

Note that if we calculate  $f(X - c_i)$  and then  $f(X - c_j)$  then it's the same as calculate  $f(X - c_j)$  and then  $f(X - c_i)$ , so  $f$  operates on some inputs more than once. We can save what we already calculated.

We can initialize an array  $A$  with indices from 0 to  $X$ , each index  $i$  corresponding to amount  $i$  that we need to pay. and  $A[i]$  will be the minimal number of coins we can pay for amount  $i$ . From  $f$  we know that index 0 contains 0.

Instead of  $\infty$  in the first condition of  $f$ , we can say  $(\Delta)$   $X+1$  for the practical case, since the smallest coin can be 1, so the maximum coins we can use to get to  $X$  is  $X$ .