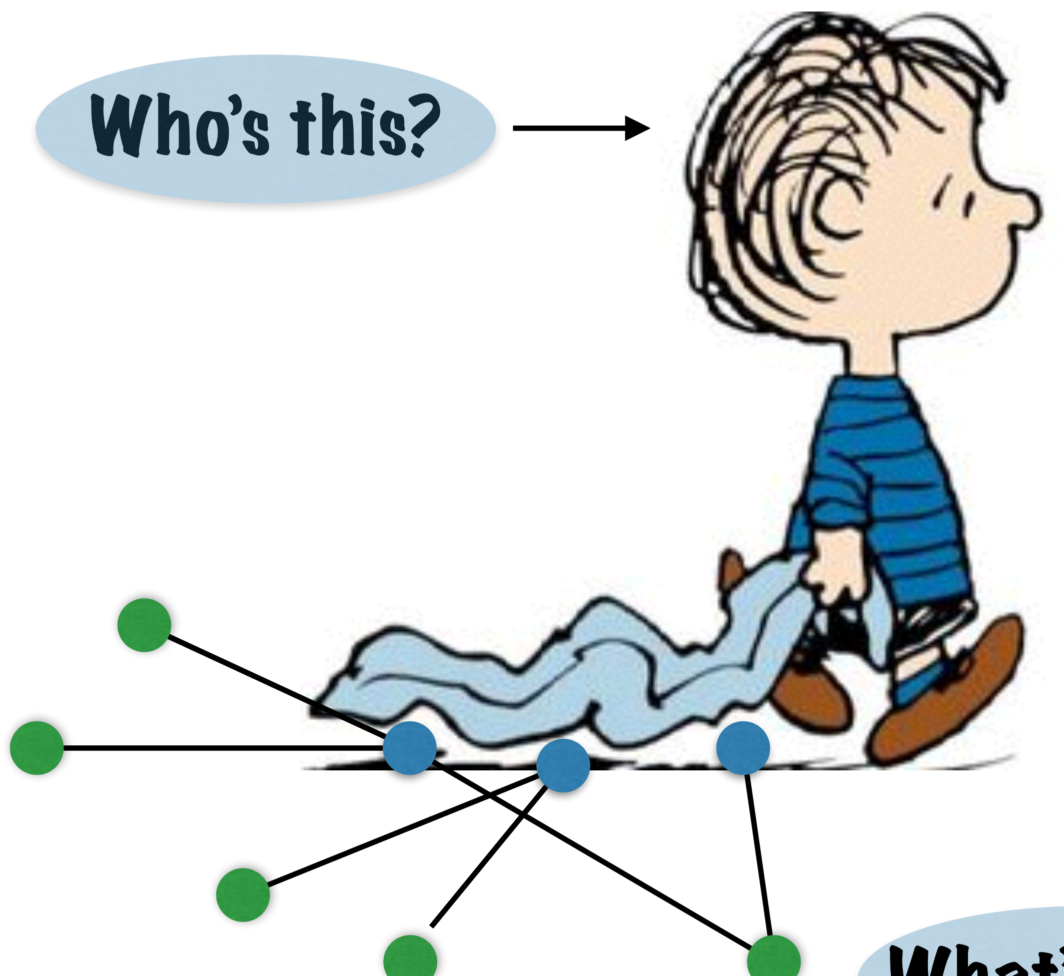
Approximation algorithms, vertex cover, and linear programming

Who's this?



$$\min c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

such that

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n & \geq b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n & \geq b_2 \\ \cdots & \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n & \geq b_m \\ \forall i: \quad 0 \leq x_i \leq 1 & \\ \forall i: \quad x_i \ \underline{\text{real number}} & \end{cases}$$

What's that?

Quality of output?

