ISt Ü6

4) Unbiased estimators

$\hat{a}, \hat{b}$ ... unbiased estimators of unknown parameters a, b

a) $\alpha, \beta \in \mathbb{R}$ $\quad \alpha\hat{a} + \beta\hat{b}$ ... unbiased estimator of $\alpha a + \beta b$?

we know $\mathbb{E}(\hat{a}) = a$ and $\mathbb{E}(\hat{b}) = b$

$\Rightarrow \mathbb{E}(\alpha\hat{a} + \beta\hat{b}) = \alpha\,\mathbb{E}(\hat{a}) + \beta\,\mathbb{E}(\hat{b}) = \alpha\hat{a} + \beta\hat{b}$ $\quad \Rightarrow$ unbiased estimator

b) $\hat{a}^2$ ... unbiased estimator of a^2?

$b(\hat{a}^2) = \mathbb{E}(\hat{a}^2) - a^2 = \mathbb{E}(\hat{a}^2) - (\mathbb{E}(\hat{a}))^2 = \mathbb{V}ar(\hat{a})$

which is only $=0$ if $\hat{a}$ is fixed at a. Otherwise $\hat{a}^2$ is not an unbiased estimator.

c) $\mathbb{E}(a) = \mathbb{E}(\{15^2, 17^2, 16^2, 16^2, 17^2, 14^2\}) = \frac{15^2+17^2+16^2+16^2+17^2+14^2}{6}$

$= \frac{1511}{6}$

$a = \frac{1511}{6}$ is an unbiased estimator of the area.