ANA Üg

4.) $f\begin{pmatrix}x\\y\\z\end{pmatrix} = \sin(3x+yz)$ ges: partielle Ableitungen bis 3. Grad

z_0	x_0	x_1	x_2	x_3
y_0	$\sin(3x+yz)$	$\cos(3x+yz)\cdot 3$	$-\sin(3x+yz)\cdot 9$	$-\cos(3x+yz)\cdot 27$
y_1	$\cos(3x+yz)\cdot z$	$-\sin(3x+yz)\cdot 3z$	$-\cos(3x+yz)9z$	
y_2	$-\sin(3x+yz)\cdot z^2$	$-\cos(3x+yz)\cdot 3z^2$		
y_3	$-\cos(3x+yz)\cdot z^3$			

z_1	x_0	x_1	x_2
y_0	$\cos(3x+yz)\cdot y$	$-\sin(3x+yz)\cdot 3y$	$-\cos(3x+yz)\cdot 9y$
y_1	$-\sin(3x+yz)\cdot zy+\cos(3x+yz)$	$-3(\cos(3x+yz)\cdot zy+\sin(3x+yz))$	
y_2	$-z(\cos(3x+yz)\cdot zy+\sin(3x+yz))-\sin(3x+yz)\cdot z$ $= -z^2y\cos(3x+yz)-2z\sin(3x+yz)$		

z_2	x_0	x_1
y_0	$-\sin(3x+yz)\cdot y^2$	$-\cos(3x+yz)\cdot 3y^2$
y_1	$-(\cos(3x+yz)\cdot z\cdot y^2+\sin(3x+yz)\cdot 2y)$	

z_3	x_0
y_0	$-\cos(3x+yz)\cdot y^3$

ges: $df\begin{pmatrix}x\\y\\z\end{pmatrix}(v)$ und $d^2f\begin{pmatrix}x\\y\\z\end{pmatrix}(v,w)$ $\quad v=\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}$ $\quad w=\begin{pmatrix}w_1\\w_2\\w_3\end{pmatrix}$

$$df\begin{pmatrix}x\\y\\z\end{pmatrix}(v) = \frac{\partial}{\partial v} f\begin{pmatrix}x\\y\\z\end{pmatrix} = v_1\cdot\cos(3x+yz)\cdot 3 + v_2\cdot\cos(3x+yz)\cdot z + v_3\cdot\cos(3x+yz)\cdot y$$

$$d^2f\begin{pmatrix}x\\y\\z\end{pmatrix}(v,w) = \sum_{l_1,l_2=1}^{3} v_{l_1}\cdot w_{l_2}\cdot \frac{\partial^2}{\partial x_{l_1}\partial x_{l_2}} f\begin{pmatrix}x\\y\\z\end{pmatrix}$$

$$= v_1\cdot w_1 \frac{\partial^2}{\partial x\partial x} f\begin{pmatrix}x\\y\\z\end{pmatrix} + v_1\cdot w_2 \frac{\partial^2}{\partial x\partial y} f\begin{pmatrix}x\\y\\z\end{pmatrix} + v_1\cdot w_3 \frac{\partial^2}{\partial x\partial z} f\begin{pmatrix}x\\y\\z\end{pmatrix} + v_2\cdot w_1 \frac{\partial^2}{\partial y\partial x} f\begin{pmatrix}x\\y\\z\end{pmatrix}$$

$$+ v_2\cdot w_2 \frac{\partial^2}{\partial y\partial y} f\begin{pmatrix}x\\y\\z\end{pmatrix} + v_2\cdot w_3 \frac{\partial^2}{\partial y\partial z} f\begin{pmatrix}x\\y\\z\end{pmatrix} + v_3 w_1 \frac{\partial^2}{\partial z\partial x} f\begin{pmatrix}x\\y\\z\end{pmatrix} + v_3 w_2 \frac{\partial^2}{\partial z\partial y} f\begin{pmatrix}x\\y\\z\end{pmatrix} + v_3 w_3 \frac{\partial^2}{\partial z\partial z} f\begin{pmatrix}x\\y\\z\end{pmatrix}$$

$$= v_1 w_1\cdot(-\sin(3x+yz)\cdot 9) + v_1 w_2 (-\sin(3x+yz)\cdot 3z) + v_1\cdot w_3(-\sin(3x+yz)\cdot 3y) + v_2\cdot w_1(-\sin(3x+yz)\cdot 3z)$$

$$+ v_2\cdot w_2(-\sin(3x+yz)\cdot z^2) + v_2\cdot w_3(-\sin(3x+yz)\cdot zy+\cos(3x+yz)) + v_3\cdot w_1(-\sin(3x+yz)\cdot 3y)$$

$$+ v_3 w_2(-\sin(3x+yz)zy+\cos(3x+yz)) + v_3 w_3(-\sin(3x+yz)\cdot y^2)$$

...

ANA Ü9

4.) ... ges: Taylorpolynom in $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ mit $q=3$ und Anschlussstelle $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$$T_3\begin{pmatrix} x \\ y \\ z \end{pmatrix} = f\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \sum_{l=1}^{3} \frac{1}{l!} d^l f\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \underbrace{\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \dots, \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}\right)}_{l\text{-mal}}$$

$$= 0 + df\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} + \frac{1}{2} d^2 f\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}, \begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) + \frac{1}{6} d^3 f\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}, \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \begin{pmatrix} x \\ y \\ z \end{pmatrix}\right)$$

$$= 3x \cdot \cos(0) + \frac{1}{2}\left(-9x^2 \cdot \sin(0) + y \cdot z \cdot \cos(0) + z \cdot y \cdot \cos(0)\right) + \frac{1}{6}\left(-27x^3 \cdot \cos(0) - 3x y \cdot z \sin(0)\right)$$

$$= 3x + yz - 4{,}5\, x^3$$