MAS Ü10

3.) $f:[a,b]\to\mathbb{R}$ $\quad f \ldots$ stetig

zz: $\int_{[a,b]} f \, d\lambda = \int_a^b f(x)\,dx$

$n\in\mathbb{N}$ $\quad t_k = a+k\frac{b-a}{n}$ $\quad \overline{f_n}(x) = \max(f_n([a+k\frac{b-a}{n}, a+(k+1)\frac{b-a}{n}]))$ für

$x\in[a+k\frac{b-a}{n}, a+(k+1)\frac{b-a}{n}]$

also Ober- und Untersummen: $\underline{f_n}(x) = \min($ ——— 〃 ——— $)$

$\lim_{n\to\infty} \underline{f_n} =: \underline{f}$ $\quad \lim_{n\to\infty} \overline{f_n} =: \overline{f}$ $\quad \underline{f_n} \le f \le \overline{f_n}$

$\lim_{n\to\infty} \int_{[a,b]} \underline{f_n}\, d\lambda = \int_{[a,b]} \underline{f}\, d\lambda$ $\quad \lim_{n\to\infty} \int_{[a,b]} \overline{f_n}\, d\lambda = \int_{[a,b]} \overline{f}\, d\lambda$

Da $f$ stetig $\Rightarrow \underline{f} = f = \overline{f}$

Also $\lim_{n\to\infty} \int \underline{f_n}\, d\lambda = \int_{[a,b]} f\, d\lambda = \lim_{n\to\infty} \int \overline{f_n}\, d\lambda$

$\qquad\quad \| \qquad\qquad\qquad\qquad\qquad \|$

$\lim_{n\to\infty} \int_a^b \underline{f_n}(x)\,dx = \int_a^b f(x)\,dx = \int_a^b \overline{f_n}(x)\,dx$