$$\begin{bmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{bmatrix}$$

A matrix

$\det(A - \lambda I) = 0$

$$A - \lambda I = \begin{bmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13-\lambda \end{bmatrix}$$

⇒ Calculate determinants

$\det(A - \lambda I) = (4-\lambda) \times m - (-2) \times m + 0 \times m - (-1) \times m$

m, m & m are 3x3 minors

⇒ remove row 1, column 1

$$m \begin{bmatrix} -9-\lambda & -2 & -4 \\ 10 & 5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{bmatrix}$$

$$m = (-9-\lambda) \times \begin{vmatrix} 5-\lambda & -10 \\ -14 & -13-\lambda \end{vmatrix} - (-2) \times \begin{vmatrix} 10 & -10 \\ -13 & -13-\lambda \end{vmatrix} + (-4) \times \begin{vmatrix} 10 & 5-\lambda \\ -13 & -14 \end{vmatrix}$$

$$= (-9-\lambda)[(5-\lambda)(-13-\lambda) - (-10)(-14)] + 2[10(-13-\lambda) - (-10)(-13)] - 4[10(-14) - (5-\lambda)(-13)]$$

$$= (-9-\lambda)[(-13-\lambda)(5-\lambda) - 140] + 2[-130 - 10\lambda - 130] - 4[-140 + 13(5-\lambda)]$$

$$= (-9-\lambda)[-65 + 13\lambda - 5\lambda + \lambda^2 - 140] + 2[-260 - 10\lambda] - 4[140 + 65 - 13\lambda]$$

$$= (-9-\lambda)[\lambda^2 + 8\lambda - 205] - 520 - 20\lambda - 4[-75 - 13\lambda]$$

$= (-9-\lambda)(\lambda^2+8\lambda-205)-520-20\lambda+300+52\lambda$
$= (-9-\lambda)(\lambda^2+8\lambda-205)-220+32\lambda$

$m = -\lambda^3-8\lambda^2+205\lambda-9\lambda^2-72\lambda+1845-220+32\lambda$
$m = -\lambda^3-17\lambda^2+165\lambda+1625$

$\Rightarrow m_2$ (remove row 2, column 1):

$$m_2 = \begin{bmatrix} 8 & -1 & -2 \\ 10 & 5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{bmatrix}$$

Expansion:

$m_2 = 8 \times \begin{vmatrix} 5-\lambda & -10 \\ -13 & -13-\lambda \end{vmatrix} - (-1) \times \begin{vmatrix} 10 & -10 \\ -13 & -13-\lambda \end{vmatrix} + (-2) \times \begin{vmatrix} 10 & 5-\lambda \\ -13 & -14 \end{vmatrix}$

$= 8[(5-\lambda)(-13-\lambda)-130] + [10(-13-\lambda)+130] - 2[10(-14) - (5-\lambda)(-13)]$

$= 8[\lambda^2+8\lambda-205] + [-130-10\lambda+130] - 2[-140+65-13\lambda]$

$= 8\lambda^2+64\lambda-1640-10\lambda-2[-75-13\lambda]$

$= 8\lambda^2+54\lambda-1640+150+26\lambda$

$m_2 = 8\lambda^2+80\lambda-1490$

$\Rightarrow m_4$ (remove row 4, colum 1

$$m_4 = \begin{bmatrix} 8 & -1 & -2 \\ -9-\lambda & -2 & -4 \\ 10 & 5-\lambda & -10 \end{bmatrix}$$

Expansion:

$m_4 = 8 \times \begin{vmatrix} -2 & -4 \\ 5-\lambda & -10 \end{vmatrix} - (-1) \times \begin{vmatrix} -9-\lambda & -4 \\ 10 & -10 \end{vmatrix} + (-2) \times \begin{vmatrix} -9-\lambda & -2 \\ 10 & 5-\lambda \end{vmatrix}$

