

1st Ü2

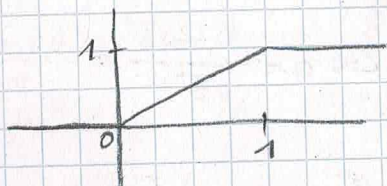
4) a) $Y \sim \exp(\lambda)$

$$f_Y(y) = \begin{cases} \lambda e^{-\lambda y} & , y \geq 0 \\ 0 & , \text{else} \end{cases}$$

$$\begin{aligned} P(Y > y) &= 1 - P(Y < y) = 1 - \int_{-\infty}^y f_Y(x) dx = 1 - \int_0^y \lambda e^{-\lambda x} dx \\ &= 1 - \lambda \left(-\frac{1}{\lambda} e^{-\lambda x} \Big|_0^y \right) = 1 - \lambda \left(-\frac{1}{\lambda} e^{-\lambda y} + \frac{1}{\lambda} e^{-\lambda \cdot 0} \right) \\ &= 1 + e^{-\lambda y} - 1 = e^{-\lambda y} \end{aligned}$$

b) $X \sim \text{uniform}(0, 1)$

cumulative distribution function of X ?



$$cdf_X(x) = \begin{cases} 0 & , x < 0 \\ x & , 0 \leq x \leq 1 \\ 1 & , x > 1 \end{cases}$$

distribution $Z = -\ln(X)$?

i) $P(Z \leq z) = P(-\ln(X) \leq z) = P(\ln(X) \geq -z) = P(X \geq \exp(-z))$

$$= 1 - P(X < \exp(-z)) = \begin{cases} 1 - e^{-z} & , \text{if } 0 \leq e^{-z} \leq 1 \Leftrightarrow z \geq 0 \\ 1 - 1 = 0 & , \text{if } e^{-z} > 1 \Leftrightarrow z < 0 \end{cases}$$

ii) $f_Z(z) = f_X(h(z)) |h'(z)| = f_X(e^{-z}) e^{-z} = \begin{cases} e^{-z} & , \text{if } 0 \leq e^{-z} \leq 1 \Leftrightarrow z \geq 0 \\ 0 & , \text{else} \end{cases}$

iii) $\int_{-\infty}^z f_Z(x) dx = \int_0^z e^{-x} dx = -e^{-x} \Big|_0^z = -e^{-z} + e^0 = 1 - e^{-z} \text{ for } z \geq 0$

$$\int_{-\infty}^z f_Z(x) dx = 0 \text{ for } z < 0$$