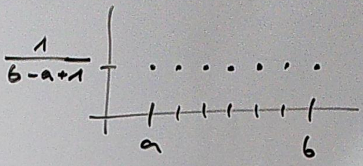
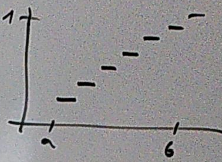
MAS Ü11

3.) diskrete Gleichverteilung

$D(a,b)$





$$\mathbb{E}(X) = \sum_{n=a}^{b} n\, \mathbb{P}(X=n) = \sum_{n=a}^{b} n \cdot \frac{1}{b-a+1}$$

$$= \frac{1}{b-a+1} \sum_{n=a}^{b} n = \frac{a+b}{2}$$

$$\mathbb{V}(X) = \mathbb{E}((X-\mathbb{E}(X))^2) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \mathbb{E}(X^2) - \frac{(a+b)^2}{4}$$

$$\mathbb{E}(X^2) = \sum_{n=a}^{b} n^2\, \mathbb{P}(X=n) = \sum_{n=a}^{b} n^2 \frac{1}{b-a+1} = \frac{1}{b-a+1} \sum_{n=a}^{b} n^2 = \frac{1}{b-a+1}\left(\sum_{n=1}^{b} n^2 - \sum_{n=1}^{a-1} n^2\right)$$

$$= \left(\frac{b(b+1)(2b+1)}{6} - \frac{(a-1)a(2a-2+1)}{6}\right)\frac{1}{b-a+1} = \frac{2b^3+b^2+2b^2+b-(2a^3-2a^2+a^2-2a^2+2a-a)}{6b-6a+6}$$

$$= \frac{2b^3+3b^2+b-2a^3+3a^2-a}{6b-6a+6}$$

$$\mathbb{E}(X^2) - \frac{a^2+2ab+b^2}{4} = \frac{4b^3+6b^2+2b-4a^3+6a^2-2a}{12b-12a+12} - \frac{(3a^2+6ab+3b^2)(b-a+1)}{12(b-a+1)}$$

$$= \frac{4b^3+6b^2+2b-4a^3+6a^2-2a-(3a^2b+6ab^2+3b^3-3a^3-6a^2b-3ab^2+3a^2+6ab+3b^2)}{12b-12a+12}$$

$$= \frac{b^3+3b^2+2b-a^3+3a^2-2a+3a^2b-3ab^2-6ab}{12b-12a+12} = \frac{(a-b-2)(a-b)}{12}$$