DGA Ü2

3.) $F_n = \begin{cases} 0 & , n=0 \\ 1 & , n=1 \\ 2F_{n-1} + 3F_{n-2} + n & , n>1 \end{cases}$

a) $F_n = 2F_{n-1} + 3F_{n-2} + n \qquad F_n - 2F_{n-1} - 3F_{n-2} = n$

Lösen wir zuerst $F_n - 2F_{n-1} - 3F_{n-2} = 0$

$\chi(\lambda) = \lambda^2 - 2\lambda - 3 = (\lambda+1)(\lambda-3) \quad \Rightarrow F_n^h := (-1)^n c + 3^n d$

Ansatz $F_n^p := an + b$

$F_n^p - 2F_{n-1}^p - 3F_{n-2}^p = an + b - 2(a(n-1)+b) - 3(a(n-2)+b)$

$= an + b - 2an + 2a - 2b - 3an + 6a - 3b = -4an + 8a - 4b = n$

$\Rightarrow -4a = 1 \quad \Rightarrow a = -\frac{1}{4} \qquad \Rightarrow 8a - 4b = 0 \quad \Rightarrow 4b = -2 \quad \Rightarrow b = -\frac{1}{2}$

$F_n^p = -\frac{1}{4}n - \frac{1}{2}$

$F_n = F_n^h + F_n^p = (-1)^n c + 3^n d - \frac{1}{4}n - \frac{1}{2}$

$F_0 = c + d - \frac{1}{2} = 0 \qquad \Leftrightarrow c = \frac{1}{2} - d$

$F_1 = -c + 3d - \frac{1}{4} - \frac{1}{2} = -c + 3d - \frac{3}{4} = 1 \qquad \Leftrightarrow -\frac{1}{2} + d + 3d = \frac{7}{4}$

$\Leftrightarrow 4d = \frac{9}{4} \qquad \Leftrightarrow d = \frac{9}{16} \qquad \Rightarrow c = -\frac{1}{16}$

$\Rightarrow F_n = (-1)^{n+1}\frac{1}{16} + 3^n \frac{9}{16} - \frac{1}{4}n - \frac{1}{2}$

b) ges: Rekursion $A(n)$ für Anzahl Additionen des Algorithmus

$A(0) = 0 \qquad A(1) = 0 \qquad A(n) = A(n-1) + A(n-2) + 2$