DGA Ü9

1)a)

$P = \{\binom{0}{3}, \binom{0}{8}, \binom{1}{5}, \binom{2}{0}, \binom{3}{2}, \binom{4}{6}, \binom{4}{9}, \binom{5}{3}, \binom{6}{0}, \binom{8}{5}, \binom{10}{7}, \binom{11}{3}, \binom{12}{5}\}$ $|P| = 13$

$P_l = \{\binom{0}{3}, \binom{0}{8}, \binom{1}{5}, \binom{2}{0}, \binom{3}{2}, \binom{4}{6}, \binom{4}{9}\}$ $P_r = \{\binom{5}{3}, \binom{6}{0}, \binom{8}{5}, \binom{10}{7}, \binom{11}{3}, \binom{12}{5}\}$

$\hookrightarrow P = \{\binom{0}{3}, \binom{0}{8}, \binom{1}{5}, \binom{2}{0}, \binom{3}{2}, \binom{4}{6}, \binom{4}{9}\}$ $|P| = 7$

$P_l = \{\binom{0}{3}, \binom{0}{8}, \binom{1}{5}, \binom{2}{0}\}$ $P_r = \{\binom{3}{2}, \binom{4}{6}, \binom{4}{9}\}$

$\hookrightarrow P = \{\binom{0}{3}, \binom{0}{8}, \binom{1}{5}, \binom{2}{0}\}$ $|P| = 4$

$P_l = \{\binom{0}{3}, \binom{0}{8}\}$ $P_r = \{\binom{1}{5}, \binom{2}{0}\}$

$\hookrightarrow P = \{\binom{0}{3}, \binom{0}{8}\}$ $|P| = 2 \longrightarrow HP = \{\binom{0}{3}, \binom{0}{8}\}$

$\hookrightarrow P = \{\binom{1}{5}, \binom{2}{0}\}$ $|P| = 2 \longrightarrow HP = \{\binom{1}{5}, \binom{2}{0}\}$

$\longrightarrow HP = \{\binom{0}{3}, \binom{0}{8}, \binom{1}{5}, \binom{2}{0}\}$

$\hookrightarrow P = \{\binom{3}{2}, \binom{4}{6}, \binom{4}{9}\}$ $|P| = 3 \longrightarrow HP = \{\binom{3}{2}, \binom{4}{6}, \binom{4}{9}\}$

$\longrightarrow HP = \{\binom{0}{3}, \binom{0}{8}, \binom{2}{0}, \binom{3}{2}, \binom{4}{6}, \binom{4}{9}\}$

$\hookrightarrow P = \{\binom{5}{3}, \binom{6}{0}, \binom{8}{5}, \binom{10}{7}, \binom{11}{3}, \binom{12}{5}\}$ $|P| = 6$

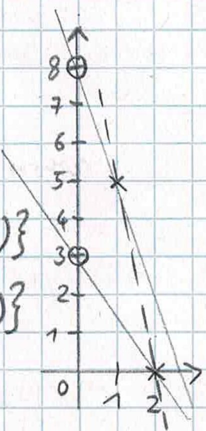
$P_l = \{\binom{5}{3}, \binom{6}{0}, \binom{8}{5}\}$ $P_r = \{\binom{10}{7}, \binom{11}{3}, \binom{12}{5}\}$

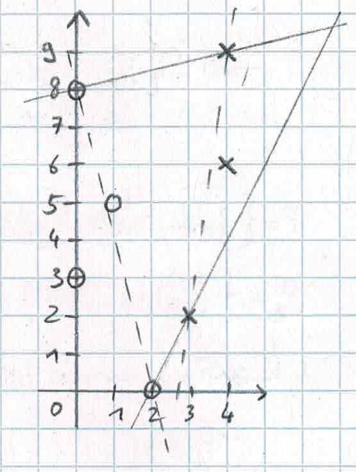
$\hookrightarrow P = \{\binom{5}{3}, \binom{6}{0}, \binom{8}{5}\}$ $|P| = 3 \longrightarrow HP = \{\binom{5}{3}, \binom{6}{0}, \binom{8}{5}\}$

