ANA Ü1

9.) zz: $\lim\limits_{x \to \infty} \frac{\exp(x)}{x^n} = +\infty \quad \forall n \in \mathbb{N} \cup \{0\}$

Sei $n \in \mathbb{N} \cup \{0\}$ bel.

$$\lim_{x \to +\infty} \frac{\exp(x)}{x^n} = \lim_{x \to +\infty} \frac{1}{x^n} \cdot \sum_{k=0}^{\infty} \frac{x^k}{k!} = \lim_{x \to +\infty} \sum_{k=0}^{\infty} \frac{x^k}{x^n \cdot k!}$$

Sei $M > 0$ bel. Da $\forall k \in \mathbb{N} \cup \{0\} : \frac{x^k}{x^n \cdot k!} \geq 0$ ist

$$\sum_{k=0}^{\infty} \frac{x^k}{x^n \cdot k!} \geq \frac{x^0}{x^n \cdot 0!} = \frac{1}{x^n \cdot 1} = \frac{1}{x^n}$$

Wähle $x = \exp(-\frac{1}{n} \cdot \ln(M+1))$, dann ist

$$\frac{1}{x^n} = x^{-n} = \exp(-\tfrac{1}{n} \cdot \ln(M+1))^{-n} = \exp(-\tfrac{1}{n} \cdot (-n) \cdot \ln(M+1))$$

$$= \exp(\ln(M+1)) = M+1 > M$$

$$\Rightarrow \lim_{x \to +\infty} \frac{\exp(x)}{x^n} = +\infty$$

$$\lim_{x \to -\infty} x^n \cdot \exp(x) = \lim_{x \to +\infty} (-x)^n \cdot \exp(-x)$$

1. Fall $\exists k \in \mathbb{N} \cup \{0\} : n = 2k$

$$\lim_{x \to \infty} (-x)^n \cdot \exp(-x) = \lim_{x \to \infty} x^n \cdot \frac{1}{\exp(x)} = \frac{1}{\lim\limits_{x \to \infty} \frac{\exp(x)}{x^n}} = 0$$

2. Fall $\exists k \in \mathbb{N} \cup \{0\} : n = 2k+1$

$$\lim_{x \to \infty} (-x)^n \cdot \exp(-x) = \lim_{x \to \infty} -1 \cdot x^n \cdot \frac{1}{\exp(x)} = -\lim_{x \to \infty} \frac{x^n}{\exp(x)} = 0$$

$$\Rightarrow \lim_{x \to -\infty} x^n \cdot \exp(x) = 0$$

$$\lim_{y \to 0+} y\,(\ln(y))^n = \lim_{x \to -\infty} \exp(x)\,(\ln(\exp(x)))^n = \lim_{x \to -\infty} \exp(x) \cdot x^n = 0$$

$\square$