9. $\langle K, +, \cdot \rangle$ .. Körper $x \in K \setminus \{0\}$ $p, q \in \mathbb{Z}$

zz: $x^{-p} = \frac{1}{x^p}$

Fallunterscheidung: $p \in \mathbb{N}$ $p = 1$: $x^{-1} = \frac{1}{x^1}$ ✓

$p+1$: $x^{-(p+1)} = \frac{1}{x^{p+1}}$ ✓

2. Fall: $p = 0$: $x^{-0} = x^0 = 1 = \frac{1}{1} = \frac{1}{x^0}$ ✓

3. Fall: $-p \in \mathbb{N}$: $p = -1$: $x^{-(-1)} = x$ $\frac{1}{x^{-1}} = \frac{1}{\frac{1}{x^1}} = \frac{x^1}{1} = x$ ✓

$p-1$: $x^{-(p-1)} = \frac{1}{x^{p-1}}$ ✓

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9.) ... zz: $x^p x^q = x^{p+q}$

o.B.d.A: $p \leq q$ Fallunterscheidung: 1. Fall: $p \in \mathbb{N} \Rightarrow q \in \mathbb{N}$

$p=1$: $x^1 \cdot x^q = x \cdot x^q = x^{q+1}$ ✓

$p+1$: $x^{p+1} x^q = x^p \cdot x^q \cdot x = x^{p+q} \cdot x = x^{p+1+q}$ ✓

2. Fall $p = 0 \Rightarrow q \in \mathbb{N} \cup \{0\}$

$x^0 \cdot x^q = 1 \cdot x^q = x^q = x^{0+q}$ ✓

3. Fall $p \in -\mathbb{N}$

$p=-1$: $x^{-1} x^q = \frac{1}{x} \cdot x^q = \frac{1}{x} \cdot x \cdot x^{q-1} = x^{q-1}$ ✓

$p-1$: $x^{p-1} x^q = x^{-1} \cdot x^p \cdot x^q = x^{-1} \cdot x^{p+q} = x^{p-1+q}$ ✓

zz: $(x^p)^q = x^{p \cdot q}$

o.B.d.A: $p \leq q$ Fallunterscheidung: 1. Fall $p \in \mathbb{N} \Rightarrow q \in \mathbb{N}$

$q=1$: $(x^p)^1 = x^p = x^{p \cdot 1}$ ✓

$q+1$: $x^{p \cdot (q+1)} = x^{p \cdot q + p} = x^{p \cdot q} x^p = (x^p)^q \cdot x^p = (x^p)^{q+1}$ ✓

2. Fall: $q = 0$: $(x^p)^0 = 1 = x^0 = x^{0 \cdot p}$ ✓

3. Fall: $q \in -\mathbb{N}$:

$q=-1$: $(x^p)^{-1} = \frac{1}{x^p} = x^{-p} = x^{p \cdot (-1)}$ ✓

$q-1$: $x^{p \cdot (q-1)} = x^{p \cdot q - p} = x^{p \cdot q} x^{-p} = (x^p)^q \cdot x^{-p} = (x^p)^{q-1}$ ✓