IST Ü8

2) $X_1, \dots, X_n$ ... random sample from population with pdf

$$f_\theta(x) = \begin{cases} \frac{\theta}{x^2}, & \theta \le x \\ 0, & \text{otherwise} \end{cases} \qquad \theta > 0 \text{ ... unknown}$$

Factorization Theorem $T(X)$ ... sufficient statistic for θ iff

$$\exists g(t,\theta)\ \exists h(x): f_\theta(x) = g(T(x),\theta)\cdot h(x)$$

pdf of $X = (X_1, \dots, X_n)$ is $\prod_{i=1}^n f_\theta(x_i)$ as they are i.i.d

$$f_\theta(x) = \prod_{i=1}^n f_\theta(x_i) = \prod_{i=1}^n \frac{\theta}{x_i^2} 1\!\!1_{[\theta,\infty)}(x_i) = 1\!\!1_{[\theta,\infty)}(\min_i x_i)\,\theta^n \prod_{i=1}^n \frac{1}{x_i^2}$$

$f_\theta(x) = g(T(x),\theta)\cdot h(x)$ for $h(x) = \prod_{i=1}^n \frac{1}{x_i^2}$

$T(x) = \min_i x_i$

$g(T(x),\theta) = \theta^n 1\!\!1_{[\theta,\infty)}(\min_i x_i)$

$\Rightarrow T(x)$ is a sufficient statistic for θ