ANA Ü3

1.) $n \in \mathbb{N}$ $\quad g: [0,1] \to \mathbb{R}$ $\qquad$ ges: $\|g\|_\infty$

$$x \mapsto x^n(1-x)^2$$

$$x^n\cdot(1-x)^2 = x^n - 2x\cdot x^n + x^2\cdot x^n = x^{n+2} - 2x^{n+1} + x^n$$

$$(x^{n+2} - 2x^{n+1} + x^n)' = (n+2)x^{n+1} - (2n+2)x^n + n\cdot x^{n-1}$$

$$x^{n-1}\cdot((n+2)\cdot x^2 - (2n+2)\cdot x + n) = 0 \quad \Rightarrow x^{n-1} = 0 \text{ oder}$$

$$x_{1,2} = \frac{(2n+2) \pm \sqrt{(2n+2)^2 - 4(n+2)n}}{2(n+2)} = \frac{2n+2 \pm \sqrt{\cancel{4n^2} + \cancel{8n} + 4 - \cancel{4n^2} - \cancel{8n}}}{2n+4}$$

$$= \frac{2n+2 \pm 2}{2n+4} \quad \begin{matrix} \nearrow 1 \\ \searrow \frac{2n}{2n+4} = \frac{n}{n+2} \end{matrix}$$

Bei $x=0$: $\quad g(0) = 0^n(1-0)^2 = 0$

Bei $x=1$: $\quad g(1) = 1^n(1-1)^2 = 1^n\cdot 0^2 = 0$

Bei $x = \frac{n}{n+2}$ $\quad g\left(\frac{n}{n+2}\right) = \left(\frac{n}{n+2}\right)^n\left(1 - \frac{n}{n+2}\right)^2 = \frac{n^2}{(n+2)^n}\left(\frac{n+2-n}{n+2}\right)^2$

$$= \frac{n^2}{(n+2)^n}\,\frac{4}{(n+2)^2} = \frac{4n^2}{(n+2)^{n+2}} > 0$$

Da g auf $[0,1]$ ein Minimum und Maximum haben muss und für das Maximum nur $g\left(\frac{n}{n+2}\right)$ in Frage kommt

$$\Rightarrow \|g\|_\infty = g\left(\frac{n}{n+2}\right) = \frac{4n^2}{(n+2)^{n+2}}$$

ANA Ü3

3.) $n \in \mathbb{N}$

•) $\lim\limits_{x\to 0} \frac{1}{x^n} \cdot \exp(-\frac{1}{x^2}) = \lim\limits_{x\to 0} \frac{1}{x^n \cdot \exp(\frac{1}{x^2})}$

$\lim\limits_{x\to 0} x^n \cdot \exp(\frac{1}{x^2}) = \lim\limits_{x\to 0} \exp(\ln(x^n)) \cdot \exp(\frac{1}{x^2}) = \lim\limits_{x\to 0} \exp(n\cdot \ln(x) + \frac{1}{x^2})$

$\lim\limits_{x\to 0} \frac{1}{x^2} + n\cdot \ln(x) = \lim\limits_{x\to 0} \frac{1+n\cdot\ln(x)\cdot x^2}{x^2} = \lim\limits_{x\to 0} \frac{1}{x^2}\cdot(1+n\cdot\ln(x)\cdot x^2)$

$= \lim\limits_{x\to 0} \frac{1}{x^2}\cdot(\lim\limits_{x\to 0} 1 + \lim\limits_{x\to 0} n\cdot\ln(x)\cdot x^2) = \lim\limits_{x\to 0}\frac{1}{x^2}\cdot(1+n\cdot\lim\limits_{x\to 0}\frac{\ln(x)}{\frac{1}{x^2}})$

$= \lim\limits_{x\to 0}\frac{1}{x^2}\cdot(1+n\cdot\lim\limits_{x\to 0}\frac{\frac{1}{x}}{-2\cdot\frac{1}{x^3}})$ (, da $\lim\limits_{x\to 0}\frac{1}{x^2} = +\infty$)

