

MAS 05

2.) $\Omega_1, \Omega_2 \dots$ Mengen mit $\Omega_1 \cap \Omega_2 = \emptyset$

$S_1 \dots$ Sigmaalgebra über Ω_1 $S_2 \dots$ Sigmaalgebra über Ω_2

$$S = \{A_1 \cup A_2 : A_1 \in S_1, A_2 \in S_2\}$$

zz: S ist Sigmaalgebra über $\Omega_1 \cup \Omega_2 =: \Omega$

$$\cdot) \quad \Omega = \Omega_1 \cup \Omega_2 \in S$$

$$\begin{aligned} \cdot) \quad & \text{Sei } A, B \in S \text{ bel. } A \setminus B = (A_1 \cup A_2) \setminus (B_1 \cup B_2) = (A_1 \cup A_2) \cap (B_1 \cup B_2)^c \\ &= (A_1 \cup A_2) \cap B_1^c \cap B_2^c = (A_1 \cap B_1^c \cap B_2^c) \cup (A_2 \cap B_1^c \cap B_2^c) = (A_1 \cap B_1^c) \cup (A_2 \cap B_2^c) \\ &= (A_1 \setminus B_1) \cup (A_2 \setminus B_2) \in S \end{aligned}$$

$$\cdot) \quad \text{Sei } (A_n)_{n \in \mathbb{N}} \in S \text{ bel. } \bigcup_{n \in \mathbb{N}} A_n = \bigcup_{n \in \mathbb{N}} A_{n_1} \cup A_{n_2} = \bigcup_{n \in \mathbb{N}} A_{n_1} \cup \bigcup_{n \in \mathbb{N}} A_{n_2} \in S$$

$$\in S \qquad \in S_2$$

$$A, B \in S$$

$$(A \cup B)^c = (A_1 \cup A_2)^c \cap (B_1 \cup B_2)^c = A_1^c \cap A_2^c \cap B_1^c \cap B_2^c = \emptyset$$

$$A^c \cup B^c = (A_1 \cup A_2)^c \cup (B_1 \cup B_2)^c = ((A_1 \cup A_2) \cap (B_1 \cup B_2))^c$$

$$= ((A_1 \cap B_1) \cup (A_1 \cap B_2) \cup (A_2 \cap B_1) \cup (A_2 \cap B_2))^c$$

$$= ((A_1 \cap B_1) \cup (A_2 \cap B_2))^c = (A_1 \cap B_1)^c \cap (A_2 \cap B_2)^c$$

$$= \underbrace{(A_1^c \cup B_1^c)}_{\in \Omega_1} \cap \underbrace{(A_2^c \cup B_2^c)}_{\in \Omega_2} = \emptyset$$