$= 8[(-2)(-10)-(-4)(5-\lambda)] + [(-9-\lambda)(-10)-(-4)(10)]$

$-2[(-9-\lambda)(5-\lambda)-(-2)(10)]$
$=8[20+20-4\lambda]+[90+10\lambda+40]-2[(-9-\lambda)(5-\lambda)+20]$
$=8[40-4\lambda]+[130+10\lambda]-2[-45+9\lambda-5\lambda+\lambda^2+20]$
$=320-32\lambda+130+10\lambda-2[\lambda^2+4\lambda-25]$
$=450-22\lambda-2\lambda^2-8\lambda+50$

$m_4 = 2\lambda^2-30\lambda+500$

$\Rightarrow$ Find λ

$\lambda \approx -15.24$
$\lambda_2 \approx -3.76$
$\lambda_3 \approx 3.0$
$\lambda_4 \approx 3.0$

$\lambda = -15.24$ (largest in absolute value)

Eigen vector V for $\lambda = -2$

$(A-\lambda I)V = 0$
substituting $\lambda = -2$

$(A-(-2)I)V = 0$
$(A+2I)V = 0$

$$\begin{bmatrix} 6 & 8 & -1 & -2 \\ -2 & -7 & -2 & -4 \\ 0 & 10 & 7 & -10 \\ -1 & -13 & -14 & -11 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ Row reduction

$R \to \frac{R}{6}$

$$\begin{bmatrix} 1 & 4/3 & -1/6 & -1/3 \\ -2 & -7 & -2 & -4 \\ 0 & 10 & 7 & -10 \\ -1 & -13 & -14 & -11 \end{bmatrix}$$

Step 2:

$$\begin{bmatrix} 1 & 4/3 & -1/6 & -1/3 \\ 0 & -13/3 & -7/3 & -14/3 \\ 0 & 10 & 7 & -10 \\ 0 & -35/3 & -85/6 & -35/3 \end{bmatrix}$$

Final eigenvector V

$V = [2, -1, 0, 1]^T$

$\Rightarrow (A + 2I)v = 0$

$\Rightarrow$ Eigenvalue importance

1) $\lambda = -2$
2) $\lambda_2 = -2$
3) $\lambda_3 = -4$
4) $\lambda_4 = -5$

$\Rightarrow$ Absolute values:

$|\lambda| = 2$

$|\lambda_2| = 2$

$|\lambda_3| = 4$

$|\lambda_4| = 5$

Sum of absolute values $= 13$

$\Rightarrow$ Importance percentages:

λ importance $= (2/13) \times 100\,\%$

$= 15.38\%$ (approximated)

* Eigenvalue $\lambda = -2$
* Eigen vector $v = [2, -1, 0, 1]^T$
* Importance: 15.38%

$$A = \begin{pmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{pmatrix}$$

$\det(A - \lambda I) = 0$

$$\begin{pmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -\lambda-13 \end{pmatrix} = 0$$

1st. $4-\lambda \begin{pmatrix} -9-\lambda & -2 & -4 \\ 10 & 5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{pmatrix}$

2nd. $8 \begin{pmatrix} -2 & -2 & -4 \\ 0 & 5-\lambda & -10 \\ -1 & -14 & -13-\lambda \end{pmatrix}$

3rd. $-2(-1) \begin{pmatrix} -2 & -9-\lambda & -4 \\ 0 & 10 & -10 \\ -1 & 13 & -13-\lambda \end{pmatrix}$

4th. $-2 \begin{pmatrix} -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -14 \end{pmatrix}$

$\Rightarrow (4-\lambda)(-\lambda^3 - 17\lambda^2 + 165\lambda + 1695) - 8(-2\lambda^2 - 22\lambda + 370) - 1(\lambda^3 + 390) +$
$2(\lambda^2 + 23\lambda + 275)$

$\Rightarrow \lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500 = 0$

$\lambda_1 = -21.125$

$\lambda_2 = -5.604$

· Eigen vector.

