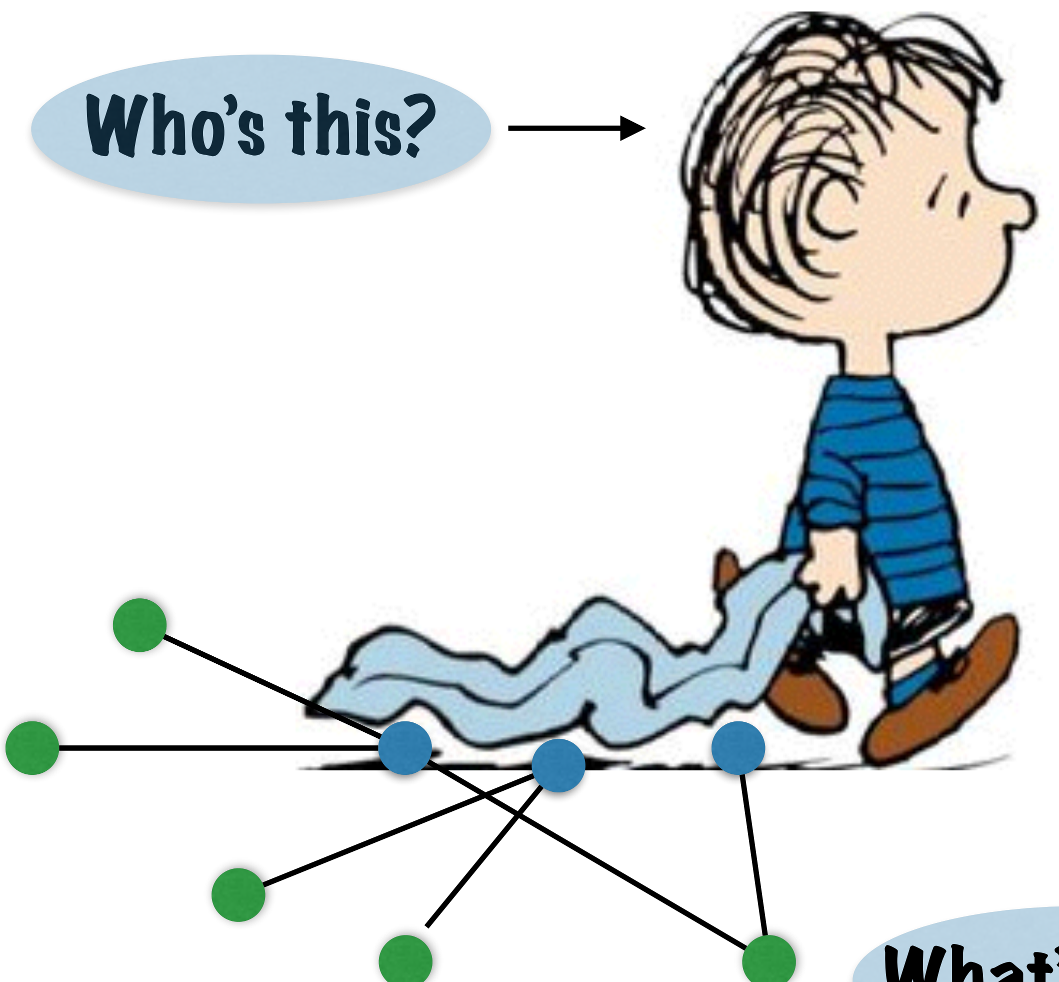
Approximation algorithms, vertex cover, and linear programming



$$
\begin{aligned}
&\min c_1x_1 + c_2x_2 + \cdots + c_nx_n \\
&\ \text{such that} \\
&\begin{cases}
a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n & \geq b_1 \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n & \geq b_2 \\
\cdots & \\
a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n & \geq b_m \\
\forall i: \quad 0 \leq x_i \leq 1 & \\
\forall i: \quad x_i \ \underline{\text{real number}} &
\end{cases}
\end{aligned}
$$

What's that?

The method

Meta approximation algorithm

- Find IP
- Solve LP relaxation
- Round solution to integers

Analysis

- correct: does it satisfy conditions?
- efficient: polynomial runtime?
- good: value of output solution within factor of optimal value?

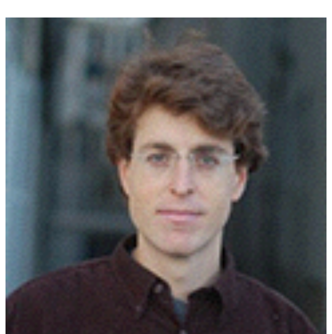
How good is it?

