ANA Ü3

4.) ·) $\lim_{x\to\frac{\pi}{2}-} \frac{1+\frac{1}{\cos(x)}}{1+\tan(x)} = \lim_{x\to\frac{\pi}{2}-} \frac{-(\cos(x))^{-2}\cdot(-\sin(x))}{\frac{1}{(\cos(x))^2}}$ $\left(, \text{da } \lim_{x\to\frac{\pi}{2}-} \tan(x) = +\infty\right)$

$= \lim_{x\to\frac{\pi}{2}-} \frac{\sin(x)\cdot\frac{1}{(\cos(x))^2}}{\frac{1}{(\cos(x))^2}} = \lim_{x\to\frac{\pi}{2}-} \sin(x) = 1$

·) $\lim_{x\to 0+} (1+x)^{\frac{1}{x}} = \lim_{x\to 0+} \exp\left(\frac{1}{x}\cdot\ln(1+x)\right)$

$\lim_{x\to 0+} \frac{\ln(1+x)}{x} = \lim_{x\to 0+} \frac{\frac{1}{1+x}\cdot 1}{1}$ $\left(, \text{da } \lim_{x\to 0+} \ln(1+x) = \ln(1) = 0 \text{ und } \lim_{x\to 0+} x = 0\right)$

$= \lim_{x\to 0+} \frac{1}{1+x} = 1$

$\Rightarrow \lim_{x\to 0+} \exp\left(\frac{\ln(1+x)}{x}\right) = \exp(1) = e$

·) $\lim_{x\to+\infty} \left(\frac{x+2}{x-1}\right)^x = \lim_{x\to+\infty} \exp\left(x\cdot\ln\left(\frac{x+2}{x-1}\right)\right)$

$\lim_{x\to+\infty} \frac{\ln\left(\frac{x+2}{x-1}\right)}{\frac{1}{x}} = \lim_{x\to+\infty} \frac{\frac{x-1}{x+2}\cdot\left(\frac{x+2}{x-1}\right)'}{-\frac{1}{x^2}}$ $\left(, \text{da } \lim_{x\to+\infty} \frac{1}{x} = 0 \text{ und } \lim_{x\to+\infty} \ln\left(\frac{x+2}{x-1}\right) = \lim_{x\to+\infty} \ln\left(\frac{1+\frac{2}{x}}{1-\frac{1}{x}}\right) = \lim_{x\to+\infty} \ln(1) = 0\right)$

$= \lim_{x\to+\infty} \frac{(x-1)}{(x+2)}\cdot\frac{1\cdot(x-1)+1\cdot(x+2)}{(x-1)^2}\cdot\frac{1}{-\frac{1}{x^2}}$

$= \lim_{x\to+\infty} -\frac{x^2\cdot(x-1+x+2)}{(x-1)(x+2)} = \lim_{x\to+\infty} -\frac{-3x^2}{x^2+2x-x-2} = \lim_{x\to+\infty} \frac{3x^2}{x^2+x-2}$

$= \lim_{x\to+\infty} \frac{3}{1+\frac{1}{x}-\frac{2}{x^2}} = 3$

$\Rightarrow \lim_{x\to+\infty} \exp\left(x\cdot\ln\left(\frac{x+2}{x-1}\right)\right) = \exp(3) = e^3$