ANA Ü3

1.) $n \in \mathbb{N} \quad n \geq 2 \qquad \prod_{k=1}^{n} \left(1 + \frac{1}{n+k}\right) = 2 - \frac{1}{n+1}$

$$n=2: \quad \prod_{k=1}^{2} \left(1 + \frac{1}{2+k}\right) = \left(1 + \frac{1}{2+1}\right) \cdot \left(1 + \frac{1}{2+2}\right) = \frac{4}{3} \cdot \frac{5}{4} = \frac{20}{12} = \frac{5}{3}$$

$$2 - \frac{1}{2+1} = \frac{6}{3} - \frac{1}{3} = \frac{5}{3} \qquad \checkmark$$

$$n+1: \quad zz: \prod_{k=1}^{n+1} \left(1 + \frac{1}{n+1+k}\right) = 2 - \frac{1}{n+1+1}$$

$$\prod_{k=1}^{n+1} \left(1 + \frac{1}{n+1+k}\right) = \prod_{k=2}^{n+2} \left(1 + \frac{1}{n+1+k-1}\right) = \prod_{k=2}^{n} \left(1 + \frac{1}{n+k}\right) \cdot \left(1 + \frac{1}{n+n+1}\right) \cdot \left(1 + \frac{1}{n+n+2}\right)$$

$$= \prod_{k=2}^{n} \left(1 + \frac{1}{n+k}\right) \cdot \left(\frac{\cancel{2n+2}}{2n+1} \cdot \frac{2n+3}{\cancel{2n+2}}\right) = \prod_{k=2}^{n} \left(1 + \frac{1}{n+k}\right) \cdot \left(\frac{2n+3}{2n+1}\right)$$

$$= \prod_{k=1}^{n} \left(1 + \frac{1}{n+k}\right) \left(\frac{2n+3}{2n+1}\right) \left(1 + \frac{1}{n+1}\right)^{-1} = \prod_{k=1}^{n} \left(1 + \frac{1}{n+k}\right) \left(\frac{2n+3}{2n+1}\right) \left(\frac{n+1+1}{n+1}\right)^{-1}$$

$$= \left(2 - \frac{1}{n+1}\right) \left(\frac{2n+3}{2n+1}\right) \left(\frac{n+1}{n+2}\right) = \frac{\cancel{(2n+2-1)} \cdot (2n+3) \cancel{(n+1)}}{\cancel{(n+1)} \cdot \cancel{(2n+1)} (n+2)} = \frac{2n+3}{n+2}$$

$$2 - \frac{1}{n+2} = \frac{2n+4-1}{n+2} = \frac{2n+3}{n+2} \qquad \checkmark \quad \square$$