ANA Ü5

3.) $\int_{-4}^{1} 5 \cdot \frac{e^x-1}{e^x+1}\, dx$

$$\int 5 \cdot \frac{e^x-1}{e^x+1} dx = 5 \cdot \int \frac{e^x-1}{e^x+1} dx = 5\left(\int \frac{e^x}{e^x+1} dx - \int \frac{1}{e^x+1} dx\right)$$

$$\left[\begin{array}{llll} u = e^x+1 & x = \ln(u-1) & \frac{du}{dx} = e^x & dx = \frac{1}{e^x} du \\ m = e^x & x = \ln(m) & \frac{dm}{dx} = e^x & dx = \frac{1}{e^x} dm \end{array} \right.$$

$$= 5 \cdot \left(\int \frac{\cancel{e^x}}{u} \frac{1}{\cancel{e^x}} du - \int \frac{1}{m+1} \frac{1}{e^x} dm\right) = 5 \cdot \left(\ln(u) - \int \frac{1}{m(m+1)} dm\right)$$

$$= 5 \cdot \left(\ln(e^x+1) - \int \frac{1}{m} - \frac{1}{m+1} dm\right) = 5 \cdot \left(\ln(e^x+1) - \int \frac{1}{m} dm + \int \frac{1}{m+1} dm\right)$$

$$= 5 \cdot (\ln(e^x+1) - \ln(m) + \ln(m+1)) = 5(\ln(e^x+1) - \ln(e^x) + \ln(e^x+1))$$

$$= 5 \cdot (2\ln(e^x+1) - x) = 10 \ln(e^x+1) - 5x$$

$$\int_{-4}^{1} 5 \cdot \frac{e^x-1}{e^x+1} dx = (10 \ln(e^1+1) - 5 \cdot 1) - (10 \cdot \ln(e^{-4}+1) - 5 \cdot (-4))$$

$$= 10 \ln(e+1) - 5 - (10 \cdot \ln(e^{-4}+1) + 20)$$

$$= 10 \ln(e+1) - 10 \ln(e^{-4}+1) - 25 \approx -12{,}049$$

$\int_{1}^{5} \frac{x-\sqrt{x}}{x+\sqrt{x}} dx$

$$\int \frac{x-\sqrt{x}}{x+\sqrt{x}} dx = \int \frac{x-u}{x+u} 2\sqrt{x}\, du \qquad u = \sqrt{x} \quad x = u^2 \quad \frac{du}{dx} = \frac{1}{2\sqrt{x}} \quad dx = 2\sqrt{x}\, du$$

$$= 2\int \frac{u^2-u}{u^2+u} u\, du = 2\int \frac{u(u-1)}{u+1} du \qquad \begin{array}{l} v = u+1 \\ u = v-1 \end{array} \quad \frac{du}{dv} = 1 \quad du = dv$$

$$= 2\int \frac{(v-1)(v-2)}{v} dv = 2\int \frac{v^2-2v-v+2}{v} dv = 2\int \frac{v^2-3v+2}{v} dv$$

$$= 2\left(\int \frac{v^2}{v} dv - 3\int \frac{v}{v} dv + 2\int \frac{1}{v} dv\right) = 2\left(\int v\, dv - 3\int 1\, dv + 2\int \frac{1}{v} dv\right)$$

$$= 2\left(\frac{v^2}{2} - 3v + 2\ln(v)\right) = v^2 - 6v + 4\ln(v) = (u+1)^2 - 6(u+1) + 4\ln(u+1)$$

$$= (\sqrt{x}+1)^2 - 6(\sqrt{x}+1) + 4\ln(\sqrt{x}+1) = x + 2\sqrt{x} + 1 - 6\sqrt{x} - 6 + 4\ln(\sqrt{x}+1)$$

$$= x - 4\sqrt{x} - 5 + 4\ln(\sqrt{x}+1)$$

$$\int_{1}^{5} \frac{x-\sqrt{x}}{x+\sqrt{x}} = 5 - 4\sqrt{5} - 5 + 4\ln(\sqrt{5}+1) - (1 - 4\sqrt{1} - 5 + 4\ln(\sqrt{1}+1)) \approx 0{,}98058$$