

1st Ü3

5) $X_1, X_2 \sim \text{Exp}(\lambda) \dots \text{iid}$ $\lambda \dots \text{rate}$ $f_{X_1}(x) = f_{X_2}(x) = \begin{cases} \lambda e^{-\lambda x} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$

$$Y = X_1 + X_2 \quad f_Y?$$

according to the theorem

$$\begin{aligned} f_{X_1+X_2}(y) &= \int_{-\infty}^{\infty} f_{X_1}(y-x_2) f_{X_2}(x_2) dx_2 & y-x_2 > 0 \quad x_2 > 0 \Rightarrow y > x_2 > 0 \\ &= \int_0^y \lambda e^{-\lambda(y-x_2)} \lambda e^{-\lambda x_2} dx_2 \\ &= \lambda^2 \int_0^y e^{-\lambda(y-x_2+x_2)} dx_2 \\ &= \lambda^2 \int_0^y e^{-\lambda y} dx_2 \\ &= \lambda^2 e^{-\lambda y} x_2 \Big|_0^y \\ &= \lambda^2 y e^{-\lambda y} \quad (\text{which is Gamma distribution } \Gamma(2, \lambda)) \end{aligned}$$