ANA Ü9

3.) $f\binom{x}{y} = x^3 - 2x^2y^2 + 4xy^3 + y^4 + 10$

ges: $\frac{\partial^{k+l}}{\partial x^k \partial y^l}$ für $k,l = 0,1,2$

$\frac{\partial}{\partial x} f\binom{x}{y} = \frac{d}{dx} x^3 - 2x^2y^2 + 4xy^3 + y^4 + 10 = 3x^2 - 4xy^2 + 4y^3 =: f_{x_1}\binom{x}{y}$

$\frac{\partial}{\partial x} f_{x_1}\binom{x}{y} = \frac{d}{dx} 3x^2 - 4xy^2 + 4y^3 = 6x - 4y^2 =: f_{x_2}\binom{x}{y}$

$\frac{\partial}{\partial y} f\binom{x}{y} = \frac{d}{dy} x^3 - 2x^2y^2 + 4xy^3 + y^4 + 10 = -4x^2y + 12xy^2 + 4y^3 =: f_{y_1}\binom{x}{y}$

$\frac{\partial}{\partial y} f_{y_1}\binom{x}{y} = \frac{d}{dy} -4x^2y + 12xy^2 + 4y^3 = -4x^2 + 24xy + 12y^2 =: f_{y_2}\binom{x}{y}$

$\frac{\partial}{\partial y} f_{x_1}\binom{x}{y} = \frac{d}{dy} 3x^2 - 4xy^2 + 4y^3 = -8xy + 12y^2 =: f_{x_1y_1}\binom{x}{y}$

$\frac{\partial}{\partial y} f_{x_2}\binom{x}{y} = \frac{d}{dy} 6x - 4y^2 = -8y =: f_{x_2y_1}\binom{x}{y}$

$\frac{\partial}{\partial x} f_{y_2}\binom{x}{y} = \frac{d}{dx} -4x^2 + 24xy + 12y^2 = -8x + 24y =: f_{x_1y_2}\binom{x}{y}$

$\frac{\partial}{\partial x} f_{x_1y_2}\binom{x}{y} = \frac{d}{dx} -8x + 24y = -8 =: f_{x_2y_2}\binom{x}{y}$

$\frac{\partial^{k+l}}{\partial x^k \partial y^l} = f_{x_k y_l}\binom{x}{y}$ $\quad f, f_{x_1}, f_{y_1}$ stetig partiell diffbar

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ges: $d f\binom{x}{y} v_1$, $d^2 f\binom{x}{y}(v_1, v_2)$

$d f\binom{x}{y} = \left(\frac{\partial f}{\partial x_j}\binom{x}{y}\right)_{j=1,2} = \left(f_{x_1}\binom{x}{y}, f_{y_1}\binom{x}{y}\right)^T$

$\left(f_{x_1}\binom{x}{y}, f_{y_1}\binom{x}{y}\right)^T \cdot \binom{v_a}{v_b} = v_a \cdot (3x^2 - 4xy^2 + 4y^3) + v_b(-4x^2y + 12xy^2 + 4y^3)$

$d^2 f\binom{x}{y}(v_1, v_2) = \frac{\partial}{\partial v_1}\frac{\partial}{\partial v_2} f\binom{x}{y} = \sum_{l_1=1}^{2} v_{1,l_1} \sum_{l_2=1}^{2} v_{2,l_2} \frac{\partial^2 f}{\partial x_{l_1} x_{l_2}}\binom{x}{y}$

$= \sum_{l_1=1}^{2} v_{1,l_1}\left(v_{2,1} f_{x_{l_1},y_1}\binom{x}{y} + v_{2,2} f_{x_{l_1},y_2}\binom{x}{y}\right) = v_{1,1}\left(v_{2,1} f_{x_1y_1}\binom{x}{y} + v_{22} f_{x_1y_2}\binom{x}{y}\right)$

$+ v_{1,2}\left(v_{2,1} f_{x_2y_1}\binom{x}{y} + v_{22} f_{x_2,y_2}\binom{x}{y}\right) =$

$= v_{11} \cdot v_{21} \cdot (-8xy + 12y^2) + v_{11} v_{22} \cdot (-8x + 24y) + v_{12} v_{21}(-8y) + v_{12} v_{22}(-8)$

...

ANA Ü9

3.) ... ges: Taylorpolynom in $\binom{x}{y}$ mit $q=2$ und Anschlussstelle $\binom{0}{0}$

$$T_2(x) = f\binom{0}{0} + \sum_{l=1}^{2} \frac{1}{l!} d^l f\binom{0}{0} \underbrace{\left(x-\binom{0}{0}, \dots, x-\binom{0}{0}\right)}_{l\text{-Mal}} \qquad x = \binom{x_1}{x_2}$$

$$= 10 + df\binom{0}{0}x + \frac{1}{2} d^2 f\binom{0}{0}(x,x)$$

$$= 10 + x_1 \cdot 0 + x_2 \cdot 0 + \frac{1}{2}\left(0 + x_2 \cdot x_2(-8)\right) = 10 - 4x_2^2$$