**Optimization Models for Capital Budgeting**

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**March 20, 2019**

1. **INTRODUCTION**

We consider a capital budgeting problem for a nuclear generation station, with possible extension to a larger fleet of plants. Due to limited resources, we can only select a subset from a number of candidate capital projects. Our goal is to maximize overall net present value (NPV), or a variant of this objective when we incorporate uncertainty in project cost–and project revenue–streams. In doing so, we must respect resource limits and capture key structural and stochastic dependencies of the system. Example projects include upgrading a steam turbine, refurbishing or replacing a set of reactor coolant pumps, and replacing a set of feed-water heaters. Selecting an individual project has multiple facets and implications.

**Rewards**: Selecting a project can improve revenue, e.g., upgrading a steam turbine may lead to an uprate in plant capacity resulting in larger revenue from selling power. Replacing a key system component can improve reliability, increasing revenue due to a reduction in forced outages and reducing operations and maintenance (O&M) costs. Choosing to perform minimum maintenance versus refurbishing a component versus replacing and improving a system can produce “reward” streams over years which can be negative or positive depending on the selection.

**Resources and liabilities**: Selecting a project in year induces multiple cost streams in year and in subsequent years, where we interpret “cost” broadly to include commitment of critical resources, including: (i) capital costs, (ii) O&M costs, (iii) time and labor-hours during a planned outage, (iv) personnel, installation & maintenance equipment, space, and more. Within these categories, resources can be placed in further subcategories, each with its own budget, due to a plant’s organizational structure so that there are multiple “colors” of money within capital costs, within O&M costs, within personnel availability, etc.

**Planned outage**: Nuclear power plants have planned outages at regular intervals (e.g., every 18 months) often in the fall and spring to be well-prepared for winter and summer peaks in load. While refueling only takes a fraction of a two-month (say) period without power production, maintenance projects may be deferred until an outage. Moreover, an outage can provide the only possible time period in which to carry out certain types of projects. Because of lost revenue, an operator seeks to limit downtime. As a result, this provides a special type of resource constraint limiting project selection due to multiple projects competing for time, space, personnel, and equipment during an outage.

**Synergies**: Selecting a project may require replacing a structure, system, or component (SSC) during a planned outage of the plant. Depending on the physical location of an SSC in the plant and its relationship to other components, selecting one project may reduce the cost of selecting another project (e.g., time or know-how required to implement the project) if they are selected at the same time or close in proximity. For example, if a plant has two units, selecting a project for one unit in a spring outage (e.g., replacement of a condensate cooler and a set of feed-water heaters) may be followed by the same activity in the fall outage in the second unit, at reduced cost.

**Options**: The goal of selecting a project is typically to improve or maintain a particular function that the plant performs, and there may be multiple ways to carry out the task. A project may be performed over a three-year period, say, years , or the start of the project could instead be two years hence with project implementation over years. Alternatively, at increased cost and increased benefit, it may be possible to complete the project in two years, or . When selecting a project to uprate plant capacity, we may have two options that increase capacity by 3% or 6%. In all these cases, we can perform the project in at most *one way*, from a collection of multiple options. We represent this by cloning a “project” into multiple project-option pairs, and adding a constraint saying that we can select at most one of from this set of options.

**Non-selection**: Not selecting a project also has implications, inducing growth in O&M costs in future years, a decrease in plant production, an increase in forced outages, and even risking a premature end to plant life. Thus, not selecting a project can be seen as one more “option” as to how a larger project is executed, expanding the list just discussed. Selection is of the “do nothing” option is reflected in both liability streams and reward streams.

**Uncertainty**: One limitation of traditional optimization models for capital budgeting is that they do *not* account for uncertainty in reward and cost streams associated with individual projects, they do not account for uncertainty in resource availability in future years. Projects can incur cost over-runs, especially when projects are large, performed infrequently, and when there is uncertainty regarding technical viability, external contractors, and/or suppliers of requisite parts and materials. Occasionally, projects are performed ahead of schedule and with cost savings. Planned budgets for capital improvements can be cut and key personnel may be lost. Or, there may be surprise windfalls in budgets for maintenance activities due to decreased costs for “unplanned” maintenance.

