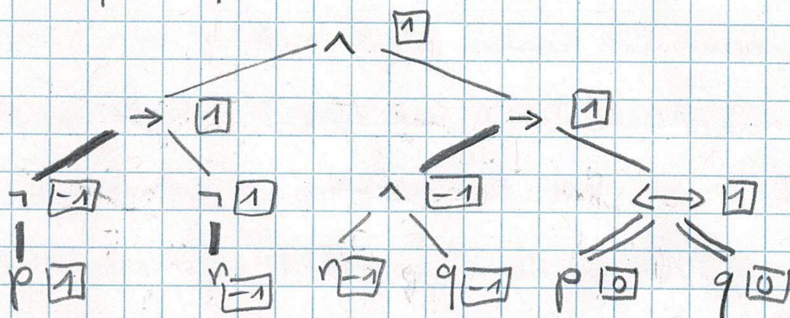
Automated Deduction HW1.1

$(\neg p \rightarrow \neg r) \wedge ((r \wedge q) \rightarrow (p \leftrightarrow q))$

a) Which atoms are pure?



As p and q appear in a eqiv they are not pure.

r is negatively pure as seen in diagram.

b) Compute clausal NF with naming and polarity optimization

		polarity	
n_0	$(\neg p \rightarrow \neg r) \wedge ((r \wedge q) \rightarrow (p \leftrightarrow q))$	1	$n_0 \rightarrow (n_1 \wedge n_2)$
n_1	$\neg p \rightarrow \neg r$	1	$n_1 \rightarrow (n_3 \rightarrow n_4)$
n_2	$(r \wedge q) \rightarrow (p \leftrightarrow q)$	1	$n_2 \rightarrow (n_5 \rightarrow n_6)$
n_3	$\neg p$	-1	$(\neg p) \rightarrow n_3$
n_4	$\neg r$	1	$n_4 \rightarrow (\neg r)$
n_5	$r \wedge q$	-1	$(r \wedge q) \rightarrow n_5$
n_6	$p \leftrightarrow q$	1	$n_6 \rightarrow (p \leftrightarrow q)$

$C = \{n_0, \neg n_0 \vee n_1, \neg n_0 \vee n_2, \neg n_1 \vee \neg n_3 \vee n_4, \neg n_2 \vee \neg n_5 \vee n_6,$
$p \vee n_3, \neg n_4 \vee \neg r, \neg r \vee \neg q \vee n_5, \neg n_6 \vee \neg p \vee q, \neg n_6 \vee p \vee \neg q\}$

c) SAT?

first unit propagation: we set $n_0 \equiv \top$ resulting clauses are

$\{n_1, n_2, \neg n_1 \vee \neg n_3 \vee n_4, \neg n_2 \vee \neg n_5 \vee n_6, p \vee n_3, \neg n_4 \vee \neg r, \neg r \vee \neg q \vee n_5, \neg n_6 \vee \neg p \vee q, \neg n_6 \vee p \vee \neg q\}$

we set $n_1 \equiv \top$ and propagate and then set $n_2 \equiv \top$ and propagate and arrive at

$\{\neg n_3 \vee n_4, \neg n_5 \vee n_6, p \vee n_3, \neg n_4 \vee \neg r, \neg r \vee \neg q \vee n_5, \neg n_6 \vee \neg p \vee q, \neg n_6 \vee p \vee \neg q\}$

no further unit propagation is possible, so we look into polarities: r is negatively pure so we set $r \equiv \bot$ and arrive at $\{\neg n_3 \vee n_4, \neg n_5 \vee n_6, p \vee n_3, \neg n_6 \vee \neg p \vee q, \neg n_6 \vee p \vee \neg q\}$. now n_4 is positively pure $(n_4 \equiv \top) \Rightarrow \{\neg n_5 \vee n_6, p \vee n_3, \neg n_6 \vee \neg p \vee q, \neg n_6 \vee p \vee \neg q\}$. $n_3 \equiv \top$ as it is positively pure $\{\neg n_5 \vee n_6, \neg n_6 \vee \neg p \vee q, \neg n_6 \vee p \vee \neg q\}$. $n_5 \equiv \bot$ (neg. pure) $\Rightarrow \{\neg n_6 \vee \neg p \vee q, \neg n_6 \vee p \vee \neg q\}$ and $n_6 \equiv \bot$ (neg. pure) $\Rightarrow \{\}$ therefore SAT with model $I(n_0) = I(n_1) = I(n_2) = I(n_3) = I(n_4) = \top$ and $I(n_5) = I(n_6) = I(r) = \bot$ now we have a free choice and set $I(p) = I(q) = \bot$.

