ANA Ü3

5.) $f: [1, +\infty) \to \mathbb{R}$

$$x \mapsto \begin{cases} (x-1)^{(x-1)} & , x>1 \\ 1 & , x=1 \end{cases}$$

bei 1 stetig?
bei 1 rechtsseitig differenzierbar?

$$\lim_{x\to 1+} f(x) = \lim_{x\to 1+} (x-1)^{(x-1)} = \lim_{x\to 1+} \exp((x-1)\cdot \ln(x-1))$$

$$= \exp\left(\lim_{x\to 1+} \frac{\ln(x-1)}{\frac{1}{(x-1)}}\right)$$

$$\lim_{x\to 1+} \frac{1}{x-1} = \lim_{y\to 0+} \frac{1}{y} = +\infty$$

$$= \exp\left(\lim_{x\to 1+} \frac{\frac{1}{x-1}\cdot(1)}{-\frac{1}{(x-1)^2}\cdot 1}\right)$$

$$= \exp\left(\lim_{x\to 1+} -\frac{(x-1)^2}{(x-1)}\right) = \exp\left(\lim_{x\to 1+} -(x-1)\right) = \exp\left(\lim_{x\to 1+} -x+1\right)$$

$$= \exp(0) = 1 \qquad \Rightarrow \text{bei } 1 \text{ stetig}$$

$$\lim_{t\to 1+} \frac{f(t)-f(1)}{t-1} = \lim_{t\to 1+} \frac{(t-1)^{(t-1)}-1}{t-1} = \lim_{x\to 0+} \frac{x^x-1}{x}$$

$$\boxed{\lim_{x\to 0+} x^x - 1 = 1-1=0}$$

$$= \lim_{x\to 0+} \frac{(x^x)'-0}{1} = \lim_{x\to 0+} (\exp(x\cdot\ln(x)))' = \lim_{x\to 0+} \exp(x\cdot\ln(x))\cdot\left(1\cdot\ln(x) + x\cdot\frac{1}{x}\right)$$

$$= \lim_{x\to 0+} \exp(x\cdot\ln(x))\cdot\ln(x) + \exp(x\cdot\ln(x)) = \lim_{x\to 0+} x^x\cdot\ln(x) + x^x$$

$$= \lim_{x\to 0+} x^x \cdot \lim_{x\to 0+} \ln(x) + \lim_{x\to 0+} x^x = 1\cdot\lim_{x\to 0+} \ln(x) + 1 = -\infty$$