$= \lim\limits_{x\to 0}\frac{1}{x^2}\cdot(1+n\cdot\lim\limits_{x\to 0}\frac{x^3}{-2x}) = \lim\limits_{x\to 0}\frac{1}{x^2}\cdot(1+n\cdot 0) = \lim\limits_{x\to 0}\frac{1}{x^2} = +\infty$

$\Rightarrow \lim\limits_{x\to 0}\exp(\frac{1}{x^2}+n\cdot\ln(x)) = +\infty \quad \Rightarrow \lim\limits_{x\to 0}\frac{1}{x^n\cdot\exp(\frac{1}{x^2})} = 0$

•) $\lim\limits_{x\to +\infty}\frac{x\cdot\ln(x)}{x^2-1} = \lim\limits_{x\to\infty}\frac{1\cdot\ln(x)+x\cdot\frac{1}{x}}{2x}$ (, da $\lim\limits_{x\to\infty} x^2-1 = +\infty$)

$= \lim\limits_{x\to\infty}\frac{\ln(x)+1}{2x} = \lim\limits_{x\to\infty}\frac{\frac{1}{x}}{2}$ (, da $\lim\limits_{x\to\infty} 2x = +\infty$)

$= \lim\limits_{x\to\infty}\frac{1}{2x} = 0$

•) $\lim\limits_{x\to 0}\frac{1-\cos(nx)}{\sin(n^2x^2)} = \lim\limits_{x\to 0}\frac{0+\sin(nx)\cdot n}{\cos(n^2x^2)\cdot n^2\cdot 2\cdot x}$ (, da $\lim\limits_{x\to 0} 1-\cos(nx) = 0$ und $\lim\limits_{x\to 0}\sin(n^2x^2) = 0$)

$= \lim\limits_{x\to 0}\frac{\sin(nx)}{2nx\cdot\cos(n^2x^2)} = \lim\limits_{x\to 0}\frac{\cos(nx)\cdot n}{2n(1\cdot\cos(n^2x^2)+x\cdot(-\sin(n^2x^2))\cdot n^2\cdot 2\cdot x)}$

(, da $\lim\limits_{x\to 0}\sin(nx) = 0$ und $\lim\limits_{x\to 0} 2nx\cdot\cos(n^2x^2) = 0$ (Nullfolge mal beschränkte Folge))

$= \lim\limits_{x\to 0}\frac{\cos(nx)}{2\cdot\cos(n^2x^2) - 4n^2x^2\cdot\sin(n^2x^2)} = \frac{1}{2-0} = \frac{1}{2}$

ANA Ü3

4.) •) $\lim\limits_{x \to \frac{\pi}{2}-} \frac{1+\frac{1}{\cos(x)}}{1+\tan(x)} = \lim\limits_{x \to \frac{\pi}{2}-} \frac{-(\cos(x))^{-2} \cdot (-\sin(x))}{\frac{1}{(\cos(x))^2}}$ $\Big($, da $\lim\limits_{x \to \frac{\pi}{2}-} \tan(x) = +\infty\Big)$

$= \lim\limits_{x \to \frac{\pi}{2}-} \frac{\sin(x) \cdot \frac{1}{(\cos(x))^2}}{\frac{1}{(\cos(x))^2}} = \lim\limits_{x \to \frac{\pi}{2}-} \sin(x) = 1$

•) $\lim\limits_{x \to 0+} (1+x)^{\frac{1}{x}} = \lim\limits_{x \to 0+} \exp\left(\frac{1}{x} \cdot \ln(1+x)\right)$

$\lim\limits_{x \to 0+} \frac{\ln(1+x)}{x} = \lim\limits_{x \to 0+} \frac{\frac{1}{1+x} \cdot 1}{1}$ $\Big($, da $\lim\limits_{x \to 0+} \ln(1+x) = \ln(1) = 0$ und $\lim\limits_{x \to 0+} x = 0\Big)$

