LINAG ÜM

9.4.6. a) $\sigma : \mathbb{R}^{3\times 1} \times \mathbb{R}^{3\times 1} \to \mathbb{R}$ ... symmetrische Bilinearform E... kanonische Basis

$$\sigma(E,E) = \begin{pmatrix} 0 & -3 & 2 \\ -3 & 1 & 1 \\ 2 & 1 & 0 \end{pmatrix} \qquad C = (c_1, c_2, c_3) = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

$c_1 = e_1 - e_3$ $\sigma(c_1,c_1) = \sigma(e_1-e_3, e_1-e_3) = \sigma(e_1,e_1) - \sigma(e_1,e_3) - \sigma(e_3,e_1) + \sigma(e_3,e_3)$

$= -2-2 = -4$

$c_2 = e_1 + 2e_2$ $\sigma(c_2,e_2) = \sigma(e_1+2e_2, e_1+2e_2) = \sigma(e_1,e_1) + 2\sigma(e_1,e_2) + 2\sigma(e_2,e_1) + 4\sigma(e_2,e_2)$

$= 0 + 4(-3) + 4\cdot 1 = -8$

$c_3 = e_1 + e_2 + e_3$ $\sigma(c_3,c_3) = \sigma(e_1+e_2+e_3, e_1+e_2+e_3) = \sigma(e_1,e_1) + 2\sigma(e_1,e_2) + 2\sigma(e_1,e_3)$

$+ \sigma(e_2,e_2) + 2\sigma(e_2,e_3) + \sigma(e_3,e_3) = 0 + 2(-3) + 2\cdot 2 + 1 + 2\cdot 1 + 0 = 1$

$\sigma(c_1,c_2) = \sigma(e_1-e_3, e_1+2e_2) = \sigma(e_1,e_1) + 2\sigma(e_1,e_2) - \sigma(e_1,e_3) - 2\sigma(e_2,e_3)$

$= 0 + 2(-3) - 2 - 2\cdot 1 = -10$

$\sigma(c_1,c_3) = \sigma(e_1-e_3, e_1+e_2+e_3) = \sigma(e_1,e_1) + \sigma(e_1,e_2) + \sigma(e_1,e_3) - \sigma(e_1,e_3) - \sigma(e_2,e_3) - \sigma(e_3,e_3)$

$= 0 - 3 - 1 - 0 = -4$

$\sigma(c_2,c_3) = \sigma(e_1+2e_2, e_1+e_2+e_3) = \sigma(e_1,e_1) + \sigma(e_1,e_2) + \sigma(e_1,e_3) + 2\sigma(e_1,e_2) + 2\sigma(e_2,e_2) + 2\sigma(e_2,e_3)$

$= 0 + (-3) + 2 + 2(-3) + 2 + 1 = -3$

$$\Rightarrow \sigma(C,C) = \begin{pmatrix} -4 & -10 & -4 \\ -10 & -8 & -3 \\ -4 & -3 & 1 \end{pmatrix}$$