Statistical signal processing (5CTA0)

Lecture 2, part A

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Part 1: Random variables and Random Signals

Part 1

Random Variables and
Random Signals

Lecture 1: Probability and Random Variables
Lecture 2: Random vectors, Random processes and random signals
Lecture 3: Rational signal models

Part 1: Random variables and Random Signals

Part 1

Random Variables and Random Signals

Lecture 2: Random vectors, Random processes and random signals

Part A: Pairs of random variables

Part B: Random vectors, random processes and random signals

Pairs of random variables

Lecture 2, Part A

Outline

- Functions of random variables
- Pairs random variables
  - Joint probability distribution and expected values
- Conditional probability distributions
- Marginalization
- Central limit theorem

Functions of random variables

- What happens when I manipulate a random variable?
  - The probability distribution as well as the statistical properties may change

If X is a random a variable and g is a function, then

DISCRETE RVs:

$$E[g(X)] = \sum_{x \epsilon S_X} g(x) p_X(x)$$

CONTINOUS RVs:

$$E[g(X)] = \int_{-\infty}^{+\infty} g(x) p_X(x) dx$$

Functions of random variables

- What happens when I manipulate a random variable?
  - The probability distribution as well as the statistical properties may change

Example: $g(x) = x^m$ $\Rightarrow$ $E[X^m] = \sum_{x \epsilon S_X} x^m p_X(x)$

Example: $g(x) = Y = aX + b$

a, b deterministic constants

$$E[g(x)] = E[aX + b] = aE[X] + b = a\mu_x + b$$

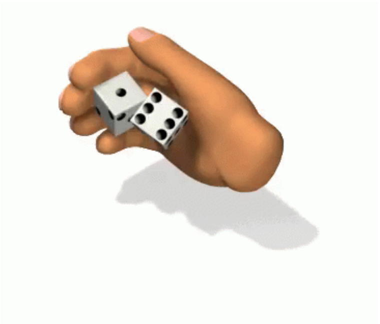
$$var[g(x)] = \mathrm{var}[aX + b] = a^2 \mathrm{var}[X]$$

Pairs of random variables: introduction

- A stochastic process may involve multiple random variables
  - Multiple random variables associated to the same stochastic process are called **joint** random variables
  - The **probability model** of joint random variables contains properties of the individual random variables and the relationships among them

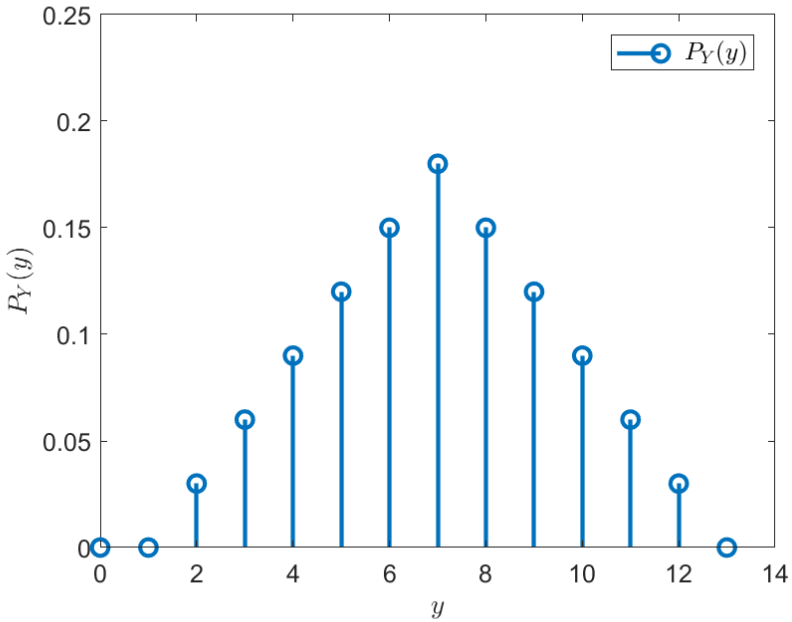
Pairs of random variables: example

Random process: rolling two dice



Y = [sum of the upward faces]
D_1 = number on first die
D_2= number on second die

$Y = D_1 + D_2$



Pairs of random variables: CDF

The joint CDF for a pair of random variables X and Y is given by

$$P_{X,Y}(x,y) = \Pr[X \le x, Y \le y]$$

Properties

1. $0 \le P_{X,Y}(x,y) \le 1$
2. $P_{X,Y}(\infty,\infty) = 1$ and $P_{X,Y}(x,-\infty) = P_{X,Y}(-\infty,y) = 0$
3. $P_X(x) = P_{X,Y}(x,\infty)$ and $P_Y(y) = P_{X,Y}(\infty,y)$
4. CDF is non-decreasing, i.e., for $x \le x_0$ and $y \le y_0$ it holds $P_{X,Y}(x,y) \le P_{X,Y}(x_0,y_0)$

Pairs of random variables: PMF and PDF

The joint PMF for a pair of **discrete** random variables X and Y is given by

$$p_{X,Y}(x,y) = \Pr[X = x, Y = y]$$

The joint PDF for a pair of **continuous** random variables X and Y is given by

$$p_{X,Y}(x,y) = \frac{\partial^2 P_{X,Y}(x,y)}{\partial x \partial y}$$

Statistical description of random vectors

The **covariance** is a measure of the dependence between two random variables

$$\mathrm{Cov}[X,Y] = \mathrm{E}[(X-\mu_X)(Y-\mu_y)] =$$

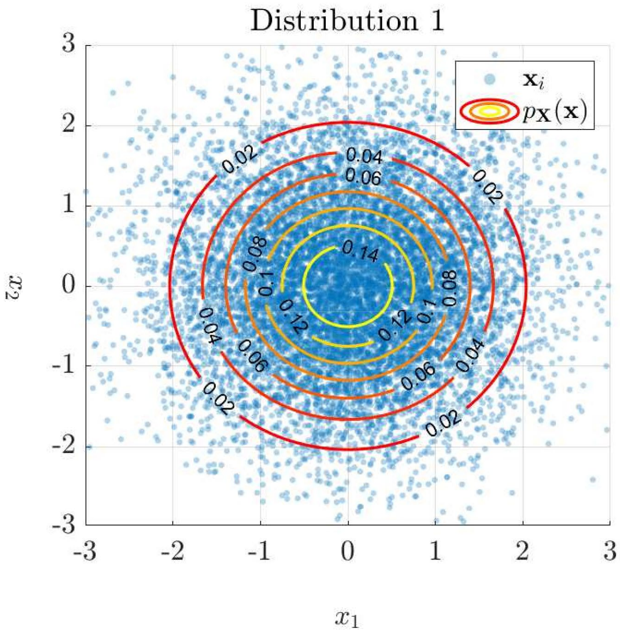
DISCRETE RV

$$= \sum_{y\in S_y}\sum_{x\in S_x}(x-\mu_x)(y-\mu_y)\,p_{X,Y}(x,y)$$

CONTINUOUS RV

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x-\mu_x)(y-\mu_y)\,p_{X,Y}(x,y)\,dxdy$$

