ANA Ü5

1.) $f(x) = e^x$ $\quad g(x) = x^2$ $\quad$ zz: f, g auf ganz $\mathbb{R}$ konvex

$h(x) = \ln(x)$ $\quad$ zz: h ist auf $\mathbb{R}^+$ konkav

Aus dem 3. Bsp des letzten Übungsblatt wissen wir:

Falls f eine reellwertige, stetige Funktion auf einem Intervall I ist und im Inneren von I ableitbar ist, dann ist f genau dann konvex falls f' monoton wachsend ist.

$f'(x) = e^x$, da e^x auf ganz $\mathbb{R}$ monoton wachsend ist, ist f auf ganz $\mathbb{R}$ konvex.

$g'(x) = 2x$, da $2x$ auf ganz $\mathbb{R}$ monoton wachsend, folgt, dass g auf ganz $\mathbb{R}$ konvex.

Aus dem 1. Bsp des letzten Übungsblatt wissen wir:

Eine Funktion f heißt konkav, wenn $-f$ konvex ist.

$\tilde{h}(x) = -\ln(x)$ $\quad \tilde{h}'(x) = -\frac{1}{x}$, da $-\frac{1}{x}$ auf $\mathbb{R}^+$ monoton wachsend ist, ist $\tilde{h}$ auf $\mathbb{R}^+$ konvex und damit h auf $\mathbb{R}^+$ konkav.

ANA Ü5

3.) $\int_{-4}^{1} 5\cdot\frac{e^x-1}{e^x+1}\,dx$

$$\int 5\cdot\frac{e^x-1}{e^x+1}\,dx = 5\cdot\int\frac{e^x-1}{e^x+1}\,dx = 5\left(\int\frac{e^x}{e^x+1}\,dx - \int\frac{1}{e^x+1}\,dx\right)$$

$$\left[\begin{array}{llll} u=e^x+1 & x=\ln(u-1) & \frac{du}{dx}=e^x & dx=\frac{1}{e^x}du \\ m=e^x & x=\ln(m) & \frac{dm}{dx}=e^x & dx=\frac{1}{e^x}dm \end{array}\right.$$

$$=5\cdot\left(\int\frac{\cancel{e^x}}{u}\,\frac{1}{\cancel{e^x}}du - \int\frac{1}{m+1}\,\frac{1}{e^x}\,dm\right) = 5\cdot\left(\ln(u) - \int\frac{1}{m(m+1)}\,dm\right)$$

$$=5\cdot\left(\ln(e^x+1) - \int\frac{1}{m}-\frac{1}{m+1}\,dm\right) = 5\cdot\left(\ln(e^x+1) - \int\frac{1}{m}\,dm + \int\frac{1}{m+1}\,dm\right)$$

$$=5\cdot(\ln(e^x+1)-\ln(m)+\ln(m+1)) = 5(\ln(e^x+1)-\ln(e^x)+\ln(e^x+1))$$

$$=5\cdot(2\ln(e^x+1)-x) = 10\ln(e^x+1)-5x$$

$$\int_{-4}^{1} 5\cdot\frac{e^x-1}{e^x+1}\,dx = (10\ln(e^1+1)-5\cdot 1)-(10\cdot\ln(e^{-4}+1)-5\cdot(-4))$$

$$=10\ln(e+1)-5-(10\cdot\ln(e^{-4}+1)+20)$$

$$=10\ln(e+1)-10\ln(e^{-4}+1)-25 \approx -12{,}049$$

$\int_{1}^{5}\frac{x-\sqrt{x}}{x+\sqrt{x}}\,dx$

$$\int\frac{x-\sqrt{x}}{x+\sqrt{x}}\,dx = \int\frac{x-u}{x+u}\,2\sqrt{x}\,du \qquad u=\sqrt{x} \quad x=u^2 \quad \frac{du}{dx}=\frac{1}{2\sqrt{x}} \quad dx=2\sqrt{x}\,du$$

$$=2\int\frac{u^2-u}{u^2+u}\,u\,du = 2\int\frac{u(u-1)}{u+1}\,du \qquad \begin{array}{l} v=u+1 \\ u=v-1 \end{array} \quad \frac{du}{dv}=1 \quad du=dv$$

$$=2\int\frac{(v-1)(v-2)}{v}\,dv = 2\int\frac{v^2-2v-v+2}{v}\,dv = 2\int\frac{v^2-3v+2}{v}\,dv$$

