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The spin-1 Ising spin glass: a renormalization-group approach

Amadeu Albino Jr, Fernando D Nobre and Francisco A da Costa

Departamento de Física Teórica e Experimental, Universidade Federal de Rio Grande do Norte, Campus Universitário, Caixa Postal 1641, 59072-970 Natal, RN, Brazil

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Abstract. The nearest-neighbour-interaction spin-1 Ising spin glass, in the presence of a random crystal field, is considered on diamond hierarchical lattices of fractal dimensions $d = 2$, 3 and 4. The coupling constants and crystal fields follow Gaussian probability distributions, which are taken as independent, at the beginning of the iteration process. By monitoring simultaneously the evolution of two probability distributions, associated respectively with the renormalized coupling constants and crystal fields, the phase diagrams of the model are obtained. A spin-glass phase, at finite temperatures, is found for hierarchical lattices with $d = 3$ and 4, but not for $d = 2$. Two distinct attractors characterized by zero effective coupling constants are detected. Following the usual procedure, i.e. associating an equilibrium phase with each basin of attraction, one obtains two phases with absence of magnetic order, namely, a zero-spin phase (where the spins prefer the 0 state) and a ± 1-spin phase (where the spins prefer ± 1 states at random).

1. Introduction

Spin glasses [1–3] have been, throughout the last decades, among the most fascinating systems in the physics of disordered magnets. Despite the large effort dedicated to this problem, a satisfactory theory for the description of short-range spin glasses is still lacking.

Most of the progress has been achieved in the spin-$\frac{1}{2}$ Ising case, either at the mean-field level, through the exact solution of the infinite-range-interaction model (the so-called Sherrington–Kirkpatrick (SK) [4] model), or through approximation techniques applied to nearest-neighbour-interaction systems. The mean-field solution presents quite remarkable properties, such as a phase transition in the presence of an external magnetic field, signalled by the Almeida–Thouless (AT) [5] line; below the AT line the free-energy landscape becomes highly nontrivial, characterized by many minima, which are not related by global inversion symmetries. The validity of the SK model for the description of real systems is a question which remains under investigation [1]. As concerns short-range-interaction models, different works [6–12] agree that the lower critical dimension d_l lies in the range $2 \leqslant d_l \leqslant 3$, in such a way that there is a finite-temperature phase transition in three dimensions, but not in two. This conclusion was reached through zero-temperature domain-wall arguments [7, 8], powerful numerical simulations [9, 10, 12] and extensive high-temperature series expansions [11]. The result $2 \leqslant d_l \leqslant 3$ was first achieved by Southern and Young (SY) [6], almost a decade before its general acceptance, through a Migdal–Kadanoff renormalization-group (MKRG) approach, based on diamond-like unit cells. For a diamond cell corresponding to a fractal dimension $d = 3$, SY estimated the critical temperatures in the case of symmetric distributions, as $k_B T_c/J = 1.05 \pm 0.02$ ($\pm J$ distribution) and $k_B T_c/J = 0.88 \pm 0.02$ (Gaussian distribution of width J). The most accurate techniques yield, for a cubic lattice, the estimates

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$k_B T_c/J \simeq 1.2$ in the former case [9–11], whereas $0.9 < k_B T_c/J < 1.0$ in the latter [10]. For the $\pm J$ Ising spin glass, the best estimate available is probably due to recent numerical simulations ($k_B T_c/J = 1.11 \pm 0.04$ [12]), which coincides, within the error bars, with the one of SY. Why, for $d = 3$, a simple approximation, like the MKRG, provides critical-temperature estimates in good agreement with the (commonly considered) best techniques is a question not yet understood. Surprisingly, such a procedure seems to work better for spin glasses than for simple ferromagnets, at least as concerns critical-temperature estimates. In fact, the MKRG approach has been extensively used recently in the study of the controversial nature of the spin-glass phase of the nearest-neighbour-interaction spin-$\frac{1}{2}$ Ising spin glass [13].

In order to achieve a better understanding of spin glasses, many other models have been proposed, such as spin-S ($S > \frac{1}{2}$) Ising, Potts and vector spin-glass models [2, 3]. Much less is known about such systems, from which many controversies, concerning different phases, characterized by new classes of order parameters, have emerged. In particular, the spin-1 Ising spin glass has been considered, mostly at the mean-field level, through generalizations of the SK model [14–20]. The phase diagram, for the system in the presence of a crystal field and a symmetric coupling probability distribution, presents two phases (paramagnetic and spin glass), separated by a phase boundary composed by a second-order part (high temperatures) and a first-order one (low temperatures), which meet at a tricritical point. The first-order critical line has been the object of some controversy: whereas Mottishaw and Sherrington [16] find a line characterized by a considerable reentrance, da Costa *et al* [17] find a different critical frontier, presenting insignificant reentrance effects. However, very little is known about the nearest-neighbour-interaction spin-1 Ising spin glass. Li *et al* [21] have considered this system, for symmetric coupling probability distributions, through zero-temperature domain-wall techniques, in two dimensions, and for both two- and three-dimensional cases, within a MKRG approach. The MKRG phase diagram, in the three-dimensional case, for the system in the presence of a crystal field, yields a critical line with no reentrance, in better agreement with the mean-field predictions of da Costa *et al* [17], as compared with those of Mottishaw and Sherrington [16].