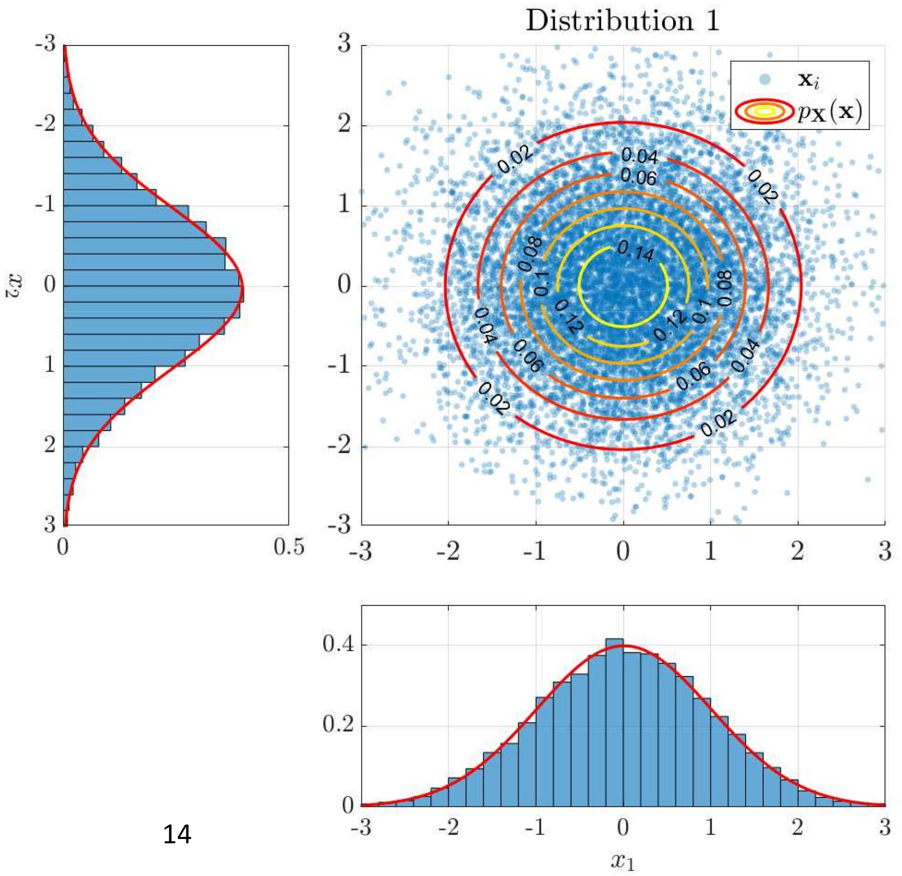
Example



$$X_1 \sim N(0,1)$$
$$X_2 \sim N(0,1)$$

$$p^1_{X_1,X_2}(x_1, x_2)$$

Example



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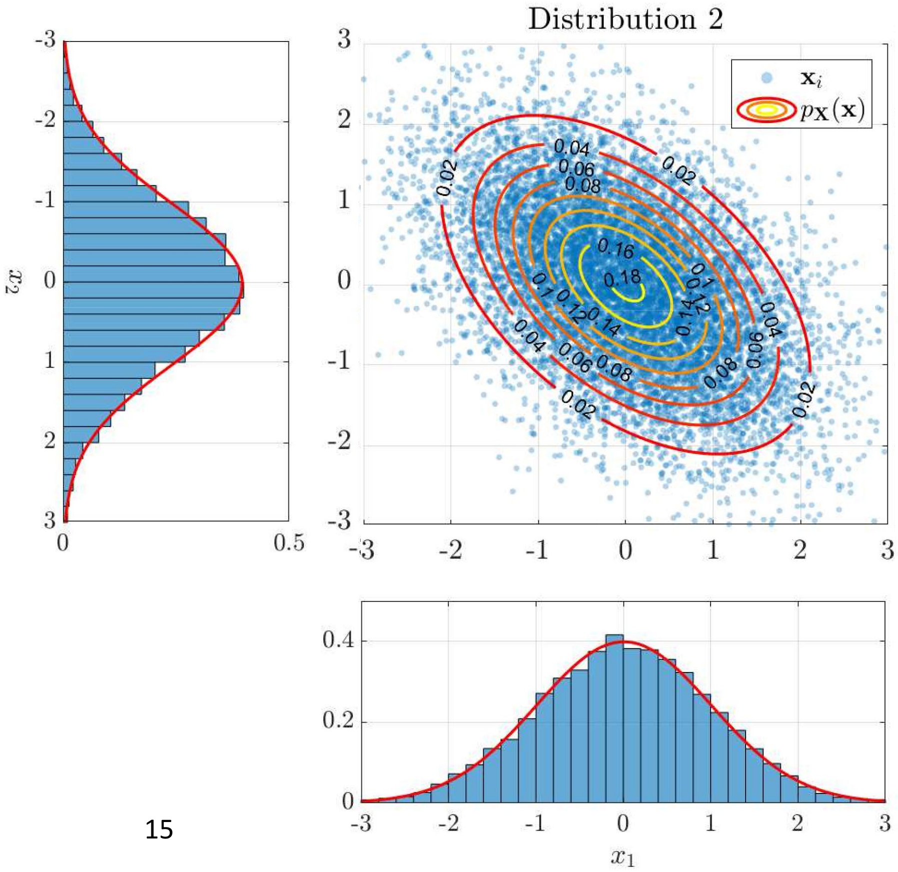
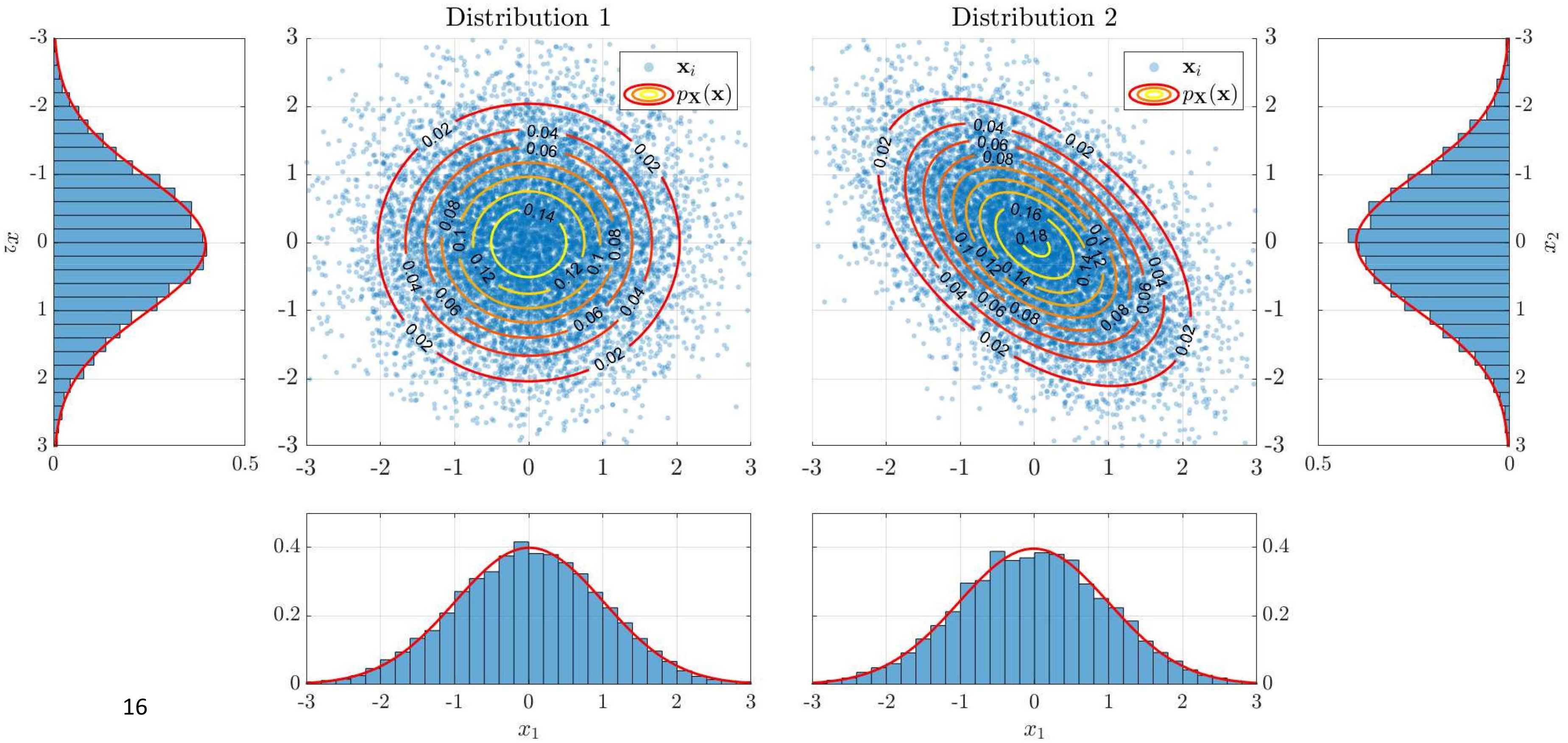


$$X_1 \sim N(0,1)$$
$$X_2 \sim N(0,1)$$

$$p^2_{X_1,X_2}(x_1,x_2)$$

Example



Distribution 1
Distribution 2
$\mathbf{x}_i$
$p_{\mathbf{X}}(\mathbf{x})$
x_1
x_2

Covariance and correlation

The **covariance** is a measure of the dependence between two random variables

$$\mathrm{Cov}[X,Y] = \mathrm{E}[(X-\mu_X)(Y-\mu_y)] =$$

$$= \mathrm{E}[XY] - \mu_X\mu_Y = r_{X,Y} - \mu_X\mu_Y$$

With $r_{x,y}$ called **correlation** $\quad r_{X,Y} = \mathrm{E}[XY]$

X, Y	Real-valued random variables

Covariance and correlation (general)

The **covariance** is a measure of the dependence between two random variables

$$\mathrm{Cov}[X,Y] = \mathrm{E}[(X-\mu_X)(Y-\mu_y)^*] =$$

$$= \mathrm{E}[XY^*] - \mu_X \mu_Y^* = r_{X,Y} - \mu_X \mu_Y^*$$

With $r_{x,y}$ called **correlation** $\quad r_{X,Y} = \mathrm{E}[XY^*]$

X,Y	Complex-valued random variables

Uncorrelated and orthogonal random variables

Two random variables are uncorrelated if

$$\mathrm{Cov}[X,Y]=0$$

And orthogonal if

$$r_{X,Y}=0$$

Correlation coefficient

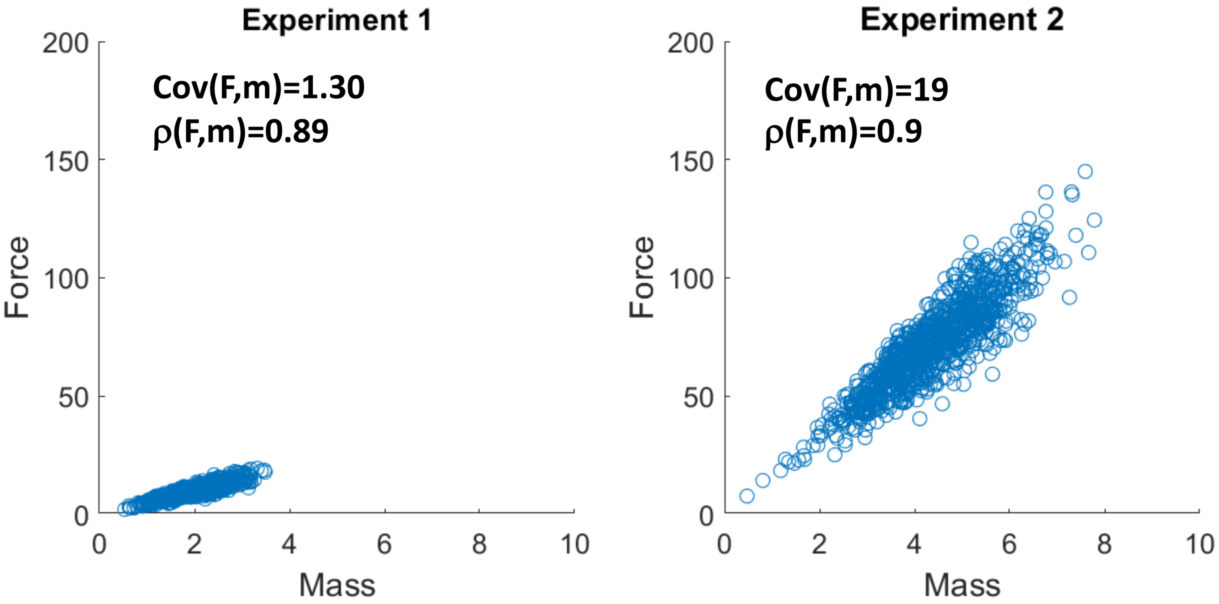
To express the relationship between two random variables independently from their variance, we use the **normalized correlation coefficient**

$$\rho_{X,Y} = \frac{\mathrm{Cov}[X,Y]}{\sqrt{\mathrm{Var}[X]\mathrm{Var}[Y]}} = \frac{\mathrm{Cov}[X,Y]}{\sigma_X \sigma_Y}.$$

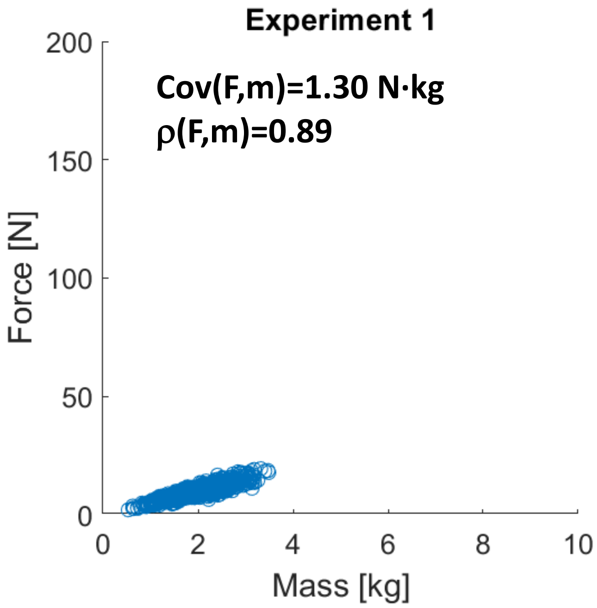
The correlation coefficient is bounded between -1 and 1

