CAGD Ü2

17) $b_0, b_1, b_2 \in \mathbb{R}^2$ $\qquad b(t) = \sum_{i=0}^{2} B_i^n(t)\, b_i = (1-t)^2 b_0 + 2t(1-t) b_1 + t^2 b_2$

$$= (1-2t+t^2) b_0 + (2t-2t^2) b_1 + t^2 b_2$$

$$\dot{b}(t) = (-2+2t) b_0 + (2-4t) b_1 + (2t) b_2$$

$$v = b_0 - 2b_1 + b_2 \qquad \begin{pmatrix} b_0^x - 2b_1^x + b_2^x \\ b_0^y - 2b_1^y + b_2^y \end{pmatrix}$$

$$0 = \dot{b}(t) \cdot v = \left((-2+2t)b_0^x + (2-4t)b_1^x + 2t b_2^x,\ (-2+2t)b_0^y + (2-4t)b_1^y + 2t b_2^y\right)$$

$$= (-2+2t){b_0^x}^2 - 2(-2+2t)b_0^x b_1^x + (-2+2t)b_0^x b_2^x + (2-4t)b_0^x b_1^x - 2(2-4t){b_1^x}^2 + (2-4t)b_1^x b_2^x + 2t b_0^x b_2^x - 4t b_1^x b_2^x + 2t {b_2^x}^2 +$$

$$(-2+2t){b_0^y}^2 - 2(-2+2t)b_0^y b_1^y + (-2+2t)b_0^y b_2^y + (2-4t)b_0^y b_1^y - 2(2-4t){b_1^y}^2 + (2-4t)b_1^y b_2^y + 2t b_0^y b_2^y - 4t b_1^y b_2^y + 2t {b_2^y}^2$$

$$= 2\left((-1+t)b_0 + (1-2t)b_1 + t b_2\right) \cdot (b_0 - 2b_1 + b_2)$$

$$= 2\left((-1+t)b_0 \cdot b_0 - 2(-1+t)b_0 \cdot b_1 + (-1+t)b_0 b_2 + (1-2t)b_0 \cdot b_1 - 2(1-2t)b_1 \cdot b_1 + (1-2t)b_1 \cdot b_2 + t b_0 b_2 - 2t b_1 b_2 + t b_2 \cdot b_2\right)$$

$$= 2\left(t(b_0 \cdot b_0 - 2b_0 b_1 + b_0 b_2 - 2b_0 b_1 + 4b_1 \cdot b_1 - 2b_1 \cdot b_2 + b_0 b_2 - 2b_1 b_2 + b_2 \cdot b_2) +\right.$$

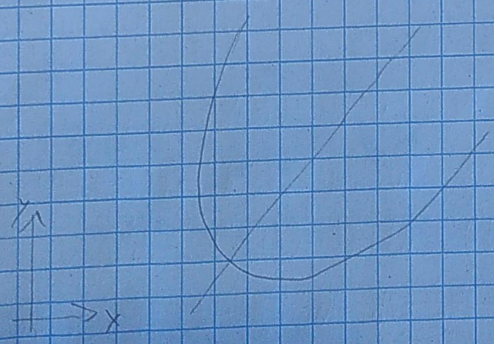
$$\left.(-b_0 \cdot b_0 + 2b_0 b_1 - b_0 b_2 + b_0 b_1 - 2b_1 b_1 + b_1 \cdot b_2)\right) = 0$$

$$= 2\left(t(b_0 \cdot b_0 - 4b_0 \cdot b_1 + 2b_0 \cdot b_2 + 4b_1 \cdot b_1 - 4b_1 \cdot b_2 + b_2 \cdot b_2) + (-b_0 \cdot b_0 + 3b_0 \cdot b_1 - b_0 \cdot b_2 - 2b_1 \cdot b_1 + b_1 \cdot b_2)\right)$$

$$\Rightarrow \quad t = \frac{b_0 \cdot b_0 - 3b_0 \cdot b_1 + b_0 \cdot b_2 + 2b_1 \cdot b_1 - b_1 \cdot b_2}{b_0 \cdot b_0 - 4b_0 \cdot b_1 + 2b_0 \cdot b_2 + 4b_1 \cdot b_1 - 4b_1 \cdot b_2 + b_2 \cdot b_2}$$

