ANA Ü13

4.) $\gamma_1 : [0,2\pi] \to \mathbb{C}$, $t \mapsto -1+\frac{1}{2}\exp(it)$ $\qquad$ $\gamma_2 : [0,2\pi] \to \mathbb{C}$, $t \mapsto 1+\frac{1}{2}\exp(it)$ $\qquad$ $\gamma_3 : [0,2\pi] \to \mathbb{C}$, $t \mapsto 4\exp(it)$

$f(z) = \frac{1}{z+1} + \frac{1}{z-1}$ $\qquad$ ges: $\int_{\gamma_1} f(\zeta)\,d\zeta + \int_{\gamma_2} f(\zeta)\,d\zeta$

Satz 11.6.12 (Cauchysche Integralformel)

$f: D \to Y$ $\quad$ $D \subseteq \mathbb{C}$ ... offen $\quad$ $w \in \mathbb{C}$ $\quad$ $\rho > 0$ mit $K_\rho(w) \subseteq D$

$z \in U_\rho(w)$ $\quad$ $\gamma : [0,2\pi] \to D$ $\quad$ $t \mapsto w + \rho \cdot \exp(it)$

$\Rightarrow f(z) = \frac{1}{2\pi i} \int_\gamma \frac{f(\zeta)}{\zeta - z}\,d\zeta$

$$\int_{\gamma_1} f(\zeta)\,d\zeta + \int_{\gamma_2} f(\zeta)\,d\zeta = \int_{\gamma_1} \frac{1}{\zeta+1} + \frac{1}{\zeta-1}\,d\zeta + \int_{\gamma_2} \frac{1}{\zeta+1} + \frac{1}{\zeta-1}\,d\zeta$$

$$= \int_{\gamma_1} \frac{1}{\zeta+1}\,d\zeta + \int_{\gamma_1} \frac{1}{\zeta-1}\,d\zeta + \int_{\gamma_2} \frac{1}{\zeta+1}\,d\zeta + \int_{\gamma_2} \frac{1}{\zeta-1}\,d\zeta$$

$w = -1$ $\quad$ $\rho = \frac{1}{2}$ $\quad$ $z = -1$ $\quad$ $\gamma(t) = -1 + \frac{1}{2}\cdot\exp(it) = \gamma_1(t)$ $\quad$ $f \equiv 1$

$\Rightarrow 2\pi i f(z) = \int_\gamma \frac{f(\zeta)}{\zeta - z}\,d\zeta \iff 2\pi i = \int_{\gamma_1} \frac{1}{\zeta-(-1)}\,d\zeta$

gleich mit $w = 1$ $\quad$ $z = 1$ $\quad$ $\gamma = \gamma_2$ $\quad \Rightarrow 2\pi i = \int_{\gamma_2} \frac{1}{\zeta - 1}\,d\zeta$

$$= 2\pi i + \int_{\gamma_1} \frac{1}{\zeta-1}\,d\zeta + \int_{\gamma_2} \frac{1}{\zeta+1}\,d\zeta + 2\pi i \qquad \frac{\frac{\zeta+1}{\zeta-1}}{\zeta+1} = \frac{\zeta+1}{(\zeta+1)(\zeta-1)} = \frac{1}{\zeta-1}$$

$$= 4\pi i + \int_{\gamma_1} \frac{\frac{\zeta+1}{\zeta-1}}{\zeta+1}\,d\zeta + \int_{\gamma_2} \frac{\frac{\zeta-1}{\zeta+1}}{\zeta-1}\,d\zeta$$

$w = -1$ $\quad$ $\rho = \frac{1}{2}$ $\quad$ $z = -1$ $\quad$ $\gamma = -1 + \frac{1}{2}\exp(it) = \gamma_1(t)$ $\quad$ $f(\zeta) = \frac{\zeta+1}{\zeta-1}$

$\Rightarrow 2\pi i f(z) = \int_\gamma \frac{f(\zeta)}{\zeta - z}\,d\zeta \iff 2\pi i \underbrace{\frac{-1+1}{-1-1}}_{=0} = \int_{\gamma_1} \frac{\frac{\zeta+1}{\zeta-1}}{\zeta-(-1)}$

gleich mit $w = 1$ $\quad$ $z = 1$ $\quad$ $\gamma = \gamma_2$ $\quad$ $f(\zeta) = \frac{\zeta-1}{\zeta+1}$

$\Rightarrow 2\pi i \underbrace{\frac{1-1}{1+1}}_{=0} = \int_{\gamma_2} \frac{\frac{\zeta-1}{\zeta+1}}{\zeta-1}\,d\zeta$

$= 4\pi i + 0 + 0 = 4\pi i$

...

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4.)... ges: $\int_{\gamma_3} f(\zeta)\,d\zeta$

$$\int_{\gamma_3} \frac{1}{\zeta+1} + \frac{1}{\zeta-1}\,d\zeta = \int_{\gamma_3} \frac{1}{\zeta+1}\,d\zeta + \int_{\gamma_3} \frac{1}{\zeta-1}\,d\zeta$$

$f(\zeta) = 1 \quad w = 0 \quad \rho = 4 \quad z = -1 \quad \gamma(t) = 4\exp(it) = \gamma_3(t)$

$\Rightarrow 2\pi i\, f(-1) = \int_{\gamma} \frac{1}{\zeta-(-1)}\,d\zeta \iff 2\pi i = \int_{\gamma_3} \frac{1}{\zeta+1}\,d\zeta$

gleich mit $z = 1$

$\Rightarrow 2\pi i\, f(1) = \int_{\gamma} \frac{1}{\zeta-1}\,d\zeta \iff 2\pi i = \int_{\gamma_3} \frac{1}{\zeta-1}\,d\zeta$

$= 2\pi i + 2\pi i = 4\pi i$

ges: Skizze