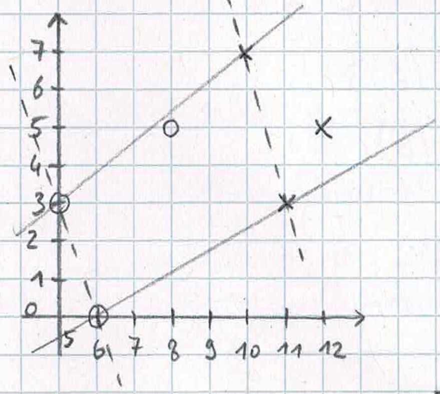
$\hookrightarrow P = \{\binom{10}{7}, \binom{11}{3}, \binom{12}{5}\}$ $|P| = 3 \longrightarrow HP = \{\binom{10}{7}, \binom{11}{3}, \binom{12}{5}\}$

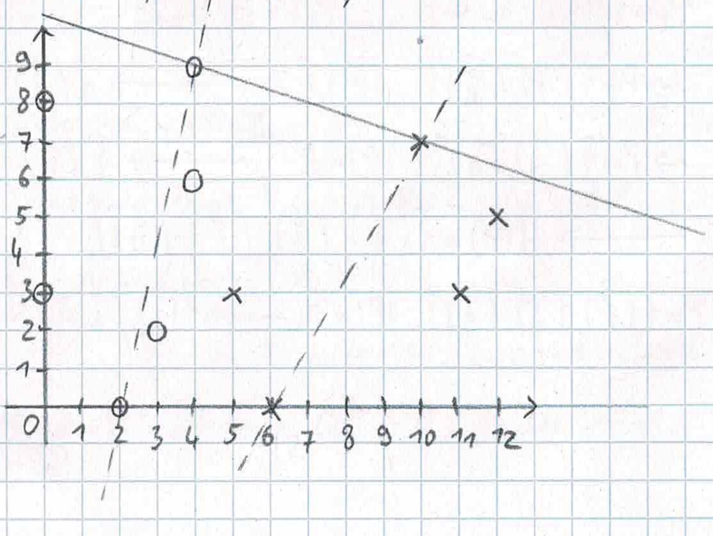
$\longrightarrow HP = \{\binom{5}{3}, \binom{6}{0}, \binom{10}{7}, \binom{11}{3}, \binom{12}{5}\}$

$\longrightarrow HP = \{\binom{0}{3}, \binom{0}{8}, \binom{2}{0}, \binom{4}{9}, \binom{6}{0}, \binom{10}{7}, \binom{11}{3}, \binom{12}{5}\}$





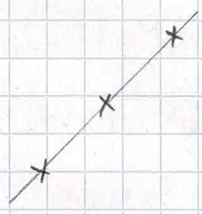




DGA Üg

1)b)

```
if (|P| < 3) {
    return P;
} else if (|P| == 3) {
    if ((P[3].y-P[2].y)(P[2].x-P[1].x)==(P[2].y-P[1].y)(P[3].x-P[2].x)) {
        return {min_x(P), max_x(P)};
    else {
        return P;
    }
}
```