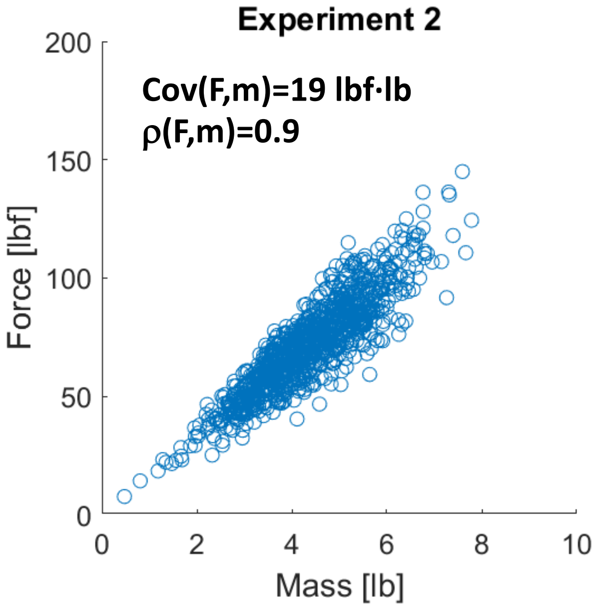
$$-1 \leq \rho_{X,Y} \leq 1.$$

Example: force measurement



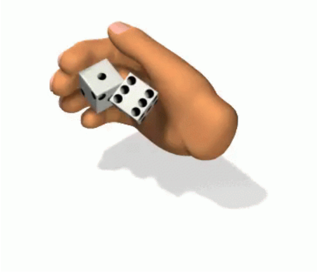
Example: force measurement





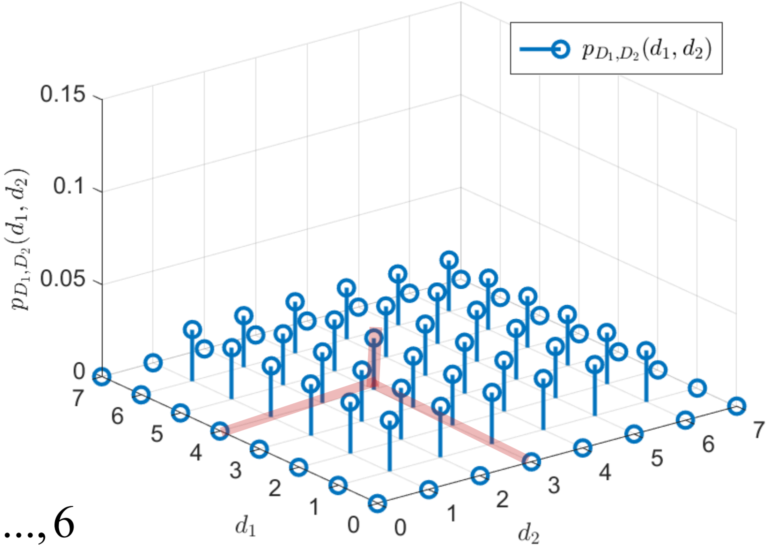
Joint probability distribution: example

Random process: rolling two dice



D_1 = number on first die
D_2= number on second die

$$p_{D_1,D_2}(d_1,d_2) = \begin{cases} 1/36 & \textit{for} \quad d_1 = 1,\ldots,6;\ d_2 = 1,\ldots,6 \\ 0 & \textit{elsewhere} \end{cases}$$



Conditional probability distributions

Given the a priori knowledge that an event set B, with probability $\Pr[B]$, has occurred we can define

Conditional probability function: $$p_{X,Y}(x,y|B) = \begin{cases} \dfrac{p_{X,Y}(x,y)}{\Pr[B]} & (x,y) \in B \\ 0 & elsewhere \end{cases}$$

The event set B becomes the sample space!

Conditional probability distributions: example

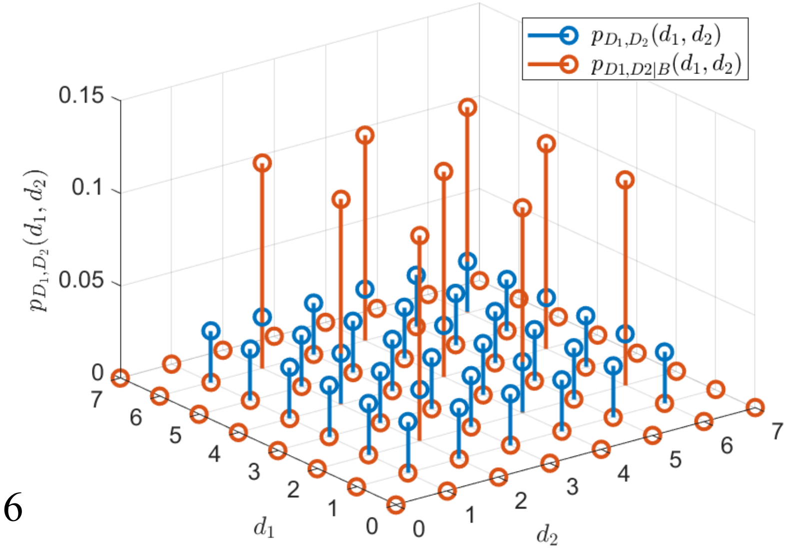
Random process: rolling two dice

D_1 = number on first die
D_2= number on second die

$$p_{D_1,D_2}(d_1,d_2) = 1/36 \quad for \quad d_1 = 1,\ldots,6;\ d_2 = 1,\ldots,6$$

B={d_1 even, d_2 even}

$$p_{D_1,D_2|B}(d_1,d_2) = \begin{cases} \frac{1/36}{\boxed{1/4}} & for \quad d_1 = 2,4,6;\ d_2 = 2,4,6 \\ 0 & elsewhere \end{cases}$$



d_1 and d_2 are independent: $\Pr[B] = \Pr[\{d_1\ even\}]\,\Pr[\{d_2\ even\}] = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$

Conditional probability distributions

Given that Y is observed, we can define the following conditional probability distributions

Conditional PMF: $$p_{X|Y}(x \mid y) = \Pr[X = x \mid Y = y] = \frac{\Pr[X = x, Y = y]}{\Pr[Y = y]} = \frac{p_{X,Y}(x, y)}{p_Y(y)}.$$

Conditional PDF: $$p_{X|Y}(x \mid y) = \frac{p_{X,Y}(x, y)}{p_Y(y)}$$

Marginalization

To obtain the probability function of a random variable of interest, we "marginalize" over all other random variables

Discrete RVs

$$p_X(x) = \sum_{y \in S_Y} p_{X,Y}(x, y)$$

Continuous RVs

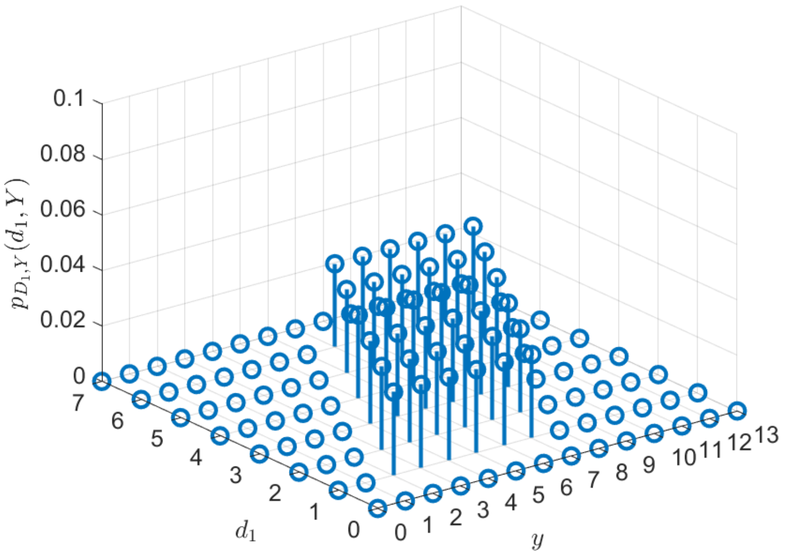
$$p_X(x) = \int_{-\infty}^{\infty} p_{X,Y}(x, y)\mathrm{d}y.$$

Joint random variables: example

Random process: rolling two dice

D_1 = number on first die
D_2= number on second die
$Y = D_1 + D_2$

$$p_{D_1,Y}(d_1, y) = 1/36 \quad \textit{for} \quad d_1 = 1,\ldots,6;\ y = 1 + d_1,\ldots,6 + d_1$$



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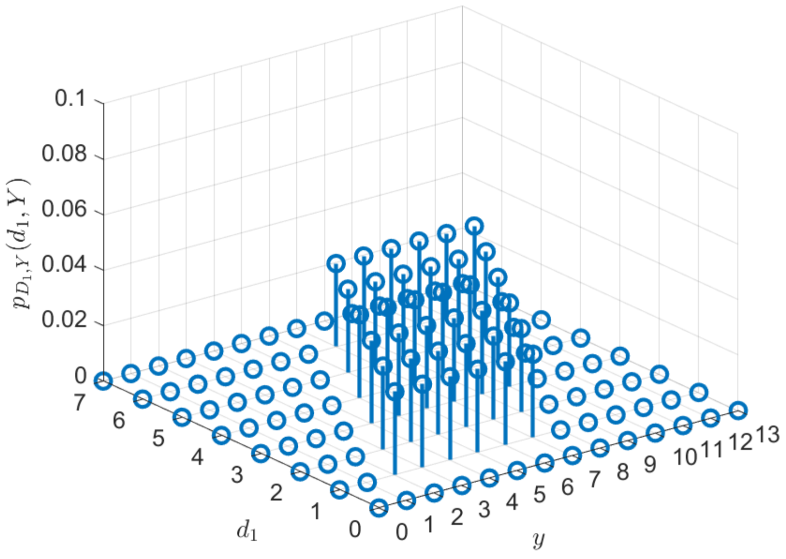
$D_1 = d_1$

$$p_{Y|D_1}(y \mid d_1) = \frac{p_{D_1,Y}(d_1, y)}{p_{D_1}(d_1)}$$



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Marginalization: example

Random process: rolling two dice

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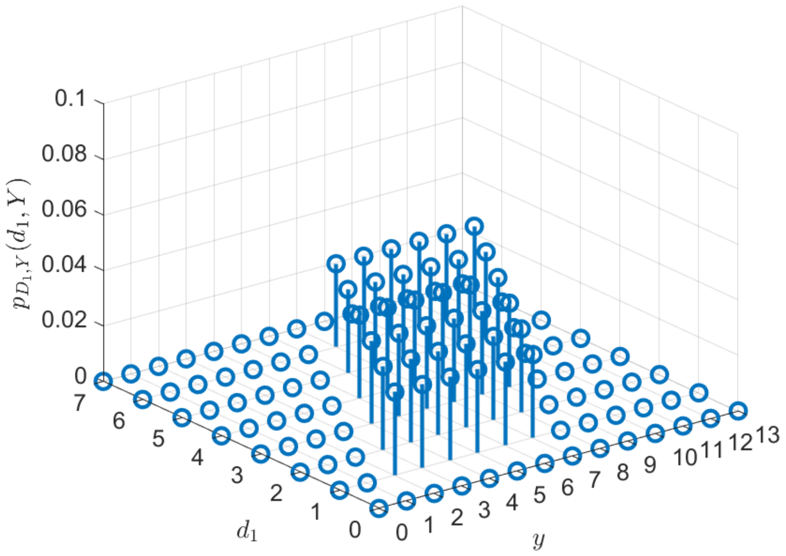
$$p_{D_1,Y}(d_1, y) = 1/36 \quad \mathit{for} \quad d_1 = 1,\ldots,6;\ y = 1 + d_1,\ldots,6 + d_1$$

