

MAS 011

5.) Poissonverteilung $P(X)$ $P(\{n\}) = \frac{\lambda^n}{n!} e^{-\lambda}$

$$\begin{aligned} E(X) &= \sum_{n=0}^{\infty} n \cdot \frac{\lambda^n}{n!} \cdot e^{-\lambda} = e^{-\lambda} \sum_{n=1}^{\infty} n \cdot \frac{\lambda^n}{n!} = e^{-\lambda} \lambda \sum_{n=1}^{\infty} \frac{\lambda^{n-1}}{(n-1)!} = \\ &= e^{-\lambda} \lambda \cdot \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} = e^{-\lambda} \lambda \cdot e^{\lambda} = \lambda \end{aligned}$$

$$\begin{aligned} E(X^2) &= \sum_{n=0}^{\infty} n^2 \cdot \frac{\lambda^n}{n!} e^{-\lambda} = e^{-\lambda} \sum_{n=1}^{\infty} n \cdot \frac{\lambda^n}{(n-1)!} = e^{-\lambda} \left(\sum_{n=1}^{\infty} (n-1) \frac{\lambda^n}{(n-1)!} + \sum_{n=1}^{\infty} \frac{\lambda^n}{(n-1)!} \right) \\ &= e^{-\lambda} \sum_{n=2}^{\infty} \frac{\lambda^n}{(n-2)!} + e^{-\lambda} \sum_{n=1}^{\infty} \frac{\lambda^n}{(n-1)!} = e^{-\lambda} \lambda^2 \sum_{n=2}^{\infty} \frac{\lambda^{n-2}}{(n-2)!} + e^{-\lambda} \lambda \sum_{n=1}^{\infty} \frac{\lambda^{n-1}}{(n-1)!} \\ &= \lambda^2 + \lambda \end{aligned}$$

$$V(X) = E(X^2) - (E(X))^2 = \lambda^2 + \lambda - \lambda^2 = \lambda$$