

# LINAG ÜM

4.6.5 B

V...VR / R

$f \in L(V, V)$

$$B = (b_1, b_2)$$

$$\tilde{B} = (\tilde{b}_1, \tilde{b}_2)$$

$$B^* \circ f \circ (B^*)^{-1} : \mathbb{R}^{2 \times 1} \rightarrow \mathbb{R}^{2 \times 1}$$

$$\langle B^*, \tilde{b}_1 \rangle = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

$$\langle B^*, \tilde{b}_2 \rangle = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mapsto \begin{pmatrix} x_1 + 3x_2 \\ 3x_1 + x_2 \end{pmatrix}$$

a) ges:  $\langle B^*, f(B) \rangle$ ,  $\langle \tilde{B}^*, f(B) \rangle$ ,  $\langle \tilde{B}^*, f(\tilde{B}) \rangle$

$$\langle B^*, \tilde{B} \rangle = \begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix}$$

$$\langle \tilde{B}^*, B \rangle = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$

$$\begin{array}{ccc} \mathbb{R}^{2 \times 1} & \xrightarrow{\langle \tilde{B}^*, f(\tilde{B}) \rangle} & \mathbb{R}^{2 \times 1} \\ \tilde{B}^* \uparrow & & \uparrow \tilde{B}^* \\ V & \xrightarrow{f} & V \\ B^* \downarrow & & \downarrow B^* \\ \mathbb{R}^{2 \times 1} & \xrightarrow{\langle B^*, f(B) \rangle} & \mathbb{R}^{2 \times 1} \end{array}$$

$$\langle B^*, f(B) \rangle = \begin{pmatrix} 1+3 \cdot 0 & 0+3 \cdot 1 \\ 3 \cdot 1+0 & 3 \cdot 0+1 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}$$

$$\begin{aligned} \langle \tilde{B}^*, f(B) \rangle &= \tilde{B}^* \circ (B^*)^{-1} \circ B^* \circ f \circ (B^*)^{-1} = \langle \tilde{B}^*, B \rangle \cdot \langle B^*, f(B) \rangle \\ &= \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} -2 & -2 \\ -1 & 1 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \langle \tilde{B}^*, f(\tilde{B}) \rangle &= \langle \tilde{B}^*, B \rangle \cdot \langle B^*, f(B) \rangle \cdot \langle B^*, \tilde{B} \rangle \\ &= \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & -2 \end{pmatrix} \end{aligned}$$

