ANA Ü3

9.) $\alpha \in \mathbb{R}, a \in \mathbb{R}, a > 0$

$$\int x^\alpha dx = \int \exp(\ln(x))^\alpha dx = \int (e^{\ln(x)})^\alpha dx = \int e^{\alpha \cdot \ln(x)} dx$$

$$\left[\begin{array}{lll} u = \alpha \cdot \ln(x) & \ln(x) = \frac{u}{\alpha} & x = e^{\frac{u}{\alpha}} \\ \frac{du}{dx} = \frac{\alpha}{x} & du = \frac{\alpha}{x} \cdot dx & dx = du \cdot \frac{x}{\alpha} \end{array}\right]$$

$$\int e^u \cdot \frac{x}{\alpha} \cdot du = \int e^u \frac{e^{\frac{u}{\alpha}}}{\alpha} du = \frac{1}{\alpha} \cdot \int e^{u + \frac{u}{\alpha}} du = \frac{1}{\alpha} \cdot \int e^{u \cdot (\frac{\alpha+1}{\alpha})} du$$

$$\left[\begin{array}{ll} m = u \cdot (\frac{\alpha+1}{\alpha}) & u = m \cdot \frac{\alpha}{\alpha+1} \\ \frac{dm}{du} = \frac{\alpha+1}{\alpha} & du = dm \cdot \frac{\alpha}{\alpha+1} \end{array}\right]$$

$$\frac{1}{\alpha} \int e^m \frac{\alpha}{\alpha+1} dm = \frac{\alpha}{\alpha \cdot (\alpha+1)} \cdot \int e^m dm = \frac{1}{\alpha+1} e^m = \frac{1}{\alpha+1} \cdot e^{u(\frac{\alpha+1}{\alpha})}$$

$$= \frac{1}{\alpha+1} \cdot e^{\alpha \cdot \ln(x) \cdot \frac{\alpha+1}{\alpha}} = \frac{1}{\alpha+1} \cdot e^{\ln(x)(\alpha+1)} = \frac{1}{\alpha+1} (e^{\ln(x)})^{(\alpha+1)} = \frac{1}{\alpha+1} \cdot x^{(\alpha+1)} + C$$

$$\int a^x dx = \int \exp(\ln(a))^x dx = \int e^{x \cdot \ln(a)} dx$$

$$u = x \cdot \ln(a) \qquad x = \frac{u}{\ln(a)}$$

$$\frac{du}{dx} = \ln(a) \qquad dx = \frac{du}{\ln(a)}$$

$$\int e^u \frac{1}{\ln(a)} \cdot du = \frac{1}{\ln(a)} \cdot \int e^u du = \frac{1}{\ln(a)} e^u = \frac{1}{\ln(a)} \cdot e^{x \cdot \ln(a)}$$

$$= \frac{1}{\ln(a)} \cdot \exp(\ln(a))^x = \frac{1}{\ln(a)} \cdot a^x + C$$