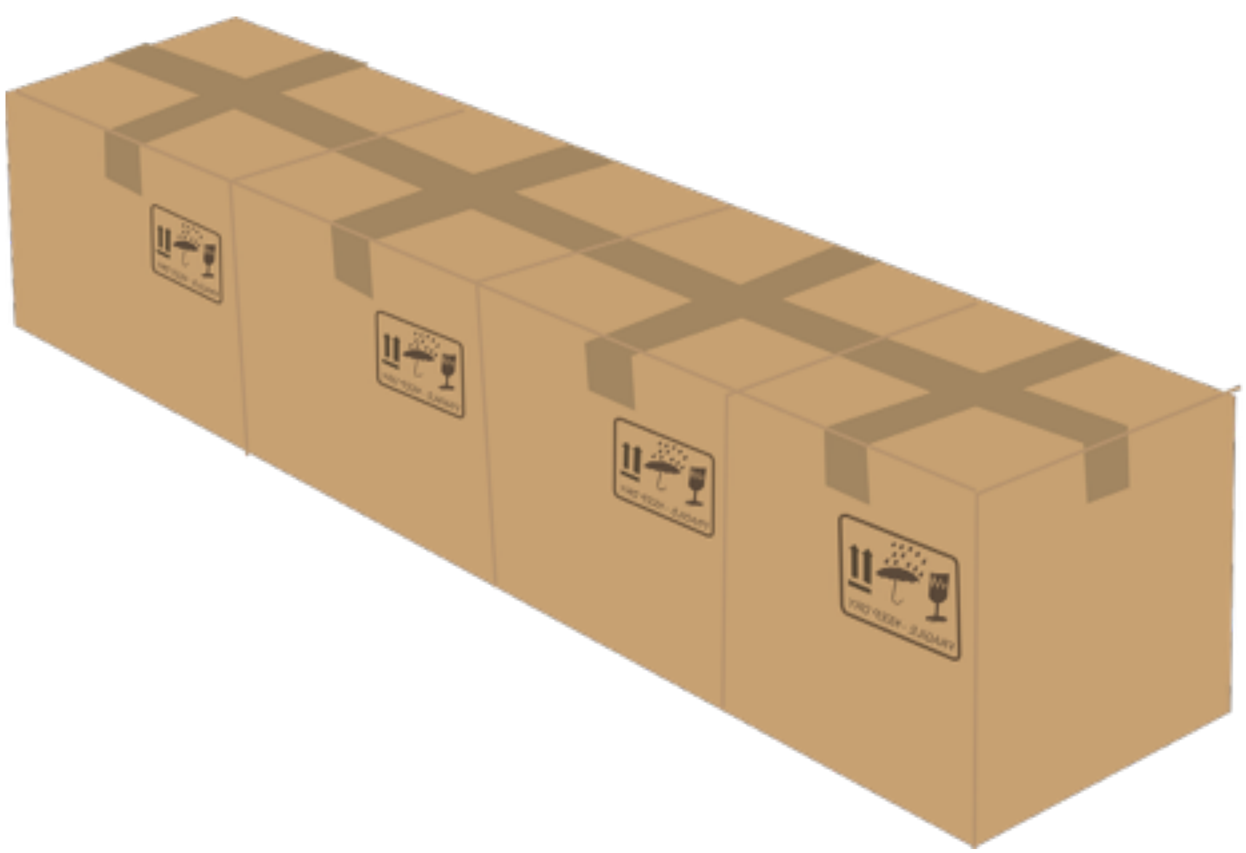
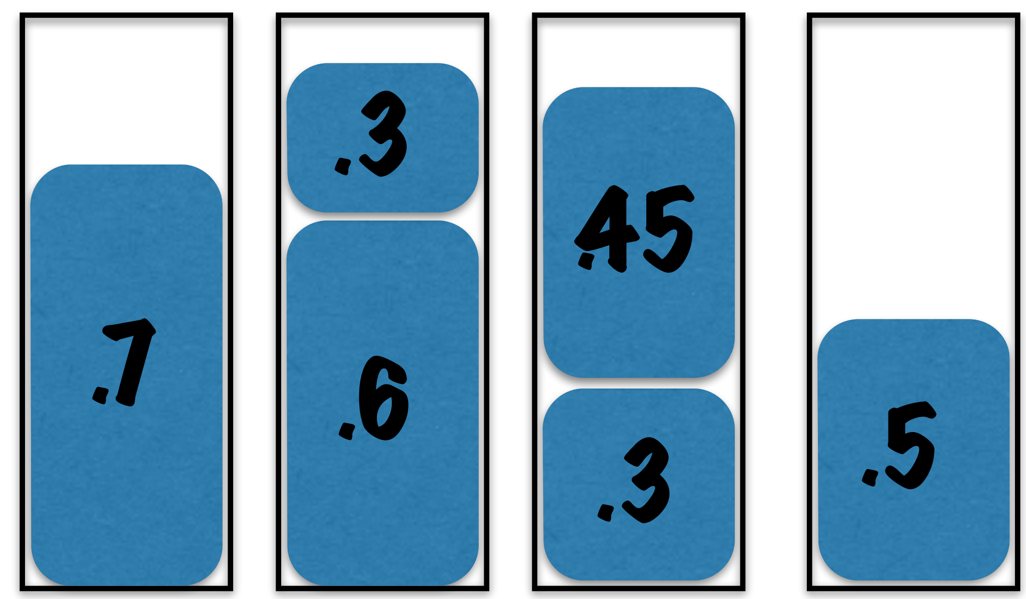
