Knapsack and rounding

Approximation schemes

The general algorithm

1. Let $N = 1000\,n$
2. Let $v'_i \leftarrow \left\lfloor v_i \times \frac{N}{\max_j v_j} \right\rfloor$
3. Apply DP to $B, (s_i, v'_i)_i$
4. Output corresponding items

Theorem: this gives a solution to knapsack with value at least .999 OPT and with runtime O(poly(n))

Approximation scheme : family of algorithms: One for each $\epsilon > 0$ runtime polynomial in input output value is near-optimal:

$$|\text{Value(Output)-OPT}| \leq \epsilon \times \text{OPT}$$

Theorem: knapsack has an approximation scheme

How ϵ comes in

Q: The smaller ϵ, the closer to OPT. Why not let it go to 0 and find the exact OPT in O(poly(n))?

A: Runtime increases as ϵ goes to zero. Runtime has N with $N=n/\epsilon$ so it goes to infinity.

Method:

1. Simplify the input
2. Design algorithm for “simple” inputs

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