ANA Ü12

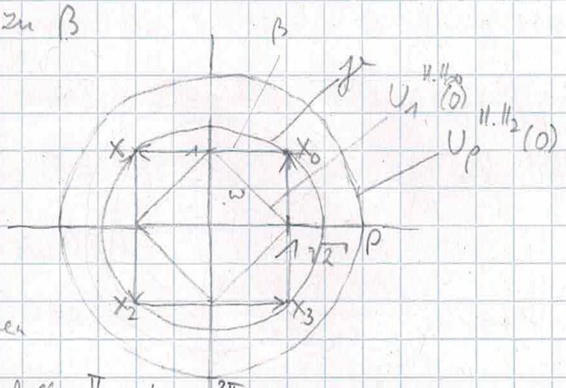
3.) $x_0 = \begin{pmatrix}1\\1\end{pmatrix}$ $x_1 = \begin{pmatrix}-1\\1\end{pmatrix}$ $x_2 = \begin{pmatrix}-1\\-1\end{pmatrix}$ $x_3 = \begin{pmatrix}1\\-1\end{pmatrix}$ $\gamma : [\frac{\pi}{4}, 2\pi + \frac{\pi}{4}] \longrightarrow \mathbb{R}^2$

$t \longmapsto \begin{pmatrix}\sqrt{2}\cos(t)\\ \sqrt{2}\sin(t)\end{pmatrix}$

$\rho > \sqrt{2}$ $w \in U_1^{\|.\|_\infty}(0) \subseteq \mathbb{R}^2$

$\beta : [\frac{\pi}{4}, 2\pi + \frac{\pi}{4}] \to \mathbb{R}^2$ α ... linear $\beta = \overrightarrow{x_0x_1}, \overrightarrow{x_1x_2}, \overrightarrow{x_2x_3}, \overrightarrow{x_3x_0} \circ \alpha$

zz: γ ist in $U_\rho^{\|.\|_2}(0) \setminus \{w\}$ homotop zu β



$$\gamma\left(\frac{\pi}{4}\right) = \begin{pmatrix}\sqrt{2}\cos(\frac{\pi}{4})\\ \sqrt{2}\sin(\frac{\pi}{4})\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix} = x_0$$

$$\gamma\left(2\pi + \frac{\pi}{4}\right) = \begin{pmatrix}\sqrt{2}\cos(2\pi + \frac{\pi}{4})\\ \sqrt{2}\sin(2\pi + \frac{\pi}{4})\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix} = x_0$$

$\Rightarrow \gamma$ und $\overrightarrow{x_0x_1}, \overrightarrow{x_1x_2}, \overrightarrow{x_2x_3}, \overrightarrow{x_3x_0}$ haben gleichen Anfangs- und Endpunkt

$$\beta : [\tfrac{\pi}{4}, 2\pi + \tfrac{\pi}{4}] \to \mathbb{R}^2 \qquad t \longmapsto \begin{cases} \begin{pmatrix}\tan(t-\frac{\pi}{2})\\ 1\end{pmatrix} & \text{falls } \frac{\pi}{4} \le t < \frac{3\pi}{4} \\ \begin{pmatrix}-1\\ \tan(t)\end{pmatrix} & \text{falls } \frac{3\pi}{4} < t \le \pi + \frac{\pi}{4} \\ \begin{pmatrix}\tan(t-\frac{\pi}{2})\\ -1\end{pmatrix} & \text{falls } \pi + \frac{\pi}{4} < t \le \pi + \frac{3\pi}{4} \\ \begin{pmatrix}1\\ \tan(t)\end{pmatrix} & \text{falls } \pi + \frac{3\pi}{4} < t \le 2\pi + \frac{\pi}{4} \end{cases}$$

$\Gamma : [\frac{\pi}{4}, 2\pi + \frac{\pi}{4}] \times [0,1] \longrightarrow \mathbb{R}^2$

$$(s,t) \longmapsto \beta(s) + t\cdot(\gamma(s) - \beta(s)) = t\cdot\gamma(s) + (1-t)\beta(s)$$

$\Gamma(s,0) = \beta(s) + 0\cdot(\gamma(s) - \beta(s)) = \beta(s)$ $\Gamma(s,1) = \beta(s) + 1(\gamma(s) - \beta(s)) = \gamma(s)$

Γ ist als Zusammensetzung stetiger Funktionen stetig.

zz: $\forall s \in [\frac{\pi}{4}, 2\pi + \frac{\pi}{4}]$ $\forall t \in [0,1] : \|\Gamma(s,t)\|_\infty \ge 1$

$\|\Gamma(s,t)\|_\infty \ge \|\beta(s)\|_\infty$, da $\Gamma(s,t) = \beta(s) + t(\gamma(s) - \beta(s))$

$\|\beta(s)\|_\infty = \max(|\tan(t)|, |1|) \ge 1$ oder

$\|\beta(s)\|_\infty = \max(|\tan(t - \frac{\pi}{2})|, |1|) \ge 1$

$\Rightarrow \gamma$ ist zu β homotop