**Antennas and radiation and fun**

Constants and useful relations:

Maxwell’s equations:

Charge conservation

A

Duality:

Helmholtz equations:

Boundary conditions:

Potentials:

When subindex means that and subindex means that

Where is the magnetic vector potential and is the electric vector potential

If we find , we can find the following fields:

If we find , we can find the following fields:

Free space and Green’s function:

We note that is for receiving antenna and is for transmitting antenna

The radiation condition:

For the general case of:

we get that is given by:

Where we assume that the source is finite and can be contained inside a finite volume with surface of

Term is the convolution of Green’s function and the source function, and it depicts the transmitted field from the antenna

Term is the received field that hits the antenna. That is why we can nullify it when the antenna is transmitting! Also, this term is dependent of the surface . If all sources are included in , then the surface integral vanishes

For vector fields the radiation condition takes the following form (far field approx.):

If the wave function obeys the radiation condition, then:

Using the notations above we write the following Radiation Integrals:

Infinitesimal electric dipole 1 (Hertzian dipole):

The source:

The charge distribution:

The magnetic vector potential:

From here we can no longer solve the integral analytically and we shall use the following approx.:

When and   
If the source was , we would get the same without any approximation

Now we rewrite with the spherical coordinates:

And from this , we can calculate the EM fields completely:

Far field approximation:

We take our Green’s function:

and we approximate it as such:

where

This approx. is valid when the following conditions are met:

where is the diameter of the smallest sphere that can contain the entire antenna relative to axes origin

We note that the third condition can be , and by this condition we get a maximal error of

Here, the maximal error is bounded by:

By this approx. we get that the magnetic vector potential is given by:

Under our 3 conditions we also approx.:

From here we can find the approx. fields:

**From this point**

The Poyting vector in the far field in real, propagating and directed in the direction:

Infinitesimal electric dipole 2:

The FF fields:

\* The far field boundary for the infinitesimal dipole is

Radiation intensity:

The total power radiated by the antenna:

where is steradian

Infinitesimal electric dipole 3:

The radiation intensity is given by:

The total power radiated is given by:

\* The radiated power is proportional to , that can be very small. Therefore, to achieve significant radiation, should be very large, which heats the antenna and introduces more problems

Radiation patterns:

Power pattern:

Infinitesimal electric dipole 4:

Field pattern:

Infinitesimal electric dipole 5:

\*

Beamwidth:

Diagram

Description automatically generated

The bandwidth is usually defined for the main lobe, and if no specification is made, is usually assumed

Infinitesimal electric dipole 6:

Sidelobe level:

The level of the first sidelobe relative to the main lobe, usually measured in :

Usually, this value is negative

Directivity:

Narrow Beam higher

Wide Beam smaller

\* If is taken, then the units are called

Infinitesimal electric dipole 7:

Radiation resistance and antenna impedance:

Antenna’s impedance:

Resistance of the antenna:

where:

is a model that averages all the antenna’s losses into it, and:

is the power actually radiated into the free space. is the dissipated power in the loss mechanism .

Together:

Infinitesimal electric dipole 8:

Efficiency and gain:

Radiation efficiency:

Another factor that Amir does not like:

Overall efficiency:

The gain of the antenna:

\* Practical definition will be:

The gain is also measured in

Effective radiated power:

Polarization (TX mode)

We first choose the following system of coordinates:

“vertical” -

“horizontal” -

Under this system we will define the polarization of the electric field:

The temporal representation of the electric filed:

\* The polarization is always defined with respect to the direction of propagation

From here we obtain:

And:

\* as our reference

By using a lot of useless trigonometry, we get the following relation, an ellipse:

Diagram

Description automatically generated

Axial ratio:

\* when , we at linear polarization

\* when , we at circular polarization

We can express the angle the following way:

The angular velocity:

This angular velocity is positive (Right-handed polarization) when , and is negative when

Linear polarization: If the forementioned ellipse becomes:

Circular polarization: Right- and left-handed polarization are obtained when and when , and the angular velocity is constant

The antenna in RX mode

The reciprocity theorem of Lorentz: Given a volume enclosed by the surface , containing two sets of sources that give two sets of EM fields, we get:

The open circuit voltage at the antenna port:

where refers to the current in the antenna’s ports at TX mode and refers to the source of the at TX mode as well, even though is a parameter of RX mode!

We also assume the transmitter and the receiver are far enough to use FF approx., so the incident wave at the antenna is given by:

Effective length of the antenna:

Now we can rewrite the open circuit voltage

\* is proportional to the vector FF of the antenna in TX mode:

Infinitesimal electric dipole 9:

\* This effective length is very small, since

Diagram, schematic

Description automatically generated

Polarization loss factor:

The power received at the load:

Where is in fact and is the available power

\* if , i.e., the impedances are matched, we get:

Effective aperture: Under our previous assumptions, we say the incident Poynting vector at the antenna has the amplitude of

From here we write:

Moreover, the antenna effective aperture in RX mode is proportional to the gain in TX mode according to the relation:

Effective aperture without losses:

Infinitesimal electric dipole 10:

The aperture efficiency:

\* is considered a high value

Friis’s formula:

Diagram

Description automatically generated

Linear wire antennas

Angular dependence of line source:

Infinitesimal electric dipole 11:

Tapered distributions

Equivalence Theorem - Given a volume enclosed by a closed surface . In the absence of sources within , the electromagnetic field in can be viewed as the outcome of the following equivalent surface on :

Wire anthennas

An induced current distribution on PEC wire is transformed into line source distribution in free space:

For a small dipole:

**\*\* The usefulness of dipoles is determined mainly by their impedance behavior \*\***

Chart

Description automatically generated

\* For integer multiples of the antenna becomes an open circuit and does not radiate (practically trying to feed the antenna at the point where ).

\* Best case is where

\* Resonant frequency is defined where

\* Thicker dipole wider bandwidth lower resonant frequency.

\* FAT IS GOOD

**TO DO - add on half wave dipole**

Option 1:

Option 2: and ,

Option 3: and ,

**Example : Uniform Rectangular Aperture (with option 2)**

And we need to remember:

- plane ( - plane)

- plane ( - plane)

Effective aperture

Directivity

**----------------------------------------------------------------------------**

**Example : Open-ended Waveguide ()**

- plane ( - plane)  
   
 - plane ( - plane)

**----------------------------------------------------------------------------**

**Example : Circular Aperture**

Where is the zero order Bessel function:

Assuming uniform illumination, :

Practically, the **uniform distribution** provides the **highest aperture efficiency** ():

**Example: Open-ended waveguide**

Again, we choose and so that only wave will be excitated.

**Horn Antennas**

**Trickim shtikim**

Shape

Description automatically generated with medium confidence

