Homework 1

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Problem 1

$$p(t) = det(A - t \cdot \mathrm{Id}) = \begin{vmatrix} a_{11} - t & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} - t & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} - t & a_{n2} & \cdots & a_{nn} - t \end{vmatrix}$$

Problem 2

The Gamma function is defined as

$$\Gamma(x) := \lim_{n \to \infty} \frac{n! n^x}{x(x+1)\cdots(x+n)}$$

There holds the Weierstraß product representation

$$\frac{1}{\Gamma(x)} = x \cdot e^{Cx} \cdot \prod_{k=1}^{\infty} (1 + \frac{x}{k}) e^{-k/k} \qquad \text{with} \qquad C := \lim_{n \to \infty} (\sum_{k=1}^{n} \frac{1}{k} - \ln n)$$

Problem 3

Let $f, g : \mathbb{R} \to \mathbb{R}$ be given functions given by

$$f(x) := \begin{cases} -1 & \text{if } x < -\frac{\pi}{2}, \\ \sin(x) & \text{if } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}, \\ 1 & \text{if } x > \frac{\pi}{2}. \end{cases} \quad \text{and} \quad g(x) := \begin{cases} 1 & \text{if } x \in \mathbb{Q}, \\ 0 & \text{if } x \in \mathbb{R} \setminus \mathbb{Q}. \end{cases}$$

Problem 4

For $q \in \mathbb{R}$, it holds that

$$\lim_{n\to\infty} q^n = \begin{cases} +\infty & \text{if } q > 1, \\ 1 & \text{if } q = 1, \\ 0 & \text{if } -1 < q < 1, \\ \nexists & \text{if } q \leq -1. \end{cases}$$

Problem 5

$$A := \begin{pmatrix} \alpha & 2\alpha & 3\alpha & \cdots & n\alpha \\ 0 & \alpha & 2\alpha & \ddots & \vdots \\ 0 & 0 & \ddots & \ddots & 3\alpha \\ \vdots & \ddots & \ddots & \ddots & 2\alpha \\ 0 & \cdots & 0 & 0 & \alpha \end{pmatrix} \in \mathbb{R}^{n\times n}_{\text{tria}}$$