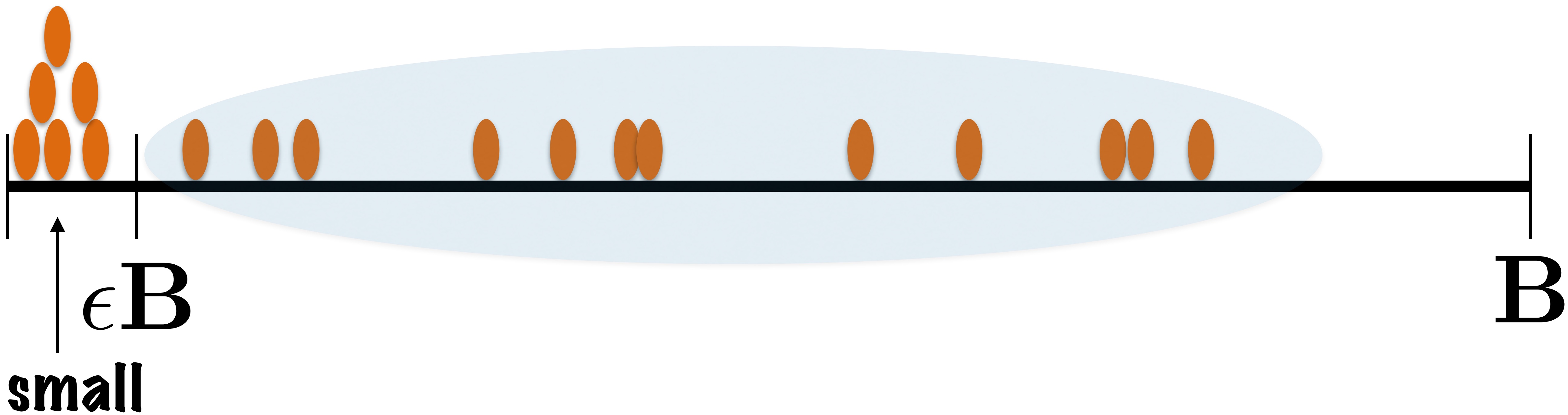
Bin packing, linear programming and rounding



