DGA Ü2

4.) n ... Wochen G_n ... gesund, nicht immun K_n ... krank I_n ... immun

$K_n = \frac{1}{p} \cdot G_{n-1}$ $\quad I_n = I_{n-1} + K_{n-1}$ $\quad G_n = G_{n-1} - K_n$

$I_1 = 0$ $\quad G_1 = p^N$

ges: explizite Formeln für K_n, I_n, G_n

$G_n = G_{n-1} - K_n = G_{n-1}(1 - \frac{1}{p}) = G_{n-2}(1 - \frac{1}{p})^2 = \ldots = G_1(1 - \frac{1}{p})^{n-1}$

$K_n = \frac{1}{p} \cdot G_{n-1} = \frac{1}{p} G_1 (1 - \frac{1}{p})^{n-2}$

$I_n = I_{n-1} + K_{n-1} = I_{n-2} + K_{n-2} + K_{n-1} = \ldots = I_1 + \sum_{i=1}^{n-1} K_{n-i} = I_1 + \sum_{i=1}^{n-1} \frac{1}{p} G_1 (1 - \frac{1}{p})^{n-i-2}$

$= I_1 + \frac{1}{p} G_1 \sum_{i=1}^{n-1} (1 - \frac{1}{p})^{n-i-2}$

Wenn $K_1 = 1$

folgt $G_n = p^N (1 - \frac{1}{p})^{n-1}$

$K_n = \frac{1}{p} \cdot p^N (1 - \frac{1}{p})^{n-2} = p^{N-1} (1 - \frac{1}{p})^{n-2}$

$I_n = \frac{1}{p} p^N \sum_{i=1}^{n-1} (1 - \frac{1}{p})^{n-i-2} = p^{N-1} (1 - \frac{1}{p})^{n-2} \sum_{i=1}^{n-1} (1 - \frac{1}{p})^{-i}$