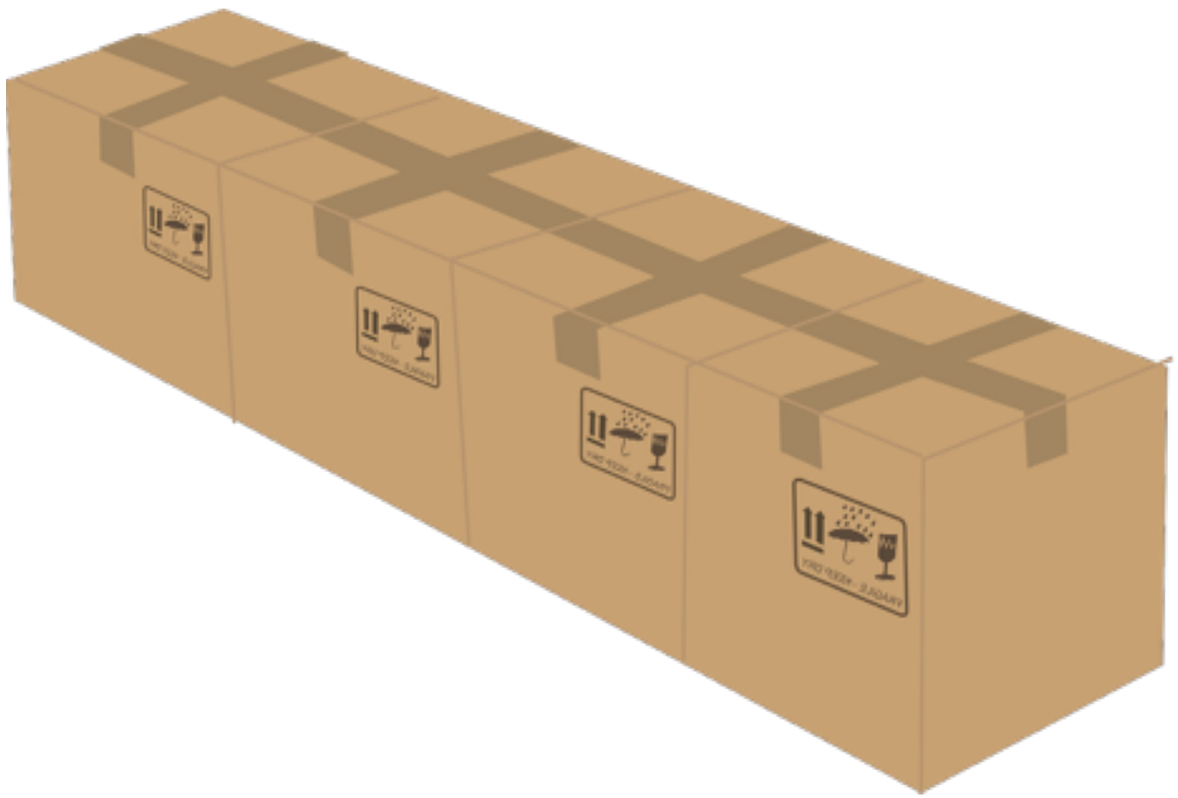
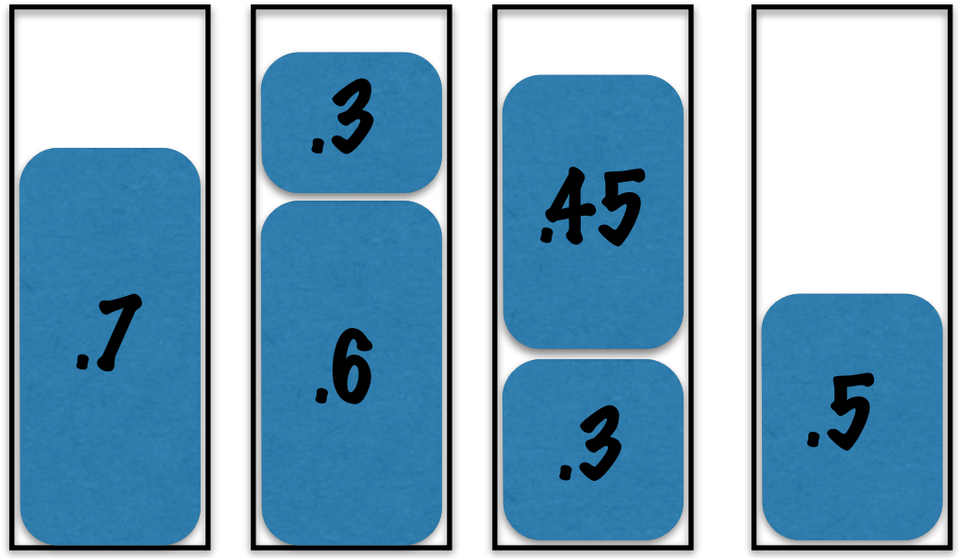
Bin packing, linear programming and rounding





