ANA Ü8

8.) $I: \mathbb{R} \to \mathbb{R}$ ges: $I'(\alpha)$

$$\alpha \mapsto \int_{-\exp(\alpha)}^{\alpha^2} \cos(\alpha t^2)\,dt$$

$f: \mathbb{R} \to \mathbb{R}^3$ $g: \mathbb{R}^3 \to \mathbb{R}$

$$\alpha \mapsto \begin{pmatrix} -\exp(\alpha) \\ \alpha^2 \\ \alpha \end{pmatrix} \qquad \begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \int_x^y \cos(z t^2)\,dt$$

$\Rightarrow I(\alpha) = (g \circ f)(\alpha) \quad \forall \alpha \in \mathbb{R}$

$$df(\alpha) = \begin{pmatrix} -\exp(\alpha) \\ 2\alpha \\ 1 \end{pmatrix} \qquad dg\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{dg}{dx}\begin{pmatrix} x \\ y \\ z \end{pmatrix} \\ \frac{dg}{dy}\begin{pmatrix} x \\ y \\ z \end{pmatrix} \\ \frac{dg}{dz}\begin{pmatrix} x \\ y \\ z \end{pmatrix} \end{pmatrix} = \begin{pmatrix} -\cos(zx^2) \\ \cos(zy^2) \\ -\int_x^y t^2 \sin(zt^2)\,dt \end{pmatrix}$$

$$\frac{d}{dx} \int_x^y \cos(zt^2)\,dt = \frac{d}{dx} \int_x^y h(t)\,dt = \frac{d}{dx}\left(H(y) - H(x)\right) = \frac{d}{dx} H(x) = -h(x)$$

$$\frac{d}{dy} \int_x^y \cos(zt^2)\,dt = \frac{d}{dy} \int_x^y h(t)\,dt = \frac{d}{dy}\left(H(y) - H(x)\right) = h(y)$$

$$\frac{d}{dz} \int_x^y \cos(zt^2)\,dt = \int_x^y \frac{d}{dz} \cos(zt^2)\,dt = \int_x^y -\sin(zt^2) \cdot t^2 dt\;dt = -\int_x^y t^2 \sin(zt^2)\,dt$$

$$I'(\alpha) = (g \circ f)'(\alpha) = dg(f(\alpha))\; df(\alpha)$$

$$= \begin{pmatrix} -\cos(\alpha(-\exp(\alpha))^2) \\ \cos(\alpha(\alpha^2)^2) \\ -\int_{-\exp(\alpha)}^{\alpha^2} t^2 \sin(zt^2)\,dt \end{pmatrix}^T \cdot \begin{pmatrix} -\exp(\alpha) \\ 2\alpha \\ 1 \end{pmatrix} = \begin{pmatrix} -\cos(\alpha \cdot \exp(2\alpha)) \\ \cos(\alpha^5) \\ -\int_{-\exp(\alpha)}^{\alpha^2} t^2 \sin(zt^2)\,dt \end{pmatrix}^T \cdot \begin{pmatrix} -\exp(\alpha) \\ 2\alpha \\ 1 \end{pmatrix}$$

$$= \exp(\alpha)\cos(\alpha \cdot \exp(2\alpha)) + 2\alpha \cos(\alpha^5) - \int_{-\exp(\alpha)}^{\alpha^2} t^2 \sin(zt^2)\,dt$$