ANA Ü8

7.) $p \in \mathbb{N}, p \geq 2$ $\quad f: \mathbb{R}^p \setminus \{0\} \to \mathbb{R}$

Für $p=2$ $\quad f(x) = \ln \|x\|_2$ $\quad$ sonst $f(x) = \frac{1}{(2-p)\|x\|_2^{p-2}}$

zz: $\operatorname{grad} f(x) (:= (df(x))^T) = \frac{1}{\|x\|_2^p} x$

$p=2$: $\quad df(x) = \left(\frac{\partial f}{\partial x_j}(x)\right)_{j=1,2} = \left(\frac{\partial f}{\partial x_1}(x) \quad \frac{\partial f}{\partial x_2}(x)\right)$

$$\frac{\partial f}{\partial x_2}\begin{pmatrix}x\\y\end{pmatrix} = \frac{d}{dy} \ln\left(\sqrt{x^2+y^2}\right) = \frac{d}{dy} \frac{1}{2} \ln(x^2+y^2) = \frac{1}{2} \cdot \frac{1}{x^2+y^2} \cdot 2y = \frac{y}{x^2+y^2}$$

$$\frac{\partial f}{\partial x_1}\begin{pmatrix}x\\y\end{pmatrix} = \frac{d}{dx} \ln\left(\sqrt{x^2+y^2}\right) = \frac{d}{dx} \frac{1}{2} \ln(x^2+y^2) = \frac{1}{2} \frac{1}{x^2+y^2} \cdot 2x = \frac{x}{x^2+y^2}$$

$$\Rightarrow df\begin{pmatrix}x\\y\end{pmatrix} = \left(\frac{x}{x^2+y^2} \quad \frac{y}{x^2+y^2}\right)$$

$$\frac{1}{\|\binom{x}{y}\|_2^2}\begin{pmatrix}x\\y\end{pmatrix} = \frac{1}{\sqrt{x^2+y^2}^2}\begin{pmatrix}x\\y\end{pmatrix} = \frac{1}{x^2+y^2}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}\frac{x}{x^2+y^2}\\ \frac{y}{x^2+y^2}\end{pmatrix} = (df(x))^T$$

$p>2$: $\quad df(x) = \left(\frac{\partial f}{\partial x_1}(x) \quad \dots \quad \frac{\partial f}{\partial x_p}(x)\right)$

$$\frac{\partial f}{\partial x_i}(x) = \frac{d}{dx_i} \frac{1}{(2-p)\|x\|_2^{p-2}} = \frac{d}{dx_i} \frac{1}{(2-p)\sqrt{x_1^2+\dots+x_p^2}^{(p-2)}} = \frac{1}{2-p} \frac{d}{dx_i} \frac{1}{(x_1^2+\dots+x_p^2)^{\frac{p}{2}-1}}$$

$$= \frac{1}{2-p} \cdot -\left(\frac{p-2}{2}\right) \cdot \frac{1}{(x_1^2+\dots+x_p^2)^{\frac{p}{2}}} \cdot 2x_i = x_i \frac{1}{\sqrt{x_1^2+\dots+x_p^2}^p} = x_i \cdot \frac{1}{\|x\|_2^p}$$

$$df(x) = \left(x_1 \cdot \frac{1}{\|x\|_2^p} \quad \dots \quad x_p \cdot \frac{1}{\|x\|_2^p}\right)$$

$$\frac{1}{\|x\|_2^p} x = \begin{pmatrix}\frac{x_1}{\|x\|_2^p}\\ \vdots \\ \frac{x_p}{\|x\|_2^p}\end{pmatrix} = df(x)^T$$