DGA Ü3

5.) $M=(ab)_{10}=10a+b \qquad N=(cd)_{10}=10c+d$

$A=ac \qquad B=bd \qquad C=(a-b)(d-c)$

$\Rightarrow MN = (ab)_{10}\cdot(cd)_{10} = 100A+10A+10B+B+10C$

a) zz: $(ab)_{10}\cdot(cd)_{10} = 100A+10A+10B+B+10C$

$100A+10A+10B+B+10C = 100ac+10ac+10bd+bd+10(a-b)(d-c)$

$= 100ac+\cancel{10ac}+\cancel{10bd}+bd+10ad-\cancel{10ac}-\cancel{10bd}+10bc$

$= 10a(10c+d)+b(10c+d) = (10c+d)(10a+b) = (ab)_{10}\cdot(cd)_{10}$

b) ges: ähnlicher Algorithmus für n-stellige Zahlen

$M=10^{\lfloor\frac{n}{2}\rfloor}a+b \qquad N=10^{\lfloor\frac{n}{2}\rfloor}c+d$

$A=ac \qquad B=bd \qquad C=(a-b)(d-c)$

$\Rightarrow MN = 10^{2\lfloor\frac{n}{2}\rfloor}A+10^{\lfloor\frac{n}{2}\rfloor}A+10^{\lfloor\frac{n}{2}\rfloor}B+B+10^{\lfloor\frac{n}{2}\rfloor}C$

$= 10^{2\lfloor\frac{n}{2}\rfloor}ac+\cancel{10^{\lfloor\frac{n}{2}\rfloor}ac}+\cancel{10^{\lfloor\frac{n}{2}\rfloor}bd}+bd+10^{\lfloor\frac{n}{2}\rfloor}(ad-\cancel{ac}-\cancel{bd}+bc)$

$= 10^{2\lfloor\frac{n}{2}\rfloor}ac+bd+10^{\lfloor\frac{n}{2}\rfloor}ad+10^{\lfloor\frac{n}{2}\rfloor}bc$

$= 10^{\lfloor\frac{n}{2}\rfloor}a(10^{\lfloor\frac{n}{2}\rfloor}c+d)+b(10^{\lfloor\frac{n}{2}\rfloor}c+d)$

$= (10^{\lfloor\frac{n}{2}\rfloor}a+b)(10^{\lfloor\frac{n}{2}\rfloor}c+d) = MN$

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Algorithm(M, N, n):
    if n == 1: return M * N;
    a = M // 10**(n/2);   b = M % 10**(n/2);
    c = N // 10**(n/2);   d = N % 10**(n/2);
    A = Algorithm(a, c);  B = Algorithm(b, d);
    C = Algorithm(a-b, d-c);
    return 10**n * A + 10**(n/2) * A + 10**(n/2) * B + B + 10**(n/2) * C;
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c) Angenommen $n=2^k, k\in\mathbb{N}$ $\quad T(n)$... Anzahl einstelliger Multiplikationen von zwei n-stelligen Zahlen. $T(1)=1 \qquad T(n)=3T(\frac{n}{2})+4 \qquad a=3 \quad b=2 \quad f(n)=4$

$n^{\log_b(a-\varepsilon)} = n^{\log_2(3-1)} = n \qquad f(n)=4=O(n) \Rightarrow T(n)=\Theta(n^{\log_2 3})$