IST ÜB

4) $X_1, \dots, X_n$ ... random sample from population with pdf $f_\theta(x) = \begin{cases} \frac{2x}{\theta^2}, & 0 < x < \theta \\ 0, & \text{otherwise} \end{cases}$

$\theta > 0$ ... unknown find minimal sufficial statistic

$$f_\theta(x) = \frac{2x}{\theta^2} \mathbb{1}_{(0,\theta)}(x)$$

$$f_\theta(x) = \prod_{i=1}^{n} f_\theta(x) = \frac{2^n}{\theta^{2n}} \mathbb{1}_{(0,\infty)}(\min_i x_i)\, \mathbb{1}_{(-\infty,\theta)}(\max_i x_i) \prod_{i=1}^{n} x_i$$

$$\frac{f_\theta(x)}{f_\theta(y)} = \frac{\mathbb{1}_{(0,\infty)}(\min_i x_i)\, \mathbb{1}_{(-\infty,\theta)}(\max_i x_i) \prod_{i=1}^{n} x_i}{\mathbb{1}_{(0,\infty)}(\min_i y_i)\, \mathbb{1}_{(-\infty,\theta)}(\max_i y_i) \prod_{i=1}^{n} y_i}$$ is constant as a function of θ iff

$$\mathbb{1}_{(-\infty,\theta)}(\max_i x_i) = \mathbb{1}_{(-\infty,\theta)}(\max_i y_i) \quad \rightarrow T(x) = \max_i x_i \text{ ... minimal sufficient statistic}$$