ALG Ü9

2010) $\mathcal{V}$ .. Varietät mit $\exists X \in \mathcal{V} : |X| \geq 2$ $\quad F_i \in \mathcal{V}$ .. frei über (B_i, ι_i) $\quad i = 1,2 \quad B_1 \cap B_2 = \emptyset$

(F, j_1, j_2) .. Koprodukt von F_1 und F_2

zz: a) $(j_1 \circ \iota_1)(B_1) \cap (j_2 \circ \iota_2)(B_2) = \emptyset$

Angenommen $\exists a \in (j_1 \circ \iota_1)(B_1) \cap (j_2 \circ \iota_2)(B_2) \neq \emptyset$

$$\begin{tikzcd} B_1 \arrow[r, "\iota_1"] \arrow[drr, "f_1"'] & F_1 \arrow[r, "j^1"] \arrow[dr, dotted, "\varphi_1"'] \arrow[dr, "g_1"] & F \arrow[d, dotted, "\varphi"] & F_2 \arrow[l, "j^2"'] \arrow[dl, "g_2"'] \arrow[dl, dotted, "\varphi_2"] & B_2 \arrow[l, "\iota_2"'] \arrow[dll, "f_2"] \\ & & X & & \end{tikzcd}$$

$\forall X \, \forall f_1 \, \exists! \varphi_1 \qquad \forall X \, \forall f_2 \, \exists! \varphi_2$

$\forall X \, \forall g_1, g_2 \, \exists! \varphi$

Sei $X \in \mathcal{V}$ mit $x_1, x_2 \in X$, $x_1 \neq x_2$ bel.

$f_1 : B_1 \to X, \; b \mapsto x_1$

$f_2 : B_2 \to X, \; b \mapsto x_2$

$\Rightarrow \exists! \varphi_1 : F_1 \to X \quad \exists! \varphi_2 : F_2 \to X$

$\varphi_1 \circ \iota_1 = f_1 \wedge \varphi_2 \circ \iota_2 = f_2$

$\Rightarrow \exists! \varphi : F \to X : \varphi_1 = \varphi \circ j_1 \wedge \varphi_2 = \varphi \circ j_2$

$\Rightarrow \varphi \circ j_1 \circ \iota_1 = f_1 \wedge \varphi \circ j_2 \circ \iota_2 = f_2$

$a_1 \in B_1, a_2 \in B_2$ mit $(j_1 \circ \iota_1)(a_1) = a = (j_2 \circ \iota_2)(a_2)$

$\Rightarrow f_1(a_1) = x_1 = \varphi((j_1 \circ \iota_1(a_1)) = \varphi(a)$; $f_2(a_2) = x_2 = \varphi(j_2 \circ \iota_2(a_2)) = \varphi(a)$

$\Rightarrow \varphi(a) = x_1 \neq x_2 = \varphi(a)$ ↯ $\qquad \Rightarrow (j_1 \circ \iota_1)(B_1) \cap (j_2 \circ \iota_2)(B_2) = \emptyset$

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b) $B = B_1 \dot\cup B_2 \qquad \iota := (j_1 \circ \iota_1) \dot\cup (j_2 \circ \iota_2) \quad \Rightarrow F$ .. frei über (B, ι)

Sei $X \in \mathcal{V}$ bel. Sei $f : B \to X$ bel. $\quad f_1 := f|_{B_1}, \; f_2 := f|_{B_2}$

$\Rightarrow \exists! \varphi_i : F_i \to X$ mit $f_i = \varphi_i \circ \iota_i \quad i = 1,2$

$$\begin{tikzcd} B \arrow[r, "\iota"] \arrow[dr, "f"'] & F \arrow[d, dotted, "\varphi"] \arrow[dl, phantom, "="] \\ & X \end{tikzcd}$$

$\Rightarrow \exists! \varphi : F \to X$ mit $\varphi_i = \varphi \circ j_i \quad i = 1,2$

$\Rightarrow f_i = \varphi \circ j_i \circ \iota_i = \varphi \circ \iota \quad i = 1,2$

$$\begin{tikzcd} B_i \arrow[r, "\iota_i"] \arrow[dr, "f_i"'] & F_i \arrow[d, dotted, "\varphi_i"] \arrow[dl, phantom, "="] \\ & X \end{tikzcd}$$

Also $\forall X \in \mathcal{V} \; \forall f : B \to X \; \exists! \varphi : F \to X : f = \varphi \circ \iota$

$\Rightarrow F$ ist frei über (B, ι)

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