Can we do better than Next Fit?

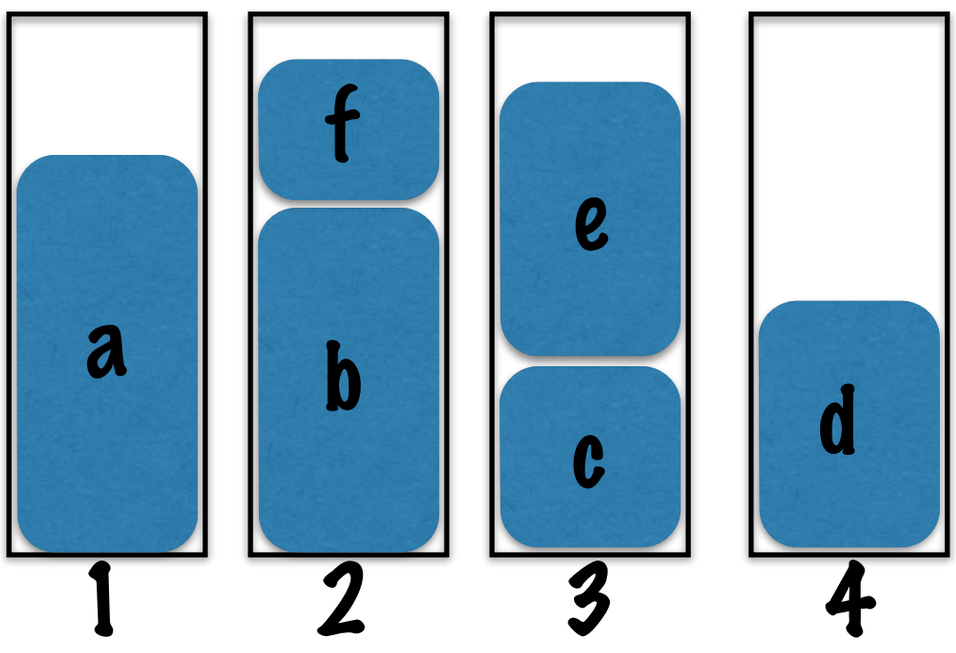
First tool: linear programming relaxation

An integer program

Given n items and K unit bins, is there a packing?

Variables: $x_{ij} \in \{0, 1\}$

$x_{ij} = 1$ **iff item i is placed in bin j**



$$x_{a1} = x_{b2} =$$

$$x_{f2} = x_{c3} =$$

$$x_{e3} = x_{d4} = 1,$$

$x_{ij} = 0$ **otherwise**

An integer program

Constraint: every item must go somewhere

Item b must go into bin 1,2,3 or 4

$$x_{b1} + x_{b2} + x_{b3} + x_{b4} = 1$$

An integer program

Constraint: must not exceed bin capacity

Item sizes in bin j sum to at most 1.

$$x_{aj}s_a + x_{bj}s_b + \cdots + x_{fj}s_f \leq 1$$

Integer program

n items, K bins

$x_{ij} =$ whether item i goes into bin j

$\forall i: \ \sum_j x_{ij} = 1$

$\forall j: \ \sum_i x_{ij} s_i \leq 1$

$\forall i, j: \ x_{ij} \in \{0, 1\}$

feasible iff items can be packed into K bins

Linear programming relaxation

$$n \text{ items, } K \text{ bins}$$
$$\forall i: \ \sum_j x_{ij} = 1$$
$$\forall j: \ \sum_i x_{ij} s_i \leq 1$$
$$\forall i,j: \ 0 \leq x_{ij} \leq 1$$

Algorithm: use the LP relaxation to pack items (somehow)

How good is the relaxation?

