

# Problem D


## Digital Content Protection

**Problem ID:** contentprotectio

**CPU Time limit:** 1 second

**Memory limit:** 1024 MB

**Source:** Rocky Mountain Regional Contest (RMRC) 2013

**License:** 

Dan is working for a digital content protection company, which is responsible for the content protection of blu-ray discs based on a standard called Anti Content Misuse (ACM).

The ACM standard works as follows. Assume there are  $2^n$  blu-ray drives/players. We represent these  $2^n$  drives as the leaves of a complete binary tree of height  $n$ , so that each root-to-leaf path consists of  $n$  edges. Each node  $u$  in this binary tree is assigned an identifier number and contains a random key  $k_u$ . The identifier numbers are assigned as follows. The root,  $r$ , is assigned 1. In addition, the left and right children of an internal node having number  $i$  are assigned numbers  $2i$  and  $2i + 1$ , respectively. This scheme assigns a distinct number to each node in the tree. The keys contained in the nodes are unknown to blu-ray users, but they are available to blu-ray drive manufacturers. Each blu-ray player is assigned the identifier number  $i$  ( $2^n \leq i \leq 2^{n+1} - 1$ ) of its corresponding leaf in the tree. A manufacturer of blu-ray drives embeds the keys associated with the nodes in the path from the root to leaf number  $i$  in player number  $i$ .

To encrypt the content of a blu-ray disc, the company in charge creates a random key  $k$  called the master key. First, they encrypt  $k$  with the key  $k_r$  (recall  $r$  is the root node of binary tree) and write it on the disc as a header. Then, they encrypt the content with  $k$ , and write the encrypted data on the blu-ray disc. A blu-ray drive first decrypts the header using key  $k_r$  embedded in it and recovers the master key  $k$  and then, decrypts the content using the key  $k$ .

Unfortunately, the keys embedded in a set of blu-ray drives,  $R$ , are exposed by hackers and published on the web. As a result, we cannot encrypt the master key  $k$  using any of these exposed keys. For example, since all blu-ray drives contain  $k_r$ , the encryption scheme above does not work any more. There is a solution oversight for this situation in the ACM standard. At the cost of a larger header, the industry can safely encrypt the content of a new blu-ray disc. They carefully choose a subset of unexposed keys  $K$  in the binary tree such that all blu-ray drives, except for drives in  $R$ , have at least one of the keys in  $K$ . They encrypt the master key  $k$  with each key  $k' \in K$  and put the result in the header (i.e., there are  $|K|$  ciphertexts in the header). Now, each active blu-ray drive can decrypt at least one of the ciphertexts in the header and can recover the master key  $k$ . Dan needs your help to determine a subset of keys  $K$  with minimum cardinality (which results in the smallest header) given the identifiers of hacked drives.

### Input

The input consists of a single test case. A test case consists of two lines. The first line contains two integers  $n$  and  $|R|$ , where  $1 \leq n \leq 62$  and  $1 \leq |R| \leq 1\,000$ .  $|R|$  is the cardinality of  $R$ , the set of exposed drives. The second line contains  $|R|$  integers, which are the identifiers of exposed blu-ray drives. You can assume that there is at least one blu-ray drive not hacked.

### Output

Display the identifiers of nodes corresponding to the keys in  $K$ , satisfying the above requirements and having minimum cardinality, in increasing order and separated with single spaces.

#### Sample Input 1

```
2 1
5
```

#### Sample Output 1

```
3 4
```

#### Sample Input 2

```
3 3
10 11 12
```

#### Sample Output 2

```
4 7 13
```