Quiz week 3

Started: 20 Sep at 22:16

Quiz instructions

Question 1 — 1 pts

Regarding minimum variance unbiased estimator (MVUE) and Cramer-Rao Lower Bound (CRLB), which of the following statement is **FALSE.**

- ◯ MVUE does not always exist. Even it exists, we may not be able to find it.
- ◯ If an estimator exists whose variance equals the CRLB, then it must be the MVUE.
- ◯ If no estimator has a variance that equals the CRLB, the MVUE doesn't exist.

Question 2 — 1 pts

The Cramer-Rao Lower Bound (CRLB) provides an lower bound on the variance of the estimate . Regarding CRLB, which of the following statement is **FALSE.**

- ◯ In CRLB, the regularity condition is violated if the region of integration depends on the parameter θ.
- ◯ The variance of estimator $\mathbf{var}\left(\hat{\theta}\right)$ is always larger than $\mathcal{I}(\theta)^{-1}$, where $\mathcal{I}(\theta) = \mathrm{E}\left[\frac{\partial^2 \ln p(\theta;\mathbf{x})}{\partial\theta^2}\right]$ represents the Fisher information.
- ◯ If the Fisher information from each single observation x_n is $i(\theta)$, the Fisher information from N such identical but independent observations is $\mathcal{I}(\theta) = Ni(\theta)$.
- ◯ An efficient estimator $\hat{\theta} = g(\mathbf{x})$ may be found if $\frac{\partial \ln p(\mathbf{x};\theta)}{\partial\theta} = \mathcal{I}(\theta)(g(x) - \theta)$.

Question 3

1 pts

Let $x_0, x_1, \ldots, x_{N-1}$ be IID and uniformly distributed in the interval $[0, A]$, i.e., $x_n \sim \text{Uniform}(0, A)$. The unknown parameter A determines the length of the interval. The PDF of the observations is

$$p(\mathbf{x}; A) = \begin{cases} \frac{1}{A^N}, & \text{for } 0 \leq x_n \leq A \quad n = 0, 1, \ldots N-1, \\ 0, & \text{else.} \end{cases}$$

Check whether the CRLB for an estimate of A exists and if so, calculate it.

- ○ $\text{Var}(g(\mathbf{x})) \geq \frac{N}{A^2}$
- ○ $\text{Var}(g(\mathbf{x})) \geq \frac{N^2}{A^2}$
- ○ $\text{Var}(g(\mathbf{x})) \geq -\frac{N}{A^2}$
- ○ The CRLB does not exist.

Question 4

1 pts

Which of the following statements about the maximum likelihood estimator is **FALSE**?

- ○ The maximum likelihood is asymptotically unbiased.
- ○ If the likelihood function has a maximum, it is unique.
- ○ The maximum likelihood estimator is the value of θ that maximized the likelihood function $p(\mathbf{x}; \theta)$ for a given observation $\mathbf{x}$.
- ○ If an efficient estimator exists, it is also the maximum likelihood estimator.

Question 5

1 pts

If we observe N independent and identically distributed samples x_n from $\mathrm{Binomial}(M, q)$ distribution with the probabilities $p(x_n; q) = \binom{M}{x_n} q^{x_n} (1-q)^{M-x_n}$, which of the following expression correctly describes the log-likelihood function $\ln p(\mathbf{x}; q)$.

- There is not enough information to calculate.
- $\ln p(\mathbf{x}; q) = \sum_{n=0}^{N-1} \ln \binom{M}{x_n} + \ln \frac{q}{1-q} \sum_{n=0}^{N-1} x_n + \ln(1-q) MN$
- $\ln p(\mathbf{x}; q) = \sum_{n=0}^{N-1} \ln \binom{M}{x_n} + \ln \frac{1-q}{q} \sum_{n=0}^{N-1} x_n + \ln(1-q) MN$
- $\ln p(\mathbf{x}; q) = \sum_{n=0}^{N-1} \ln \binom{M}{x_n} + \ln \frac{q}{1-q} \sum_{n=0}^{N-1} x_n + \ln(q)(M-1)N$

Question 6

1 pts

Continue with above question, which of the following expression is the correct maximum likelihood estimate of q.

- $\hat{q}_{\mathrm{ML}} = \frac{\sum_{n=0}^{N-1} x_n}{MN}$
- $\hat{q}_{\mathrm{ML}} = \frac{\sum_{n=0}^{N-1} x_n}{MN-1}$
- There is not enough information to calculate.
- $\hat{q}_{\mathrm{ML}} = \frac{\sum_{n=0}^{N-1} x_n}{M(N-1)}$
- $\hat{q}_{\mathrm{ML}} = \frac{\sum_{n=0}^{N-1} x_n}{(M-1)N}$

Question 7

1 pts

Regarding efficient estimators for linear models, which of the following statement is **FALSE.**

- ◯ The efficient estimator is available when the signal model is linear.
- ◯ If the model is linear with additive white Gaussian noise of variance σ^2, the covariance of the estimate is proportional to σ^2.
- ◯ When colored Gaussian noise is added to the linear signal model, the noise covariance of MVUE can be expressed as $\mathbf{C}_{\hat{\boldsymbol{\theta}}} = (\mathbf{H}^T\mathbf{C}^{-1}\mathbf{H})^{-1}$. Here, $\mathbf{C}$ is the covariance matrix of the colored noise.

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