$$p_{D_1}(d_1) = \sum_{y \in S_Y} p_{D_1,Y}(d_1, y)$$



Marginalization: example

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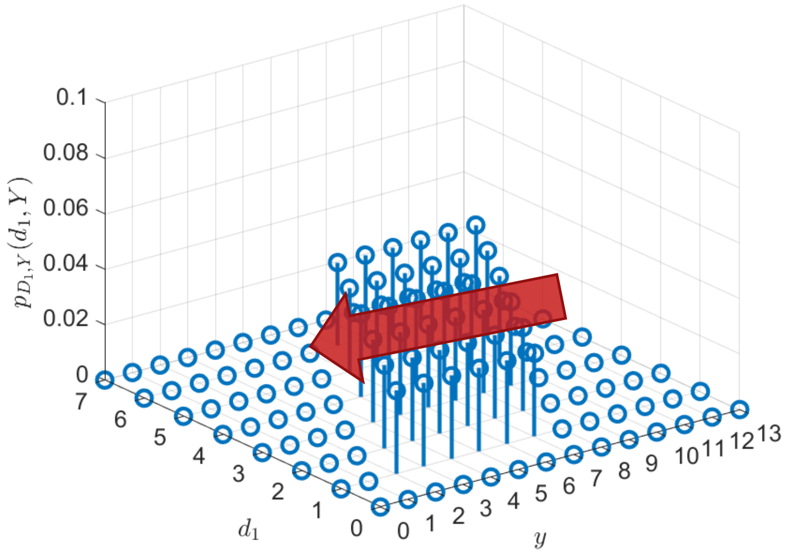

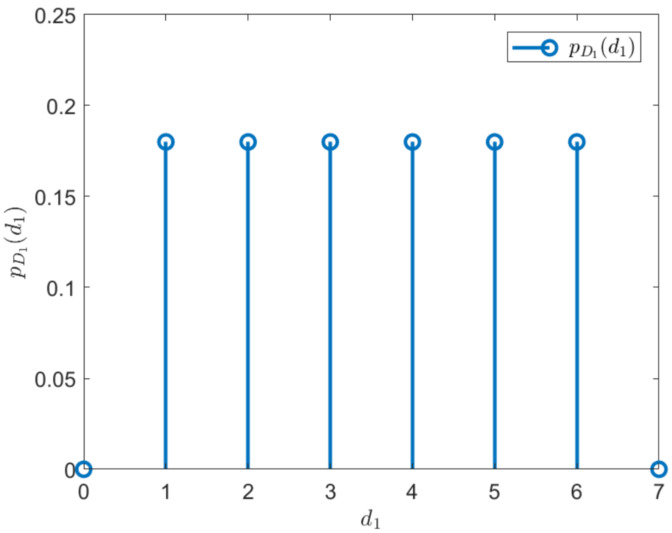

Marginalization: example

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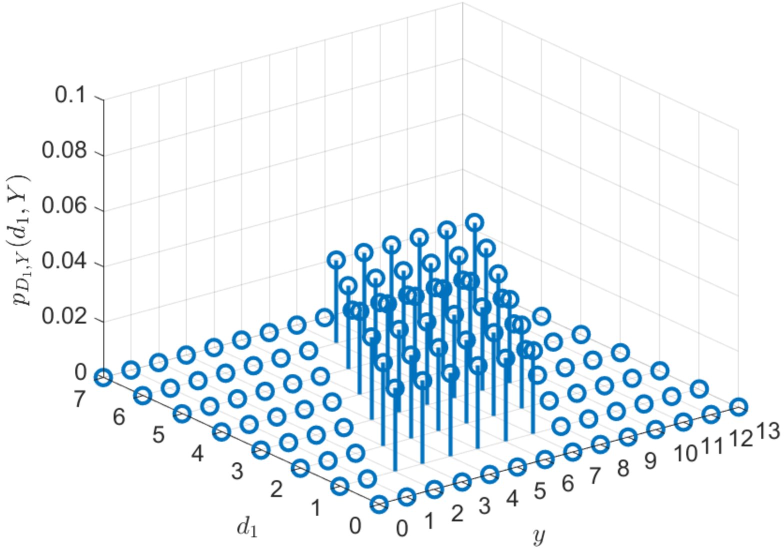
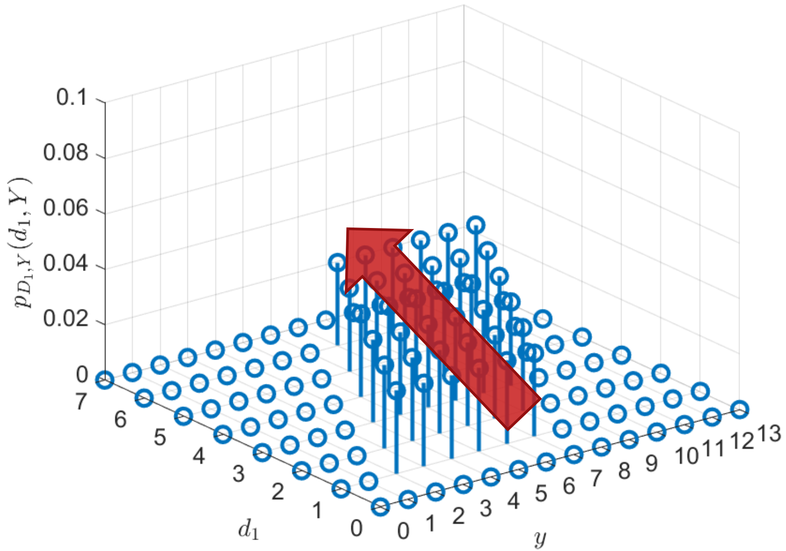
$$p_{D_1}(d_1) = \sum_{y \in S_Y} p_{D_1,Y}(d_1,y)$$

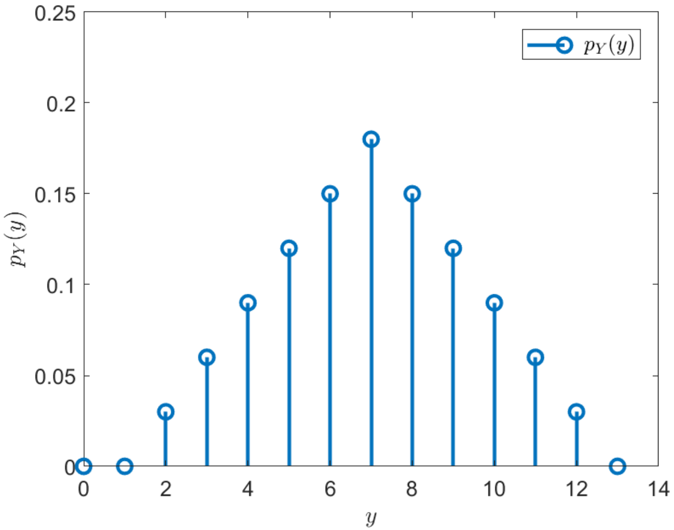
$$p_Y(y) = \sum_{d_1 \in S_{D_1}} p_{D_1,Y}(d_1,y)$$



Marginalization: example

$$p_Y(y) = \sum_{d_1 \in S_{D_1}} p_{D_1,Y}(d_1, y)$$





Marginalization: example

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$$p_{D_1}(d_1) = \sum_{y \in S_Y} p_{D_1,Y}(d_1,y)$$

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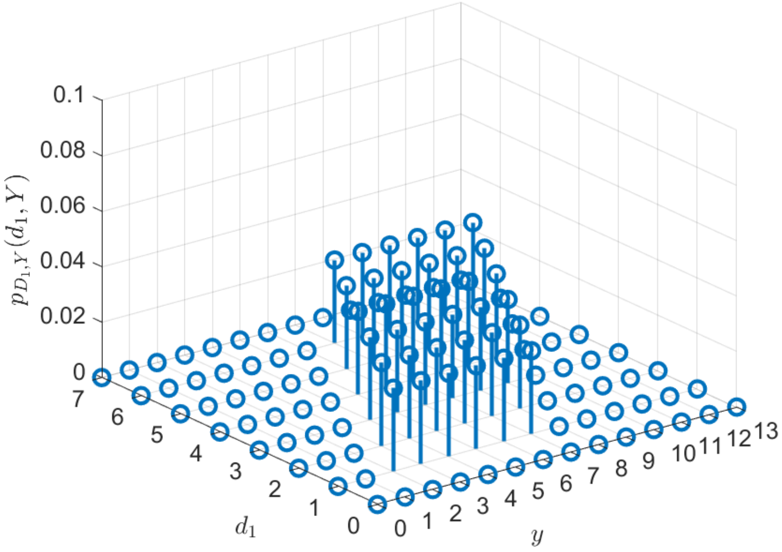

Conditional PMF: example

Random process: rolling two dice

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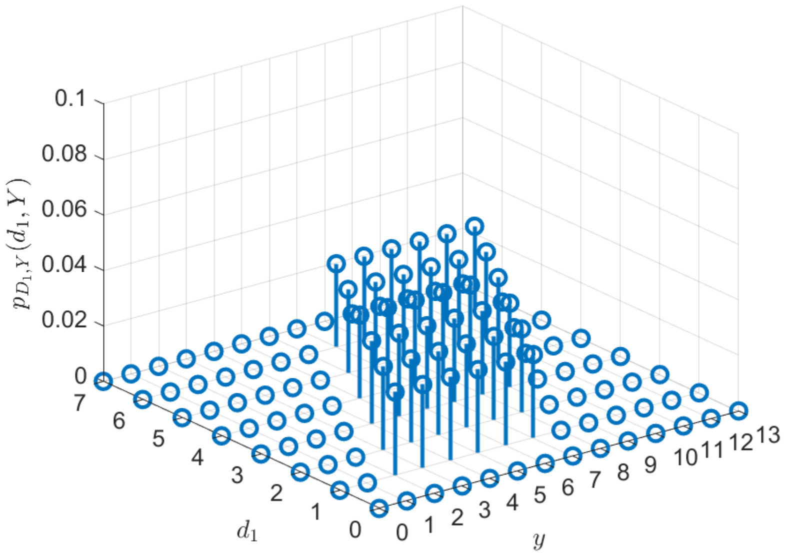
$Y = D_1 + D_2$

$$p_{D_1,Y}(d_1, y) = 1/36 \quad \textit{for} \quad d_1 = 1,\ldots,6;\ y = 1 + d_1,\ldots,6 + d_1$$

