Proposed Kalman Filter for WASP / INS integration

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***Abstract*—This paper outlines a proposed error-state (indirect) kalman filter for WASP / INS integration.**

I. Introduction

THERE are a number of design choices available when constructing a filter for integrating multiple sensors. In this case the error-state (indirect) Kalman filter is proposed, such that the estimated variables are the errors in position, velocity and orientation calculated from INS integration relative to the true values. Consistent with this approach, the measurement presented to the filter contains an observation of the position error, calculated as the difference between the INS integrated position and the noisy observation of the true position provided from WASP. The main reasons for adopting this formulation are: 1. There is no need to define a motion model, so the same filter can be used for both vehicle and pedestrian applications without modification; 2. The output from the system is position, velocity, acceleration and orientation integrated directly from the error-compensated IMU. Since the filter is outside of this INS loop, it can be run at lower update frequency than the IMU, whilst still preserving high dynamics output; 3. Since orientation error angles should be small, we can accurately linearise the system dynamics using small angle approximations. These approximations would not be valid if the Kalman filter state including orientation rather than orientation error.

There are two ways the Kalman filter error estimate can be used in a tracking system of this type. It can be fed forward to correct the INS position estimate without updating the INS (feedforward architecture), or it can be fed back to the INS to correct its next starting position as well (feedback architecture). The proposed system uses the feedback architecture, since the approximations used to propagate the orientation error in the filter process model are only valid for small rotations. The feedback architecture has the advantage of ensuring orientation errors remain small. This occurs because every time the mean of the Kalman filter error state is fed back to the INS, the state mean (but not the state covariance) can be reset to zero.

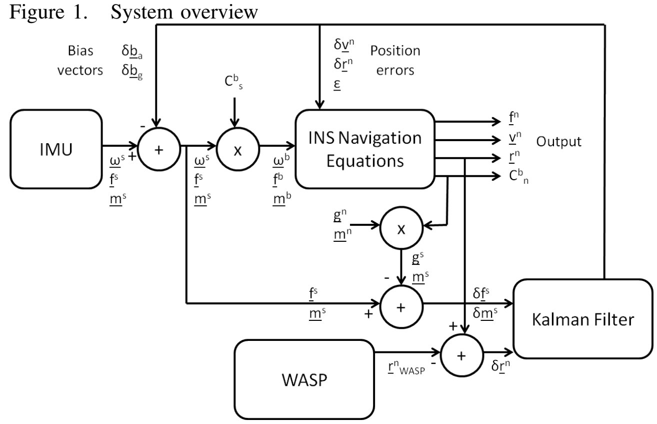
In addition to the positioning errors, the proposed Kalman filter state also contains estimates for the drift bias in the IMU gyros and accelerometers, which can be fed back to correct the sensor outputs before INS integration. As such, the proposed design corrects for accumulated drift errors whilst also estimating and correcting the bias causing the buildup of drift errors. In the literature this design is sometimes labeled as a dual kalman filter since it simultaneously estimates both the state and parameters of the system. It is anticipated that this augmentation will perform the same function as the outer loop process used by [1] to estimate gyro drift bias. Consistent with [2], the measurement vector includes an observation of the difference between the measured and estimated accelerometer and magnetometer data, which allows gyro bias to be estimated, whilst accelerometer bias can be estimated from WASP position observations. Constant turn-on biases and scale biases have not been modeled, and are therefore assumed to be corrected in an initial calibration step if necessary.

Another key consideration is the parametrisation of orientation errors in the filter. This filter uses the Euler angle parametrisation. The quaternion orientation representation has the advantage of dynamics that are linear and non-singular (it avoids gimbal-lock). However, assuming orientation errors will remain small, gimbal-lock cannot occur and the Euler angle parameterisation is well approximated by linear dynamics. The Euler angle parametrisation has therefore been chosen because it has no redundant parameters and keeps the Kalman filter state as small as possible.

In the GPS / INS integration community, filters are often characterised according to whether there is loose or tight coupling between GPS and INS measurements. This filter can be characterised as loosely coupled, in the sense that the measurement required from the WASP system is a location observation, rather than a set of ranges to WASP beacons. This disadvantage of this approach is that WASP measurements cannot be used if there are less than 3 WASP beacons in range, however the measurement updates are somewhat simplified relative to a tightly coupled filter. It is anticipated that if the loosely coupled filter can be validated then the measurement function can be modified to implement tight coupling at a later date.

An overview of the system is provided in the figure below. All symbols are defined in the following sections.

