1.10.1 a) 1) Ja, z.B. $(\{0\},+)$

2) Nein, da jeder Körper zumindest zwei (unterschiedliche!) neutrale Element enthalten muss (für jede Operation eines.)

b) $\mathbb{Z}_2$:

+	0	1
0	0	1
1	1	0

·	0	1
0	0	0
1	0	1

$\mathbb{Z}_3$:

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

·	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

$\mathbb{Z}_5$:

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

·	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

c) $x^2 + x + \bar{1} = \bar{0}$

$\mathbb{Z}_2$: $x=\bar{0}$: $\bar{0}\cdot\bar{0}+\bar{0}+\bar{1} = \bar{0}+\bar{0}+\bar{1} = \bar{0}+\bar{1} = \bar{1}$

$x=\bar{1}$: $\bar{1}\cdot\bar{1}+\bar{1}+\bar{1} = \bar{1}+\bar{1}+\bar{1} = \bar{0}+\bar{1} = \bar{1}$

$\mathbb{Z}_3$: $x=\bar{0}$: $\bar{0}\cdot\bar{0}+\bar{0}+\bar{1} = \bar{0}+\bar{0}+\bar{1} = \bar{0}+\bar{1} = \bar{1}$

$\boxed{x=\bar{1}:}$ $\bar{1}\cdot\bar{1}+\bar{1}+\bar{1} = \bar{1}+\bar{1}+\bar{1} = \bar{2}+\bar{1} = \underline{\bar{0}}$

$x=\bar{2}$: $\bar{2}\cdot\bar{2}+\bar{2}+\bar{1} = \bar{1}+\bar{2}+\bar{1} = \bar{0}+\bar{1} = \bar{1}$

$\mathbb{Z}_5$: $x=\bar{0}$: $\bar{0}\cdot\bar{0}+\bar{0}+\bar{1} = \bar{0}+\bar{0}+\bar{1} = \bar{0}+\bar{1} = \bar{1}$

$x=\bar{1}$: $\bar{1}\cdot\bar{1}+\bar{1}+\bar{1} = \bar{1}+\bar{1}+\bar{1} = \bar{2}+\bar{1} = \bar{3}$

$x=\bar{2}$: $\bar{2}\cdot\bar{2}+\bar{2}+\bar{1} = \bar{4}+\bar{2}+\bar{1} = \bar{1}+\bar{1} = \bar{2}$

$x=\bar{3}$: $\bar{3}\cdot\bar{3}+\bar{3}+\bar{1} = \bar{4}+\bar{3}+\bar{1} = \bar{2}+\bar{1} = \bar{3}$

$x=\bar{4}$: $\bar{4}\cdot\bar{4}+\bar{4}+\bar{1} = \bar{1}+\bar{4}+1 = \bar{0}+\bar{1} = \bar{1}$

Die Lösungsformel funktioniert in $\mathbb{C}$ immer. In $\mathbb{R}$ muss $\frac{p^2}{4} \geq q$ sein, damit es eine Lösung gibt.

$$x = \frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$