

ANA Ü9

$$3.) f\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = x^3 - 2x^2y^2 + 4xy^3 + y^4 + 10$$

$$\text{ges: } \frac{\partial^{k+l}}{\partial x^k \partial y^l} f \text{ für } k, l = 0, 1, 2$$

$$\frac{\partial}{\partial x} f\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \frac{d}{dx} x^3 - 2x^2y^2 + 4xy^3 + y^4 + 10 = 3x^2 - 4xy^2 + 4y^3 =: f_{x_1}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right)$$

$$\frac{\partial}{\partial x} f_{x_1}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \frac{d}{dx} 3x^2 - 4xy^2 + 4y^3 = 6x - 4y^2 =: f_{x_2}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right)$$

$$\frac{\partial}{\partial y} f\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \frac{d}{dy} x^3 - 2x^2y^2 + 4xy^3 + y^4 + 10 = -4x^2y + 12xy^2 + 4y^3 =: f_{y_1}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right)$$

$$\frac{\partial}{\partial y} f_{y_1}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \frac{d}{dy} -4x^2y + 12xy^2 + 4y^3 = -4x^2 + 24xy + 12y^2 =: f_{y_2}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right)$$

$$\frac{\partial}{\partial y} f_{x_1}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \frac{d}{dy} 3x^2 - 4xy^2 + 4y^3 = -8xy + 12y^2 =: f_{x_2y_1}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right)$$

$$\frac{\partial}{\partial y} f_{x_2}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \frac{d}{dy} 6x - 4y^2 = -8y =: f_{x_2y_1}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right)$$

$$\frac{\partial}{\partial x} f_{y_2}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \frac{d}{dx} -4x^2 + 24xy + 12y^2 = -8x + 24y =: f_{x_1y_2}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right)$$

$$\frac{\partial}{\partial x} f_{x_1y_2}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \frac{d}{dx} -8x + 24y = -8 =: f_{x_2y_2}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right)$$

$$\frac{\partial^{k+l}}{\partial x^k \partial y^l} f = f_{x_k y_l}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) \quad f, f_{x_1}, f_{y_1} \text{ stetig partiell diffbar}$$

$$\text{ges: } d f\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) v_1, \quad d^2 f\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) (v_1, v_2)$$

$$d f\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \left(\frac{\partial f}{\partial x_j}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) \right)_{j=1,2} = (f_{x_1}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right), f_{y_1}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right))^T$$

$$(f_{x_1}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right), f_{y_1}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right))^T \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = v_1 \cdot (3x^2 - 4xy^2 + 4y^3) + v_2 \cdot (-4x^2y + 12xy^2 + 4y^3)$$

$$d^2 f\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) (v_1, v_2) = \frac{\partial}{\partial v_1} \frac{\partial}{\partial v_2} f\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \sum_{l_1=1}^2 v_{1,l_1} \sum_{l_2=1}^2 v_{2,l_2} \frac{\partial^2 f}{\partial x_{l_1} \partial x_{l_2}}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right)$$

$$= \sum_{l_1=1}^2 v_{1,l_1} (v_{2,1} f_{x_{l_1} y_1}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) + v_{2,2} f_{x_{l_1} y_2}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right)) = v_{1,1} (v_{2,1} f_{x_1 y_1}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) + v_{2,2} f_{x_1 y_2}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right)) + v_{1,2} (v_{2,1} f_{x_2 y_1}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) + v_{2,2} f_{x_2 y_2}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right)) =$$

$$= v_{1,1} v_{2,1} \cdot (-8xy + 12y^2) + v_{1,1} v_{2,2} \cdot (-8x + 24y) + v_{1,2} v_{2,1} \cdot (-8y) + v_{1,2} v_{2,2} \cdot (-8)$$

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3.) ... ges: Taylorpolynom in $\begin{pmatrix} x \\ y \end{pmatrix}$ mit $q=2$ und Anschlussstelle $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$T_2(x) = f\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \sum_{l=1}^2 \frac{1}{l!} d^l f\begin{pmatrix} 0 \\ 0 \end{pmatrix} \underbrace{(x - \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \dots, x - \begin{pmatrix} 0 \\ 0 \end{pmatrix})}_{l\text{-Mal}} \quad x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= 10 + df\begin{pmatrix} 0 \\ 0 \end{pmatrix} x + \frac{1}{2} d^2 f\begin{pmatrix} 0 \\ 0 \end{pmatrix} (x, x)$$

$$= 10 + x_1 \cdot 0 + x_2 \cdot 0 + \frac{1}{2} (0 + x_2 \cdot x_2 (-8)) = 10 - 4x_2^2$$