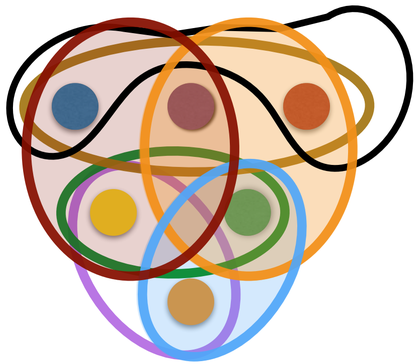
Set cover, linear programming and randomized rounding



How good is it?

It depends...

$$\mathbf{Value(Output)} = \sum_{S \textbf{ in cover}} c_S$$

How good is it on average?

$$\mathbf{E}[\sum_{\mathbf{S}} \mathbf{1(S \text{ in cover})c_S}] = ?$$

Linearity of expectation

$$\mathbf{E}[\mathbf{A} + \mathbf{B}] = \mathbf{E}[\mathbf{A}] + \mathbf{E}[\mathbf{B}]$$

$$\mathbf{E}[\sum_{\mathbf{S}} \mathbf{1}(\mathbf{S\ in\ cover})\mathbf{c_S}] =$$
$$\sum_{\mathbf{S}} \mathbf{E}[\mathbf{1}(\mathbf{S\ in\ cover})\mathbf{c_S}]$$

$$\mathbf{E}[\lambda \mathbf{X}] = \lambda \mathbf{E}[\mathbf{X}]$$

$$\sum_{\mathbf{S}} \mathbf{E}[\mathbf{1}(\mathbf{S} \textbf{ in cover}) \mathbf{c_S}] = \sum_{\mathbf{S}} \mathbf{E}[\mathbf{1}(\mathbf{S} \textbf{ in cover})] \mathbf{c_S}$$

$$\mathbf{E}[\mathbf{1}(\mathbf{S\ in\ cover})] = \Pr[\mathbf{S\ in\ cover})]$$

$$\Pr[\mathbf{S \text{ in cover}})] = \mathbf{x_S}$$

Together

$$\mathbf{E[Value(Output)] = \sum_S x_S c_S}$$

Value of the linear program!

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