MAS Ü11

5.) Poissonverteilung $P(\lambda)$ $\qquad \mathbb{P}(\{n\}) = \frac{\lambda^n}{n!} e^{-\lambda}$

$$\mathbb{E}(X) = \sum_{n=0}^{\infty} n \cdot \frac{\lambda^n}{n!} \cdot e^{-\lambda} = e^{-\lambda} \sum_{n=1}^{\infty} n \cdot \frac{\lambda^n}{n!} = e^{-\lambda} \lambda \sum_{n=1}^{\infty} \frac{\lambda^{n-1}}{(n-1)!} =$$

$$= e^{-\lambda} \lambda \cdot \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} = e^{-\lambda} \lambda \cdot e^{\lambda} = \lambda$$

$$\mathbb{E}(X^2) = \sum_{n=0}^{\infty} n^2 \cdot \frac{\lambda^n}{n!} e^{-\lambda} = e^{-\lambda} \sum_{n=1}^{\infty} n \cdot \frac{\lambda^n}{(n-1)!} = e^{-\lambda} \left(\sum_{n=1}^{\infty} (n-1) \frac{\lambda^n}{(n-1)!} + \sum_{n=1}^{\infty} \frac{\lambda^n}{(n-1)!} \right)$$

$$= e^{-\lambda} \sum_{n=2}^{\infty} \frac{\lambda^n}{(n-2)!} + e^{-\lambda} \sum_{n=1}^{\infty} \frac{\lambda^n}{(n-1)!} = e^{-\lambda} \lambda^2 \sum_{n=2}^{\infty} \frac{\lambda^{n-2}}{(n-2)!} + e^{-\lambda} \lambda \sum_{n=1}^{\infty} \frac{\lambda^{n-1}}{(n-1)!}$$

$$= \lambda^2 + \lambda$$

$$\mathbb{V}(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \lambda^2 + \lambda - \lambda^2 = \lambda$$