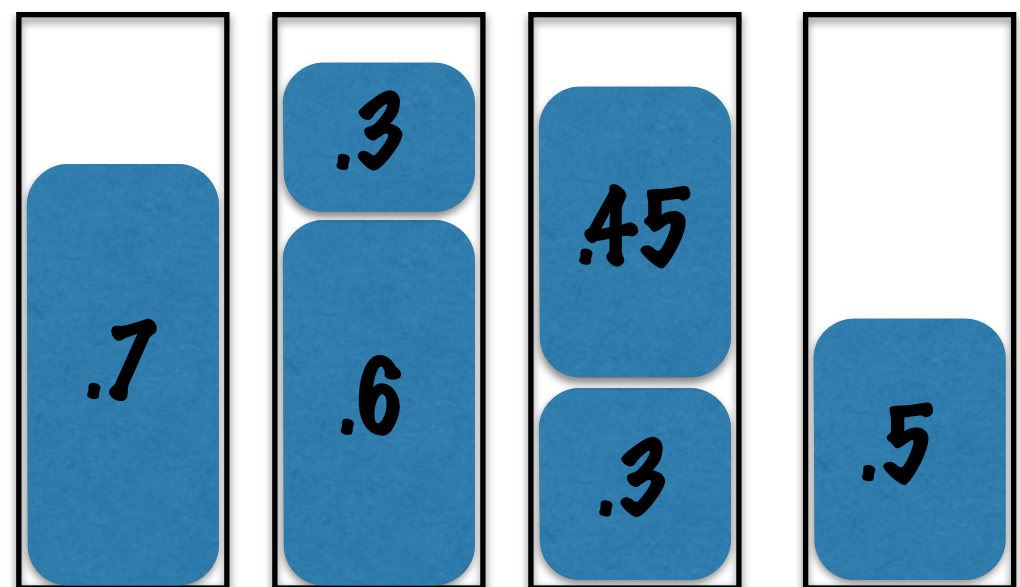
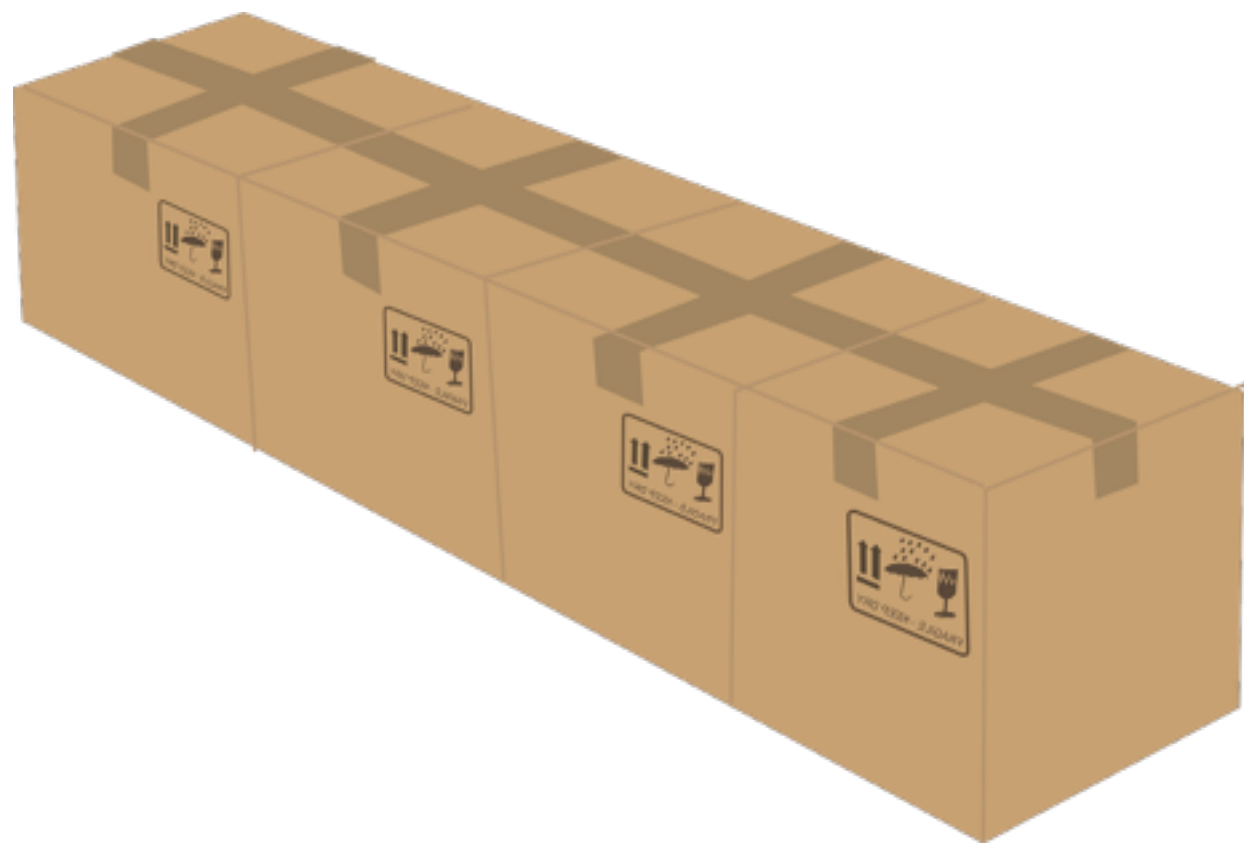