$= \lim\limits_{x \to 0+} \frac{1}{1+x} = 1$

$\Rightarrow \lim\limits_{x \to 0+} \exp\left(\frac{\ln(1+x)}{x}\right) = \exp(1) = e$

•) $\lim\limits_{x \to +\infty} \left(\frac{x+2}{x-1}\right)^x = \lim\limits_{x \to +\infty} \exp\left(x \cdot \ln\left(\frac{x+2}{x-1}\right)\right)$

$\lim\limits_{x \to +\infty} \frac{\ln\left(\frac{x+2}{x-1}\right)}{\frac{1}{x}} = \lim\limits_{x \to +\infty} \frac{\frac{x-1}{x+2} \cdot \left(\frac{x+2}{x-1}\right)'}{-\frac{1}{x^2}}$ $\Big($, da $\lim\limits_{x \to +\infty} \frac{1}{x} = 0$ und $\lim\limits_{x \to +\infty} \ln\left(\frac{x+2}{x-1}\right) = \lim\limits_{x \to +\infty} \ln\left(\frac{1+\frac{2}{x}}{1-\frac{1}{x}}\right) = \lim\limits_{x \to +\infty} \ln(1) = 0\Big)$

$= \lim\limits_{x \to +\infty} \frac{\frac{\cancel{(x-1)}}{(x+2)} \cdot \frac{1 \cdot (x-1) - 1 \cdot (x+2)}{(x-1)^{\cancel{2}}}}{-\frac{1}{x^2}}$

$= \lim\limits_{x \to +\infty} - \frac{x^2 \cdot (x-1-x-2)}{(x-1)(x+2)} = \lim\limits_{x \to +\infty} - \frac{-3x^2}{x^2+2x-x-2} = \lim\limits_{x \to +\infty} \frac{3x^2}{x^2+x-2}$

$= \lim\limits_{x \to +\infty} \frac{3}{1+\frac{1}{x}-\frac{2}{x^2}} = 3$

$\Rightarrow \lim\limits_{x \to +\infty} \exp\left(x \cdot \ln\left(\frac{x+2}{x-1}\right)\right) = \exp(3) = e^3$

ANA Ü3

5.) $f: [1, +\infty) \to \mathbb{R}$

$$x \mapsto \begin{cases} (x-1)^{(x-1)} & , x > 1 \\ 1 & , x = 1 \end{cases}$$

bei 1 stetig?

bei 1 rechtsseitig differenzierbar?

$$\lim_{x \to 1+} f(x) = \lim_{x \to 1+} (x-1)^{(x-1)} = \lim_{x \to 1+} \exp((x-1) \cdot \ln(x-1))$$

$$= \exp\left(\lim_{x \to 1+} \frac{\ln(x-1)}{\frac{1}{(x-1)}} \right)$$

$$\lim_{x \to 1+} \frac{1}{x-1} = \lim_{y \to 0+} \frac{1}{y} = +\infty$$

$$= \exp\left(\lim_{x \to 1+} \frac{\frac{1}{x-1} \cdot (1)}{-\frac{1}{(x-1)^2} \cdot 1} \right)$$

$$= \exp\left(\lim_{x \to 1+} -\frac{(x-1)^2}{(x-1)} \right) = \exp\left(\lim_{x \to 1+} -(x-1) \right) = \exp\left(\lim_{x \to 1+} -x+1 \right)$$

$$= \exp(0) = 1 \qquad \Rightarrow \text{bei } 1 \text{ stetig}$$

$$\lim_{t \to 1+} \frac{f(t) - f(1)}{t-1} = \lim_{t \to 1+} \frac{(t-1)^{(t-1)} - 1}{t-1} = \lim_{x \to 0+} \frac{x^x - 1}{x}$$

$$\boxed{\lim_{x \to 0+} x^x - 1 = 1 - 1 = 0}$$

$$= \lim_{x \to 0+} \frac{(x^x)' - 0}{1} = \lim_{x \to 0+} (\exp(x \cdot \ln(x)))' = \lim_{x \to 0+} \exp(x \cdot \ln(x)) \cdot (1 \cdot \ln(x) + x \cdot \frac{1}{x})$$

$$= \lim_{x \to 0+} \exp(x \cdot \ln(x)) \cdot \ln(x) + \exp(x \cdot \ln(x)) = \lim_{x \to 0+} x^x \cdot \ln(x) + x^x$$

$$= \lim_{x \to 0+} x^x \cdot \lim_{x \to 0+} \ln(x) + \lim_{x \to 0+} x^x = 1 \cdot \lim_{x \to 0+} \ln(x) + 1 = -\infty$$

