ANA Ü6

10.) $A_1 : \mathbb{R}^2 \to \mathbb{R}^2 \qquad \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}$

$\|A_1\| = \sup \{ \|A_1 x\|_1 : x \in \mathbb{R}^2, \|x\|_\infty \leq 1 \}$

$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad \|x\|_\infty = \max(|x_1|, |x_2|) \leq 1$

$\|A_1 x\|_1 = \left\| \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right\|_1 = \left\| \begin{pmatrix} 2x_1 - x_2 \\ x_2 \end{pmatrix} \right\|_1 = |2x_1 - x_2| + |x_2|$

$x_1 \in [-1, 1], \quad 2x_1 \in [-2, 2], \quad 2x_1 - x_2 \in [-3, 3] \quad x_2 \in [-1, 1]$

$|2x_1 - x_2| + |x_2| \in [0, 4]$

$\Rightarrow \|A_1\| = 4$

$A_2 : \mathbb{R}^3 \to \mathbb{R} \qquad (-2 \;\; 0 \;\; 2)$

$\|A_2\| = \sup \{ \|A_2 x\|_1 : x \in \mathbb{R}^3, \|x\|_\infty \leq 1 \}$

$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad \|x\|_\infty = \max(|x_1|, |x_2|, |x_3|) \leq 1$

$\|A_2 x\|_1 = \left\| (-2 \;\; 0 \;\; 2) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \right\|_1 = \left\| \begin{pmatrix} -2x_1 \\ 0 \\ 2x_3 \end{pmatrix} \right\|_1 = |-2x_1| + |2x_3|$

$= 2|x_1| + 2|x_3| \in [0, 4]$

$\Rightarrow \|A_2\| = 4$

$A_3 : \mathbb{R} \to \mathbb{R}^3 \qquad \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix}$

$\|A_3\| = \sup \{ \|A_3 x\|_1 : x \in \mathbb{R}, \|x\|_\infty \leq 1 \}$

$x = (x_1) \quad \|x\|_\infty = \max(|x_1|) = |x_1| \leq 1$

$\|A_3 x\|_1 = \left\| \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix} (x_1) \right\|_1 = \left\| \begin{pmatrix} 5x_1 \\ 0 \\ x_1 \end{pmatrix} \right\|_1 = |5x_1| + |x_1| = 6|x_1| \in [0, 6]$

$\Rightarrow \|A_3\| = 6$