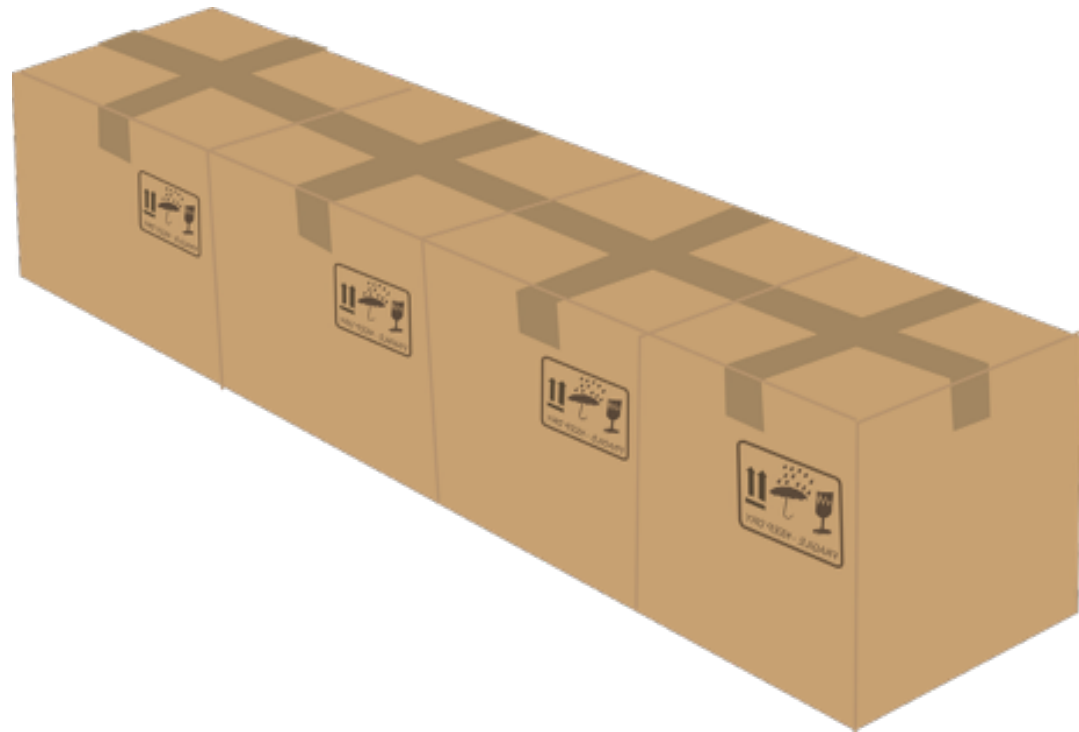


