# IST Ü5

(1) $f, g$ ... probability density functions $\qquad X$ ... random variable with pdf $f$

prove $\mathbb{E}\left(\log \frac{f(X)}{g(X)}\right) \geq 0$ !

We know that $\log(x)$ is a concave function, therefore $-\log(x)$ is convex.

$\mathbb{E}(\log \frac{f(X)}{g(X)}) = \mathbb{E}(-\log \frac{g(X)}{f(X)}) \geq -\log(\mathbb{E}(\frac{g(X)}{f(X)}))$ according to Jensen's inequality.

$\mathbb{E}(\frac{g(X)}{f(X)}) = \int_{-\infty}^{+\infty} \frac{g(x)}{f(x)} f(x)\,dx = \int_{-\infty}^{+\infty} g(x)\,dx = 1$

$\Rightarrow -\log(\mathbb{E}(\frac{g(X)}{f(X)})) = -\log(1) = 0$ which shows that $\mathbb{E}(\log(\frac{f(X)}{g(X)})) \geq 0$

$\mathbb{E}(\log(\frac{f(X)}{g(X)})) = \mathbb{E}(\log(f(X)) - \log(g(X))) = \mathbb{E}(\log(f(X))) - \mathbb{E}(\log(g(X)))$

$\Rightarrow \mathbb{E}(\log(f(X))) - \mathbb{E}(\log(g(X))) \geq 0$

$\Rightarrow \mathbb{E}(\log(f(X))) \geq \mathbb{E}(\log(g(X)))$ and obviously for $f = g$ there holds equality.

Therefore $\mathbb{E}(\log(g(X)))$ is maximized when $g = f$.