MAS Ü10

5.) $F(x) = \begin{cases} 1-e^{-x} & \text{wenn } x>0 \\ 0 & \text{sonst} \end{cases}$ ges: $\int x \, d\mu_F(x)$

$$\int x \, d\mu_F(x) = \int_0^\infty \mu_F([x>y])dy - \int_0^\infty \mu_F([x<-y])dy$$

$$= \int_0^\infty \mu_F(\{x\in\mathbb{R} : x>y\})\,dy - \int_0^\infty \mu_F(\{x\in\mathbb{R} : x<-y\})\,dy$$

$$\int_0^\infty \mu_F(]y,+\infty[)\,dy = \lim_{z\to+\infty} \int_0^\infty F(z)-F(y) = \lim_{z\to+\infty}\int_0^\infty 1-e^{-z}-1+e^{-y}\,dy$$

$$= \lim_{z\to+\infty}\int_0^\infty \frac{1}{e^y} - \underbrace{\frac{1}{e^z}}_{\to 0}\,dy = \int_0^\infty \frac{1}{e^y}\,dy = -e^{-y}\Big|_0^\infty = 0-(-1) = 1$$

$$\int_0^\infty \mu_F(]-\infty,-y[)\,dy = \lim_{z\to-\infty}\int_0^\infty \underbrace{F(-y)}_{=0} - \underbrace{F(z)}_{=0}\,dy = \int_0^\infty 0\,dy = 0$$

$$\Rightarrow \int x\,d\mu_F(x) = 1$$