Previous perturbed weighting function

$$\boldsymbol{v} + \frac{\varpi h^2}{2\mu}\mathbf{F}^{-T}\nabla q$$

Implemented in the previous code

$$\int_{\Omega_0} \frac{\partial \widetilde{\mathcal{W}}(\mathbf{F}, p, \alpha)}{\partial \mathbf{F}} : \nabla \otimes \boldsymbol{v}\, dV - \int_{\partial_N \Omega_0} \boldsymbol{g}_0 \cdot \boldsymbol{v}\, dA = 0,$$

$$\int_{\Omega_0} \left(-\sqrt{b(\alpha)}\,(J-1) - \frac{p}{\kappa}\right) q\, dV - \frac{\varpi h^2}{2\mu} \sum_{e=1}^{n_{el}} \int_{\Omega_0^e} \underline{\sqrt{b(\alpha)} J \mathbf{C}^{-1} : [\nabla p \otimes \nabla q]}\, dV = 0,$$

NOT same as Eq. (25)

$$\int_{\Omega_0} \frac{\partial \widetilde{\mathcal{W}}(\mathbf{F}, p, \alpha)}{\partial \alpha} \beta\, dV + \frac{\mathcal{G}_c}{c_w \ell} \int_{\Omega_0} \left(\frac{\partial w(\alpha)}{\partial \alpha}\beta + \ell^2 \nabla\alpha \cdot \nabla\beta\right) dV \geq 0.$$

Current perturbed weighting function

$$\boldsymbol{v}+\frac{\varpi h^2}{2\mu}\sqrt{b(\alpha)}\mathbf{F}^{-T}\nabla q$$

$$\int_{\Omega_0}\frac{\partial\widetilde{\mathcal{W}}(\mathbf{F},p,\alpha)}{\partial\mathbf{F}}:\nabla\otimes\boldsymbol{v}\,dV-\int_{\partial_N\Omega_0}\boldsymbol{g}_0\cdot\boldsymbol{v}\,dA=0,$$

$$\int_{\Omega_0}\left(-\sqrt{b(\alpha)}\,(J-1)-\frac{p}{\kappa}\right)q\,dV-\frac{\varpi h^2}{2\mu}\sum_{e=1}^{n_{el}}\int_{\Omega_0^e}\sqrt{b(\alpha)}J\mathbf{C}^{-1}:\left[\nabla\left(\sqrt{b(\alpha)}p\right)\otimes\nabla q\right]\,dV$$

$$+\frac{\varpi h^2}{2\mu}\sum_{e=1}^{n_{el}}\int_{\Omega_0^e}\left[\frac{\partial\mathcal{W}\left(\mathbf{F}\right)}{\partial\mathbf{F}}\nabla a(\alpha)\right]\cdot\left(\sqrt{b(\alpha)}\mathbf{F}^{-\mathbf{T}}\nabla q\right)\,dV=0,$$

$$\int_{\Omega_0}\frac{\partial\widetilde{\mathcal{W}}(\mathbf{F},p,\alpha)}{\partial\alpha}\beta\,dV+\frac{\mathcal{G}_c}{c_w\ell}\int_{\Omega_0}\left(\frac{\partial w(\alpha)}{\partial\alpha}\beta+\ell^2\nabla\alpha\cdot\nabla\beta\right)\,dV\geq 0.$$