$\boxed{?}$

$$\ddot{b}(t) = (b_0 - 2b_1 + b_2)t^2 + (-2b_0 + 2b_1)t + b_0$$



CAGD Ü3

18) $pr: \mathbb{R}^3 \to \mathbb{R}^2 \qquad (x,y,z) \mapsto (x,y)$

$b(t) = \sum_{i=0}^{n} B_i^n(t) b_i \qquad b_0, \dots, b_n \in \mathbb{R}^3$

$pr(b(t)) = \left(\sum_{i=0}^{n} B_i^n(t) b_i^x, \sum_{i=0}^{n} B_i^n(t) b_i^y \right)$

$\hat{b}(t) := \sum_{i=0}^{n} B_i^n(t)\, pr(b_i) = \left(\sum_{i=0}^{n} B_i^n(t) b_i^x, \sum_{i=0}^{n} B_i^n(t) b_i^y \right) = pr(b(t))$

$\square$

CAGD 03

19)

$$b(t) = \sum_{i=0}^{3} B_i^3(t)\, b_i$$

$$b_0^0 = b_0, \quad b_1^0 = b_1, \quad b_2^0 = b_2, \quad b_3^0 = b_3$$

$$b_0^1 = (1-t)\, b_0^0 + t\, b_1^0 = b_0 - t b_0 + t b_1 \qquad b_1^1 = (1-t) b_1^0 + t b_2^0 = b_1 - t b_1 + t b_2$$

$$b_2^1 = (1-t)\, b_2^0 + t\, b_3^0 = b_2 - t b_2 + t b_3$$

$$b_0^2 = (1-t)\, b_0^1 + t\, b_1^1 = b_0 - t b_0 + t b_1 - t b_0 + t^2 b_0 - t^2 b_1 + t b_1 - t^2 b_1 + t^2 b_2 = b_0 - 2t b_0 + 2t b_1 + t^2 b_0 - 2t^2 b_1 + t^2 b_2$$

$$b_1^2 = (1-t)\, b_1^1 + t\, b_2^1 = b_1 - t b_1 + t b_2 - t b_1 + t^2 b_1 - t^2 b_2 + t b_2 - t^2 b_2 + t^2 b_3 = b_1 - 2t b_1 + 2t b_2 + t^2 b_1 - 2t^2 b_2 + t^2 b_3$$

$$b_0^3 = (1-t)\, b_0^2 + t\, b_1^2 = b_0 - 2t b_0 + 2t b_1 + t^2 b_0 - 2t^2 b_1 + t^2 b_2 - t b_0 + 2t^2 b_0 - 2t^2 b_1 - t^3 b_0 + 2t^3 b_1 - t^3 b_2 +$$

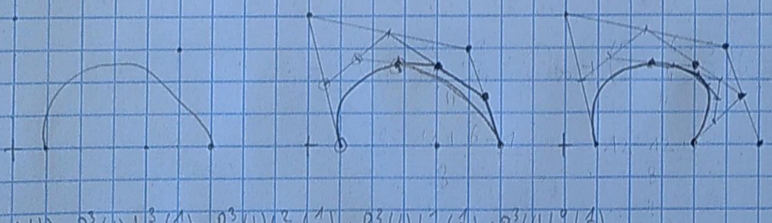
$$+ t b_1 - 2t^2 b_1 + 2t^2 b_2 + t^3 b_1 - 2t^3 b_2 + t^3 b_3$$

$$= b_0 - 3t b_0 + 3t b_1 + 3t^2 b_0 - 6t^2 b_1 + 3t^2 b_2 - t^3 b_0 + 3t^3 b_1 - 3t^3 b_2 + t^3 b_3$$

$$c(t) := b(\tfrac{1}{2} t) = \sum_{i=0}^{3} B_i^3(t)\, b_0^i(\tfrac{1}{2}) \qquad d(t) := b((1-\tfrac{1}{2})t + \tfrac{1}{2}) = b(\tfrac{1}{2}t + \tfrac{1}{2}) = \sum_{i=0}^{3} B_i^3(t)\, b_i^{n-i}(\tfrac{1}{2})$$

$$c(t) = B_0^3(t)\, b_0 + B_1^3(t)\left(\frac{b_0 + b_1}{2}\right) + B_2^3(t)\left(\frac{b_0 + 2b_1 + b_2}{4}\right) + B_3^3(t)\left(\frac{b_0 + 3b_1 - 3b_2 + b_3}{8}\right)$$

$$\left(b_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad b_1 = \begin{pmatrix} 0 \\ 4 \end{pmatrix} \quad b_2 = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \quad b_3 = \begin{pmatrix} 0 \\ 6 \end{pmatrix} \quad b_4 = \begin{pmatrix} 0 \\ 4 \end{pmatrix} \right)$$

