Lab 1: Circuit analysis methods

Circuit Theory and Electronics Fundamentals
Master's programme in Engineering Physics, Técnico, University of Lisbon

Diogo Costa | 96187
Ana Sousa | 96508
Isabel Alexandre | 96537

March 23, 2021

Contents

1 Introduction

The objective of this laboratory assignment was to study a circuit made up of a two voltage sources (one dependent), two current sources (one dependent) and seven resistors. It can be seen in the Figure **??** below.

In Section **??**, a theoretical analysis of the circuit is presented. It is followed by a simulation of the circuit, in Section **??**, the results of which are compared to the theoretical results in the final section, Section **??**. Here, some considerations are also made about this study, the results obtained and whether the goals were achieved.

In order to conduct the theoretical study, the tool **Octave** was employed. The simulation was run in **ngspice**. In order to numerically access the results, a **Python** *script* was also used.

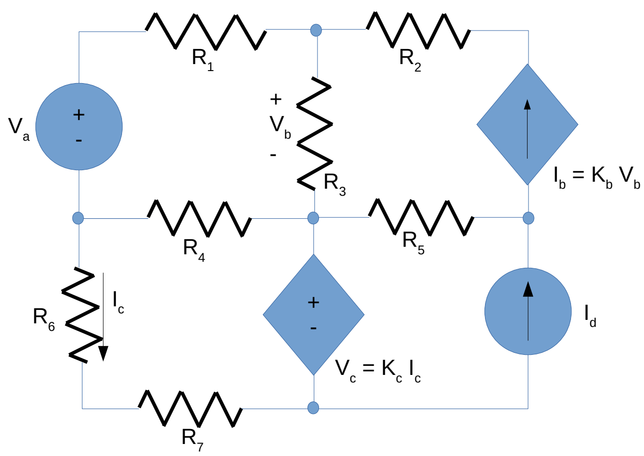

Figure 1: Circuit considered

2 Theoretical Analysis

In this section, the circuit depicted in Figure **??** is analysed with two different methods, so as to determine how it behaves, theoretically, both in terms of current in each branch and potential difference between nodes.

2.1 Mesh analysis method

The first method considered is the Mesh analysis method, considering the four meshes present in the circuit. Each mesh is given a label (α, β, γ and δ) and an arbitrary direction for the current to flow in, as the below figure shows.

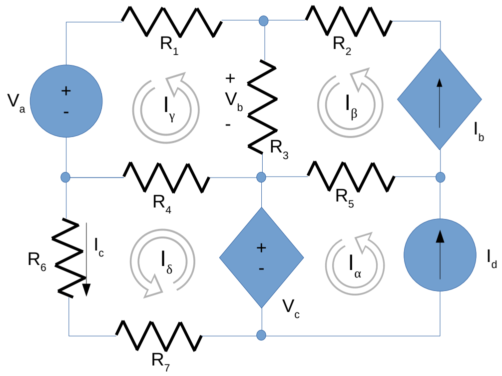

Figure 2: Mesh analysis

By inspection:

$$\begin{cases} I_\alpha = I_d \\ I_\beta = I_b \end{cases} \tag{1}$$

Applying KVL to the two remaining meshes:

$$\begin{cases} V_a + R_4(I_\gamma + I_\delta) + R_3(I_\gamma - I_\beta) + R_1 I_\gamma = 0 \\ V_c + R_7 I_\delta + R_6 I_\delta + R_4(I_\delta + I_\gamma) = 0 \end{cases} \tag{2}$$

The conditional sources behave as:

$$\begin{cases} I_b = k_b V_b \\ V_c = k_c I_c = -k_c I_\delta \end{cases} \tag{3}$$

Lastly, by Ohm's Law

$$V_b = R_3(I_\beta - I_\gamma) \tag{4}$$

Manipulating these equations and solving them using **Octave** yields:

Name	**Value**
$I_\alpha\ (A)$	0.001000
$I_\beta\ (A)$	-0.000251
$I_\gamma\ (A)$	-0.000240
$I_\delta\ (A)$	-0.000969
$I_c\ (A)$	0.000969
$I_b\ (A)$	-0.000251
$V_2\ (V)$	5.070727
$V_3\ (V)$	4.825468
$V_4\ (V)$	4.304579
$V_5\ (V)$	4.860091
$V_6\ (V)$	8.720760
$V_7\ (V)$	-2.939898
$V_8\ (V)$	-1.950198
$V_b\ (V)$	-0.034623
$V_c\ (V)$	7.799989
height	

Table 1: Theoretical results

2.2 Node analysis method

The second method used was the Node analysis method. With this method, it was possible to determine the voltages in the 8 nodes of the circuit. In figure **??**, the directions of the currents chosen to do this analysis are represented, as well as the numerical labels that were given to the nodes.