$$=2\left(\int\frac{v^2}{v}\,dv - 3\int\frac{v}{v}\,dv + 2\int\frac{1}{v}\,dv\right) = 2\left(\int v\,dv - 3\int 1\,dv + 2\int\frac{1}{v}\,dv\right)$$

$$=2\left(\frac{v^2}{2}-3v+2\ln(v)\right) = v^2-6v+4\ln(v) = (u+1)^2-6(u+1)+4\ln(u+1)$$

$$=(\sqrt{x}+1)^2-6(\sqrt{x}+1)+4\ln(\sqrt{x}+1) = x+2\sqrt{x}+1-6\sqrt{x}-6+4\ln(\sqrt{x}+1)$$

$$=x-4\sqrt{x}-5+4\ln(\sqrt{x}+1)$$

$$\int_{1}^{5}\frac{x-\sqrt{x}}{x+\sqrt{x}} = 5-4\sqrt{5}-5+4\ln(\sqrt{5}+1)-(1-4\sqrt{1}-5+4\ln(\sqrt{1}+1)) \approx 0{,}98058$$

ANA Ü5

4.) $\int_{\frac{\pi}{8}}^{\frac{\pi}{2}} \frac{1}{\sin(x)}\,dx$

$\int \frac{1}{\sin(x)}dx = \int \frac{\sin(x)}{(\sin(x))^2}dx = \int \frac{\sin(x)}{1-(\cos(x))^2}$ $\quad \left[u = \cos(x) \quad \frac{du}{dx} = -\sin(x) \quad dx = \frac{1}{-\sin(x)}du \right.$

$= \int \frac{\sin(x)}{1-u^2}\,\frac{1}{-\sin(x)}du = -\int \frac{1}{1-u^2}du = -\int \frac{1}{(1+u)(1-u)}du$

$\frac{1}{(1+u)(1-u)} = \frac{A}{1+u} + \frac{B}{1-u} = \frac{A(1-u)+B(1+u)}{(1+u)(1-u)} = \frac{A+B+(B-A)u}{(1+u)(1-u)}$

$\Rightarrow A+B=1 \qquad B-A=0 \quad \Rightarrow A=B \quad \Rightarrow A=B=\frac{1}{2}$

$\Rightarrow \frac{1}{(1+u)(1-u)} = \frac{1}{2}\cdot\left(\frac{1}{1+u}\right) + \frac{1}{2}\cdot\left(\frac{1}{1-u}\right)$

$-\int \frac{1}{(1+u)(1-u)}du = -\int \frac{1}{2}\cdot\left(\frac{1}{1+u}+\frac{1}{1-u}\right)du = -\frac{1}{2}\cdot\left(\int\frac{1}{1+u}du - \int\frac{1}{u-1}du\right)$

$= -\frac{1}{2}\cdot\left(\ln(1+u) - \ln(u-1)\right) = -\frac{1}{2}\cdot\left(\ln(1+\cos(x)) - \ln(\cos(x)-1)\right)$

$= -\frac{1}{2}\cdot\left(\ln\left(\frac{1+\cos(x)}{\cos(x)-1}\right)\right) = \frac{1}{2}\cdot\left(\ln\left(\frac{\cos(x)-1}{\cos(x)+1}\right)\right) = \frac{1}{2}\cdot\ln\left(\frac{(\cos(x))^2-1}{(\cos(x)+1)^2}\right)$

$= \frac{1}{2}\cdot\ln\left(\frac{(\sin(x))^2}{(\cos(x)+1)^2}\right) = \ln\left(\frac{\sin(x)}{\cos(x)+1}\right)$

$\int_{\frac{\pi}{8}}^{\frac{\pi}{2}} \frac{1}{\sin(x)}dx = \ln\left(\frac{\sin(\frac{\pi}{2})}{\cos(\frac{\pi}{2})+1}\right) - \ln\left(\frac{\sin(\frac{\pi}{8})}{\cos(\frac{\pi}{8})+1}\right) \approx 1{,}615$

$\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{1}{(\sin(x))^2\cdot(\cos(x))^4}dx$

$\int \frac{1}{(\sin(x))^2\cdot(\cos(x))^4}dx = \int \frac{1}{(\sin(x))^2}\cdot\left(\frac{1}{(\cos(x))^2}\right)^2 dx = \int \frac{(\sin(x))^2+(\cos(x))^2}{(\sin(x))^2}\cdot\left(\frac{(\sin(x))^2+(\cos(x))^2}{(\cos(x))^2}\right)^2 dx$

