ANA Ü8

3.) $f:\mathbb{R}^2\to\mathbb{R}^2$

$$\binom{\xi}{\eta}\mapsto\begin{pmatrix}\xi^2\eta\sin(\xi\eta)\\ \frac{\xi}{\xi^2+\eta^2+1}\end{pmatrix}$$

ges: partielle Ableitungen

$$\frac{\partial}{\partial x_1}f\binom{\xi}{\eta}=\lim_{s\to0}\frac{1}{s}\left(f\left(\binom{\xi}{\eta}+\binom{s}{0}\right)-f\binom{\xi}{\eta}\right)=\lim_{s\to0}\frac{1}{s}\left(\begin{pmatrix}(\xi+s)^2\eta\sin((\xi+s)\eta)\\ \frac{\xi+s}{(\xi+s)^2+\eta^2+1}\end{pmatrix}-\begin{pmatrix}\xi^2\eta\sin(\xi\eta)\\ \frac{\xi}{\xi^2+\eta^2+1}\end{pmatrix}\right)$$

$$\lim_{s\to0}\frac{(\xi+s)^2\eta\sin((\xi+s)\eta)-\xi^2\eta\sin(\xi\eta)}{s}=\lim_{s\to0}\eta\left(2(\xi+s)\cdot\sin((\xi+s)\eta)+(\xi+s)^2\cos((\xi+s)\eta)\cdot\eta\right)$$

$$=\eta(2\xi\sin(\xi\eta)+\xi^2\eta\cos(\xi\eta))=\xi\eta(2\sin(\xi\eta)+\xi\eta\cos(\xi\eta))$$

$$\lim_{s\to0}\left(\frac{(\xi+s)(\xi^2+\eta^2+1)-\xi((\xi+s)^2+\eta^2+1)}{s((\xi+s)^2+\eta^2+1)(\xi^2+\eta^2+1)}\right)=\lim_{s\to0}\frac{(\xi^2+\eta^2+1)-\xi(2(\xi+s))}{(\xi^2+\eta^2+1)((\xi+s)^2+\eta^2+1)+s(2(\xi+s))}$$

$$=\frac{\xi^2+\eta^2+1-2\xi^2}{(\xi^2+\eta^2+1)(\xi^2+\eta^2+1)}=\frac{-\xi^2+\eta^2+1}{(\xi^2+\eta^2+1)^2}$$

$$\Rightarrow\frac{\partial}{\partial x_1}f\binom{\xi}{\eta}=\begin{pmatrix}\xi\eta(2\sin(\xi\eta)+\xi\eta\cos(\xi\eta))\\ \frac{-\xi^2+\eta^2+1}{(\xi^2+\eta^2+1)^2}\end{pmatrix}$$

$$\frac{\partial}{\partial x_2}f\binom{\xi}{\eta}=\lim_{s\to0}\frac{1}{s}\left(f\left(\binom{\xi}{\eta}+\binom{0}{s}\right)-f\binom{\xi}{\eta}\right)=\lim_{s\to0}\frac{1}{s}\left(\begin{pmatrix}\xi^2(\eta+s)\sin(\xi(\eta+s))\\ \frac{\xi}{\xi^2+(\eta+s)^2+1}\end{pmatrix}-\begin{pmatrix}\xi^2\eta\sin(\xi\eta)\\ \frac{\xi}{\xi^2+\eta^2+1}\end{pmatrix}\right)$$

$$\lim_{s\to0}\frac{\xi^2(\eta+s)\sin(\xi(\eta+s))-\xi^2\eta(\sin(\xi\eta))}{s}=\lim_{s\to0}\xi^2\left(\sin(\xi(\eta+s))+(\eta+s)\cos(\xi(\eta+s))\cdot\xi\right)$$

$$=\xi^2(\sin(\xi\eta)+\xi\eta\cos(\xi\eta))$$

$$\lim_{s\to0}\frac{1}{s}\left(\frac{\xi(\xi^2+\eta^2+1)-\xi(\xi^2+(\eta+s)^2+1)}{(\xi^2+(\eta+s)^2+1)(\xi^2+\eta^2+1)}\right)=\lim_{s\to0}\frac{-\xi(2(\eta+s))}{(\xi^2+\eta^2+1)(2(\eta+s))}=\frac{-2\xi\eta}{(\xi^2+\eta^2+1)2\eta}=\frac{\xi}{\xi^2+\eta^2+1}$$

$$\Rightarrow\frac{\partial}{\partial x_2}f\binom{\xi}{\eta}=\begin{pmatrix}\xi^2(\sin(\xi\eta)+\xi\eta\cos(\xi\eta))\\ \frac{\xi}{\xi^2+\eta^2+1}\end{pmatrix}$$

ges: Matrixdarstellung von $df(x)$

$$\begin{pmatrix}\xi\eta(2\sin(\xi\eta)+\xi\eta\cos(\xi\eta)) & \xi^2(\sin(\xi\eta)+\xi\eta\cos(\xi\eta))\\ -\frac{\xi^2+\eta^2+1}{(\xi^2+\eta^2+1)^2} & \frac{\xi}{\xi^2+\eta^2+1}\end{pmatrix}$$

ges: $\frac{\partial f}{\partial v}(x)$ für $v=\binom{1}{1}$

$$\frac{\partial f}{\partial v}(x)=\begin{pmatrix}\xi\eta(2\sin(\xi\eta)+\xi\eta\cos(\xi\eta))+\xi^2(\sin(\xi\eta)+\xi\eta\cos(\xi\eta))\\ -\frac{\xi^2+\eta^2+1}{(\xi^2+\eta^2+1)^2}+\frac{\xi}{\xi^2+\eta^2+1}\end{pmatrix}$$

für $v=\binom{1}{-1}$

$$\begin{pmatrix}\xi\eta(2\sin(\xi\eta)+\xi\eta\cos(\xi\eta))-\xi^2(\sin(\xi\eta)+\xi\eta\cos(\xi\eta))\\ -\frac{\xi^2+\eta^2+1}{(\xi^2+\eta^2+1)^2}-\frac{\xi^2}{\xi^2+\eta^2+1}\end{pmatrix}$$