**Correctness:**

Claim:

For all two subsequences $(a_n)_{n\in\mathbb{N}}, (b_m)_{m\in\mathbb{N}}$ of characters. s.t $1 \leq n, m$

$f(n, m)$ returns the LCS of let $(a_n)$ and $(b_m)$

Proof:(by induction on $n$ and $m$)

Base case:

**(3)** Assume $n = 0\ or\ m = 0$. Then f returns 0 , as required.

**(4)** Assume $n = 1\ and\ m = 1$.

if $a_1 = b_1$ then f returns $1 + f(0,0)\ =_{(3)}\ 1 + 0 = 1$ , as required.

Let $(a_n)_{n\in\mathbb{N}}, (b_m)_{m\in\mathbb{N}}$ be any two subsequences of characters.

**(*)** Assume that the claim holds for any $k < n,\ l < m$.

Assume that $a_n = b_m$.

**(5)** Therefore, we count 1 and add to that the return value of $f(n-1, m-1)$.

From (*), $f(n-1, m-1)$ returns the LCS of

$(a_1, a_2, \dots, a_{n-1})$ and $(b_1, b_2, \dots, b_{m-1})$, say that this value is $r$, then of course that the LCS of $(a_n)$ and $(b_m)$ is 1 + r. From here and (5) we get what is required.

Assume that $a_n \neq b_m$.

Therefore, the LCS of $(a_n)$ and $(b_m)$ can only be at

**(6)** $(a_1, a_2, \dots, a_{n-1})$ and $(b_1, b_2, \dots, b_m)$ or at

**(7)** $(a_1, a_2, \dots, a_n)$ and $(b_1, b_2, \dots, b_{m-1})$ .

From (*), $f(n-1, m)$ return the LCS of the subsequence at (6), say this value is $r_1$

and $f(n, m-1)$ returns the LCS of the subsequence at (7), say this value is $r_2$

Of course, the result is the maximum value between $r_1$ and $r_2$.

That means, the result is $\max(f(n-1, m), f(n, m-1)$. As required.

We assumed that the claim holds for any $k < n,\ l < m$ and shows that the claim holds for $n$ and $m$. Therefore, we completed the inductive step.

Conclusion: By mathematical induction, it's proved that the claim holds for all $1 \leq n, m$.

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