MAS Ü4

5.) zz: A, B, C ... unabhängig $\Rightarrow A^c, B^c, C^c$ ... unabhängig

A, B, C ... unabhängig $\Leftrightarrow P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$

$$P(A^c \cap B^c \cap C^c) = 1 - P((A^c \cap B^c \cap C^c)^c) = 1 - P(A \cup B \cup C)$$
$$= 1 - (P(A) + P(B \cup C) - P(A \cap (B \cup C))) = 1 - (P(A) + P(B) + P(C) - P(B \cap C)$$
$$- P((A \cap B) \cup (A \cap C))) = 1 - (P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C)$$
$$+ P((A \cap B) \cap (A \cap C))) = 1 - P(A) - P(B) - P(C) + P(A \cap B) + P(A \cap C) + P(B \cap C)$$
$$- P(A \cap B \cap C) = 1 - P(A) - P(B) - P(C) + P(A) \cdot P(B) + P(A) \cdot P(C) + P(B) \cdot P(C)$$
$$- P(A) \cdot P(B) \cdot P(C) = (1 - P(B) - P(A) + P(A) \cdot P(B))(1 - P(C))$$
$$= (1 - P(A)) \cdot (1 - P(B)) \cdot (1 - P(C)) = P(A^c) \cdot P(B^c) \cdot P(C^c)$$

$\square$