LINAG Ü1

7.3.3. $b_1 = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ $b_2 = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$ $b_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ $B = (b_1, b_2, b_3)$ .. Basis von $\mathbb{R}^{3\times 1}$

$f \in GL(\mathbb{R}^{3\times 1})$ $f(b_1) = b_2$ $f(b_2) = b_3$ $f(b_3) = b_1$

a) ges: $\langle E^*, f(E)\rangle$, $\langle E^*, f^3(E)\rangle$ sowie jeweils Determinante

$$\left(\begin{array}{ccc} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \\ \hline 2 & 1 & 3 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{array}\right) \rightsquigarrow \left(\begin{array}{ccc} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ \hline 1 & -2 & 3 \\ 1 & -1 & 2 \\ 0 & 0 & 1 \end{array}\right) \rightsquigarrow \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 1 & -3 & 5 \\ 1 & -2 & 3 \\ 0 & 0 & 1 \end{array}\right) \Rightarrow \langle E^*, f(E)\rangle = \begin{pmatrix} 1 & -3 & 5 \\ 1 & -2 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

$\underline{\det(\langle E^*, f(E)\rangle)} = (-2) + 0 + 0 - 0 - 0 - (-3) = 3 - 2 = 1$

$\langle E^*, f^3(E)\rangle = (\langle E^*, f(E)\rangle)^3$

$$\begin{array}{ccc|ccc} & & & 1 & -3 & 5 \\ & & & 1 & -2 & 3 \\ & & & 0 & 0 & 1 \\ \hline 1 & -3 & 5 & -2 & 3 & 1 \\ 1 & -2 & 3 & -1 & 1 & 2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \qquad \begin{array}{ccc|ccc} & & & 1 & -3 & 5 \\ & & & 1 & -2 & 3 \\ & & & 0 & 0 & 1 \\ \hline -2 & 3 & 1 & 1 & 0 & 0 \\ -1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}$$

$\boxed{\text{oder auch } \langle E^*, f^3(E)\rangle \text{ muss gleich } E_3 \text{ sein, da } f^3 = id}$

$\Rightarrow \langle E^*, f^3(E)\rangle = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\Rightarrow \det(\langle E^*, f^3(E)\rangle) = 1$

b) $f, f^3 \in SL(\mathbb{R}^{3\times 1})$, da $f, f^3 \in GL(\mathbb{R}^{3\times 1})$ und $\det(f) = 1$ und $\det(f^3) = 1$.

$\langle B^*, f(B)\rangle = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ $\langle B^*, f^3(B)\rangle = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\det(\langle B^*, f(B)\rangle) = 1$ $\det(\langle B^*, f^3(B)\rangle) = 1$

$\Rightarrow f, f^3 \in SL(\mathbb{R}^{3\times 1})$