MAS Ü8

1.) $(\mathbb{R}, \mathcal{B}, \mathbb{P})$ $\mathbb{P}$ .. Standardnormalverteilung

ges: $\mathbb{P}(]-\infty; 1,6])$, $\mathbb{P}(]-1,1; 1,8[)$, $\mathbb{P}([1,4; \infty[)$

$\mathbb{P}(]-\infty, 1,6]) = \Phi_{0,1}(1,6) = 0,94520$

$\mathbb{P}(]-1,1; 1,8[) = \Phi_{0,1}(1,8) - \Phi_{0,1}(-1,1) = \Phi_{0,1}(1,8) - (1 - \Phi_{0,1}(1,1)) = 0,96407 - (1 - 0,86433)$

$= 0,8284$

$\mathbb{P}([1,4; \infty[) = 1 - \mathbb{P}(]-\infty, 1,4[) = 1 - \Phi_{0,1}(1,4) = 1 - 0,91924 = 0,08076$

3.) $\mathbb{P} = B(6; 0,6)$

ges: $\mathbb{P}(]-\infty, 2])$, $\mathbb{P}(]1,5[)$, $\mathbb{P}(]3, \infty[)$

$\mathbb{P}(\{0\}) = \binom{6}{0} 0,6^0 (1-0,6)^{6-0} = 0,4^6 = 0,004096$

$\mathbb{P}(\{1\}) = \binom{6}{1} 0,6^1 (1-0,6)^{6-1} = 6 \cdot 0,6 \cdot 0,4^5 = 0,036864$

$\mathbb{P}(\{2\}) = 15 \cdot 0,6^2 \cdot 0,4^4 = 0,13824$

$\mathbb{P}(\{3\}) = 20 \cdot 0,6^3 \cdot 0,4^3 = 0,27648$

$\mathbb{P}(\{4\}) = 15 \cdot 0,6^4 \cdot 0,4^2 = 0,31104$

$\mathbb{P}(\{5\}) = 6 \cdot 0,6^5 \cdot 0,4^1 = 0,186624$

$\mathbb{P}(\{6\}) = 1 \cdot 0,6^6 \cdot 0,4^0 = 0,046656$

$\mathbb{P}(]-\infty, 2]) = \mathbb{P}(\{0\}) + \mathbb{P}(\{1\}) + \mathbb{P}(\{2\}) = 0,1792$

$\mathbb{P}(]1,5[) = \mathbb{P}(\{2\}) + \mathbb{P}(\{3\}) + \mathbb{P}(\{4\}) = 0,72576$

$\mathbb{P}(]3, \infty[) = \mathbb{P}(\{4\}) + \mathbb{P}(\{5\}) + \mathbb{P}(\{6\}) = 0,54432$

4.) $\mathbb{P} = P(3,6)$

ges: $\mathbb{P}(]-\infty, 2])$, $\mathbb{P}(]1,5[)$, $\mathbb{P}(]3, \infty[)$

$\mathbb{P}(\{0\}) = \frac{3,6^0}{0!} \cdot e^{-3,6} = 0,0273237$

$\mathbb{P}(\{1\}) = \frac{3,6^1}{1!} \cdot e^{-3,6} = 0,0983654$

$\mathbb{P}(\{2\}) = \frac{3,6^2}{2!} \cdot e^{-3,6} = 0,177058$

$\mathbb{P}(\{3\}) = 0,212469$

$\mathbb{P}(\{4\}) = 0,191222$

$\mathbb{P}(\{5\}) = 0,13768$

$\mathbb{P}(\{6\}) = 0,0826081$

$\mathbb{P}(]-\infty, 2]) = \mathbb{P}(\{0\}) + \mathbb{P}(\{1\}) + \mathbb{P}(\{2\}) = 0,3027471$

$\mathbb{P}(]1,5[) = \mathbb{P}(\{2\}) + \mathbb{P}(\{3\}) + \mathbb{P}(\{4\}) = 0,580749$

$\mathbb{P}(]3, \infty[) = \mathbb{P}(\{4\}) + \mathbb{P}(\{5\}) + \mathbb{P}(\{6\}) = 0,4115101$

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5.) $\mathbb{P}$ ... Normalverteilung mit $\mu = 4 \quad \sigma^2 = 25$

ges: $\mathbb{P}(]-\infty, 7])$ $\quad \mathbb{P}(]3,\infty[)$ $\quad \mathbb{P}(\{x : |x| > 6\})$

$\mathbb{P}(]-\infty,7]) = \Phi_{4,5}(7) = \Phi_{0,1}\left(\frac{7-4}{5}\right) = \Phi_{0,1}(0,6) = 0,72575$

$\mathbb{P}(]3,\infty[) = 1 - \mathbb{P}(]-\infty,3]) = 1 - \Phi_{4,5}(3) = 1 - \Phi_{0,1}\left(\frac{3-4}{5}\right) = 1 - \Phi_{0,1}(-0,2)$

$= 1 - (1 - \Phi_{0,1}(0,2)) = \Phi_{0,1}(0,2) = 0,57926$

$\mathbb{P}(\{x : |x| > 6\}) = 1 - \mathbb{P}([-6,6]) = 1 - (\Phi_{4,5}(6) - \Phi_{4,5}(-6)) = 1 - \left(\Phi_{0,1}\left(\frac{6-4}{5}\right) - \Phi_{0,1}\left(\frac{-6-4}{5}\right)\right)$