Automated Deductions HW 1.2.

Let A be a propositional formulae and p be an atom of A. Assume that p only occurs with negative polarity in A.

a) Assume $I \vDash A$ and define I' to be the interpretation $I \cup \{p \mapsto 0\}$. Show that $I' \vDash A$.

We have that $\bot \rightarrow p$ is a tautology, therefore $I \vDash (\bot \rightarrow p)$.

The polarity of p in A is negative. Using the monotonic replacement lemma with $B := \bot$ and $B' := p$ we have that $I \vDash A \rightarrow A_p^\bot$.

As $I \vDash A$ it follows that $I \vDash A_p^\bot$ and since I and I' agree on all values except maybe p, but there are no occurences of p in $A_p^\bot$ we have that $I' \vDash A_p^\bot$. By using the equivalent replacement lemma and $I' \vDash p \leftrightarrow \bot$ (per definition) this results in $I' \vDash A$.

b) Show that A is satisfiable if and only if $A_p^\bot$ is satisfiable

$\Rightarrow$ Assume A is SAT. Therefore $\exists I$ ... Interpretation such that $I \vDash A$ from the first part of the above proof (using monotonic replacement) it follows that $I \vDash A_p^\bot$, meaning there $\exists$ an Interpretation $\Rightarrow A_p^\bot$ is SAT.

$\Leftarrow$ Assume $A_p^\bot$ is SAT. Once again $\exists I$ ... Interpretation : $I \vDash A_p^\bot$

Define $I' := I \cup \{p \mapsto 0\}$. $I \vDash A_p^\bot$ and $A_p^\bot$ does not contain the atom p, I and I' agree on all values except maybe p $\Rightarrow I' \vDash A_p^\bot$. By definition of I' $I' \vDash p \leftrightarrow \bot$. Equivalent replacement lemma gives us $I' \vDash A$ $\Rightarrow A$ is SAT

$\square$

Automated Deduction HW 1.3.

4 children; exactly 1 ate the cake; exactly 1 does not tell the truth

Alex: Either Martin or Lea does not tell the truth. Julian: I did not eat the cake.

Lea: Alex did not eat the cake Martin: Either Lea or Julian ate the cake.

a) represent this puzzle as a set of propositional formulas and explain their intended meaning.

AL ... Alex is lying JL ... Julian is lying LL ... Lea is lying ML ... Martin is lying

AC ... Alex ate the cake JC ... Julian ate the cake LC ... Lea ate the cake MC ... Martin ate the cake

exactly 1 does not tell the truth: $AL \leftrightarrow (\neg JL \wedge \neg LL \wedge \neg ML)$ $JL \leftrightarrow (\neg AL \wedge \neg LL \wedge \neg ML)$

$LL \leftrightarrow (\neg AL \wedge \neg JL \wedge \neg ML)$ $ML \leftrightarrow (\neg AL \wedge \neg JL \wedge \neg LL)$

exactly 1 ate the cake: $AC \leftrightarrow (\neg JC \wedge \neg LC \wedge \neg MC)$ $JC \leftrightarrow (\neg AC \wedge \neg LC \wedge \neg MC)$

$LC \leftrightarrow (\neg AC \wedge \neg JC \wedge \neg MC)$ $MC \leftrightarrow (\neg AC \wedge \neg JC \wedge \neg LC)$

Alex's statement: $(\neg AL) \leftrightarrow (ML \vee LL)$ Julian's statement: $(\neg JL) \leftrightarrow (\neg JC)$

Lea's statement: $(\neg LL) \leftrightarrow (\neg AC)$ Martin's statement: $(\neg ML) \leftrightarrow (LC \vee JC)$

b) classify propositional formulas; encode in DIMACS format; solve with MiniSat.

$(\neg AL) \leftrightarrow (ML \vee LL) \equiv (AL \vee ML \vee LL) \wedge (\neg(ML \vee LL) \vee (\neg AL))$

$\equiv (AL \vee ML \vee LL) \wedge ((\neg ML \wedge \neg LL) \vee (\neg AL)) \equiv (AL \vee ML \vee LL) \wedge (\neg ML \vee \neg AL) \wedge (\neg LL \vee \neg AL)$

$(\neg JL) \leftrightarrow (\neg JC) \equiv (JL \vee (\neg JC)) \wedge (JC \vee (\neg JL))$

$(\neg LL) \leftrightarrow (\neg AC) \equiv (LL \vee \neg AC) \wedge (\neg LL \vee AC)$

