Automated Deduction Compendium SS2023

Ida Hönigmann

May 7, 2023

1 Introduction, SAT Solving

1.0.1 Proposition, Formulas

Def (Proposition)**.** *Proposition is a statement that can be either true or false.*

Def (Propositional formula, Atom, Connective)**.** *Atoms are boolean variables (e.g. p,q).*

1. *Atoms are formulas.*
2. $\top$, $\bot$ *are formulas.*
3. *If* A *is a formula, then* $\neg A$ *is a formula.*
4. *If* $A_1, ..., A_n$ *are formulas, then* $(A_1 \wedge ... \wedge A_n)$ *and* $(A_1 \vee ... \vee A_n)$ *are formulas.*
5. *If* A *and* B *are formulas, then* $A \rightarrow B$ *and* $A \leftrightarrow B$ *are formulas.*

The symbols $\top, \bot, \wedge, \vee, \neg, \rightarrow, \leftrightarrow$ *are called logical connectives.*

1.0.2 Precedence

Connective	Name	Precedence
$\top$	verum	
$\bot$	falsum	
$\neg$	negation	5
$\wedge$	conjunction	4
$\vee$	disjunction	3
$\rightarrow$	implication	2
$\leftrightarrow$	equivalence	1

1.0.3 Boolean Values, Interpretation

Def (Boolean values, Interpretation)**.** *There are two boolean vales: true* (1) *and false* (0)*.*
An interpretation for a set P *of boolean variables is a mapping* $I : P \rightarrow \{0, 1\}$*.*

1.0.4 Interpreting formulas

1. $I(\top) = 1$ and $I(\bot) = 0$
2. $I(A_1 \wedge ... \wedge A_n) = 1$ iff $I(A_i) = 1$ for all i
3. $I(A_1 \vee ... \vee A_n) = 1$ iff $I(A_i) = 1$ for some i
4. $I(\neg A) = 1$ iff $I(A) = 0$
5. $I(A_1 \rightarrow A_2) = 1$ iff $I(A_1) = 0$ or $I(A_2) = 1$
6. $I(A_1 \leftrightarrow A_2) = 1$ iff $I(A_1) = I(A_2)$

1.0.5 Safisfiable, Valid, Model

Def (Satisfiable, Model, Valid)**.** *If $I(A) = 1$ then I satisfies A and I is a model of A, denoted by $I \models A$.*
A is satisfiable if some interpretation is a model of A.
A is valid if every interpretation is a model of A.
A and B are equivalent, denoted by $A \equiv B$, if they have the same models.

1.0.6 Connection valid, satisfiable

1. A is valid iff $\neg A$ is unsatisfiable.
2. A is satisfiable iff $\neg A$ is not valid.

1.0.7 Equivalent replacement

Def (Equivalent replacement)**.** *$A[B]$ is a formula A with a fixed occurrence of subformula B. $A[B']$ is the formula A where every occurrence of B is replaced by B'.*

Lemma 1 (Equivalent Replacement)**.** *Let I be an interpretation and $I \models A_1 \leftrightarrow A_2$. Then $I \models B[A_1] \leftrightarrow B[A_2]$.*
Let $A_1 \equiv A_2$. Then $B[A_1] \equiv B[A_2]$.

1.0.8 Evaluating a formula

Algorithm 1.
```
procedure evaluate(G,I)
input: formula G, interpretation I
output: the boolean value I(G)
begin
  forall atoms p occurring in G
    if I models p
      then replace all occurrences of p in G by 1;
      else replace all occurrences of p in G by 0;
  rewrite G into a normal form using the rewrite rules
  if G = 1 then return 1 else return 0
end
```

2 Splitting, Polarities

3 CNF, DPLL, MiniSat

4 Random SAT, Horn clauses

5 First-Order Logic, Theories

6 SMT, Theory of Equality, DPLL(T)

7 Theory of Arrays, Theory Combination, Nelson-Oppen, Z3

8 First-Order Theorem Proving, TPTP, Inference Systems

9 Selection functions, Saturation, Fairness and Redundancy

10 Redundancy, First-Order Reasoning with Equality

11 Ground Superposition, Term Orderings

12 Unification and Lifting

13 Non-Ground Superposition