5.) $\langle K, +, \cdot, P \rangle$ ... angeordneter Körper $M \subseteq K$

zz: $\exists \sup(M) \iff \exists \inf(-M)$, dann gilt $-\sup(M) = \inf(-M)$

- $\exists \sup(M) \Rightarrow \exists \inf(-M)$

$\exists s \in K: \forall m \in M: m \leq s \Rightarrow \exists s \in K: \forall m \in M: -m \geq s \Rightarrow \exists s \in K: \forall m \in -M: s \leq m$

$\Rightarrow \exists \inf(-M)$

- $\exists \inf(-M) \Rightarrow \exists \sup(M)$

$\exists i \in K: \forall m \in -M: i \leq m \Rightarrow \exists i \in K: \forall m \in M: i \leq -m \Rightarrow \exists i \in K: \forall m \in M: i \geq m$

$\Rightarrow \exists \sup(M)$

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5.) - $\exists \sup(M) \wedge \exists \inf(-M) \Rightarrow -\sup(M) = \inf(-M)$

Sei $s = -\sup(M) \Rightarrow \sup(M) = -s$

$\forall m \in M: -s \geq m \Rightarrow \forall m \in M: -s - m \in P \cup \{0\} \Rightarrow \forall m \in M: -m - s \in P \cup \{0\}$

$\Rightarrow \forall m \in M: -m \geq s \Rightarrow \forall m \in -M: m \geq s \Rightarrow \forall m \in -M: s \leq m$

$\Rightarrow s = \inf(-M)$ □