ANA Ü2

3.) $f: \mathbb{R} \to \mathbb{R}$ $\quad f(t) = \begin{cases} at &, t < 2 \\ b + t^{\frac{3}{2}} &, t \geq 2 \end{cases}$

$$\lim_{t \to 2-} f(t) = \lim_{t \to 2-} at = 2a$$

$$\lim_{t \to 2+} f(t) = \lim_{t \to 2+} b + \sqrt[2]{t^3} = b + 2\sqrt{2}$$

$$\Rightarrow 2a = b + 2\sqrt{2}$$

$$\lim_{t \to 2-} \frac{f(t) - f(2)}{t - 2} = \lim_{t \to 2-} \frac{at - 2a}{t - 2} = \lim_{t \to 2-} a \cdot \frac{t-2}{t-2} = a$$

$$\lim_{t \to 2+} \frac{f(t) - f(2)}{t - 2} = \lim_{t \to 2+} \frac{b + t^{\frac{3}{2}} - (b + 2\sqrt{2})}{t - 2} = \lim_{t \to 2+} \frac{\sqrt{t^3} - 2\sqrt{2}}{t - 2}$$

$$= \lim_{t \to 2+} \frac{(t^{\frac{3}{2}})' - (2\sqrt{2})'}{t' - 2'} = \lim_{t \to 2+} \frac{\frac{3}{2} \cdot t^{\frac{1}{2}} - 0}{1 \cdot t^0 - 0} = \lim_{t \to 2+} \frac{3}{2} \cdot \sqrt[2]{t} = \frac{3\sqrt{2}}{2}$$

$$\Rightarrow a = \frac{3\sqrt{2}}{2}$$

$$2a = b + 2\sqrt{2} \iff 2\,\frac{3\sqrt{2}}{2} = b + 2\sqrt{2} \iff 3\sqrt{2} = b + 2\sqrt{2}$$

$$\iff b = 3\sqrt{2} - 2\sqrt{2} \iff b = \sqrt{2}$$

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ges: f'

$$\left(\frac{3\sqrt{2}}{2} t\right)' = \frac{3\sqrt{2}}{2}$$

$$\left(\sqrt{2} + t^{\frac{3}{2}}\right)' = \frac{3}{2} \cdot t^{\frac{3}{2} - 1} = \frac{3\sqrt{t}}{2}$$

$$\Rightarrow f'(t) = \begin{cases} \frac{3\sqrt{2}}{2} &, t < 2 \\ \frac{3\sqrt{t}}{2} &, t \geq 2 \end{cases}$$

Da f' stetig und differenzierbar ist f stetig differenzierbar und zwei mal differenziabar auf $\mathbb{R}$.