ANA Ü1

4.) $z \in \mathbb{C}$

$$\cosh(z) = \frac{\exp(z)+\exp(-z)}{2} = \frac{1}{2} \cdot \left(\sum_{n=0}^{\infty} \frac{z^n}{n!} + \sum_{n=0}^{\infty} \frac{(-z)^n}{n!} \right)$$

$$= \frac{1}{2} \cdot \left(\sum_{n=0}^{\infty} \frac{z^n}{n!} + \sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!} + \sum_{n=0}^{\infty} \frac{(-z)^{2n+1}}{(2n+1)!} \right)$$

$$= \frac{1}{2} \cdot \left(\sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!} + \sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!} + \sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!} - \sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!} \right)$$

$$= \frac{1}{2} \cdot \left(2 \cdot \sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!} \right) = \sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!}$$

Quotientenkriterium:

$$\frac{\left| \frac{z^{2(n+1)}}{(2(n+1))!} \right|}{\left| \frac{z^{2n}}{(2n)!} \right|} = \frac{(2n)! \cdot |z|^{2n+2}}{(2n+2)! \cdot |z|^{2n}} = \frac{(2n)! \cdot |z|^{2n} \cdot |z|^2}{(2n)! \cdot (2n+1) \cdot (2n+2) \cdot |z|^{2n}}$$

$$= \frac{|z|^2}{(2n+1)(2n+2)} = \frac{|z|^2}{4n^2+4n+2n+2} = \frac{|z|^2 \cdot \frac{1}{n^2}}{4 + \frac{6}{n} + \frac{2}{n^2}} \xrightarrow{n \to \infty} 0$$

$\Rightarrow R = \infty$, da für alle z konvergent

$$\sinh(z) = \frac{\exp(z)-\exp(-z)}{2} = \frac{1}{2} \cdot \left(\sum_{n=0}^{\infty} \frac{z^n}{n!} - \sum_{n=0}^{\infty} \frac{(-z)^n}{n!} \right)$$

$$= \frac{1}{2} \cdot \left(\sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!} + \sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!} - \sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!} + \sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!} \right)$$

$$= \frac{1}{2} \cdot 2 \cdot \sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!}$$

Quotientenkriterium:

$$\frac{\left| \frac{z^{2(n+1)+1}}{(2(n+1)+1)!} \right|}{\left| \frac{z^{2n+1}}{(2n+1)!} \right|} = \frac{(2n+1)! \cdot |z|^{2n+3}}{(2n+3)! \cdot |z|^{2n+1}} = \frac{(2n+1)! \cdot |z|^{2n+1} \cdot |z|^2}{(2n+1)! \cdot (2n+2) \cdot (2n+3) \cdot |z|^{2n+1}}$$

$$= \frac{|z|^2}{(2n+2)(2n+3)} = \frac{|z|^2}{4n^2+6n+4n+6} = \frac{|z|^2 \cdot \frac{1}{n^2}}{4 + \frac{10}{n} + \frac{6}{n^2}} \xrightarrow{n \to \infty} 0$$

$\Rightarrow R = \infty$

...

ANA 01

4.)... $x \in \mathbb{R}$

$$\lim_{x\to\infty} \sinh(x) = \lim_{x\to\infty} \frac{\exp(x)-\exp(-x)}{2} = \frac{1}{2}\cdot\left(\lim_{x\to\infty}\exp(x) - \lim_{x\to\infty}\exp(-x)\right)$$

$$= \frac{1}{2}\cdot\lim_{x\to\infty}\exp(x) = +\infty$$

$$\lim_{x\to-\infty} \sinh(x) = \lim_{x\to-\infty} \frac{\exp(x)-\exp(-x)}{2} = \frac{1}{2}\left(\lim_{x\to-\infty}(\exp(x)) - \lim_{x\to-\infty}\exp(-x)\right)$$

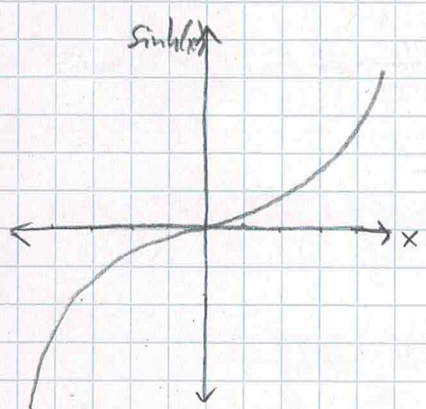
$$= -\frac{1}{2}\cdot\lim_{x\to-\infty}\exp(-x) = -\frac{1}{2}\lim_{x\to\infty}\exp(x) = -\infty$$

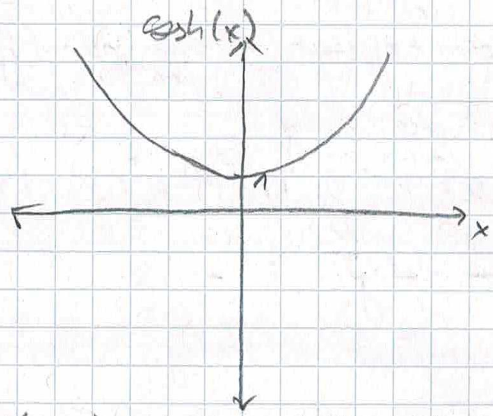
$$\lim_{x\to\infty} \cosh(x) = \lim_{x\to\infty} \frac{\exp(x)+\exp(-x)}{2} = \frac{1}{2}\left(\lim_{x\to\infty}\exp(x) + \lim_{x\to\infty}\exp(-x)\right)$$

$$= \frac{1}{2}\cdot\lim_{x\to\infty}\exp(x) = +\infty$$

$$\lim_{x\to-\infty} \cosh(x) = \lim_{x\to-\infty} \frac{\exp(x)+\exp(-x)}{2} = \frac{1}{2}\cdot\left(\lim_{x\to\infty}\exp(x) + \lim_{x\to\infty}\exp(-x)\right)$$

$$= \frac{1}{2}\cdot\lim_{x\to\infty}\exp(x) = +\infty$$





$$\cos(iz) = \frac{\exp(iiz)+\exp(-iiz)}{2} = \frac{\exp(-z)+\exp(z)}{2} = \cosh(z)$$

$$-i\cdot\sin(iz) = -i\cdot\frac{\exp(i\cdot iz)-\exp(-iiz)}{2i} = -\frac{\exp(-z)-\exp(z)}{2}$$

$$= \frac{\exp(z)-\exp(-z)}{2} = \sinh(z)$$

$\cosh(z)=0 \iff \cos(iz)=0 \iff \exists k\in\mathbb{Z}: iz = \frac{\pi}{2}+\pi\cdot k \iff \exists k\in\mathbb{Z}: z = \frac{i\pi}{2}+i\pi k$

$\sinh(z)=0 \iff -i\cdot\sin(iz)=0 \iff \exists k\in\mathbb{Z}: iz = \pi k \iff \exists k\in\mathbb{Z}: z = i\pi\cdot k$