$= \int\left(1+\frac{(\cos(x))^2}{(\sin(x))^2}\right)\cdot\left((\tan(x))^2+1\right)^2 dx = \int\left(1+\frac{1}{(\tan(x))^2}\right)\cdot\left((\tan(x))^2+1\right)^2 dx$

$\left[u = \tan(x) \quad \frac{du}{dx} = \frac{1}{(\cos(x))^2} \quad dx = (\cos(x))^2 du = \frac{1}{\frac{1}{(\cos(x))^2}}du = \frac{1}{(\tan(x))^2+1}du \right.$

$= \int\left(1+\frac{1}{u^2}\right)\cdot(u^2+1)^2\cdot\frac{1}{u^2+1}du = \int u^2+1+1+\frac{1}{u^2}du = \int u^2+\frac{1}{u^2}+2\,du$

$= \int u^2 du + \int\frac{1}{u^2}du + \int 2\,du = \frac{u^3}{3} - \frac{1}{u} + 2u = \frac{(\tan(x))^3}{3} - \frac{1}{\tan(x)} + 2\tan(x)$

$\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{1}{(\sin(x))^2\cdot(\cos(x))^4}dx = \frac{(\tan(\frac{\pi}{4}))^3}{3} - \frac{1}{\tan(\frac{\pi}{4})} + 2\tan(\frac{\pi}{4}) - \left(\frac{(\tan(\frac{\pi}{8}))^3}{3} - \frac{1}{\tan(\frac{\pi}{8})} + 2\tan(\frac{\pi}{8})\right)$

$= \frac{1}{3} - 1 + 2 - \left(\frac{(\tan(\frac{\pi}{8}))^3}{3} - \frac{1}{\tan(\frac{\pi}{8})} + 2\tan(\frac{\pi}{8})\right) \approx 2{,}895$

ANA Ü5

5.) $m, n \in \mathbb{Z}$ ges: $\int_0^{2\pi} \exp(i \cdot n \cdot t) \cdot \exp(-i \cdot m \cdot t)\, dt$

$\int \exp(int) \cdot \exp(-imt)\, dt = \int \exp(it \cdot (n-m))\, dt$

$m \neq n$: $\left[u = it(n-m) \quad \frac{du}{dt} = i(n-m) \qquad dt = \frac{1}{i(n-m)}\, du \right.$

$= \int \exp(u) \cdot \frac{1}{i(n-m)}\, du = \frac{1}{i(n-m)} \cdot \int \exp(u)\, du = \frac{1}{i(n-m)} \cdot \exp(u) = \frac{\exp(i \cdot t(n-m))}{i(n-m)}$

$\int_0^{2\pi} \exp(int) \cdot \exp(-imt)\, dt = \frac{\exp(i \cdot 2\pi (n-m))}{i(n-m)} - \frac{\exp(i \cdot 0 \cdot (n-m))}{i(n-m)}$

$= \frac{1}{i(n-m)} - \frac{1}{i(n-m)} = 0$

$m = n$: $\int \exp(it \cdot (n-m))\, dt = \int \exp(0)\, dt = \exp(0) = 1$

$\int_0^{2\pi} \sin(n \cdot t) \cdot \sin(m \cdot t)\, dt = \int_0^{2\pi} \frac{\exp(int) - \exp(-int)}{2i} \cdot \frac{\exp(imt) - \exp(-imt)}{2i}\, dt$

$= \int_0^{2\pi} \frac{1}{-4} \cdot (\exp(int) - \exp(-int)) \cdot (\exp(imt) - \exp(-imt)))\, dt$

$= -\frac{1}{4} \left(\int_0^{2\pi} \exp(int) \cdot \exp(imt) - \int_0^{2\pi} \exp(int) \cdot \exp(-imt) - \int_0^{2\pi} \exp(imt) \cdot \exp(-int) + \int_0^{2\pi} \exp(-int) \exp(-imt) \right)$

$m \neq n$: $-\frac{1}{4} \cdot (0 - 0 - 0 + 0) = 0$

$m = n$: $-\frac{1}{4} \cdot \left(\int_0^{2\pi} \exp(it(2n))\, dt - 1 - 1 + \int_0^{2\pi} \exp(-it(2n))\, dt \right)$

