

1st Ü7

5) show  $\text{Poi}(\lambda)$  with unknown  $\lambda > 0$  belongs to the exponential family

$$f_{\lambda}(k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

in order to belong to the exponential family we need the following form

$$f_{\lambda}(k) = h(k) c(\lambda) e^{w(\lambda) + t(k)} \quad h(k), c(\lambda) \geq 0$$

The following transformations achieve this form

$$f_{\lambda}(k) = \frac{\lambda^k}{k!} e^{-\lambda} = \frac{1}{k!} e^{\ln(\lambda^k)} e^{-\lambda} = \frac{1}{k!} e^{-\lambda} e^{k \ln(\lambda)}$$

$$\Rightarrow h(k) = \frac{1}{k!} \geq 0; \quad c(\lambda) = e^{-\lambda} \geq 0; \quad w(\lambda) = \ln(\lambda); \quad t(k) = k$$

$\Rightarrow \text{Poi}(\lambda)$  belongs to the exponential family