2.) w∈ C y: La, 6] → C gerdlessen, 155 d w €y (La, 6]) n (7, w):= 27: 5 5-w of 5 ... Umlant zahl van je im w 82: n(y,w)eZ Jands = Sy's) gister ds, da stw fir Stw Rolomorph $g(t) := \int_{a}^{b} \frac{1}{s} \frac{1}{s} ds = g'(s) = \frac{1}{s} \frac{1}{s} \frac{1}{s} \frac{1}{s}$ off exp (-g(+))(p(+)-w) = exp(-g(+)). (-g(+))(p(+)-w) + exp(-g(+)) p'(+) =exp(-g(+)) = (g(+)+w) +exp(-g(+)) g'(+) = exp (-g (+)) gr ((+) - exp (-g (+)) gr ((+) = 0 => 3 c ++ e c : exp (= (+))(y(+)+w) = C (=> y(+))-w) = C · exp(g(+)) $C \cdot \exp(g(a)) = y(a) - w = y(b) - w = C \cdot \exp(g(b))$ Da y(a)-w + O (da w &y([a,6])) => C + 0 $C \cdot exp(g(a)) = C \cdot exp(g(b)) \Leftrightarrow exp(g(a)) = exp(g(b))$ (=) $g(a) - g(b) \in 2\pi i \mathbb{Z}$ $g(a) = S \frac{n'(s)}{n'(s) - n'(s)} = 0$ => g(6) = \$ \frac{1}{y(s)} = \ds = \frac{1}{5} = \ds \in 2\frac{1}{3} = \frac{1}{5} = => n(y, w) = 27 i 5 5-w ds EZ

ANA UN3 3) ZZ: exp: C -> C, cs: C -> C, sin: C -> C sind holomorph exp(x+iy) = exp(x)(as(y) + i sinly) = exp(x)as(y)+i exp(x)sin(y) d Relexp) = d exp(x).cos(y) = exp(x).cos(y) dinlexp = d exp(x). sin(y) = exp(x) cos(y) d Relexp) = d exp(x) cs(y) = exp(x) sin(y) $\frac{\partial \operatorname{Im}(\exp)}{\partial x} = \frac{\partial}{\partial x} \exp(x) \sin(y) = -\exp(x) \sin(y)$ (exp(x+iy)) = exp(x+iy) ... steling => holomorph Stammfunktion:exp cos (x+iy)=cos (x)cos (iy)-sin(x): cin(iy) = cos (x) cosh(y)-icin(x) cinh(y) dretas) = of cos(x). coshly) = -sin(x). coshly) = of -sin(x). cinhly) = dlm(cos dre(ss) = d cos(x) cosh(y) = cos(x) sinh(y) = -dx - sin(x) sinh(y) = -dx (cos(xtiy))'= -sin(xtiy)...stelig = sholomorph Stammfunktion: sin sin(x+iy) = sin(x) cos(iy) + cos(x) sin(iy) = sin(x) cosh(y)+i cos(x) sin(y) The (sin) = of sin(x) ash(y) = cos(x) sxl(y) = of cos(x) sinh(y) = of m(sin) ore (sin) = of sin(x) cosh(y)=-sin(x) sinh(y) = - ox cos(x) sinh(y) = -ox (an(xeig)) - as (x+ig) ... steling = sholomorph Sternyfunktion: -ss ges: Stammfuntion von 22 (cos (2))2 [2 (cos(2))2 dz = 52 2 (cos(27)+1) dz = = (52 cos(22) dz + 52 dz $=\frac{1}{2}\int \left(\frac{\sin(2z)}{2}\right)^{\frac{1}{2}} z^{2} dz + \frac{1}{2}\frac{z^{3}}{3} = \frac{1}{2}\left(\frac{\sin(2z)}{2}\right) z^{2} - \int \frac{\sin(2z)}{2} \cdot 2z dz + \frac{2^{3}}{6}$ = 23 + 22 sin(22) - 25 sin(22) · 2 d2 - 23 + 22 sin(22) 15(-cos (22)) · 2 d2 $=\frac{\xi^3}{6} + \frac{\xi^2 \sin(2\xi)}{4} + \frac{\cos(2\xi)}{4} + \frac{1}{\xi} + \frac{1}{\xi} + \frac{\cos(2\xi)}{2} d\xi = \frac{\xi^3}{6} + \frac{\xi^2 \sin(2\xi)}{4} + \frac{\xi \cos(2\xi)}{4} - \frac{1}{\xi} \int \cos(2\xi) d\xi$ $= \frac{2^{3}}{6} + \frac{2^{2} \sin(2z)}{4} + \frac{2 \cos(2z)}{4} - \frac{4}{9} \int \cos(\omega) \frac{1}{2} d\omega \qquad \qquad \int \omega = 2 \frac{1}{2} d\omega = 2 d\omega = \frac{1}{2} d\omega$ $= \frac{2^{3}}{6} + \frac{2^{2} \sin(2z)}{4} + \frac{2 \cos(2z)}{4} - \frac{1}{9} \sin(\omega) = \frac{2^{3}}{6} + \frac{2^{2} \sin(2z)}{4} - \frac{1}{2} \sin(2z) = \frac{1}{2} d\omega$ (62²-3) cin(2z) +62 cos(22) +423

