2.) $z = a + ib \in \mathbb{C}$ $\quad w = c + id$

zz: z hat Quadratwurzeln, also $\exists w$, sodass $w^2 = z$

$$(c+id)^2 = (c+id)(c+id) = (c^2-d^2) + i(cd+cd) = (c^2-d^2) + i(2cd)$$

$$c^2-d^2 + i\,2cd = a + ib$$

$$\Rightarrow c^2-d^2 = a \qquad 2cd = b$$

$$(c^2-d^2)^2 + (2cd)^2 = a^2+b^2$$

$$c^4 - 2c^2d^2 + d^4 + 4c^2d^2 = a^2+b^2$$

$$c^4 + 2c^2d^2 + d^4 = a^2+b^2$$

$$(c^2+d^2)^2 = a^2+b^2$$

$$c^2+d^2 = \sqrt{a^2+b^2}$$

$c^2-d^2 = a$

$\Leftrightarrow d^2 = c^2-a$

$$c^2 + (c^2-a) = \sqrt{a^2+b^2}$$

$$2c^2 = \sqrt{a^2+b^2} + a$$

$$c^2 = \frac{1}{2}\left(\sqrt{a^2+b^2} + a\right)$$

$$c = \pm\sqrt{\frac{\sqrt{a^2+b^2}+a}{2}}$$

$$c^2+d^2 = \sqrt{a^2+b^2}$$

$$a + d^2 + d^2 = \sqrt{a^2+b^2}$$

$$2d^2 = \sqrt{a^2+b^2} - a$$

$$d^2 = \frac{\sqrt{a^2+b^2}-a}{2}$$

$$d = \pm\sqrt{\frac{\sqrt{a^2+b^2}-a}{2}}$$

$$\sqrt{a+ib} = \pm\sqrt{\frac{\sqrt{a^2+b^2}+a}{2}} + i\left(\pm\sqrt{\frac{\sqrt{a^2+b^2}-a}{2}}\right)$$

8

2.) ... Wie viele Lösungen gibt es?

Da $b = 2cd$ muss b das gleiche Vorzeichen wie cd haben.

$\Rightarrow$ Es gibt 2 Lösungen.

ges: $\sqrt{i} = \sqrt{0 + i \cdot 1} = \pm\sqrt{\frac{\sqrt{0^2+1^2}+0}{2}} + i\left(\pm\sqrt{\frac{\sqrt{0^2+1^2}-0}{2}}\right) = \pm\sqrt{\frac{1}{2}} + i\left(\pm\sqrt{\frac{1}{2}}\right)$

$\Rightarrow \sqrt{i} = \sqrt{\frac{1}{2}} + i\sqrt{\frac{1}{2}}$ und $\sqrt{i} = -\sqrt{\frac{1}{2}} + i\left(-\sqrt{\frac{1}{2}}\right)$

ges: $\sqrt{3 - i2} = \pm\sqrt{\frac{\sqrt{3^2+(-2)^2}+3}{2}} + i\left(\pm\sqrt{\frac{\sqrt{3^2+(-2)^2}-3}{2}}\right) = \pm\sqrt{\frac{\sqrt{13}+3}{2}} + i\left(\pm\sqrt{\frac{\sqrt{13}-3}{2}}\right)$

$\Rightarrow \sqrt{3 - i2} = \sqrt{\frac{\sqrt{13}+3}{2}} + i\left(-\sqrt{\frac{\sqrt{13}-3}{2}}\right) \approx 1{,}817 + i(-0{,}55)$

$\sqrt{3 - i2} = -\sqrt{\frac{\sqrt{13}+3}{2}} + i\sqrt{\frac{\sqrt{13}-3}{2}} \approx -1{,}817 + i(0{,}55)$