ALG Ü8

297) $t(x_1,\dots,x_n)$, $t'(x_1,\dots,x_n)$ ... Terme in fester Sprache L

$\mathcal{V}$ ... Varietät zu Sprache L $\qquad$ $F\in\mathcal{V}$ ... frei über $\underset{X}{\{b_1,\dots,b_n\}}$ in $\mathcal{V}$

(a) In F gilt $t(b_1,\dots,b_n)=t'(b_1,\dots,b_n)$

(b) $\forall C\in\mathcal{V}: C\models t\approx t'$

(c) $F\models t\approx t'$ $\qquad$ zz: (a) $\Leftrightarrow$ (b) $\Leftrightarrow$ (c)

(b) $\Rightarrow$ (c) klar, da $F\in\mathcal{V}$

(c) $\Rightarrow$ (a) $F\models t\approx t' \Rightarrow \forall f_1,\dots,f_n\in F: t(f_1,\dots,f_n)=t'(f_1,\dots,f_n)$

Da $\{b_1,\dots,b_n\}\subseteq F \Rightarrow t(b_1,\dots,b_n)=t'(b_1,\dots,b_n)$

(a) $\Rightarrow$ (b) Sei $C\in\mathcal{V}$ bel.

$$\begin{tikzcd} X \arrow[r, "id_X"] \arrow[dr, "j"'] & F \arrow[d, dashed, "\varphi"] \\ & C \end{tikzcd}$$

Sei $c_1,\dots,c_n\in C$ bel.

$j: X\to C$, $\quad b_i\mapsto c_i \ \forall i\in\{1,\dots,n\}$ $\Rightarrow \exists!\varphi: F\to C$ ... Hom. $j=\varphi\circ id_X=\varphi$

$t(c_1,\dots,c_n)=t(j(b_1),\dots,j(b_n))=t(\varphi(b_1),\dots,\varphi(b_n))\overset{*}{=}\varphi(t(b_1,\dots,b_n))=\dots$

$*$ folgt aus Prop. 2.3.3.21

$\dots=\varphi(t'(b_1,\dots,b_n))\overset{*}{=}t'(\varphi(b_1),\dots,\varphi(b_n))=t'(c_1,\dots,c_n)$

$\Rightarrow C\models t\approx t'$

$\square$