LINAG Ü10

4.4.1 α) $\quad B = (b_1, b_2, b_3) \quad \tilde{B} = (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) \quad V \ldots$ VR über $\mathbb{R}$

$$\langle \tilde{B}^*, B \rangle = \begin{pmatrix} 1 & -3 & -3 \\ 1 & -1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

a) $x \in V \quad \langle B^*, x \rangle = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad \langle \tilde{B}^*, x \rangle = \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{pmatrix}$

ges: Transformationsformeln für beide Koordinatenwechsel

Aus den Zeilen von $\langle \tilde{B}^*, B \rangle$ kann man ablesen:

$\tilde{x}_1 = x_1 - 3x_2 - 3x_3 \qquad \tilde{x}_2 = x_1 - x_2 \qquad \tilde{x}_3 = x_1 + x_3$

Die zu $\langle \tilde{B}^*, B \rangle$ inverse Matrix ist $\langle B^*, \tilde{B} \rangle$.

$$\left(\begin{array}{ccc} 1 & -3 & -3 \\ 1 & -1 & 0 \\ 1 & 0 & 1 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right) \rightsquigarrow \begin{array}{c} \begin{array}{ccc} +\mathrm{II} & \cdot(-1) & \end{array} \\ \left(\begin{array}{ccc} -2 & 3 & -3 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ \hline 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{array}\right) \end{array} \rightsquigarrow \begin{array}{c} \begin{array}{ccc} & & -\mathrm{I} \end{array} \\ \left(\begin{array}{ccc} -2 & 3 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ \hline 1 & 0 & -1 \\ 1 & -1 & -1 \\ 0 & 0 & 1 \end{array}\right) \end{array} \rightsquigarrow \begin{array}{c} \begin{array}{ccc} -\mathrm{III} & \mathrm{II} & \mathrm{I} \end{array} \\ \left(\begin{array}{ccc} 1 & 3 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 1 & 0 & 1 \\ 1 & -1 & 1 \\ -1 & 0 & 0 \end{array}\right) \end{array} \rightsquigarrow \begin{array}{c} \begin{array}{ccc} & -3\cdot\mathrm{I} & +2\cdot\mathrm{I} \end{array} \\ \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 1 & -3 & 3 \\ 1 & -4 & 3 \\ -1 & 3 & -2 \end{array}\right) \end{array}$$

$$\boxed{\begin{matrix} 1 & -3 & 3 \\ 1 & -4 & 3 \\ -1 & 3 & -2 \end{matrix}} = \langle B^*, \tilde{B} \rangle$$

Aus den Zeilen von $\langle B^*, \tilde{B} \rangle$ kann man ablesen:

$x_1 = \tilde{x}_1 - 3\tilde{x}_2 + 3\tilde{x}_3 \qquad x_2 = \tilde{x}_1 - 4\tilde{x}_2 + 3\tilde{x}_3 \qquad x_3 = -\tilde{x}_1 + 3\tilde{x}_2 - 2\tilde{x}_3$

b) Aus den Spalten von $\langle \tilde{B}^*, B \rangle$ kann man ablesen:

$b_1 = 1\tilde{b}_1 + 1\tilde{b}_2 + 1\tilde{b}_3 \qquad b_2 = -3\tilde{b}_1 - 1\tilde{b}_2 \qquad b_3 = -3\tilde{b}_1 + 1\tilde{b}_3$

Aus den Spalten von $\langle B^*, \tilde{B} \rangle$ kann man ablesen:

$\tilde{b}_1 = 1b_1 + 1b_2 - 1b_3 \qquad \tilde{b}_2 = -3b_1 - 4b_2 + 3b_3 \qquad \tilde{b}_3 = 3b_1 + 3b_2 - 2b_3$

c) Aus den Zeilen von $\langle \tilde{B}^*, B \rangle$ kann man ablesen:

$\tilde{b}_1^* = 1b_1^* - 3b_2^* - 3b_3^* \qquad \tilde{b}_2^* = 1b_1^* - 1b_2^* \qquad \tilde{b}_3^* = 1b_1^* + 1b_3^*$

Aus den Zeilen von $\langle B^*, \tilde{B} \rangle$ kann man ablesen:

$b_1^* = 1\tilde{b}_1^* - 3\tilde{b}_2^* + 3\tilde{b}_3^* \qquad b_2^* = 1\tilde{b}_1^* - 4\tilde{b}_2^* + 3\tilde{b}_3^* \qquad b_3^* = -1\tilde{b}_1^* + 3\tilde{b}_2^* - 2\tilde{b}_3^*$