scratch_1.py

```python
import numpy as np

if __name__ == "__main__":
    A = np.matrix([[1, 0, -1, 0], [0, 1, 0, -1], [0, 1, 2, 1], [0, 0, 0, 2]])

    print(A)

    B = np.identity(4) - A
    for i in range(1, 3):
        C = (2 * np.identity(4)) - A
        for k in range(1, 3):
            print((i, k))
            print(B * C, end="\n\n")
            C = C * C
        B = B * B

```

LINAG Ü9

8.7.15. β)

$V \ldots VR/\mathbb{R} \quad f \in L(V,V) \quad B \ldots$ Basis von V

$$A := \langle B^*, f(B)\rangle = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

a)

$$\det\begin{pmatrix} 1-X & 0 & -1 & 0 \\ 0 & 1-X & 0 & -1 \\ 0 & 1 & 2-X & 1 \\ 0 & 0 & 0 & 2-X \end{pmatrix} = (2-X)\cdot\det\begin{pmatrix} 1-X & 0 & -1 \\ 0 & 1-X & 0 \\ 0 & 1 & 2-X \end{pmatrix} = (2-X)\cdot(1-X)^2\cdot(2-X) = (1-X)^2(2-X)^2$$

$\Rightarrow \chi_f(X) = (1-X)^2(2-X)^2$

b) ges: J-NF

$$rg(A-1\cdot E_4) = rg\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} = 3 \qquad rg((A-1\cdot E_4)^2) = rg\begin{pmatrix} 0 & -1 & -1 & -1 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} = 2$$

$$rg(A-2E_4) = rg\begin{pmatrix} -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} = 2$$

$\lambda = 1$:

	0	1	2	3	
r	4	3	2	2	…
u		1	1	0	…
k		0	1	0	…

$\lambda = 2$:

	0	1	2	
r	4	2	2	…
u		2	0	…
k		2	0	…

$$\Rightarrow \text{J-NF von } A = \begin{pmatrix} \boxed{\begin{matrix} 1 & 1 \\ 0 & 1 \end{matrix}} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \begin{matrix} \boxed{2} & 0 \\ 0 & \boxed{2} \end{matrix} \end{pmatrix}$$

c) ges: Basis C von V mit $\langle C^*, f(C)\rangle =$ J-NF von A

$$(A-1\cdot E_4)x = 0 \iff \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \iff x_3 = 0,\ x_4 = 0,\ x_2 = 0$$

Lösungsraum $\left[\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}\right]$

$$(A-1\cdot E_4)^2 x = 0 \iff \begin{pmatrix} 0 & -1 & -1 & -1 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \iff x_4 = 0,\ x_2 = -x_3$$

Lösungsraum $\left[\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}\right]$

$$(A-2\cdot E_4)\cdot x = 0 \iff \begin{pmatrix} -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$\iff x_2 = -x_4 \quad x_1 = -x_3 \quad$ LSG $\left[\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}\right]$

$$\Rightarrow C = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

d) ges: $\mu_f(X)$

$$(1-A)(2-A) = \begin{pmatrix} 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad (1-A)(2-A)^2 = \begin{pmatrix} 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad (1-A)^2(2-A) = \mathcal{O}$$

$\Rightarrow (1-A)^2(2-A) = \mu_f(X)$ (siehe Python Code)