ANA Ü11

2.) $\gamma : [0, 2\pi] \to \mathbb{R}^3$ $\qquad$ ges: $\int_\gamma ((x^2+5y+3yz)dx + (5x+3xz-2)dy + (3xy-4z)dz)$

$t \mapsto \begin{pmatrix} \sin(t) \\ \cos(t) \\ t \end{pmatrix}$

$\gamma \in C^1[0,2\pi]$ mit $\gamma'(t) = \begin{pmatrix} \cos(t) \\ -\sin(t) \\ 1 \end{pmatrix}$ und Φ ist stetig $\quad$ Satz 11.2.5

$$\int_0^{2\pi} \left(\sin^2(t)+5\cos(t)+3\cos(t)\cdot t \quad 5\sin(t)+3\sin(t)\cdot t-2 \quad 3\sin(t)\cos(t)-4t\right)\begin{pmatrix} \cos(t) \\ -\sin(t) \\ 1 \end{pmatrix} dt$$

$$= \int_0^{2\pi} \sin^2(t)\cdot\cos(t)+5\cos^2(t)+3\cos^2(t)\cdot t-5\sin^2(t)-3\sin^2(t)\cdot t+2\sin(t)+3\sin(t)\cos(t)-4t \quad dt$$

$$= \int_0^{2\pi}\sin^2(t)\cdot\cos(t)dt+5\cdot\int_0^{2\pi}\cos^2(t)dt+3\cdot\int_0^{2\pi}\cos^2(t)\cdot t\,dt-5\int_0^{2\pi}\sin^2(t)dt-3\int_0^{2\pi}\sin^2(t)\cdot t\,dt+2\int_0^{2\pi}\sin(t)dt$$

$$+3\int_0^{2\pi}\sin(t)\cdot\cos(t)dt-4\int_0^{2\pi}t\,dt$$

- $\int\sin^2(t)\cdot\cos(t)\,dt = \int u^2\cdot\cos(t)\cdot\frac{1}{\cos(t)}du \quad [u=\sin(t) \quad \frac{du}{dt}=\cos(t) \quad dt=\frac{1}{\cos(t)}du$

$= \int u^2\,du = \frac{u^3}{3} = \frac{\sin^3(t)}{3} \quad \Rightarrow \int_0^{2\pi}\sin^2(t)\cdot\cos(t)dt = 0$

- $\int\cos^2(t)dt = \int\frac{1}{2}(1+\cos(2t))dt = \frac{1}{2}\cdot(\int 1\,dt+\int\cos(2t)dt) = \frac{1}{2}(t+\int\cos(2t)dt)$

$= \frac{1}{2}(t+\int\cos(u)\cdot\frac{1}{2}du) = \frac{1}{2}t+\frac{1}{4}\int\cos(u)du \qquad [u=2t \quad \frac{du}{dt}=2 \quad dt=\frac{1}{2}du$

$= \frac{1}{2}t+\frac{1}{4}\sin(u) = \frac{1}{2}t+\frac{1}{4}\sin(2t) \qquad \int_0^{2\pi}\cos^2(t)dt = \pi$

- $\int\cos^2(t)\cdot t\,dt = \int\frac{1}{2}(\cos(2t)+1)\cdot t\,dt = \frac{1}{2}(\int\cos(2t)\cdot t\,dt+\int t\,dt)$

$= \frac{1}{2}(\int\cos(u)\cdot\frac{u}{2}\cdot\frac{1}{2}du+\frac{t^2}{2}) = \frac{1}{4}(\frac{1}{2}\int\cos(u)\cdot u\,du+t^2) \quad [u=2t \quad \frac{du}{dt}=2 \quad dt=\frac{1}{2}du$

$= \frac{1}{4}(\frac{1}{2}u\cdot\sin(u)-\int\sin(u)du+t^2) = \frac{1}{4}(\frac{1}{2}\cdot 2t\cdot\sin(2t)+\cos(2t)+t^2) = \frac{1}{4}(\sin(2t)\cdot t+\cos(2t)+t^2)$

$\int_0^{2\pi}\cos^2(t)\cdot t\,dt = \frac{1}{8}+\pi^2-\frac{1}{8} = \pi^2$

- $\int\sin^2(t)dt = \int\frac{1}{2}-\frac{1}{2}\cos(2t)dt = \frac{1}{2}\int 1\,dt-\frac{1}{2}\int\cos(2t)dt = \frac{1}{2}(t-\int\cos(u)\frac{1}{2}du) \quad [u=2t \quad \frac{du}{dt}=2 \quad dt=\frac{1}{2}du$

$= \frac{1}{2}(t-\frac{1}{2}\sin(u)) = \frac{1}{2}(t-\frac{1}{2}\sin(2t)) \qquad \int_0^{2\pi}\sin^2(t)dt = \pi$

- $\int\sin^2(t)\cdot t\,dt = \int\frac{1}{2}(1-\cos(2t))\cdot t\,dt = \frac{1}{2}(\int t\,dt-\int\cos(2t)\cdot t\,dt) = \frac{1}{2}(\frac{t^2}{2}-(\frac{1}{4}\cos(2t)+\frac{1}{2}t\sin(2t))$

$= \frac{t^2}{4}-\frac{1}{8}\cos(2t)-\frac{1}{4}t\sin(2t) \qquad \int_0^{2\pi}\sin^2(t)\cdot t\,dt = \pi^2$

- $\int\sin(t)\cdot\cos(t)dt = \int\sin(t)u(-\frac{1}{\sin(t)})du = -\int u\,du = -\frac{u^2}{2} \quad [u=\cos(t) \quad \frac{du}{dt}=-\sin(t) \quad dt=-\frac{1}{\sin(t)}du$

$= -\frac{\cos^2(t)}{2} \qquad \int_0^{2\pi}\sin(t)\cdot\cos(t)\,dt = 0$

$\Rightarrow \int_0^{2\pi}\dots dt = 0+5\pi+3\cdot\pi^2-5\pi-3\pi^2+2\cdot 0+3\cdot 0-4\cdot 2\pi^2 = -8\pi^2$