$(\neg ML) \leftrightarrow (LC \vee JC) \equiv (ML \vee LC \vee JC) \wedge (\neg(LC \vee JC) \vee \neg ML) \equiv (ML \vee LC \vee JC) \wedge (\neg LC \vee \neg MC) \wedge (\neg JC \vee \neg MC)$

$AL \leftrightarrow \neg(JL \vee LL \vee ML) \equiv (\neg AL \vee (\neg JL \wedge \neg LL \wedge \neg ML)) \wedge (AL \vee JL \vee LL \vee ML) \equiv$

$(\neg AL \vee \neg JL) \wedge (\neg AL \vee \neg LL) \wedge (\neg AL \vee \neg ML) \wedge (AL \vee JL \vee LL \vee ML)$

same with the others of the same structure. Naming AL: 1, JL: 2, LL: 3, ML: 4, AC: 5, JC: 6, LC: 7 and MC: 8 gives us the CNF in DIMACS format given in the separate text file.

c) MiniSat gives us two possible Interpretations:

$I = \{AL \mapsto 0, JL \mapsto 0, LL \mapsto 0, ML \mapsto 1, AC \mapsto 0, JC \mapsto 0, LC \mapsto 0, MC \mapsto 1\}$

$I' = \{AL \mapsto 1, JL \mapsto 0, LL \mapsto 0, ML \mapsto 0, AC \mapsto 0, JC \mapsto 0, LC \mapsto 1, MC \mapsto 0\}$

Automated Deduction HW 1.4.

$p_0 \vee \neg p_1 \vee \neg p_2$; $p_0 \vee \neg p_1 \vee p_2$; $p_1 \vee \neg p_2 \vee p_4$; $\neg p_0 \vee \neg p_1 \vee \neg p_2$

$p_0 \vee \neg p_1 \vee p_4$; $p_2 \vee p_3 \vee p_4$; $\neg p_1 \vee p_2 \vee p_4$; $p_2 \vee \neg p_3 \vee \neg p_4$

$p_1 \vee p_4$; $p_0 \vee p_2 \vee p_3$; $\neg p_0 \vee p_2 \vee \neg p_3$; $p_0 \vee p_2 \vee p_3$

a) $I = \{p_0 \mapsto 0, p_1 \mapsto 1, p_2 \mapsto 0, p_3 \mapsto 1, p_4 \mapsto 0\}$

What is the propability GSAT will choose p_i for flipping?

currently I does not satisfy the clauses (e.g. $p_0 \vee \neg p_1 \vee p_4 \equiv \bot$)

after flipping p_0 all but $\neg p_1 \vee p_2 \vee p_4$; $\neg p_0 \vee p_2 \vee \neg p_3$ will be satisfied $\Rightarrow 12-2=10$ are sat

flipping p_1 instead: $p_1 \vee p_4$ is not sat $\Rightarrow 12-1=11$ are satisfied

flipping p_2 instead: $p_0 \vee \neg p_1 \vee \neg p_2$; $p_0 \vee \neg p_1 \vee p_4$; $\Rightarrow 12-2=10$ are satisfied

flipping p_3 instead: $p_0 \vee \neg p_1 \vee p_4$; $p_0 \vee \neg p_1 \vee p_2$; $p_2 \vee p_3 \vee p_4$; $p_0 \vee p_2 \vee p_3$; $\neg p_1 \vee p_2 \vee p_4$; $p_0 \vee p_2 \vee p_3$ are all not sat $\Rightarrow 12-6=6$ are satisfied

flipping p_4 insted: $p_0 \vee \neg p_1 \vee p_2$; $p_2 \vee \neg p_3 \vee \neg p_4$ $\Rightarrow 12-2=10$ are satisfied

Therefore the probability of GSAT flipping p_1 are 100% and probability of any other being flipped are 0%.

b) as in a) but with WSAT instead of GSAT

currently the following clauses are not satisfied by I:

$$\boxed{\begin{array}{l} p_0 \vee \neg p_1 \vee p_4 \\ p_0 \vee \neg p_1 \vee p_2 \\ \neg p_1 \vee p_2 \vee p_4 \end{array}}$$

out of these we randomly choose one (all with probability $\frac{1}{3}$) then we choose a random variable out of the chosen clause. We calculate the probabilities:

$p(p_0 \text{ is chosen}) = \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3} = \frac{2}{9}$ $\quad p(p_1 \text{ is chosen}) = \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3} = \frac{3}{9} = \frac{1}{3}$

$p(p_2 \text{ is chosen}) = \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3} = \frac{2}{9}$ $\quad p(p_3 \text{ is chosen}) = 0$ $\quad p(p_4 \text{ is chosen}) = \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3} = \frac{2}{9}$