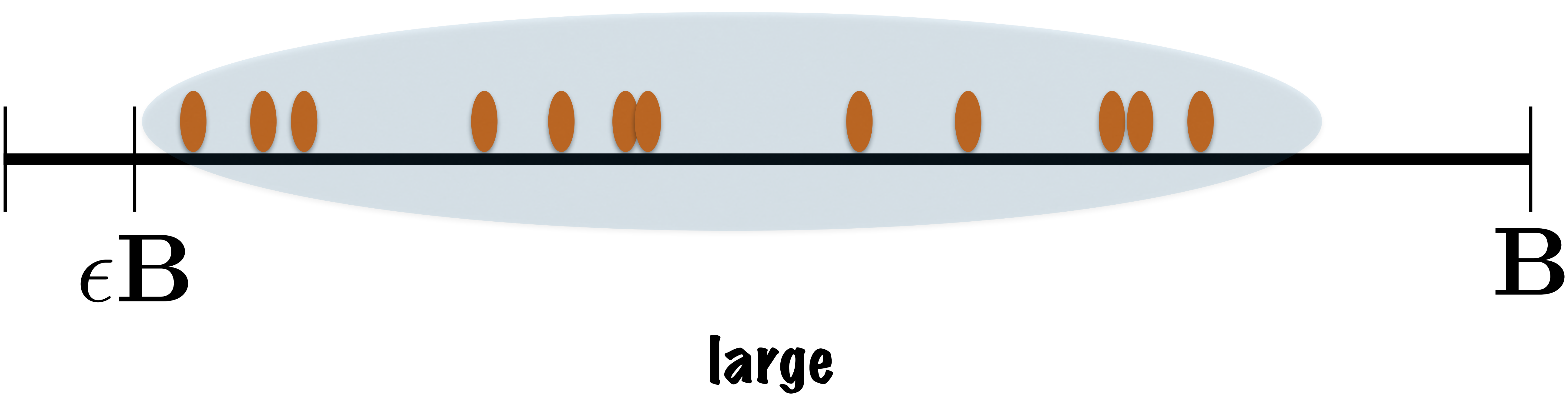


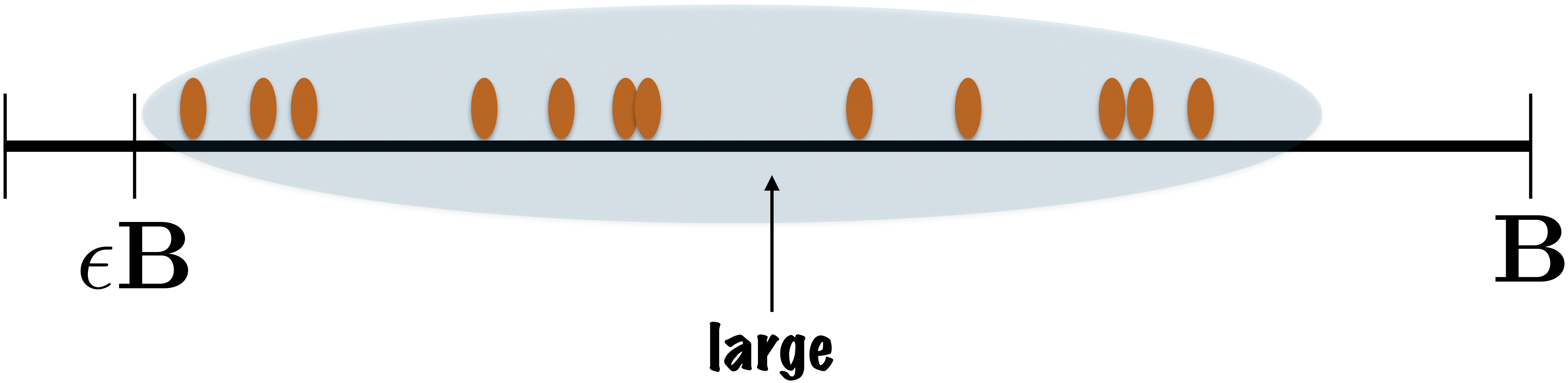
General algorithm

Set aside: sizes < cap. * ϵ (small)
Sort remaining sizes
Make groups of cardinality $\mathbf{n} \times \epsilon^{\mathbf{2}}$
Round up to max size in group
Solve rounded problem U
Greedily add small sizes

