ANA Ü3

7.) $f(x) = x^3 - \frac{48}{x} \quad , x \neq 0$

- Nullstellen

$f(x) = 0 \iff x^3 - \frac{48}{x} = 0 \iff x^3 = \frac{48}{x} \iff x^4 = 48 \iff x = \sqrt[4]{2^4 \cdot 3}$

$\iff x = \pm 2 \cdot \sqrt[4]{3}$

$\Rightarrow$ Nullstellen bei $2\sqrt[4]{3}$, $-2\sqrt[4]{3}$

- Extrema und Wendepunkte

$f'(x) = 3x^2 - 48 \cdot (-1) \cdot \frac{1}{x^2} = 3x^2 + \frac{48}{x^2} = \frac{3x^4 + 48}{x^2}$

$f'(x) = 0 \iff 3x^4 + 48 = 0 \iff x^4 = \frac{48}{3} \iff x = \sqrt[4]{16} \iff x = \sqrt[4]{2^4}$

$\Rightarrow$ Extremum (?) bei $2, -2$

$f''(x) = 6x + (-2) \cdot 48 \cdot \frac{1}{x^3} = 6x - 96 \cdot \frac{1}{x^3}$

$f''(2) = 12 - 96 \cdot \frac{1}{8} = 12 - 12 = 0 \quad \Rightarrow$ Wendepunkt

$f''(-2) = -12 - 96 \cdot \frac{1}{-8} = -12 + 12 = 0 \quad \Rightarrow$ Wendepunkt

$\lim\limits_{x \to 0-} f(x) = \lim\limits_{x \to 0-} x^3 - \frac{48}{x} = \lim\limits_{x \to 0-} (x^3) - 48 \cdot \lim\limits_{x \to 0-} (\frac{1}{x}) = -48 \cdot \lim\limits_{x \to 0-} \frac{1}{x} = \infty$

$\lim\limits_{x \to 0+} f(x) = \lim\limits_{x \to 0+} x^3 - \frac{48}{x} = \lim\limits_{x \to 0+} (x^3) - 48 \cdot \lim\limits_{x \to 0+} (\frac{1}{x}) = -48 \cdot \lim\limits_{x \to 0+} \frac{1}{x} = -\infty$

$\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} x^3 - \frac{48}{x} = \lim\limits_{x \to \infty} (x^3) - 48 \cdot \lim\limits_{x \to \infty} \frac{1}{x} = \lim\limits_{x \to \infty} x^3 = \infty$

$\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} x^3 - \frac{48}{x} = \lim\limits_{x \to -\infty} (x^3) - 48 \cdot \lim\limits_{x \to -\infty} \frac{1}{x} = \lim\limits_{x \to -\infty} x^3 = -\infty$

- monoton wachsend und fallend

$f'(x) > 0 \iff 3x^2 + \frac{48}{x^2} > 0 \iff 3x^2 > \frac{48}{x^2} \iff 3x^4 > 48$

$\iff x^4 > 16 \iff x > 2$ oder $x < -2$

$f'(x) < 0 \iff 3x^2 + \frac{48}{x^2} < 0 \iff 3x^2 < \frac{48}{x^2} \iff 3x^4 < 48$

$\iff x^4 < 16 \iff -2 < x < 2$

$\Rightarrow$ auf $(-\infty, -2)$ streng monoton $\nearrow$, auf $(-2, 2)$ streng monoton $\searrow$

und auf $(2, +\infty)$ streng monoton $\nearrow$