<u>ISt</u> Ü3

3) $X, Y\ldots$ random variables

$$f(x,y) = \begin{cases} c(x+2y) & , 0<y<1 \text{ and } 0<x<2 \\ 0 & , \text{otherwise} \end{cases}$$

a) find c and the marginal distribution of Y

$$1 = \int_0^1 \int_0^2 f(x,y)\,dx\,dy = \int_0^1 \int_0^2 c(x+2y)\,dx\,dy = c\int_0^1 2+4y\,dy$$

$$= c(2+2) = 4c \qquad \Rightarrow c = \frac{1}{4}$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_0^2 \frac{1}{4}(x+2y)\,dx = \frac{1}{4}(2+4y) = \frac{1}{2}+y \quad \text{for } y \in (0,1)$$

and 0 elsewhere

b) joint cdf of X and Y

$$F(x,y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f(s,t)\,dt\,ds = \int_0^x \int_0^y \frac{1}{4}(s+2t)\,dt\,ds = \frac{1}{4}\int_0^x sy+y^2\,ds$$

$$= \frac{1}{4}\left(\frac{1}{2}x^2y + xy^2\right) = \frac{x^2y}{8} + \frac{xy^2}{4}$$

c) marginal distribution of X and pdf of $Z = \frac{9}{(X+1)^2}$?

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_0^1 \frac{1}{4}(x+2y)\,dy = \frac{1}{4}(x+1) = \frac{x+1}{4}$$

$X\ldots$ continous random variable with pdf f_X

$g: \mathbb{R} \to \mathbb{R}^+$, $x \mapsto \frac{9}{(x+1)^2}$ $\qquad g^{-1}: \mathbb{R}^+ \to \mathbb{R}$, $x \mapsto \frac{3}{\sqrt{x}} - 1$ $\quad$ .. differentiable $\quad (g^{-1})'(x) = -\frac{3}{2}x^{-\frac{3}{2}}$

$$Z = g(X) \quad \Rightarrow f_Z(z) = f_X(g^{-1}(z))\,|(g^{-1})'(z)| = f_X\left(\frac{3}{\sqrt{z}}-1\right)\left|-\frac{3}{2}z^{-\frac{3}{2}}\right|$$

$$= \frac{\frac{3}{\sqrt{z}} \cancel{-1+1}}{4}\left|\frac{3}{2}z^{-\frac{3}{2}}\right| = \frac{3}{4}\cdot\frac{1}{\sqrt{z}}\,\frac{3}{2}\,\frac{1}{\sqrt{z}^3} = \frac{9}{8}\,\frac{1}{\sqrt{z}^4} = \frac{9}{8z^2}$$