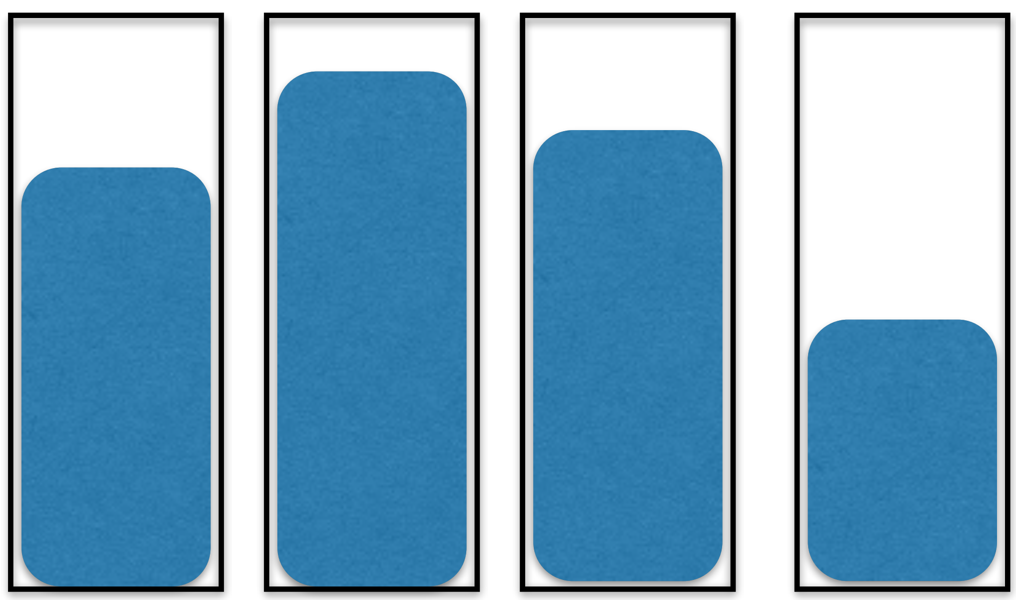


Set aside: sizes < B ϵ (small)

