## Stochastic Capital Budgeting

**David Morton and Ivilina Popova**

**August 19, 2019**

## The deterministic capital budgeting model of previously developed allows for multiple options in how we select a particular project. For example, we might select a project via Plan A, Plan B, Plan C, or not select the project at all. In addition, the deterministic model allows for multiple types of resources (e.g., capital budgets and operations and maintenance budgets), and further allows for piggy-backing constraints.

## Our initial stochastic capital budgeting model illustrates the ideas of prioritization without the additional features of multiple types of resources, piggy-backing, and multiple options for selecting each project. The former two features integrate with the prioritization scheme in a straightforward way, as we will describe below. The latter-most feature proves to have subtle interactions with the notion of prioritization, and we discuss that in some detail in this section. The model sketched here is new and, to our knowledge, has not appeared in the literature. Even though the notation has been sketched above, we develop the full model here so that this section is self-contained, given that it specified our “full” mathematical model for stochastic capital budgeting.

*Indices and sets:*

candidate projects

options for selecting project , e.g., initiate project in year or and in a

standard (three year) or in an expedited (two year) manner

must-do projects; e.g., projects that must be done due to regulatory or safety reasons

even if their NPV is negative

option for project can be selected only if option is selected for project , i.e.,

piggy-backing

types of resources, e.g., capital funds, O&M funds, labor-hours, time during outage

time periods (years)

scenarios

*Data:*

NPV (revenue less financial cost) of selecting project via option under scenario

available budget for a resource of type in year under scenario

consumption of resource of type in year if project is performed via option under scenario

probability mass of scenario

*Decision variables:*

*Optimization model formulation*

, (1m)

## We maximize net present value of the selected portfolio of projects in (1a). For simplicity, in what follows we will say that variable means that project is higher priority than even though the variable definition allows for ties, i.e., the projects being the same priority. Constraint (1b) indicates that either project is higher priority than project or vice versa, and further allows both (i.e., a tie). Constraint (1c) indicates that if project is higher priority than project ( then if we select the lower priority project under some option then we must also select the higher priority project; if then the constraint is vacuous. Constraint (1d) requires that we be within budget in each time period, for each resource type, under each scenario. Constraint (1e) defines binary variable and simultaneously ensures that we select project via at most one option. Constraint (1f) ensures that we select all must-do projects. Constraint (1g) captures piggy-backing conditions. Constraints (1h)-(1i) require that we produce a total ordering of the projects rather than allowing for ties. If we remove constraints (1h)-(1i) then it will not change the optimal NPV that we obtain, but including the constraints can facilitate easier parsing of the solutions. Constraint (1j) is a type of consistency constraint with respect to the notion of options; the constraint matters only when project is higher priority than project (In this case, if we select Plan A for the lower priority project then we must select plan A for the higher priority project. If we select Plan B for the lower priority project then we can select Plan A or Plan B for the higher priority project. And, if we select Plan C for the lower priority project then we can select Plan A, B, or C for the higher priority project. Inclusion of constraint (1j) is “optional” and reflects how the decision maker prefers to interpret the notion of priorities. Constraints (1k) and (1l) taken together indicate that, for each project separately, we cannot mix use of Plans A, B, and C across different scenarios. For example, if for project #4, we select Plan B under any scenario then we must use Plan B for project #4 (or not select the project) under all other scenarios.

We implement the above model with the data for the 16 projects portfolio. In order to observe the behavior of the solution depending on including different constraints, we first solve the problem with uncertain budgets. Using an Uniform distribution U[19,40] for the annual budget, we create the following 10 scenarios:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| S1 | S2 | S3 | S4 | S5 | S6 | S7 | S8 | S9 | S10 |
| 19.00 | 22.15 | 24.25 | 26.35 | 28.45 | 30.55 | 32.65 | 34.75 | 36.85 | 40.00 |

We use the same scenarios for all five years.

First, we solve the stochastic optimization problem including constraints (1b), (1d) and (1e). This is the minimal set of constraints ensuring the budget is met and negative NPV projects, if they are MustDo projects, are allowed to be selected. The following table shows the optimal solution with an optimal objective value of 266.707M:



As it can be seen all MustDo projects are selected, but the solutions “jump” between different plans and switch on and off between scenarios. Note for example project, Service water system upgrade. For the first scenario S1 = 19M budget, optimal solution is Plan A. However, for the second and forth scenarios this project is not selected. Clearly this is not a desired behavior of the solution.

Next, we solve the stochastic optimization problem by adding constraints (1c) and (1h). The following table shows the optimal solution with an optimal objective value of 264.994M:



Note, that the undesired behavior of not switching the solution on and off between scenarios is gone, but there is still “jumping” between different plans. For example, see project Replace moisture separator reheater. For S1 it starts with Plan A, S2 jumps to Plan C, then back to Plan A and later even goes to Plan B. remember that Plan A starts now, Plan B delays one year and Plan C delays two years. It is impossible to switch to Plan C and later decide to go back to Plan A since this will imply the ability to reverse time.

To remedy this, we include constraint (1j). The following table shows the optimal solution with an optimal objective value of 241.623M:



Note that the solution behavior improved but there is still jumping between different plans. By incorporating constraints (1k) and (1l) we fix this problem. The following table shows the optimal solution with an optimal objective value of 225.708M:



Now, once a particular plan is selected, the solution stays there for the selected scenarios.

We rerun the full stochastic optimization model with different uniform budget distributions for the five years:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Capital budget [M$]** | | | | |
| **Y1** | **Y2** | **Y3** | **Y4** | **Y5** |
| U[20,23] | U[34,38] | U[17,22] | U[20,25] | U[18,24] |

Similarly, to the first budget scenarios, we create a set of 10 budget scenarios:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Budget | S1 | S2 | S3 | S4 | S5 | S6 | S7 | S8 | S9 | S10 |
| year1 | 20.0000 | 20.3333 | 20.6667 | 21.0000 | 21.3333 | 21.6667 | 22.0000 | 22.3333 | 22.6667 | 23.0000 |
| year2 | 34.0000 | 34.4444 | 34.8889 | 35.3333 | 35.7778 | 36.2222 | 36.6667 | 37.1111 | 37.5556 | 38.0000 |
| year3 | 17.0000 | 17.5556 | 18.1111 | 18.6667 | 19.2222 | 19.7778 | 20.3333 | 20.8889 | 21.4444 | 22.0000 |
| year4 | 20.0000 | 20.5556 | 21.1111 | 21.6667 | 22.2222 | 22.7778 | 23.3333 | 23.8889 | 24.4444 | 25.0000 |
| year5 | 18.0000 | 18.6667 | 19.3333 | 20.0000 | 20.6667 | 21.3333 | 22.0000 | 22.6667 | 23.3333 | 24.0000 |

The following table shows the the optimal solution with an optimal objective value of 221.736M:

