IST Ü1

(3) (a) A, B ... independent events show A^c, B ... independent

Definition A, B ... independent $:\Longleftrightarrow \mathbb{P}(A \cap B) = \mathbb{P}(A)\,\mathbb{P}(B)$

$\mathbb{P}(A^c \cap B) = \mathbb{P}(B \setminus A) = \mathbb{P}(B) - \mathbb{P}(A \cap B) = \mathbb{P}(B) - \mathbb{P}(A)\,\mathbb{P}(B)$

$= (1 - \mathbb{P}(A))\,\mathbb{P}(B) = \mathbb{P}(A^c)\,\mathbb{P}(B)$

(b) $A \subseteq B$ can A, B be independent?

For $B = \Omega$ it holds that $\mathbb{P}(A \cap B) = \mathbb{P}(A) = \mathbb{P}(A) \cdot 1 = \mathbb{P}(A) \cdot \mathbb{P}(B)$

(c) A, B ... independent B, C ... independent is A, C ... independent?

For $B = \Omega$ and $\emptyset \neq A = C \neq \Omega$ it holds that

$\mathbb{P}(A \cap B) = \mathbb{P}(A) = \mathbb{P}(A) \cdot 1 = \mathbb{P}(A) \cdot \mathbb{P}(B)$

$\mathbb{P}(B \cap C) = \mathbb{P}(C) = \mathbb{P}(C) \cdot 1 = \mathbb{P}(B) \cdot \mathbb{P}(C)$

$\mathbb{P}(A \cap C) = \mathbb{P}(A) \neq \mathbb{P}(A)^2 = \mathbb{P}(A) \cdot \mathbb{P}(C)$ e.g. for $\mathbb{P}(A) = \mathbb{P}(C) = \frac{1}{2}$