Motivated by the good critical-temperature estimates for the spin-$\frac{1}{2}$ case, in the present work we consider the nearest-neighbour-interaction spin-1 Ising spin-glass model, in the presence of a random crystal field, defined on diamond hierarchical lattices. The coupling constants and crystal fields are distributed according to Gaussian probability distributions with nonzero means. The model is studied through the MKRG approach, which is exact for pure systems on such lattices [22–24] and expected to yield good approximations for random systems. The purpose of the present work is to show new results for the spin-1 Ising spin glass; several phase diagrams, not considered in [21], are presented, for lattices with fractal dimensions $d = 2$, 3 and 4. In particular, two phases associated with distinct zero-effective-coupling-constant attractors are found; to our knowledge, such phases have never been identified in such a model. In the next section we define the model and the formalism; in section 3 we exhibit and discuss our phase diagrams; finally, in section 4 we present our conclusions.

2. The model and formalism

Let us consider the spin-1 Ising spin glass, defined through the Hamiltonian

$$\mathcal{H} = -\sum_{\langle ij \rangle} J_{ij} S_i S_j - \sum_i D_i S_i^2 \qquad (S_i = 0, \pm 1) \tag{2.1}$$

where the sum $\sum_{\langle ij\rangle}$ is restricted to nearest-neighbour pairs of spins on a diamond hierarchical lattice, generated in such a way that, at each step, a bond is replaced by a diamond-like cell. Herein, we shall restrict ourselves to unit cells containing L parallel branches, each with two bonds in series (scaling factor $b = 2$), whose fractal dimension is given by $d = [\ln(2L)]/(\ln 2)$. More specifically, we shall deal with the cases $L = 2$, 4 and 8, which correspond, respectively, to $d = 2$, 3 and 4. These cells are characteristic of the MKRG approach, which is an exact procedure for pure systems defined on diamond hierarchical lattices [22–24] and is expected to represent a good approximation for random systems on such lattices.

The coupling constants $\{J_{ij}\}$ and the crystal fields $\{D_i\}$ are quenched random variables following Gaussian probability distributions,

$$P(J_{ij}) = \frac{1}{\sqrt{2\pi J^2}} \exp[-(J_{ij} - J_0)^2/2J^2] \tag{2.2a}$$

$$P(D_i) = \frac{1}{\sqrt{2\pi D^2}} \exp[-(D_i - D_0)^2/2D^2]. \tag{2.2b}$$

The renormalization-group (RG) procedure works inversely to the generation of the lattice, i.e. it transforms the diamond cells into elementary bonds. It is a well known fact that the model defined in equation (2.1) is ambiguous under such an RG transformation, in the sense that one obtains more equations than variables to work with [25]. One possible way to overcome this difficulty is to introduce new terms in the Hamiltonian, in order to obtain a closed space of parameters. However, it is important to study the model in its simpler forms, like the one defined in equation (2.1), or even in the case of no crystal fields, i.e. $D_i = 0$; this is justifiable, based on the fact that very little is known about the nearest-neighbour-interaction spin-1 Ising spin glass. Therefore, in order to study the model defined in equation (2.1) through an RG approach, one should decide on the best choice of RG equations to deal with. In the appendix we define our RG transformation and discuss the mathematical consistency of the set of equations; one sees that the equations obtained by fixing the external spins of the diamond cell to $S_i = S_j = 0$ is only consistent with the remaining equations at the paramagnetic fixed point. If one is restricted to the equations which may be valid throughout the whole range of the parameters space, one obtains the recursion relations (see the appendix)

$$K'_{ij} = \sum_{l=1}^{L} K'_l = \frac{1}{2}\sum_{l=1}^{L} \ln\left\{\frac{1 + 2[\exp(\Delta_l)]\cosh(K_{il} + K_{lj})}{1 + 2[\exp(\Delta_l)]\cosh(K_{il} - K_{lj})}\right\} \tag{2.3}$$

$$\Delta'_i = L\Delta_i + \sum_{l=1}^{L} \ln\left\{\frac{(1 + 2[\exp(\Delta_l)]\cosh(K_{il} + K_{lj}))(1 + 2[\exp(\Delta_l)]\cosh(K_{il} - K_{lj}))}{(1 + 2[\exp(\Delta_l)]\cosh(K_{lj}))^2}\right\} \tag{2.4a}$$

$$\Delta'_j = L\Delta_j + \sum_{l=1}^{L} \ln\left\{\frac{(1 + 2[\exp(\Delta_l)]\cosh(K_{il} + K_{lj}))(1 + 2[\exp(\Delta_l)]\cosh(K_{il} - K_{lj}))}{(1 + 2[\exp(\Delta_l)]\cosh(K_{il}))^2}\right\}. \tag{2.4b}$$

In the equations above, K'_{ij}, K_{il} and K_{lj} represent, respectively, the renormalized and two original dimensionless exchange energies ($\{K_{ij}\} \equiv \beta\{J_{ij}\}$), associated with a given path l ($l = 1, 2, \ldots, L$) connecting the external sites i and j, whereas Δ'_i, Δ'_j, Δ_i, Δ_j and Δ_l represent the renormalized and original dimensionless crystal fields ($\{\Delta_i\} \equiv \beta\{D_i\}$), associated with the external and internal sites of a given path l of the diamond cell.

