Statistical signal processing (5CTA0)

Student Led Tutorials

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Exercise 6.1: Given the biased estimate $\hat{\rho}[\tau]$ for the autocorrelation:

$$\hat{\rho}[\tau] = \begin{cases} \frac{1}{N}\sum_{k=\tau}^{N-1} x[k]x[k-\tau] & 0 \leq \tau \leq N-1 \\ \hat{\rho} & -(N-1) \leq \tau \leq 0 \\ 0 & \text{elsewhere} \end{cases} \tag{1}$$

Show that the Periodogram $\hat{P}(e^{j\theta})$ can be expressed as the FTD of $\hat{\rho}[\tau]$.

Exercise 6.2: In order for the psd estimate to be nonnegative when using the Blackman-Tukey approach, the used correlation lag window $w[\tau]$ must have a nonnegative Fourier transform. Thus

$$\hat{P}_{BT}(e^{j\theta}) \geq 0 \Rightarrow W(e^{j\theta}) \geq 0 \tag{2}$$

(a) Formulate a procedure to generate a symmetric correlation lag window $w[\tau]$ of length $2N+1$ with nonnegative Fourier transform.

(b) Verify the procedure of a) when using a rectangular window as the correlation lag window $w[\tau]$.

Exercise 6.3: Signal $x[k]$ is AR(1) and is described by the difference equation: $x[k] = i[k] - \frac{3}{4}x[k-1]$. In this equation $i[k]$ is white (innovation) noise with zero mean and variance $\sigma_i^2 = \frac{7}{16}$.

(a) Calculate the autocorrelation function $\rho[\tau] = \mathrm{E}\{x[k]x[k-\tau]\}$.

(b) Calculate the PSD $\hat{P}(e^{j\theta})$ via the 'indirect' correlogram method in which the PSD is obtained as the FTD of the windowed AC:

$$\hat{\rho}[\tau] = \begin{cases} \rho[\tau] & \text{for } \tau = -1, 0, 1 \\ 0 & \text{elsewhere} \end{cases} \tag{3}$$

Explain why this correlogram is not a valid PSD.

(c) Now we use the unbiased AC estimate:

$$\hat{\rho}[\tau] = \frac{1}{N-|\tau|}\sum_{k=\tau}^{N-1} x[k]x[k-|\tau|], \text{for} |\tau| = 0, 1, \cdots, N-1 \tag{4}$$

Calculate $E\{\hat{P}(e^{j\theta})\}$ and show that this can lead to a negative spectral estimate.

Exercise 6.4: Consider random signal $x[k]$ that is generated by filtering innovation $i[k]$ (white noise sequence with zero mean and variance $\sigma_i^2 = 1$ with the filter $H(z) = \frac{1-\frac{2}{3}z^{-1}}{1-\frac{1}{2}z^{-1}}$.

(a) Calculate the autocorrelation of this random singal.

(b) Now suppose this model is generated by a AR(1) process. Please find the AR model parameters.

(c) Provide a sketch of the $P_{AR1}(e^{j\theta})$ obtained from the AR(1) model.