IST Ü1

(1) tel 452 * * * * all digits are equally likely

probability tel contains seven distinct digits?

$\frac{7}{10}$ ... probability first new digit has not yet occured

$\frac{6}{10}$ ... —"— second —"—

$\frac{5}{10}$ ... —"— third —"—

$\frac{4}{10}$ ... —"— fourth —"—

$$\Rightarrow \frac{7}{10} \cdot \frac{6}{10} \cdot \frac{5}{10} \cdot \frac{4}{10} = \frac{840}{10000} = 0{,}084$$

(2) drawer of socks 7 black, 8 blue, 9 green

2 socks are chosen randomly

(a) probability they match?

$\frac{7}{24} \cdot \frac{6}{23}$ ... probability of choosing two black socks

$\frac{8}{24} \cdot \frac{7}{23}$ ... —"— blue —"—

$\frac{9}{24} \cdot \frac{8}{23}$ ... —"— green —"—

$$\Rightarrow \frac{7}{24} \cdot \frac{6}{23} + \frac{8}{24} \cdot \frac{7}{23} + \frac{9}{24} \cdot \frac{8}{23} = \frac{42+56+72}{552} = \frac{170}{552} = 0{,}308$$

(b) probability of black pair?

$$\Rightarrow \frac{7}{24} \cdot \frac{6}{23} = \frac{42}{552} = 0{,}076$$

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(3) (a) $A, B \ldots$ independent events   show $A^c, B \ldots$ independent

Definition $A, B \ldots$ independent $:\Leftrightarrow \mathbb{P}(A \cap B) = \mathbb{P}(A)\,\mathbb{P}(B)$

$\mathbb{P}(A^c \cap B) = \mathbb{P}(B \setminus A) = \mathbb{P}(B) - \mathbb{P}(A \cap B) = \mathbb{P}(B) - \mathbb{P}(A)\,\mathbb{P}(B)$

$= (1 - \mathbb{P}(A))\,\mathbb{P}(B) = \mathbb{P}(A^c)\,\mathbb{P}(B)$

(b) $A \subseteq B$ can A, B be independent?

For $B = \Omega$ it holds that $\mathbb{P}(A \cap B) = \mathbb{P}(A) = \mathbb{P}(A) \cdot 1 = \mathbb{P}(A) \cdot \mathbb{P}(B)$

(c) $A, B \ldots$ independent $B, C \ldots$ independent   is $A, C \ldots$ independent?

For $B = \Omega$ and $\emptyset \neq A = C \neq \Omega$ it holds that

$\mathbb{P}(A \cap B) = \mathbb{P}(A) = \mathbb{P}(A) \cdot 1 = \mathbb{P}(A) \cdot \mathbb{P}(B)$

$\mathbb{P}(B \cap C) = \mathbb{P}(C) = \mathbb{P}(C) \cdot 1 = \mathbb{P}(B) \cdot \mathbb{P}(C)$

$\mathbb{P}(A \cap C) = \mathbb{P}(A) \neq \mathbb{P}(A)^2 = \mathbb{P}(A) \cdot \mathbb{P}(C)$ e.g. for $\mathbb{P}(A) = \mathbb{P}(C) = \frac{1}{2}$

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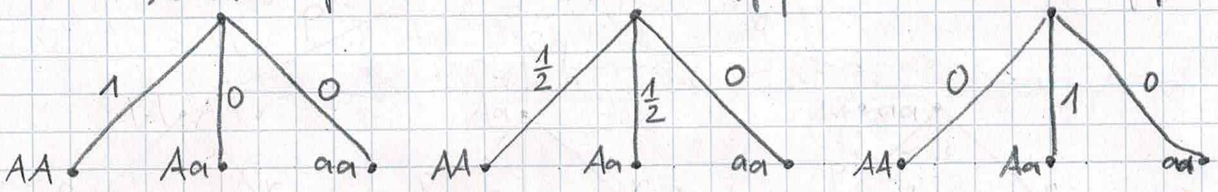
(4) genetic model

(a) AA & Aa — AA $\quad 1 \cdot \frac{1}{2} = 0{,}5$
$\quad\quad\quad\quad$ — Aa $\quad 1 \cdot \frac{1}{2} = 0{,}5$

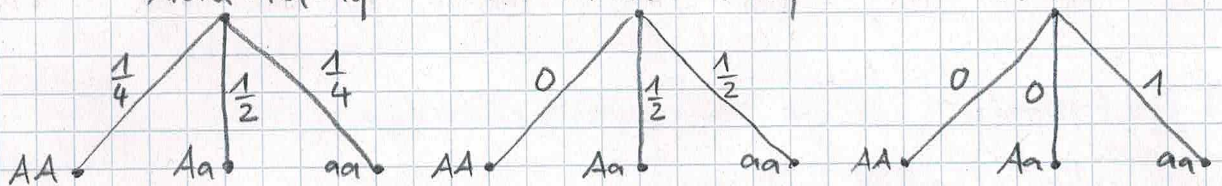
(b) first generation $\quad$ AA…p $\quad$ Aa…2q $\quad$ aa…r $\quad$ with $p+2q+r=1$

probability of second and third generation?

