

1st Q2

3)

$$f(x) = \begin{cases} ax^2 e^{-bx^2} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$$

$$b = \frac{m}{2kT}$$

$$1 = \int_{-\infty}^{\infty} f(x) dx = \int_0^{\infty} ax^2 e^{-bx^2} dx = a \int_0^{\infty} \underbrace{x}_{u'} \cdot \underbrace{(xe^{-bx^2})}_{v'} dx$$

$$u = a \left( x \cdot \left( -\frac{e^{-bx^2}}{2b} \right) \right) \Big|_0^{\infty} - \int_0^{\infty} 1 \cdot \left( -\frac{e^{-bx^2}}{2b} \right) dx$$

$$= a \left( -\frac{1}{2b} x e^{-bx^2} \right) \Big|_0^{\infty} + \frac{1}{2b} \int_0^{\infty} e^{-bx^2} dx$$

$$= \frac{a}{2b} \left( \lim_{x \rightarrow \infty} (-x e^{-bx^2}) \right) + \int_0^{\infty} \frac{1}{\sqrt{b}} e^{-y^2} dy$$

$$\begin{cases} y = \sqrt{b} x \\ dx = \frac{1}{\sqrt{b}} dy \end{cases}$$

$$= \frac{a}{2b} (0)$$

$$+ \frac{1}{\sqrt{b}} \frac{\sqrt{\pi}}{2}$$

$$= \frac{a \sqrt{\pi}}{4b \sqrt{b}}$$

$$\Rightarrow 4b \sqrt{b} = a \sqrt{\pi} \quad \Leftrightarrow a = \frac{4b \sqrt{b}}{\sqrt{\pi}} = \frac{4b \sqrt{b \pi}}{\pi}$$