

# LINAlg Ü3

1.11.2 a)  $(G, \cdot)$  ... kommutative Gruppe

$$\Psi: G \rightarrow G \quad \text{für } \forall n \in \mathbb{N}$$

$$x \mapsto x^n$$

zz:  $\Psi$  Homomorphismus

Seien  $a, b \in G$  beliebig.  $\Psi(a) \cdot \Psi(b) = a^n \cdot b^n = (a \cdot b)^n = \Psi(a \cdot b)$

b)  $\Psi: \mathbb{C}^x \rightarrow \mathbb{C}^x$  Homomorphismus  
 $x \mapsto x^4$

ges:  $\ker \Psi$

neutrales Element bzgl. von  $\mathbb{C}^x$ :  $1+0i$

$$\ker \Psi = \{a+bi : (a+bi)^4 = 1+0i\} = \{[1, 0], [1, 2 \cdot \frac{\pi}{4}], [1, 4 \cdot \frac{\pi}{4}], [1, 6 \cdot \frac{\pi}{4}]\}$$

$$= \{1+0i, 0+1i, -1+0i, 0-1i\} = \{1, -1, i, -i\}$$

c)  $\Psi^{-1}(\{16\}) = \{x \in \mathbb{C}^x : x^4 = 16\} = \{2, -2, 2i, -2i\}$

$$\Psi^{-1}(\{-1\}) = \{x \in \mathbb{C}^x : x^4 = -1\}$$

$$x^4 = -1 \Leftrightarrow x^4 + 1 = 0 \Leftrightarrow x^4 - i^2 = 0$$

$$\Leftrightarrow (x^2 + i)(x^2 - i) = 0 \Leftrightarrow (x^2 - (-i))(x - \sqrt{-i})(x + \sqrt{-i}) = 0$$

$$\Leftrightarrow (x + \sqrt{-i}) \cdot (x - \sqrt{-i}) \cdot (x - \sqrt{i}) \cdot (x + \sqrt{i}) = 0$$

$$\Rightarrow (x + \sqrt{-i}) = 0$$

$$(x - \sqrt{-i}) = 0$$

$$x = -\sqrt{-i}$$

$$x = \sqrt{-i}$$

$$(x - \sqrt{i}) = 0$$

$$(x + \sqrt{i}) = 0$$

$$x = \sqrt{i}$$

$$x = -\sqrt{i}$$

$$\Rightarrow \Psi^{-1}(\{-1\}) = \{-\sqrt{-i}, \sqrt{-i}, \sqrt{i}, -\sqrt{i}\}$$