LINAG Ü13

11.5.7. $\mathbb{C}^{3\times 1}$ mit kanonischem unitären Skalarprodukt $a=\begin{pmatrix}1\\i\\0\end{pmatrix}$

U ... zu a orthogonaler UR

a) ges: Gleichung von U und Orthonormalbasis von U

$U = a^\perp = \left\{\begin{pmatrix}x\\y\\z\end{pmatrix} \in \mathbb{C}^{3\times 1} : \begin{pmatrix}1\\i\\0\end{pmatrix}\cdot\begin{pmatrix}x\\y\\z\end{pmatrix} = x - iy = 0\right\} \Rightarrow x = iy$ ... Gleichung von U

$\overline{b}_1 := \begin{pmatrix}1\\-i\\0\end{pmatrix} \in U$, da $1 + i\cdot i = 1 - 1 = 0$ $\quad \overline{b}_2 := \begin{pmatrix}0\\0\\1\end{pmatrix} \in U$, da $0 - i\cdot 0 = 0$

$(\overline{b}_1, \overline{b}_2)$ sind offensichtlich l. u.

$\hat{b}_1 := \overline{b}_1 \qquad \hat{b}_2 := \overline{b}_2 - \frac{\hat{b}_1\cdot\overline{b}_2}{\hat{b}_1\cdot\hat{b}_1}\hat{b}_1 = \overline{b}_2 - \frac{0}{1-i\cdot i}\hat{b}_1 = \overline{b}_2 - \frac{0}{2}\hat{b}_1 = \overline{b}_2$

$b_1 := \frac{\hat{b}_1}{\|\hat{b}_1\|} = \frac{\hat{b}_1}{\sqrt{2}} = \left(\frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0\right)^T \qquad b_2 := \frac{\hat{b}_2}{\|\hat{b}_2\|} = \frac{\hat{b}_2}{\sqrt{1}} = \hat{b}_2 = (0, 0, 1)^T$

b) ges: $p: \mathbb{C}^{3\times 1} \to U$ ... Orthogonalprojektion in Form $\langle E^*, p(E)\rangle$

$e_1 = \begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}\frac{1}{2}\\\frac{i}{2}\\0\end{pmatrix} + \begin{pmatrix}\frac{1}{2}\\-\frac{i}{2}\\0\end{pmatrix} = \frac{1}{2}a + \frac{1}{\sqrt{2}}b_1 \qquad p(e_1) = \frac{1}{\sqrt{2}}b_1 = \left(\frac{1}{2}, -\frac{i}{2}, 0\right)^T$

$e_2 = \begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}-\frac{i}{2}\\\frac{1}{2}\\0\end{pmatrix} + \begin{pmatrix}\frac{i}{2}\\\frac{1}{2}\\0\end{pmatrix} = -\frac{i}{2}a + \frac{i}{\sqrt{2}}b_1 \qquad p(e_2) = \frac{i}{\sqrt{2}}b_1 = \left(\frac{i}{2}, \frac{1}{2}, 0\right)^T$

$e_3 = \begin{pmatrix}0\\0\\1\end{pmatrix} = b_2 \qquad p(e_3) = b_2 = (0, 0, 1)^T$

$\Rightarrow \langle E^*, p(E)\rangle = \begin{pmatrix}\frac{1}{2} & \frac{i}{2} & 0\\ -\frac{i}{2} & \frac{1}{2} & 0\\ 0 & 0 & 1\end{pmatrix}$

c) ges: $p(c)$ mit $c = \begin{pmatrix}i\\i\\i\end{pmatrix}$ $\quad$ ges: Länge von $c - p(c)$

$c = i\cdot e_1 + i\cdot e_2 + i\cdot e_3 \Rightarrow p(c) = i\cdot p(e_1) + i\cdot p(e_2) + i\,p(e_3) = i\begin{pmatrix}\frac{1}{2}\\-\frac{i}{2}\\0\end{pmatrix} + i\begin{pmatrix}\frac{i}{2}\\\frac{1}{2}\\0\end{pmatrix} + i\begin{pmatrix}0\\0\\1\end{pmatrix}$

$= \begin{pmatrix}\frac{i}{2} - \frac{1}{2}\\\frac{1}{2} + \frac{i}{2}\\i\end{pmatrix} = \begin{pmatrix}\frac{1}{2}(i-1)\\\frac{1}{2}(1+i)\\i\end{pmatrix}$

$\|c - p(c)\| = \left\|\begin{pmatrix}i - (\frac{i}{2} - \frac{1}{2})\\i - (\frac{1}{2} + \frac{i}{2})\\i - i\end{pmatrix}\right\| = \left\|\begin{pmatrix}\frac{i}{2} + \frac{1}{2}\\\frac{i}{2} - \frac{1}{2}\\0\end{pmatrix}\right\| = \sqrt{\left(\frac{i}{2}+\frac{1}{2}\right)\left(-\frac{i}{2}+\frac{1}{2}\right) + \left(\frac{i}{2}-\frac{1}{2}\right)\left(-\frac{i}{2}-\frac{1}{2}\right) + 0\cdot 0}$

$= \sqrt{\frac{1}{2} + \frac{1}{2}} = \sqrt{1} = 1$ ... Länge von $c - p(c)$