LINAG Ü12

G9 $V=\mathbb{R}^{2\times 1}$ $E=(e_1,e_2)$ ... kanonische Basis $\sigma: V\times V\to V$

$$\sigma(E,E)=\begin{pmatrix}1&0\\0&-1\end{pmatrix}=:A$$

$$\langle E^*, B\rangle = T$$

1.) ges: alle Basen mit $\sigma(B,B)=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$

Transformationsmatrix $T=\begin{pmatrix}a&b\\c&d\end{pmatrix}$ $\cancel{T^T\cdot E\cdot T=B}$ $\underline{T^T\cdot A\cdot T=A}$

$$T^T\cdot A\cdot T=\begin{pmatrix}a&c\\b&d\end{pmatrix}\cdot\begin{pmatrix}1&0\\0&-1\end{pmatrix}\cdot\begin{pmatrix}a&b\\c&d\end{pmatrix}=\begin{pmatrix}a^2-c^2&ab-cd\\ab-cd&b^2-d^2\end{pmatrix}=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

Falls $b=0$: $ab-cd=0 \Leftrightarrow -cd=0 \Rightarrow c=0 \vee d=0$ wenn $d=0$: $b^2-d^2=b^2\neq -1$ ↯

Falls $d=0$: $ab-cd=0 \Leftrightarrow ab=0 \Rightarrow a=0\vee b=0$ wenn $a=0$: $a^2-c^2=-c^2\neq 1$ ↯

Also ist beides der gleiche Fall. $b^2-d^2=0\neq -1$ ↯

Falls $b\neq 0 \wedge d\neq 0$: $ab-cd=0 \Leftrightarrow a=\frac{cd}{b} \Leftrightarrow c=\frac{ab}{d}$

$$a^2-c^2=\frac{c^2d^2}{b^2}-c^2=\frac{c^2(d^2-b^2)}{b^2}=1 \quad [\,b^2-d^2=-1 \Rightarrow d^2-b^2=1$$

$$\frac{c^2}{b^2}=1 \Leftrightarrow b^2=c^2 \Leftrightarrow |b|=|c|$$

$$a^2-c^2=a^2-\frac{a^2b^2}{d^2}=\frac{a^2(d^2-b^2)}{d^2}=\frac{a^2}{d^2}=1 \Leftrightarrow a^2=d^2 \Leftrightarrow |d|=|a|$$

Also $T=\begin{pmatrix}a&b\\b&a\end{pmatrix}$ $T=\begin{pmatrix}a&b\\b&-a\end{pmatrix}$ $T=\begin{pmatrix}a&b\\-b&a\end{pmatrix}$ oder $T=\begin{pmatrix}a&b\\-b&-a\end{pmatrix}$.

1. Fall $a^2-b^2=1$ $a\cdot b-b\cdot a=0$ ✓ $b^2-a^2=-1 \Leftrightarrow a^2-b^2=1$

$\Rightarrow b^2=a^2-1 \Leftrightarrow b=\pm\sqrt{a^2-1}$

2. Fall $a^2-b^2=1$ $a\cdot b+b\cdot a=0 \Rightarrow a=0 \vee (b=0$ ↯$) \Rightarrow T$ ... nicht regulär

3. Fall $a^2-b^2=1$ $a\cdot b+b\cdot a=0$ — " — $\pm a^2 \pm a^2-1$

4. Fall $a^2-b^2=1$ $a\cdot b-b\cdot a=0$ ✓ $b^2=a^2-1$ $b=\pm\sqrt{a^2-1}$

$\Rightarrow T=\begin{pmatrix}a&\sqrt{a^2-1}\\\sqrt{a^2-1}&a\end{pmatrix}$ $T=\begin{pmatrix}a&\mp\sqrt{a^2-1}\\-\sqrt{a^2-1}&a\end{pmatrix}$ $T=\begin{pmatrix}a&\sqrt{a^2-1}\\-\sqrt{a^2-1}&-a\end{pmatrix}$ $T=\begin{pmatrix}a&\mp\sqrt{a^2-1}\\\sqrt{a^2-1}&-a\end{pmatrix}$

$$\cancel{T^T E\cdot T=\begin{pmatrix}a^2+c^2&ab+cd\\ab+cd&b^2+d^2\end{pmatrix}=\begin{pmatrix}a^2+a^2-1&\pm a\sqrt{a^2-1}\pm\sqrt{a^2-1}\cdot a\\\pm a\sqrt{a^2-1}\pm\sqrt{a^2-1}\cdot a&a^2-1+a^2\end{pmatrix}}$$

2.) zz: $\exists U^+, U^-$ wie in 9.10.3. $\exists \tilde U^+, \tilde U^-$ wie in 9.10.3 verschieden

$U^+=[\begin{pmatrix}1\\0\end{pmatrix}]$ $U^-=[\begin{pmatrix}0\\1\end{pmatrix}]$ $\tilde U^+=[\begin{pmatrix}2\\\sqrt3\end{pmatrix}]$ $\tilde U^-=[\begin{pmatrix}\sqrt3\\2\end{pmatrix}]$ $(a=2)$

3.) ges: $U_1, U_2\subseteq V$ $V=U_1\oplus U_2$ $\forall v\in U_1^\times: \sigma(v,v)>0$ $\forall v\in U_2^\times: \sigma(v,v)>0$

$U_1 \overset{a=2}{=} [\begin{pmatrix}2\\\sqrt3\end{pmatrix}]$ $U_2\overset{a=3}{=}[\begin{pmatrix}3\\\sqrt8\end{pmatrix}]$ $k\cdot\begin{pmatrix}2\\\sqrt3\end{pmatrix}=\begin{pmatrix}3\\\sqrt8\end{pmatrix} \Rightarrow 2k=3 \Leftrightarrow k=\frac32$

$\frac32\cdot\sqrt3 \neq 2\sqrt2 \Rightarrow U_1\cap U_2=\{0\}$ $\dim U_1+U_2=2 \Rightarrow U_1\oplus U_2=V$