$D_1 = d_1$

$$p_{Y|D_1}(y \mid d_1) = \frac{p_{D_1,Y}(d_1, y)}{p_{D_1}(d_1)}$$

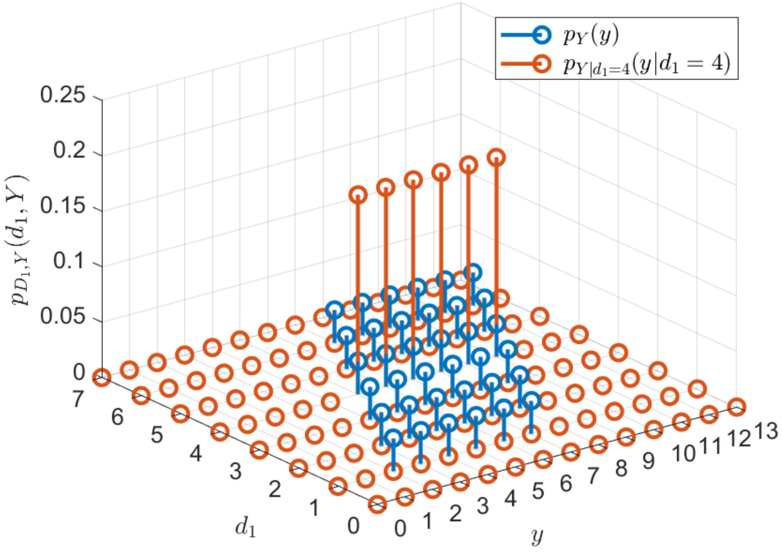


Conditional PMF: example

$$p_{D_1,Y}(d_1,y)=1/36 \quad for \quad d_1=1,\ldots,6;\ y=1+d_1,\ldots,6+d_1$$

$D_1 = d_1$

$$p_{Y|D_1}(y\,|\,d_1=4)=\frac{p_{D_1,Y}(d_1,y)}{p_{D_1}(d_1=4)}$$



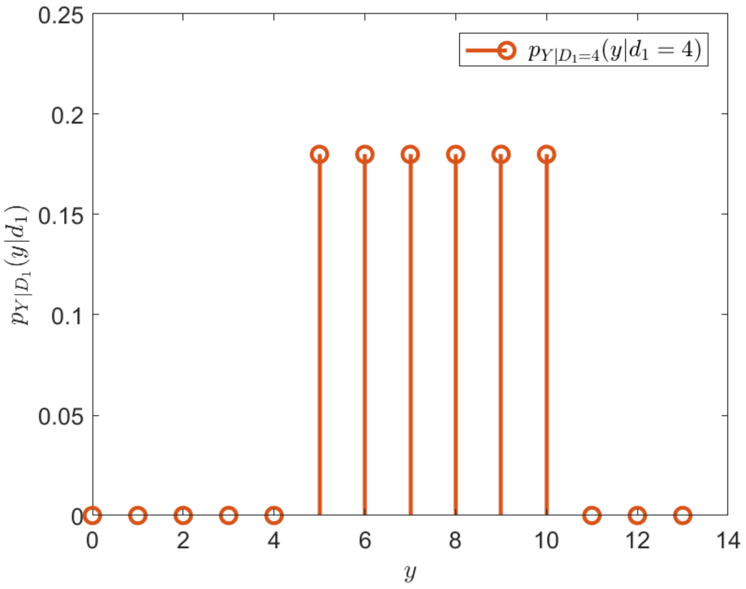
Conditional PMF : example

$$p_{D_1,Y}(d_1,y)=1/36 \quad for \quad d_1=1,\dots,6;\ y=1+d_1,\dots,6+d_1$$

$D_1 = d_1$

$$p_{Y|D_1}(y\,|\,d_1=4)=\frac{p_{D_1,Y}(d_1,y)}{p_{D_1}(d_1=4)}$$

$$p_{Y|D_1}(y\,|\,d_1)=\begin{cases}\dfrac{p_{D_1,Y}(d_1,y)}{p_{D_1}(d_1)}=\dfrac{1/36}{1/6} & for \quad y=1+d_1,\dots,6+d_1\\ 0 & elsewhere\end{cases}$$



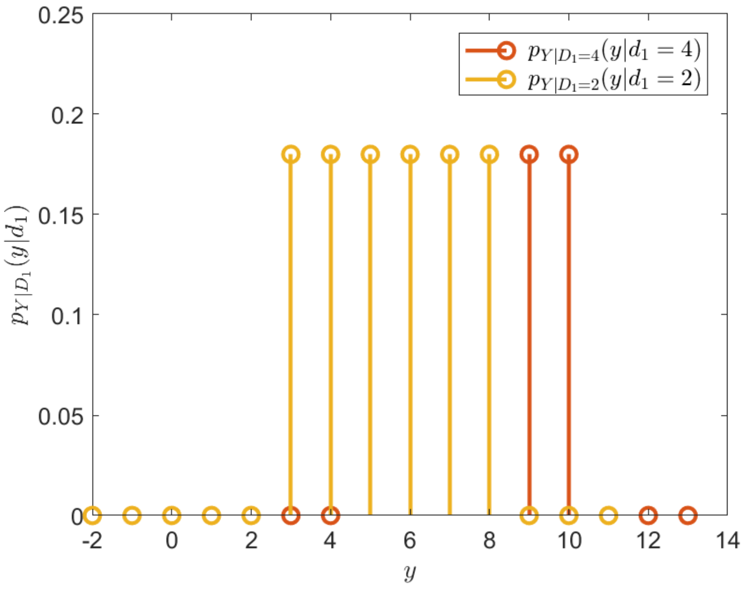
Conditional PMF: example

$$p_{D_1,Y}(d_1, y) = 1/36 \quad for \quad d_1 = 1,\ldots,6;\ \color{red}{y = 1 + d_1,\ldots,6 + d_1}$$

$D_1 = d_1$

$$p_{Y|D_1}(y \mid d_1 = 4) = \frac{p_{D_1,Y}(d_1, y)}{p_{D_1}(d_1 = 4)}$$

$$p_{Y|D_1}(y \mid d_1) = \begin{cases} \dfrac{p_{D_1,Y}(d_1, y)}{p_{D_1}(d_1)} = \dfrac{1/36}{1/6} & for \quad y = 1 + d_1,\ldots,6 + d_1 \\ 0 & elsewhere \end{cases}$$



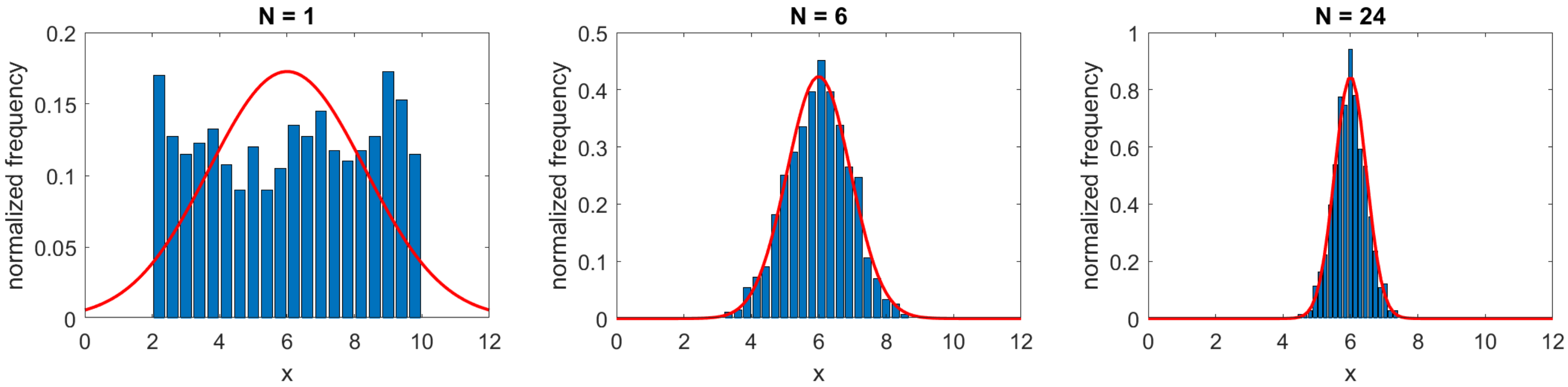
Central limit theorem (classic)

Let $X_1, X_2, \dots, X_N$ be a set of N **independent identically-distributed (*i.i.d*)** random variables and each X_i has an arbitrary probability distribution $p(x_1, x_2, \dots, x_n)$ with finite mean $\mu_i = \mu$ and finite standard deviation $\sigma_i = \sigma$.

If the sample size N is **"sufficiently large"**, then the CDF of the sum converges to a Gaussian CDF

Examples

The normalized sum of a **sufficient** number of *i.i.d* random variables tends to a Gaussian distribution.



X~Uniform(2,10)

Uniform distribution

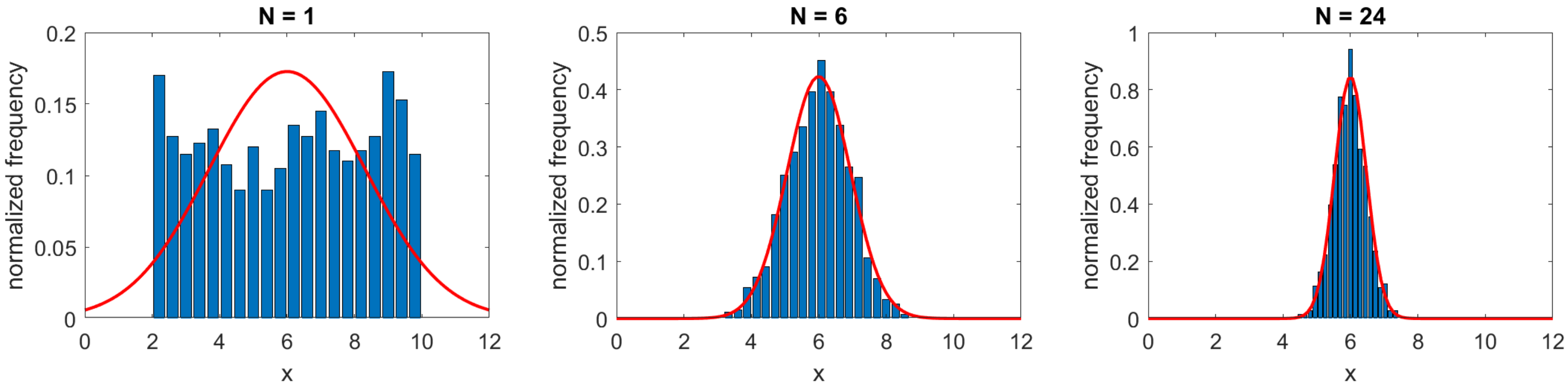
- Continous or discrete **uniform distribution**: used to model equally likely events

X~**Uniform**(a,b)

$$p_X(x) = \begin{cases} \dfrac{1}{b-a} & a \leq x \leq b \\ 0 & otherwise \end{cases}$$

Examples

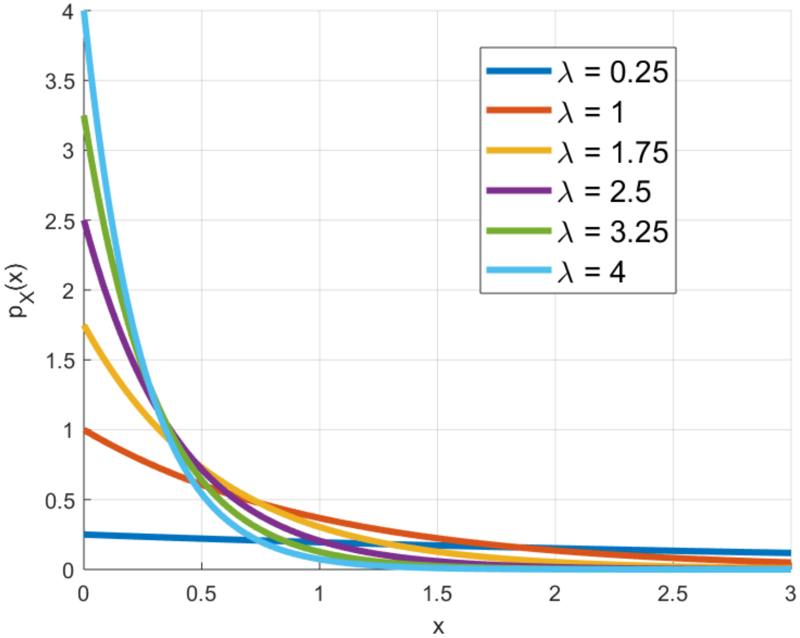
The normalized sum of a **sufficient** number of *i.i.d* random variables tends to a Gaussian distribution.



