ANA Ü 13

7.) $D \subseteq \mathbb{C}$ ... offen $\quad f: D \to \mathbb{C}$ ... holomorph

$\bar{D} := \{\bar{z} : z \in D\} \quad f^*: \bar{D} \to \mathbb{C} \quad z \mapsto \overline{f(\bar{z})}$

zz: f^* ist holomorph

D ... offen $\Rightarrow \forall x \in D \; \exists \varepsilon > 0 \; U_\varepsilon(x) \subseteq D \Rightarrow \forall y \in \bar{D} \; \exists \varepsilon > 0 \; U_\varepsilon(y) \subseteq \bar{D}$

$\Rightarrow \bar{D}$ ... offen

f ... holomorph $\Rightarrow \frac{\partial Re(f)}{\partial x} = \frac{\partial Im(f)}{\partial y} \wedge \frac{\partial Re(f)}{\partial y} = -\frac{\partial Im(f)}{\partial x}$

$\Rightarrow \frac{\partial Re(f^*)}{\partial x} = \frac{\partial -Im(f^*)}{\partial -y} \wedge \frac{\partial Re(f^*)}{\partial -y} = -\frac{\partial -Im(f^*)}{\partial x}$

$\Rightarrow \frac{\partial Re(f^*)}{\partial x} = \frac{\partial Im(f^*)}{\partial y} \wedge -\frac{\partial Re(f^*)}{\partial y} = \frac{\partial Im(f^*)}{\partial x}$

$\Rightarrow f^*$ ist holomorph