ANA Ü6

2.) $\int_{-1}^{1} \frac{x}{x^6+i}\,dx$

$f(x) = \frac{x}{x^6+i}$ $\qquad f(-x) = \frac{-x}{(-x)^6+i} = -\frac{x}{x^6+i} = -f(x)$

$$\int_{-1}^{1} f(x)\,dx = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx = -\int_{0}^{-1} f(x)\,dx + \int_{0}^{1} f(x)\,dx$$

$\Big[\, u = -x \quad \frac{du}{dx} = -1 \quad dx = -du$

$$= -\int_{0}^{-(-1)} f(-u)\,(-1)\,du + \int_{0}^{1} f(x)\,dx = \int_{0}^{1} f(-u)\,du + \int_{0}^{1} f(x)\,dx$$

$$= -\int_{0}^{1} f(u)\,du + \int_{0}^{1} f(x)\,dx = 0$$

...

ANA Ü6

2.) ... 6)

Behauptung: $\int_0^1 x^m (\ln(x))^n \, dx = \frac{(-1)^n \, n!}{(m+1)^{n+1}}$

$n=0$: $\int_0^1 x^m (\ln(x))^0 dx = \int_0^1 x^m \, dx = \frac{x^{m+1}}{m+1}\Big|_0^1 = \frac{1}{m+1} - \frac{0}{m+1} = \frac{1}{m+1}$

$$\frac{(-1)^0 \, 0!}{(m+1)^{0+1}} = \frac{1}{m+1} \quad \checkmark$$

$n+1$: $\int_0^1 x^m (\ln(x))^{n+1} dx = \int_0^1 \left(\frac{x^{m+1}}{m+1}\right)' (\ln(x))^{n+1} dx$

$$= \frac{x^{m+1}}{m+1} (\ln(x))^{n+1}\Big|_0^1 - \int_0^1 \frac{x^{m+1}}{m+1} \cdot (n+1) \cdot (\ln(x))^n \cdot \frac{1}{x} \, dx$$

$$= \frac{x^{m+1}}{m+1} (\ln(x))^{n+1}\Big|_0^1 - \frac{n+1}{m+1} \int_0^1 x^m \cdot (\ln(x))^n \, dx$$

$$= \frac{x^{m+1}}{m+1} (\ln(x))^{n+1}\Big|_0^1 - \frac{n+1}{m+1} \cdot \frac{(-1)^n \, n!}{(m+1)^{n+1}}$$

$$\frac{x^{m+1}}{m+1} (\ln(x))^{n+1}\Big|_0^1 = \frac{1}{m+1} \cdot 0 - \lim_{\alpha \to 0+} \frac{\alpha^{m+1}}{m+1} (\ln(\alpha))^{n+1}$$

$$= \frac{1}{m+1} \cdot \lim_{\alpha \to 0+} \frac{-(\ln(\alpha))^{n+1}}{\frac{1}{\alpha^{m+1}}} = \frac{1}{m+1} \cdot \lim_{\alpha \to 0+} \frac{-(n+1) \cdot \ln(\alpha)^n \cdot \frac{1}{\alpha}}{-(m+1) \cdot \frac{1}{\alpha^{m+2}}}$$

$$= \frac{1}{m+1} \cdot \frac{n+1}{-(m+1)} \cdot \lim_{\alpha \to 0+} \frac{-(\ln(\alpha))^n}{\frac{1}{\alpha^{m+1}}} = \ldots = \frac{1}{m+1} \cdot \frac{n+1}{-(m+1)} \cdot \ldots \cdot \lim_{\alpha \to 0+} \frac{-(\ln(\alpha))^1}{\frac{1}{\alpha^{m+1}}}$$

$$= \frac{1}{m+1} \cdot \frac{n+1}{-(m+1)} \cdot \ldots \cdot \lim_{\alpha \to 0+} \frac{-\frac{1}{\alpha}}{-(m+1) \cdot \frac{1}{\alpha^{m+2}}} = \frac{1}{m+1} \cdot \frac{n+1}{-(m+1)} \cdot \ldots \cdot \frac{1}{m+1} \cdot \lim_{\alpha \to 0+} \alpha^{m+1} = 0$$

$$\frac{x^{m+1}}{m+1} (\ln(x))^{n+1}\Big|_0^1 - \frac{n+1}{m+1} \cdot \frac{(-1)^n \, n!}{(m+1)^{n+1}} = (-1)^{n+1} \cdot \frac{(n+1)!}{(m+1)^{n+2}}$$

$$\Rightarrow \int_0^1 x^m (\ln(x))^n dx = (-1)^n \frac{n!}{(m+1)^{n+1}}$$