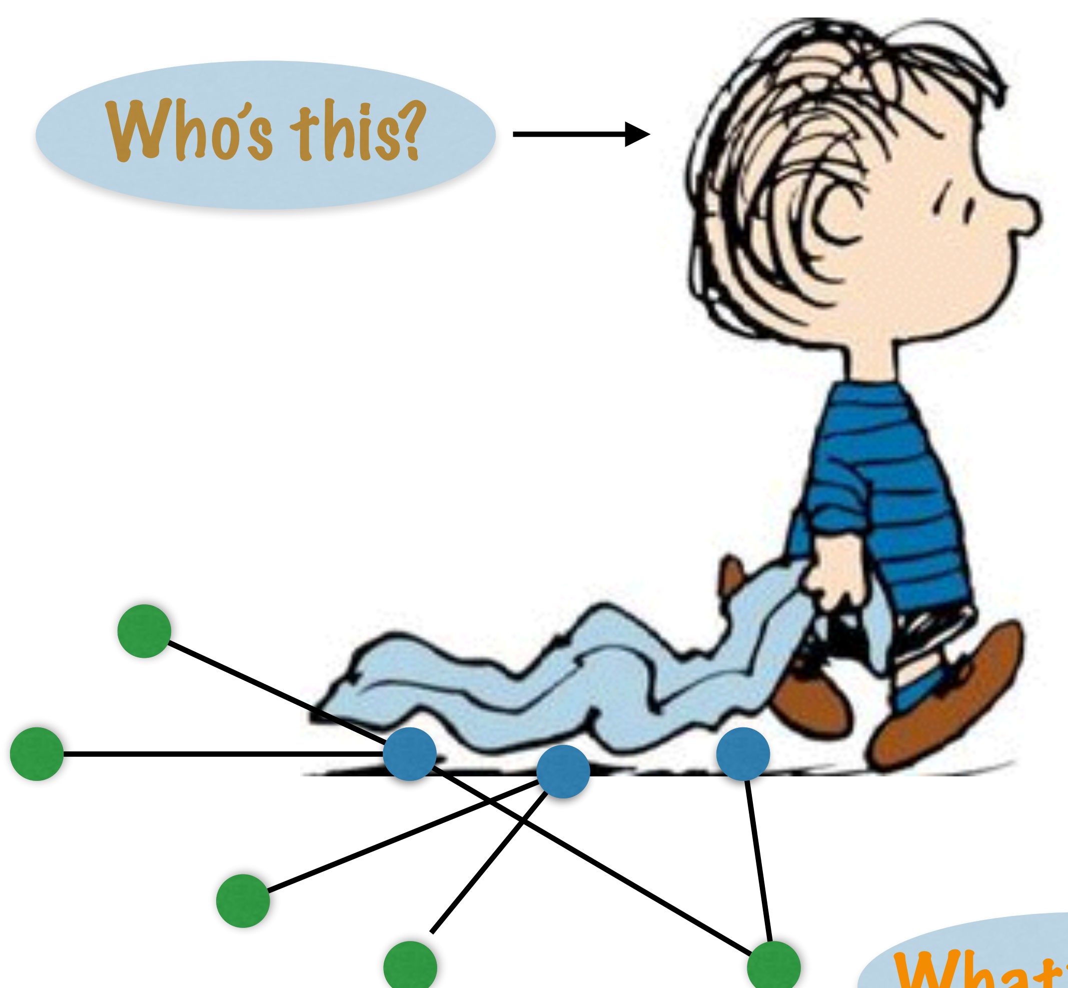
Approximation algorithms, vertex cover, and linear programming

Who's this?



$$\min c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

such that

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n & \geq b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n & \geq b_2 \\ \ldots & \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n & \geq b_m \\ \forall i: \quad 0 \leq x_i \leq 1 & \\ \forall i: \quad x_i \ \underline{\text{real number}} & \end{cases}$$

What's that?

