Need to show:

$$\frac{\hat{q}_{1-\frac{\alpha}{2}}}{\sqrt{n}} = \hat{\theta}^{*}_{1-\frac{\alpha}{2}} - \hat{\theta}_n$$

$$\hat{q}_{\frac{\alpha}{2}} = \inf\left\{x : G_n(x) \geq \frac{\alpha}{2}\right\} = \inf\left\{x : \frac{1}{B}\sum_{b=1}^{B} I\left[\sqrt{n}(\hat{\theta}_n^{*b} - \hat{\theta}_n) \leq x\right] \geq \frac{\alpha}{2}\right\}$$

$$= \inf\left\{x : \sum_{b=1}^{B} I\left[\sqrt{n}(\hat{\theta}_n^{*b} - \hat{\theta}_n) \leq x\right] \geq \frac{B\alpha}{2}\right\}$$

$$= \sqrt{n} \cdot \inf\left\{x : \frac{1}{B}\sum_{b=1}^{B} I\left[\hat{\theta}_n^{*b} - \hat{\theta}_n \leq x\right] \geq \frac{\alpha}{2}\right\}$$

$$= \sqrt{n}\Big(\inf\left\{x : \frac{1}{B}\sum_{b=1}^{B} I\left[\hat{\theta}_n^{*b} \leq x\right] \geq \frac{\alpha}{2}\right\} - \hat{\theta}_n = \sqrt{n} \cdot \left(\hat{\theta}^{*}_{\frac{\alpha}{2}} - \hat{\theta}_n\right)$$

Therefore:

$$\begin{cases} \hat{q}_{\frac{\alpha}{2}} = \sqrt{n}\left(\hat{\theta}^{*}_{\frac{\alpha}{2}} - \hat{\theta}_n\right) \\ \hat{q}_{1-\frac{\alpha}{2}} = \sqrt{n}\left(\hat{\theta}^{*}_{1-\frac{\alpha}{2}} - \hat{\theta}_n\right) \end{cases} \Rightarrow CI = \left[2\hat{\theta}_n - \hat{\theta}^{*}_{1-\frac{\alpha}{2}},\ 2\hat{\theta}_n - \hat{\theta}^{*}_{\frac{\alpha}{2}}\right]$$