Statistical signal processing
5CTA0

Estimation theory - Introduction

Application I - Radar

- Estimate range, velocity, and angle to enable self driving cars

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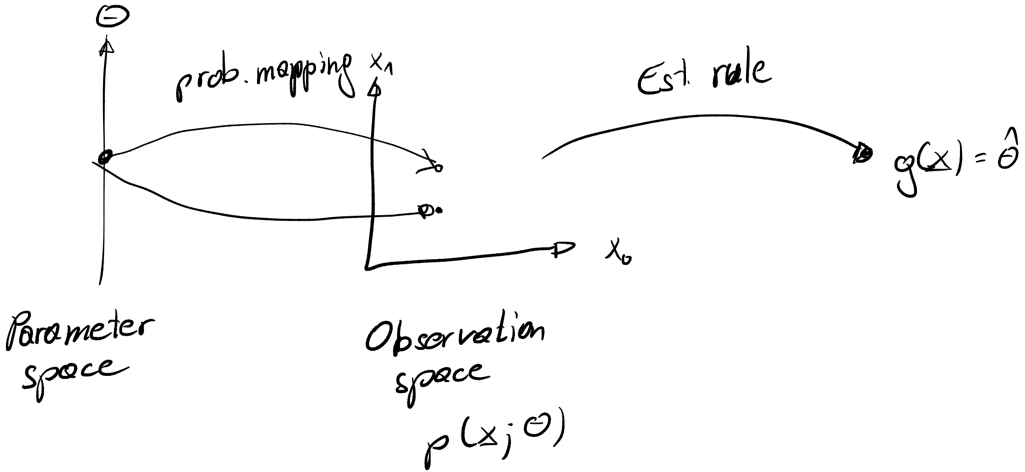
Application II - Ultrasound

- Range and angle to create image of fetus or organs

Application III - Wireless communication

- Estimate timing, carrier phase, and carrier frequency offset to synchronize Tx and Rx

Estimation Problem



Example: Estimation of a DC voltage

- Obervation model:

$$x_n = A + w_n, \qquad w_n \sim \mathcal{N}(0, \sigma^2)$$

$$p(\mathbf{x}; \theta) = \frac{1}{(2\pi\sigma^2)^{N/2}} \exp\left(-\frac{1}{2\sigma^2}\sum_{n=0}^{N-1}(x_n - A)^2\right)$$

- Estimators:

$$\left.\begin{aligned} \hat{A}_1 &= \frac{1}{N}\sum_{n=0}^{N-1} x_n \\ \hat{A}_2 &= x_i, \qquad 0 \le i \le N-1 \\ \hat{A}_3 &= \frac{1}{N-2}\left(2x_0 + \sum_{n=1}^{N-2} x_n - 2x_{N-1}\right) \end{aligned}\right\} \text{optimal?}$$

Example: Estimation of a DC voltage

- Unbiasedness: $E[\hat{\theta}] = \theta$
- $\mathrm{E}[\hat{A}_1] = \frac{1}{N} \sum_{n=0}^{N-1} A + \underbrace{E[w_n]}_{=0} = \frac{1}{\cancel{N}} \cancel{N} A = A$ ✓
- $\mathrm{E}[\hat{A}_2] = A + E[w_i] = A$ ✓
- $\mathrm{E}[\hat{A}_2] = \frac{1}{N-2} \left(2A + (N-2)A - 2A \right) = \frac{1}{N-2} (N-2) A = A$ ✓

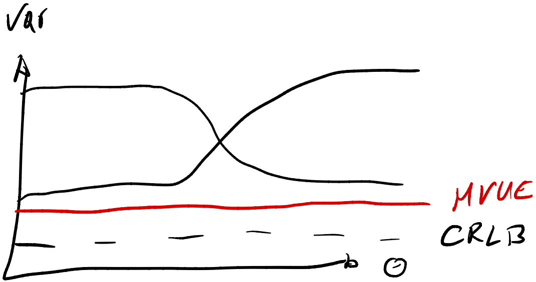
Example: Estimation of a DC voltage

$$\mathrm{Var}[x] = E[x^2] - E^2[x] \Rightarrow E[x^2] = \mathrm{Var}[x] + E^2[x]$$

- Variance: variability of estimator
- $\mathrm{Var}[\hat{A}_1] = \frac{1}{N^2} E\left[\sum_n \sum_i x_n x_i\right] - A^2 = \frac{1}{N^2}\left[\sum_n \overbrace{E[x_n^2]}^{\sigma^2 + A^2} + \sum_n \sum_{i \neq n} \overbrace{E[x_n]}^{A} \, \overbrace{E[x_i]}^{A}\right] - A^2$

  $= \frac{1}{N^{\cancel{2}}}\left[\cancel{N}(\sigma^2 + \cancel{A^2}) + \cancel{N}(N - \cancel{1}) A^2\right] - A^2 = \frac{1}{N}\left[\sigma^2 + NA^2\right] - A^2 = \frac{\sigma^2}{N}$
- $\mathrm{Var}[\hat{A}_2] = \sigma^2$
- $\mathrm{Var}[\hat{A}_2] = \frac{N+6}{(N-2)^2} \sigma^2$

Performance bound



Outline of Part II

- Cramér Rao Lower Bound
- Maximum likelihood estimation
- Linear models
- Least squares estimation
- Bayesian estimation
- Numerical Methods