Integer program

$$\min c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

such that

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n & \geq b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n & \geq b_2 \\ \ldots & \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n & \geq b_m \\ \forall i: \quad 0 \leq x_i \leq 1 & \\ \forall i: \quad x_i \text{ integer} & \end{cases}$$

NP-hard

Linear program

$$\min c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

$$\text{such that}$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n & \geq b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n & \geq b_2 \\ \cdots & \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n & \geq b_m \\ \forall i: \quad 0 \leq x_i \leq 1 & \\ \forall i: \quad x_i \ \underline{\text{real number}} & \end{cases}$$

polynomial time

Two ways to present

$$\min c_1x_1 + c_2x_2 + \cdots + c_nx_n$$
such that
$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n & \geq b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n & \geq b_2 \\ \ldots & \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n & \geq b_m \\ \forall i: \quad 0 \leq x_i \leq 1 & \\ \forall i: \quad x_i \ \underline{\text{real number}} & \end{cases}$$

Linear program

$$\min c \cdot x \text{ such that}$$
$$\begin{cases} Ax \geq b \\ x \in [0,1]^n \\ x \text{ vector of } \mathbb{R}^n \end{cases}$$

Same linear program

Integer vs. linear programs

IP

$$\min c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

such that

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n & \geq b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n & \geq b_2 \\ \cdots & \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n & \geq b_m \\ \forall i: \quad 0 \leq x_i \leq 1 & \\ \forall i: \quad x_i \text{ integer} & \end{cases}$$

NP-hard

LP

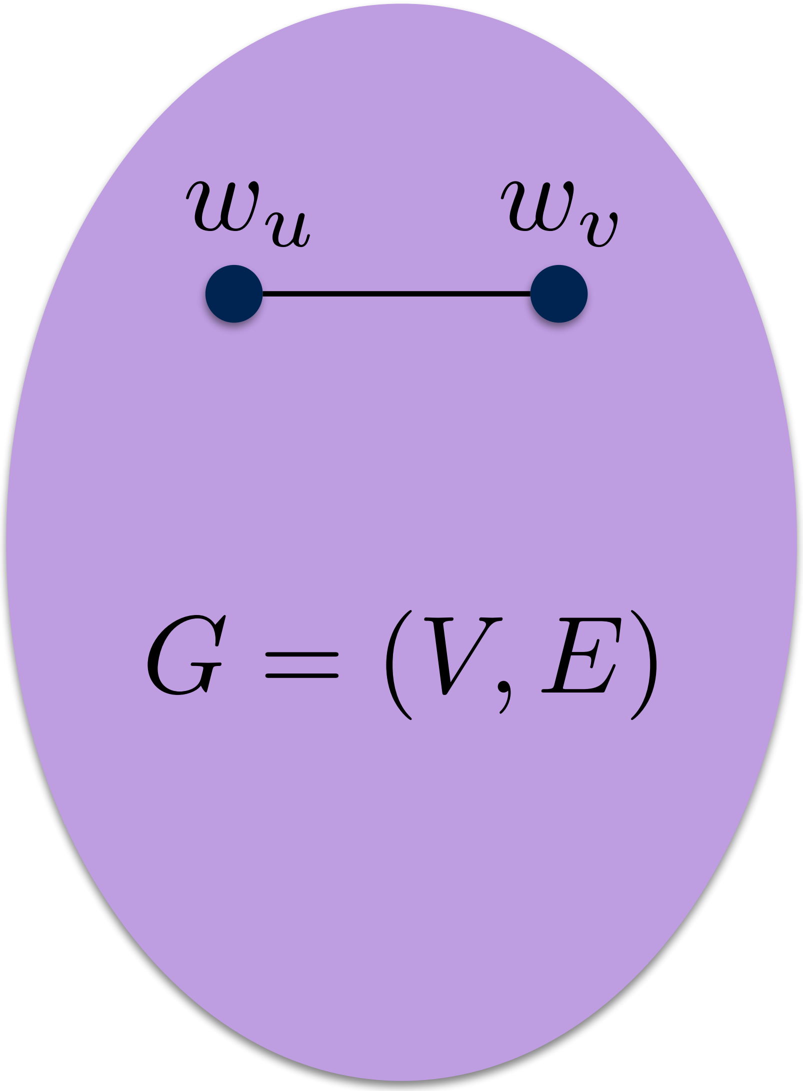
$$\min c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

such that

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n & \geq b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n & \geq b_2 \\ \cdots & \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n & \geq b_m \\ \forall i: \quad 0 \leq x_i \leq 1 & \\ \forall i: \quad x_i \ \underline{\text{real number}} & \end{cases}$$