ANA Ü3

7.) $f(x) = x^3 - \frac{48}{x}$, $x \neq 0$

· Nullstellen

$f(x) = 0 \iff x^3 - \frac{48}{x} = 0 \iff x^3 = \frac{48}{x} \iff x^4 = 48 \iff x = \sqrt[4]{2^4 \cdot 3}$

$\iff x = \pm 2 \cdot \sqrt[4]{3}$

$\Rightarrow$ Nullstellen bei $2\sqrt[4]{3}$, $-2\sqrt[4]{3}$

· Extrema und Wendepunkte

$f'(x) = 3x^2 - 48 \cdot (-1) \cdot \frac{1}{x^2} = 3x^2 + \frac{48}{x^2} = \frac{3x^4 + 48}{x^2}$

$f'(x) = 0 \iff 3x^4 + 48 = 0 \iff x^4 = \frac{48}{3} \iff x = \sqrt[4]{16} \iff x = \sqrt[4]{2^4}$

$\Rightarrow$ Extremum (?) bei $2, -2$

$f''(x) = 6x + (-2) \cdot 48 \cdot \frac{1}{x^3} = 6x - 96 \cdot \frac{1}{x^3}$

$f''(2) = 12 - 96 \cdot \frac{1}{8} = 12 - 12 = 0$ $\Rightarrow$ Wendepunkt

$f''(-2) = -12 - 96 \cdot \frac{1}{-8} = -12 + 12 = 0$ $\Rightarrow$ Wendepunkt

$\lim\limits_{x \to 0-} f(x) = \lim\limits_{x \to 0-} x^3 - \frac{48}{x} = \lim\limits_{x \to 0-} (x^3) - 48 \cdot \lim\limits_{x \to 0-} \left(\frac{1}{x}\right) = -48 \cdot \lim\limits_{x \to 0-} \frac{1}{x} = \infty$

$\lim\limits_{x \to 0+} f(x) = \lim\limits_{x \to 0+} x^3 - \frac{48}{x} = \lim\limits_{x \to 0+} (x^3) - 48 \cdot \lim\limits_{x \to 0+} \left(\frac{1}{x}\right) = -48 \cdot \lim\limits_{x \to 0+} \frac{1}{x} = -\infty$

$\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} x^3 - \frac{48}{x} = \lim\limits_{x \to \infty} (x^3) - 48 \cdot \lim\limits_{x \to \infty} \frac{1}{x} = \lim\limits_{x \to \infty} x^3 = \infty$

$\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} x^3 - \frac{48}{x} = \lim\limits_{x \to -\infty} (x^3) - 48 \cdot \lim\limits_{x \to -\infty} \frac{1}{x} = \lim\limits_{x \to -\infty} x^3 = -\infty$

· monoton wachsend und fallend

$f'(x) > 0 \iff 3x^2 + \frac{48}{x^2} > 0 \iff 3x^2 > \frac{48}{x^2} \iff 3x^4 > 48$

$\iff x^4 > 16 \iff x > 2$ oder $x < -2$

$f'(x) < 0 \iff 3x^2 + \frac{48}{x^2} < 0 \iff 3x^2 < \frac{48}{x^2} \iff 3x^4 < 48$

