MAS Ü11

6.) Normalverteilung $N(\mu, \sigma^2)$

Für $\mu = 0 \quad \sigma = 1$: $\mathbb{E}(X) = \int_{-\infty}^{\infty} x \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x e^{-\frac{x^2}{2}} = 0$, da punktsymmetrisch

Sonst $\mathbb{E}(Y) = \mathbb{E}(\sigma X + \mu) = \sigma\, \mathbb{E}(X) + \mu = \mu$

Wolframalpha

Für $\mu = 0 \quad \sigma = 1$: $\mathbb{E}(X^2) = \int_{-\infty}^{\infty} x^2 \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x^2 \cdot e^{-\frac{x^2}{2}} dx \stackrel{\vdots}{=} \frac{\sqrt{2\pi}}{\sqrt{2\pi}} = 1$

$$\mathbb{V}(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = 1 - 0 = 1$$

Sonst: $\mathbb{E}(Y^2) = \mathbb{E}((\sigma X + \mu)^2) = \mathbb{E}(\sigma^2 X^2 + 2\sigma X \mu + \mu^2)$

$= \sigma^2\, \mathbb{E}(X^2) + 2\sigma\mu\, \mathbb{E}(X) + \mu^2 = \sigma^2 + \mu^2$

$$\mathbb{V}(Y) = \mathbb{E}(Y^2) - (\mathbb{E}(Y))^2 = \sigma^2 + \mu^2 - \mu^2 = \sigma^2$$