recurrence relation:

let $(a_n)_{n\in\mathbb{N}}, (b_m)_{m\in\mathbb{N}}$ be two subsequences of some characters.

We need to somehow iterate the two subsequences and:

1. do something if we iterate out of the bounds of some subsequences;

2. count 1 if two current elements are the same and check CS at the rest of the sequences entries;

3. do something else if they not the same;

We can start iterating some subsequence from the first element or the last element.

Lets say that we iterating starting from the last element.

Handle 1

If we point to an index less than one then we need to return 0 for the current recursive call, since there's no CS starting from an index that is less than one.

Handle 2

If there exists some i, j s.t $a_i = b_j$ then we need to count 1 and check if there's a CS at

$(a_1, a_2, \dots, a_{i-1})$ and $(b_1, b_2, \dots, b_{j-1})$

Handle 3

If we get that $a_i \neq b_j$.

(1) In this case there's a possibility that there exists some a_k for some $k < i$ s.t $a_k = b_j$ and therefore there exists some CS at

$(a_1, a_2, \dots, a_k)$ and $(b_1, b_2, \dots, b_j)$

(2) or, we can argue the same about symmetric case between (a_n) and (b_m)

(1) and (2) are two distinct subproblems of our original problems, therefore we should check their return values and compare them, and take the bigger result.

From the all the above, we can say that our recurrence relation is:

$$f(n,m) = \begin{cases} 0 & , \quad n < 1 \; or \; m < 1 \\ 1 + f(n-1, m-1) & , \quad a_n = b_m \\ \max(f(n-1,m), f(n,m-1) & , \quad a_n \neq b_m \end{cases}$$

Author: Idan Damri