$\iff x^4 < 16 \iff -2 < x < 2$

$\Rightarrow$ auf $(-\infty, -2)$ streng monoton $\nearrow$, auf $(-2, 2)$ streng monoton $\searrow$

und auf $(2, +\infty)$ streng monoton $\nearrow$

ANA Ü3

9.) $\alpha \in \mathbb{R}, a \in \mathbb{R}, a > 0$

$$\int x^\alpha dx = \int \exp(\ln(x))^\alpha dx = \int (e^{\ln(x)})^\alpha dx = \int e^{\alpha \cdot \ln(x)} dx$$

$$\left[\begin{array}{lll} u = \alpha \cdot \ln(x) & \ln(x) = \frac{u}{\alpha} & x = e^{\frac{u}{\alpha}} \\ \frac{du}{dx} = \frac{\alpha}{x} & du = \frac{\alpha}{x} \cdot dx & dx = du \cdot \frac{x}{\alpha} \end{array}\right]$$

$$\int e^u \cdot \frac{x}{\alpha} \cdot du = \int e^u \, \frac{e^{\frac{u}{\alpha}}}{\alpha} du = \frac{1}{\alpha} \cdot \int e^{u + \frac{u}{\alpha}} du = \frac{1}{\alpha} \cdot \int e^{u \cdot \left(\frac{\alpha+1}{\alpha}\right)} du$$

$$\left[\begin{array}{ll} m = u \cdot \left(\frac{\alpha+1}{\alpha}\right) & u = m \cdot \frac{\alpha}{\alpha+1} \\ \frac{dm}{du} = \frac{\alpha+1}{\alpha} & du = dm \cdot \frac{\alpha}{\alpha+1} \end{array}\right]$$

$$\frac{1}{\alpha} \int e^m \frac{\alpha}{\alpha+1} dm = \frac{\alpha}{\alpha \cdot (\alpha+1)} \cdot \int e^m dm = \frac{1}{\alpha+1} e^m = \frac{1}{\alpha+1} \cdot e^{u\left(\frac{\alpha+1}{\alpha}\right)}$$

$$= \frac{1}{\alpha+1} \cdot e^{\alpha \cdot \ln(x) \cdot \frac{\alpha+1}{\alpha}} = \frac{1}{\alpha+1} \cdot e^{\ln(x)(\alpha+1)} = \frac{1}{\alpha+1} \left(e^{\ln(x)}\right)^{(\alpha+1)} = \frac{1}{\alpha+1} \cdot x^{(\alpha+1)} + C$$

$$\int a^x dx = \int \exp(\ln(a))^x dx = \int e^{x \cdot \ln(a)} dx$$

$$u = x \cdot \ln(a) \qquad x = \frac{u}{\ln(a)}$$

$$\frac{du}{dx} = \ln(a) \qquad dx = \frac{du}{\ln(a)}$$

$$\int e^u \frac{1}{\ln(a)} \cdot du = \frac{1}{\ln(a)} \cdot \int e^u du = \frac{1}{\ln(a)} e^u = \frac{1}{\ln(a)} \cdot e^{x \cdot \ln(a)}$$

$$= \frac{1}{\ln(a)} \cdot \exp(\ln(a))^x = \frac{1}{\ln(a)} \cdot a^x + C$$

<u>ANA</u> Ü3

2.) $\tan(x): \mathbb{R}\setminus(\frac{\pi}{2}+\pi\mathbb{Z}) \to \mathbb{R}$

$$x \longmapsto \frac{\sin(x)}{\cos(x)}$$

•) ges $\tan'$, $\tan''$

$$(\tan(x))' = \left(\frac{\sin(x)}{\cos(x)}\right)' = \frac{\cos(x)\cdot\cos(x) - \sin(x)\cdot(-\sin(x))}{(\cos(x))^2} = \frac{(\sin(x))^2 + (\cos(x))^2}{(\cos(x))^2}$$

$$= \frac{1}{(\cos(x))^2}$$

$$(\tan(x))'' = \left(\frac{1}{(\cos(x))^2}\right)' = \frac{0\cdot(\cos(x))^2 - 1\cdot((\cos(x))^2)'}{((\cos(x))^2)^2} = -\frac{2\cos(x)\cdot(-\sin(x))}{(\cos(x))^4}$$

$$= \frac{2\sin(x)}{(\cos(x))^3}$$

•) ges: Nullstellen von tan

$$\tan(x) = 0 \iff \frac{\sin(x)}{\cos(x)} = 0 \iff \sin(x) = 0 \iff \exists k \in \mathbb{Z}: x = \pi k$$

