ANA Ü10

5) $\sum_{n=2}^{\infty} \left(\sum_{m=2}^{\infty} \frac{1}{m^n} \right)$

$$\sum_{m=2}^{\infty} \left(\sum_{n=2}^{\infty} \frac{1}{m^n} \right) = \sum_{m=2}^{\infty} \left(\sum_{n=0}^{\infty} \frac{1}{m^n} \right) - 1 - \frac{1}{m} \Big) = \sum_{m=2}^{\infty} \Big(\sum_{n=0} \left(\frac{1}{m} \right)^n \Big) - 1 - \frac{1}{m} \Big)$$

da $\frac{1}{m} < 1$

$$= \sum_{m=2} \left(\frac{m}{m-1} - 1 - \frac{1}{m} \right) = \sum_{m=2} \left(\frac{m^2 - m^2 + m - m + 1}{m^2 - m} \right) = \sum_{m=2} \frac{1}{m^2 m}$$

$$= \sum_{m=2}^{\infty} \frac{1}{m \cdot (m-1)} = \sum_{m=1}^{\infty} \frac{1}{(m+1) \cdot m} \overset{\text{Teleskop-reihe}}{=} 1$$

$\Rightarrow \sum_{m=2}^{\infty} \left(\sum_{n=2}^{\infty} \frac{1}{m^n} \right)$ konvergiert gegen 1

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6) $|z| < 1$ zz: $\sum_{n=0}^{\infty} (n+1) z^n = (1-z)^{-2}$

$$\sum_{n=0}^{\infty} (n+1) z^n = \sum_{n=0}^{\infty} \left(\sum_{k=0}^{n} 1 \right) \cdot z^n = \sum_{n=0}^{\infty} \sum_{k=0}^{n} z^n = \sum_{n=0}^{\infty} \sum_{k=0}^{n} z^k \cdot z^{n-k}$$

$$= \sum_{n=0}^{\infty} z^n \cdot \sum_{n=0}^{\infty} z^n = \frac{1}{1-z} \cdot \frac{1}{1-z} = \frac{1}{(1-z)^2} = (1-z)^{-2}$$