ANA Ü5

5.) $m, n \in \mathbb{Z}$ ges: $\int_0^{2\pi} \exp(i\cdot n\cdot t)\cdot \exp(-i\cdot m\cdot t)\,dt$

$$\int \exp(int)\cdot\exp(-imt)\,dt = \int \exp(it\cdot(n-m))\,dt$$

$m \neq n$: $\left[\; u = it(n-m) \quad \frac{du}{dt} = i(n-m) \qquad dt = \frac{1}{i(n-m)}\,du\right.$

$$= \int \exp(u)\cdot\frac{1}{i(n-m)}\,du = \frac{1}{i(n-m)}\cdot\int\exp(u)\,du = \frac{1}{i(n-m)}\cdot\exp(u) = \frac{\exp(i\cdot t(n-m))}{i(n-m)}$$

$$\int_0^{2\pi}\exp(int)\cdot\exp(-imt)\,dt = \frac{\exp(i\cdot 2\pi(n-m))}{i(n-m)} - \frac{\exp(i\cdot 0\cdot(n-m))}{i(n-m)}$$

$$= \frac{1}{i(n-m)} - \frac{1}{i(n-m)} = 0$$

$m = n$: $\int\exp(it\cdot(n-m))\,dt = \int\exp(0)\,dt = \exp(0) = 1$

$$\int_0^{2\pi}\sin(n\cdot t)\cdot\sin(m\cdot t)\,dt = \int_0^{2\pi}\frac{\exp(int)-\exp(-int)}{2i}\cdot\frac{\exp(imt)-\exp(-imt)}{2i}\,dt$$

$$= \int_0^{2\pi}\frac{1}{-4}\cdot(\exp(int)-\exp(-int))\cdot(\exp(imt)-\exp(-imt)))\,dt$$

$$= -\frac{1}{4}\left(\int_0^{2\pi}\exp(int)\exp(imt) - \int_0^{2\pi}\exp(int)\exp(-imt) - \int_0^{2\pi}\exp(imt)\exp(-int) + \int_0^{2\pi}\exp(-int)\exp(-imt)\right)$$

$m \neq n$: $-\frac{1}{4}\cdot(0-0-0+0) = 0$

$m = n$: $-\frac{1}{4}\cdot\left(\int_0^{2\pi}\exp(it(2n))\,dt - 1 - 1 + \int_0^{2\pi}\exp(-it(2n))\,dt\right)$

$\int\exp(it(2n))\,dt = \quad u = 2itn \quad \frac{du}{dt} = 2in \quad dt = \frac{1}{2in}\,du$

$= \int\exp(u)\frac{1}{2in}\,du = \frac{1}{2in}\exp(u) = \frac{1}{2in}\exp(2itn)$

$\int_0^{2\pi}\exp(2itn)\,dt = \frac{1}{2in}\cdot\exp(2i2\pi n) - \frac{1}{2in}\exp(2i0n) = 0$

analog für $\int_0^{2\pi}\exp(-it2n)\,dt$

$$= -\frac{1}{4}\cdot(0-1-1+0) = -\frac{1}{4}\cdot(-2) = \frac{1}{2}$$

$$\int_0^{2\pi}\cos(n\cdot t)\cdot\cos(m\cdot t)\,dt = \int_0^{2\pi}\frac{\exp(int)+\exp(-int)}{2}\cdot\frac{\exp(imt)+\exp(-imt)}{2}\,dt$$

$$= \frac{1}{4}\cdot\int_0^{2\pi}\exp(int)\exp(imt)+\exp(int)\exp(-imt)+\exp(-int)\exp(imt)+\exp(-int)\exp(-imt)\,dt$$

$m \neq n$: $\frac{1}{4}\cdot(0+0+0+0) = 0$ $\qquad$ $m = n$: $\frac{1}{4}\cdot(0+1+1+0) = \frac{1}{2}$

''

ANA Ü5

5.) ... $\int_0^{2\pi} \sin(nt)\cdot\cos(mt)\,dt = \int_0^{2\pi} \frac{\exp(int)-\exp(-int)}{2i}\cdot\frac{\exp(imt)+\exp(-imt)}{2}\,dt$

$= \frac{1}{4i}\int_0^{2\pi} \exp(int)\exp(imt)+\exp(int)\exp(-imt)-\exp(-int)\cdot\exp(imt)-\exp(-int)\exp(-imt)\,dt$

$m\neq n: \frac{1}{4i}\cdot(0)=0$ $\qquad$ $m=n: \frac{1}{4i}(0+1-1-0)=0$