LINAG Ü12

9.10.3 $\quad A = \begin{pmatrix} 0 & i & 2i \\ -i & 0 & -i \\ -2i & i & 0 \end{pmatrix} \in \mathbb{C}^{3\times 3}$

ges: $P \in GL_3(\mathbb{C})$ mit $B := P^T A P$ ist eine zu A kongruente Matrix in Normalform

$$\left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 0 & i & 2i \\ -i & 0 & -i \\ -2i & i & 0 \end{array}\right) \overset{s}{\rightsquigarrow} \overset{-2\cdot II}{\left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \\ \hline 0 & i & 0 \\ -i & 0 & -i \\ -2i & i & -2i \end{array}\right)} \overset{z}{\rightsquigarrow} \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \\ \hline 0 & i & 0 \\ -i & 0 & -i \\ 0 & i & 0 \end{array}\right) \overset{s}{\rightsquigarrow} \overset{-I}{\left(\begin{array}{ccc} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \\ \hline 0 & i & 0 \\ -i & 0 & 0 \\ 0 & i & 0 \end{array}\right)} \overset{z}{\rightsquigarrow} \left(\begin{array}{ccc} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \\ \hline 0 & i & 0 \\ -i & 0 & 0 \\ 0 & 0 & 0 \end{array}\right) \overset{s}{\rightsquigarrow} \overset{\cdot i}{\left(\begin{array}{ccc} 1 & 0 & -1 \\ 0 & i & -2 \\ 0 & 0 & 1 \\ \hline 0 & -1 & 0 \\ -i & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)} \overset{z}{\rightsquigarrow} \left(\begin{array}{ccc} 1 & 0 & -1 \\ 0 & i & -2 \\ 0 & 0 & 1 \\ \hline 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$P = \begin{pmatrix} 1 & 0 & -1 \\ 0 & i & -2 \\ 0 & 0 & 1 \end{pmatrix} \qquad P^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & i & 0 \\ -1 & -2 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \operatorname{diag}(\square, 0)$$

ges: $Q \in GL_3(\mathbb{C})$ mit $C := \overline{Q}^T A Q$ ist eine zu A hermitesch kongruente Matrix in Normalform

$$\left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 0 & i & 2i \\ -i & 0 & -i \\ -2i & i & 0 \end{array}\right) \overset{s}{\rightsquigarrow} \overset{-2II}{\left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \\ \hline 0 & i & 0 \\ -i & 0 & -i \\ -2i & i & -2i \end{array}\right)} \overset{z}{\rightsquigarrow} \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \\ \hline 0 & i & 0 \\ -i & 0 & -i \\ 0 & i & 0 \end{array}\right) \overset{s}{\rightsquigarrow} \overset{-I}{\left(\begin{array}{ccc} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \\ \hline 0 & i & 0 \\ -i & 0 & 0 \\ 0 & i & 0 \end{array}\right)} \overset{z}{\rightsquigarrow} \left(\begin{array}{ccc} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \\ \hline 0 & i & 0 \\ -i & 0 & 0 \\ 0 & 0 & 0 \end{array}\right) \overset{s}{\rightsquigarrow} \overset{+(-i)II}{\left(\begin{array}{ccc} 1 & 0 & -1 \\ -i & 1 & -2 \\ 0 & 0 & 1 \\ \hline 1 & i & 0 \\ -i & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)} \overset{z}{\rightsquigarrow} \left(\begin{array}{ccc} 1 & 0 & -1 \\ -i & 1 & -2 \\ 0 & 0 & 1 \\ \hline 2 & i & 0 \\ -i & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\overset{s}{\rightsquigarrow} \overset{-\frac{1}{2}i\,I}{\left(\begin{array}{ccc} 1 & -\frac{1}{2}i & -1 \\ -i & \frac{1}{2} & -2 \\ 0 & 0 & 1 \\ \hline 2 & 0 & 0 \\ -i & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{array}\right)} \overset{z}{\rightsquigarrow} \left(\begin{array}{ccc} 1 & -\frac{1}{2}i & -1 \\ -i & \frac{1}{2} & -2 \\ 0 & 0 & 1 \\ \hline 2 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{array}\right) \overset{s}{\rightsquigarrow} \overset{\cdot\frac{1}{\sqrt{2}}}{\left(\begin{array}{ccc} \frac{1}{\sqrt{2}} & -\frac{1}{2}i & -1 \\ -\frac{i}{\sqrt{2}} & \frac{1}{2} & -2 \\ 0 & 0 & 1 \\ \hline \frac{2}{\sqrt{2}} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{array}\right)} \overset{z}{\rightsquigarrow} \left(\begin{array}{ccc} \frac{1}{\sqrt{2}} & -\frac{1}{2}i & -1 \\ -\frac{i}{\sqrt{2}} & \frac{1}{2} & -2 \\ 0 & 0 & 1 \\ \hline 1 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{array}\right) \overset{s}{\rightsquigarrow} \overset{\cdot i\sqrt{2}}{\left(\begin{array}{ccc} \frac{1}{\sqrt{2}} & \frac{\sqrt{2}}{2} & -1 \\ -\frac{i}{\sqrt{2}} & i\frac{\sqrt{2}}{2} & -2 \\ 0 & 0 & 1 \\ \hline 1 & 0 & 0 \\ 0 & -\frac{1}{2}i\sqrt{2} & 0 \\ 0 & 0 & 0 \end{array}\right)} \overset{z}{\rightsquigarrow} \left(\begin{array}{ccc} \frac{1}{\sqrt{2}} & \frac{\sqrt{2}}{2} & -1 \\ -\frac{i}{\sqrt{2}} & i\frac{\sqrt{2}}{2} & -2 \\ 0 & 0 & 1 \\ \hline 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$Q = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{\sqrt{2}}{2} & -1 \\ -i\frac{1}{\sqrt{2}} & i\frac{\sqrt{2}}{2} & -2 \\ 0 & 0 & 1 \end{pmatrix} \qquad \overline{Q}^T = \begin{pmatrix} \frac{1}{\sqrt{2}} & +i\frac{1}{\sqrt{2}} & 0 \\ \frac{\sqrt{2}}{2} & -i\frac{\sqrt{2}}{2} & 0 \\ -1 & -2 & 1 \end{pmatrix} \qquad C = \operatorname{diag}(1, -1, 0)$$