LINAG 08

2.8.10 $U_1, U_2, U_3 \subseteq \mathbb{R}^{3\times 1}$ mit $U_1 \cap U_2 = U_2 \cap U_3 = U_3 \cap U_1 = \{0\}$

und keiner direkten Summe $U_1 + U_2 + U_3$

$U_1 = [\{\begin{pmatrix}1\\0\\0\end{pmatrix}\}]$ $U_2 = [\{\begin{pmatrix}0\\1\\0\end{pmatrix}\}]$ $U_3 = [\{\begin{pmatrix}1\\1\\0\end{pmatrix}\}]$

$U_1 \cap U_2 = \{v : x_1 \cdot \begin{pmatrix}1\\0\\0\end{pmatrix} = v = x_2 \cdot \begin{pmatrix}0\\1\\0\end{pmatrix}\}$

$$x_1 \cdot \begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}x_1\\0\\0\end{pmatrix} = v = \begin{pmatrix}0\\x_2\\0\end{pmatrix} = x_2 \cdot \begin{pmatrix}0\\1\\0\end{pmatrix}$$

$$\begin{pmatrix}x_1\\0\\0\end{pmatrix} = \begin{pmatrix}0\\x_2\\0\end{pmatrix} \Rightarrow x_1 = x_2 = 0$$

$\Rightarrow U_1 \cap U_2 = \{0\}$

$U_2 \cap U_3 = \{v : x_2 \cdot \begin{pmatrix}0\\1\\0\end{pmatrix} = v = x_3 \cdot \begin{pmatrix}1\\1\\0\end{pmatrix}\}$

$$\begin{pmatrix}0\\x_2\\0\end{pmatrix} = \begin{pmatrix}x_3\\x_3\\0\end{pmatrix} \Rightarrow x_3 = x_2 = 0$$

$\Rightarrow U_2 \cap U_3 = \{0\}$

$U_3 \cap U_1 = \{v : x_3 \cdot \begin{pmatrix}1\\1\\0\end{pmatrix} = v = x_1 \cdot \begin{pmatrix}1\\0\\0\end{pmatrix}\}$

$$\begin{pmatrix}x_3\\x_3\\0\end{pmatrix} = \begin{pmatrix}x_1\\0\\0\end{pmatrix} \Rightarrow x_3 = x_1 = 0$$

$\Rightarrow U_3 \cap U_1 = \{0\}$

$v_1 = \begin{pmatrix}1\\0\\0\end{pmatrix} \in U_1$ $v_2 = \begin{pmatrix}0\\1\\0\end{pmatrix} \in U_2$ $v_3 = \begin{pmatrix}1\\1\\0\end{pmatrix} \in U_3$

$$v_1 + v_2 = \begin{pmatrix}1\\0\\0\end{pmatrix} + \begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}1\\1\\0\end{pmatrix} = v_3$$

$\Rightarrow$ Summe ist nicht direkt