6) (i) $(x_n)_{n\in\mathbb{N}}$ $x_n = 2 + (-1)^n \cdot \frac{3}{n}$ $d(x,y) = |x-y|$

Beh: $(x_n)_{n\in\mathbb{N}} \to x = 2$

zz: $\forall \varepsilon > 0 \; \exists N \in \mathbb{N} \; \forall n \geq N: d(x_n, x) \leq \varepsilon$

Sei $\varepsilon > 0$ bel. Wähle $N = ?$

$$\left[\left| 2 + (-1)^n \cdot \frac{3}{n} - 2 \right| = \left| (-1)^n \cdot \frac{3}{n} \right| = \left| \frac{3}{n} \right| = \frac{3}{n} \leq \varepsilon \iff 3 \leq \varepsilon \cdot n \iff \frac{3}{\varepsilon} \leq n \right]$$

$N = \lceil \frac{3}{\varepsilon} \rceil$ Sei $n \geq N$ bel.

$$\left| 2 + (-1)^n \cdot \frac{3}{n} - 2 \right| = \left| (-1)^n \cdot \frac{3}{n} \right| = \frac{3}{n} \overset{n \geq N}{\leq} \frac{3}{N} = \frac{3}{\lceil \frac{3}{\varepsilon} \rceil} \leq \frac{3}{\frac{3}{\varepsilon}} = \varepsilon$$

$\square$

(ii) $(x_n)_{n\in\mathbb{N}}$ $x_n = (-1)^n + \frac{1}{n^6}$ $d(x,y) = |x-y|$

Beh: divergiert

zz: $\forall x \in \mathbb{R} \; \exists \varepsilon > 0 \; \forall N \in \mathbb{N} \; \exists n \geq N: d(x_n, x) \geq \varepsilon$

Sei $x \in \mathbb{R}$ bel. Wähle $\varepsilon = 1 - |x|$ Sei $N \in \mathbb{N}$ bel. Wähle $n \geq N$ so dass n gerade.

$$d(x_n, x) = \left| (-1)^n + \frac{1}{n^6} - x \right| \geq \left| \left| (-1)^n + \frac{1}{n^6} \right| - |x| \right|$$

$$= \left| \left| 1 + \frac{1}{n^6} \right| - |x| \right| = \left| 1 + \frac{1}{n^6} - |x| \right| \geq 1 + \frac{1}{n^6} - |x| = \frac{1}{n^6} + 1 - |x| \geq 1 - |x| = \varepsilon$$

$\square$