ANA Ü12

7.) $f: D \to \mathbb{C}$ $\quad z \in D \quad$ f ... ist bei z komplex differenzierbar

zz: $\det df(z) = |f'(z)|^2$

$$\det df = \det \begin{pmatrix} \frac{\partial Re(f)}{\partial x} & \frac{\partial Im(f)}{\partial x} \\ \frac{\partial Re(f)}{\partial y} & \frac{\partial Im(f)}{\partial y} \end{pmatrix} = \frac{\partial Re(f)}{\partial x} \cdot \frac{\partial Im(f)}{\partial y} - \frac{\partial Re(f)}{\partial y} \cdot \frac{\partial Im(f)}{\partial x}$$

$$= \frac{\partial Re(f)}{\partial x} \cdot \frac{\partial Re(f)}{\partial x} + \frac{\partial Im(f)}{\partial x} \cdot \frac{\partial Im(f)}{\partial x} = \left(\frac{\partial Re(f)}{\partial x}\right)^2 + \left(\frac{\partial Im(f)}{\partial x}\right)^2$$

Cauchy-Riemansche Differenzialgleichungen

$$|f'(z)|^2 = \left|\frac{\partial f}{\partial x}(z)\right|^2 = \left|\frac{\partial Re(f)}{\partial x} + i\,\frac{\partial Im(f)}{\partial x}\right|^2 = \sqrt{\left(\frac{\partial Re(f)}{\partial x}\right)^2 + \left(\frac{\partial Im(f)}{\partial x}\right)^2}^{\,2}$$

$$= \left(\frac{\partial Re(f)}{\partial x}\right)^2 + \left(\frac{\partial Im(f)}{\partial x}\right)^2$$