hat offensichtlich konstanten Aufwand $\Rightarrow \Theta(1)$

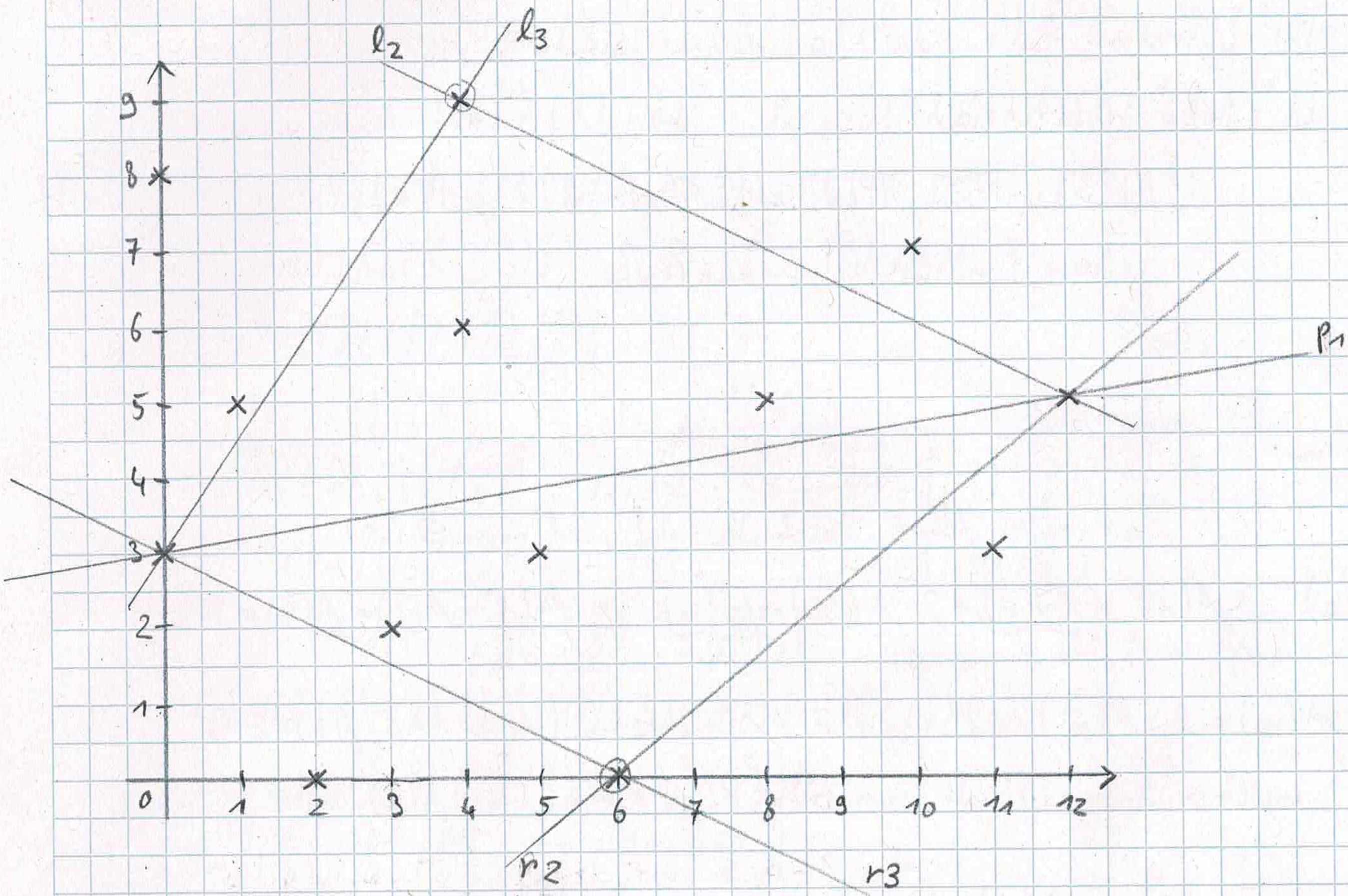
1)c) $A(n) = \underbrace{\Theta(n)}_{ii} + \underbrace{2 \cdot A(\frac{n}{2})}_{iii} + \underbrace{\Theta(n)}_{iv}$ $A(1) = A(2) = A(3) = \Theta(1)$

