

1st Ü3

2) random points on a line

X_1, X_2, X_3 independently, uniformly chosen on line of length L
probability X_2 lies between X_1 and X_3 ?

$$X_1, X_2, X_3 \sim U(0, L) \text{ iid}$$

$$\begin{aligned} P(X_1 < X_2 < X_3) &= \int_0^L f(x_1) P(X_1 < X_2 < X_3) dx_1 = \int_0^L f(x_1) \int_{x_1}^L f(x_2) P(X_1 < x_2 < X_3) dx_2 dx_1 \\ &= \int_0^L f(x_1) \int_{x_1}^L f(x_2) \int_{x_2}^L f(x_3) dx_3 dx_2 dx_1 = \int_0^L \frac{1}{L} \int_{x_1}^L \frac{1}{L} \int_{x_2}^L \frac{1}{L} dx_3 dx_2 dx_1 = \frac{1}{L^3} \int_0^L \int_{x_1}^L \int_{x_2}^L 1 dx_3 dx_2 dx_1 \\ &= \frac{1}{L^3} \int_0^L \int_{x_1}^L (L - x_2) dx_2 dx_1 = \frac{1}{L^3} \int_0^L L^2 - \frac{L^2}{2} - Lx_1 + \frac{x_1^2}{2} dx_1 \\ &= \frac{1}{L^3} \left(\frac{L^3}{2} - \frac{L^3}{2} + \frac{L^3}{6} \right) = \frac{1}{6} \end{aligned}$$

$$P(X_3 < X_2 < X_1) = \frac{1}{6}$$

$$\Rightarrow P(X_2 \text{ lies between } X_1 \text{ and } X_3) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$