$= 1 - (\Phi_{0,1}(0,4) - \Phi_{0,1}(-2)) = 1 - (\Phi_{0,1}(0,4) - (1 - \Phi_{0,1}(2))) = 1 - (\Phi_{0,1}(0,4) - 1 + \Phi_{0,1}(2))$

$= 1 - \Phi_{0,1}(0,4) + 1 - \Phi_{0,1}(2) = 2 - 0,65542 - 0,97725 = 0,36733$

ges: $c \in \mathbb{R}$ mit $\mathbb{P}(]-\infty, c]) = 0,9$

$0,9 = \mathbb{P}(]-\infty, c]) = \Phi_{4,5}(c) = \Phi_{0,1}\left(\frac{c-4}{5}\right)$

$\Phi_{0,1}(1,29) = 0,90147 \quad \Rightarrow \frac{c-4}{5} = 1,29 \iff c - 4 = 6,45 \iff c = 10,45$

6.) $\mathbb{P} = E(2)$ ... Exponentialverteilung

ges: $\mathbb{P}([0,1;1,3])$, $\mathbb{P}(]0,5;\infty[)$ und $c \in \mathbb{R}$ mit $\mathbb{P}(]-\infty,c]) = \frac{1}{4} \,/\, \frac{1}{2} \,/\, \frac{3}{4}$

$\mathbb{P}([0,1;1,3]) = F(1,3) - F(0,1) = 1 - e^{-2\cdot 1,3} - (1 - e^{-2\cdot 0,1}) = e^{-0,2} - e^{-2,6} = 0,744457$

$\mathbb{P}(]0,5;\infty[) = 1 - \mathbb{P}(]-\infty,0,5]) = 1 - F(0,5) = 1 - (1 - e^{-2\cdot 0,5}) = e^{-1} = 0,367879$

$\cdot\ \mathbb{P}(]-\infty,c]) = F(c) = 1 - e^{-2\cdot c}$

$\bullet)\ 1 - e^{-2c} = \frac{1}{4} \iff e^{-2c} = \frac{3}{4} \iff -2c = \ln\left(\frac{3}{4}\right)$

$\iff c = -\frac{\ln\left(\frac{3}{4}\right)}{2} \iff c = 0,143841$

$\bullet)\ 1 - e^{-2c} = \frac{1}{2} \iff e^{-2c} = \frac{1}{2} \iff -2c = \ln\left(\frac{1}{2}\right)$

$\iff c = -\frac{\ln\left(\frac{1}{2}\right)}{2} \iff c = 0,346573$

$\bullet)\ 1 - e^{-2c} = \frac{3}{4} \iff e^{-2c} = \frac{1}{4} \iff -2c = \ln\left(\frac{1}{4}\right)$

$\iff c = -\frac{1}{2} \cdot \ln\left(\frac{1}{4}\right) \iff c = 0,693147$

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7.) $\mathbb{P} = \Gamma(3,1) \ldots$ Gammaverteilung

ges: Verteilungsfunktion und $\mathbb{P}([1,2])$

$$\mathbb{P}(x) = \frac{x^{3-1} \cdot 1^3}{\Gamma(3)} \cdot e^{-1x} = \frac{x^2}{\int_0^\infty t^{3-1} \cdot e^{-t}\, dt} \cdot \frac{1}{e^x}$$

$$\int t^2 \cdot e^{-t}\, dt = -e^{-t} \cdot t^2 + 2 \int e^{-t} \cdot t\, dt = -e^{-t} \cdot t^2 + 2\left(-e^{-t} \cdot t + \int e^{-t}\, dt\right)$$

$$= -e^{-t} \cdot t^2 - 2e^{-t} \cdot t - 2e^{-t} = -e^{-t} \cdot (t^2 + 2t + 2)$$

$$\lim_{\beta \to \infty} \int_0^\beta t^2 \cdot e^{-t}\, dt = \lim_{\beta \to \infty} -e^{-\beta}(\beta^2 + 2\beta + 2) - (-e^0 \cdot 2) = \lim_{\beta \to \infty} -\frac{\beta^2 + 2\beta + 2}{e^\beta} + 2$$

$$= \lim_{\beta \to \infty} -\frac{2\beta + 2}{e^\beta} + 2 = \lim_{\beta \to \infty} -\frac{2}{e^\beta} + 2 = 0 + 2 = 2$$

$$\mathbb{P}(x) = \frac{x^2}{2e^x} \qquad F(x) = \int_0^x \frac{t^2}{2e^t}\, dt$$

$$\int \frac{1}{2} \cdot t^2 \cdot e^{-t}\, dt = \frac{1}{2} \cdot (-e^{-t} \cdot (t^2 + 2t + 2))$$

$$\int_0^x \frac{1}{2} \cdot t^2 \cdot e^{-t}\, dt = -\frac{1}{2} e^{-x} \cdot (x^2 + 2x + 2) + \frac{1}{2} \cdot e^{-0}(2) = -\frac{1}{2} \cdot e^{-x}(x^2 + 2x + 2) + 1 = F(x)$$

$$\mathbb{P}([1,2]) = F(2) - F(1) = -\frac{1}{2} \cdot e^{-2}(4 + 4 + 2) + 1 - \left(-\frac{1}{2} \cdot e^{-1}(1 + 2 + 2) + 1\right)$$

$$= -5 \cdot e^{-2} + \cancel{1} + \frac{5}{2} \cdot e^{-1} - \cancel{1} = 0{,}243022$$