ANA Ü4

g.) $\int_1^e \frac{\ln(x)}{x}\,dx$

$\int \frac{1}{x}\cdot\ln(x)\,dx \qquad v=\ln(x) \qquad \frac{dv}{dx}=\frac{1}{x} \qquad dx = x\cdot dv$

$\int \frac{1}{x}\cdot v\cdot x\,dv = \int v\,dv = \frac{v^2}{2} = \frac{\ln(x)^2}{2}$

$\int_1^e \frac{\ln(x)}{x}\,dx = \frac{\ln(e)^2}{2} - \frac{\ln(1)^2}{2} = \frac{1}{2} - 0 = \frac{1}{2}$

$\int_1^2 \frac{1}{x^3+x}\,dx$

$\int \frac{1}{x^3+x}\,dx = \int \frac{x^2+1-x^2}{x\cdot(x^2+1)}\,dx = \int \frac{x^2+1}{x\cdot(x^2+1)} - \frac{x^2}{x\cdot(x^2+1)}\,dx = \int \frac{1}{x} - \frac{x}{x^2+1}\,dx$

$= \int \frac{1}{x}\,dx - \int \frac{x}{x^2+1}\,dx = \ln(x) - \int \frac{x}{x^2+1}\,dx$

$v = x^2+1 \qquad \frac{dv}{dx} = 2x \qquad dx = \frac{1}{2x}\cdot dv$

$\ln(x) - \int \frac{x}{v}\cdot\frac{1}{2x}\,dv = \ln(x) - \frac{1}{2}\cdot\int \frac{1}{v}\,dv = \ln(x) - \frac{1}{2}\cdot\ln(v)$

$= \ln(x) - \frac{1}{2}\cdot\ln(x^2+1)$

$\int_1^2 \frac{1}{x^3+x}\,dx = \ln(2) - \frac{1}{2}\cdot\ln(2^2+1) - \left(\ln(1) - \frac{1}{2}\cdot\ln(1^2+1)\right)$

$= \ln(2) - \frac{\ln(5)}{2} - \left(0 - \frac{1}{2}\cdot\ln(2)\right) = \ln(2) - \frac{\ln(5)}{2} + \frac{1}{2}\cdot\ln(2)$

$= \frac{3}{2}\cdot\ln(2) - \frac{1}{2}\cdot\ln(5)$