Set cover, linear programming and randomized rounding
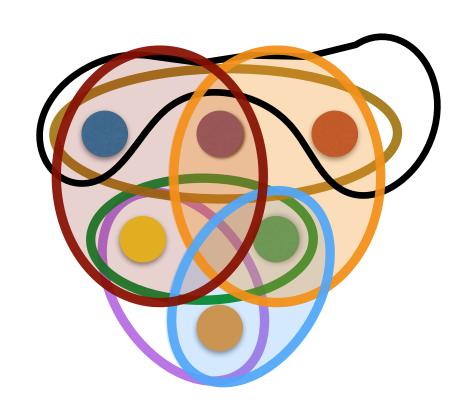
Linear programming relaxation

$$\min \sum_{\mathbf{S}} \mathbf{c_S x_S}$$

such that

$$\begin{cases} \sum_{S:e \in S} x_S \geq 1 & \forall e \\ 0 \leq x_S \leq 1 & \forall S \end{cases}$$

How do we round the LP solution?

Randomized Rounding: An algorithm

$x_i = .9$: should probably go to 1

$x_i = .1$: should probably go to 0

Cannot fix a threshold

Idea: randomized rounding

$x_i = .8 \implies$ round to 1
w.p. 80%

New rounding algorithm

For each set S
with probability x_S
put S in the cover

Analysis

Is it efficient?
Is the output a cover?
How good is it?

Analysis

Is it efficient?

Yes

Analysis

Is the output a cover?
Maybe, maybe not

Analysis

How good is it?
It depends...

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