Correctness

(*) Claim: (assume we're given a sequence C_n of positive integers.
For any $X \in \mathbb{N}$. [redacted]
f returns the minimal number of coins we can use from C_n, to pay the amount X.

Proof (by induction on X):

base cases:

Assume $X=0$.
f returns 0, as required

Assume $X=1$
if $\exists c_i \in C_n$ s.t $X-c_i=0$ then $f(X-c_i)=f(0)=0$
therefore f returns $1+f(0)=1$ as required
else $\forall c_i \in C_n$ $1<c_i$ and $X-c_i<0$
therefore f returns $1+f(X-c_i)=1+\infty=\infty$
as required.

Let $X \in \mathbb{N}$ s.t $1<|X|$ [redacted]

(**) Assume (*) for any $l<$ X
We start with the third condition of f.
f computes all n possibilities of the first (there are n coins) act of payment

Let $c_i \in C_n$ $1 \leq c_i$. Therefore $X-c_i<X$.
From (**) $f(X-c_i)$ returns the correct result, that is, the minimal number of coins we can pay an amount of $X-c_i$ (given that the first coin we used for payment is c_i) therefore, the minimum between all the results that f returns for the first coin we used is the correct answer. Assuming that for some $1 \leq k \leq n$ $f(X-c_k) \leq f(X-c_i)$ $\forall_{1 \leq k \neq i \leq n}$ we only need to add 1 to that result, for using the coin c_k, that's exactly the final result that f returns.
We assumed (*) for any $k<$ X and showed for X, as required.