ANA Ü12

1.) $\gamma : [0,1] \to \mathbb{R}^2$, $t \mapsto \begin{pmatrix} 1+t^2 \\ 1+t^3 \end{pmatrix}$ ... stetiger Weg $\quad \Phi : (1,+\infty)\times\mathbb{R} \to L(\mathbb{R}^2,\mathbb{R}) \cong \mathbb{R}^{1\times 2}$

$\begin{pmatrix} x \\ y \end{pmatrix} \mapsto (\sin(\sqrt{x-1}), xy)$

ges: $l(\gamma)$ und $\int_\gamma \Phi(x)\,dx$

$\gamma'(t) = \begin{pmatrix} 2t \\ 3t^2 \end{pmatrix}$ ... stetig $\Rightarrow \gamma \in C^1[0,1]$ Nach Satz 11.1.8. gilt nun

$$l(\gamma) = \int_0^1 \|\gamma'(x)\|_2\,dx = \int_0^1 \left\|\begin{pmatrix} 2x \\ 3x^2 \end{pmatrix}\right\|_2 dx = \int_0^1 \sqrt{4x^2+9x^4}\,dx = \int_0^1 x\cdot\sqrt{4+9x^2}\,dx$$

$\int x\sqrt{4+9x^2}\,dx = \int x\sqrt{u}\,\frac{1}{18x}\,du \qquad \left[u = 4+9x^2 \quad \frac{du}{dx} = 18x \quad dx = \frac{1}{18x}\,du\right.$

$= \frac{1}{18}\int \sqrt{u}\,du = \frac{1}{18}\cdot\frac{2}{3}u^{\frac{3}{2}} = \frac{1}{27}(4+9x^2)^{\frac{3}{2}}$

$$= \frac{1}{27}(4+9x^2)^{\frac{3}{2}}\Big|_0^1 = \frac{1}{27}\cdot 13^{\frac{3}{2}} - \frac{1}{27}\cdot 4^{\frac{3}{2}} \approx 1{,}4397$$

Φ ist auf $(1,+\infty)\times\mathbb{R}$ stetig. Nach Satz 11.2.5 gilt nun

$$\int_\gamma \Phi(x)\,dx = \int_0^1 \Phi(\gamma(t))\cdot\gamma'(t)\,dt = \int_0^1 \left(\sin(\sqrt{1+t^2-1}), (1+t^2)(1+t^3)\right)\cdot\begin{pmatrix} 2t \\ 3t^2 \end{pmatrix} dt$$

$$= \int_0^1 2\sin(t)\cdot t + 3t^2(1+t^3+t^2+t^5)\,dt = \int_0^1 2\sin(t)t + 3t^2+3t^5+3t^4+3t^7\,dt$$

$$= 2\int_0^1 \sin(t)t\,dt + 3\left(\int_0^1 t^2\,dt + \int_0^1 t^5\,dt + \int_0^1 t^4\,dt + \int_0^1 t^7\,dt\right)$$

$\int \sin(t)\cdot t\,dt = -t\cdot\cos(t) + \int\cos(t)\,dt = -\cos(t)\cdot t + \sin(t)$

$$\Rightarrow \int_\gamma \Phi(x)\,dx = 2\cdot(-\cos(1)+\sin(1) - 0) + 3\left(\frac{1}{3}+\frac{1}{6}+\frac{1}{5}+\frac{1}{8}\right) = \frac{99}{40} + 2\sin(1) - 2\cos(1) \approx 3{,}07734$$