LINAG ÜM

9.5.2. σ ... orthosymmetrische ω-Sesquilinearform auf V mit $\omega^2 = id$

$M \subseteq V$ $\quad M^\perp := \{y \in V \mid \forall x \in M: y \perp x\}$ $\quad$ orthosymmetrisch: $x \perp y \Leftrightarrow y \perp x$

a) zz: $M \subseteq M^{\perp\perp}$

$$M^{\perp\perp} = \{z \in V \mid \forall y \in \{y \in V \mid \forall x \in M: y \perp x\}: z \perp y\}$$

Sei $m \in M$ bel. Sei $y \in M^\perp$ bel. $m \perp y$, da $\forall x \in M: x \perp y$

b) zz: $M \subseteq N \Rightarrow N^\perp \subseteq M^\perp$

Sei $n \in N^\perp$ bel. D.h. $\forall x \in N: n \perp x$, da $M \subseteq N \Rightarrow \forall x \in M: n \perp x \Rightarrow n \in M^\perp$

c) zz: $M \subseteq N \Rightarrow M^{\perp\perp} \subseteq N^{\perp\perp}$

Sei $m \in M^{\perp\perp}$ bel. D.h. $\forall x \in M^\perp: m \perp x$, da $N^\perp \subseteq M^\perp$ (siehe b)

$\Rightarrow \forall x \in N^\perp: m \perp x \Rightarrow m \in N^{\perp\perp}$

d) zz: $M^{\perp\perp\perp} = M^\perp$

$\subseteq$ Sei $m \in M^{\perp\perp\perp}$ bel. D.h. $\forall x \in M^{\perp\perp}: m \perp x$, da $M \subseteq M^{\perp\perp}$ gilt $\forall x \in M: m \perp x$

$\Rightarrow m \in M^\perp$

$\supseteq$ Nach a ist $(M^\perp) \subseteq (M^\perp)^{\perp\perp}$ also $M^\perp \subseteq M^{\perp\perp\perp}$

e) zz: $M^{\perp\perp\perp\perp} = M^{\perp\perp}$

$\subseteq$ Sei $m \in M^{\perp\perp\perp\perp}$ bel. D.h. $\forall x \in M^{\perp\perp\perp}: m \perp x$ $\quad M^\perp \subseteq M^{\perp\perp\perp} \Rightarrow \forall x \in M^\perp: m \perp x$

$\Rightarrow m \in M^{\perp\perp}$

$\supseteq$ Nach a ist $(M^{\perp\perp}) \subseteq (M^{\perp\perp})^{\perp\perp}$

f) zz: $M = M^{\perp\perp} \Leftrightarrow \exists N \subseteq V: M = N^\perp$

$\Rightarrow$ $N = M^{\perp\perp\perp} \Rightarrow N^\perp = M^{\perp\perp\perp\perp} \underset{\text{nach e)}}{=} M^{\perp\perp} \underset{\text{nach Vorraussetzung}}{=} M$

$\Leftarrow$ Nach d) ist $N^\perp = N^{\perp\perp\perp} \Rightarrow M = M^{\perp\perp}$

g) zz: $M \subseteq N^\perp \Leftrightarrow N \subseteq M^\perp$

$\Rightarrow$ $\forall n \in N^\perp\ \forall x \in N: n \perp x$ $\quad$ aus $M \subseteq N^\perp$ folgt $\forall n \in M\ \forall x \in N: n \perp x$

$\Leftrightarrow \forall x \in N\ \forall n \in M: x \perp n \Rightarrow N \subseteq M^\perp$

$\Leftarrow$ genauso

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9.4.6. a) $\beta : \mathbb{R}^{3\times 1} \times \mathbb{R}^{3\times 1} \to \mathbb{R}$ ... symmetrische Bilinearform E.. kanonische Basis

$$\beta(E,E) = \begin{pmatrix} 0 & -3 & 2 \\ -3 & 1 & 1 \\ 2 & 1 & 0 \end{pmatrix} \qquad C = (c_1, c_2, c_3) = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

$c_1 = e_1 - e_3$ $\beta(c_1, c_1) = \beta(e_1 - e_3, e_1 - e_3) = \beta(e_1, e_1) - \beta(e_1, e_3) - \beta(e_3, e_1) + \beta(e_3, e_3)$

$= -2 - 2 = -4$

$c_2 = e_1 + 2e_2$ $\beta(c_2, e_2) = \beta(e_1 + 2e_2, e_1 + 2e_2) = \beta(e_1, e_1) + 2\beta(e_1, e_2) + 2\beta(e_2, e_1) + 4\beta(e_2, e_2)$

$= 0 + 4(-3) + 4 \cdot 1 = -8$

$c_3 = e_1 + e_2 + e_3$ $\beta(c_3, c_3) = \beta(e_1 + e_2 + e_3, e_1 + e_2 + e_3) = \beta(e_1, e_1) + 2\beta(e_1, e_2) + 2\beta(e_1, e_3)$

$+ \beta(e_2, e_2) + 2\beta(e_2, e_3) + \beta(e_3, e_3) = 0 + 2(-3) + 2 \cdot 2 + 1 + 2 \cdot 1 + 0 = 1$

$\beta(c_1, c_2) = \beta(e_1 - e_3, e_1 + 2e_2) = \beta(e_1, e_1) + 2\beta(e_1, e_2) - \beta(e_1, e_3) - 2\beta(e_2, e_3)$

$= 0 + 2(-3) - 2 - 2 \cdot 1 = -10$

$\beta(c_1, c_3) = \beta(e_1 - e_3, e_1 + e_2 + e_3) = \beta(e_1, e_1) + \beta(e_1, e_2) + \beta(e_1, e_3) - \beta(e_1, e_3) - \beta(e_2, e_3) - \beta(e_3, e_3)$

$= 0 - 3 - 1 - 0 = -4$

$\beta(c_2, c_3) = \beta(e_1 + 2e_2, e_1 + e_2 + e_3) = \beta(e_1, e_1) + \beta(e_1, e_2) + \beta(e_1, e_3) + 2\beta(e_1, e_2) + 2\beta(e_2, e_2) + 2\beta(e_2, e_3)$

$= 0 + (-3) + 2 + 2(-3) + 2 + 1 = -3$

$$\Rightarrow \beta(C, C) = \begin{pmatrix} -4 & -10 & -4 \\ -10 & -8 & -3 \\ -4 & -3 & 1 \end{pmatrix}$$