Figure 1. System overview



Before outlining the system in detail, first define three frames of reference: the sensor frame (s-frame), the body frame (b-frame) and the local navigation frame (n-frame). The INS navigation equations are developed in the n-frame using north-east-down (NED) coordinates with components designated with subscripts x, y and z respectively. The (assumed constant) rotation matrix that transforms a vector in the s-frame into the b-frame is given by C_s^b.

II. Sensor models

The accelerometers and the gyros are considered to operate according to the same sensor model, which assumes that the sensor outputs are corrupted by a high frequency white noise process and a drifting bias term. The measurement equations for the accelerometer and gyro are therefore given by:

$$\underline{\tilde{f}}^s = \underline{f}^s + \underline{b}_a + \underline{n}_a \tag{1}$$

$$\underline{\tilde{\omega}}^s = \underline{\omega}^s + \underline{b}_g + \underline{n}_g \tag{2}$$

where $\widetilde{f}^s$ and $\tilde{\omega}^s$ are the sensor measurements in the s-frame, f^s and ω^s are the true measurements, $\underline{b}$ is the drift-rate bias and $\underline{n}$ is sensor noise, assumed to be Gaussian white noise with covariance N_a and N_g respectively. The drift-rate bias $\underline{b}$ is not a static quantity, but is assumed to be a first order Gauss-Markov process with correlation time T and white process noise $\underline{\mu}$, with process noise covariance U_a and U_g respectively. The two noise processes are assumed to be uncorrelated. Thus the drift-rate bias propagates as:

$$\dot{\underline{b}}_a = -\frac{1}{T_a}\underline{b}_a + \underline{\mu}_a \tag{3}$$

$$\dot{\underline{b}}_g = -\frac{1}{T_g}\underline{b}_g + \underline{\mu}_g \tag{4}$$

III. INS navigation equations

The INS navigation equations are used to integrate gyro and accelerometer data to determine the position, velocity and orientation of the IMU in the n-frame. Neglecting the curvature of the earth's surface, fluctuations in gravitational acceleration, the Coriolis effect and the rotation of the navigation frame, the simplified INS navigation equations expressed in continuous time are as follows ([3]):

$$\begin{bmatrix} \dot{\underline{r}}^n \\ \dot{\underline{v}}^n \\ \dot{C}_b^n \end{bmatrix} = \begin{bmatrix} \underline{v}^n \\ C_b^n \underline{f}^b + \underline{g}^n \\ C_b^n \left[\underline{\omega}^b \times\right] \end{bmatrix} \tag{5}$$

where:

$\underline{g}^n$is the 3-component gravity vector in the n-frame

$\underline{f}^b$is the 3-component accelerometer force vector in the b-frame

$\underline{\omega}^b = [\omega_x^b \ \omega_y^b \ \omega_z^b]^T$is the body angular velocity vector which represents the turn rate of the b-frame with respect to the n-frame expressed in body axes

$\left[\underline{\omega}^b \times\right]$ is the skew-symmetric matrix form of the body angular rate vector (also known as the cross product operator) given by:

$$\left[\underline{\omega}^b \times\right] = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \tag{6}$$

1) Discrete time approximations with corrections: Since the output of the proposed system is the directly integrated INS position, velocity and orientation, the INS navigation equations must be approximated in discrete time using bias compensated sensor measurements. In addition, the INS navigation equations must be corrected for navigation errors estimated by the Kalman filter. Firstly, the bias compensated sensor measurements in the b-frame at t_k are calculated as:

$$\hat{\underline{f}}_k^b = C_s^b \left[\tilde{\underline{f}}_k^s - \underline{b}_a\right] \tag{7}$$

$$\hat{\underline{\omega}}_k^b = C_s^b \left[\widetilde{\underline{\omega}}_k^s - \underline{b}_g\right] \tag{8}$$

Assuming $\hat{\underline{\omega}}_k^b$ is constant during the update interval, the compensated rotation vector over the time interval t_k to t_k+1 is given by:

$$\hat{\underline{\sigma}}_{k+1} = \hat{\underline{\omega}}_{k+1}.dt \tag{9}$$

The new body orientation matrix is now given by:

$$C_{b,k+1}^n = \left[I - \left[\underline{\varepsilon}_k \times\right]\right]^T C_{b,k}^n e^{\left[\hat{\underline{\sigma}}_{k+1} \times\right]} \tag{10}$$

where:

$\left[\hat{\underline{\sigma}}_{k+1} \times\right]$ is the cross product of the compensated rotation vector over the time interval t_k to t_{k+1} (as previously defined)

$\left[I - \underline{\varepsilon}_k \times\right]^T$is the orientation correction estimated from the Kalman filter with $\underline{\varepsilon}$ as defined in IV-B1.