The RG scheme may now be carried out by following numerically the evolution of the probability distributions associated with the exchange energies and crystal fields. Let us define $P^{(K)}(K'_{ij})$ and $P^{(\Delta)}(\Delta'_i)$ as the probability distributions associated, respectively, with

the renormalized exchange energies in equation (2.3) and with the symmetrical renormalized crystal fields,

$$\frac{1}{2}(\Delta'_i + \Delta'_j) = \frac{L}{2}(\Delta_i + \Delta_j) + \sum_{l=1}^{L} \ln \left\{ \frac{(1 + 2[\exp(\Delta_l)]\cosh(K_{il} + K_{lj}))(1 + 2[\exp(\Delta_l)]\cosh(K_{il} - K_{lj}))}{(1 + 2[\exp(\Delta_l)]\cosh(K_{il}))(1 + 2[\exp(\Delta_l)]\cosh(K_{lj}))} \right\}. \tag{2.5}$$

Such probability distributions will describe the evolution of the renormalized random variables $\{K'_{ij}\}$ and $\{\Delta'_i\}$ within the renormalization process. Operationally, $P^{(K)}(K'_{ij})$ and $P^{(\Delta)}(\Delta'_i)$ are represented by two pools of M real numbers each ($\{K'_I\}$ and $\{\Delta'_I\}$, $I = 1, 2, \ldots, M$) from which one may compute, at each renormalization step,

$$\sigma_n^{(K)} = \frac{1}{M}\sum_{I=1}^{M}(K'_I)^n \tag{2.6a}$$

$$\sigma_n^{(\Delta)} = \frac{1}{M}\sum_{I=1}^{M}(\Delta'_I)^n \tag{2.6b}$$

which should approach the moments of $P^{(K)}(K'_{ij})$ and $P^{(\Delta)}(\Delta'_i)$, respectively.

The process starts by creating two initial pools of M real numbers each, produced from Gaussian random-number generators [26]. Such initial pools emulate the probability distributions in equations (2.2). The first RG iteration consists of M operations, where in each of them one picks at random, $2L$ numbers from the pool of bonds and $(L + 2)$ numbers from the pool of crystal fields, assigned, respectively, to the bonds and sites of the diamond cell, in order to generate the renormalized exchange energies and crystal fields, according to equations (2.3) and (2.5). After that, one obtains two new pools of numbers, representing the probability distributions $P^{(K)}(K'_{ij})$ and $P^{(\Delta)}(\Delta'_i)$. The whole procedure is repeated, by picking up numbers from the new pools, in order to generate the renormalized random variables at the next step. As usual in the RG approach, each phase is dominated by its own attractor, characterized by well defined limits of the moments in equations (2.6).

In the next section we present and discuss our phase diagrams.

3. Results

The pools of real numbers, emulating the probability distributions $P^{(K)}(K'_{ij})$ and $P^{(\Delta)}(\Delta'_i)$, associated with the renormalized variables in equations (2.3) and (2.5), are dominated by the behaviour of the lowest-order moments, as described below.

(i) *Pool of renormalized bonds.* All odd moments follow the behaviour of $\sigma_1^{(K)}$, whereas the even ones behave according to $\sigma_2^{(K)}$.

(ii) *Pool of renormalized crystal fields.* All odd moments follow the behaviour of $\sigma_1^{(\Delta)}$ (whose absolute value always increases under successive renormalizations), whereas all even moments increase at each iteration.

Therefore, in order to characterize the different phases of the present model, one needs to monitor only three moments, namely, $\sigma_1^{(K)}$, $\sigma_2^{(K)}$ and $\sigma_1^{(\Delta)}$. Depending on the fractal dimension d, one may find the ferromagnetic (F), spin-glass (SG), 0-spin (P_0) and ± 1-spin (P_1) attractors,

characterized by

$$\begin{array}{llll} \sigma_1^{(K)} \to \infty & \sigma_2^{(K)} \to \infty & \sigma_1^{(\Delta)} \to \infty & \text{(F attractor)} \\ \sigma_1^{(K)} \to 0 & \sigma_2^{(K)} \to \infty & \sigma_1^{(\Delta)} \to \infty & \text{(SG attractor)} \\ \sigma_1^{(K)} \to 0 & \sigma_2^{(K)} \to 0 & \sigma_1^{(\Delta)} \to -\infty & (\mathrm{P}_0 \text{ attractor}) \\ \sigma_1^{(K)} \to 0 & \sigma_2^{(K)} \to 0 & \sigma_1^{(\Delta)} \to \infty & (\mathrm{P}_1 \text{ attractor}). \end{array}$$

The variables in our phase diagrams are directly related to such moments,

$$\beta J_0 = \sigma_1^{(K)} \qquad \beta J = [\sigma_2^{(K)}]^{1/2} \qquad \beta D_0 = \sigma_1^{(\Delta)}. \tag{3.1}$$

