ANA Ü10

8) i) $\lim\limits_{x\to+\infty} \sqrt{x}\left(\sqrt{x+1}-\sqrt{x}\right)$

$$\sqrt{x}\left(\sqrt{x+1}-\sqrt{x}\right)=\sqrt{x^2+x}-\sqrt{x^2}=\frac{\left(\sqrt{x^2+x}-\sqrt{x^2}\right)\left(\sqrt{x^2+x}+\sqrt{x^2}\right)}{\left(\sqrt{x^2+x}+\sqrt{x^2}\right)}$$

$$=\frac{x^2+x-x^2}{\sqrt{x^2+x}+\sqrt{x^2}}=\frac{x}{\sqrt{x^2+x}+x}=\frac{\frac{x}{x}}{\frac{\sqrt{x^2+x}}{x}+\frac{x}{x}}=\frac{1}{\sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}}+1}=\frac{1}{\sqrt{1+\frac{1}{x}}+1}$$

$$\lim_{x\to+\infty}\frac{1}{\sqrt{1+\frac{1}{x}}+1}=\frac{1}{\sqrt{1+0}+1}=\frac{1}{2}$$

ii) $\lim\limits_{x\to 0}\frac{x^2}{|x|+x^2}$

$$\frac{x^2}{|x|+x^2}=\frac{\frac{x^2}{x^2}}{\frac{|x|}{x^2}+\frac{x^2}{x^2}}=\frac{1}{\frac{|x|}{|x^2|}+1}=\frac{1}{\left|\frac{x}{x^2}\right|+1}=\frac{1}{\frac{1}{|x|}+1}$$

$$\lim_{x\to 0}\frac{1}{\frac{1}{|x|}+1}=0, \quad \text{da } \lim_{x\to 0}\frac{1}{|x|}=+\infty$$

iii) $\lim\limits_{x\to\xi}\frac{x^k-\xi^k}{x-\xi}\overset{?}{=}k\cdot\xi^{k-1}$ für $\xi\in\mathbb{R}$ und $k\in\mathbb{N}$

$$x^k-\xi^k=(x-\xi)\sum_{j=0}^{k-1}x^{k-1-j}\xi^j$$

$$\Rightarrow \frac{x^k-\xi^k}{x-\xi}=\sum_{j=0}^{k-1}x^{k-1-j}\xi^j$$

$$\lim_{x\to\xi}\sum_{j=0}^{k-1}x^{k-1-j}\xi^j=\sum_{j=0}^{k-1}\xi^{k-1-j}\xi^j=\sum_{j=0}^{k-1}\xi^{k-1-j+j}=k\cdot\xi^{k-1}$$

$\square$