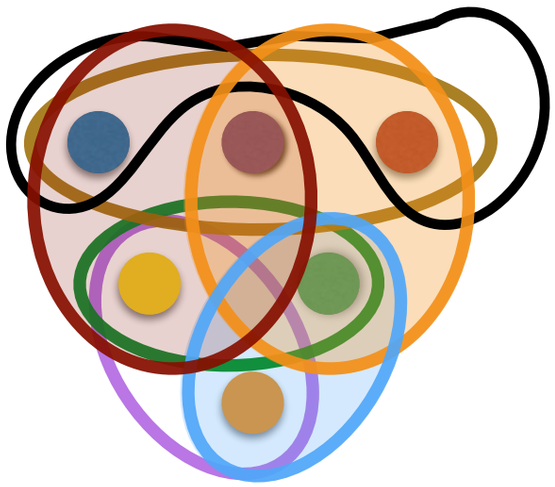
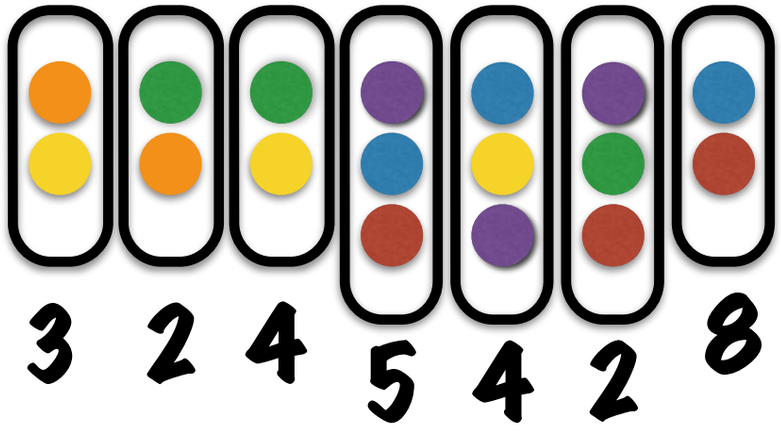
Set cover, linear programming and randomized rounding



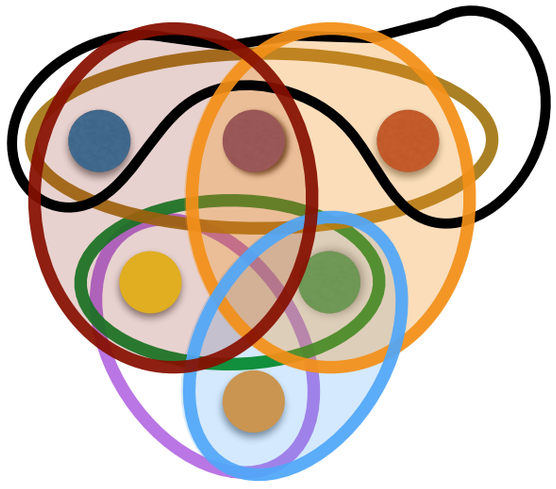
The set cover problem

Elements

Subsets with costs



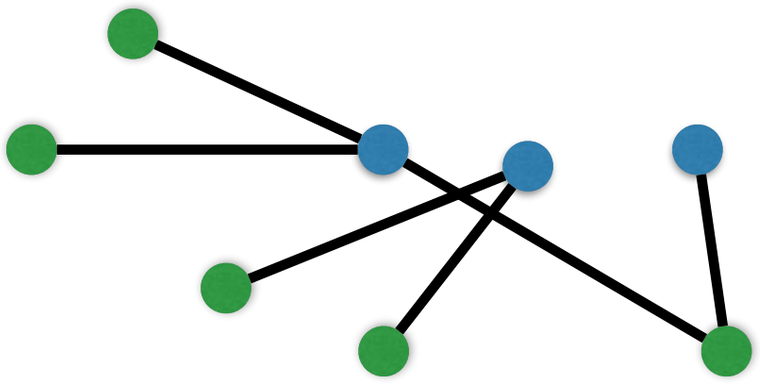
Choose subsets
Cover elements
at min cost



Cost=4+2+2=8

Does this ring a bell?

Vertex cover



Cover all edges with the fewest vertices

edges = elements

vertices = sets

Integer program for Set cover

Variable for subset S:

$$x_S = 1$$

iff S in cover

Constraint for element i:

$$\sum_{S:e \in S} x_S \geq 1$$

Objective:

$$\min \sum_S c_S x_S$$

Linear programming relaxation

$$\min \sum_{\mathbf{S}} \mathbf{c_S x_S}$$

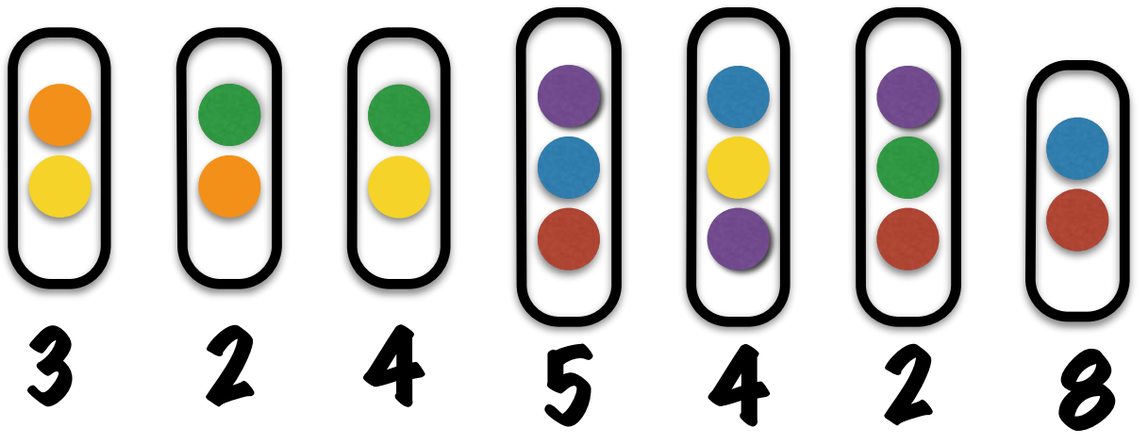
$$\textbf{such that}$$

$$\begin{cases} \sum_{S:e \in S} x_S \geq 1 & \forall e \\ 0 \leq x_S \leq 1 & \forall S \end{cases}$$

Rounding

Like vertex cover:
round to 1 iff $x_u \geq 1/2$

Fails



$$x_S : \frac{1}{3}, \frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{2}, \frac{1}{3}$$

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