Bin packing, linear programming and rounding

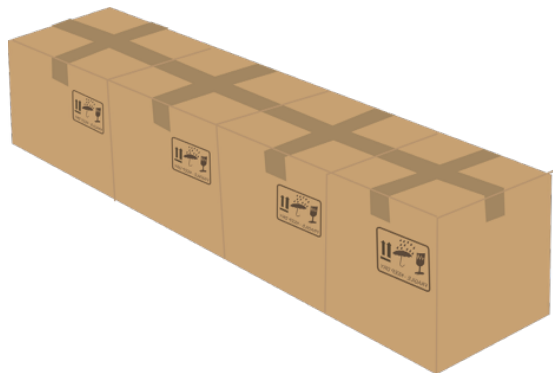


The bin packing problem

**Pack your items
using
as few bins as possible**



**Given n items
item i has size $s_i < 1$
pack items into the fewest
unit capacity bins**

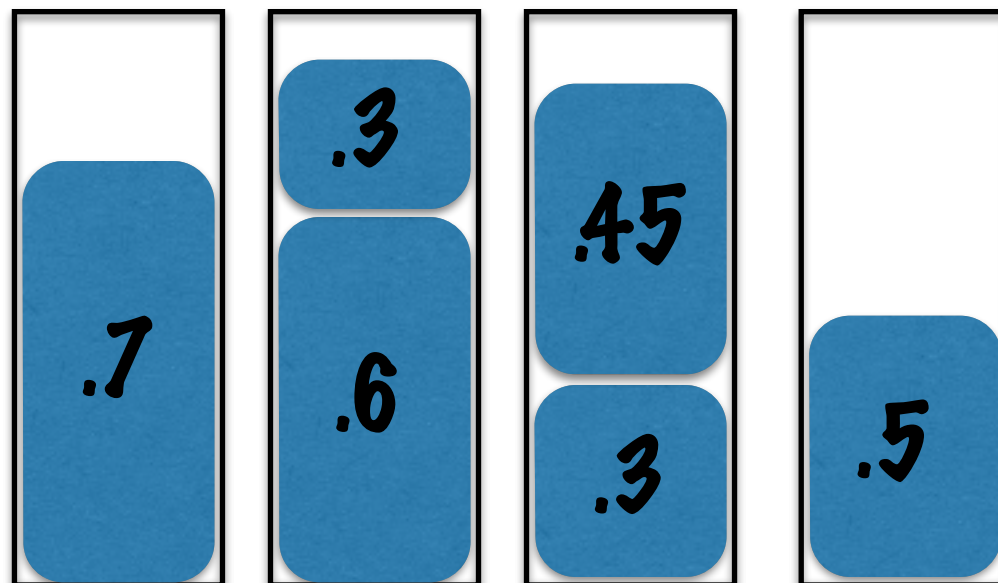


The Next Fit algorithm

**One bin at a time:
If next item does not fit,
close the bin and
open a new bin**

"Next Fit" algorithm

$s_1 = .7$
 $s_2 = .6$
 $s_3 = .3$
 $s_4 = .3$
 $s_5 = .45$
 $s_6 = .5$



can this instance
be packed better?

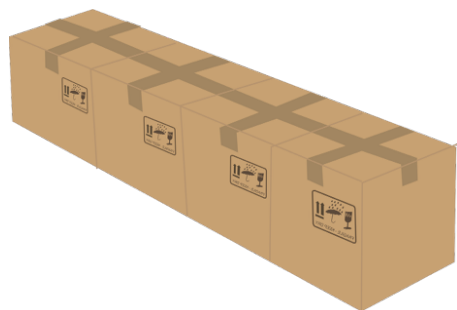
How good is Next Fit?

Capacity 100. Items:

91 50 21 90 39 43 54 95 25 14 25 61 35
28 97 16 18 42 99 95 12 7 21 23 42 50 77
100 79 36 31 71 35 1 63 6 13 92 58 73 72
32 37 62 54 18 25 9 52 93

Next Fit:

91	50	21	90	39	43	54	95	25	14	25	61	35	28	97	16	18	42	99	95	12	7	21	23	
42	50	77	100	79	36	31	71	35	1	63	6	13	92	58	73	72	32	37	62	54	18	25	9	52
93																								



used 31 bins

91	50	21	90	39	43	54	95	25	14	25	61	35	28	97	16	18	42	99	95	12	7	21	23	
42	50	77	100	79	36	31	71	35	1	63	6	13	92	58	73	72	32	37	62	54	18	25	9	52
93																								

bin 7: (25+14+25)
 next item: 61, but
 $(25+14+25) + 61 > 100$
 so, close bin 7, open bin 8,
 put item 61 in bin 8.



91	50	21	90	39	43	54	95	25	14	25	61	35	28	97	16	18	42	99	95	12	7	21	23	
42	50	77	100	79	36	31	71	35	1	63	6	13	92	58	73	72	32	37	62	54	18	25	9	52
93																								

In general:

(items in bin $2i-1$) + next item > 100

(items in bins $2i-1$ or $2i$) > 100

31 bins: total item sizes $> 15 * 100$



91	50	21	90	39	43	54	95	25	14	25	61	35	28	97	16	18	42	99	95	12	7	21	23	
42	50	77	100	79	36	31	71	35	1	63	6	13	92	58	73	72	32	37	62	54	18	25	9	52
93																								

In general:
k bins by Next Fit
Total item sizes $> (k-1)/2 * 100$



91	50	21	90	39	43	54	95	25	14	25	61	35	28	97	16	18	42	99	95	12	7	21	23	
42	50	77	100	79	36	31	71	35	1	63	6	13	92	58	73	72	32	37	62	54	18	25	9	52
93																								

What about OPT?

Total item sizes $< OPT * 100$



91	50	21	90	39	43	54	95	25	14	25	61	35	28	97	16	18	42	99	95	12	7	21	23	
42	50	77	100	79	36	31	71	35	1	63	6	13	92	58	73	72	32	37	62	54	18	25	9	52
93																								

**Combining:
k bins by Next Fit**

$$OPT * 100 > (k-1)/2 * 100$$

$$\#(\text{bins of Next Fit}) < 2 * OPT + 1$$



Asymptotic 2 approximation

Is this tight?

**Example with
OPT=501 bins,
Next Fit=1000 bins**

What about non-asymptotic?

**Distinguishing between
 $OPT=2$ and $OPT=3$
is NP-hard**

What have we learned?

**1. Crude algorithms can give
good bounds**

**Message: first try the
simplest algorithm**

**2. Analysis: for intuition, first
execute it on some concrete
examples**

Bin packing, linear programming and rounding

