

ALG Ü3

126) $M = \{1, 2, 3, 4, 5\}$

θ .. Äquivalenzrelation definiert durch $\{\{1, 2\}, \{3, 4\}, \{5\}\}$

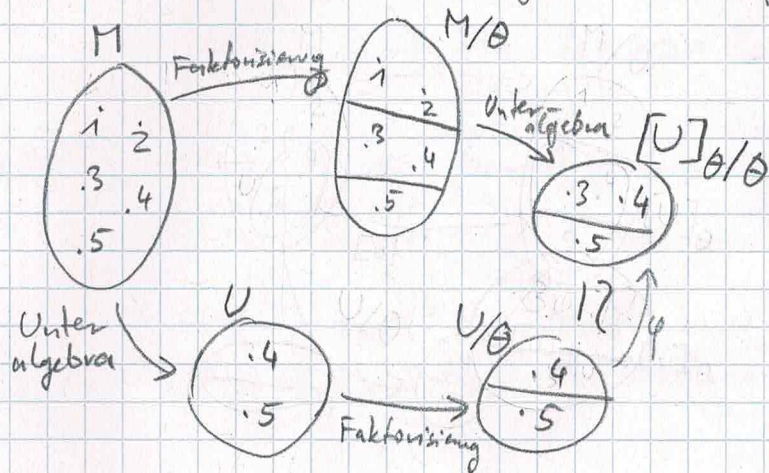
$U = \{4, 5\}$

ges: $\mathcal{M} = (M, w_1, w_2)$

sodass

neben Äquivalenz nicht 2~3

θ Kongruenz, U... Unteralgebra und Isomorphismus



$\varphi: U/\theta \rightarrow [U]_{\theta}/\theta$

$\varphi([4]_{\theta/\theta}) = [4]_{\theta} (= \{3, 4\})$

$\varphi([5]_{\theta/\theta}) = [5]_{\theta} (= \{5\})$

$w_1(a) = \begin{cases} 2, & \text{falls } a=1 \\ 1, & a=2 \\ 4, & a=3 \\ 4, & a=4 \\ 5, & a=5 \end{cases}$

$w_2(a) = \begin{cases} 1, & \text{falls } a=1 \\ 2, & a=2 \\ 5, & a=3 \\ 5, & a=4 \\ 5, & a=5 \end{cases}$

offenbar $w_1(U) \subseteq U, w_2(U) \subseteq U$

$1 \sim 2 \Rightarrow w_1(1) \sim w_1(2) \Leftrightarrow 1 \sim 2 \checkmark$

$3 \sim 4 \Rightarrow w_1(3) \sim w_1(4) \Leftrightarrow 4 \sim 4 \checkmark \quad w_2(3) \sim w_2(4) \Leftrightarrow 5 \sim 5 \checkmark$

$2 \sim 3 \Rightarrow w_2(2) \sim w_2(3) \rightarrow 2 \sim 5 \rightarrow w_1(2) \sim w_1(5) \Rightarrow 1 \sim 5$

$\Rightarrow w_1(2) \sim w_1(3) \Rightarrow 1 \sim 4 \quad \text{also } 1 \sim 2 \sim 3 \sim 4 \sim 5 \checkmark$