A bad example

n items, K bins

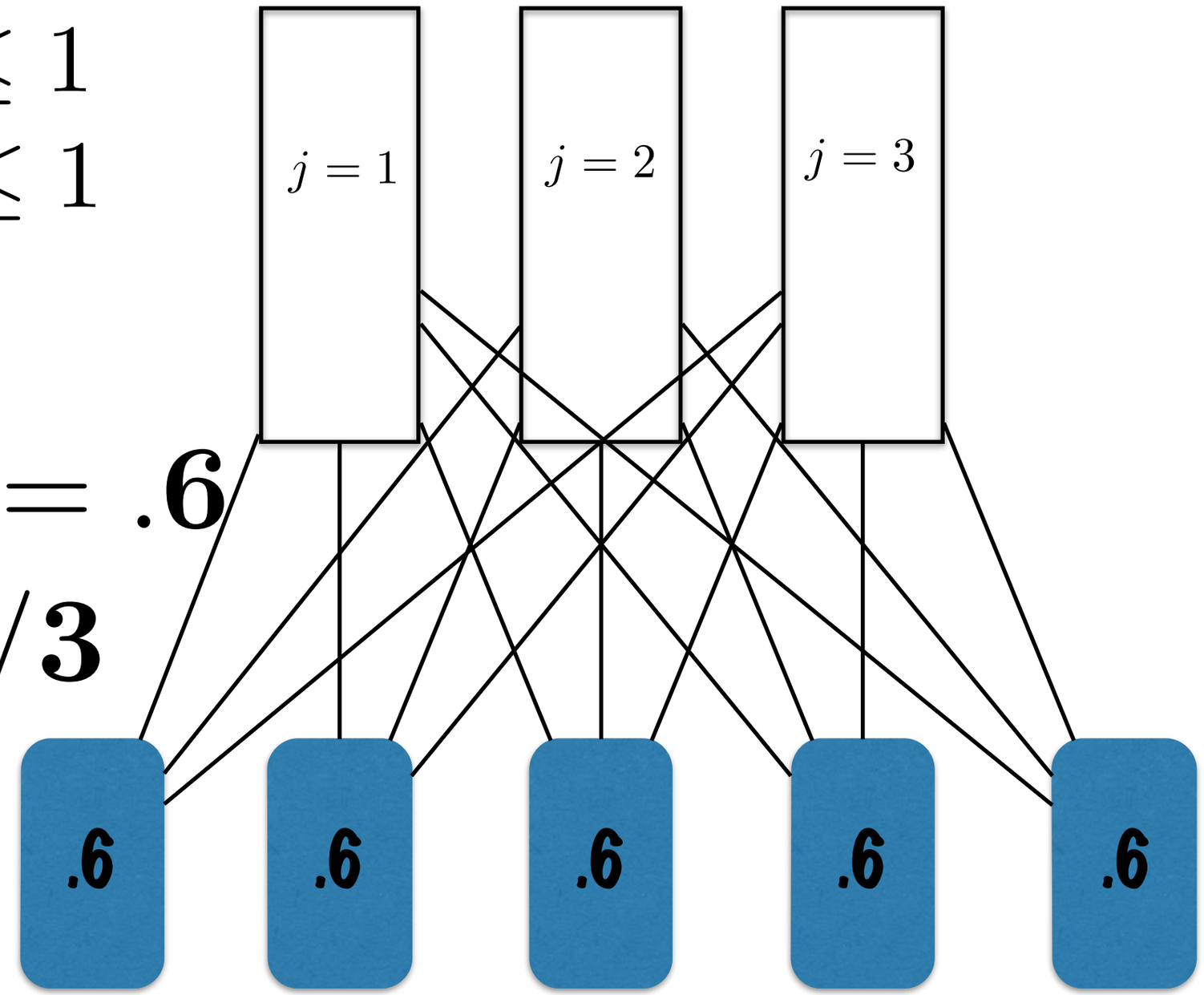
$$\forall i: \ \sum_j x_{ij} = 1$$
$$\forall j: \ \sum_i x_{ij} s_i \leq 1$$
$$\forall i,j: \ 0 \leq x_{ij} \leq 1$$

$$\mathbf{s_1 = \ldots = s_5 = .6}$$

$$\mathbf{\forall i,j \ \ x_{ij} = 1/3}$$

LP: 3 bins

OPT: 5 bins



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