IST Ü11

1) Test power in the z-test

$X_1, \ldots, X_n$ ...iid random $N(\mu, \sigma^2)$ variables

$H_0 : \mu = \mu_0$

a) compute test power of left-sided z-test; express cdf of $N(0,1)$ depending on μ_0, μ, σ, n and significance level α

$H_1 : \mu < \mu_0$

$$\text{power} = \mathbb{P}_\mu(\text{reject } H_0) = \mathbb{P}_\mu(z < -z_\alpha)$$

$$z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \qquad \mathbb{P}(N(0,1) \leq z_\alpha) = \alpha$$

$$\Rightarrow \text{power} = \mathbb{P}_\mu\left(\underbrace{\frac{\bar{X} - \mu}{\sigma/\sqrt{n}}}_{N(0,1)} + \frac{\mu - \mu_0}{\sigma/\sqrt{n}} < -z_\alpha \right) = \mathbb{P}_\mu\left(N(0,1) < -z_\alpha - \frac{\mu - \mu_0}{\sigma/\sqrt{n}} \right)$$

$$= \mathbb{P}\left(N(0,1) < -\left(z_\alpha + \frac{\mu - \mu_0}{\sigma/\sqrt{n}}\right) \right) = \Phi\left(\frac{\mu_0 - \mu}{\sigma/\sqrt{n}} - z_\alpha \right)$$

b) comment on the impact of μ_0, μ, σ, n and α on the test power

the power is increasing when ...

... $\mu_0 - \mu$ is increasing

... α is decreasing

... σ is decreasing

... n is increasing