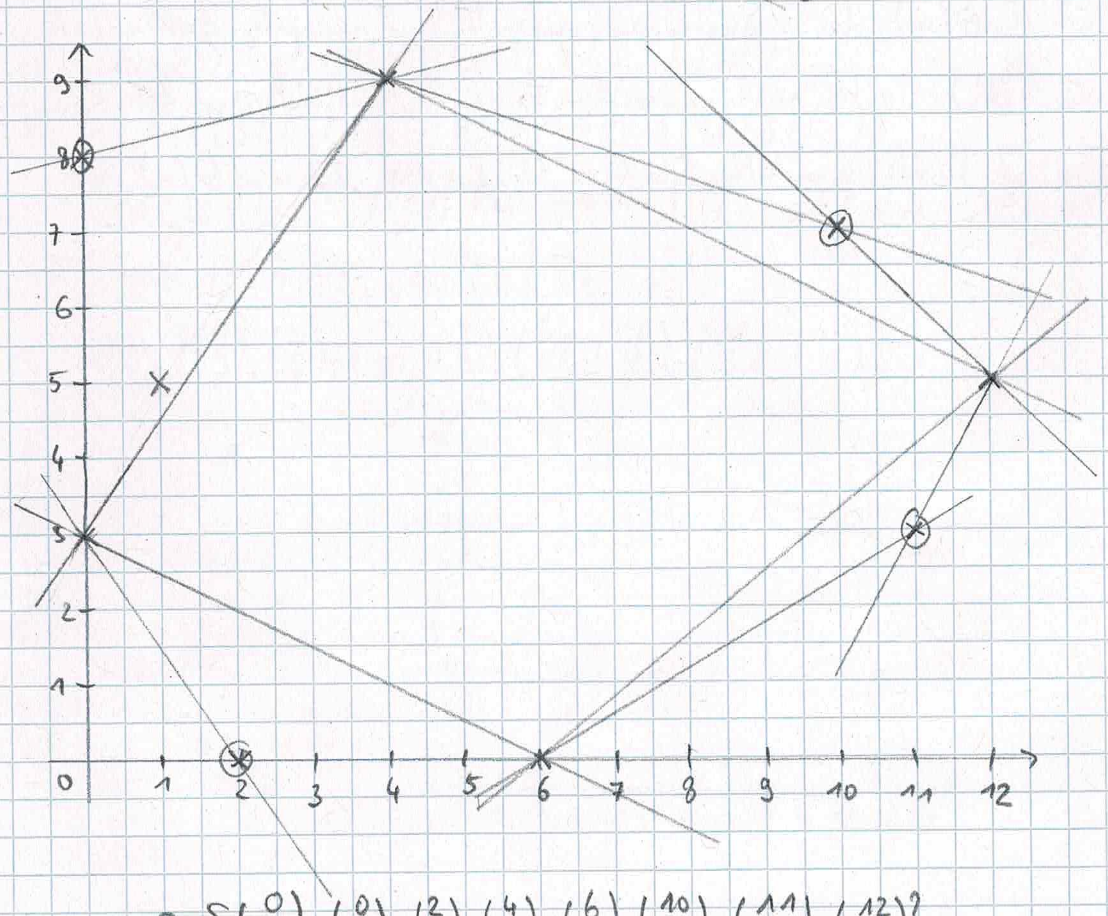
$A(n) = 2 \cdot A(\frac{n}{2}) + 2n$ Mit Mastertheorem $(a=2, b=2, f(n)=2n)$ ergibt sich aus $f(n) = 2n = \Theta(n^{\log_2 2}) = \Theta(n)$, dass $A(n) = \Theta(n \log(n))$.

iv hat Aufwand $\Theta(n)$, da Suche nach maximaler und minimaler y-Koordinate hat Aufwand $\Theta(n)$, Suche nach passenden Tangente hat Aufwand $\Theta(n)$. Überprüfen ob Punkte innerhalb Fläche liegen hat Aufwand $\Theta(n)$.

DGA Ü9

2a) $P = \{ \binom{0}{3}, \binom{0}{8}, \binom{1}{5}, \binom{2}{0}, \binom{3}{2}, \binom{4}{6}, \binom{4}{9}, \binom{5}{3}, \binom{6}{0}, \binom{8}{5}, \binom{10}{7}, \binom{11}{3}, \binom{12}{5} \}$

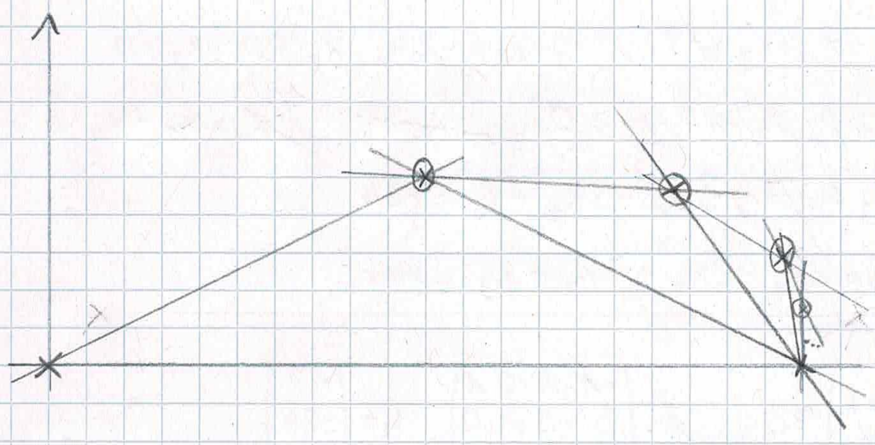




$\Rightarrow HP = \{ \binom{0}{3}, \binom{0}{8}, \binom{2}{0}, \binom{4}{9}, \binom{6}{0}, \binom{10}{7}, \binom{11}{3}, \binom{12}{5} \}$

DGA Ü9

2b) Idee: schlechtester Fall ist: es liegt kein Punkt innerhalb des Dreiecks und das für jeden Rekursionsschritt. Die Höhe der Dreiecke darf dabei nur so hoch sein, dass die x-Koordinate der Punkte nicht zu groß wird.



