Quiz week 4

Started: 27 Sep at 20:11

Quiz instructions

Question 1 — 1 pts

Regarding least squares estimation (LSE), which of the following statement is **False.**

- ○ By minimizing the cost function $J(\theta) = \sum_{n=0}^{N-1}(x[n] - s[n;\theta])^2$, the LSE estimate $\hat{\boldsymbol{\theta}}_{LS}$ can always be determined.
- ◉ Suppose the model is linear as form $\boldsymbol{s}[n;\boldsymbol{\theta}] = \mathbf{H}\boldsymbol{\theta}$, the LSE can be formulated as $\hat{\boldsymbol{\theta}}_{LS} = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{x}$.
- ○ As long as one signal model can further reduce the squared error, it presents a more accurate estimation.

Question 2 — 1 pts

Consider a fitting example where the model is $s_n(\theta) = An^2 + Bn$ for $n = 0, 1, 2, 3, 4$. We can use a weighted LSE to fit the model to a 5-samples data $\mathbf{x} = [-0.2, 1.3, 3, 5.2, 8.1]^T$ with specific weighting matrix $\mathbf{W} = \mathrm{diag}([2, 2, 2, 1, 1])$. Here, diag denotes a diagonal matrix.

Construct the observation matrix $\mathbf{H}$.

$\mathbf{H}$=

Question 3

1 pts

In the same situation as question 2, please use Matlab to calculate the weighted LSE $\hat{\boldsymbol{\theta}}_{\text{WLS}} = [A, B]^T$. Which of the following estimation is correct?

- ○ $\hat{\boldsymbol{\theta}}_{\text{WLS}} = [0.245, 0.994]^T$
- ○ $\hat{\boldsymbol{\theta}}_{\text{WLS}} = [0.255, 0.994]^T$.
- ○ $\hat{\boldsymbol{\theta}}_{\text{WLS}} = [0.245, 1.004]^T$.
- ○ $\hat{\boldsymbol{\theta}}_{\text{WLS}} = [0.255, 1.004]^T$.
- ○ Non of the above answers is correct.

Question 4

1 pts

Consider an example of estimating DC level in AWGN noise. The conditional PDF is $p(x|A) = \frac{1}{(2\pi\sigma^2)^{N/2}} \exp\left(-\frac{1}{2\sigma^2}\sum_{n=0}^{N-1}(x_n - A)^2\right)$.

The prior PDFs of A is assumed to be a Gaussian distribution with mean value of μ_A : $p(A) = \frac{1}{\sqrt{2\pi\alpha}} \exp\left[-\frac{1}{2\alpha}(A - \mu_A)^2\right]$.

Which of the following expression correctly describes the maximum posterior estimator $\hat{A}_{\text{MAP}} = \arg\max_A p(A|\mathbf{x})$?

- $\hat{A}_{\text{MAP}} = \frac{\sum_{n=0}^{N-1} x_n + \frac{\mu_A}{\alpha}}{N + \frac{1}{\alpha}}$
- $\hat{A}_{\text{MAP}} = \frac{\frac{\sum_{n=0}^{N-1} x_n}{\sigma^2} + \frac{\mu_A}{\alpha}}{\frac{N}{\sigma^2} + \frac{1}{\alpha}}$
- $\hat{A}_{\text{MAP}} = \frac{\sum_{n=0}^{N-1} x_n}{N} + \frac{\mu_A}{\sigma}$
- $\hat{A}_{\text{MAP}} = \frac{\frac{\sum_{n=0}^{N-1} x_n}{\sigma} + \frac{\mu_A}{\alpha}}{\frac{N}{\sigma} + \frac{1}{\alpha}}$

Question 5

1 pts

Consider a posterior PDF of parameter θ given the evidence x: $p(\theta|x) = \begin{cases} x^2 \theta e^{-\theta x} & \text{if } x > 0 \text{ and } \theta \geq 0 \\ 0 & \text{else} \end{cases}$, which of the following expression is the correct Bayesian minimum mean squared error estimation?

Hint: There is an known indefinite integral $\int y^2 e^{cy} dy = e^{cy}\left(\frac{y^2}{c} - \frac{2y}{c^2} + \frac{2}{c^3}\right)$.

- $\hat{\theta}_{\text{mmse}} = \frac{1}{x}$
- $\hat{\theta}_{\text{mmse}} = \frac{2}{x}$
- $\hat{\theta}_{\text{mmse}} = 1$
- $\hat{\theta}_{\text{mmse}} = 2$

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