3a) $\Omega = \mathbb{N}$ $\quad \mathcal{A} = \{A \subseteq \Omega : |A| < \infty \vee |A^c| < \infty\}$

$\mu_n(A) = \frac{1}{n} \cdot |A \cap \{1, \dots, n\}|$ $\qquad$ z.z.: $\forall n \in \mathbb{N}$: μ_n ... Maß

Sei $n \in \mathbb{N}$ bel.

•) $\mu_n(\emptyset) = \frac{1}{n} \cdot |\emptyset \cap \{1, \dots, n\}| = \frac{1}{n} \cdot 0 = 0$

•) Sei $A \in \mathcal{A}$ bel.

$\mu_n(A) = \underbrace{\frac{1}{n}}_{>0} \cdot \underbrace{|A \cap \{1, \dots, n\}|}_{\geq 0} \geq 0$

•) Sei $(A_n)_{n \in \mathbb{N}}$ aus $\mathcal{A}$ paarweise disjunkt bel.

$\mu_n\left(\dot\bigcup_{\tilde n \in \mathbb{N}} A_{\tilde n}\right) = \frac{1}{n} \cdot \left|\dot\bigcup_{\tilde n \in \mathbb{N}} (A_{\tilde n}) \cap \{1, \dots, n\}\right| = \frac{1}{n} \cdot \left|\dot\bigcup_{\tilde n \in \mathbb{N}} (A_{\tilde n} \cap \{1, \dots, n\})\right| = \frac{1}{n} \cdot \sum_{\tilde n \in \mathbb{N}} |A_{\tilde n} \cap \{1, \dots, n\}|$

$= \sum_{\tilde n \in \mathbb{N}} \frac{1}{n} \cdot |A_{\tilde n} \cap \{1, \dots, n\}| = \sum_{\tilde n \in \mathbb{N}} \mu_n(A_{\tilde n})$

$\Rightarrow \mu_n$ ist ein Maß

$\mu := \lim_{n \in \mathbb{N}} \mu_n$ $\qquad \mu(\mathbb{N}) = \lim_{n \to \infty} \mu_n(\mathbb{N}) = \lim_{n \to \infty} \frac{1}{n} \cdot |\mathbb{N} \cap \{1, \dots, n\}| = \lim_{n \to \infty} \frac{1}{n} \cdot |\{1, \dots, n\}| = \lim_{n \to \infty} \frac{n}{n} = 1$

$\mu\left(\dot\bigcup_{k \in \mathbb{N}} \{k\}\right) = \sum_{k \in \mathbb{N}} \mu(\{k\}) = \sum_{k \in \mathbb{N}} \lim_{n \in \mathbb{N}} \mu_n(\{k\}) = \sum_{k \in \mathbb{N}} \lim_{n \in \mathbb{N}} \frac{1}{n} \cdot |\{k\} \cap \{1, \dots, n\}| \leq \sum_{k \in \mathbb{N}} \lim_{n \in \mathbb{N}} \frac{1}{n} = 0$ $\lightning$

b) $\Omega = \mathbb{N}$ $\quad \mathcal{A} = \mathcal{P}(\mathbb{N})$ $\quad \mu_n(A) = \frac{|A|}{n}$ $\qquad$ Sei $n \in \mathbb{N}$ bel.

•) $\mu_n(\emptyset) = \frac{|\emptyset|}{n} = 0$

•) Sei $A \in \mathcal{A}$ bel. $\quad \mu_n(A) = \frac{|A|}{n} \geq 0$

•) Sei $(A_n)_{n \in \mathbb{N}}$ aus $\mathcal{A}$ paarweise disjunkt bel.

$\mu_n\left(\dot\bigcup_{\tilde n \in \mathbb{N}} A_{\tilde n}\right) = \frac{\left|\dot\bigcup_{\tilde n \in \mathbb{N}} A_{\tilde n}\right|}{n} = \frac{1}{n} \cdot \sum_{\tilde n \in \mathbb{N}} |A_{\tilde n}| = \sum_{\tilde n \in \mathbb{N}} \frac{|A_{\tilde n}|}{n} = \sum_{\tilde n \in \mathbb{N}} \mu_n(A_{\tilde n})$ $\Rightarrow \mu_n$ ist ein Maß

$\mu := \lim_{n \in \mathbb{N}} \mu_n$ $\qquad \mu(\mathbb{N}) = \lim_{n \in \mathbb{N}} \mu_n(\mathbb{N}) = \lim_{n \in \mathbb{N}} \frac{|\mathbb{N}|}{n} = 1$

$\mu\left(\dot\bigcup_{k \in \mathbb{N}} \{k\}\right) = \sum_{k \in \mathbb{N}} \mu(\{k\}) = \sum_{k \in \mathbb{N}} \lim_{n \in \mathbb{N}} \frac{|\{k\}|}{n} = \sum_{k \in \mathbb{N}} \lim_{n \in \mathbb{N}} \frac{1}{n} = 0$ $\lightning$ $\Rightarrow \mu$ ist kein Maß