ANA Ü3

10.) $w \in \mathbb{C}$

$$\int (x^3+2x^2-3)e^{2x-4}dx = \int x^3\cdot e^{2x}\cdot\frac{1}{e^4} + 2x^2\cdot e^{2x}\cdot\frac{1}{e^4} - 3e^{2x}\cdot\frac{1}{e^4}dx$$

$$= \frac{1}{e^4}\cdot\int x^3\cdot e^{2x}dx + 2\cdot\frac{1}{e^4}\cdot\int x^2\cdot e^{2x}dx - 3\frac{1}{e^4}\cdot\int e^{2x}dx$$

$$\left[u=2x \quad x=\frac{u}{2} \quad \frac{du}{dx}=2 \quad dx=\frac{du}{2}\right]$$

$$= \frac{1}{e^4}\cdot\int\left(\frac{1}{2}\cdot u\right)^3\cdot e^u\cdot\frac{1}{2}du + 2\cdot\frac{1}{e^4}\cdot\int\left(\frac{1}{2}\cdot u\right)^2\cdot e^u\cdot\frac{1}{2}du - 3\frac{1}{e^4}\int e^u\cdot\frac{1}{2}du$$

$$= \frac{1}{16e^4}\int u^3\cdot e^u du + \frac{1}{4e^4}\int u^2\cdot e^u du - \frac{3}{2e^4}\int e^u du$$

$$\int u^3(e^u)'du = e^u\cdot u^3 - \int e^u\cdot 3u^2du = e^u\cdot u^3 - 3\cdot\left(\int (e^u)'\cdot u^2du\right)$$

$$= e^u\cdot u^3 - 3\cdot\left(e^u\cdot u^2 - \int (e^u)'2u\,du\right) = e^u\cdot u^3 - 3\cdot\left(e^u\cdot u^2 - 2\cdot\left(e^u\cdot u - \int e^u\cdot 1\,du\right)\right)$$

$$= e^u\cdot u^3 - 3\cdot(\underbrace{e^u\cdot u^2 - 2e^u\cdot u + 2e^u}_{=\int e^u\cdot u^2}) = e^u\cdot u^3 - 3e^u\cdot u^2 + 6e^u\cdot u - 6e^u$$

$$\frac{1}{16\cdot e^4}(e^u\cdot u^3 - 3e^u u^2 + 6e^u\cdot u - 6e^u) + \frac{1}{4\cdot e^4}(e^u\cdot u^2 - 2e^u\cdot u + 2e^u) - \frac{3}{2e^4}e^u$$

$$= \frac{1}{16\cdot e^4}(e^{2x}\cdot(2x)^3 - 3e^{2x}\cdot(2x)^2 + 6\cdot e^{2x}(2x) - 6e^{2x}) + \frac{1}{4e^4}(e^{2x}(2x)^2 - 2e^{2x}(2x) + 2e^{2x}) - \frac{3}{2e^4}e^{2x}$$

$$= \frac{1}{16e^4}\cdot(8e^{2x}\cdot x^3 - 12e^{2x}x^2 + 12e^{2x}x - 6e^{2x}) + \frac{1}{4e^4}(4e^{2x}x^2 - 4e^{2x}x + 2e^{2x}) - \frac{3}{2e^4}e^{2x}$$

$$= \frac{1}{2}e^{2x-4}x^3 - \frac{3}{4}e^{2x-4}x^2 + \frac{3}{4}e^{2x-4}x - \frac{3}{8}e^{2x-4} + e^{2x-4}x^2 - e^{2x-4}x + \frac{1}{2}e^{2x-4} - \frac{3}{2}e^{2x-4}$$

$$= \frac{4}{8}e^{2x-4}\cdot x^3 - \frac{6}{8}e^{2x-4}x^2 + \frac{6}{8}e^{2x-4}x - \frac{3}{8}e^{2x-4} + \frac{8}{8}e^{2x-4}x^2 - \frac{8}{8}e^{2x-4}x + \frac{4}{8}e^{2x-4} - \frac{12}{8}e^{2x-4}$$

$$= \frac{1}{8}\cdot e^{2x-4}(4x^3 - 6x^2 + 6x - 3 + 8x^2 - 8x + 4 - 12)$$

$$= \frac{1}{8}\cdot e^{2x-4}(4x^3 + 2x^2 - 2x - 11) + C$$

...

ANA Ü3

10.) ... $\int x^3 \cdot \exp(wx)\, dx$ $\qquad w \in \mathbb{C}$

Falls $w=0$ $\qquad \int x^3 \cdot e^0 dx = \int x^3 dx = \frac{x^4}{4} + C$

Sonst

$$u = w \cdot x \qquad x = \frac{u}{w} \qquad \frac{du}{dx} = w \qquad dx = \frac{du}{w}$$

$$\int x^3 \cdot e^{wx} dx = \int \left(\frac{u}{w}\right)^3 \cdot e^u \cdot \frac{1}{w} \cdot du = \frac{1}{w^4} \cdot \int u^3 \cdot e^u du$$

$$= \frac{1}{w^4} \cdot (e^u \cdot u^3 - 3e^u \cdot u^2 + 6e^u \cdot u - 6e^u)$$

$$= \frac{1}{w^4} \cdot (e^{wx} \cdot (wx)^3 - 3e^{wx}(wx)^2 + 6e^{wx}(wx) - 6e^{wx})$$

$$= \frac{e^{wx}}{w^4} (w^3 \cdot x^3 - 3w^2 \cdot x^2 + 6w \cdot x - 6) + C$$