ANA Ü6

4.) $\lim_{a \to 0} \int_0^1 \frac{1}{1+a(\sin(x))^2} dx$

$f_n(x) := \frac{1}{1+\frac{1}{n}(\sin(x))^2}$ $f(x) := 1$

Szz: f_n konvergiert gleichmäßig gegen f

$d_\infty(f_n, f) = \sup\{d(f_n(x), f(x)) : x \in [0,1]\}$

$|f_n(x) - f(x)| = \left| \frac{1}{1+\frac{1}{n}(\sin(x))^2} - 1 \right| = \left| \frac{1-(1+\frac{1}{n}(\sin(x))^2)}{1+\frac{1}{n}(\sin(x))^2} \right|$

$= \left| \frac{-\frac{1}{n}(\sin(x))^2}{1+\frac{1}{n}(\sin(x))^2} \right| = \left| \frac{\frac{1}{n}(\sin(x))^2}{1+\frac{1}{n}(\sin(x))^2} \right| \xrightarrow{n\to\infty} 0$

$\Rightarrow \lim_{n\to\infty} \int_0^1 f_n(x)\,dx = \int_0^1 f(x)\,dx$ laut Satz 8.7.2.

$\int_0^1 f(x)\,dx = \int_0^1 1\,dx = 1$