2. Round the LP solution

$$\mathbf{z_u} \in \{0, 1\}$$

3. Output $\{\mathbf{u} : \mathbf{z_u} = 1\}$

Output cost $= \sum_{\mathbf{u}} \mathbf{w_u z_u}$

1. Solve the LP

$$(\mathbf{x}^*_{\mathbf{u}})$$

2. Round the LP solution

$$\mathbf{z}_{\mathbf{u}} = \begin{cases} 1 & \text{if } \mathbf{x}^*_{\mathbf{u}} \geq \mathbf{1/2} \\ 0 & \text{otherwise} \end{cases}$$

Observe: $\mathbf{z}_{\mathbf{u}} \leq \mathbf{2x}^*_{\mathbf{u}}$

1. Solve the LP

IP min: x(u)=1 iff u in optimum vertex cover

$$\min \sum_{\mathbf{u}} \mathbf{w_u x_u}$$
$$\mathbf{x_u} + \mathbf{x_v} \geq 1$$
$$\mathbf{x_u} \in \{0, 1\}$$

LP min: x*(u)

$$\min \sum_{\mathbf{u}} \mathbf{w_u x_u^*}$$
$$\mathbf{x_u^*} + \mathbf{x_v^*} \geq 1$$
$$\mathbf{0} \leq \mathbf{x_u^*} \leq \mathbf{1}$$

The LP is a relaxation of the IP

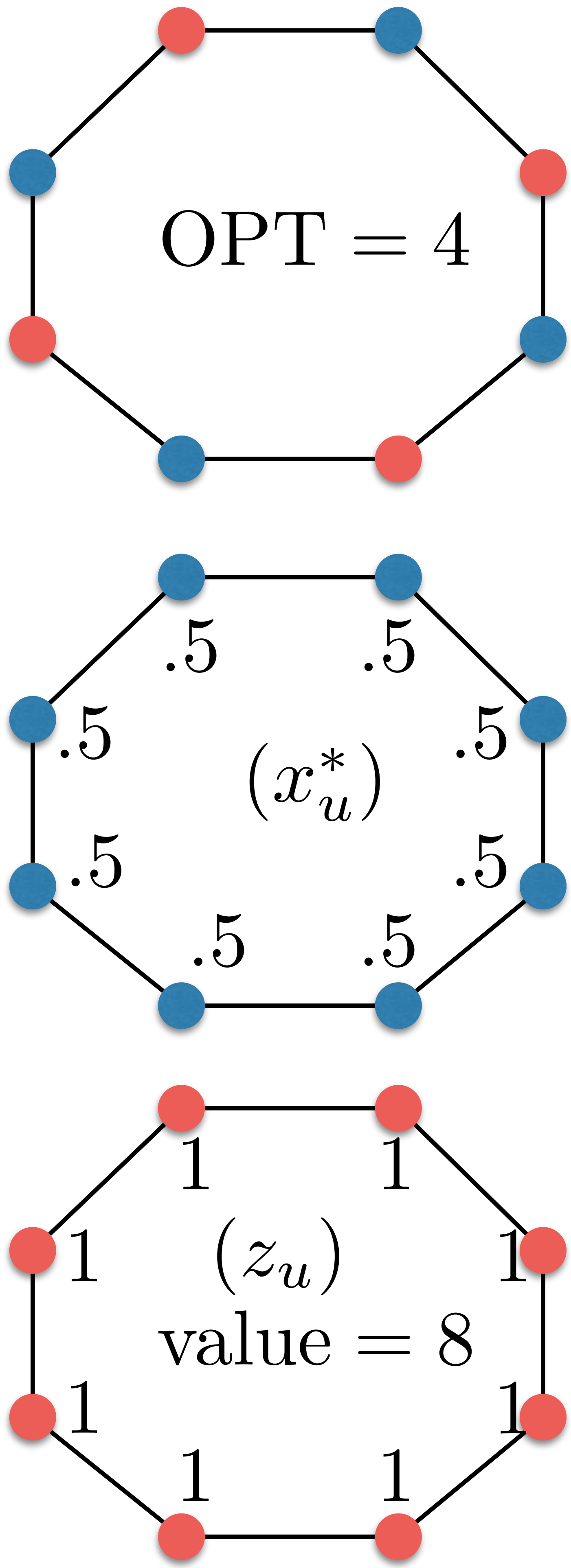
$$\sum_{\mathbf{u}} \mathbf{w_u x_u^*} \leq \mathrm{OPT}$$

Combine

$$\text{Output cost} = \sum_{\mathbf{u}} \mathbf{w_u z_u}$$
$$\leq \mathbf{2} \sum_{\mathbf{u}} \mathbf{w_u x_u^*}$$
$$\leq \mathbf{2}\mathrm{OPT}$$

Thm: output is a vertex cover of value at most 2 OPT

Is the analysis tight?



