3.) $z, w \in \mathbb{C}$ $\quad z = a + ib \quad w = c + id$

- zz: $|zw| = |z| \cdot |w|$

Zwischenbew: $z \cdot \overline{z} = |z|^2$ $\quad z \cdot \overline{z} = (a+ib)(a-ib) = (a^2+b^2) + i(ab-ab) = a^2+b^2$

$$= \sqrt{a^2+b^2}^2 = |z|^2$$

$\overline{z \cdot w} = \overline{z} \cdot \overline{w}$ $\quad \overline{z \cdot w} = \overline{(a+ib) \cdot (c+id)} = \overline{(ac-bd) + i(ad+bc)} = (ac-bd) + i(-ad-bc)$

$$= (a+i(-b)) \cdot (c+i(-d)) = \overline{(a+ib)} \cdot \overline{(c+id)} = \overline{z} \cdot \overline{w}$$

$$|z \cdot w|^2 = (z \cdot w) \cdot \overline{(z \cdot w)} = z \cdot w \cdot \overline{z} \cdot \overline{w} = z \cdot \overline{z} \cdot w \cdot \overline{w} = |z|^2 \cdot |w|^2$$

$$\Rightarrow |z \cdot w| = |z| \cdot |w|$$

- zz: $|z+w| \leq |z| + |w|$

Zwischenbew: $\overline{z+w} = \overline{z} + \overline{w}$ $\quad \overline{z+w} = \overline{(a+ib) + (c+id)} = \overline{(a+c + i(b+d))} = (a+c) + i(-b-d)$

$$= (a + i(-b)) + (c + i(-d)) = \overline{(a+ib)} + \overline{(c+id)} = \overline{z} + \overline{w}$$

$(w \cdot z) + \overline{(w \cdot z)} = 2 \cdot \mathrm{Re}(w \cdot z)$ $\quad wz + \overline{wz} = (a+ib) \cdot (c+id) + (a+i(-b)) \cdot (c+i(-d))$

$$= (a \cdot c - b \cdot d) + i(ad+bc) + (ac-bd) + i(-ad-bc) = 2\,\mathrm{Re}(w \cdot z)$$

3.) ... zz: $|z+w| \le |z|+|w|$ $z=a+ib$ $w=c+id$

Bew: $|z+w|^2 = (z+w)\overline{(z+w)} = (z+w)\cdot(\bar{z}+\bar{w}) = z\cdot\bar{z} + z\cdot\bar{w} + w\cdot\bar{z} + w\cdot\bar{w}$

$= |z|^2 + |w|^2 + z\cdot\bar{w} + \overline{z\cdot\bar{w}} = |z|^2 + |w|^2 + 2\cdot Re(z\cdot\bar{w})$

$\le |z|^2 + |w|^2 + 2\cdot|z\cdot\bar{w}| = |z|^2 + |w|^2 + 2\cdot|z|\cdot|\bar{w}| = (|z|+|w|)^2$

$\Rightarrow |z+w| \le |z|+|w|$ ✓

zz: $||z|-|w|| \le |z-w|$

Bew: $|z| = |z-w+w| = |(z-w)+w| \le |z-w|+|w|$

$\Rightarrow |z|-|w| \le |z-w|$

o.B.d.A: $|z| \geq |w| \Rightarrow |z|-|w| > 0 \Rightarrow |z|-|w| = ||z|-|w||$

$\Rightarrow ||z|-|w|| \le |z-w|$