$A(n) = A(n-1) + n \qquad A(1) = 1$

$\Rightarrow A(n) = n + A(n-1) = n + n-1 + A(n-2) + \ldots + 1 = \frac{n(n+1)}{2} = O(n^2)$

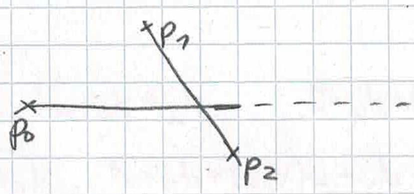
DGA Ü9

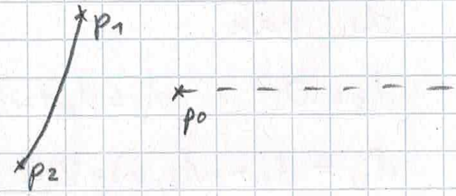
4) $p_0=(x_0,y_0)$ $\{p_i=(x_i,y_i): x_i \geq x_0, y_i=y_0\}$ $p_1, p_2 \in \mathbb{R}^2$

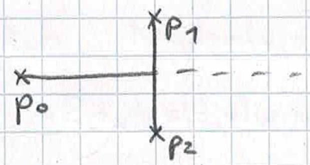
a) schneidet der rechte Strahl die Strecke $\overline{p_1 p_2}$ in $O(1)$

Idee: verwende Segment-Intersect aus VO mit "verkürzter" Strecke $\overline{p_0 p_3}$

für ein geeignetes $p_3 \in \mathbb{R}^2$





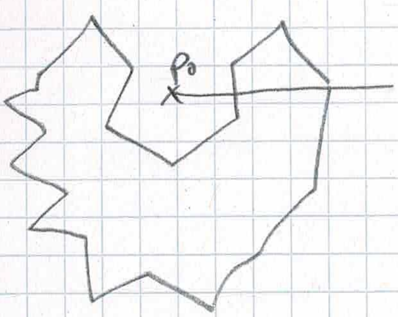


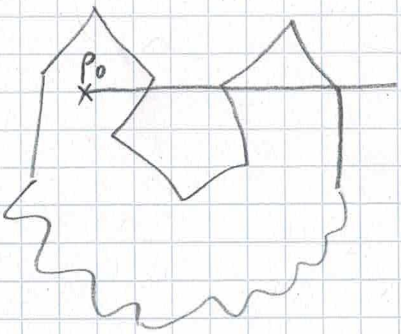
SEGMENT-INTERSECT$(p_0, (\max(x_1,x_2), y_0), p_1, p_2)$ hat $O(1)$

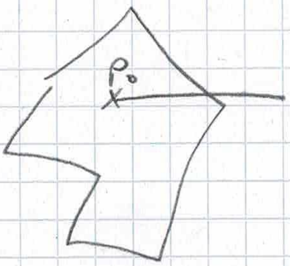
b) liegt p_0 im Inneren eines einfachen Polygons? in $\Theta(n)$

```
Algorithm(p0, edges=[(p1,p2),(p2,p3),...,(pn,p1)]) {
    cnt=0;
    for edge in edges {
        if INTERSECT(Strahl von p0, edge) {
            cnt++;
        }
    }
    return not(cnt%2);
}
```

Aufwand ist offenbar $\Theta(n)$

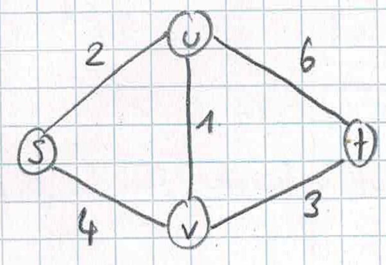






DGA Üg

6)



ges: kürzeste Weg von s nach t als Problem der linearen Optimierung

d_t max!

$d_s = 0$ $\quad d_s \leq d_u + w(u,s) = d_u + 2$ $\quad d_2 \leq d_v + w(v,s) = d_v + 4$

$d_u \leq d_s + w(s,u) = 2$ $\quad d_u \leq d_v + w(v,u) = d_v + 1$ $\quad d_u \leq d_t + w(t,u) = d_t + 6$

$d_v \leq d_s + w(s,v) = 4$ $\quad d_v \leq d_u + w(u,v) = d_u + 1$ $\quad d_v \leq d_t + w(t,v) = d_t + 3$

$d_t \leq d_u + w(u,t) = d_u + 6$ $\quad d_t \leq d_v + w(v,t) = d_v + 3.$