$\lambda_2 = -5.604$

$$A - \lambda_2 I = \begin{pmatrix} 9.604 & 8 & -1 & -2 \\ -2 & -3.396 & -2 & -4 \\ 0 & 10 & 10.604 & -10 \\ -1 & -13 & -14 & -7.396 \end{pmatrix}$$

$(A - \lambda I)v = 0$

$$\begin{pmatrix} 9.604 & 8 & -1 & -2 \\ -2 & -3.396 & -2 & -4 \\ 0 & 10 & 10.604 & -10 \\ -1 & -13 & -14 & -7.396 \end{pmatrix} \times (0.104)$$

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ -2 & -3.396 & -2 & -4 & 0 \\ 0 & 10 & 10.604 & -10 & 0 \\ -1 & -13 & -14 & -7.396 & 0 \end{array}\right) \times 2 \quad R_2-(2)R_1$$

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ 0 & -1.730 & -2.208 & -4.416 & 0 \\ 0 & 10 & 10.604 & -10 & 0 \\ -1 & -13 & -14 & -7.396 & 0 \end{array}\right) \times (1)$$

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ 0 & 1 & 2.206 & -4.416 & 0 \\ 0 & 0 & 10.604 & -10 & 0 \\ 0 & -12.167 & -14.104 & -7.604 & 0 \end{array}\right) (-0.578)$$

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ 0 & 1 & 1.276 & 2.553 & 0 \\ 0 & 10 & 10.604 & -10 & 0 \\ 0 & -12.167 & -14.104 & -7.604 & 0 \end{array}\right) 10$$

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ 0 & 1 & 1.276 & 2.553 & 0 \\ 0 & 0 & -2.160 & -35.529 & 0 \\ 0 & -12.167 & -14.104 & -7.604 & 0 \end{array}\right) [12.167]$$

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ 0 & 1 & 1.276 & 2.553 & 0 \\ 0 & 0 & 1 & 16.446 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \times (-0.463)$$

$$\left(\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ 0 & 1 & 1.276 & 2.553 & 0 \\ 0 & 0 & 1 & 16.446 & 0 \\ 0 & 0 & 1.426 & 23.457 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 0.833 & 0 & 1.504 & 0 \\ 0 & 1 & 0 & -18.439 & 0 \\ 0 & 0 & 1 & 16.446 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) (-0.833) = \left(\begin{array}{cccc|c} 1 & 0 & 0 & 16.864 & 0 \\ 0 & 1 & 0 & -18.439 & 0 \\ 0 & 0 & 1 & 16.446 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{aligned} x_1 \quad\quad\quad + 16.864 x_4 &= 0 \\ x_2 \quad\quad - 18.439 x_4 &= 0 \\ x_3 + 16.446 x_4 &= 0 \end{aligned}$$

$x_3 = -16.446 x_4$, $x_2 = 18.439 x_4$ $x_1 = -16.864 x_4$

$$x = \begin{pmatrix} -16.864 x_4 \\ 18.439 x_4 \\ -16.446 x_4 \\ x_4 \end{pmatrix} \quad x_4 \begin{pmatrix} -16.864 \\ 18.439 \\ -16.446 \\ 1 \end{pmatrix} \quad V = \begin{pmatrix} -16.864 \\ 18.439 \\ -16.446 \end{pmatrix}$$

Date

M T W

$$A = \begin{bmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ 1 & -13 & 14 & -13 \end{bmatrix}$$

$$V_3 = \begin{bmatrix} 0.025 \\ -0.335 \\ -0.229 \\ -0.915 \end{bmatrix}$$

λ total $= -12.99$

$$\lambda_3 = \frac{-21.12}{-12.99} \times 100 = 162.10\%$$

$\lambda_3 = -21.12$ (Eigen value)

Eigen vector (V_3) $\begin{bmatrix} 0.025 \\ -0.335 \\ -0.222 \\ -0.915 \end{bmatrix}$

162.10% of total variance.

Date

$$A = \begin{bmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{bmatrix}$$

$$V_4 = \begin{bmatrix} 0.563 \\ -0.616 \\ 0.549 \\ -0.033 \end{bmatrix}$$

$$\lambda total = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \approx -12.99$$

$$\lambda_4 = \left(\frac{-5.60}{-12.99} \right) \times 100 = 43.11\%$$

Eigen value $(\lambda_4) \rightarrow \lambda_4 = -5.60$

Eigen Vector (V_4) $\begin{bmatrix} 0.563 \\ -0.616 \\ 0.549 \\ -0.033 \end{bmatrix}$

43.11% of total variance