MAS Ü7

6.) $F(x_1, x_2) = \min(x_1, x_2)$

zz: F ist eine 2-D Verteilungsfunktion

[Satz 3.3 : F... Verteilungsfunktion $\Leftrightarrow$ F... rechtsstetig $\wedge\ \forall a,b \in \mathbb{R}^2, a \leq b \Rightarrow \Delta(a,b)F \geq 0$

· rechtsstetig

Sei $(x_1, x_2) \in \mathbb{R}^2$ bel.

$$\lim_{y_1 \to x_1} F(y_1, x_2) = \lim_{y_1 \to x_1} \min(y_1, x_2) = \min(x_1, x_2)$$

für $\lim_{x_2 \to x_2}$ genauso

· $\forall \underbrace{(a_1, a_2)}_{a}, \underbrace{(b_1, b_2)}_{b} \in \mathbb{R}^2, a_1 \leq b_1, a_2 \leq b_2 : \Delta(a,b)F \geq 0$

$$\Delta(a,b)F = \mu(]a,b]) = F(b) - F(a) = \min(b_1, b_2) - \min(a_1, a_2)$$

1. Fall $b_1 \leq b_2$

$$\min(a_1, a_2) \leq a_1 \leq b_1 = \min(b_1, b_2)$$

2. Fall $b_2 \leq b_1$

$$\min(a_1, a_2) \leq a_2 \leq b_2 = \min(b_1, b_2)$$

$$\Rightarrow \min(b_1, b_2) - \min(a_1, a_2) \geq 0$$

also $\Delta(a,b)F \geq 0$

$\Rightarrow$ F ist eine Verteilungsfunktion

ges: $\mu_F(]\binom{0}{0}, \binom{1}{1}]) = F(1,1) - F(0,1) - F(1,0) + F(0,0) = 1$

ges: $\mu_F(\{(x,x) : 0 < x \leq 1\})$

$$G = F|_{\{(x,x): x \in \mathbb{R}\}} \qquad G(x) = x \qquad \mu_F|_{\{(x,x): x \in \mathbb{R}\}} = \mu_G$$

$$\mu_G(]0,1]) = G(1) - G(0) = 1$$

$$\Rightarrow \mu_F(\{(x,x): 0 < x \leq 1\}) = 1$$

?

$$\mu_F(\{(x,x): 0 < x \leq 1\}) = \lim_{n\to\infty} \sum_{k=1}^{2^n} \mu_F\left(\left]\begin{pmatrix}\frac{k-1}{2^n} \\ \frac{k-1}{2^n}\end{pmatrix}, \begin{pmatrix}\frac{k}{2^n} \\ \frac{k}{2^n}\end{pmatrix}\right]\right) =$$

$$= \lim_{n\to\infty} \sum_{k=1}^{2^n} F\left(\frac{k}{2^n}, \frac{k}{2^n}\right) - F\left(\frac{k-1}{2^n}, \frac{k}{2^n}\right) - F\left(\frac{k}{2^n}, \frac{k-1}{2^n}\right) + F\left(\frac{k-1}{2^n}, \frac{k-1}{2^n}\right)$$

$$= \lim_{n\to\infty} \sum_{k=1}^{2^n} \frac{k}{2^n} - \frac{k-1}{2^n} - \cancel{\frac{k-1}{2^n}} + \cancel{\frac{k-1}{2^n}} = \lim_{n\to\infty} \sum_{k=1}^{2^n} \frac{1}{2^n} = \lim_{n\to\infty} \frac{2^n}{2^n} = 1$$