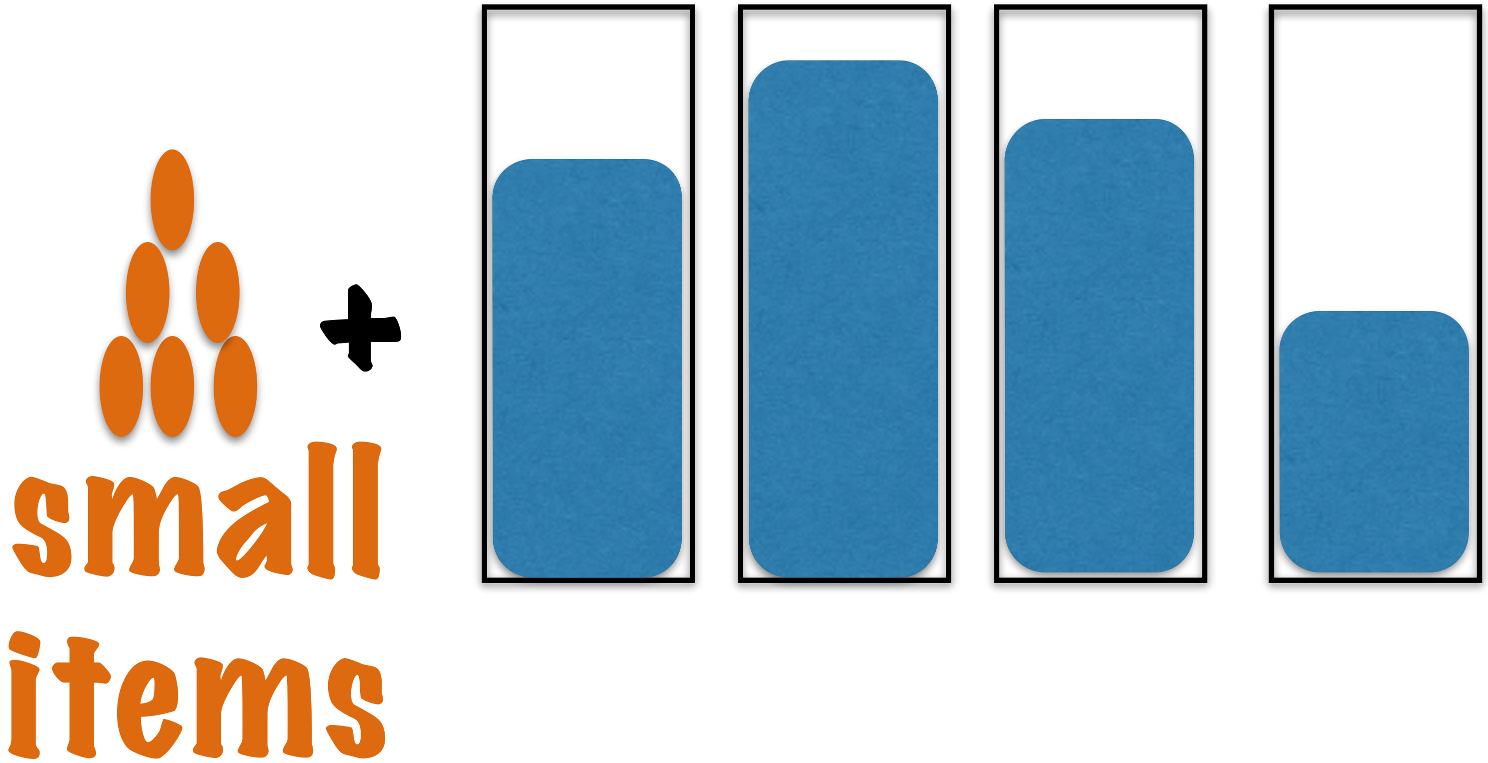




Solve for remaining sizes

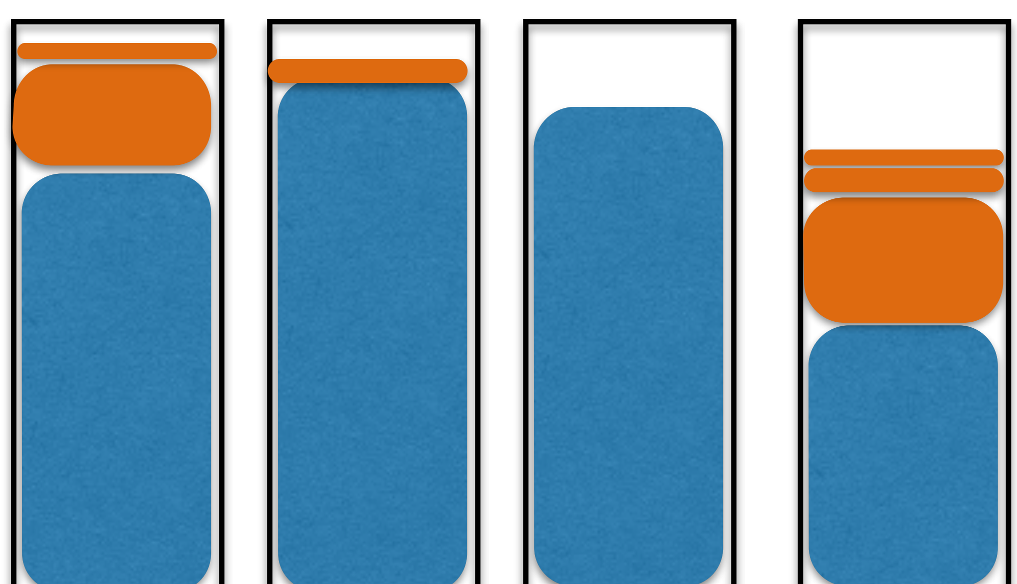


Packing of large items



Packing of large items

Greedily add small sizes



Analysis

$$\textbf{Input } \mathbf{I} = \mathbf{S} \cup \mathbf{L}$$

Case 1

No new bins opened by S: then

$$
\begin{aligned}
\mathrm{Value}(\mathrm{Output}) &= \\
\mathrm{Value}(\text{packing of } L) \\
\leq (1+\epsilon) \cdot \mathbf{OPT(L)} \\
\leq (1+\epsilon) \cdot \mathbf{OPT(I)}
\end{aligned}
$$

Case 2

Some new bin opened by S: then all bins except last are filled to B times

$$\geq 1 - \epsilon$$

$$(1/\mathrm{B}) \sum \mathrm{s_i} \geq (\#\mathrm{bins} - 1)(1 - \epsilon)$$

$$(1/\mathrm{B}) \sum \mathrm{s_i} \leq \mathrm{OPT}$$

$$\mathrm{Value(Output)} \leq \frac{1}{1-\epsilon}\mathrm{OPT} + 1$$

Theorem

Algorithm, in polynomial time gives packing s.t. Value(Output) <

$$\mathbf{OPT}(1+\mathbf{O}(\epsilon))+1$$

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