$\int \exp(it(2n))\, dt = \qquad u = 2itn \quad \frac{du}{dt} = 2in \quad dt = \frac{1}{2in}\, du$

$= \int \exp(u) \frac{1}{2in}\, du = \frac{1}{2in} \exp(u) = \frac{1}{2in} \exp(2itn)$

$\int_0^{2\pi} \exp(2itn)\, dt = \frac{1}{2in} \cdot \exp(2i 2\pi n) - \frac{1}{2in} \exp(2i0n) = 0$

analog für $\int_0^{2\pi} \exp(-it2n)\, dt$

$= -\frac{1}{4} \cdot (0 - 1 - 1 + 0) = -\frac{1}{4} \cdot (-2) = \frac{1}{2}$

$\int_0^{2\pi} \cos(n \cdot t) \cdot \cos(m \cdot t)\, dt = \int_0^{2\pi} \frac{\exp(int) + \exp(-int)}{2} \cdot \frac{\exp(imt) + \exp(-imt)}{2}\, dt$

$= \frac{1}{4} \cdot \int_0^{2\pi} \exp(int) \exp(imt) + \exp(int) \cdot \exp(-imt) + \exp(-int) \exp(imt) + \exp(-int) \exp(-imt)\, dt$

$m \neq n$: $\frac{1}{4} \cdot (0 + 0 + 0 + 0) = 0 \qquad m = n$: $\frac{1}{4} \cdot (0 + 1 + 1 + 0) = \frac{1}{2}$

ANA Ü5

5.) ... $\int_0^{2\pi} \sin(nt)\cdot\cos(mt)\,dt = \int_0^{2\pi} \frac{\exp(int)-\exp(-int)}{2i}\cdot\frac{\exp(imt)+\exp(-imt)}{2}\,dt$

$= \frac{1}{4i}\int_0^{2\pi} \exp(int)\exp(imt)+\exp(int)\exp(-imt)-\exp(-int)\cdot\exp(imt)-\exp(-int)\exp(-imt)\,dt$

$m\neq n: \frac{1}{4i}\cdot(0)=0 \qquad m=n: \frac{1}{4i}(0+1-1-0)=0$

ANA Ü5

6.) $f : [a,b) \to [0,+\infty)$ $\quad \forall x \in [a,b) : f|_{[a,x]}$ ... Riemann-Integrierbar

zz: $\left(\exists C > 0 \; \forall x \in [a,b) : \int_a^x f(t)\,dt \leq C\right) \Rightarrow \left(\int_a^b f(t)\,dt \text{ ... konvergiert}\right)$

$\int_a^b f(t)\,dt$ ... konvergiert $\Leftrightarrow \lim_{\beta \to b-} \int_a^\beta f(t)\,dt$ existiert

Offensichtlich gilt $\forall x,y \in [a,b), x<y : \int_a^x f(t)\,dt \leq \int_a^y f(t)\,dt$, da f nach $\mathbb{R}^+$ abbildet. $\Rightarrow \left(\int_a^\beta f(t)\,dt\right)_{\beta \in [a,b)}$ ist monoton wachsend. Da das Netz auch durch C nach oben beschränkt ist muss der Grenzwert existieren. $\Rightarrow \int_a^b f(t)\,dt$ konvergiert $\square$

ANA Ü5

9.) $\alpha \in \mathbb{R}\setminus\{0\}$ Für welche α existieren: $\int_0^1 x^\alpha dx$, $\int_1^{+\infty} x^\alpha dx$, $\int_0^{+\infty} x^\alpha dx$

$$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}$$

$$\int_0^1 x^\alpha dx = \frac{1^{\alpha+1}}{\alpha+1} = \frac{1}{\alpha+1}$$ existiert für alle $\alpha \in \mathbb{R}\setminus\{0,-1\}$

$$\int_1^{+\infty} x^\alpha dx = \lim_{\beta\to+\infty} \int_1^\beta x^\alpha dx = \lim_{\beta\to+\infty} \frac{\beta^{\alpha+1}}{\alpha+1} - \frac{1^{\alpha+1}}{\alpha+1} = \lim_{\beta\to+\infty} \frac{\beta^{\alpha+1}-1}{\alpha+1}$$

1\. Fall $\alpha = -1$: $\lim_{\beta\to+\infty} \frac{\beta^{-1+1}-1}{-1+1} = \lim_{\beta\to+\infty} \frac{1-1}{0} \Rightarrow$ existiert nicht

