ANA Ü7

8.) $T, A \in \mathbb{R}^{m \times m}$ $\quad T \ldots$ invertierbar

zz: $T^{-1} \cdot \exp(A) \cdot T = \exp(T^{-1} \cdot A \cdot T)$

$$\exp(A) := \sum_{n=0}^{\infty} \frac{1}{n!} A^n$$

$$T^{-1} \cdot \exp(A) \cdot T = T^{-1} \cdot \left(\sum_{n=0}^{\infty} \frac{1}{n!} A^n\right) \cdot T = \sum_{n=0}^{\infty} \left(\frac{1}{n!} T^{-1} A^n T\right)$$

$$\exp(T^{-1} A \cdot T) = \sum_{n=0}^{\infty} \frac{1}{n!} (T^{-1} A \cdot T)^n$$

$$(T^{-1} A \cdot T)^n = T^{-1} A \underbrace{T \cdot T^{-1}}_{=I} \cdot A \cdot \underbrace{T \ldots}_{=I} \underbrace{\ldots T^{-1}}_{=I} \cdot A \cdot T = T^{-1} \cdot A^n \cdot T$$

$$\Rightarrow \exp(T^{-1} A \cdot T) = T^{-1} \cdot \exp(A) \cdot T$$

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ges: $\exp(A)$, falls $A = \operatorname{diag}(a_1, a_2, \ldots, a_m)$

$$A^k = \operatorname{diag}(a_1^k, a_2^k, \ldots, a_m^k)$$

$$\exp(A) = \sum_{n=0}^{\infty} \frac{1}{n!} A^n$$

$$= \sum_{j=0}^{\infty} \operatorname{diag}\left(\frac{1}{j!} a_1^j, \frac{1}{j!} a_2^j, \ldots, \frac{1}{j!} a_m^j\right)$$

$$= \operatorname{diag}\left(\sum_{j=0}^{\infty} \frac{1}{j!} a_1^j, \ldots, \sum_{j=0}^{\infty} \frac{1}{j!} a_m^j\right)$$

$$= \operatorname{diag}(\exp(a_1), \ldots, \exp(a_m))$$

$$\begin{array}{c|c} & \begin{matrix} a_1 & 0 & \cdots & 0 \\ 0 & a_2 & & \vdots \\ \vdots & & \ddots & \\ 0 & \cdots & & a_m \end{matrix} \\ \hline \begin{matrix} a_1 & 0 & \cdots & 0 \\ 0 & a_2 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & a_m \end{matrix} & \begin{matrix} a_1^2 & 0 & \cdots & 0 \\ 0 & a_2^2 & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & & a_m^2 \end{matrix} \end{array}$$

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ges: $\exp\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ $\quad A := \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

$$\exp(A) = \sum_{n=0}^{\infty} \frac{1}{n!} A^n = \frac{1}{0!} A^0 + \frac{1}{1!} \cdot A^1$$

$$= 1 \cdot I + 1 \cdot A = I + A$$

$$\begin{array}{c|c} & \begin{matrix} 0 & 1 \\ 0 & 0 \end{matrix} \\ \hline \begin{matrix} 0 & 1 \\ 0 & 0 \end{matrix} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \end{array}$$