Method

- Output can be related to LP value
- OPT can be bounded by LP value
- Combine

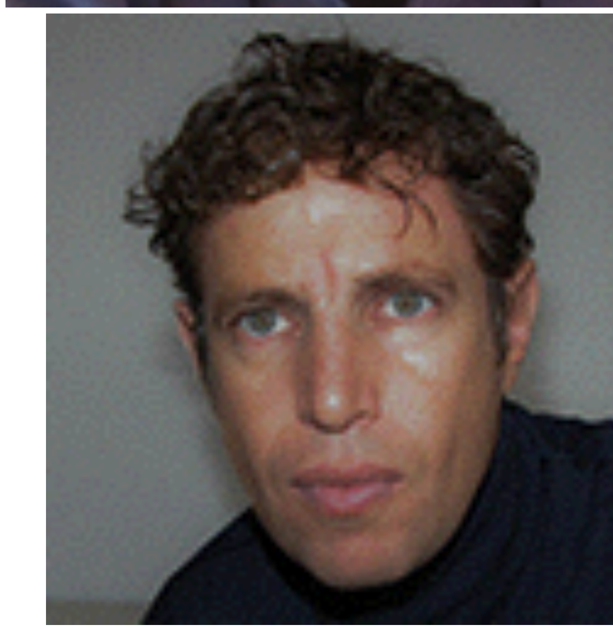
Message: for analysis, focus on LP value.

Karp (1972)
NP-complete



Khot
Regev
(2003)
Conditional <2 hard

Papadimitriou
Yannakakis:
1.0001 hard (1991)



Dinur
Safra:
1.36 hard (2002)

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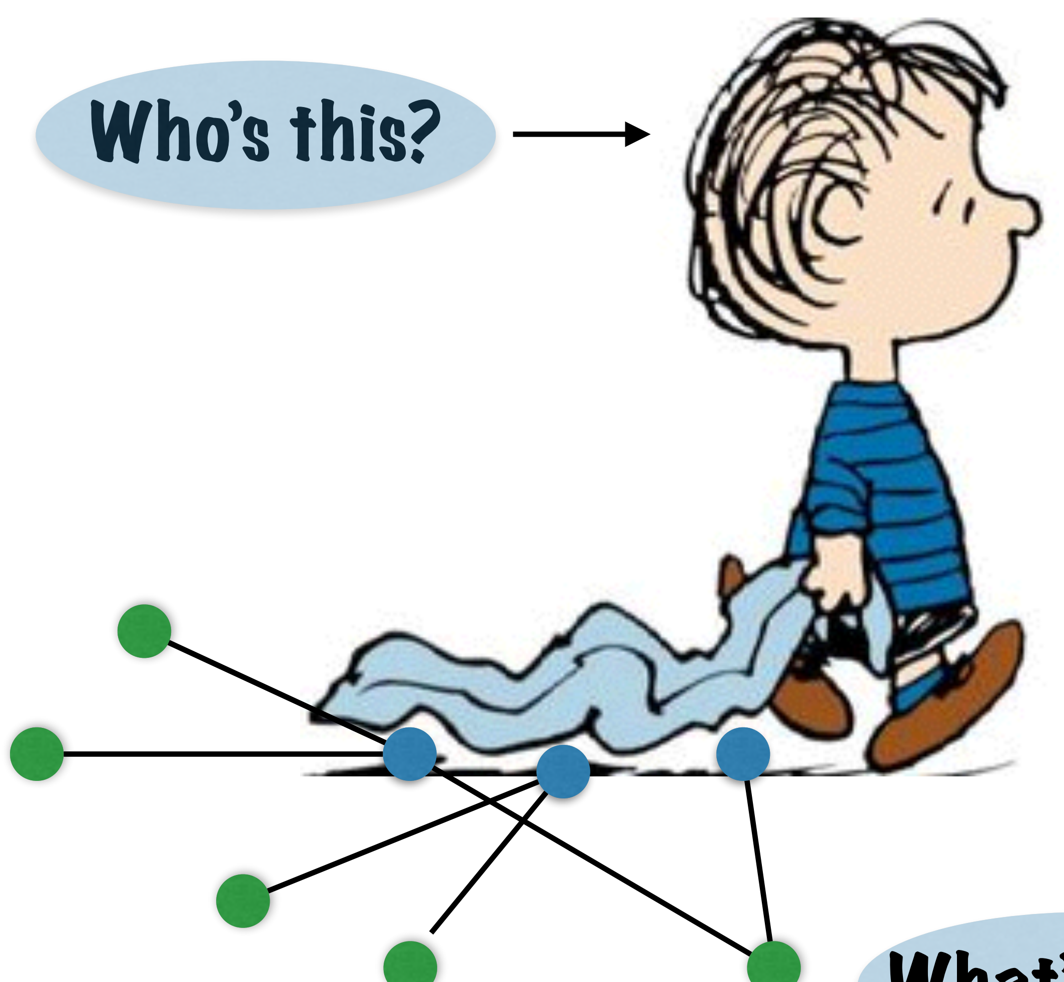
Who's this?



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