forum for assistance." Could you please advise? — 2
- Strugling to understand the "Convergence in probability and variance" question — The definition of convergence in probability is the probability of the distance between X_n and X being greater than a small epsilon, goes to zero as n goes to infinity. My ques... — 9
- format, question 1 Phi(c), can c be negative? — It is not taking my negative c answer...; and please add note to know if it only takes positive values if needed — 3
- About Modes of convergence questions — Are the random variables i.i.d.? Are X and Y independent? We haven't really talked about any other types in this unit, but I shouldn't really assume. — 2
- Question about the last item of the first exercise — 2
- Will the solution for homework be release after the due date? — As the title asks — 2
- Links for Lecture 3 and 4 — I don't see the links for lecture 3 and 4. There is only one link for the homework — 3

8. Some examples of convergence

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Rescaled Poisson random variables

2/2 points (graded)

For $n \geq 1$, let X_n be a Poisson random variable with parameter $1/n$. Compute

$\mathbf{P}(X_n = 0) =$ exp(-1/n)

$\exp\left(-\frac{1}{n}\right)$

What can you conclude?

- $X_n \to 0$ in probability, but nX_n does not converge in probability
- $X_n \to 0$ in probability, $nX_n \to 0$ in probability, and $\mathbb{E}[(nX_n)^2]$ converges.
- $X_n \to 0$ and $nX_n \to 0$ in probability, but $\mathbb{E}[(nX_n)^2]$ does not converge.

STANDARD NOTATION

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Limit of rescaled Binomials

1.0/1 point (graded)

Let X_n be a binomial random variable with parameters n and $p = \lambda/n$, where λ is a fixed positive number.

Let $k \in \mathbb{N}$ be fixed. As $n \to \infty$, the probability mass function $\mathbf{P}(X_n = k)$ converges to a number that only depends on λ and k. What is the limit?

(If necessary, enter **fact** to indicate the factorial function. For instance, **fact(10)** denotes $10!$. Note that **fact(10)** may not be rendered correctly by the parser, but do not worry, the grader will work independently. If you want proper rendering, enclose the factorial by extra parentheses, i.e. **(fact(10))**.

$\lim_{n\to\infty} \mathbf{P}(X_n = k) =$ lambda^k/(fact(k))*e^(-lambda)

$\frac{\lambda^k}{fact(k)} \cdot e^{-\lambda}$

(*Food for thought:* What can you conclude?)

STANDARD NOTATION

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Anyone else finding the homework much harder than the lectures?	I have really struggled with this, despite following the lectures easily enough, and getting nearly full marks in the other 3 modules. Is anyone else finding this? It feels like there...	54 new
Limit of rescaled Binomials (Food for thought: What can you conclude?)	I solved the problem by WolframAlpha, but I failed to understand its means. Since it's more important to understand what the answer means than just solve it, could someon...	5
Food for thought	Binomial and poison distribution are related.	2
Question about the notation in #1		2
Limit of Rescaled Binomial: Hint	Look at Mitx: 6.431x (Probability - The Science of data and uncertainty). Unit 9, Lecture 21: The Bernoulli Process. The very last video in the series contains a Hint ;-)...	12
Not showing answers till due date.	I understand the rational behind it. But as often answers from the previous parts of a question are require to work out the answer for the following parts, this means that for ...	5
Availability of homework	Hi There! Is it possible to download a pdf containing the excercises/homework questions in a bundle? Do you guys provide it?	4
Second question: Answer not depending on lamda ?	I was solving the second problem, and I found a solution of the limit, depending on lamda and k (as the problem statement implies). However, when submitting the answer, t...	3
does grader not recognise e^ on part a?	I tried the same answer once in exp() format and once in e^() format. Only exp() format was recognised. However e^ notation is generally accepted in all other parts of the ed...	22
Unable to submit answer for part "Limit of rescaled Binomials"	I am getting below message everytime I try to submit the answer Invalid Input: Could not parse <xxx> as a formula Is it possible for TA to check my answer. My answer contai...	3
Re: Limit of rescaled Binomials		6
Confusing notation for "Limit of rescaled Binomials"		1
"Rescaled Poisson random variables": can we make a statement as a conclusion of the first computation ?	Once we have computed [mathjaxinline]P(X_n = 0)[/mathjaxinline], we can know its limit but can we really make a statement about convergence of [mathjaxinline]X_n[/mathja...	2
Tehnical problem for the first question	I answered for the first part and was marked as correct but I got the 2nd answer wrong. When correcting for the second answer (without modifying the first) I got an x for the...	1