The two attractors with zero effective coupling constants are associated, respectively, with basins of attraction where the spins prefer $S_i = 0$ (P_0 attractor), and $S_i = \pm 1$ at random (P_1 attractor). Herein, we shall follow the standard RG procedure, associated with each attractor a distinct equilibrium phase. Therefore, our phase diagrams may display, besides the usual F and SG phases, two other phases (P_0 and P_1). From the thermodynamical point of view, such phases are identified by the magnetization (m), the Edwards–Anderson spin-glass order parameter [2, 3] (q) and the quadrupolar order parameter (p),

$$m = \frac{1}{N}\sum_i [\langle S_i \rangle_T]_{J,D} \qquad q = \frac{1}{N}\sum_i [\langle S_i \rangle_T^2]_{J,D} \qquad p = \frac{1}{N}\sum_i [\langle S_i^2 \rangle_T]_{J,D} \tag{3.2}$$

where $\langle\rangle_T$ and $[]_{J,D}$ represent thermal and disorder averages, respectively. The F phase, which usually occurs for sufficiently high values of $\sigma_1^{(K)}$, is characterized by strong correlations between the spins, in such a way that $m \neq 0$, $q \neq 0$, $p \neq 0$. For dimensions above the spin-glass lower critical dimension d_l, an SG phase usually appears at low temperatures and small values of the ratio $\sigma_1^{(K)}/\sigma_2^{(K)}$, with the spins frozen at random in one of the $S_i = 0, \pm 1$ states, leading to $m = 0$, $q \neq 0$, $p \neq 0$. Since in phase P_0 (P_1) the spins prefer the state $S_i = 0$ (states $S_i = \pm 1$), in the thermodynamic limit one expects $p = 0$ ($p > 0$). Phase P_0, which should occur for sufficiently negative values of the first moment of the crystal-field probability distribution ($\sigma_1^{(\Delta)}$), is essentially a frozen phase and may be extended down to zero temperature; trivially one obtains $m = q = 0$ along P_0. Due to thermal effects, throughout phase P_1 the spins are uncorrelated and should fluctuate in time among the possible states, leading to $m = q = 0$ as well; however, at very high temperatures, all states are equally probable, in such a way that $p \to 2/3$, whereas at low temperatures, the preference of the ± 1 states leads to $p \to 1$.

Let us start our analysis by looking at the case $D_i = 0$; although this is not an invariant subspace (see the appendix), one may consider a crude approximation by imposing $\Delta_i = 0$ at all renormalization steps. In doing that, one sees from the appendix that equations (A.2*c*), (A.2*d*) and (A.2*e*) become mathematically inconsistent (except at the paramagnetic fixed point, $K_{ij} = 0$); dealing with equations (A.2*a*) and (A.2*b*), one obtains recursion relation (2.3) (with $\Delta_l = 0$) for the renormalized bonds. The different phases are dominated by

$$\begin{array}{lll} \sigma_1^{(K)} \to \infty & \sigma_2^{(K)} \to \infty & \text{(F attractor)} \\ \sigma_1^{(K)} \to 0 & \sigma_2^{(K)} \to \infty & \text{(SG attractor)} \\ \sigma_1^{(K)} \to 0 & \sigma_2^{(K)} \to 0 & \text{(P attractor).} \end{array}$$

Our phase diagrams for $D_i = 0$ are exhibited in figure 1, for hierarchical lattices with fractal dimensions $d = 3$ and 4 (the phase diagram for $d = 2$ presents a single critical frontier separating phases P and F). Analogous to what happens for the spin-$\frac{1}{2}$ Ising spin glass, one obtains an SG phase for $d = 3$ and 4, but not for $d = 2$, which leads to the result that the lower critical dimension d_l should lie in the range $2 \leqslant d_l \leqslant 3$.

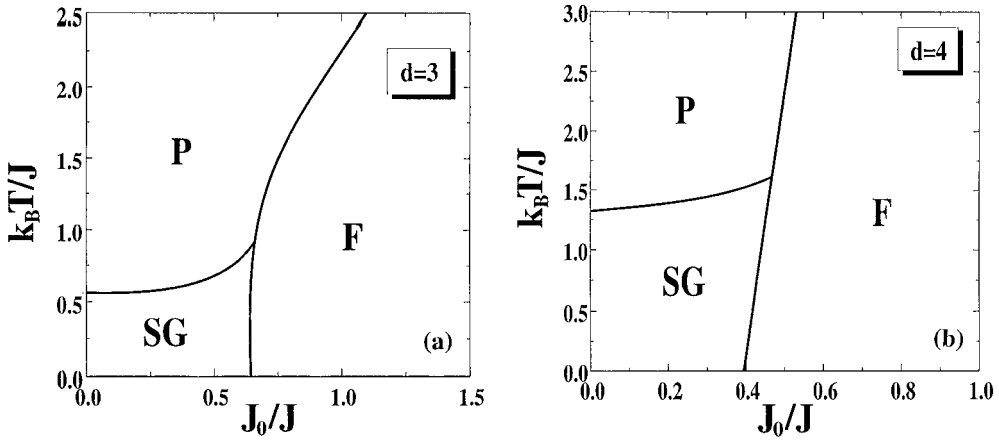

Figure 1. The phase diagram for $D_i = 0$ of the spin-1 Ising spin glass on hierarchical lattices with fractal dimensions d: (a) $d = 3$; (b) $d = 4$.

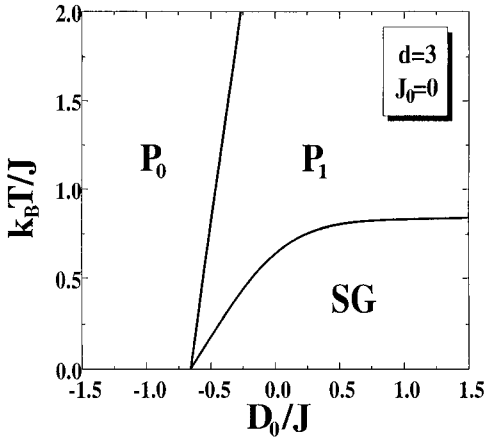

Figure 2. The plane $J_0 = 0$ (invariant under the present RG transformation) for $d = 3$.