Yes

How good is that?

Typical performance (hearsay):
within 10% of optimum

How do we know?

Can compare output value to

$$\sum_{\mathbf{u}} \mathbf{w}_{\mathbf{u}} \mathbf{x}^*_{\mathbf{u}}$$

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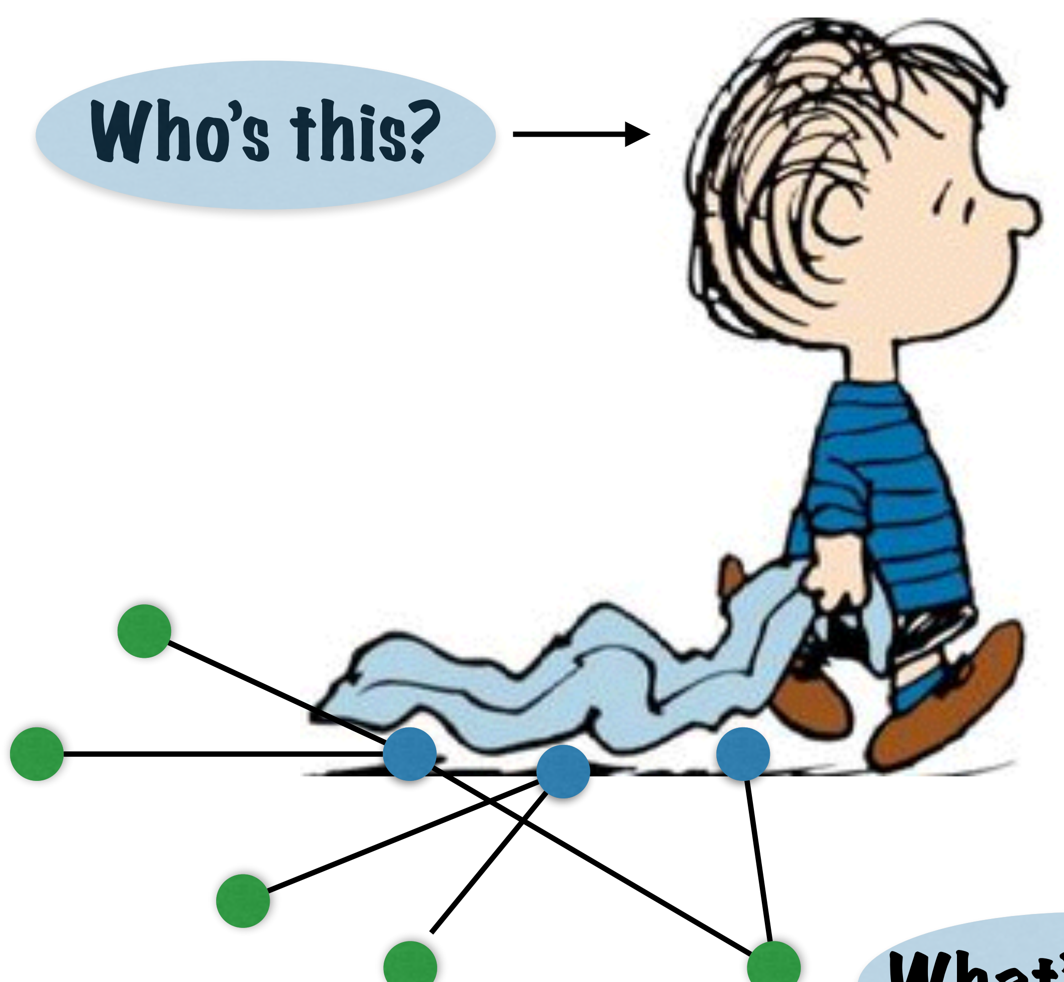

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