MAS Ü12

2.) X ... standardnormalverteilt

a) $t \in \mathbb{R}$ ges: $\mathbb{E}(e^{tX})$

$$\mathbb{E}(e^{tX}) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx = \int_{-\infty}^{\infty} e^{tx} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{tx - \frac{x^2}{2}} dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\frac{1}{2}t^2 - \left(\frac{1}{\sqrt{2}}x - \frac{1}{\sqrt{2}}t\right)^2} dx$$

$$\left[\begin{aligned} tx - \frac{x^2}{2} &= -\left(\frac{1}{2}x^2 - tx + \frac{1}{2}t^2 - \frac{1}{2}t^2\right) \\ &= -\left(\left(\frac{1}{\sqrt{2}}x - \frac{1}{\sqrt{2}}t\right)^2 - \frac{1}{2}t^2\right) \\ &= \frac{1}{2}t^2 - \left(\frac{1}{\sqrt{2}}x - \frac{1}{\sqrt{2}}t\right)^2 \end{aligned}\right.$$

$$u = \frac{x-t}{\sqrt{2}} \quad \frac{du}{dx} = \frac{1}{\sqrt{2}} \quad dx = \sqrt{2}\,du$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\frac{1}{2}t^2 - u^2} \sqrt{2}\,du$$

$$= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{\frac{1}{2}t^2 - u^2} du$$

$$= \frac{1}{\sqrt{\pi}} \left(\int_{-\infty}^{\infty} e^{\frac{1}{2}t^2} \cdot e^{-u^2} du\right) = \frac{1}{\sqrt{\pi}} e^{\frac{1}{2}t^2} \int_{-\infty}^{\infty} e^{-u^2} du = \frac{1}{\sqrt{\pi}} \cdot e^{\frac{1}{2}t^2} \cdot \sqrt{\pi} = e^{\frac{1}{2}t^2}$$

b) ges: obere Schranke für $P(X \geq x)$ $(x > 0)$

Nach Markov-Ungleichung:

$$P(X \geq c) \leq \frac{\mathbb{E}(X)}{c}$$

$$P(e^{tX} \geq e^{tc}) \leq \frac{\mathbb{E}(e^{tX})}{e^{tc}} = \frac{e^{\frac{1}{2}t^2}}{e^{tc}}$$

ges: kleinste Schranke

$$f(t) = \frac{e^{\frac{1}{2}t^2}}{e^{tc}} = e^{\frac{1}{2}t^2 - tc}$$

$$f'(t) = (t - c) \cdot \underbrace{e^{\frac{1}{2}t^2 - tc}}_{\text{für kein } t \in \mathbb{R} \text{ wird das Null}} \quad \Rightarrow f'(t) = 0 \iff t = c$$