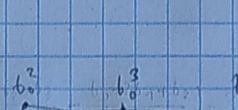


$$d(t) = B_0^3(t)\, b_0^3(\tfrac{1}{2}) + B_1^3(t)\, b_1^2(\tfrac{1}{2}) + B_2^3(t)\, b_2^1(\tfrac{1}{2}) + B_3^3(t)\, b_3^0(\tfrac{1}{2})$$

stetig ? $\qquad c(1) = b(\tfrac{1}{2}) = d(0)$ ✓

stetig diffbar ? $\qquad \dot{c}(1) = 3(b_0^3(\tfrac{1}{2}) - b_0^2(\tfrac{1}{2}))$ ✓

$$\dot{d}(0) = 3(b_1^2(\tfrac{1}{2}) - b_0^3(\tfrac{1}{2}))$$



$$b_0^3(\tfrac{1}{2}) - b_0^2(\tfrac{1}{2}) = b_0 - \tfrac{3}{2} b_0 + \tfrac{3}{2} b_1 + \tfrac{3}{4} b_0 - \tfrac{6}{4} b_1 + \tfrac{3}{4} b_2 - \tfrac{1}{8} b_0 + \tfrac{3}{8} b_1 - \tfrac{3}{8} b_2 + \tfrac{1}{8} b_3 - (b_0 - \tfrac{2}{2} b_0 + \tfrac{2}{2} b_1 + \tfrac{1}{4} b_0 - \tfrac{2}{4} b_1 + \tfrac{1}{4} b_2) =$$

$$= (1 - \tfrac{3}{2} + \tfrac{3}{4} - \tfrac{1}{8} - 1 + \tfrac{2}{2} - \tfrac{1}{4}) b_0 + (\tfrac{3}{2} - \tfrac{6}{4} + \tfrac{3}{8} - \tfrac{2}{2} + \tfrac{2}{4}) b_1 + (\tfrac{3}{4} - \tfrac{3}{8} - \tfrac{1}{4}) b_2 + \tfrac{1}{8} b_3 = -\frac{b_0}{8} - \frac{b_1}{8} + \frac{b_2}{8} + \frac{b_3}{8}$$

$$b_1^2(\tfrac{1}{2}) - b_0^3(\tfrac{1}{2}) = b_1 - \tfrac{2}{2} b_1 + \tfrac{2}{2} b_2 + \tfrac{1}{4} b_1 - \tfrac{2}{4} b_2 + \tfrac{1}{4} b_3 - (b_0 - \tfrac{3}{2} b_0 + \tfrac{3}{2} b_1 + \tfrac{3}{4} b_0 - \tfrac{6}{4} b_1 + \tfrac{3}{4} b_2 - \tfrac{1}{8} b_0 + \tfrac{3}{8} b_1 - \tfrac{3}{8} b_2 + \tfrac{1}{8} b_3) =$$

$$= (-1 + \tfrac{3}{2} - \tfrac{3}{4} + \tfrac{1}{8}) b_0 + (1 - 1 + \tfrac{1}{4} - \tfrac{3}{2} + \tfrac{6}{4} - \tfrac{3}{8}) b_1 + (1 - \tfrac{2}{4} - \tfrac{3}{4} + \tfrac{3}{8}) b_2 + (\tfrac{1}{4} - \tfrac{1}{8}) b_3 = -\frac{b_0}{8} - \frac{b_1}{8} + \frac{b_2}{8} + \frac{b_3}{8}$$

...

CAGD Ü3

20) Basis der Bernsteinpolynome $= \{ B_0^3(t), B_1^3(t), B_2^3(t), B_3^3(t) \}$

$$B_0^3(t) = \binom{3}{0} t^0 (1-t)^3 = (1-t)^3 = 1 - 3t + 3t^2 - t^3$$

$$B_1^3(t) = \binom{3}{1} t^1 (1-t)^2 = 3t(1-t)^2 = 3t - 6t^2 + 3t^3$$

$$B_2^3(t) = \binom{3}{2} t^2 (1-t)^1 = 3t^2(1-t) = 3t^2 - 3t^3$$

$$B_3^3(t) = \binom{3}{3} t^3 (1-t)^0 = t^3$$

$$\Rightarrow t^3 = B_3^3(t) \qquad t^2 = \frac{B_2^3(t)}{3} + B_3^3(t) \qquad t = \frac{B_1^3(t)}{3} + 2\left(\frac{B_2^3(t)}{3} + B_3^3(t)\right) - B_3^3(t) = \frac{B_1^3(t)}{3} + \frac{2}{3}B_2^3(t) + B_3^3(t)$$

