Ant Colony Optimisation (ACO) Algorithm

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1 Inspiration for Algorithm

As the name suggests this algorithm is inspired by the behaviour of ants. When ants move they release a chemical called pheromone which other ants in the colony can detect. The higher the concentration of pheromone the more likely it is that an ant will follow the path [2].

Consider Figure 1, where N is an ants nest and F is a source of food.

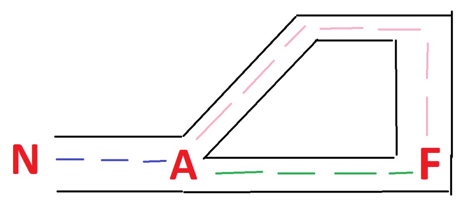


Figure 1: Possible paths of ants from nest to food

When the colony of ants leave the nest the only way they can go is to point A. However, once they get to point A they can either take the orange, upwards path or the green, straight path. As this is the first time they have left the nest there will be no pheromone on either path. Due to this, there is an equal chance that an ant takes the orange path as there is the green path, hence we will assume half the colony goes either way. As the green route is much shorter than the orange route those ants will get there first. When they have got their food they will then need to return back to their nest. As they originally, recently travelled on the green route, there will be a high concentration of pheromone on this path whereas there will be no pheromone on the end of the orange path as the ants that took that path have not reached that point yet. This means all those ants will again take the green route releasing even more pheromone on the path back to point A and then back along the blue path to their nest.

Eventually, the ants that took the orange path will reach point F. Once they have taken the food from the food source they will also want to return to the nest. Unlike the other half of the colony, there will now be concentrations of pheromone on both the green path and the orange path. However, there have now been two journeys on the green path (by the first half of ants there and back) and one journey on the orange path (by the second half) meaning the concentration of pheromone will be stronger on the green path than the orange path (assuming the evaporation of pheromone is negligible). Because of this, it is more likely for an individual ant to take the green route than the orange route, hence the majority of the ants will follow the green path to A then the blue path to N.

When the colony of ants next leave the nest and get to point A the majority of ants will take the green path instead of the orange path. The reason for this is due to the higher concentration of pheromone on this path meaning the probability of an ant taking this path is much higher than the orange path. As the ants carry on travelling from point N to F the ratio of pheromone on the green path compared to the orange path will continue to increase and in turn will the probability of choosing the green path. Because of this, eventually, the green path (the shortest path) will become the established path that the ants take.

2 ACO for Travelling Salesman Problem

To set up the Ant Colony Algorithm for the Travelling Salesman Problem (TSP) we will assume there are n cities and that L distinct ants start from any of the cities n randomly [1]. The differences between the 'ants' that we use for this algorithm and the normal ants that we saw in the previous example are:

- The ants have a memory so will not visit a city that they have already visited as this would produce an infeasible solution.
- The ants know the distance between any two cities and so will tend to choose the most nearby city if everything is the same (such as the level of pheromone).

The probability that ant k chooses city j from city i is given by the following formula:

$$P_{i,j}^k = \begin{cases} \frac{(\tau_{i,j})^\alpha \cdot (\eta_{i,j})^\beta}{\sum_{s \in allowed_k} [(\tau_{i,j})^\alpha \cdot (\eta_{i,j})^\beta]} & \text{if } x = j \in allowed_k \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

To understand this formula, we must define what each parameter means:

- $\tau_{i,j}$ is the concentration intensity of pheromone between city i and city j.
- α is a parameter which determines the impact of the concentration intensity of pheromone in decision making.
- $\eta_{i,j}$ is the visibility of city j from city i and is equal to $1/d_{i,j}$ where $d_{i,j}$ is the distance between city i and j.
- β is a parameter which determines the impact of the visibility in decision making.
- $allowed_k$ is the set of all cities that ant k has not yet visited.

Once ant k has made n iterations this means a complete tour is created. After a complete tour is created the levels of pheromones are updated so that a shorter path has a higher level of pheromones than a longer path. The levels of pheromones are updated using the following formulas:

$$\tau_{i,j}(t+1) = (1-\rho)\tau_{i,j}(t) + \Delta\tau_{i,j} \quad (2)$$

Formula 2 tells us that the level of pheromone between city i and city j at iteration $t+1$ is determined by the level at iteration t multiplied by evaporation constant, $\rho \in [0, 1]$, plus the change in the level of pheromone.

If the evaporation constant is 0, this means no pheromone on the path evaporates between iteration t and $t+1$, whereas if it is 1 then all pheromone is evaporated between t and $t+1$.