•) zz: tan ist auf jedem Intervall in $\mathbb{R}\setminus(\frac{\pi}{2}+\pi\mathbb{Z})$ streng monoton wachsend

$$(\tan(x))' = \frac{1}{(\cos(x))^2} = \left(\frac{1}{\cos(x)}\right)^2 > 0, \quad \Rightarrow \text{streng monoton wachsend}$$

•) ges: $\lim\limits_{t\to\frac{\pi}{2}-} \tan(t)$, $\lim\limits_{t\to-\frac{\pi}{2}+} \tan(t)$

$$\lim_{t\to\frac{\pi}{2}-} \tan(t) = \lim_{t\to\frac{\pi}{2}-} \frac{\sin(t)}{\cos(t)} = \lim_{t\to\frac{\pi}{2}-} \sin(t) \cdot \lim_{t\to\frac{\pi}{2}-} \frac{1}{\cos(t)} = 1\cdot \lim_{x\to 0+} \frac{1}{x} = \infty$$

$$\lim_{t\to\frac{\pi}{2}+} \tan(t) = \lim_{t\to-\frac{\pi}{2}+} \frac{\sin(t)}{\cos(t)} = \lim_{t\to-\frac{\pi}{2}+} \sin(t) \cdot \lim_{t\to-\frac{\pi}{2}+} \frac{1}{\cos(t)} = -1\cdot \lim_{x\to 0+} \frac{1}{x} = -\infty$$

$$\Rightarrow \tan: (-\tfrac{\pi}{2}, \tfrac{\pi}{2}) \to \mathbb{R} \text{ ... bijektiv}$$

•) ges: $(\arctan)'$

$y = \tan(x) \qquad x = \arctan(y)$

$$\arctan'(y) = \arctan'(\tan(x)) = \frac{1}{\tan'(x)} = (\cos(x))^2 = \cos(\arctan(y))^2$$

(Satz 7.1.12)

ANA Ü3

10.) $w \in \mathbb{C}$

$$\int (x^3+2x^2-3)e^{2x-4}dx = \int x^3 \cdot e^{2x} \cdot \frac{1}{e^4} + 2x^2 \cdot e^{2x} \cdot \frac{1}{e^4} - 3e^{2x} \cdot \frac{1}{e^4} dx$$

$$= \frac{1}{e^4} \cdot \int x^3 \cdot e^{2x} dx + 2 \cdot \frac{1}{e^4} \cdot \int x^2 \cdot e^{2x} dx - 3 \frac{1}{e^4} \cdot \int e^{2x} dx$$

$$\left[u = 2x \quad x = \frac{u}{2} \quad \frac{du}{dx} = 2 \quad dx = \frac{du}{2} \right]$$

$$= \frac{1}{e^4} \cdot \int \left(\frac{1}{2} \cdot u\right)^3 \cdot e^u \cdot \frac{1}{2} du + 2 \cdot \frac{1}{e^4} \cdot \int \left(\frac{1}{2} \cdot u\right)^2 \cdot e^u \cdot \frac{1}{2} du - 3 \frac{1}{e^4} \int e^u \cdot \frac{1}{2} du$$

$$= \frac{1}{16e^4} \int u^3 \cdot e^u du + \frac{1}{4e^4} \int u^2 \cdot e^u du - \frac{3}{2e^4} \int e^u du$$

$$\int u^3 (e^u)' du = e^u \cdot u^3 - \int e^u \cdot 3u^2 du = e^u \cdot u^3 - 3 \cdot \left(\int (e^u)' \cdot u^2 du\right)$$