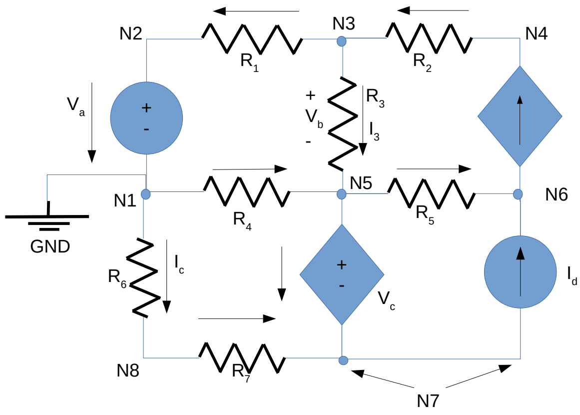

Figure 3: Node analysis

The node N1 was considered the ground, so:

$$\begin{cases} V_1 = 0\ V \\ V_2 = V_a \end{cases} \tag{5}$$

By applying KCL, it is possible to obtain the following equations:

$$\begin{cases} \frac{V_8}{R_6} + \frac{V_5}{R_4} + \frac{V_3 - V_a}{R_1} = 0 \\ -\frac{V_8}{R_6} - \frac{V_8 - V_7}{R_7} = 0 \\ -\frac{V_3 - V_a}{R_1} - \frac{V_3 - V_5}{R_3} + \frac{V_4 - V_3}{R_2} = 0 \\ -\frac{V_4 - V_3}{R_2} + K_b(V_3 - V_5) = 0 \\ -K_b(V_3 - V_5) + \frac{V_5 - V_6}{R_5} + I_d = 0 \end{cases} \tag{6}$$

By analysing the dependent voltage source, it is also possible to conclude that

$$V_5 - V_7 + K_c\frac{V_8}{R_6} = 0 \tag{7}$$

Once again, using **Octave** and the former equations, is possible to determine the voltages in the nodes and the unknown currents and voltages of the circuit:

Name	**Value**
$V_2\ (V)$	5.070727
$V_3\ (V)$	4.825468
$V_4\ (V)$	4.304579
$V_5\ (V)$	4.860091
$V_6\ (V)$	8.720760
$V_7\ (V)$	-2.939898
$V_8\ (V)$	-1.950198
$V_b\ (V)$	-0.034623
$I_b\ (V)$	-0.000251
$I_c\ (V)$	0.000969
$V_c\ (V)$	7.799989
height	

Table 2: Theoretical results

The Mesh analysis and the Node analysis are equivalent so, as expected, they produced the same results.

3 Simulation Analysis

3.1 Operating Point Analysis

After analysing the circuit we proceeded to simulate it using **ngspice** in order to compare the results with the theoretical predictions. The results are shown in Table **??**.

Name	Value
n2 (V)	5.070727e+00
n3 (V)	4.827131e+00
n4 (V)	4.332704e+00
n5 (V)	4.827131e+00
n6 (V)	8.648396e+00
n7 (V)	-2.91996e+00
n8 (V)	-1.93697e+00
Vc (V)	7.747092e+00
r1 (A)	-2.38700e-04
r2 (A)	-2.38700e-04
r3 (A)	1.057915e-11
r4 (A)	-1.20099e-03
r5 (A)	-1.23889e-03
r6 (A)	9.622908e-04
r7 (A)	9.622908e-04
Ib (A)	2.387000e-04

Table 3: Operating point. The 'n' variables represent the potencial difference determined at the nodes, compared to GND. The 'r' variables correspond to the current flowing through each resistor, as depicted in Figure **??**.

4 Conclusion

This laboratory assignment had as the main objective the analysis of the circuit in Figure **??**. That goal was achieved by performing a theoretical analysis, using Octave, and a simulation, using Ngspice tool.

V	V_t	V_s	$\|V_t - V_s\|$	$Error(\%)$[1]
$V_2\ (V)$	5.070727	5.070727	0.0	0
$V_3\ (V)$	4.825468	4.827131	0.001662999999999748	0.04
$V_4\ (V)$	4.304579	4.332704	0.02812499999999929	0.7000000000000001
$V_5\ (V)$	4.860091	4.827131	0.0329600000000001	0.7000000000000001
$V_6\ (V)$	8.72076	8.648396	0.07236400000000032	0.9
$V_7\ (V)$	-2.939898	-2.91996	0.01993799999999979	0.7000000000000001
$V_8\ (V)$	-1.950198	-1.93697	0.01322800000000018	0.7000000000000001
height				

Table 4: Theoretical results and Simulation results

[1]The error presented here is rounded up to one significant digit, for ease of interpretation.

As one can see there's a slight deviation in the values compared with previous calculations. We proceeded to determine, numerically, how significant these deviations are, as shown in Table **??**. The theoretical results (V_t) do not match perfectly the simulation results (V_s). However, the error is quite small, always inferior to 1%.

The circuit is "simple" and contains only linear components, so large deviations were not anticipated. These can be due to any number of causes, including but not limited to:

- floating point arithmetics;
- different numerical precisions in the different tools employed;
- propagated erros made when solving the linear system of equations.

All in all, the results obtained were satisfactory.