$$1 = B_0^3(t) + 3\left(\frac{B_1^3(t)}{3} + \frac{2}{3}B_2^3(t) + B_3^3(t)\right) - 3\left(\frac{B_2^3(t)}{3} + B_3^3(t)\right) + B_3^3(t)$$

$$= B_0^3(t) + B_1^3(t) + B_2^3(t) + B_3^3(t)$$ (wussten wir auch vorher schon)

$$\Rightarrow f(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$= a_0\left(B_0^3(t) + B_1^3(t) + B_2^3(t) + B_3^3(t)\right) + a_1\left(\frac{1}{3}B_1^3(t) + \frac{2}{3}B_2^3(t) + B_3^3(t)\right) + a_2\left(\frac{1}{3}B_2^3(t) + B_3^3(t)\right) + a_3 B_3^3(t)$$

$$= (a_0) B_0^3(t) + \left(a_0 + \frac{a_1}{3}\right) B_1^3(t) + \left(a_0 + \frac{2a_1}{3} + \frac{a_2}{3}\right) B_2^3(t) + (a_0 + a_1 + a_2 + a_3) B_3^3(t) \qquad \square$$

---

19)... 2. stetig diffbar?

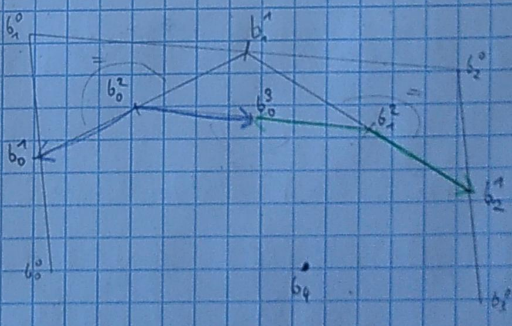
$$\left.\begin{aligned} \ddot{c}(1) &= 3\cdot 2\left(b_0^3(\tfrac{1}{2}) - 2b_0^2(\tfrac{1}{2}) + b_0^1(\tfrac{1}{2})\right) \\ \ddot{d}(0) &= 3\cdot 2\left(b_2^1(\tfrac{1}{2}) - 2b_1^2(\tfrac{1}{2}) + b_0^3(\tfrac{1}{2})\right) \end{aligned}\right\} = \quad \checkmark$$

$$b_0^3 - 2b_0^2 + b_0^1 = (b_0^3 - b_0^2) + (b_0^1 - b_0^2) =$$

$$= (b_0^3 - b_0^2) + (b_0^2 - b_1^1) = b_0^3 - b_1^1$$

$$b_2^1 - 2b_1^2 + b_0^3 = (b_2^1 - b_1^2) + (b_0^3 - b_1^2) =$$

$$= (b_0^3 - b_1^2) + (b_1^2 - b_1^1) = b_0^3 - b_1^1$$



3. stetig diffbar?

Nein, weil $\dddot{d}(0)$ vom letzten Kontrollpunkt b_4 abhängt und somit für $b_3 \neq b_4$ die Ableitung verschieden ist.

CAGD Ü3

21) $c(t) := (1-3t^2+2t^3)p_0 + (t-2t^2+t^3)v_0 + (3t^2-2t^3)p_1 + (-t^2+t^3)v_1$

wobei $p_0, p_1, v_0, v_1 \in \mathbb{R}^d$ ges: $c(0)$, $c(1)$, $\dot{c}(0)$, $\dot{c}(1)$

$c(0) = (1)p_0 = p_0$ $c(1) = \underbrace{(1-3+2)}_{=0}p_0 + \underbrace{(1-2+1)}_{=0}v_0 + \underbrace{(3-2)}_{=1}p_1 + \underbrace{(-1+1)}_{=0}v_1 = p_1$

$\dot{c}(t) = (-6t+6t^2)p_0 + (1-4t+3t^2)v_0 + (6t-6t^2)p_1 + (-2t+3t^2)v_1$

$\dot{c}(0) = v_0$ $\dot{c}(1) = \underbrace{(-6+6)}_{=0}p_0 + \underbrace{(1-4+3)}_{=0}v_0 + \underbrace{(6-6)}_{=0}p_1 + \underbrace{(-2+3)}_{=1}v_1 = v_1$

$\square$