ANA Ü13

3.) zz: $\exp: \mathbb{C} \to \mathbb{C}$, $\cos: \mathbb{C} \to \mathbb{C}$, $\sin: \mathbb{C} \to \mathbb{C}$ sind holomorph

$$\exp(x+iy) = \exp(x)(\cos(y) + i\sin(y)) = \exp(x)\cos(y) + i\exp(x)\sin(y)$$

$$\frac{\partial \operatorname{Re}(\exp)}{\partial x} = \frac{\partial}{\partial x}\exp(x)\cdot\cos(y) = \exp(x)\cdot\cos(y)$$

$$\frac{\partial \operatorname{Im}(\exp)}{\partial y} = \frac{\partial}{\partial y}\exp(x)\cdot\sin(y) = \exp(x)\cdot\cos(y)$$

$$\frac{\partial \operatorname{Re}(\exp)}{\partial y} = \frac{\partial}{\partial y}\exp(x)\cos(y) = -\exp(x)\cdot\sin(y)$$

$$-\frac{\partial \operatorname{Im}(\exp)}{\partial x} = -\frac{\partial}{\partial x}\exp(x)\sin(y) = -\exp(x)\sin(y)$$

$(\exp(x+iy))' = \exp(x+iy)$ ... stetig $\Rightarrow$ holomorph Stammfunktion: exp

$$\cos(x+iy) = \cos(x)\cos(iy) - \sin(x)\sin(iy) = \cos(x)\cdot\cosh(y) - i\sin(x)\sinh(y)$$

$$\frac{\partial \operatorname{Re}(\cos)}{\partial x} = \frac{\partial}{\partial x}\cos(x)\cdot\cosh(y) = -\sin(x)\cdot\cosh(y) = \frac{\partial}{\partial y} -\sin(x)\cdot\sinh(y) = \frac{\partial \operatorname{Im}(\cos)}{\partial y}$$

$$\frac{\partial \operatorname{Re}(\cos)}{\partial y} = \frac{\partial}{\partial y}\cos(x)\cdot\cosh(y) = \cos(x)\cdot\sinh(y) = -\frac{\partial}{\partial x} -\sin(x)\sinh(y) = -\frac{\partial \operatorname{Im}(\cos)}{\partial x}$$

$(\cos(x+iy))' = -\sin(x+iy)$ ... stetig $\Rightarrow$ holomorph Stammfunktion: sin

$$\sin(x+iy) = \sin(x)\cos(iy) + \cos(x)\sin(iy) = \sin(x)\cosh(y) + i\cos(x)\sinh(y)$$

$$\frac{\partial \operatorname{Re}(\sin)}{\partial x} = \frac{\partial}{\partial x}\sin(x)\cosh(y) = \cos(x)\cosh(y) = \frac{\partial}{\partial y}\cos(x)\sinh(y) = \frac{\partial \operatorname{Im}(\sin)}{\partial y}$$

$$\frac{\partial \operatorname{Re}(\sin)}{\partial y} = \frac{\partial}{\partial y}\sin(x)\cosh(y) = -\sin(x)\sinh(y) = -\frac{\partial}{\partial x}\cos(x)\sinh(y) = -\frac{\partial \operatorname{Im}(\sin)}{\partial x}$$

$(\sin(x+iy))' = \cos(x+iy)$ ... stetig $\Rightarrow$ holomorph Stammfunktion: $-\cos$

ges: Stammfunktion von $z^2(\cos(z))^2$

$$\int z^2(\cos(z))^2\,dz = \int z^2\,\tfrac{1}{2}(\cos(2z)+1)\,dz = \tfrac{1}{2}\left(\int z^2\cos(2z)\,dz + \int z^2\,dz\right)$$

$$= \frac{1}{2}\int\left(\frac{\sin(2z)}{2}\right)'\cdot z^2\,dz + \frac{1}{2}\frac{z^3}{3} = \frac{1}{2}\left(\frac{\sin(2z)}{2}z^2 - \int\frac{\sin(2z)}{2}\cdot 2z\,dz\right) + \frac{z^3}{6}$$

$$= \frac{z^3}{6} + \frac{z^2\sin(2z)}{4} - \frac{1}{2}\int\sin(2z)\cdot z\,dz = \frac{z^3}{6} + \frac{z^2\sin(2z)}{4} - \frac{1}{2}\int\left(-\frac{\cos(2z)}{2}\right)'\cdot z\,dz$$

$$= \frac{z^3}{6} + \frac{z^2\sin(2z)}{4} + \frac{\cos(2z)}{4}z + \frac{1}{2}\int -\frac{\cos(2z)}{2}\,dz = \frac{z^3}{6} + \frac{z^2\sin(2z)}{4} + \frac{z\cos(2z)}{4} - \frac{1}{4}\int\cos(2z)\,dz$$

$$= \frac{z^3}{6} + \frac{z^2\sin(2z)}{4} + \frac{z\cos(2z)}{4} - \frac{1}{4}\int\cos(u)\,\tfrac{1}{2}\,du \qquad \left[u = 2z \quad \frac{du}{dz} = 2 \quad dz = \tfrac{1}{2}du\right.$$

$$= \frac{z^3}{6} + \frac{z^2\sin(2z)}{4} + \frac{z\cos(2z)}{4} - \frac{1}{8}\sin(u) = \frac{z^3}{6} + \frac{z^2\sin(2z)}{4} + \frac{z\cos(2z)}{4} - \frac{\sin(2z)}{8}$$

$$= \frac{(6z^2-3)\sin(2z) + 6z\cos(2z) + 4z^3}{24}$$