X~Uniform(2,10)

Exponential distribution

- Continuous **exponential distribution**: used to model the amount of time until some specific event occurs

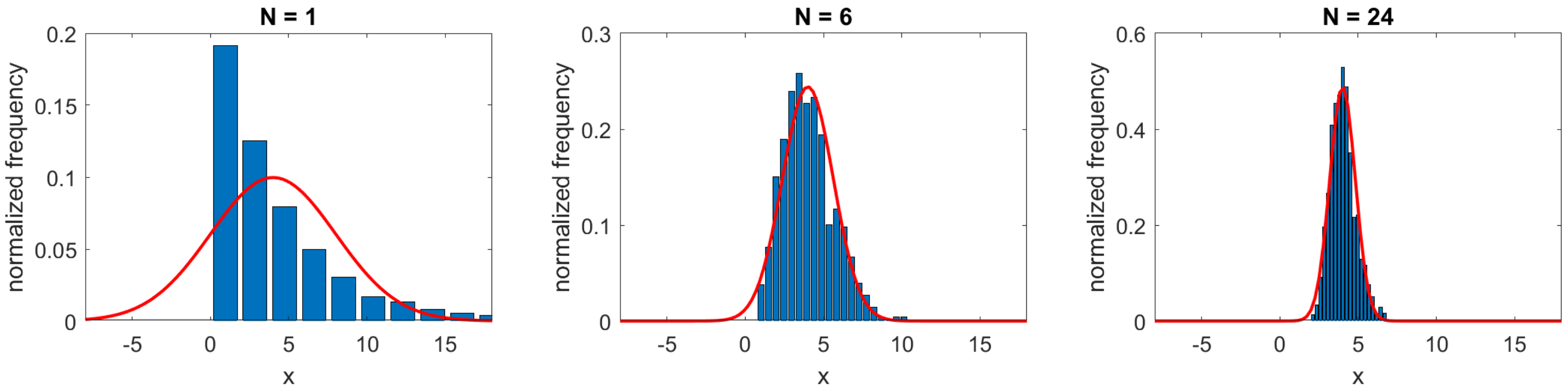


X~**Exponential**(λ)

$$p_X(x) = \begin{cases} \lambda e^{-\lambda x} & if\ x \geq 0 \\ 0 & otherwise \end{cases} \qquad \lambda > 0$$

Example

The normalized sum of a sufficient number of i.i.d random variables tends to a Gaussian distribution.



X~Exponential(4)

TU/e

Central limit theorem (Lyapunov)

Let $X_1, X_2, \ldots, X_N$ be a set of N **independent** and each X_i has an **arbitrary probability distribution** $p(x_1, x_2, \ldots, x_N)$ with finite mean μ_i and finite standard deviation σ_i.

If the sample size N is **sufficiently large**, and the **Lyapunov condition is satisfied**, then the CDF of the sum converges to a Gaussian CDF

$$X = \frac{1}{N}\sum_{i=1}^{N} X_i$$

$$P_X \xrightarrow{N} \mathcal{N}(\mu_X, \sigma_X)$$

with:

$$\mu_X = \frac{1}{N}\sum_{i=1}^{N} \mu_i$$

$$\sigma_X^2 = \frac{1}{N^2}\sum_{i=1}^{N} \sigma_i^2$$

Lyapunov condition

Given $X_1, X_2, \dots, X_N$ a set of N **independent** with finite mean μ_i and finite standard deviation σ_i, and defining:

$$s_n{}^2 = \sum_{i=1}^{n} \sigma_i{}^2$$

For some $\delta>0$, the **Lyapunov condition** is a satisfied when

$$\lim_{n\to\infty} \frac{1}{s_n{}^{2+\delta}} \sum_{i=1}^{n} E\left[\,|X_i - \mu_i|^{2+\delta}\right] = 0$$

Lyapunov condition

Given $X_1, X_2, \dots, X_N$ a set of N **independent** with finite mean μ_i and finite standard deviation σ_i, and defining:

$$s_n^{\ 2} = \sum_{i=1}^{n}$$

For some δ>0

The Lyapunov condition puts a limit on the rate of growth of the moments

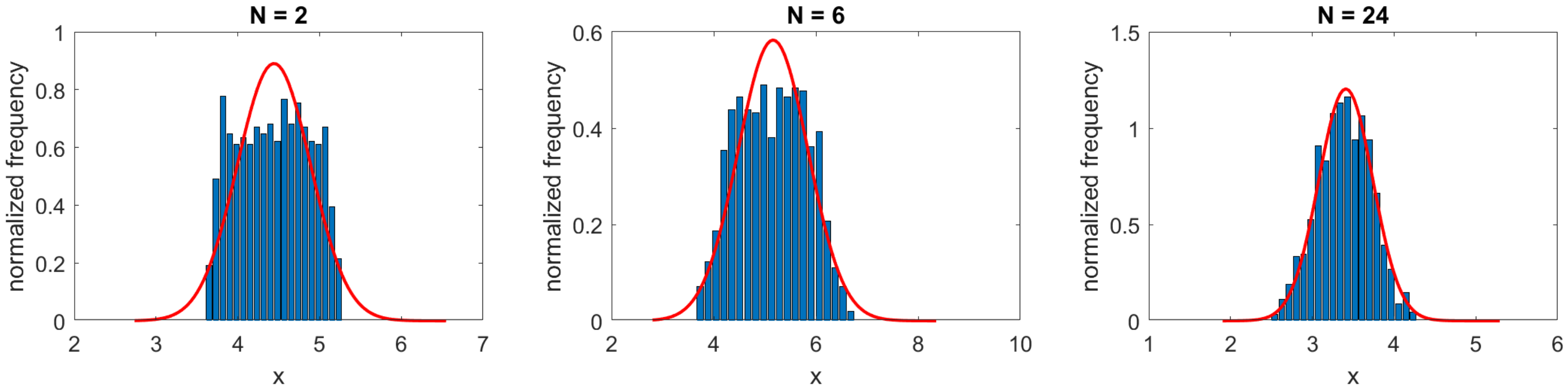
$$\lim_{n\to\infty} \frac{1}{s_n^{\ 2+\delta}} \sum_{i=1}^{n} E\left[| X_i - \mu_i |^{2+\delta} \right] = 0$$

In simple words..

- All CLT formulations essentially state that the sum of multiple random variables converges to a gaussian distribution, provided that:
    - The number of random variables is “sufficiently large”
    - They are not too correlated
    - The variables are not too large (Lyapunov quantifies this by imposing a condition on the moments)

“Sufficiently large” number condition

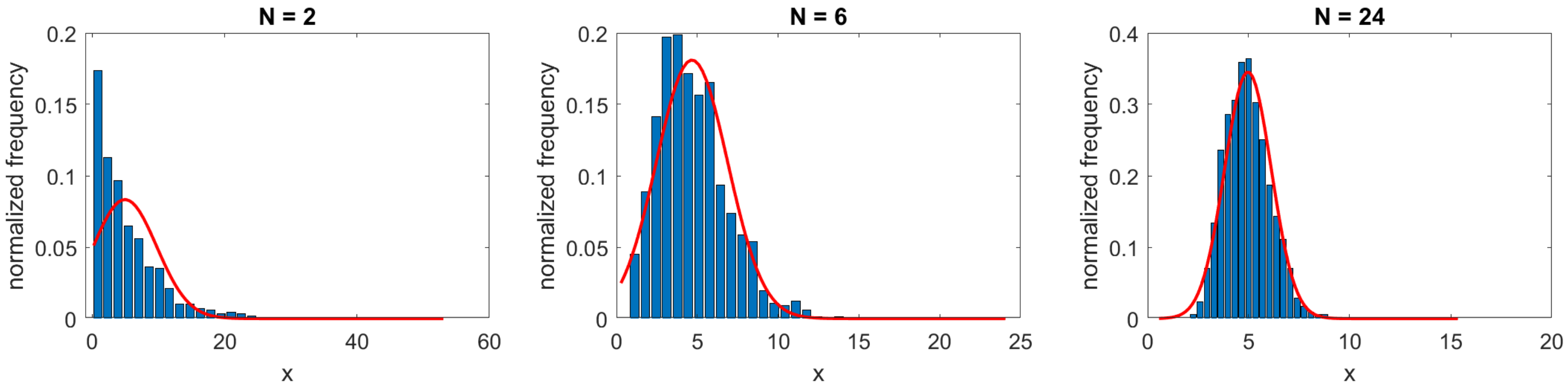
The normalized sum of a sufficient number of **independent** random variables tends to a Gaussian distribution.



