Series 12

Ida Hönigmann

June 23, 2021

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1 Problem 1

Theorem 1 *Let $n \in \mathbb{N}$ and $A, B \subset \mathbb{R}^n$ open subsets with compact closure $\bar{A}, \bar{B}$ and $A \cap B = \emptyset$. We define the boundary of the sets as $\partial A := \bar{A} \setminus A$ and $\partial B := \bar{B} \setminus B$. Then, there holds for the distances of the two sets that $dist(A, B) = dist(\partial A, \partial B)$, where we define for arbitrary sets $C, D \subset \mathbb{R}^n$*

$$dist(C, D) := \inf\{||x - y||_2 : x \in C, y \in D\} \tag{1}$$

Proof TODO ■

2 Problem 2

$$A = \begin{pmatrix} \beta_0 & -\gamma_1 & 0 & \cdots & 0 \\ -\gamma_1 & \beta_1 & -\gamma_2 & \ddots & \vdots \\ 0 & -\gamma_2 & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & -\gamma_n \\ 0 & \cdots & 0 & -\gamma_n & \beta_n \end{pmatrix} \in \mathbb{R}^{(n+1)\times(n+1)}_{\text{sym}}$$

3 Problem 3

The .toc file stores information regarding the table of contents. It is created at the end of the latex compile process, which means one has to compile multiple times before the toc shows up in the document correctly.

Why is the .toc file created at end document?

4 Problem 4

Theorem 2 *If $A \in \mathbb{R}^{n \times n}$ is a matrix with $\sum_{j,k=1}^{n} x_j A_{jk} x_k > 0$ for all $x \in \mathbb{R}^n \setminus \{0\}$ then A is regular.*

Proof Let $A \in \mathbb{R}^{n \times n}$ be an arbitrary matrix with $\sum_{j,k=1}^{n} x_j A_{jk} x_k > 0$ for all $x \in \mathbb{R}^n \setminus \{0\}$. Let $x \in \mathbb{R}^n \setminus \{0\}$ be an arbitrary vector.

$$\begin{aligned} 0 < \sum_{j,k=1}^{n} x_j A_{jk} x_k &= \sum_{j,k=1}^{n} x_j (e_j^T A e_k) x_k \\ &= \sum_{j,k=1}^{n} (x_j e_j^T) A (e_k x_k) \\ &= \left(\sum_{j=1}^{n} (x_j e_j^T) \right) A \left(\sum_{k=1}^{n} (x_k e_k) \right) \\ &= x^T A x \end{aligned}$$

Therefore A is positive definite, which implies that $\det A > 0$. So A must be regular. ■

5 Problem 5

- ♠ A
- ♠ B
- ♠ C

6 Problem 6

6.1 Subsection 1

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6.2 Subsection 2

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7 Problem 7

$$\begin{aligned}\int_{-1}^{1}\sqrt{1-x^2}dx &= \left[x\sqrt{1-x^2}\right]_{x=-1}^{1} - \int_{-1}^{1}\frac{x(-2x)}{2\sqrt{1-x^2}}dx \\ &= \left[x\sqrt{1-x^2}\right]_{x=-1}^{1} + \int_{-1}^{1}\frac{dx}{\sqrt{1-x^2}} - \int_{-1}^{1}\frac{1-x^2}{\sqrt{1-x^2}}dx \\ &= \left[x\sqrt{1-x^2}+\arcsin x\right]_{x=-1}^{1} - \int_{-1}^{1}\sqrt{1-x^2}dx\end{aligned}$$

8 Problem 8

Theorem 3 *Let I be a nonempty open interval. Then it holds for $f, g \in C^\infty(I)$ and $n \in \mathbb{N}$ that*

$$(fg)^{(n)} = \sum_{k=0}^{n} \binom{n}{k} f^{(k)} g^{(n-k)}$$

Proof TODO ■

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