ANA UM3 4.) $y_1: [0, 2\pi] \rightarrow C$ $y_2: [0, 2\pi] \rightarrow C$ $y_3: [0, 2\pi] \rightarrow C$ $+ \mapsto -1 + \frac{1}{2} \exp(i+)$ $+ \mapsto 1 + \frac{1}{2} \exp(i+)$ $+ \mapsto 4 \exp(i+)$ ++>4exp(it) f(z) = 2+1 + 2 1 ges = Sf(5) of + Sf(6) of 5 Salz 11.6.12 (Canchysche Sutegralformel) J: D -> Y D = C... offen w & C p>0 mit Kp(w) ED ZEUplw) y:[0,27] -> D + -> w+p·exp(i+) => f(2) = 2 1 1 5 - 2 25 = S = 1 dS + S = 1 dS + S = 1 dS + S = 1 dS $[\alpha = -1] p = \frac{1}{2} = -1$ $y(t) = -1 + \frac{1}{2} \cdot exp(it) = y_1(A)$ f = 1=> 2n i f(z) = 5 \$ (5) d5 (=> 2n i = 5 = (1) d5 gleich mit w=1 z=1 y=yz -> 2 = i= 5 5-1 d5 $= 2\pi i + (\frac{1}{3} - i) + (\frac{1}{3} + i) + (\frac{$ = 47 i + 5 5-1 d5 + 5 5+1 d5 w = -1 $P = \frac{1}{2}$ z = -1 $y = -1 + \frac{1}{2} \exp(4) = y - (4)$ $f(5) = \frac{3+1}{5-1}$ gleich mit w= 1 2= 1 1= y2 1 (5)= 5-1 => 2n 1+1 = 5 3+1 de = 471 +0 +0 = 4 11

4)... ges: 5 /(5) 05 5 3+1 + 5-1 d5 = 5 3+1 d5 + 5 3-1 d5 $\begin{cases}
f(S) = 1 & w = 0 & p = 4 & z = -1 & y(t) = 4 \exp(it) = g(s)(t) \\
= 2\pi i f(-1) = 5 & f(-1) & dS & (z) \\
= 2\pi i f(-1) = 5 & f(-1) & dS & (z) &$ I gleich mit 2=1 1 => 24 i f(1) = 53-1 d5 (=> 2n i = 5 3-1 d5 = 201 + 201 - 401 ges: Skitze

ANA 13/13 7:) DEC...offen J: D -> C...holomorph D:= { = : 2 CD3 } J* D > C 2 H> J (2) 22: 1 ist holomorph D. offen => VxED JEDO VE(x) & D => VyED JEDO VE(y) & D => D...offen f. holomorph => dx = dm(f) A drelf) = - dm(f) $\Rightarrow \frac{\partial \text{Re}(f)}{\partial x} = \frac{\partial -\text{Im}(f^*)}{\partial -\text{y}} \wedge \frac{\partial \text{Re}(f^*)}{\partial -\text{y}} = -\frac{\partial -\text{Im}(f^*)}{\partial x}$ $= \frac{\partial Re(J^{*})}{\partial x} = \frac{\partial Im(J^{*})}{\partial y} = \frac{\partial Im(J^{*})}{\partial x}$ =) for ist holoncorph