X~Uniform(a_i,b_i)

"Sufficiently large" number condition

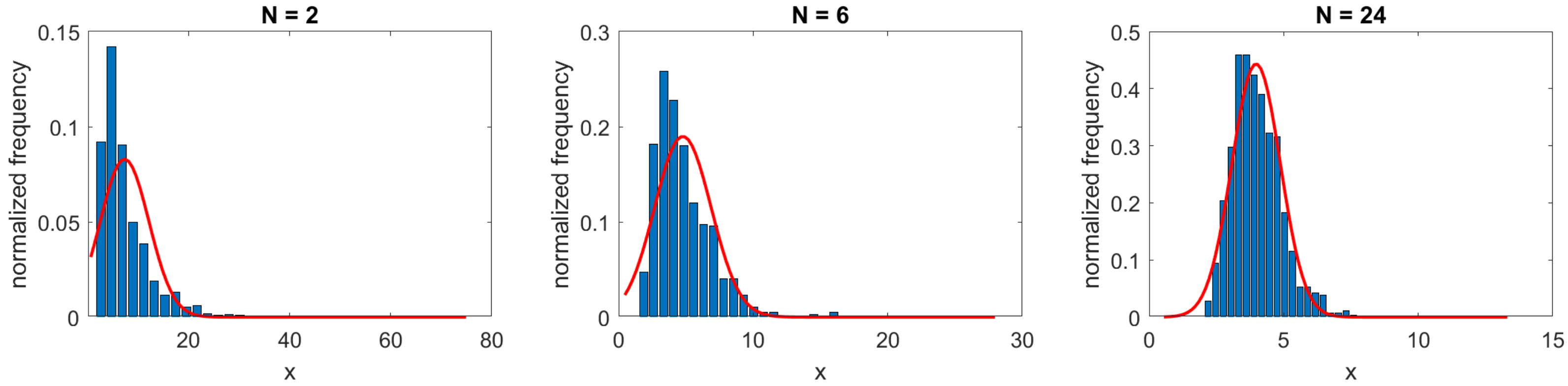
The normalized sum of a sufficient number of **independent** random variables tends to a Gaussian distribution.



X~Exponential(λ_i)

“Sufficiently large” number condition

The normalized sum of a sufficient number of **independent** random variables tends to a Gaussian distribution.



X ~ mix of uniform and exponential

“Sufficiently large” number condition

The
vari

$\downarrow N$

- Symmetric distribution
- Identically distributed

$\uparrow N$

- Skewed distribution
- Same distribution, different parameters
- Different distributions

X ~ mix of uniform and exponential

Application CLT

- If we sum multiple random variables, however distributed, we can apply the central limit theorem to obtain the full probability model (Gaussian with known expected value and variance)

- In practice: any random variable resulting from the sum of multiple random variables (e.g., multiple noise sources affecting a measurement) can be considered gaussian distributed

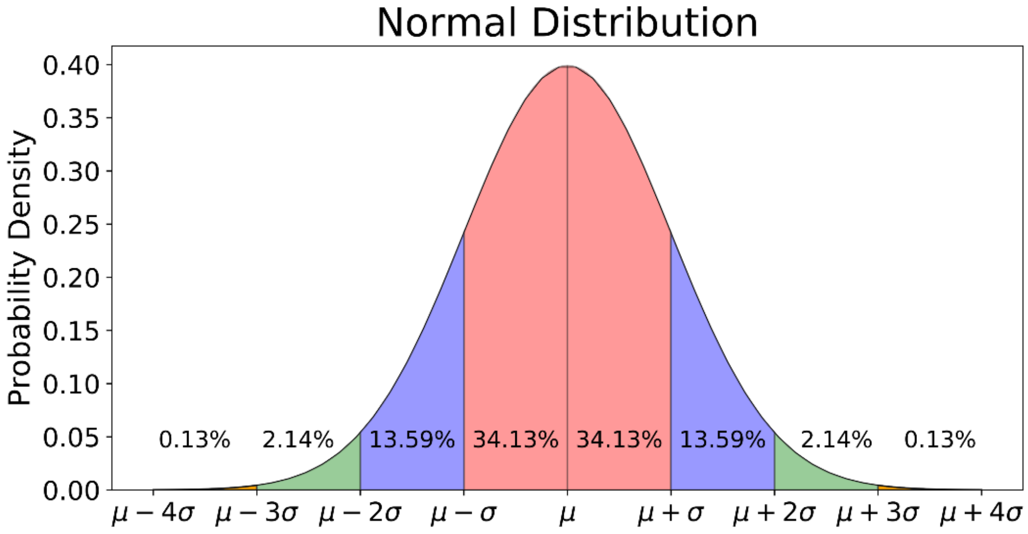
- From the obtained Gaussian probability model, we can perform easy an accurate calculations

Gaussian (normal) distribution

$\boldsymbol{N\ (\mu, \sigma^2)}$:

$$p_x(x) = \frac{1}{\sqrt{2\pi\boldsymbol{\sigma}^2}} e^{-\frac{(x-\boldsymbol{\mu})^2}{2\sigma^2}}$$

- μ: mean
- σ: standard deviation



- $\Pr[-\sigma < T < \sigma]$~68.2%
- $\Pr[-2\sigma < T < 2\sigma]$~95.4%
- $\Pr[-3\sigma < T < 3\sigma]$~99.7%

Gaussian (normal) distribution: Q-function

- The **standard** normal distribution, also called **z-distribution** is a normal distribution with *zero-mean* and *unit variance* (standard deviation)

$$Z = \frac{X - \mu_X}{\sigma_X}$$

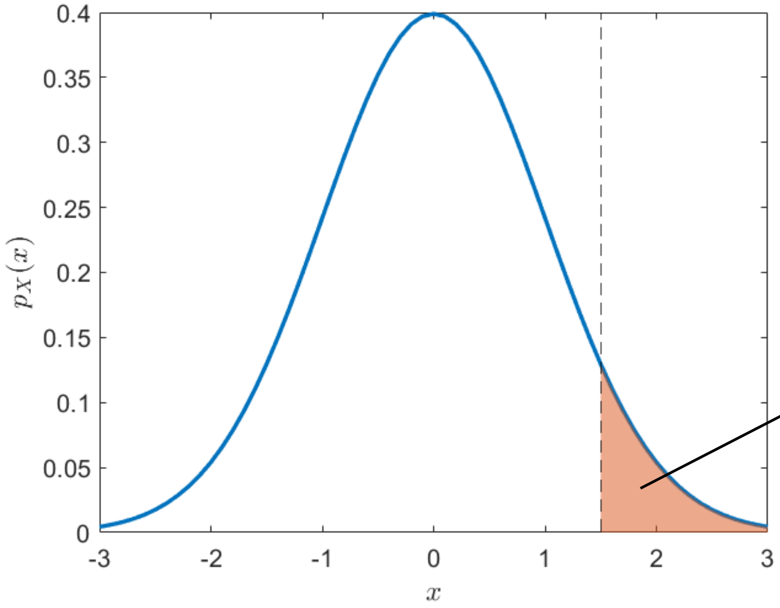
- X, gaussian random variable
- μ_X, mean of X
- σ_X, standard deviation

- The **Q-function** is the tail distribution function of the standard normal distribution

$$Q(x) = \Pr[X > x] = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{u^2}{2}} du$$

Gaussian (normal) distribution: Q-function

$$Q(x) = \Pr[X > x] = \frac{1}{\sqrt{2\pi 1^2}} \int_x^\infty e^{-\frac{(u-0)^2}{2\cdot 1^2}} du$$

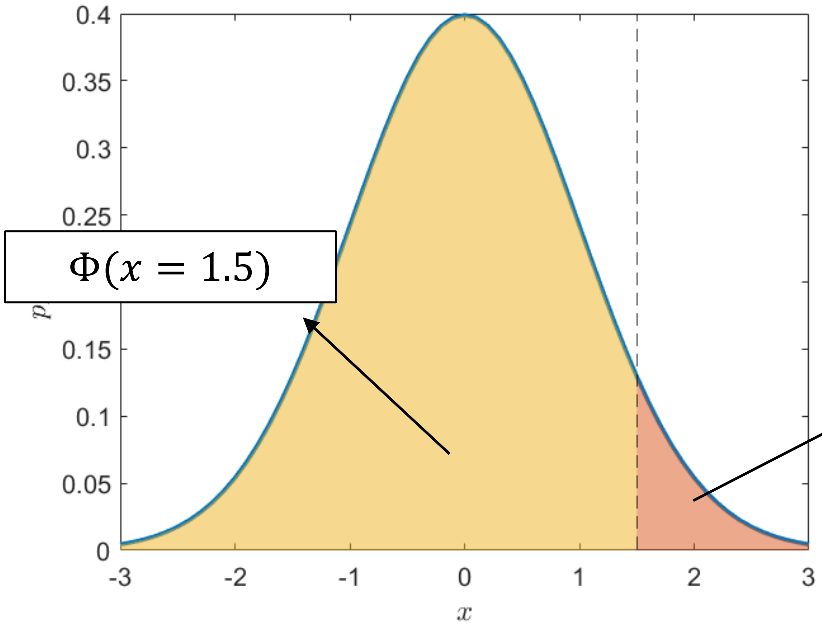


$Q(x = 1.5)$

Gaussian (normal) distribution: Q-function

$$Q(x) = \Pr[X > x] = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{u^2}{2}} du \; = 1 - Q(-x) = 1 - \Phi(x)$$

CDF of standard normal distribution



MATLAB functions

```
qfunc(x)
qfuncinv(x)
normcdf(x)
normcdf(x, mu, sigma)
```

Application CLT: example

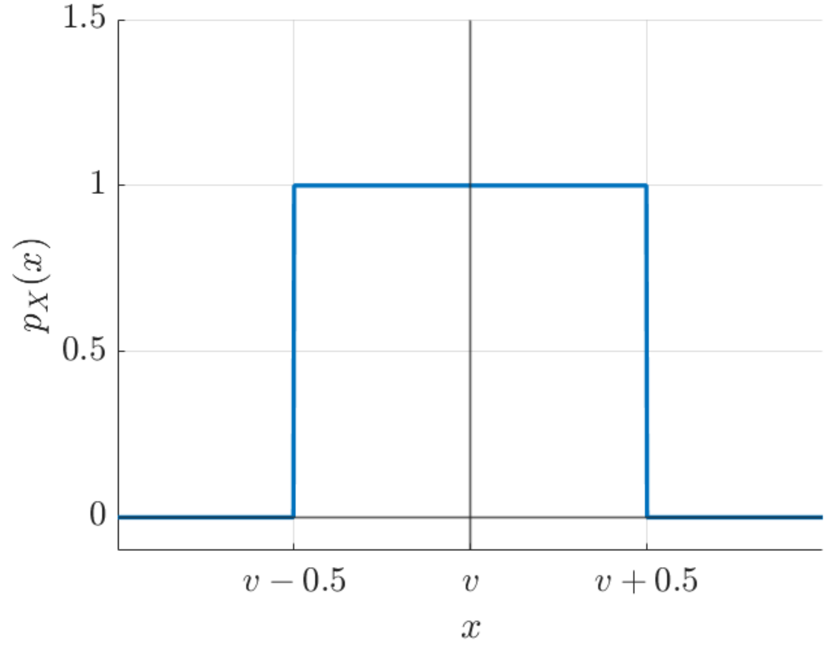
- A compact disc (CD) contains digitized samples of an acoustic waveform.

- In a CD player with a "one-bit digital to analog converter," each digital sample is represented to an **accuracy of ±0.5 mV**.
- The CD player "oversamples" the waveform by making **eight independent measurements** corresponding to each sample.
- The CD player obtains a waveform sample by calculating the average (sample mean) of the eight measurements.

What is the probability that the error in the waveform sample is greater than 0.1 mV?