Let us know consider the cases $D_i \neq 0$, described by the evolution of the random variables in the recursive equations (2.3) and (2.5). For $d = 2$ there is no spin-glass attractor, and the only phases possible, at finite temperatures, are the F, P_0 and P_1 ones. Slices of the phase diagram for fixed average values of the initial distribution of crystal fields yield a single critical frontier between phase F and P_0 (P_1) for small (large) enough initial values of D_0/J. The invariant plane $J_0 = 0$ exhibits a single critical frontier between phase P_0 and P_1.

The most relevant case is the approach of a cubic lattice through the diamond hierarchical lattice of fractal dimension $d = 3$. Let us first consider the plane $J_0 = 0$ (invariant under the present RG), shown in figure 2; there is no ferromagnetic phase and the boundaries P_0–P_1 and P_1–SG meet at zero temperature. One sees no low-temperature re-entrance in the SG phase; this result is in better agreement with the mean-field phase diagram of da Costa *et al* [17] than with the one of Mottishaw and Sherrington [16]. Except for the phases P_0 and P_1, the phase diagram shown in figure 2 is qualitatively analogous to the one by Li *et al* [21] (cf figure 5 of [21]), which used a different RG approach for the spin-1 Ising spin glass. In figure 3 we show

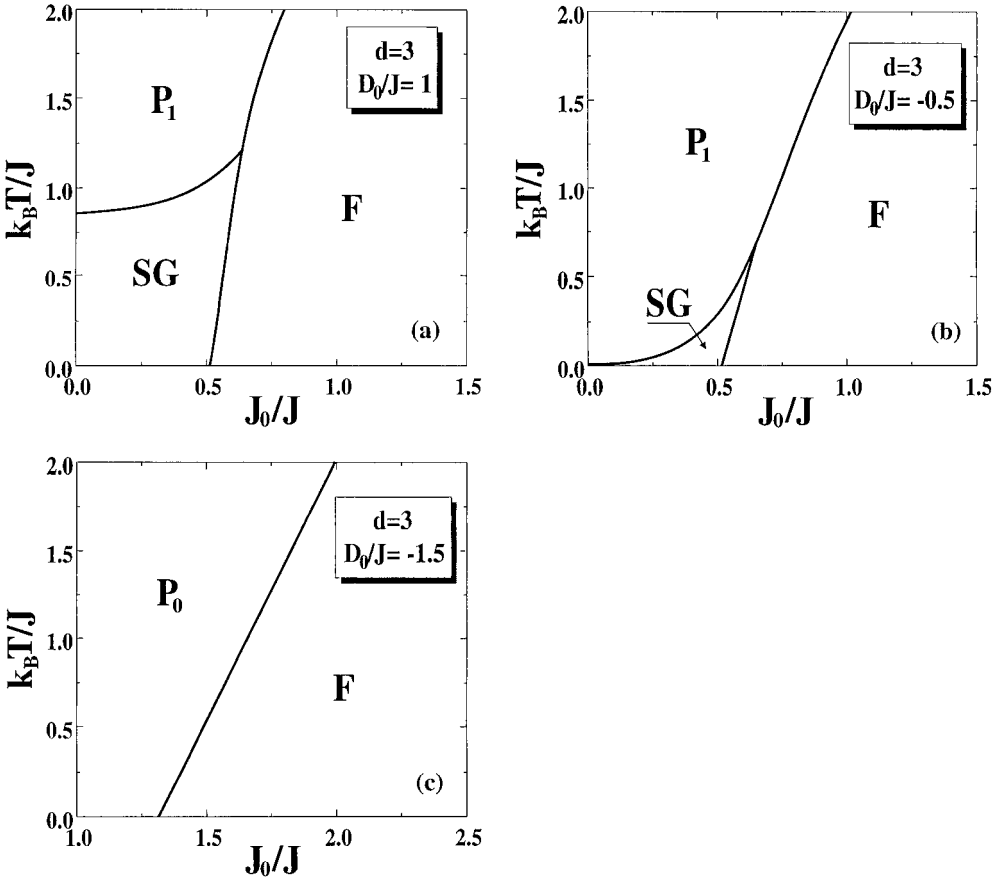

Figure 3. Slices of the phase diagram for $d = 3$, for typical values of the average value of the initial distribution of crystal fields. (a) $D_0/J = 1.0$; (b) $D_0/J = -0.5$; (c) $D_0/J = -1.5$.

slices of the phase diagram for typical choices of the average value of the initial distribution of crystal fields; it should be mentioned that such planes are noninvariant under the present RG transformation. One notices that the SG phase is suppressed by decreasing D_0, in such a way that for sufficiently negative values of D_0 one finds only phases P_0 and F (see figure 3(c)). In figure 4 we show a three-dimensional part of the phase diagram; for the sake of clarity, the boundary separating phases P_0 and P_1 is not exhibited.

