ANA Ü1

2.) $z, w \in \mathbb{C}$

$$\cos(z+w) = \frac{\exp(i(z+w)) + \exp(-i(z+w))}{2} = \frac{\exp(iz)\cdot\exp(iw) + \exp(-iz)\exp(-iw)}{2}$$

$$= \frac{2\exp(iz)\cdot\exp(iw) + 2\exp(-iz)\cdot\exp(-iw)}{4}$$

$$= \frac{\exp(iz)\cdot\exp(iw) + \exp(iz)\cdot\exp(-iw) + \exp(-iz)\cdot\exp(iw) + \exp(-iz)\cdot\exp(-iw)}{4} +$$

$$\frac{\exp(iz)\cdot\exp(iw) - \exp(iz)\cdot\exp(-iw) - \exp(-iz)\cdot\exp(iw) + \exp(-iz)\cdot\exp(-iw)}{4}$$

$$= \frac{\exp(iz)+\exp(-iz)}{2} \cdot \frac{\exp(iw)+\exp(-iw)}{2} - \frac{\exp(iz)-\exp(-iz)}{2i} \cdot \frac{\exp(iw)-\exp(-iw)}{2i}$$

$$= \cos(z)\cdot\cos(w) - \sin(z)\cdot\sin(w)$$

$$\sin(z)\cdot\cos(w) + \cos(z)\cdot\sin(w)$$

$$= \frac{\exp(iz)-\exp(-iz)}{2i} \cdot \frac{\exp(iw)+\exp(-iw)}{2} + \frac{\exp(iz)+\exp(-iz)}{2} \cdot \frac{\exp(iw)-\exp(-iw)}{2i}$$

$$= \frac{\exp(iz)\cdot\exp(iw) + \exp(iz)\cdot\exp(-iw) - \exp(-iz)\cdot\exp(iw) - \exp(-iz)\cdot\exp(-iw)}{4i} +$$

$$\frac{\exp(iz)\cdot\exp(iw) - \exp(iz)\cdot\exp(-iw) + \exp(-iz)\cdot\exp(iw) - \exp(-iz)\cdot\exp(-iw)}{4i}$$

$$= \frac{2\exp(iz)\cdot\exp(iw) - 2\exp(-iz)\cdot\exp(-iw)}{4i} = \frac{\exp(i(z+w)) - \exp(-i(z+w))}{2i}$$

$$= \sin(z+w)$$