ANA Ü7

8.) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[4]{n^3+n}}$

$$\frac{1}{\sqrt[4]{n^3+n}} = \frac{\frac{1}{\sqrt[4]{n^3}}}{\sqrt[4]{\frac{n^3}{n^3}+\frac{n}{n^3}}} = \frac{\frac{1}{\sqrt[4]{n^3}}}{\sqrt[4]{1+\frac{1}{n^2}}} = \frac{\sqrt[4]{\left(\frac{1}{n}\right)^3}}{\sqrt[4]{1+\frac{1}{n^2}}}$$

$$\lim_{n\to\infty} \frac{\sqrt[4]{\left(\frac{1}{n}\right)^3}}{\sqrt[4]{1+\frac{1}{n^2}}} = \frac{\sqrt[4]{0^3}}{\sqrt[4]{1+0}} = \frac{0}{1} = 0$$

$$\frac{1}{\sqrt[4]{n^3+n}} - \frac{1}{\sqrt[4]{(n+1)^3+n+1}} = \frac{1}{\sqrt[4]{n^3+n}} - \frac{1}{\sqrt[4]{n^3+3n^2+3n+1+n+1}}$$

$$= \frac{1}{\sqrt[4]{n^3+n}} - \frac{1}{\sqrt[4]{n^3+3n^2+4n+2}} \qquad \boxed{\sqrt[4]{n^3+n} < \sqrt[4]{n^3+3n^2+4n+2}}$$

$$\Rightarrow \frac{1}{\sqrt[4]{n^3+n}} > \frac{1}{\sqrt[4]{n^3+3n^2+4n+2}} \quad \Rightarrow \text{monoton fallend}$$

$\Rightarrow$ laut Leibnitz Kriterium konvergent

$\sum_{n=1}^{\infty} \frac{(-1)^n}{(n+1)n^{1+(-1)^n}}$ bei n gerade: $\frac{1}{(n+1)n^2}$ bei n ungerade: $\frac{-1}{n+1}$

$$\frac{\left|\frac{-1}{n+1}\right|}{\left|\frac{1}{(n+1)\cdot n^2}\right|} = \frac{(n+1)\cdot n^2}{n+1} = n^2$$

, d.h. für alle geraden n ist $\left|\frac{a_{n+1}}{a_n}\right| \geq 1$

$\Rightarrow$ divergiert, laut Quotientenkriterium