For a fractal dimension $d = 4$ most of the phase diagrams are qualitatively similar to those of the case $d = 3$. In figure 5 we exhibit the invariant plane $J_0 = 0$; in analogy to what happens for $d = 3$, the SG phase displays no reentrant behaviour at low temperatures and the basic difference from the phase diagram shown in figure 2 is that the boundary P_0–P_1 meets the SG critical frontier at a finite temperature. A slice of the phase diagram for an average value of the initial distribution of crystal fields $D_0/J = 0$ is presented in figure 6. Within the present RG approach, diamond hierarchical lattices with $d > 4$ are expected to yield, qualitatively, similar phase diagrams to those of the cases $d = 3$ and 4.

It is interesting to compare the qualitative properties of the spin-$\frac{1}{2}$ and spin-1 Ising spin glasses; the inclusion of the state $S_i = 0$ may bring significant changes to the well known physical behaviour of the spin-$\frac{1}{2}$ Ising spin glass as we describe below.

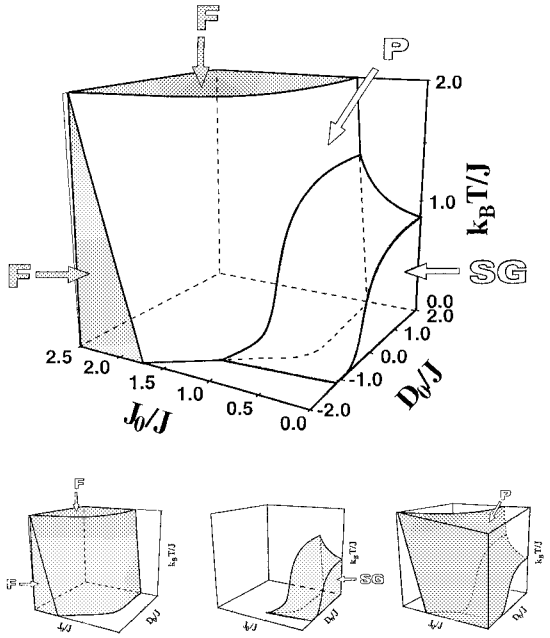

Figure 4. A part of the phase diagram for the hierarchical lattice with fractal dimension $d = 3$. For clarity, the phases P_0 and P_1 are combined into a single phase (P). In the smaller blocks each phase is represented separately.

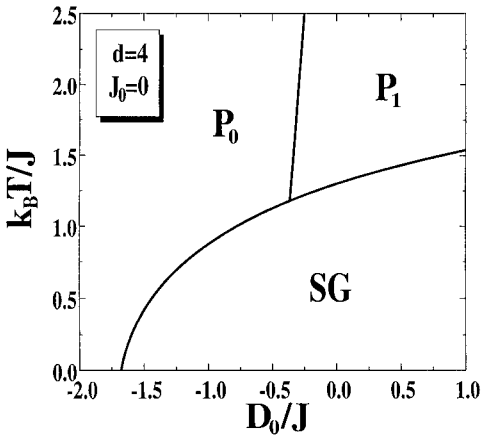

Figure 5. The plane $J_0 = 0$ (invariant under the present RG transformation) for $d = 4$.

(i) For the spin-$\frac{1}{2}$ case the crystal field introduced in equation (2.1) is irrelevant, leading to an additive constant in the Hamiltonian; in the present problem a positive (negative) crystal field favours the $S_i = \pm 1$ states ($S_i = 0$ state).

(ii) An enhancement of the paramagnetic order is expected by the inclusion of the state $S_i = 0$. For $D_i = 0$, such an effect is illustrated in figures 1(a) and 1(b), where the P–SG critical frontier appears at lower temperatures, as compared with the corresponding critical frontier of the spin-$\frac{1}{2}$ case [6]. The paramagnetic phase may become even more pronounced by the

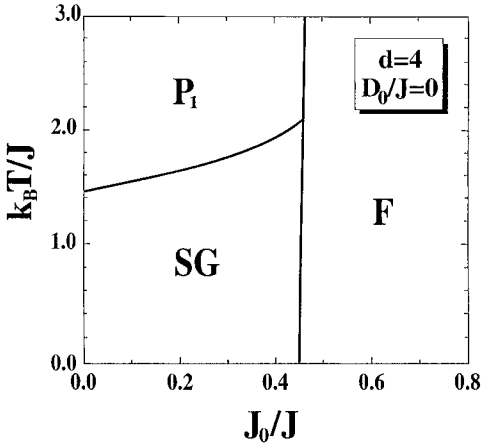

Figure 6. Slice of the phase diagram for $d = 4$, for an average value of the initial distribution of crystal fields $D_0/J = 0$.

introduction of a crystal field, as observed in the mean-field treatment of the spin-1 Ising spin glass, where one obtains a paramagnetic phase that extends down to zero temperature within a certain range of values of the crystal field [14–17]. Within the present approach, for certain values of the average of the crystal-field probability distribution, two distinct phases with absence of magnetic order were found; in some cases, such phases prolong to zero temperature as well.

(iii) The spin-glass lower critical dimension does not seem to be significantly affected by the inclusion of the state $S_i = 0$. The present work is in agreement with a previous RG analysis [21], leading to $2 \leqslant d_l \leqslant 3$ for the spin-1 Ising spin glass, in analogy to what happens for the corresponding spin-$\frac{1}{2}$ model [6–12]. Although the spin-glass phase may occur at lower temperatures, as compared to the one of the spin-$\frac{1}{2}$ case, due to a weakening of the long-range spin-glass correlations among the spins at states $S_i = \pm 1$, the lattice dimension seems to be determinant for the occurrence of spin-glass order at finite temperatures.

