LINAG Ü7

8.5.7 (3) $\quad t \in \mathbb{R} \qquad A_t = \begin{pmatrix} 1 & 0 & 0 \\ t & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$ Für welche t ist A_t diagonalisierbar?

$$\chi_{A_t}(X) = \det\begin{pmatrix} 1-X & 0 & 0 \\ t & 1-X & 0 \\ 0 & 1 & 3-X \end{pmatrix} = (1-X)^2(3-X)$$

$$B = P^{-1}AP \iff PB = AP$$

$$B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix} \qquad P = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix} = \begin{pmatrix} a & b & 3c \\ d & e & 3f \\ g & h & 3i \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 0 \\ t & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \begin{pmatrix} a & b & c \\ at+d & bt+e & ct+f \\ d+3g & e+3h & f+3i \end{pmatrix}$$

$\Rightarrow 3c = c \quad d = at+d \quad e = bt+e \quad 3f = ct+f$

$g = d+3g \quad h = e+3h \quad 3i = f+3i$

$t \neq 0 \Rightarrow a=0 \quad b=0 \quad c=0 \quad d=-2g \quad e=-2h \quad f=0 \quad g=-\frac{1}{2}d \quad h=-\frac{1}{2}e \quad i=*$

also $P = \begin{pmatrix} 0 & 0 & 0 \\ -2g & -2e & 0 \\ g & h & i \end{pmatrix}$ aber P nicht regulär! (siehe erste Zeile)

$t = 0 \Rightarrow a=* \quad b=* \quad c=0, d=-2g, e=-2h, f=0, g=-\frac{1}{2}d, h=-\frac{1}{2}e, i=*$

also $P = \begin{pmatrix} a & b & 0 \\ -2g & -2h & 0 \\ g & h & i \end{pmatrix}$ z.B. $\begin{pmatrix} 1 & 1 & 0 \\ -2 & -4 & 0 \\ 1 & 2 & 1 \end{pmatrix} =: P$

$$\left(\begin{array}{ccc} 1 & 1 & 0 \\ -2 & -4 & 0 \\ 1 & 2 & 1 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right) \rightsquigarrow \left(\begin{array}{ccc} 1 & -1 & 0 \\ -2 & 0 & 0 \\ 1 & 0 & 1 \\ \hline 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right) \rightsquigarrow \left(\begin{array}{ccc} 0 & 1 & 0 \\ -2 & 0 & 0 \\ 1 & 0 & 1 \\ \hline -1 & 2 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{array}\right) \rightsquigarrow \left(\begin{array}{ccc} 0 & 1 & 0 \\ -2 & 0 & 0 \\ 0 & 0 & 1 \\ \hline -1 & 2 & 0 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{array}\right) \rightsquigarrow \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 2 & \frac{1}{2} & 0 \\ -1 & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 1 \end{array}\right) = P^{-1}$$

$$\begin{pmatrix} 2 & \frac{1}{2} & 0 \\ -1 & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & \frac{1}{2} & 0 \\ -1 & -\frac{1}{2} & 0 \\ 0 & \frac{3}{2} & 3 \end{pmatrix}, \qquad \begin{pmatrix} 2 & \frac{1}{2} & 0 \\ -1 & -\frac{1}{2} & 0 \\ 0 & \frac{3}{2} & 3 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ -2 & -4 & 0 \\ 1 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

... Probe geglückt!

$\Rightarrow A_t$ ist nur für $t=0$ diagonalisierbar