polynomial time

Vertex cover linear program



Constraints:

$$\forall u \in V : 0 \leq x_u \leq 1$$

$$\forall \{u, v\} \in E : x_u + x_v \geq 1$$

Objective: $\min \sum_u w_u x_u$

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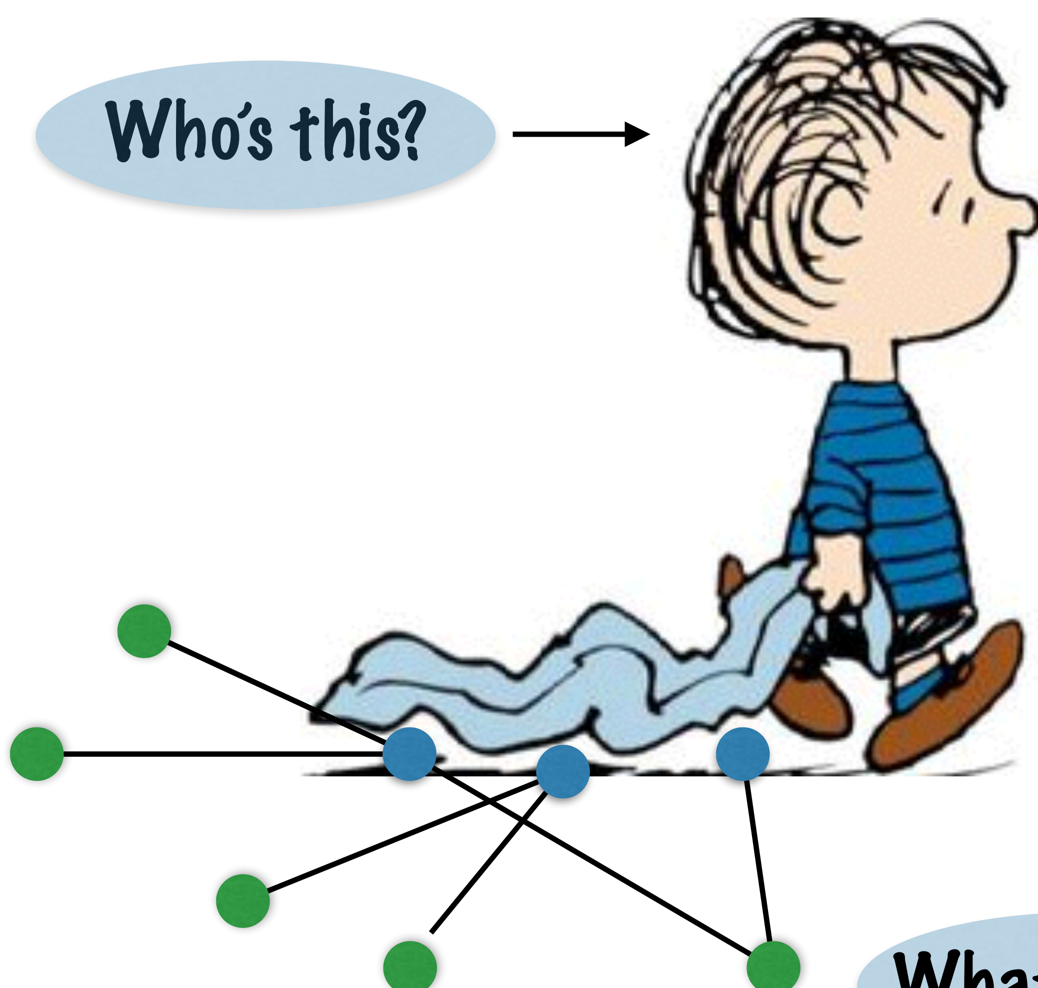

$$
\begin{aligned}
&\min c_1x_1 + c_2x_2 + \cdots + c_nx_n \\
&\text{ such that} \\
&\begin{cases}
a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n & \geq b_1 \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n & \geq b_2 \\
\cdots & \\
a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n & \geq b_m \\
\forall i: \quad 0 \leq x_i \leq 1 & \\
\forall i: \quad x_i \ \underline{\text{real number}} &
\end{cases}
\end{aligned}
$$

What's that?