1. **A DETERMINISTIC OPTIMIZATION MODEL**

In what follows, we begin by specifying data requirements and a mathematical model for the capital budgeting problem assuming deterministic point forecasts for project cost streams and revenue streams. We use an integer programming formulation for the optimization model, which we make precise below. Then, we extend the model to handle uncertainty in costs, revenues, and resource availability.

*Indices and sets:*

candidate projects

options for selecting project , e.g., initiate project in year or and in a standard

(three year) or in an expedited (two year) manner

option for project can be selected only if option is selected for

project

types of resources, e.g., capital funds, O&M funds, labor-hours, time during outage

time periods (years)

*Data:*

reward (revenue less financial cost) in year of selecting project via option

available budget for a resource of type in year

consumption of resource of type in year if project is performed via option

discount factor

*Decision variables:*

takes value 1 if project is selected via option

*Optimization model formulation:*

The decision variables, , indicate whether we choose to do project by means. Restated, if , then we recommend doing project via option , and taken together these decision variables produce a schedule for performing projects over time. The set of available options, , includes the “do nothing” option, and the first constraint ensures that we choose exactly one option from the available set for each project. Even if we select the “do nothing” option for a project, it induces a “reward” stream, , which can may be negative, representing growing O&M costs, losses in plant efficiency, etc. as time progresses. The second structural constraint ensures that the budget of each resource is respected in each year . The third structural constraint captures piggybacking situations in which option for project (which may have cheaper costs) may be selected only if project-option pair is also selected. The objective function includes the reward stream for each project-option pair, , and the correct stream is selected by the 0-1 decision variable. The reward stream is discounted using rate , e.g., .

1. **A STOCHASTIC OPTIMIZATION MODEL**

The following model, which we have implemented in GAMS and Pyomo is an implementation of model (14a)-(14f) from Koc and Morton (2015), which we provide below. We note that this model has better computational performance than the model with additional decision variables in Koc et al. (2009). Moreover, the variables defined below are analogous to the variables in Koc et al. (2009).

*Indices and sets:*

, items(projects)

, knapsack dimensions (time periods)

, scenarios

*Data:*

= profit of item *i* under scenario (NPV)

= capacity (budget) of dimension *t* under scenario 

= resource consumption of item *i* for dimension *t* under scenario  (cost of project *i* for time period *t* under scenario )

= probability of scenario 

*Decision variables:*

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Formulation:

We note that the prioritization model just sketched can be extended in a straightforward way to include the additional features of the model in Section 2.

**4. NOTES ON PYOMO AND GAMS IMPLEMENTATIONS**

The Pyomo and GAMS code that we have provided uses a direct implementation, which avoids the stochastic programming extensive-form option that is available.

Solutions for the 41-project data with 10 budget scenarios were obtained using 3 different solvers:

1. Cplex, is the fastest of the solvers we investigated. It obtained an optimal solution in fewer than 8 seconds. The Excel file “results\_41\_projects\_cplex\_solver.xlsx” has the optimal solution in a format similar to Table 7 from Koc et al. (2009).
2. CBC, is a free solver from COIN-OR, and it solved the problem in about 12 minutes.
3. GLPK is also a free solver, but it was not able to converge after a couple of hours running, and does not appear useable for large problems.

**REFERENCES**

A. Koc and D.P. Morton, “Prioritization via stochastic optimization”, *Management Science* 61, 586-603 (2015).

A. Koc, D.P. Morton, E. Popova, S.M. Hess, E. Kee, and D. Richards, “Prioritization project selection”, *Engineering Economist* 54, 267-297 (2009).