DGA Ü3

3.) a) $T(n) = 9\,T(\frac{n}{3}) + n^2$ $a=9$ $b=3$ $f(n)=n^2$

$f(n) = n^2 = \Theta(n^2) = \Theta(n^{\log_3(9)})$

$\Rightarrow T(n) = \Theta(n^{\log_3(9)} \log(n)) = \Theta(n^2 \log(n))$

b) $T(n) = 8\,T(\frac{n}{2}) + n!$ $a=8$ $b=2$ $f(n)=n!$

$n^{\log_b a+\varepsilon} = n^{\log_2 8+\varepsilon} = O(n^{\log_2 8}) = O(n^3) = O(n!) = O(f(n))$

$a \cdot f(\frac{n}{b}) = 8 \cdot f(\frac{n}{2}) = 8 \cdot (\frac{n}{2})! \leq \frac{1}{2} n! = \frac{1}{2} \cdot f(n)$

also $f(n) = \Omega(n^{\log_b a+\varepsilon}) \wedge a f(\frac{n}{b}) \leq c f(n)$ für ein $c<1$

$\Rightarrow T(n) = \Theta(f(n)) = \Theta(n!)$

c) $T(n) = T(\frac{n}{5}) + n \log(n)$ $a=1$ $b=\frac{5}{4}$ $f(n)=n\log(n)$

$n^{\log_b(a+\varepsilon)} = n^{\log_{\frac{5}{4}}(1+\frac{1}{4})} = n = O(n \log(n))$

$a \cdot f(\frac{n}{b}) = 1 \cdot f(\frac{4}{5} n) = \frac{4}{5} n \log(\frac{4}{5} n) = \frac{4}{5} n (\log(n) + \log(\frac{4}{5}))$

$= \frac{4}{5} f(n) + \frac{4}{5} n \log(\frac{4}{5}) \approx \frac{4}{5} f(n) - 0{,}1785\, n \leq \frac{4}{5} f(n)$

$\Rightarrow T(n) = \Theta(n \log(n))$

d) $T(n) = 5\,T(\frac{n}{2}) + \log(n+1)$ $a=5$ $b=2$ $f(n)=\log(n+1)$

$n^{\log_b a-\varepsilon} = n^{\log_2(5-1)} = n^2$ $f(n) = \log(n+1) = O(n^2)$

$\Rightarrow T(n) = \Theta(n^{\log_2 5})$

e) $T(n) = T(\frac{8}{9} n) + n$ $a=1$ $b=\frac{9}{8}$ $f(n)=n$

$n^{\log_b(a+\varepsilon)} = n^{\log_{\frac{9}{8}}(1+\frac{1}{8})} = n = O(n) = O(f(n))$

$a \cdot f(\frac{n}{b}) = f(\frac{8}{9} n) = \frac{8}{9} n \leq \frac{8}{9} n = \frac{8}{9} f(n)$

$\Rightarrow T(n) = \Theta(f(n)) = \Theta(n)$

f) $T(n) = 11\,T(\frac{n}{3}) + n^{1,5}$ $a=11$ $b=3$ $f(n)=n^{1,5}$

$n^{\log_b(a-\varepsilon)} = n^{\log_3(11-2)} = n^2$ $f(n) = n^{1,5} = O(n^2)$

$\Rightarrow T(n) = \Theta(n^{\log_3 11})$