As shown in [3], there are methods for simplifying and rapidly computing truncated Taylor expansions of the above exponential. However, because of this truncation, it is necessary to re-orthogonalise the rows of the C_b^n orientation matrix at periodic intervals.

In knowledge of the gravity vector g^n, the updated velocity estimate in discrete time (including the rotation correction) can be given the following two equations:

$$\underline{v}_{k+1}^n = \underline{v}_k^n + \underline{u}_{k+1}^n - \delta\underline{v}_k^n \tag{11}$$

$$\underline{u}_{k+1}^n = C_b^n \left[\hat{f}_{k+1}^b.dt + \frac{1}{2}\hat{\omega}^b.dt \times \hat{f}_{k+1}^b.dt\right] - \underline{g}^n.dt \tag{12}$$

where:

$\underline{v}_{k+1}^n$is the velocity at t_{k+1}

$\underline{u}_{k+1}^n$is the change in velocity over the interval t_k to t_{k+1}

$\underline{g}^n.dt$ is the gravity vector correction

$\delta\underline{v}_k^n$ is the 2-component velocity error correction augmented with a zero in the z-axis

Finally, the updated position estimate can be found by trapezoidal integration of velocity:

$$\underline{r}_{k+1}^n = \underline{r}_k^n + \left[\frac{\underline{v}_k^n + \underline{v}_{k+1}^n}{2}\right] - \delta\underline{r}_k^n \tag{13}$$

where

$\underline{r}_{k+1}^n$is the position at t_{k+1}

$\delta\underline{r}_k^n$ is the 2-component position error correction augmented with a zero in the z-axis

IV. Kalman filter

A. State vector

The KF error state vector has 13 components as follows:

$$\underline{x} = \begin{bmatrix} \delta\underline{r}^n & \delta\underline{v}^n & \underline{\varepsilon} & \underline{b}_g & \underline{b}_a \end{bmatrix}^T \quad (14)$$

where:
$\delta\underline{r}^n$ is the 2-component position error vector in the n-frame
$\delta\underline{v}^n$ is the 2-component velocity error vector in the n-frame
$\underline{\varepsilon}$ is the 3-component orientation error vector in Euler angles
$\underline{b}_g$ is the 3-component gyro bias vector
$\underline{b}_a$is the 3-component accelerometer bias vector
The z (vertical) components of the position and velocity error vectors are not included in the state since, unless WASP nodes are deployed vertically, these errors are likely to be unobservable. The dynamics of the system process model are developed in continuous time according to the following first-order matrix differential equation:

$$\dot{\underline{x}} = F\underline{x} + \underline{w} \quad (15)$$

where F is the system dynamics matrix and $\underline{w}$ is process noise. Although it is necessary to convert from the continuous time formulation to discrete time in order to implement the Kalman filter, the continuous time formulation preserves the true dynamics of system in the process model. Approximations can be made in the actual implementation by choosing the appropriate cut-off for the Taylor-series expansion of the exponential solution to this differential equation, where:

$$\underline{x}_{k+1} = \Phi_k \underline{x}_k + \underline{w}_k \quad (16)$$

$$\Phi_k = e^{F_k.dt} = I + F_k.dt + \frac{(F_k.dt)^2}{2!} + \frac{(F_k.dt)^3}{3!} + ... \quad (17)$$

To determine the system dynamics matrix F, the INS error dynamics equations must first be described.

B. INS error dynamics equations

The equations describing the propagation of errors in the Kalman filter process model can be found by perturbing the INS navigation equations.

1) Orientation errors: Using small angle approximations and neglecting product terms which are also small, the estimated orientation $\widetilde{C}_b^n$ can be expressed as a transformation of the true orientation C_b^n as follows:

$$\widetilde{C}_b^n = [I - [\underline{\varepsilon}\times]]\, C_b^n \quad (18)$$

where:
$\underline{\varepsilon}$ is the orientation error vector given by $\underline{\varepsilon} = [\delta\alpha\ \delta\beta\ \delta\gamma]^T$
$\delta\alpha$is the error in rotation about the x axis
$\delta\beta$ is the error in rotation about the y axis
$\delta\gamma$ is the error in rotation about the z axis
Following the method in [3], neglecting error product terms and rotation of the n-frame, it can be shown that the orientation error propagates as a function of gyro measurement errors as follows:

$$\dot{\underline{\varepsilon}} = -C_b^n \delta\underline{\omega}^b \quad (19)$$

where $\delta\underline{\omega}^b$ is the error in gyro measurement, containing both drift-bias and high frequency noise such that

$$\delta\underline{\omega}^b = C_s^b \left[\underline{b}_g + \underline{n}_g\right]$$

