ANA Ü9

2.) $f: \mathbb{C} \to \mathbb{C}$ $\quad z = x+iy \quad u\binom{x}{y} = \operatorname{Re} f(x+iy)$

$z \mapsto \exp(z) \quad v\binom{x}{y} = \operatorname{Im} f(x+iy)$

f... stetig differenzierbar bereits bekannt

zz: $\frac{\partial u}{\partial x}\binom{x}{y} = \frac{\partial v}{\partial y}\binom{x}{y}$ und $\frac{\partial u}{\partial y}\binom{x}{y} = -\frac{\partial v}{\partial x}\binom{x}{y}$ ... Cauchy-Riemannschen Differentialgleichungen

∘ $\frac{\partial u}{\partial x}\binom{x}{y} = \frac{d}{dx} \operatorname{Re}(\exp(x+iy)) = \frac{d}{dx} \operatorname{Re}(\exp(x)(\cos(y)+i\sin(y)))$

$= \frac{d}{dx} \exp(x)\cos(y) = \cos(y)\cdot\exp(x)$

∘ $\frac{\partial v}{\partial y}\binom{x}{y} = \frac{d}{dy} \operatorname{Im}(\exp(x+iy)) = \frac{d}{dy} \operatorname{Im}(\exp(x)(\cos(y)+i\sin(y)))$

$= \frac{d}{dy} \exp(x)\sin(y) = \cos(y)\cdot\exp(x)$

∘ $\frac{\partial u}{\partial y}\binom{x}{y} = \frac{d}{dy} \operatorname{Re}(\exp(x+iy)) = \frac{d}{dy} \exp(x)\cdot\cos(y) = -\sin(y)\cdot\exp(x)$

∘ $-\frac{\partial v}{\partial x}\binom{x}{y} = -\frac{d}{dx} \operatorname{Im}(\exp(x+iy)) = -\frac{d}{dx} \exp(x)\sin(y) = -\sin(y)\cdot\exp(x)$

$f: \mathbb{C}\setminus\{0\} \to \mathbb{C} \quad z \mapsto \frac{1}{z}$

| f... stetig diffbar bereits bekannt

$\frac{1}{x+iy} = \frac{x-iy}{(x+iy)(x-iy)} = \frac{x-iy}{x^2+y^2} = \frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}$

∘ $\frac{\partial u}{\partial x}\binom{x}{y} = \frac{d}{dx} \operatorname{Re}\left(\frac{1}{x+iy}\right) = \frac{d}{dx} \frac{x}{x^2+y^2}$

$= \frac{x^2+y^2-x2x}{(x^2+y^2)^2} = \frac{-x^2+y^2}{x^4+2x^2y^2+y^4}$

∘ $\frac{\partial v}{\partial y}\binom{x}{y} = \frac{d}{dy} \operatorname{Im}\left(\frac{1}{x+iy}\right) = \frac{d}{dy} -\frac{y}{x^2+y^2} = -\frac{x^2+y^2-y2y}{(x^2+y^2)^2} = \frac{-x^2+y^2}{x^4+2x^2y^2+y^4}$

∘ $\frac{\partial u}{\partial y}\binom{x}{y} = \frac{d}{dy} \operatorname{Re}\left(\frac{1}{x+iy}\right) = \frac{d}{dy} \frac{x}{x^2+y^2} = \frac{-x2y}{(x^2+y^2)^2} = \frac{-2xy}{x^4+2x^2y^2+y^4}$

∘ $-\frac{\partial v}{\partial x}\binom{x}{y} = -\frac{d}{dx} \operatorname{Im}\left(\frac{1}{x+iy}\right) = -\frac{d}{dx} -\frac{y}{x^2+y^2} = \frac{-y2x}{x^4+2x^2y^2+y^4}$

$\Rightarrow \exp(z), \frac{1}{z}$ sind holomorph

$f: \mathbb{C} \to \mathbb{C} \quad z \mapsto z$

· $\frac{\partial u}{\partial x}\binom{x}{y} = \frac{d}{dx} \operatorname{Re}(x+iy) = \frac{d}{dx} x = 1 \qquad \frac{\partial v}{\partial y}\binom{x}{y} = \frac{d}{dy} \operatorname{Im}(x+iy) = \frac{d}{dy} y = 1$

· $\frac{\partial u}{\partial y}\binom{x}{y} = \frac{d}{dy} \operatorname{Re}(x+iy) = \frac{d}{dy} x = 0 \qquad -\frac{\partial v}{\partial x}\binom{x}{y} = -\frac{d}{dx} \operatorname{Im}(x+iy) = -\frac{d}{dx} y = 0$ $\Rightarrow z \mapsto z$ ist holomorph