ANA U8

4.) $x \in \mathbb{R}$ $\sum_{n=1}^{\infty} \frac{3x^n}{2+x^{4n}}$ für welche x konvergent?

Quotientenkriterium:

$$\frac{\left|\frac{3x^{n+1}}{2+x^{4(n+1)}}\right|}{\left|\frac{3x^n}{2+x^{4n}}\right|} = \frac{\frac{3\cdot|x|^{n+1}}{2+x^{4(n+1)}}}{\frac{3\cdot|x|^n}{2+x^{4n}}} = \frac{3\cdot|x|^{n+1}\cdot(2+x^{4n})}{3\cdot|x|^n\cdot(2+x^{4(n+1)})} = \frac{|x|\cdot(2+x^{4n})}{2+x^{4n+4}}$$

- Fallunterscheidung: $x>1$

$$\frac{|x|\cdot(2+x^{4n})}{2+x^{4n+4}} = \frac{2x+x^{4n+1}}{2+x^{4n+4}} = \frac{\frac{2}{x^{4n+3}}+\frac{1}{x^3}}{\frac{2}{x^{4n+4}}+1}$$

$$\lim_{n\to\infty} \frac{\frac{2}{x^{4n+3}}+\frac{1}{x^3}}{\frac{2}{x^{4n+4}}+1} = \frac{\frac{1}{x^3}}{1} = \frac{1}{x^3}$$

$\Rightarrow$ bei $x>1$ konvergent

- 2. Fall $x<-1$

$$\frac{-x\cdot(2+x^{4n})}{2+x^{4n+4}} = \frac{-2x-x^{4n+1}}{2+x^{4n+4}} = \frac{\frac{-2}{x^{4n+3}}-\frac{1}{x^3}}{\frac{2}{x^{4n+4}}+1}$$

$$\lim_{n\to\infty} \frac{\frac{2}{x^{4n+3}}-\frac{1}{x^3}}{\frac{2}{x^{4n+4}}+1} = \frac{-\frac{1}{x^3}}{1} = -\frac{1}{x^3}$$

$\Rightarrow$ bei $x<-1$ konvergent

- 3. Fall $-1<x<1$

$$\lim_{n\to\infty} \frac{2x\pm x^{4n+1}}{2+x^{4n+4}} = \frac{2x}{2} = x < 1$$

$\Rightarrow$ bei $-1<x<1$ konvergent

- 4. Fall $|x|=1$

$$\frac{1\cdot(2+1^{4n})}{2+1^{4n+4}} = \frac{2+1}{2+1} = 1$$

$\Rightarrow$ bei $|x|=1$ divergent

□