LINAG Ü4

G4 1) K ... Körper $z, x, y \in K$ $\quad y \neq 0 \neq z \quad \frac{x}{y} := x \cdot y^{-1}$

zz: $\frac{x}{y} \cdot \frac{y}{z} = \frac{x}{z} \qquad \frac{x}{z} + \frac{y}{-z} = \frac{x-y}{z}$

$$\frac{x}{y} \cdot \frac{y}{z} = (x \cdot y^{-1}) \cdot (y \cdot z^{-1}) = x \cdot y^{-1} \cdot y \cdot z^{-1} = x \cdot z^{-1} = \frac{x}{z}$$

$$\frac{x}{z} + \frac{y}{-z} = (x \cdot z^{-1}) + (y \cdot (-z)^{-1}) = (x \cdot z^{-1}) + (y \cdot (-(z^{-1}))) = (x \cdot z^{-1}) + (-y \cdot z^{-1})$$

$$= z^{-1} \cdot (x + (-y)) = (x-y) \cdot z^{-1} = \frac{x-y}{z}$$

2) $A = u + [b]$ ... eindimensionaler affiner Raum $\quad q \in A \quad q_1 \in K$ sodass gilt

$q = u + q_1 \cdot b \qquad y, p, v \in A, \; q \neq v \qquad \frac{y-v}{q-v} := TV(y, q, v)$

zz: $q_1 \neq v_1$ mit $v_1 \in K$ sodass $v = u + v_1 \cdot b$

Angenommen $q_1 = v_1 \Rightarrow q = u + q_1 \cdot b = u + v_1 \cdot b = v$ ↯ zu $q \neq v$

$\Rightarrow q_1 \neq v_1$

zz: $\frac{y-v}{q-v} = \frac{y_1 - v_1}{q_1 - v_1}$ mit $y_1 \in K$ sodass $y = u + y_1 \cdot b$

$\frac{y-v}{q-v} = TV(y,q,v)$ also $x \in K$, sodass $y = v + x(q-v)$

$$v + \frac{y_1-v_1}{q_1-v_1}(q-v) = v + \frac{y_1-v_1}{q_1-v_1} q - \frac{y_1-v_1}{q_1-v_1} v = u + v_1 \cdot b + \frac{y_1-v_1}{q_1-v_1}(u + q_1 b) - \frac{y_1-v_1}{q_1-v_1}(u + v_1 b)$$

$$= u + v_1 b + \cancel{\frac{y_1-v_1}{q_1-v_1} u} + q_1 \frac{y_1-v_1}{q_1-v_1} b - \cancel{\frac{y_1-v_1}{q_1-v_1} u} - v_1 \frac{y_1-v_1}{q_1-v_1} b$$

$$= u + \left(v_1 + \frac{y_1-v_1}{q_1-v_1} \cdot q_1 - \frac{y_1-v_1}{q_1-v_1} v_1\right) b = u + \left(v_1 + \frac{q_1(y_1-v_1)}{q_1-v_1} + \frac{v_1(y_1-v_1)}{-(q_1-v_1)}\right) b$$

$$= u + \left(v_1 + \frac{q_1(y_1-v_1) - v_1(y_1-v_1)}{q_1-v_1}\right) b = u + \left(v_1 + \frac{(y_1-v_1)(q_1-v_1)}{(q_1-v_1)}\right) b = u + (v_1 + y_1 - v_1) b$$

$$= u + y_1 \cdot b = y \qquad \Rightarrow x = \frac{y_1-v_1}{q_1-v_1}$$

3) a) $x, u, p \in A \quad u \neq p \qquad$ zz: $TV(x, u, p) = 1 - TV(x, p, u)$

$$TV(x, u, p) = \frac{x-p}{u-p} = \frac{x_1-p_1}{u_1-p_1} = \frac{-(x_1-p_1)}{-(u_1-p_1)} = \frac{p_1-x_1}{p_1-u_1} = \frac{p_1-u_1-x_1+u_1}{p_1-u_1}$$

$$= \frac{p_1-u_1}{p_1-u_1} - \frac{x_1-u_1}{p_1-u_1} = 1 - \frac{x_1-u_1}{p_1-u_1} = 1 - \frac{x-u}{p-u} = 1 - TV(x, p, u)$$

c) zz: $TV(x, p, u) = TV(y, p, u) \cdot TV(x, y, u)$ für $y \neq u \neq p$

$$TV(y, p, u) \cdot TV(x, y, u) = \frac{y-u}{p-u} \cdot \frac{x-u}{y-u} = \frac{y_1-u_1}{p_1-u_1} \cdot \frac{x_1-u_1}{y_1-u_1} = \frac{x_1-u_1}{p_1-u_1} = \frac{x-u}{p-u} = TV(x, p, u)$$