“A New Ordering for Efficient Sphere Decoding” Constrained ILS Reduction

Consider the ILS problem, $\min_{x \epsilon X} \|Ax - y\|$. Define $G = (A^{-1})^T$ and G_i references the i^{th} column of G. The algorithm starts with all elements of x unfixed and an ‘index’ set $Index = 1...m$ which will be used to reference columns of G.

In the first iteration, we find the column of G which maximizes the distance to the second nearest integer point. Meaning that for each $i \epsilon Index$ we compute $a = \arg\min_{x \epsilon X} \left|y^T G_i - x\right|$ which gives the $a = x_m$ that minimizes the partial residual assuming that we choose column i to be the m^{th} column in the final matrix A. We then compute $b = \arg\min_{x \epsilon X \backslash a} \left|y^T G_i - x\right|$ which gives the $b = x_m$ that has the second smallest partial residual. We compute the distance between the target vector y and the affine set defined by fixing $x_m = b$ as $dist = \left\|(1/(G_i^T G_i)(G_i G_i^T))(y - A_i * b)\right\|$ where the term $1/(G_i^T G_i)(G_i G_i^T)$ is just the orthogonal projector that projects onto the i^{th} colum of G. We want to keep track of which value of i gives the maximum value for $dist$.

After completing the above process we should have values for a, b and i, where i was the index giving the maximum value for $dist$. We then set the m^{th} entry in a permutation vector to move the i^{th} column to the m^{th}, record the value of a because it will be the m^{th} entry of the babai point and remove i from the Index set $Index_i = []$.

Since we have now fixed x_m we must project and shift the target vector as follows, $y = (y - A_i * a) - (1/(G_i^T G_i)(G_i G_i^T))(y - A_i * a)$. This is equivalent to applying the constraint $x_m = a$.

We must also project all of the remaining columns of G as follows, $\forall j \epsilon Index \quad G_j = G_j - (1/(G_i^T G_i)(G_i G_i^T)) * G_j$.

The algorithm proceeds to repeat the above process m times. So that in each iteration there is one less column of G in the Index set. We are finished when the index set is empty.