ANA Ü8

5.) $\sum_{n=1}^{\infty} \frac{1}{n}\left(e-\left(1+\frac{1}{n}\right)^n\right)$ $\qquad$ $\left(1+\frac{1}{n}\right)^n < e < \left(1+\frac{1}{n}\right)^{n+1}$

$$\sum_{n=1}^{\infty} \frac{1}{n}\left(e-\left(1+\frac{1}{n}\right)^n\right) < \sum_{n=1}^{\infty} \frac{1}{n}\left(\left(1+\frac{1}{n}\right)^{n+1} - \left(1+\frac{1}{n}\right)^n\right)$$

$$= \sum_{n=1}^{\infty} \frac{1}{n}\left(\left(1+\frac{1}{n}\right)^n \cdot \left(1+\frac{1}{n}-1\right)\right) = \sum_{n=1}^{\infty} \frac{1}{n} \cdot \frac{1}{n} \cdot \left(1+\frac{1}{n}\right)^n = \sum_{n=1}^{\infty} \frac{1}{n^2} \cdot \left(1+\frac{1}{n}\right)^n$$

$$< \sum_{n=1}^{\infty} \frac{1}{n^2} \cdot e = e \cdot \sum_{n=1}^{\infty} \frac{1}{n^2} = e \cdot \frac{\pi^2}{6}$$ (Basel problem)

$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{n} \cdot \left(e-\left(1+\frac{1}{n}\right)^n\right)$ konvergiert