2\. Fall $\alpha < -1$: $\lim_{\beta\to+\infty} \frac{\beta^{\alpha+1}-1}{\alpha+1} = \frac{1}{\alpha+1} \cdot \lim_{\beta\to+\infty} \underbrace{\beta^{\alpha+1}}_{\to 0} - 1 = -\frac{1}{\alpha+1}$ $\Rightarrow$ existiert

3\. Fall $\alpha > -1$: $\lim_{\beta\to+\infty} \frac{\beta^{\alpha+1}-1}{\alpha+1} = +\infty$ $\Rightarrow$ existiert nicht

$$\int_0^{+\infty} x^\alpha dx = \int_0^1 x^\alpha dx + \int_1^{+\infty} x^\alpha dx = \frac{1}{\alpha+1} + \int_1^{+\infty} x^\alpha dx$$ für $\alpha \neq -1$

$\Rightarrow$ existiert falls $\alpha < -1$

$$\int_1^{+\infty} \frac{\ln(t)}{t^2} dt$$

$$\int \frac{\ln(t)}{t^2} dt = \int \ln(t)\cdot\left(-\frac{1}{t}\right)' dt = -\frac{1}{t}\cdot\ln(t) - \int -\frac{1}{t}(\ln(t))' dt$$

$$= -\frac{\ln(t)}{t} - \int -\frac{1}{t}\cdot\frac{1}{t} dt = -\frac{\ln(t)}{t} + \int \frac{1}{t^2} dt = \frac{t^{-1}}{-1} - \frac{\ln(t)}{t}$$

$$= -\frac{1}{t} - \frac{\ln(t)}{t} = \frac{-\ln(t)-1}{t}$$

$$\int_1^{+\infty} \frac{\ln(t)}{t^2} dt = \lim_{\beta\to+\infty} \int_1^\beta \frac{\ln(t)}{t^2} dt = \lim_{\beta\to+\infty} \frac{-\ln(\beta)-1}{\beta} - \frac{-\ln(1)-1}{1}$$

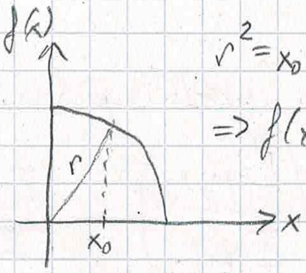
$$= \lim_{\beta\to+\infty} \frac{-\ln(\beta)-1}{\beta} - (0-1) = \lim_{\beta\to+\infty} \frac{-\ln(\beta)-1+\beta}{\beta}$$

$$= \lim_{\beta\to+\infty}\left(-\frac{\ln(\beta)}{\beta}\right) - \lim_{\beta\to+\infty}\left(\frac{1}{\beta}\right) + \lim_{\beta\to+\infty}\left(\frac{\beta}{\beta}\right) = 1 - \lim_{\beta\to+\infty} \frac{\overbrace{\ln(\beta)}^{\to+\infty}}{\underbrace{\beta}_{\to+\infty}}$$

$$= 1 - \lim_{\beta\to+\infty} \frac{\frac{1}{\beta}}{1} = 1 - \lim_{\beta\to+\infty} \frac{1}{\beta} = 1$$

ANA Ü5

8.) $f(x)=\sqrt{r^2-x^2}$ $\qquad r>0$



$r^2=x_0^2+f(x_0)^2$

$\Rightarrow f(x_0)=\sqrt{r^2-x_0^2}$

$\int f(x)dx=\int\sqrt{r^2-x^2}\,dx=$

$\left[\; x=r\cdot\sin(u) \quad u=\arcsin\left(\frac{x}{r}\right) \quad \frac{du}{dx}=\frac{1}{\sqrt{1-\left(\frac{x}{r}\right)^2}}\cdot\frac{1}{r}=\frac{1}{r\cdot\sqrt{1-\frac{x^2}{r^2}}}\right.$

$\left[\; dx=r\cdot\sqrt{1-\frac{x^2}{r^2}}\,du=r\cdot\cos(u)du\right.$, da $\cos(\arcsin(x))=\sqrt{1-x^2}$

$\int\sqrt{r^2-x^2}dx=\int\sqrt{r^2-(r\cdot\sin(u))^2}\cdot\cos(u)\cdot r\,du=\int\sqrt{r^2\cdot(1-(\sin(u))^2)}\cdot r\cdot\cos(u)du$

