# Homework 1

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## Problem 1

$$p(t) = \det(A - t \cdot \mathrm{Id}) = \begin{vmatrix} a_{11} - t & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} - t & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} - t & a_{n2} & \cdots & a_{nn} - t \end{vmatrix}$$

## Problem 2

The Gamma function is defined as

$$\Gamma(x) := \lim_{n \to \infty} \frac{n! n^x}{x(x+1) \cdots (x+n)}$$

There holds the Weierstraß product representation

$$\frac{1}{\Gamma(x)} = x \cdot e^{Cx} \cdot \prod_{k=1}^{\infty} (1 + \frac{x}{k}) e^{-k/k} \qquad \text{with} \qquad C := \lim_{n \to \infty} (\sum_{k=1}^{n} \frac{1}{k} - \ln n)$$

## Problem 3

Let $f, g : \mathbb{R} \to \mathbb{R}$ be given functions given by

$$f(x) := \begin{cases} -1 & \text{if } x < -\frac{\pi}{2}, \\ \sin(x) & \text{if } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}, \\ 1 & \text{if } x > \frac{\pi}{2}. \end{cases} \text{ and } \quad g(x) := \begin{cases} 1 & \text{if } x \in \mathbb{Q}, \\ 0 & \text{if } x \in \mathbb{R} \setminus \mathbb{Q}. \end{cases}$$

## Problem 4

For $q \in \mathbb{R}$, it holds that

$$\lim_{n\to\infty} q^n = \begin{cases} +\infty & \text{if } q > 1, \\ 1 & \text{if } q = 1, \\ 0 & \text{if } -1 < q < 1, \\ \nexists & \text{if } q \le -1. \end{cases}$$

## Problem 5

$$A := \begin{pmatrix} \alpha & 2\alpha & 3\alpha & \cdots & n\alpha \\ 0 & \alpha & 2\alpha & \ddots & \vdots \\ 0 & 0 & \ddots & \ddots & 3\alpha \\ \vdots & \ddots & \ddots & \ddots & 2\alpha \\ 0 & \cdots & 0 & 0 & \alpha \end{pmatrix} \in \mathbb{R}^{n\times n}_{\text{tria}}$$

## Problem 6

$a^3b^2c - a^3b^2d - a^3bc^2 + a^3bd^2 + a^3c^2d - a^3cd^2 - a^2b^3c + a^2b^3d + a^2bc^3 - a^2bd^3 - a^2c^3d + a^2cd^3 + ab^3c^2 - ab^3d^2 - ab^2c^3 + ab^2d^3 + ac^3d^2 - ac^2d^3 - b^3c^2d + b^3cd^2 + b^2c^3d - b^2cd^3 - bc^3d^2 + bc^2d^3$

## Problem 7

**Theorem 1** *For $a, b \in \mathbb{R}$ and a continuous function $f : (a, b) \to \mathbb{R}$, the following two assertions are equivalent:*

*(i) $f$ is uniformly continuous.*

*(ii) $f$ has a continuous extension onto the compact interval $[a, b]$, i.e., there exists a function $\hat{f} : [a, b] \to \mathbb{R}$ with $\hat{f} = f(x)$ for all $x \in (a, b)$.*

*In this case the continuous extension $\hat{f}$ is even unique.*

***Proof*** TODO ■

## Problem 8

**Theorem 2** *For real numbers $x, y \in \mathbb{R}$ and $n \in \mathbb{N}$ we have*

$$(x + y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k.$$

***Proof*** TODO ■