The change in the level of pheromone on the path between city i and j, $\Delta\tau_{i,j}$, is calculated using the following:

$$\Delta\tau_{i,j} = \sum_{k=1}^{L} \Delta\tau_{i,j}^k \quad (3)$$

where

$$\Delta\tau_{i,j}^k = \begin{cases} \frac{1}{L_k} & \text{if ant k travels on path (i,j)} \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

In equation 4, $\Delta\tau_{i,j}^{k}$ is the change in the level of pheromone on the path between city i and j because of ant k and L_K is the length of ant k's tour.

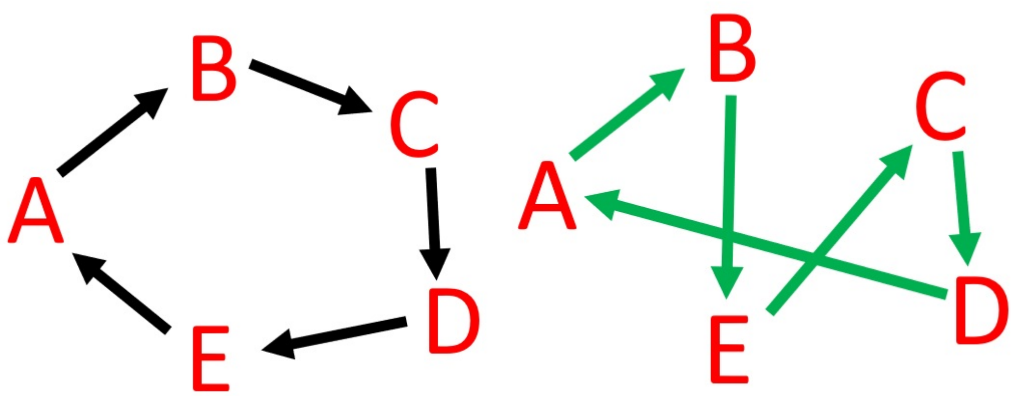
3 Example of ACO for TSP

Suppose we have 2 ants and 5 cities with Table 1 showing the distances between each city.

Table 1: Distances between the 5 cities

	B	**C**	**D**	**E**
A	2	5	15	3
B		4	7	4
C			5	4
D				6

Suppose after 1 iteration we end up with the following two tours, one for each ant:



(a) Ant 1's tour

(b) Ant 2's tour

Figure 2: The tours of the two ants after one iteration

As A-B and C-D appear in both tours, the pheromone level between those cities will be higher than the path between the cities which only appear in one of the tours, such as E-D. Whereas the path between cities which neither ant selected, such as A-D and B-D, will have no pheromone between them meaning on the 2^{nd} iteration they are less likely to be chosen. Therefore ahead of the 2^{nd} iteration, the pheromone levels are as seen in Figure 3.

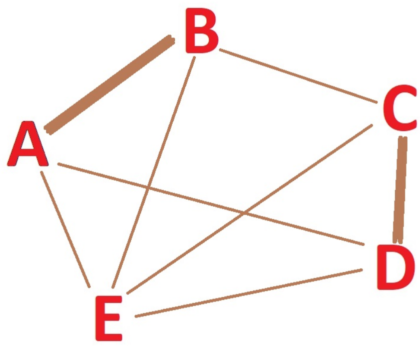


Figure 3: Pheromone levels ahead of 2^{nd} iteration

From Table 1 and Figure 3, we can make some notes. Firstly an ant starting at A is most likely to choose to go to city B first due to it being closest and having the highest level of pheromone. Assuming the ant goes from A to B, it is equally likely that the ant then goes to C or E (both more likely than going to D) as they are both the same distance away and have the same level of pheromone. Assuming the ant goes to C, the only two cities left for the ant to visit are D and E. D is closest and has a higher level pheromone than E so it is has a higher probability to go there first. If it goes to D then the only remaining city is E so the ant will go there then return to A.

If a city is further away but has a higher concentration of pheromone the city with the highest probability will depend on the values of α and β chosen in formula 1.

Eventually, as t gets larger and larger the majority of ants will all choose the same tour until if t is large enough all ants choose the same tour and this is the final tour chosen by the algorithm.

References

[1] Dorigo, M. [2004], *Ant Colony Optimization*, Cambridge, Mass.:MIT Press.

[2] Gopal Prasad Malakar [2017], 'Tutorial - Introduction to Ant Colony Optimization Algorithm n How it is applied on TSP', Available at: `https://www.youtube.com/watch?v=wfD5xlEcmuQ&t=729s&ab_channel=GopalPrasadMalakar`. (Last accessed on 02 January 2021).