IST Ü11

1) Test power in the z-test

$X_1, \dots, X_n$ ... iid random $\mathcal{N}(\mu, \sigma^2)$ variables

$H_0 : \mu = \mu_0$

a) compute test power of left-sided z-test; express cdf of $N(0,1)$ depending on μ_0, μ, σ, n and significance level α

$H_1 : \mu < \mu_0$

$$\text{power} = \mathbb{P}_\mu(\text{reject } H_0) = \mathbb{P}_\mu(z < -z_\alpha)$$

$$z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \qquad \mathbb{P}(N(0,1) \leq z_\alpha) = \alpha$$

$$\Rightarrow \text{power} = \mathbb{P}_\mu\Big(\underbrace{\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}}_{\sim N(0,1)} + \frac{\mu-\mu_0}{\sigma/\sqrt{n}} < -z_\alpha\Big) = \mathbb{P}_\mu\Big(N(0,1) < -z_\alpha - \frac{\mu-\mu_0}{\sigma/\sqrt{n}}\Big)$$

$$= \mathbb{P}_\mu\Big(N(0,1) < -\Big(z_\alpha + \frac{\mu-\mu_0}{\sigma/\sqrt{n}}\Big)\Big) = \Phi\Big(\frac{\mu_0-\mu}{\frac{\sigma}{\sqrt{n}}} - z_\alpha\Big)$$

b) comment on the impact of μ_0, μ, σ, n and α on the test power

the power is increasing when...

... $\mu_0 - \mu$ is increasing

... α is decreasing

... σ is decreasing

... n is increasing

IS I Ü11

2) Shock absorbers

Car #	Manufacturer	Competitor	Difference
1	8,8	8,4	0,4
2	10,5	10,1	0,4
3	12,5	12,0	0,5
4	9,7	9,3	0,4
5	9,6	9,0	0,6
6	13,2	13,0	0,2

We consider the difference, as the data is dependent (because the same car is used).

$H_0 : \mu_d = 0$ vs. $H_1 : \mu_d \neq 0$ $\qquad \alpha = 0,05$

$$\hat{\mu} = \frac{1}{6}(0,4+0,4+0,5+0,4+0,6+0,2) = 0,4167$$

$$S^2 = \frac{1}{6-1}\left((0,2-\hat{\mu})^2 + 3(0,4-\hat{\mu})^2 + (0,5-\hat{\mu})^2 + (0,6-\hat{\mu})^2\right) = 0,0177$$

$$z = \frac{\hat{\mu} - 0}{s/\sqrt{n}} = \frac{0,4167}{\frac{0,1329}{\sqrt{6}}} = 7,6802$$

$$z_{\alpha/2} = \Phi(0,025) = 1,96 \qquad \Rightarrow |z| > z_{\alpha/2}$$

we reject the null hypothesis

ISTÜ 11

4) Comparing Two Populations

	Schizophrenia	Normal
Sample size	41	49
Mean time	104,23	62,24
Standard deviation	62,24	16,34

a) Define the parameters of interest to the researchers

μ_s ... mean time of schizophrenic persons μ_n ... mean time of normal persons

b) Set up the null and alternative hypothesis for testing the researches theory

$H_0: \mu_s = \mu_n$ vs. $H_1: \mu_s > \mu_n$

c) p-value is reported as 0.001 $\alpha = 0{,}01$ conclusion?

as the p-value 0,001 is smaller than $\alpha = 0{,}01$ we reject the null hypothesis (accept the alternative).

d) find 99% confidence interval for target parameter

$$\mu_s - \mu_n \pm z_{\alpha/2} \sqrt{\frac{s_s^2}{n_s} + \frac{s_n^2}{n_n}} = 104{,}23 - 62{,}24 \pm z_{0{,}005} \sqrt{\frac{62{,}24^2}{41} + \frac{16{,}34^2}{49}}$$

$$= 41{,}99 \pm z_{0{,}005} \cdot 9{,}9966 = 41{,}99 \pm 2{,}58 \cdot 9{,}9966$$

gives $(16{,}1988 \; ; \; 67{,}7812)$ as the confidence intervall.

IST Ü11

5) χ^2-test for independence

	Uni A	Uni B	Uni C	Σ
calculus	10	5	5	20
algebra	10	20	10	40
probability	20	5	0	25
Σ	40	30	15	85

$\alpha = 0,05$

expected - table

	Uni A	Uni B	Uni C
calculus	$40 \cdot \frac{20}{85} = 9,4118$	$30 \cdot \frac{20}{85} = 7,0588$	$15 \cdot \frac{20}{85} = 3,5294$
algebra	$40 \cdot \frac{40}{85} = 18,8235$	$30 \cdot \frac{40}{85} = 14,1176$	$15 \cdot \frac{40}{85} = 7,0588$
probability	$40 \cdot \frac{25}{85} = 11,7647$	$30 \frac{25}{85} = 8,8235$	$15 \cdot \frac{25}{85} = 4,4118$

$\frac{(\text{observed} - \text{expected})^2}{\text{expected}}$ - table

	A	B	C
c	0,0368	0,6005	0,6128
a	4,136	2,451	1,2255
p	5,7648	1,6568	4,4118

Sum of this table is 20,8955

$\chi^2(4)$ at the 95% quantile is 9,49 and since 20,8955 is larger we reject the null hypothesis (which says the preferace for a lecture is independent from the university).