Application CLT: example

- Each digital sample is represented to an **accuracy of ±0.5 mV**
- With can model each measurement with a uniform distribution



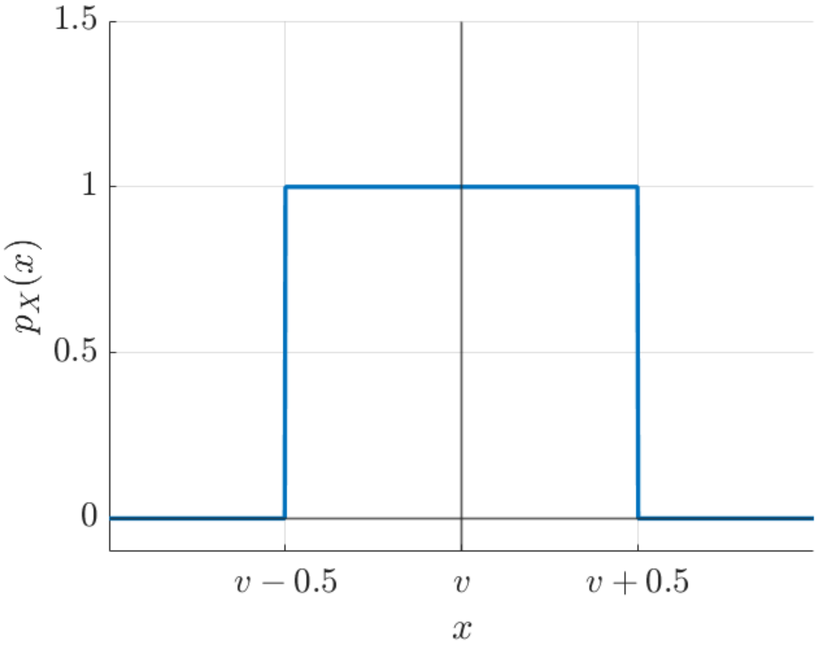
Application CLT: example

- **Approach 1: we don’t know the CLT…**
  - Calculate exact probability model
    - Eightfold convolution of the uniform PDF [out of scope]
    - Moment generating function method [out of scope]

- **Approach 2: use the CLT!**
  - Calculate the expected mean and variance of the sum of random variables
  - Model the sum as Gaussian probability function
  - Use Gaussian CDF (or Q function) to calculate probability

Solution

Measurements X_i have all uniform distribution between v-0.5 mV and v + 0.5 mV



$$E[X_i] = \frac{a+b}{2}$$

$$\mathrm{Var}[X_i] = \frac{1}{12}(b-a)^2$$

Solution

Measurements X_i have all uniform distribution between v-0.5 mV and v + 0.5 mV

$$E[X_i] = \frac{a+b}{2} = \frac{v-0.5+v+0.5}{2} = \frac{2v}{2} = v$$

$$\mathrm{Var}[X_i] = \frac{1}{12}(b-a)^2 = \frac{1}{12}(v-0.5-(v-0.5))^2 = \frac{1}{12}$$

Solution

- Measurements X_i have all uniform distribution between v-0.5 mV and v + 0.5 mV
- Output U of CD player:

$$U = \frac{1}{8} \sum_{i=1}^{8} X_i$$

- Using central limit theorem:

$$E[U] = 8E[X_i]/8 = v$$

$$\mathrm{var}[U] = 8\,\mathrm{var}[X_i]/64 = 1/8 \cdot 1/12 = 1/96$$

Output U is approximately Gaussian with mean v and variance 1/96

Solution

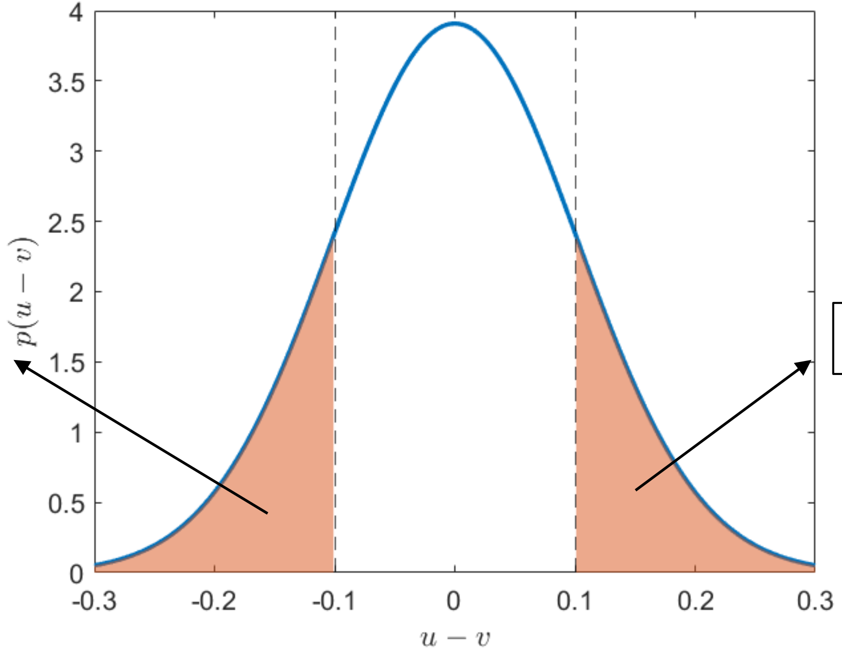
- We want to know $\Pr[|U - v| > 0.1]$
- Output $\boldsymbol{U}$ is approximately Gaussian with mean v and variance 1/96

Error random variable $\boldsymbol{U} - \boldsymbol{v}$ is approximately Gaussian with mean 0 and variance $1/96$

Solution

$$\Pr[|U - v| > 0.1]$$

$\Pr[U - v < -0.1]$



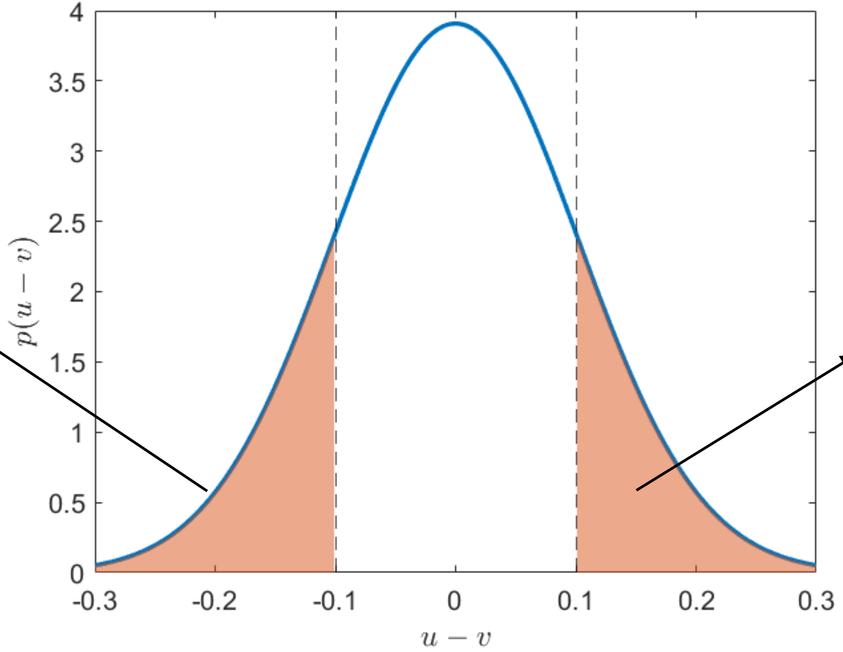
$\Pr[U - v > 0.1]$

Solution

standardization

$$\Pr[|U-v|>0.1]=2\cdot Q\left(\frac{0.1-0}{\sqrt{1/96}}\right)$$

$$\boxed{1-Q\left(-\frac{0.1-0}{\sqrt{1/96}}\right)}$$



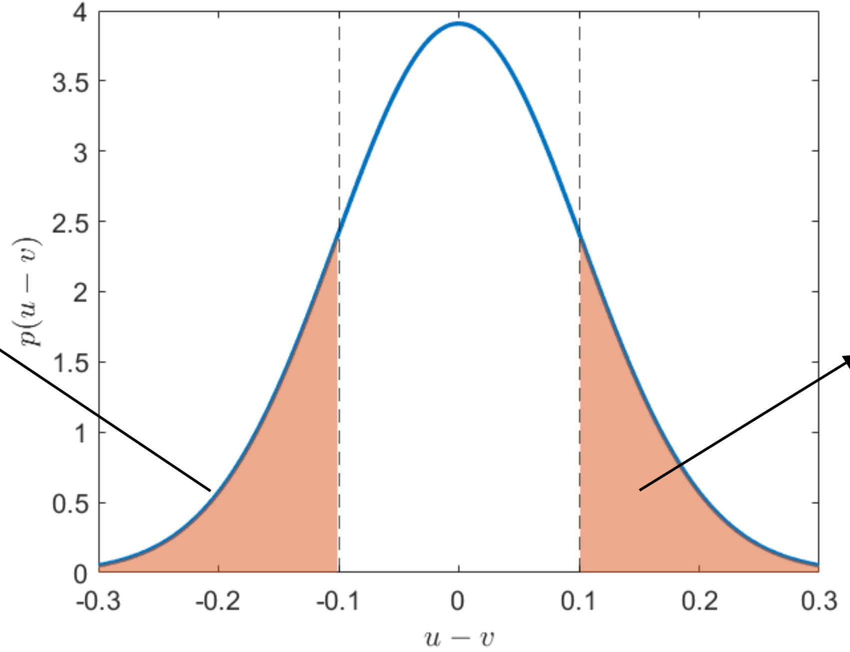
$$\boxed{Q\left(\frac{0.1-0}{\sqrt{1/96}}\right)}$$

Solution

standardization

$$\Pr[|U-v|>0.1] = 2\cdot Q\left(\frac{0.1-0}{\sqrt{1/96}}\right) = 2\cdot\left(1-\Phi\left(\frac{0.1}{\sqrt{1/96}}\right)\right) = 0.3272$$

$$1-Q\left(-\frac{0.1-0}{\sqrt{1/96}}\right)$$



$$Q\left(\frac{0.1-0}{\sqrt{1/96}}\right)$$

Wrap up (I)

- If we a apply a **function** to a **random variable,** the probability model as well as the moments may change
- Stochastic process involving **pairs of random variables** can be described by joint probability models
- Given a joint probability distribution, and an observed event or random variable, we can define **conditional probability distributions**
- The operation of **marginalization** permits calculating the probability distribution of an individual random variable, given the joint probability distribution

Wrap up (II)

- The **central limit theorem** allows to assume a gaussian probability distribution as the result of summing "many", "not-to-correlated", "not-too-large" random variables
- The **standard normal random variable** is defined as a normal random variable with zero mean and unit variance
- The **Q-function** is the tail distribution function of the standard normal distribution

Statistical signal processing (5CTA0)

Lecture 2, part A

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Electrical Engingeering, Signal Processing Systems group