ANA Ü12

2.) $\Phi : \mathbb{R}^3 \to L(\mathbb{R}^3, \mathbb{R}) \cong \mathbb{R}^{1\times 3}$ Ist Φ ein Gradientenfeld? Falls ja suche eine Stammfunktion.

i) $\Phi\begin{pmatrix}\xi\\ \eta\\ \zeta\end{pmatrix} = (1\ 1\ 1)$ $\frac{\partial \Phi}{\partial \xi} = \frac{\partial \Phi}{\partial \eta} = \frac{\partial \Phi}{\partial \zeta} = (0\ 0\ 0)$ $\Rightarrow$ Gradientenfeld

$\frac{\partial f}{\partial \xi} = 1 \Rightarrow f\begin{pmatrix}\xi\\ \eta\\ \zeta\end{pmatrix} = \int 1\, d\xi = \xi + c(\eta, \zeta)$

$\frac{\partial f}{\partial \eta} = c'(\eta, \zeta) = 1 \Rightarrow c(\eta, \zeta) = \int 1\, d\eta = \eta + c(\zeta)$ $\Rightarrow f\begin{pmatrix}\xi\\ \eta\\ \zeta\end{pmatrix} = \xi + \eta + c(\zeta)$

$\frac{\partial f}{\partial \zeta} = c'(\zeta) = 1 \Rightarrow c(\zeta) = \int 1\, d\zeta = \zeta + c$ $\Rightarrow f\begin{pmatrix}\xi\\ \eta\\ \zeta\end{pmatrix} = \xi + \eta + \zeta + c$

ii) $\Phi\begin{pmatrix}\xi\\ \eta\\ \zeta\end{pmatrix} = (-\xi, -\eta, -\zeta)$

$\frac{\partial \Phi}{\partial \xi} = (-1, 0, 0)$ $\frac{\partial \Phi}{\partial \eta} = (0, -1, 0)$ $\frac{\partial \Phi}{\partial \zeta} = (0, 0, -1)$ $\Rightarrow$ Gradientenfeld

$\frac{\partial f}{\partial \xi} = -\xi \Rightarrow f\begin{pmatrix}\xi\\ \eta\\ \zeta\end{pmatrix} = \int -\xi\, d\xi = -\frac{1}{2}\xi^2 + c(\eta, \zeta)$

$\frac{\partial f}{\partial \eta} = c'(\eta, \zeta) = -\eta \Rightarrow c(\eta, \zeta) = \int -\eta\, d\eta = -\frac{1}{2}\eta^2 + c(\zeta)$ $\Rightarrow f\begin{pmatrix}\xi\\ \eta\\ \zeta\end{pmatrix} = -\frac{1}{2}\xi^2 - \frac{1}{2}\eta^2 + c(\zeta)$

$\frac{\partial f}{\partial \zeta} = c'(\zeta) = -\zeta \Rightarrow c(\zeta) = \int -\zeta\, d\zeta = -\frac{1}{2}\zeta^2$ $\Rightarrow f\begin{pmatrix}\xi\\ \eta\\ \zeta\end{pmatrix} = -\frac{1}{2}(\xi^2 + \eta^2 + \zeta^2)$

iii) $\Phi\begin{pmatrix}\xi\\ \eta\\ \zeta\end{pmatrix} = (\eta^3, 2, \xi^2)$

$\frac{\partial \Phi}{\partial \xi} = (0, 0, 2\xi)$ $\frac{\partial \Phi}{\partial \eta} = (3\eta^2, 0, 0)$ $\frac{\partial \Phi}{\partial \zeta} = (0, 0, 0)$

$\frac{\partial \Phi}{\partial \xi} e_3 = 2\xi$ $\frac{\partial \Phi}{\partial \zeta} e_1 = 0$ $\Rightarrow$ kein Gradientenfeld