

ANA Ü8.

10. $h: (-\frac{\pi}{2}, \frac{\pi}{2}) \rightarrow \mathbb{R} \quad h(t) = (\cos(t))^{\sin(t)}$

ges: h'

$$(\cos(t))^{\sin(t)} = \exp(\sin(t) \cdot \ln(\cos(t)))$$

$$\begin{aligned} (\exp(\sin(t) \cdot \ln(\cos(t))))' &= \exp(\sin(t) \cdot \ln(\cos(t))) \cdot (\cos(t) \cdot \ln(\cos(t)) + \sin(t) \cdot \frac{1}{\cos(t)} \cdot (-\sin(t))) \\ &= (\cos(t))^{\sin(t)} \cdot (\cos(t) \cdot \ln(\cos(t)) - \tan(t) \sin(t)) \end{aligned}$$

$$f: (-\frac{\pi}{2}, \frac{\pi}{2}) \rightarrow (0,1) \times (-1,1) \quad f(t) = \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix}$$

$$g: (0,1) \times (-1,1) \rightarrow \mathbb{R}$$

$$g\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = x^y$$

$$\Rightarrow h = g \circ f$$

ges: $(g \circ f)'$

$$d(g \circ f)(t) = dg(f(t)) \cdot df(t)$$

$$df(t) = \begin{pmatrix} -\sin(t) \\ \cos(t) \end{pmatrix} = \begin{pmatrix} -\sin(x) \\ \cos(x) \end{pmatrix} \quad \text{mit } x = t$$

$$df(t) = \begin{pmatrix} -\sin(t) \\ \cos(t) \end{pmatrix}$$

$$dg\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} \frac{dg}{dx}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) \\ \frac{dg}{dy}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) \end{pmatrix} = \begin{pmatrix} y \cdot x^{y-1} \\ x^y \cdot \ln(x) \end{pmatrix}$$

$$d(g \circ f)(t) = dg(f(t)) \cdot df(t) = \begin{pmatrix} \sin(t) \cdot \cos(t)^{\sin(t)-1} \\ \cos(t)^{\sin(t)} \ln(\cos(t)) \end{pmatrix}^T \cdot \begin{pmatrix} -\sin(t) \\ \cos(t) \end{pmatrix}$$

$$= -(\sin(t))^2 \cos(t)^{\sin(t)-1} + \cos(t)^{\sin(t)+1} \ln(\cos(t))$$

$$= (\cos(t))^{\sin(t)} \cdot (\cos(t) \cdot \ln(\cos(t)) - \sin(t) \cdot \tan(t))$$