ALG Ü10

368) $p \in \mathbb{P}$ $\quad g(x) = x^{p-1} + x^{p-2} + \ldots + x + 1 = \frac{x^p - 1}{x-1}$

zz: $g(x)$ ist in $\mathbb{Z}[x]$ irreduzibel

$$r(x) = g(x+1) = \frac{(x+1)^p - 1}{x} = \frac{\left(\sum_{k=0}^{p} \binom{p}{k} x^k\right) - 1}{x} = \frac{\sum_{k=1}^{p} \binom{p}{k} x^k}{x} = \sum_{k=1}^{p} \binom{p}{k} x^{k-1}$$

$$= \sum_{k=0}^{p-1} \binom{p}{k+1} x^k = \sum_{k=0}^{p-1} \frac{p!}{(k+1)!(p-k-1)!} x^k = \underbrace{\frac{p!}{(p-1)!}}_{k=0} + \frac{p!}{2!(p-2)!} x + \ldots + \underbrace{\frac{p!}{p!} x^{p-1}}_{k=p-1}$$

$$= p + p\left(\sum_{k=1}^{p-2} \frac{(p-1)!}{(k+1)!(p-k-1)!} x^k\right) + x^{p-1}$$

Eisensteinsches Kriterium $\quad R$ ... faktorieller Ring $\quad f = \sum_{i=0}^{n} a_i x^i \in R[x]$ mit grad ≥ 1 ... primitiv

$p \in R$ ... irreduzibel ; $p \nmid a_n$, $p \mid a_i$ für $i = 0, \ldots, n-1$, $p^2 \nmid a_0$

$\Rightarrow f$ ... irreduzibel in $R[x]$

$p \geq 2$, da $p \in \mathbb{P}$

$\mathbb{Z}$ ... faktorieller Ring $\quad r \in \mathbb{Z}[x] \quad$ grad$(r) = p-1 \geq 1$ ... primitiv

p ... prim also auch irreduzibel $\quad p \nmid \underbrace{1}_{a_{p-1}} \quad p \mid a_i$ für $i = 0, \ldots, p-2$; $p^2 \nmid \underbrace{p}_{a_0}$

$\Rightarrow r$ ... irreduzibel in $\mathbb{Z}[x]$

Sei $s, t \in \mathbb{Z}[x] : s(x) \cdot t(x) = g(x)$

$\Rightarrow s(x+1) \cdot t(x+1) = g(x+1) = r(x)$. $\quad \Rightarrow s(x+1) \in E(\mathbb{Z}[x]) \vee t(x+1) \in E(\mathbb{Z}[x])$

$E(\mathbb{Z}[x]) = \{1\}$ $\quad$ o.B.d.A. $s(x+1) = 1 \Rightarrow s(x) = 1 \in E(\mathbb{Z}[x])$

$\Rightarrow g(x)$ ist irreduzibel