(iv) Within the RG approach, the state $S_i = 0$ poses new difficulties, in the sense that the recursion relations are not closed under renormalization, and further approximations are required. The possible consequences of the approximations used in obtaining the recursion relations of the present approach are discussed in the next section, where we present our main conclusions.

4. Conclusion

We have considered the nearest-neighbour-interaction spin-1 Ising spin glass on diamond hierarchical lattices of fractal dimensions $d = 2$, 3 and 4, in the presence of random crystal fields. At the beginning of the iteration process, the coupling constants and crystal fields were taken from independent Gaussian probability distributions. Using a Migdal–Kadanoff renormalization-group approach to study the system, we have shown that the lower critical dimension, above which one expects a spin-glass phase at finite temperatures, should lie in the range $2 \leqslant d_l \leqslant 3$, in analogy to what happens in the corresponding spin-$\frac{1}{2}$ model. Two distinct attractors characterized by zero effective coupling constants were found; such attractors were associated with two distinct phases with absence of magnetic order, namely, a zero-spin phase

(where the spins prefer the 0 state) and a ± 1-spin phase (with the spins favouring the ± 1 states at random). It should be mentioned that similar phases have also been found in the mean-field treatment of the $S = 1$ spin glass with p-spin interactions [27].

It is important to mention that RG approaches for spin-S ($S > \frac{1}{2}$) Ising systems are usually ambiguous, in the sense that one obtains more equations than variables to deal with. One possible way to overcome this difficulty is to introduce higher-order terms in the Hamiltonian, in such a way as to end up with a closed space of parameters, e.g. instead of dealing with the form in equation (2.1) one could use

$$\mathcal{H} = -\sum_{\langle ij \rangle} J_{ij} S_i S_j - \sum_{\langle ij \rangle} L_{ij} S_i^2 S_j^2 - \sum_i D_i S_i^2. \tag{4.1}$$

One may easily see that the introduction of a new coupling constant in the Hamiltonian of equation (2.1) yields a well defined RG framework. There are several reasons that led us to consider, throughout the present work, the form of equation (2.1), as mentioned below.

(i) It is important to have results for short-range-interaction spin glasses in their simpler Hamiltonian forms; most of the investigations for the spin-1 Ising spin glass, as defined in equation (2.1), have been carried out at the mean-field level, in the limit of infinite-range interactions [14–20].

(ii) Each new coupling constant considered in the Hamiltonian will introduce, within the RG framework, a new probability distribution to be followed numerically. In such a case, the RG parameter space will be enlarged by at least two new dimensions (associated with the two lowest moments of the corresponding distribution); this certainly will turn the problem into a much more complicated one.

(iii) It is important to mention that, at the mean-field level, the term $L_{ij} S_i^2 S_j^2$ generates a new parameter, which is nonzero for all temperatures [28]. If the mean-field phase diagram is preserved (at least qualitatively) for the present problem, such a behaviour would correspond, within the RG approach, to a single attractor, associated with the probability distribution of $\{L_{ij}\}$, i.e. no interesting new effect would occur by the inclusion of this term.

(iv) With the inclusion of higher-order terms, new fixed points, and, consequently, new phases may appear. However, the fixed points found in the present work (associated with the interactions $\{J_{ij}\}$ and crystal fields $\{D_i\}$), as well as their corresponding universality classes, should remain unaltered, although their basins of attraction may change (leading to modifications in the critical frontiers). By preserving the fixed points, the phase diagrams presented herein are not expected to undergo significant qualitative changes.

Therefore, in choosing to work with the set of most relevant equations, we believe we have captured the essential physics behind the nearest-neighbour- interaction spin-1 Ising spin glass, in the presence of a crystalline field, on diamond hierarchical lattices.

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Appendix. The recursion relations

In this appendix we will find the recursion relations for the spin-1 Ising spin glass on diamond hierarchical lattices; let us consider the model defined in equation (2.1) in its symmetrical

form,

$$\mathcal{H} = -\sum_{\langle ij \rangle} J_{ij} S_i S_j - \frac{1}{2}\sum_i D_i S_i^2 - \frac{1}{2}\sum_j D_j S_j^2. \tag{A1}$$

The decimation of the internal spin of a given one-dimensional branch l ($l = 1, 2, \ldots, L$) of the diamond cell corresponds to

$$\exp(K_l' S_i S_j + \tfrac{1}{2}\Delta_i'' S_i^2 + \tfrac{1}{2}\Delta_j'' S_j^2 + A)$$
$$= \sum_{S_l = 0, \pm 1} \exp(K_{il} S_i S_l + K_{lj} S_l S_j + \tfrac{1}{2}\Delta_i S_i^2 + \tfrac{1}{2}\Delta_l S_l^2 + \tfrac{1}{2}\Delta_j S_j^2)$$

where $\{K_{ij}\} \equiv \beta\{J_{ij}\}$ and $\{\Delta_i\} \equiv \beta\{D_i\}$. After carrying the partial trace over the spin variable S_l, one obtains the following set of equations, corresponding to the different configurations of the external spins S_i and S_j:

