Computer Aided Geometric Design Compendium WS2023

Ida Hönigmann

October 13, 2023

Orginazation

Lecture each Thursday 12:00 to 14:00 (full 2 hours).
Oral exam. Write email to fix date and time.
Problem session each Thursday 14:00 to 16:00. Mandatory attendance!
Kreuzerlübung.

1 Bezier curves

Example 1. ***Linear combination*** $\lambda a + \mu b$
TODO image
Affine combination $\lambda a + \mu b$ *and* $\lambda + \mu = 1$
TODO image
What is μ *so that* $\lambda a + \mu B$ *is on the line?*

$$\lambda a + \mu b = a + t(b - a) \implies a(\underbrace{\lambda - 1 + t}_{=0}) + b(\underbrace{\mu - t}_{=0}) = 0$$

If a, b *are linearly independent* $\implies \mu = t \wedge \lambda + \mu = 1$
Convex combination $\lambda a + \mu b$ *and* $\lambda + \mu = 1$ *and* $\lambda, \mu \geq 0$
TODO image
Line is $a + t(b - a)$ *with* $t \in [0, 1] \implies \mu, \lambda \in [0, 1]$

Definition 1 (combinations). *linear combination* $\sum_{i=1}^n \lambda_i v_i$ *with* $v_1, ..., v_n \in \mathbb{R}^d, \lambda_1, ..., \lambda_n \in \mathbb{R}$
affine combination $\sum_{i=1}^n \lambda_i v_i$ *with* $\sum_{i=1}^n \lambda_i = 1$
convex combination $\sum_{i=1}^n \lambda_i v_i$ *with* $\sum_{i=1}^n \lambda_i = 1$ *and* $\forall i : \lambda_i \geq 0$

Algorithm 1 (of de Casteljou, Bezier curve). *Given:* $b_0, ..., b_n \in \mathbb{R}^d$ *(called control points / Kontrollpunkte),* $t \in \mathbb{R}$
Recursion: $b_i^0(t) := b_i$
$b_i^j(t) := (1 - t)b_i^{j-1}(t) + tb_{i+1}^{j-1}(t)$ *for* $j = 1, ..., n$ *and* $i = 0, ..., n - j$
Result: $b(t) := b_0^n(t)$ *(called Bezier curve)*

Remark 1. *In the algorithm above often we choose* $t \in [0, 1]$.

Example 2. *TODO images*

Remark 2. *In this course* $\mathbb{N} = \{1, 2, 3, ...\}$ *and* $\mathbb{N}_0 = \{0, 1, 2, 3, ...\}$

Recap 1. $0! := 1, n! := n(n - 1)(n - 2) \cdots 1$ *for* $n \geq 1$.

$$\binom{n}{k} := \begin{cases} \frac{n!}{k!(n-k)!} & , n \geq k \geq 0 \\ 0 & , k > n \end{cases} \quad \text{for } n, k \in \mathbb{N}_0$$

Definition 2 (Bernstein polynomials). *For* $n, i \in \mathbb{N}_0$ *we define* $B_i^n(t) := \binom{n}{i} t^i (1 - t)^{n-i} \in \mathbb{R}[t]$

Remark 3. *Special cases of Bernstein polynomials*

$$i > n \implies B_i^n(t) = 0 \qquad B_i^n(0) = \begin{cases} 0, i \neq 0 \\ 1, i = 0 \end{cases}$$

$$B_i^n(1) = \begin{cases} 0, i \neq n \\ 1, i = n \end{cases} \qquad B_0^0(t) = 1$$

Theorem 1. $b_i^j(t) = \sum_{l=0}^{j} B_l^j(t) b_{i+l}$