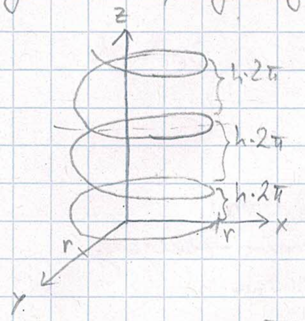
ANA Ü10

9.) $\gamma : [0, 4\pi] \to \mathbb{R}^3 \qquad \gamma(t) = \begin{pmatrix} r\cdot\cos(t) \\ r\cdot\sin(t) \\ h\cdot t \end{pmatrix} \qquad r, h > 0$ fest

ges: Skizze, Bogenlänge und $t(s)$, sodass die Bogenlänge von $\gamma(0)$ bis $\gamma(t(s))$ gleich s ist.



$r\cdot\cos(t)$, $r\cdot\sin(t)$ und $h\cdot t \in C^1[0, 4\pi]$

$$\Rightarrow l(\gamma) = \int_0^{4\pi} \|\gamma'(x)\|_2 \, dx$$

$$\gamma'(t) = \begin{pmatrix} -r\cdot\sin(t) \\ r\cdot\cos(t) \\ h \end{pmatrix}$$

$$\|\gamma'(t)\|_2 = \sqrt{(-r\cdot\sin(t))^2 + (r\cdot\cos(t))^2 + h^2} = \sqrt{r^2\cdot\sin^2(t) + r^2\cdot\cos^2(t) + h^2}$$

$$= \sqrt{r^2\cdot(\sin^2(t) + \cos^2(t)) + h^2} = \sqrt{r^2+h^2}$$

$$\int_0^{4\pi} \|\gamma'(t)\|_2 dt = \int_0^{4\pi} \sqrt{r^2+h^2}\, dt = \sqrt{r^2+h^2}\cdot 4\pi - \sqrt{r^2+h^2}\cdot 0 = 4\pi\cdot\sqrt{r^2+h^2} = l(\gamma)$$

$$l(\gamma|_{[0,s]}) = \int_0^s \sqrt{r^2+h^2}\, dt = s\cdot\sqrt{r^2+h^2}$$

$$y = s\cdot\sqrt{r^2+h^2} \qquad s = \frac{y}{\sqrt{r^2+h^2}}$$

$$t(s) := \frac{s}{\sqrt{r^2+h^2}} \qquad \gamma(t(s)) = \begin{pmatrix} r\cdot\cos\left(\frac{s}{\sqrt{r^2+h^2}}\right) \\ r\cdot\sin\left(\frac{s}{\sqrt{r^2+h^2}}\right) \\ h\cdot\frac{s}{\sqrt{r^2+h^2}} \end{pmatrix}$$

$$(\gamma\circ t)'(s) = \begin{pmatrix} -r\cdot\sin\left(\frac{s}{\sqrt{r^2+h^2}}\right)\cdot\frac{1}{\sqrt{r^2+h^2}} \\ r\cdot\cos\left(\frac{s}{\sqrt{r^2+h^2}}\right)\cdot\frac{1}{\sqrt{r^2+h^2}} \\ h\cdot\frac{1}{\sqrt{r^2+h^2}} \end{pmatrix}$$

$$\|\gamma'(s)\|_2 = \sqrt{\left(-r\cdot\sin\left(\frac{s}{\sqrt{r^2+h^2}}\right)\cdot\frac{1}{\sqrt{r^2+h^2}}\right)^2 + \left(r\cdot\cos\left(\frac{s}{\sqrt{r^2+h^2}}\right)\cdot\frac{1}{\sqrt{r^2+h^2}}\right)^2 + \left(h\cdot\frac{1}{\sqrt{r^2+h^2}}\right)^2}$$

$$= \sqrt{r^2\cdot\sin^2\left(\frac{s}{\sqrt{r^2+h^2}}\right)\cdot\frac{1}{r^2+h^2} + r^2\cdot\cos^2\left(\frac{s}{\sqrt{r^2+h^2}}\right)\cdot\frac{1}{r^2+h^2} + h^2\cdot\frac{1}{r^2+h^2}}$$

$$= \sqrt{r^2\cdot\frac{1}{r^2+h^2}\left(\sin^2\left(\frac{s}{\sqrt{r^2+h^2}}\right) + \cos^2\left(\frac{s}{\sqrt{r^2+h^2}}\right)\right) + h^2\cdot\frac{1}{r^2+h^2}} = \sqrt{\frac{1}{r^2+h^2}(r^2+h^2)} = 1$$

$$\Rightarrow \int_0^s \|(\gamma\circ t)'(x)\|_2\, dx = \int_0^s 1\, dx = s$$