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1.) $\frac{3+i6}{-2-i3} = (3+i6)\cdot(-2-i3)^{-1} = (3+i6)\cdot\left(\frac{-2}{(-2)^2+(-3)^2} + i\left(\frac{-(-3)}{(-2)^2+(-3)^2}\right)\right)$

$= (3+i6)\cdot\left(\frac{-2}{13} + i\,\frac{3}{13}\right) = \left(3\cdot\frac{-2}{13} - 6\cdot\frac{3}{13}\right) + i\left(3\cdot\frac{3}{13} + 6\cdot\frac{-2}{13}\right)$

$= \left(-\frac{6}{13} - \frac{18}{13}\right) + i\left(\frac{9}{13} - \frac{12}{13}\right) = \left(-\frac{24}{13}\right) + i\left(-\frac{3}{13}\right)$

- $(-1+i2)^{-2} = (-1+i2)^{-1}\cdot(-1+i2)^{-1} = \left(\frac{-1}{(-1)^2+2^2} + i\,\frac{-2}{(-1)^2+2^2}\right)\cdot\left(\frac{-1}{(-1)^2+2^2} + i\,\frac{-2}{(-1)^2+2^2}\right)$

$= \left(-\frac{1}{5} + i\left(-\frac{2}{5}\right)\right)\cdot\left(-\frac{1}{5} + i\left(-\frac{2}{5}\right)\right) = \left(-\frac{1}{5}\right)\cdot\left(-\frac{1}{5}\right) - \left(\left(-\frac{2}{5}\right)\cdot\left(-\frac{2}{5}\right)\right) + i\left(\left(-\frac{1}{5}\right)\cdot\left(-\frac{2}{5}\right) + \left(-\frac{2}{5}\right)\cdot\left(-\frac{1}{5}\right)\right)$

$= \frac{1}{25} - \frac{4}{25} + i\left(\frac{2}{25} + \frac{2}{25}\right) = \left(-\frac{3}{25}\right) + i\left(\frac{4}{25}\right)$

- $(1+i)^2 = (1+i)\cdot(1+i) = (1\cdot1 - 1\cdot1) + i(1\cdot1 + 1\cdot1) = 0 + i2$

- $\sum_{j=0}^{16} i^j = i^0 + i^1 + i^2 + \dots + i^{16} =$

$= 5\cdot i^0 + 4\cdot i^1 + 4\cdot i^2 + 4\cdot i^3$

$= 5\cdot 1 + 4\cdot i + 4\cdot(-1) + 4\cdot(-i)$

$= 5 + 4i - 4 - 4i = 1 + 0i$

i^0 i^1 i^2 i^3 i^4 i^5

1 i -1 $-i$ 1 i

$\cdot i$ $\cdot i$ $\cdot i$ $\cdot i$ $\cdot i$

$\Rightarrow i^0 = i^4 = i^8 = i^{12} = i^{16}$

$i^1 = i^5 = i^9 = i^{13}$

$i^2 = i^6 = i^{10} = i^{14}$

$i^3 = i^7 = i^{11} = i^{15}$