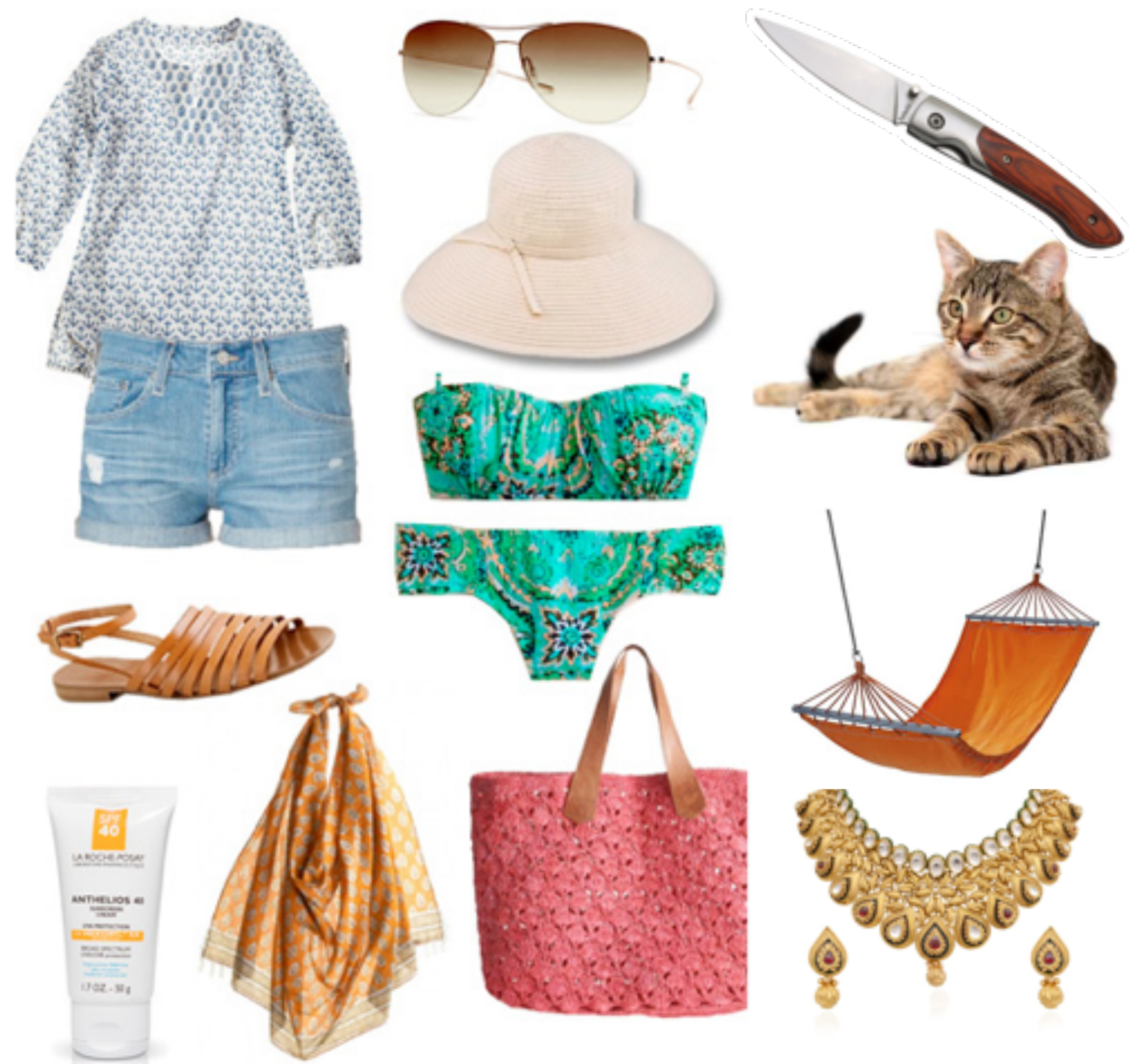
Knapsack and rounding



Analysis

- S: output items
- DP: S optimal for scaled rounded input
- S*: optimal items
- Scaling: S* optimal for scaled unrounded input

Relate S for original and modified input value

output value

$$
\begin{aligned}
\mathbf{Value(S)} &= \sum_{\mathbf{S}} \mathbf{v_i} \\
&= \frac{\mathbf{1}}{\alpha} \sum_{\mathbf{S}} (\alpha \mathbf{v_i}) \\
&\geq \frac{\mathbf{1}}{\alpha} \sum_{\mathbf{S}} \mathbf{v'_i}
\end{aligned}
$$

value for scaled&rounded input

Relate S to S*
on modified input

$$\sum_{S} v'_i \geq \sum_{S^*} v'_i$$

S optimal for scaled&rounded

Relate S* for original and modified input value

$$\mathbf{v}_\mathbf{i}' > \alpha \mathbf{v}_\mathbf{i} - 1$$

effect of rounding

Sum:

$$\sum_{\mathbf{S}^*} \mathbf{v}_\mathbf{i}' > \alpha \sum_{\mathbf{S}^*} \mathbf{v}_\mathbf{i} - \mathbf{n}$$

Combine and substitute:

$$
\begin{aligned}
\mathbf{Value(S)} &\geq \frac{1}{\alpha}\sum_{\mathbf{S}} \mathbf{v}'_{\mathbf{i}} \\
&\geq \frac{1}{\alpha}\sum_{\mathbf{S}^*} \mathbf{v}'_{\mathbf{i}} \\
&\geq \frac{1}{\alpha}\left[\alpha\sum_{\mathbf{S}^*} \mathbf{v}_{\mathbf{i}} - \mathbf{n}\right] \\
&= \mathbf{OPT} - \frac{\mathbf{n}}{\alpha} \\
&= \mathbf{OPT} - \frac{\mathbf{n}\times\max\mathbf{v}_{\mathbf{i}}}{\mathbf{N}}
\end{aligned}
$$

Lower bound OPT

Discarded items that don't fit:

$$\mathrm{OPT} \geq \max \mathbf{v_i}$$

Wrapping up

$$\mathbf{Value(S) \geq OPT - \frac{n}{N}OPT}$$

$$\mathbf{N = 100 \times n}$$

$$\mathbf{Value(S) \geq .99 \times OPT}$$

Theorem: Solution to knapsack with value at least .99 OPT and runtime O(poly(n))

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