Using PCA on EEG Data to Distinguish Sleep Stages

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***Abstract*—[TODO]**

I. Introduction

[TODO general introduction]

A. EEG Data and Sleep Stages

Ganong [2] describes typical patterns observed in electroencephalogram (EEG) data of a sleeping person. He describes the EEG patterns associated with rapid eye movement (REM) sleep and non-REM (NREM) sleep.

NREM sleep is further partitioned into four (although some only use three) stages, termed Stage 1 (S1) to Stage 4 (S4). Example EEG data of these different sleep stages can be seen in Figure **??** [TODO image]. The EEG data of these stages is characterized as follows:

S1: low-amplitude, high-frequency
S2: appearance of sleep spindles (bursts of higher amplitude, lower frequency waves)
S3: increased amplitude, lower frequency
S4: maximal amplitude, minimal frequency

In REM sleep the EEG data is that of high frequency and low amplitude patterns, resembling the data observed in alert humans.

II. Study of Literature

[Note:
first work on pca [9] and [4]
given paper [6]
when does pca fail? [11] and [12] (non-linear method)
book containing sleep phases eeg [2]
Review Paper on Sleep Stage Classification Methods [1]
papers trying to solve similar problem [13] and [10] and [7]
competition using similar data set [3]
winner of competition [5]]

A substantial body of scientific research has been devoted to exploring Principal Component Analysis (PCA). The foundation of this method was laid by Pearson [9] and Hotelling [4].

An introduction to PCA, as well as a good overview on how to derive the formula used to compute the Principal Components (PC) is given by Shlens [11]. Recent applications and variants of PCA are explored by Jolliffe et. al. [6].

Shlens discusses the limitations of PCA, as well as examples in which PCA fails [11], such as the requirement of linearly dependent data. Tenenbaum proposes a non-linear method to combat this problem[12].

Generally speaking the variables must not have third or higher order dependencies[1] between them. In some cases it is possible to reduce a problem with higher order dependencies to a second order one by applying a non-linear transformation beforehand. This method is called kernel PCA[11].

Another method for combating this problem is Independent Component Analysis (ICA) which is discussed by Naik et. al.[8].

The given problem of distinguishing sleep stages given some EEG data has been investigated by use of PCA, as well as neural networks. Some of these works are summarized below.

A review of different methods in the preprocessing, feature extraction and classification is given by Boostani et. al.[1]. They find that using a random forest classifier and entropy of wavelet coefficients as feature gives the best results.

Tăuţan et. al.[13] compare different methods of dimensionality reduction on EEG data, such as PCA, factor analysis and autoencoders. They conclude that PCA and factor analysis improves the accuracy of the model.

Putilov[10] used PCA to find boundaries between Stage 1, Stage 2 and Stage 3. Changes in the first two PC were related to changes between the Stage 1 and Stage 2, while changes in the fourth PC exhibited a change in sign at the boundary of Stage 2 and Stage 3. This suggests that changes between Stage 1 and Stage 2 are easier to detect that ones between Stage 2 and Stage 3.

Metzner et. al.[7] try to rediscover the different human-defined sleep stages. They find that using PCA on the results makes clusters apparent. These clusters could then be used as a basis for a redefinition of sleep stages.

The PhysioNet/Computing in Cardiology Challange 2018 was a competition using a similar data[3]. The goal was to identify arousal during sleep from EEG, EOG, EMG, ECG and SaO2 data given. The winning paper of this competition describes the use of a dense recurrent convolutional neural network (DRCNN) consisting of multiple dense convolutional layers, a bidirectional long-short term memory layer and a softmax output layer[5].

[1]e.g. $\mathbb{E}[x_i x_j x_k] \neq 0$ for some i, j, k assuming mean-free variables

As shown in this section, the utilization of PCA to analyze EEG data has been used with success.

III. Mathematical Basics

We define basic mathematical notation, which will be used in Section IV to define the PCA.

A. Covariance

Assume we have two sets of n observations of variables with mean 0. Let us call the first list of observations $\mathbf{a} = (a_1, ..., a_n)$ and the second $\mathbf{b} = (b_1, ..., b_n)$.

Definition 1 (covariance): Let us define the *covariance* of $\mathbf{a}$ and $\mathbf{b}$ as

$$\sigma^2_{\mathbf{ab}} := \frac{1}{n}\sum_{i=1}^{n} a_i b_i = \frac{1}{n}\mathbf{a}\cdot\mathbf{b}^T.$$

From the definition it is obvious that the covariance is symmetric, $\sigma_{\mathbf{ab}} = \sigma_{\mathbf{ba}}$.

Definition 2 (covariance matrix): Generalizing to m variables $\mathbf{X} = [\mathbf{x_1}, ..., \mathbf{x_m}]$, each having been observed n times, gives us the *covariance matrix*.

$$\mathbf{C_X} := \begin{pmatrix} \sigma_{\mathbf{x_1 x_1}} & \cdots & \sigma_{\mathbf{x_1 x_m}} \\ \vdots & \ddots & \vdots \\ \sigma_{\mathbf{x_m x_1}} & \cdots & \sigma_{\mathbf{x_m x_m}} \end{pmatrix} = \frac{1}{n}\mathbf{X}\mathbf{X}^T$$

The covariance matrix is a symmetric $m \times m$ matrix.

B. Diagonalizable Matrix

Definition 3 (Diagonalizable Matrix): A square matrix $\mathbf{A}$ is called *diagonalizable*, if there exists a invertable matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A} = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$.

Lemma 1: Every symmetric matrix is diagonalizable.

[TODO]

IV. Principal Component Analysis

V. Sleep Stages and EEG Data

VI. Data and Algorithm

1) subdivide eeg signals in the temporal domain
2) apply fft transforming into frequency domain
3) pca
4) achive dimensinality reduction
5) classification of sleep stages
6) visulisation

VII. Results

VIII. Conclusion

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