1) ... $(a_n) \xrightarrow{n\to\infty} a$ Wann gilt ...?

(i) $\liminf_{n\in\mathbb{N}} \,]-\infty, a_n] =]-\infty, a]$ $\subseteq$ wurde oben gezeigt

$\bigcup_{n\in\mathbb{N}} \bigcap_{k\geq n} \,]-\infty, a_k]$

② Sei $x \in \,]-\infty, a]$ bel.

1. Fall $x \neq a$ $\quad \varepsilon := \frac{a-x}{2}$ $\quad \exists N\in\mathbb{N}\ \forall n\geq N: |a_n - a| < \varepsilon$

$\bigcap_{n\geq N} \,]-\infty, a_n] =]-\infty, \inf_{n\geq N}(a_n)] \ni x \quad \Rightarrow x \in \bigcup_{n\in\mathbb{N}} \bigcap_{k\geq n} \,]-\infty, a_k]$

2. Fall $x = a$

Falls a_n monoton fallend: $\forall k\in\mathbb{N}: a \in \,]-\infty, a_k] \Rightarrow x=a \in \bigcup_{n\in\mathbb{N}} \bigcap_{k\geq n} \,]-\infty, a_k]$ ✓

Falls a konstant : $\forall k\in\mathbb{N}: a\in\,]-\infty, a_k]$, da $a_k = a$ $\Rightarrow$ ——"—— ✓

Falls a_n monoton wachsend: $\forall k\in\mathbb{N}: a \notin \,]-\infty, a_k]$, da $a_k < a$

$\Rightarrow x = a \notin \bigcup_{n\in\mathbb{N}} \bigcap_{k\geq n} \,]-\infty, a_k]$ ✗

(ii) $\limsup_{n\in\mathbb{N}} \,]-\infty, a_n] =]-\infty, a]$ $\subseteq$ oben gezeigt

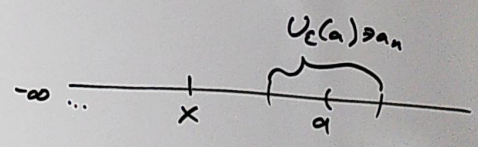
$\bigcap_{n\in\mathbb{N}} \bigcup_{k\geq n} \,]-\infty, a_k]$

② Sei $x \in \,]-\infty, a]$ bel.

1. Fall $x \neq a$ $\quad \varepsilon := \frac{a-x}{2}$ $\quad \exists N\in\mathbb{N}\ \forall n\geq N: |a_n - a| < \varepsilon$

$\bigcup_{n\geq N} \,]-\infty, a_n] =]-\infty, \sup_{n\geq N}(a_n)] \ni x$

$x \in \bigcap_{n\in\mathbb{N}} \bigcup_{k\geq n} \,]-\infty, a_k]$



2. Fall $x = a$

Falls a_n monoton fallend: $\forall k\in\mathbb{N}: a\in\,]-\infty, a_k] \Rightarrow x=a \in \bigcap_{n\in\mathbb{N}} \bigcup_{k\geq n} \,]-\infty, a_k]$

Falls a konstant : ——"——

Falls a_n monoton wachsend: $\forall k\in\mathbb{N}: a \notin \,]-\infty, a_k] \Rightarrow x=a \notin \bigcap_{n\in\mathbb{N}} \bigcup_{k\geq n} \,]-\infty, a_k]$