$$= e^u \cdot u^3 - 3 \cdot \left(e^u \cdot u^2 - \int (e^u)' 2u du\right) = e^u \cdot u^3 - 3 \cdot \left(e^u \cdot u^2 - 2 \cdot \left(e^u \cdot u - \int e^u \cdot 1 du\right)\right)$$

$$= e^u \cdot u^3 - 3 \cdot \underbrace{(e^u \cdot u^2 - 2e^u \cdot u + 2e^u)}_{= \int e^u \cdot u^2} = e^u \cdot u^3 - 3e^u \cdot u^2 + 6e^u u - 6e^u$$

$$\frac{1}{16 \cdot e^4} (e^u \cdot u^3 - 3e^u u^2 + 6e^u \cdot u - 6e^u) + \frac{1}{4 \cdot e^4} (e^u \cdot u^2 - 2e^u \cdot u + 2e^u) - \frac{3}{2e^4} e^u$$

$$= \frac{1}{16 \cdot e^4} (e^{2x} \cdot (2x)^3 - 3e^{2x} \cdot (2x)^2 + 6 \cdot e^{2x} (2x) - 6e^{2x}) + \frac{1}{4e^4} (e^{2x}(2x)^2 - 2e^{2x}(2x) + 2e^{2x}) - \frac{3}{2e^4} e^{2x}$$

$$= \frac{1}{16e^4} \cdot (8e^{2x} \cdot x^3 - 12e^{2x} x^2 + 12e^{2x} x - 6e^{2x}) + \frac{1}{4e^4} (4e^{2x} x^2 - 4e^{2x} x + 2e^{2x}) - \frac{3}{2e^4} e^{2x}$$

$$= \frac{1}{2} e^{2x-4} x^3 - \frac{3}{4} e^{2x-4} x^2 + \frac{3}{4} e^{2x-4} x - \frac{3}{8} e^{2x-4} + e^{2x-4} x^2 - e^{2x-4} x + \frac{1}{2} e^{2x-4} - \frac{3}{2} e^{2x-4}$$

$$= \frac{4}{8} e^{2x-4} \cdot x^3 - \frac{6}{8} e^{2x-4} x^2 + \frac{6}{8} e^{2x-4} x - \frac{3}{8} e^{2x-4} + \frac{8}{8} e^{2x-4} x^2 - \frac{8}{8} e^{2x-4} x + \frac{4}{8} e^{2x-4} - \frac{12}{8} e^{2x-4}$$

$$= \frac{1}{8} \cdot e^{2x-4} (4x^3 - 6x^2 + 6x - 3 + 8x^2 - 8x + 4 - 12)$$

$$= \frac{1}{8} \cdot e^{2x-4} (4x^3 + 2x^2 - 2x - 11) + C$$

...

ANA Ü3

10.) ... $\int x^3 \cdot \exp(wx)\, dx$ $\qquad w \in \mathbb{C}$

Falls $w=0$ $\qquad \int x^3 \cdot e^0 dx = \int x^3 dx = \frac{x^4}{4} + C$

Sonst

$$u = w \cdot x \qquad x = \frac{u}{w} \qquad \frac{du}{dx} = w \qquad dx = \frac{du}{w}$$

$$\int x^3 \cdot e^{wx} dx = \int \left(\frac{u}{w}\right)^3 \cdot e^u \cdot \frac{1}{w} \cdot du = \frac{1}{w^4} \cdot \int u^3 \cdot e^u du$$

$$= \frac{1}{w^4} \cdot \left(e^u \cdot u^3 - 3e^u \cdot u^2 + 6e^u \cdot u - 6e^u\right)$$

$$= \frac{1}{w^4} \cdot \left(e^{wx} \cdot (wx)^3 - 3e^{wx}(wx)^2 + 6e^{wx}(wx) - 6e^{wx}\right)$$

$$= \frac{e^{wx}}{w^4}\left(w^3 \cdot x^3 - 3w^2 \cdot x^2 + 6w \cdot x - 6\right) + C$$