LINAG Ü12

11.1.1. $K \dots$ Körper $\quad K^{2\times 1}$ mit kanonischem Skalarprodukt

Gibt es isotrope Vektoren? Anzahl der zu sich selbst orthogonalen 1-dim UR + Basis

i) $K = \mathbb{Z}_2$

- $\binom{0}{0}$ $\quad [\binom{0}{0}] \cap [\binom{0}{0}]^\perp = \{\binom{0}{0}\}$, da $[\binom{0}{0}]^\perp = \mathbb{Z}_2$
- $\binom{0}{1}$ $\quad \binom{0}{1}\cdot\binom{a}{b} = b \Rightarrow [\binom{0}{1}]^\perp = \{\binom{0}{1}\}^\perp = \{\binom{0}{0}, \binom{1}{0}\} \Rightarrow [\binom{0}{1}] \cap [\binom{0}{1}]^\perp = \{\binom{0}{0}\}$
- $\binom{1}{0}$ $\quad \binom{1}{0}\cdot\binom{a}{b} = a \Rightarrow [\binom{1}{0}]^\perp = \{\binom{0}{0}, \binom{0}{1}\} \Rightarrow [\binom{1}{0}] \cap [\binom{1}{0}]^\perp = \{\binom{0}{0}\}$
- $\binom{1}{1}$ $\quad \binom{1}{1}\cdot\binom{a}{b} = a+b \Rightarrow [\binom{1}{1}]^\perp = \{\binom{0}{0}, \binom{1}{1}\} \Rightarrow [\binom{1}{1}] \cap [\binom{1}{1}]^\perp = \{\binom{0}{0}, \binom{1}{1}\}$

$\Rightarrow \binom{1}{1}$ ist isotrop. Es gibt 1 zu sich selbst orthogonalen 1-dim UR, nämlich $[\binom{1}{1}]$.

ii) $K = \mathbb{Z}_3$

- $\binom{0}{0}$ $\quad [\binom{0}{0}] \cap [\binom{0}{0}]^\perp = \{\binom{0}{0}\}$
- $\binom{0}{1} / \binom{0}{2}$ $\quad \binom{0}{1}\cdot\binom{a}{b} = b \quad [\binom{0}{1}]^\perp = \{\binom{0}{0}, \binom{1}{0}, \binom{2}{0}\} \Rightarrow [\binom{0}{1}] \cap [\binom{0}{1}]^\perp = \{\binom{0}{0}\}$ für $\binom{0}{2}$ genauso
- $\binom{1}{0} / \binom{2}{0}$ $\quad \binom{1}{0}\cdot\binom{a}{b} = a \quad [\binom{1}{0}]^\perp = \{\binom{0}{0}, \binom{0}{1}, \binom{0}{2}\} \Rightarrow [\binom{1}{0}] \cap [\binom{1}{0}]^\perp = \{\binom{0}{0}\}$
- $\binom{1}{1}$ $\quad \binom{1}{1}\cdot\binom{a}{b} = a+b \quad [\binom{1}{1}]^\perp = \{\binom{0}{0}, \binom{1}{2}, \binom{2}{1}\} \Rightarrow [\binom{1}{1}] \cap [\binom{1}{1}]^\perp = \{\binom{0}{0}\}$
- $\binom{2}{2}$ $\quad \binom{2}{2}\cdot\binom{a}{b} = 2a+2b \quad [\binom{2}{2}]^\perp = \{\binom{0}{0}, \binom{1}{2}, \binom{2}{1}\} \Rightarrow [\binom{2}{2}] \cap [\binom{2}{2}]^\perp = \{\binom{0}{0}\}$
- $\binom{1}{2} / \binom{2}{1}$ $\quad \binom{1}{2}\cdot\binom{a}{b} = a+2b \quad [\binom{1}{2}]^\perp = \{\binom{0}{0}, \binom{1}{1}, \binom{2}{2}\} \Rightarrow [\binom{1}{2}] \cap [\binom{1}{2}]^\perp = \{\binom{0}{0}\}$

$\Rightarrow \nexists$ isotroper Vektor. Es gibt keine zu sich selbst orthogonalen 1-dim UR.

