ANA Ü8

6. $A \in \mathbb{R}^{n\times n}$ $\quad A^T = A$ $\quad f: \mathbb{R}^n \to \mathbb{R}$, $\; x \mapsto x^T A x$

zz: $df(x) = 2(Ax)^T$

$df(x) = \left(\frac{\partial f}{\partial x_j}(x)\right)_{j=1,\dots,n}$ $\qquad \frac{\partial f}{\partial x_i}(x) = \lim\limits_{s\to 0} \frac{1}{s}\left(f(x+se_i) - f(x)\right)$

laut 10.1.9, 5. $\qquad = \lim\limits_{s\to 0} \frac{1}{s}\left((x+se_i)^T A (x+se_i) - x^T A x\right)$

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \qquad x^T A \cdot x = (x_1\ x_2 \dots x_n)\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & & \vdots \\ a_{n1} & & \cdots & a_{nn} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = (x_1\ x_2 \dots x_n)\begin{pmatrix} \sum_{i=1}^n a_{1i}\cdot x_i \\ \vdots \\ \sum_{i=1}^n a_{ni}\cdot x_i \end{pmatrix}$$

$$= \left(x_1 \cdot \sum_{i=1}^n a_{1i} x_i + \dots + x_n \cdot \sum_{i=1}^n a_{ni} x_i\right)$$

bei $i=j$ steht (x_j+s)

$$\lim_{s\to 0} \frac{1}{s}\left((x+se_j)^T A (x+se_j) - x^T A x\right) = \lim_{s\to 0}\frac{1}{s}\Big(x_1 \sum_{i=1}^n a_{1i} x_i + \dots + (x_j+s)\sum_{i=1}^n a_{ji} x_i + \dots + x_n \sum_{i=1}^n a_{ni} x_i$$
$$- \big(x_1 \sum_{i=1}^n a_{1i} x_i + \dots + x_n \sum_{i=1}^n a_{ni} x_i\big)\Big) =$$

$$x_1 \cdot \Big(\sum_{\substack{i=1\\ i\neq j}}^n a_{1i} x_i + a_{1j}(x_j+s)\Big) - x_1\Big(\sum_{\substack{i=1\\ i\neq j}}^n a_{1i} x_i + a_{1j} x_j\Big) = x_1 a_{1j}(x_j+s) - x_1 a_{1j} x_j$$
$$= x_1 a_{1j} x_j + s x_1 a_{1j} - x_1 a_{1j} x_j = s x_1 a_{1j}$$

$$= \lim_{s\to 0}\frac{1}{s}\Big(s x_1 a_{1j} + \dots + s x_n a_{nj} + s \sum_{i=1}^n a_{ji} x_i\Big) = x_1 a_{1j} + \dots + x_n a_{nj} + \sum_{i=1}^n a_{ji} x_i$$

$$= \sum_{i=1}^n a_{ij} x_i + \sum_{i=1}^n a_{ji} x_i = 2 \sum_{i=1}^n a_{ij} x_i \quad \text{, da } a_{ij} = a_{ji} \text{ wegen } A^T = A$$

stetig

$$df(x) = \Big(2 \sum_{i=1}^n a_{ji} x_i\Big)_{j=1,\dots,n} = 2\cdot\Big(\sum_{i=1}^n a_{1i} x_i \ \dots \ \sum_{i=1}^n a_{ni} x_i\Big) = 2(Ax)^T$$

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ges: $\frac{\partial f}{\partial x}(x)$

$$\frac{\partial f}{\partial x}(x) = \sum_{j=1}^n x_j \frac{\partial f}{\partial x_j}(x) = df(x)\,x = 2(Ax)^T x = 2x^T A^T x \underset{A^T=A}{=} 2x^T A x = 2f(x)$$