6.4 Calculate partial derivatives of: $f(x,y) = \frac{\sin(xy)}{\cos(x+y)}$

$$\frac{\partial}{\partial x} f(x,y) = \frac{\cos(x+y) \cdot y \cdot \cos(x \cdot y) - \sin(x \cdot y)(-\sin(x+y))}{\cos(x+y)^2}$$

$$= \frac{\cos(x+y) \cdot y \cdot \cos(x \cdot y) + \sin(x \cdot y)\sin(x+y)}{\cos(x+y)^2}$$

⟺ Remember here to use the quotient rule

Now the partial derivative wrt. y

$$\frac{\partial}{\partial y} f(x,y) = \frac{\cos(x+y) \cdot x \cdot \cos(x \cdot y) - \sin(x \cdot y)(-\sin(x+y))}{\cos(x+y)^2}$$

$$= \frac{\cos(x+y) \cdot x \cdot \cos(x \cdot y) + \sin(x \cdot y) \cdot \sin(x+y)}{\cos(x+y)^2}$$

6.7 Partially differentiate and use the chain rule

$$\frac{\partial}{\partial u} f(u,v) = 1 \cdot \frac{1}{2\sqrt{u+v^2}} = \frac{1}{2}\sqrt{u+v^2}$$

$$\frac{\partial}{\partial u} f\left(\frac{1}{2}, \frac{1}{3}\right) = 0.6396$$

$$\frac{\partial}{\partial v} f(u,v) = 2v \cdot \frac{1}{2\sqrt{u+v^2}} = \frac{v}{\sqrt{u+v^2}}$$

$$\frac{\partial}{\partial v} \left(\frac{1}{2}, \frac{1}{3}\right) = 0{,}4264$$

6.9 Obtaining the first second and third derivatives

$$f(x) = 5x^4 + 3x^3 - 11x^2 + x - 7$$
$$f'(x) = 20x^3 + 9x^2 - 22x + 1$$
$$f''(x) = 60x^2 + 18x - 22$$
$$f'''(x) = 120x + 18$$

$$h(z) = 111z^3 - 121z$$
$$h'(z) = 333z^2 - 121$$
$$h''(z) = 666z$$
$$h'''(z) = 666$$

$$f(y) = y^{\frac{1}{2}} + y^{-\frac{7}{2}}$$
$$f'(y) = \frac{1}{2}y^{-\frac{1}{2}} - \frac{7}{2}y^{-\frac{9}{2}}$$
$$f''(y) = -\frac{1}{4}y^{-\frac{3}{2}} + \frac{64}{4}y^{-\frac{11}{2}}$$
$$f'''(y) = \frac{3}{8}y^{-\frac{5}{2}} - \frac{693}{8}y^{-\frac{13}{2}}$$

6.9

$f(x) = x^{-18}$

$f'(x) = -18x^{-19}$

$f''(x) = 342x^{-20}$

$f'''(x) = -6840x^{-21}$

$h(u) = \log(u) + k$

$h'(u) = u^{-1}$

$h''(u) = -u^{-2}$

$h'''(u) = 2u^{-3}$

$g(z) = \sin(z) - \cos(z)$

$g'(z) = \cos(z) + \sin(z)$

$g''(z) = -\sin(z) + \cos(z)$

$g'''(z) = -\cos(z) - \sin(z)$

Determining the gradients

$f(x,y) = 10x^2 + e^y$

$\frac{\partial}{\partial x} f(x,y) = 20x \qquad \frac{\partial}{\partial y} f(x,y) = e^y$

$\nabla f(x,y) = (20x, e^y)$

$f(x,y) = x^3 + 10xy + 4y + 20$

$\frac{\partial}{\partial x} f(x,y) = 3x^2 + 10y \qquad \frac{\partial}{\partial y} f(x,y) = 10x + 4$

$\nabla f(x,y) = (3x^2 + 10y, 10x + 4)$

$f(x,y) = \sin(x^2 + 5 + y)$

$\frac{\partial}{\partial x} f(x,y) = 2x\cos(x^2 + 5 + y) \qquad \frac{\partial}{\partial y} f(x,y) = \cos(x^2 + y + 5)$