iii) $K = GF(4)$

- $\binom{0}{0}$ $\quad [\binom{0}{0}] \cap [\binom{0}{0}]^\perp = \{\binom{0}{0}\}$
- $\binom{0}{1} / \binom{0}{a} / \binom{0}{b}$ $\quad [\binom{0}{1}]^\perp = \{\binom{0}{0}, \binom{1}{0}, \binom{a}{0}, \binom{b}{0}\} \Rightarrow [\binom{0}{1}] \cap [\binom{0}{1}]^\perp = \{\binom{0}{0}\}$
- $\binom{1}{0} / \binom{a}{0} / \binom{b}{0}$ $\quad [\binom{1}{0}]^\perp = \{\binom{0}{0}, \binom{0}{1}, \binom{0}{a}, \binom{0}{b}\} \Rightarrow [\binom{1}{0}] \cap [\binom{1}{0}]^\perp = \{\binom{0}{0}\}$
- $\binom{1}{1} / \binom{a}{a} / \binom{b}{b}$ $\quad \binom{1}{1}\binom{x}{y} = x+y = 1\cdot(x+y) \quad [\binom{1}{1}]^\perp = \{\binom{0}{0}, \binom{1}{1}, \binom{a}{a}, \binom{b}{b}\} \Rightarrow [\binom{1}{1}] \cap [\binom{1}{1}]^\perp = \{\binom{0}{0}, \binom{1}{1}, \binom{a}{a}, \binom{b}{b}\}$
- $\binom{1}{a} / \binom{a}{1}$ $\quad \binom{1}{a}\binom{x}{y} = x+ay \quad [\binom{1}{a}]^\perp = \{\binom{0}{0}, \binom{1}{b}, \binom{a}{1}, \binom{b}{a}\} \Rightarrow [\binom{1}{a}] \cap [\binom{1}{a}]^\perp = \{\binom{0}{0}\}$
- $\binom{1}{b} / \binom{b}{1}$ $\quad \binom{1}{b}\binom{x}{y} = x+by \quad [\binom{1}{b}]^\perp = \{\binom{0}{0}, \binom{1}{a}, \binom{a}{b}, \binom{b}{1}\} \Rightarrow [\binom{1}{b}] \cap [\binom{1}{b}]^\perp = \{\binom{0}{0}\}$
- $\binom{a}{b} / \binom{b}{a}$ $\quad \binom{a}{b}\binom{x}{y} = ax+by \quad [\binom{a}{b}]^\perp = \{\binom{0}{0}, \binom{1}{b}, \binom{a}{1}, \binom{b}{a}\} \Rightarrow [\binom{a}{b}] \cap [\binom{a}{b}]^\perp = \{\binom{0}{0}\}$

$\Rightarrow \binom{1}{1}, \binom{a}{a}, \binom{b}{b}$ sind isotrop. Es gibt 1 zu sich selbst orthogonalen 1-dim UR, nämlich $[\binom{1}{1}]$

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iv) $K = \mathbb{Z}_5$

- $\binom{0}{0}$ $\quad [\binom{0}{0}] \cap [\binom{0}{0}]^\perp = \{\binom{0}{0}\}$

- $\binom{0}{1} / \binom{0}{2} / \binom{0}{3} / \binom{0}{4}$ $\quad \binom{0}{1} \cdot \binom{x}{y} = y \Rightarrow [\binom{0}{1}]^\perp = \{\binom{0}{0}, \binom{1}{0}, \binom{2}{0}, \binom{3}{0}, \binom{4}{0}\}$ $\quad [\binom{0}{1}] \cap [\binom{0}{1}]^\perp = \{\binom{0}{0}\}$
  $\binom{1}{0} / \binom{2}{0} / \binom{3}{0} / \binom{4}{0}$

- $\binom{1}{1}, \binom{2}{2}, \binom{3}{3}, \binom{4}{4}$ $\quad \binom{1}{1} \cdot \binom{x}{y} = x + y = 1 \cdot (x+y)$ $\quad [\binom{1}{1}]^\perp = \{\binom{0}{0}, \binom{1}{4}, \binom{2}{3}, \binom{3}{2}, \binom{4}{1}\}$ $\quad [\binom{1}{1}] \cap [\binom{1}{1}]^\perp = \{\binom{0}{0}\}$

- $\binom{1}{2}, \binom{3}{1}$ $\quad \binom{1}{2} \cdot \binom{x}{y} = x + 2y$ $\quad [\binom{1}{2}]^\perp = \{\binom{0}{0}, \binom{1}{2}, \binom{2}{4}, \binom{3}{1}, \binom{4}{3}\}$ $\quad [\binom{1}{2}] \cap [\binom{1}{2}]^\perp = \{\binom{0}{0}, \binom{1}{2}, \binom{2}{4}, \binom{3}{1}, \binom{4}{3}\}$

- $\binom{2}{4}, \binom{4}{2}$ $\quad \binom{2}{4} \cdot \binom{x}{y} = 2x + 4y = 2 \cdot (x + 2y)$ wie oben

- $\binom{1}{3}, \binom{3}{1}$ — " — $\qquad$ - $\binom{3}{4}, \binom{4}{3}$ — " —

- $\binom{1}{4}, \binom{4}{1}$ $\quad \binom{1}{4} \cdot \binom{x}{y} = x + 4y$ $\quad [\binom{1}{4}]^\perp = \{\binom{0}{0}, \binom{1}{1}, \binom{2}{2}, \binom{3}{3}, \binom{4}{4}\} \Rightarrow [\binom{1}{4}] \cap [\binom{1}{4}]^\perp = \{\binom{0}{0}\}$

- $\binom{2}{3}, \binom{3}{2}$ $\quad \binom{2}{3} \cdot \binom{x}{y} = 2x + 3y$ $\quad [\binom{2}{3}]^\perp = \{\binom{0}{0}, \binom{1}{1}, \binom{2}{2}, \binom{3}{3}, \binom{4}{4}\} \Rightarrow [\binom{2}{3}] \cap [\binom{2}{3}]^\perp = \{\binom{0}{0}\}$

$\Rightarrow \binom{1}{2}, \binom{2}{4}, \binom{3}{1}, \binom{4}{3}, \binom{2}{1}, \binom{4}{2}, \binom{1}{3}, \binom{3}{4}$ sind isotrop. Es gibt 2 zu sich selbst isomorphe 1-dim UR, nämlich $[\binom{1}{2}]$ und $[\binom{2}{1}]$.