

Pracise Exercise on Knapsack

The solutions will be available in 1-3 weeks in one of the following modules.

Question 1

Consider the following greedy algorithm for knapsack packing.

- (a) Sort items in non-increasing order of v_i/s_i
- (b) Greedily add items until we hit an item a_L that is too big ($\sum_{k=1}^L s_L > B$)
- (c) Pick the better of $\{a_1, a_2, \dots, a_{L-1}\}$ and $\{a_L\}$.

Your task is the following.

- (a) Show that the value of the solution found by the greedy algorithm is at least half of the (unknown) optimal value as the number of items n tends to infinity. (2-Approximation)
- (b) There exists an instance (set of items) such that the optimal value reached by the greedy algorithm is half of the value reached by the optimal algorithm as the size of knapsack goes to infinity. (tightness)

Question 2

You are doing the Tour de France and you are given a map with all the n places where you can refill your water bottles. Your bottles fit 2 liters of water with which are enough for m kilometres. Your goal is to stop as few times as possible to refill your bottles, since you care about a good time for the Tour de France. Design an efficient algorithm using dynamic programming, prove its correctness and analysis its run time. Give the one-dimensional array A . And the time complexity of filling your array.