2) Velocity and position errors: Ignoring the Coriolis effect and product terms, velocity errors propagate as a function of accelerometer measurement errors, and the impact of orientation errors on accelerations:

$$\delta\dot{\underline{v}}^n = \left[\underline{f}^n \times\right] \underline{\varepsilon} + C_b^n \delta f^b \quad (20)$$

where $\delta\underline{f}^b$ is the error in accelerometer measurement, containing both drift-bias and high frequency noise such that $\delta\underline{f}^b = C_s^b\left[\underline{b}_a + \underline{n}_a\right]$
Finally, the position errors can be expressed as:

$$\delta\dot{\underline{r}}^n = \delta\underline{v}^n \quad (21)$$

C. Process model

Recalling the definition of the state equation as:

$$\dot{\underline{x}} = F\underline{x} + \underline{w}$$

with state vector:

$$\underline{x} = \begin{bmatrix} \delta\underline{r}^n & \delta\underline{v}^n & \underline{\varepsilon} & \underline{b}_g & \underline{b}_a \end{bmatrix}^T$$

we can now define:

$$F = \begin{bmatrix} 0 & I & 0 & 0 & 0 \\ 0 & 0 & \left[\underline{f}^n\times\right] & 0 & C_b^n C_s^b \\ 0 & 0 & 0 & -C_b^n C_s^b & 0 \\ 0 & 0 & 0 & diag(\frac{-1}{T_g}) & 0 \\ 0 & 0 & 0 & 0 & diag(\frac{-1}{T_a}) \end{bmatrix} \quad (22)$$

$$\underline{w} = \begin{bmatrix} 0 \\ C_b^n C_s^b \underline{n}_a \\ -C_b^n C_s^b \underline{n}_g \\ \underline{\mu}_g \\ \underline{\mu}_a \end{bmatrix} \quad (23)$$

D. Measurement model

The KF measurement vector consists of an observation of position error using WASP, and an observation of the difference between the measured and estimated accelerometer and magnetometer data (as in [2]). Conceptually, the position error observation enables position, velocity and accelerometer bias errors to be estimated. The accelerometer and magnetometer error observations enable gyro bias errors to be estimated. In practice position observations from WASP occur with a frequency of around 10Hz, relative to approximately 800Hz for the IMU accelerometer and magnetometer observations, so this component of the measurement vector will often consistent of zeros. The measurement vector is as follows:

$$\underline{z} = \begin{bmatrix} \delta\underline{r}^n \\ \delta\underline{f}^s \\ \delta\underline{m}^s \end{bmatrix} = \begin{bmatrix} \underline{r}^n - \underline{r}^n_{WASP} \\ \hat{\underline{f}}^s - C_b^s C_n^b \underline{g}^n \\ \hat{\underline{m}}^s - C_b^s C_n^b \underline{m}^n \end{bmatrix} \quad (24)$$

where

$\delta \underline{r}^n$ is the INS position error (relative to the WASP position in the n-frame)

$\delta \underline{f}^s$ is the observed gravity error vector (relative to the INS predicted gravity vector in the s-frame)

$\delta \underline{m}^s$ is the observed magnetic field error vector (relative to the INS predicted field vector in the s-frame)

Note that in accordance with [2], the bias compensated sensor measurments $\hat{\underline{f}}^s$ and $\underline{\hat{m}}^s$ should be normalised before calculating the measurement errors above, or otherwise the magnitude error will reduce the orientation precision. Using the standard observation model formulation as follows:

$$\underline{z}_k = H\underline{x}_k + \underline{v}_k \tag{25}$$

where $\underline{v}_k$is the measurement noise vector, $\underline{v}_k \sim N(0, R_k)$, then:

$$H = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & 0 & \left[C_b^s C_n^b \left[\underline{g}^n \times\right]\right] & 0 & 0 \\ 0 & 0 & \left[C_b^s C_n^b \left[\underline{m}^n \times\right]\right] & 0 & 0 \end{bmatrix} \tag{26}$$

E. Adaptive weighting

The uncertainty in the covariance parameters of the process noise Q and the measurement errors R has a substantial effect on Kalman filter performance. Since process noise Q is determined by sensor noise, it is assumed to remain relatively constant. However, since measurement error R is dependent on the application environment, an adaptive weighting method based on covariance scaling is proposed. The covariance scaling method is designed to improve filter stability by introducing a multiplication factor to the measurement error covariance. With regards to gravity and magnetic field errors, the covariance scaling factor follows [2]. With regards to the position error observation, it is proposed that the covariance scaling factor be related to the variance of the residuals from the WASP OLS position estimate.

References

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