4.) $f: \mathbb{R} \to \mathbb{R}$ $\qquad f(x) = \begin{cases} x^2 \cdot \sin\left(\frac{1}{x} + 1\right), & x > 0 \\ ax + b, & x \le 0 \end{cases}$

$$\lim_{x \to 0+} f(x) = \lim_{x \to 0+} x^2 \cdot \sin\left(\frac{1}{x} + 1\right) = 0$$

$$\lim_{x \to 0-} f(x) = \lim_{x \to 0-} ax + b = b \qquad \Rightarrow b = 0$$

$$\lim_{x \to 0+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0+} \frac{x^2 \cdot \sin\left(\frac{1}{x} + 1\right) - (ax + b)}{x} = \lim_{x \to 0+} x \cdot \sin\left(\frac{1}{x} + 1\right) - a = -a$$

$$\lim_{x \to 0-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0-} \frac{ax + b - b}{x} = \lim_{x \to 0-} \frac{ax}{x} = a$$

$$\Rightarrow a = -a \quad \Rightarrow a = 0$$

ges: f'

$$\left(x^2 \cdot \sin\left(\frac{1}{x} + 1\right)\right)' = 2x \cdot \sin\left(\frac{1}{x} + 1\right) + x^2 \cdot \left(\sin\left(\frac{1}{x} + 1\right)\right)'$$

$$= 2x \cdot \sin\left(\frac{1}{x} + 1\right) + x^2 \cdot \cos\left(\frac{1}{x} + 1\right) \cdot (-1) \cdot x^{-2}$$

$$= 2x \cdot \sin\left(\frac{1}{x} + 1\right) - x^2 \cdot \cos\left(\frac{1}{x} + 1\right) \cdot \frac{1}{x^2} = 2x \cdot \sin\left(\frac{1}{x} + 1\right) - \cos\left(\frac{1}{x} + 1\right)$$

$$f'(x) = \begin{cases} 2x \cdot \sin\left(\frac{1}{x} + 1\right) - \cos\left(\frac{1}{x} + 1\right), & x > 0 \\ 0, & x \le 0 \end{cases}$$

Da f' stetig und differenzierbar ist, ist f stetig differenzierbar und zwei Mal differenzierbar.