

3.3.10

$$U_1 = \left[\left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\} \right] \quad U_2 = \left[\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\} \right]$$

a) $p_2: \mathbb{R}^{4 \times 1} \rightarrow \mathbb{R}^{4 \times 1}$

Projektion auf U_2 in Richtung U_1

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \xrightarrow{\substack{-III \rightarrow IV \\ -III \rightarrow I}} \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \end{pmatrix} \xrightarrow{(-1) \cdot II} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{pmatrix} \xrightarrow{II \leftrightarrow I} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ -1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\Rightarrow A = \begin{pmatrix} 0 & 0 & 1 & 1 \\ -1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

b) $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = x_1 \cdot \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix} + x_2 \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_3 \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} + x_4 \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 + x_3 \\ x_2 + x_4 \\ -x_1 \\ x_1 + x_3 \end{pmatrix}$

$$v_1 = x_1 + x_2 + x_3$$

$$v_2 = x_2 + x_4$$

$$v_3 = -x_1$$

$$v_4 = x_1 + x_3$$

$$\Rightarrow x_1 = -v_3$$

$$v_1 = -v_3 + x_2 + x_3$$

$$v_2 = x_2 + x_4$$

$$v_4 = -v_3 + x_3$$

$$\Rightarrow x_3 = v_4 + v_3$$

$$v_1 = -v_3 + x_2 + v_4 + v_3$$

$$v_2 = x_2 + x_4$$

$$\Rightarrow x_2 = v_1 - v_4$$

$$v_2 = v_1 - v_4 + x_4$$

$$\Rightarrow x_4 = v_2 - v_1 + v_4$$

$$\Rightarrow v = -v_3 \cdot \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix} + (v_1 - v_4) \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + (v_4 + v_3) \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} + (v_2 - v_1 + v_4) \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} -v_3 + v_1 - v_4 \\ 0 + v_1 - v_4 \\ v_3 + 0 \\ -v_3 + 0 \end{pmatrix} + \begin{pmatrix} v_4 + v_3 + 0 \\ 0 + v_2 - v_1 + v_4 \\ 0 + 0 \\ v_4 + v_3 + 0 \end{pmatrix} = \begin{pmatrix} v_1 - v_3 - v_4 \\ v_1 - v_4 \\ v_3 \\ -v_3 \end{pmatrix} + \begin{pmatrix} v_3 + v_4 \\ -v_1 + v_2 + v_4 \\ 0 \\ v_3 + v_4 \end{pmatrix}$$

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