ANA Ü3

2.) $\tan(x): \mathbb{R}\setminus(\frac{\pi}{2}+\pi\mathbb{Z}) \to \mathbb{R}$

$$x \mapsto \frac{\sin(x)}{\cos(x)}$$

•) ges $\tan'$, $\tan''$

$$(\tan(x))' = \left(\frac{\sin(x)}{\cos(x)}\right)' = \frac{\cos(x)\cdot\cos(x) - \sin(x)\cdot(-\sin(x))}{(\cos(x))^2} = \frac{(\sin(x))^2 + (\cos(x))^2}{(\cos(x))^2}$$

$$= \frac{1}{(\cos(x))^2}$$

$$(\tan(x))'' = \left(\frac{1}{(\cos(x))^2}\right)' = \frac{0\cdot(\cos(x))^2 - 1\cdot((\cos(x))^2)'}{((\cos(x))^2)^2} = -\frac{2\cos(x)\cdot(-\sin(x))}{(\cos(x))^4}$$

$$= \frac{2\sin(x)}{(\cos(x))^3}$$

•) ges: Nullstellen von tan

$$\tan(x)=0 \Leftrightarrow \frac{\sin(x)}{\cos(x)}=0 \Leftrightarrow \sin(x)=0 \Leftrightarrow \exists k\in\mathbb{Z}: x=\pi k$$

•) zz: tan ist auf jedem Intervall in $\mathbb{R}\setminus(\frac{\pi}{2}+\pi\mathbb{Z})$ streng monoton wachsend

$$(\tan(x))' = \frac{1}{(\cos(x))^2} = \left(\frac{1}{\cos(x)}\right)^2 > 0$$

$\Rightarrow$ streng monoton wachsend

•) ges: $\lim\limits_{t\to\frac{\pi}{2}-} \tan(t)$, $\lim\limits_{t\to-\frac{\pi}{2}+} \tan(t)$

$$\lim_{t\to\frac{\pi}{2}-} \tan(t) = \lim_{t\to\frac{\pi}{2}-} \frac{\sin(t)}{\cos(t)} = \lim_{t\to\frac{\pi}{2}-} \sin(t) \cdot \lim_{t\to\frac{\pi}{2}-} \frac{1}{\cos(t)} = 1\cdot \lim_{x\to 0+} \frac{1}{x} = \infty$$

$$\lim_{t\to\frac{\pi}{2}+} \tan(t) = \lim_{t\to-\frac{\pi}{2}+} \frac{\sin(t)}{\cos(t)} = \lim_{t\to-\frac{\pi}{2}+} \sin(t) \cdot \lim_{t\to-\frac{\pi}{2}+} \frac{1}{\cos(t)} = -1\cdot \lim_{x\to 0+} \frac{1}{x} = -\infty$$

$\Rightarrow \tan: (-\frac{\pi}{2}, \frac{\pi}{2}) \to \mathbb{R}$ ... bijektiv

•) ges: $(\arctan)'$

$y = \tan(x)$ $x = \arctan(y)$

$$\arctan'(y) = \arctan'(\tan(x)) = \frac{1}{\tan'(x)} = (\cos(x))^2 = \cos(\arctan(y))^2$$

(Satz 7.1.12)