(a) $S_i = S_j = 1$ (or $S_i = S_j = -1$)

$$\exp(K_l' + \tfrac{1}{2}\Delta_i'' + \tfrac{1}{2}\Delta_j'' + A) = \{\exp[\tfrac{1}{2}(\Delta_i + \Delta_j)]\}\{1 + 2[\exp(\Delta_l)]\cosh(K_{il} + K_{lj})\} \tag{A2a}$$

(b) $S_i = -S_j = 1$ (or $S_i = -S_j = -1$)

$$\exp(-K_l' + \tfrac{1}{2}\Delta_i'' + \tfrac{1}{2}\Delta_j'' + A) = \{\exp[\tfrac{1}{2}(\Delta_i + \Delta_j)]\}\{1 + 2[\exp(\Delta_l)]\cosh(K_{il} - K_{lj})\} \tag{A2b}$$

(c) $S_i = 1$, $S_j = 0$ (or $S_i = -1$, $S_j = 0$)

$$\exp\left(\frac{\Delta_i''}{2} + A\right) = \left[\exp\left(\frac{\Delta_i}{2}\right)\right]\{1 + 2[\exp(\Delta_l)]\cosh(K_{il})\} \tag{A2c}$$

(d) $S_i = 0$, $S_j = 1$ (or $S_i = 0$, $S_j = -1$)

$$\exp\left(\frac{\Delta_j''}{2} + A\right) = \left[\exp\left(\frac{\Delta_j}{2}\right)\right]\{1 + 2[\exp(\Delta_l)]\cosh(K_{lj})\} \tag{A2d}$$

(e) $S_i = S_j = 0$

$$\exp(A) = 1 + 2\exp(\Delta_l). \tag{A2e}$$

One sees clearly that the set of equations (A.2) is not well defined, in the sense that one has five equations to be solved for the variables K_l', Δ_i'', Δ_j'' and A. However, one may easily see that equation (A.2e) is consistent with the others only at the paramagnetic fixed point, i.e. $K_{il} = K_{lj} = 0$; indeed, taking the product of the squares of equations (A.2c) and (A.2d), divided by the product of equations (A.2a) and (A.2b), one obtains

$$\exp(2A) = \frac{\{1 + 2[\exp(\Delta_l)]\cosh(K_{il})\}^2\{1 + 2[\exp(\Delta_l)]\cosh(K_{lj})\}^2}{\{1 + 2[\exp(\Delta_l)]\cosh(K_{il} + K_{lj})\}\{1 + 2[\exp(\Delta_l)]\cosh(K_{il} - K_{lj})\}}$$

which recovers (A.2e) only for $K_{il} = K_{lj} = 0$. Therefore, in order to find the recursion relations, we shall not take into account equation (A.2e); solving equations (A.2a)–(A.2d), one obtains

$$K_l' = \frac{1}{2}\ln\left\{\frac{1 + 2[\exp(\Delta_l)]\cosh(K_{il} + K_{lj})}{1 + 2[\exp(\Delta_l)]\cosh(K_{il} - K_{lj})}\right\} \tag{A3a}$$

$$\Delta_i'' = \Delta_i + \ln\left\{\frac{(1 + 2[\exp(\Delta_l)]\cosh(K_{il} + K_{lj}))(1 + 2[\exp(\Delta_l)]\cosh(K_{il} - K_{lj}))}{(1 + 2[\exp(\Delta_l)]\cosh(K_{lj}))^2}\right\} \tag{A3b}$$

$$\Delta_j'' = \Delta_j + \ln\left\{\frac{(1 + 2[\exp(\Delta_l)]\cosh(K_{il} + K_{lj}))(1 + 2[\exp(\Delta_l)]\cosh(K_{il} - K_{lj}))}{(1 + 2[\exp(\Delta_l)]\cosh(K_{il}))^2}\right\}. \tag{A3c}$$

The MKRG prescription for the renormalized coupling constant of the diamond cell incorporates the contributions from all L bonds,

$$K'_{ij} = \sum_{l=1}^{L} K'_l = \frac{1}{2}\sum_{l=1}^{L} \ln\left\{\frac{1+2[\exp(\Delta_l)]\cosh(K_{il}+K_{lj})}{1+2[\exp(\Delta_l)]\cosh(K_{il}-K_{lj})}\right\}. \tag{A4a}$$

For the renormalized fields, we shall adopt the prescription of Griffiths and Kaufman [24] which consists in adding up the contributions from all bonds connected to the given site, i.e.

$$\Delta'_i = \sum_{l=1}^{L} \Delta''_i = L\Delta_i + \sum_{l=1}^{L} \ln\left\{\frac{(1+2[\exp(\Delta_l)]\cosh(K_{il}+K_{lj}))(1+2[\exp(\Delta_l)]\cosh(K_{il}-K_{lj}))}{(1+2[\exp(\Delta_l)]\cosh(K_{lj}))^2}\right\} \tag{A4b}$$

$$\Delta'_j = \sum_{l=1}^{L} \Delta''_j = L\Delta_j + \sum_{l=1}^{L} \ln\left\{\frac{(1+2[\exp(\Delta_l)]\cosh(K_{il}+K_{lj}))(1+2[\exp(\Delta_l)]\cosh(K_{il}-K_{lj}))}{(1+2[\exp(\Delta_l)]\cosh(K_{il}))^2}\right\}. \tag{A4c}$$

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