$=\int r^2\cdot\sqrt{(\cos(u))^2}\cdot\cos(u)du=r^2\cdot\int(\cos(u))^2du=r^2\cdot\int\frac{1}{2}(\cos(u))^2+\frac{1}{2}(\cos(u))^2du$

$=r^2\cdot\int\frac{1}{2}(\cos(u))^2+\frac{1}{2}(1-(\sin(u))^2)du=r^2\cdot\int\frac{1}{2}((\cos(u))^2-(\sin(u))^2+1)du$

$=r^2\cdot\int\frac{1}{2}(\cos(u)\cos(u)-\sin(u)\sin(u)+1)du=r^2\cdot\int\frac{1}{2}(\cos(2\cdot)+1)du$

$=\frac{r^2}{2}\cdot\left(\int\cos(2u)du+\int 1\,du\right)=\frac{r^2}{2}\cdot\left(\int\cos(2u)du+u\right)$

$\left[\; m=2u \quad u=\frac{m}{2} \quad \frac{dm}{du}=2 \quad du=\frac{1}{2}dm\right.$

$=\frac{r^2}{2}\cdot\left(\int\cos(m)\cdot\frac{1}{2}dm+u\right)=\frac{r^2}{2}\cdot\left(\frac{1}{2}(-\sin(m))+u\right)=\frac{r^2}{2}\cdot\left(-\frac{\sin(2u)}{2}+u\right)$

$=\frac{r^2}{2}\cdot\left(\arcsin\left(\frac{x}{r}\right)-\frac{\sin\left(2\arcsin\left(\frac{x}{r}\right)\right)}{2}\right)$

$\int_0^r f(x)=\frac{r^2}{2}\cdot\left(\arcsin\left(\frac{r}{r}\right)-\frac{\sin\left(2\cdot\arcsin\left(\frac{r}{r}\right)\right)}{2}\right)-\frac{r^2}{2}\cdot\left(\arcsin\left(\frac{0}{r}\right)-\frac{\sin\left(2\arcsin\left(\frac{0}{r}\right)\right)}{2}\right)$

$=\frac{r^2}{2}\cdot\left(\frac{\pi}{2}-\frac{\sin\left(2\cdot\frac{\pi}{2}\right)}{2}\right)-\frac{r^2}{2}\cdot\left(0-\frac{\sin(2\cdot 0)}{2}\right)=\frac{r^2}{2}\cdot\left(\frac{\pi}{2}-0\right)-\frac{r^2}{2}\cdot(-0)$

$=\frac{r^2\cdot\pi}{4}$

$\Rightarrow$ Fläche eines Kreises mit Radius $r=4\cdot\frac{r^2\cdot\pi}{4}=r^2\cdot\pi$

$\{(x,y)\in\mathbb{R}^2: x^2+y^2\leq 1,\ y\geq x^2\}$ $\qquad x^2+y^2\leq 1 \Leftrightarrow y^2\leq 1-x^2 \Leftrightarrow y\leq\sqrt{1-x^2}$

Fläche eines Kreises mit Radius 1 $\cap$ $y\geq x^2$

$y=x^2 \wedge y=\sqrt{1-x^2} \Rightarrow y=\sqrt{1-y} \Leftrightarrow y^2=1-y \Leftrightarrow y^2+y-1=0$

$\Leftrightarrow y_{1,2}=-\frac{1}{2}\pm\sqrt{\frac{1}{4}+1}=-\frac{1}{2}\pm\sqrt{\frac{5}{4}}=-\frac{1}{2}\pm\frac{\sqrt{5}}{2}=\frac{\sqrt{5}}{2}-\frac{1}{2}$, da $y>0$ $(y\geq x^2)$

$x=\sqrt{y}=\pm\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}$

$\int_{-\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}}^{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\sqrt{1-x^2}dx=2\cdot\int_0^{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\sqrt{1-x^2}dx\approx 2\cdot 0{,}6952$ (da Symmetrisch bzgl. y-Achse)

$\int x^2dx=\frac{x^3}{3}$ $\qquad 2\cdot\int_0^{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}}x^2=2\cdot\left(\frac{\left(\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)^3}{3}-\frac{0^3}{3}\right)\approx 2\cdot 0{,}162$

$\Rightarrow$ Fläche $=2\cdot 0{,}6952-2\cdot 0{,}162\approx 1{,}0664$