AA&AA: p^2 $\quad$ AA&Aa: $4pq$ $\quad$ AA&aa: $2pr$



Aa&Aa: $4q^2$ $\quad$ Aa&aa: $4qr$ $\quad$ aa&aa: r^2



$\Rightarrow$ second generation: AA: $p^2+4pq\frac{1}{2}+4q^2\frac{1}{4} = p^2+2pq+q^2 = (p+q)^2 \; (=c^2)$

Aa: $4pq\frac{1}{2}+2pr+4q^2\frac{1}{2}+4qr\frac{1}{2} = 2(p+q)(q+r) \; (=2cd)$

aa: $4q^2\frac{1}{4}+4qr\frac{1}{2}+r^2 = q^2+2qr+r^2 = (q+r)^2 \; (=d^2)$

third generation: $\quad$ renaming $\quad c := p+q \quad d := q+r \quad c^2+2cd+d^2 = (c+d)^2 \overset{*}{=} 1$

AA&AA: c^4 $\quad$ AA&Aa: $4c^3d$ $\quad$ AA&aa: $2c^2d^2$

Aa&Aa: $4c^2d^2$ $\quad$ Aa&aa: $4cd^3$ $\quad$ aa&aa: d^4

AA: $c^4+4c^3d\cdot\frac{1}{2}+4c^2d^2\cdot\frac{1}{4} = c^4+2c^3d+c^2d^2 = c^2(c+d)^2 = c^2$

Aa: $4c^3d\cdot\frac{1}{2}+2c^2d^2+4c^2d^2\cdot\frac{1}{2}+4cd^3\cdot\frac{1}{2} = 2c^3d+2c^2d^2+2c^2d^2+2cd^3$

$= 2cd(c^2+2cd+d^2) = 2cd(c+d)^2 = 2cd$

aa: $4c^2d^2\frac{1}{4}+4cd^3\frac{1}{2}+d^4 = c^2d^2+2cd^3+d^4 = d^2(c^2+2cd+d^2) = d^2(c+d)^2$

$= d^2$

$* \; (c+d)^2 = (p+q+q+r)^2 = (p+2q+r)^2 = 1^2 = 1$

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(5) box containing $3\times HH$ $2\times TT$ $4\times HT$

(a) random coin is tossed probability of tail?

$\frac{2}{9}$ ... probability of choosing TT coin 1 ... probability of getting tail

$\frac{4}{9}$ ... probability of choosing HT coin $\frac{1}{2}$ ... probability of getting tail

$$\Rightarrow \frac{2}{9} + \frac{4}{9}\cdot\frac{1}{2} = \frac{4+4}{18} = \frac{4}{9} = 0{,}4444$$

(b) tail probability of having selected TT coin?

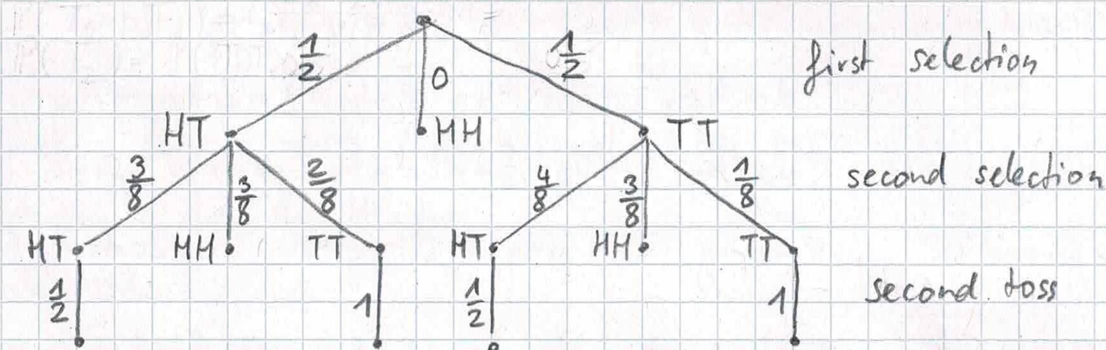
probability of having selected HT coin?

$$P(TT \mid T) = \frac{P(TT \cap T)}{P(T)} = \frac{P(TT)}{P(T)} = \frac{\frac{2}{9}}{\frac{4}{9}} = \frac{1}{2} = 0{,}5$$

$$P(HT \mid T) = \frac{P(HT \cap T)}{P(T)} = \frac{\frac{4}{9}\cdot\frac{1}{2}}{\frac{4}{9}} = \frac{\frac{4}{18}}{\frac{4}{9}} = \frac{1}{2} = 0{,}5 \quad \left(= 1 - P(TT \mid T)\right)$$

(c) first toss: tail second coin randomly selected and tossed

probability of getting tail again?



$$\Rightarrow \frac{1}{2}\cdot\frac{3}{8}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{2}{8}\cdot 1 + \frac{1}{2}\cdot\frac{4}{8}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{8}\cdot 1 = \frac{3}{32} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16}$$

$$= \frac{13}{32} = 0{,}4063$$