

Lecture 1: Boolean Algebra

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1 Introduction

Boolean algebra is the branch of algebra in which the variables store truth value. All variables are true(1) or false(0). There are 3 main operations that create the base for boolean algebra: **AND(conjunction)**, **OR(disjunction)**, and **NOT(negation)**.

2 Basic Operations

AND, denoted $x \wedge y$ or x AND y or $x \cdot y$, satisfies $x \wedge y = 1$ if $x = y = 1$, else $x \wedge y = 0$

OR, denoted $x \vee y$ or x OR y or $x + y$, satisfies $x \vee y = 0$ if $x = y = 0$, else $x \vee y = 1$

NOT, denoted $\neg x$ or NOT x or $\sim x$ or \bar{x} , satisfies $\neg x = 0$ if $x = 1$ and $\neg x = 1$ if $x = 0$, reverses truth values of operation

x	y	$x \wedge y$	$x \vee y$
0	0	0	0
1	0	0	1
0	1	0	1
1	1	1	1

x	$\neg x$
0	1
1	0

3 Secondary Operations

Material Implication, denoted $x \rightarrow y = \neg x \vee y$, if $x = 1$, then $x \rightarrow y = y$, if $x = 0$, then $x \rightarrow y = 1$

Exclusive Or, denoted $x \oplus y$ or x XOR y , $x \oplus y = 1$ if $x = 1$ & $y = 0$ or $x = 0$ & $y = 1$, else $x \oplus y = 0$, true when values are different

Equivalence, denoted $x \equiv y$, $x \equiv y = 1$ if $x = 1$ & $y = 1$, or if $x = 0$ & $y = 0$, complement of XOR, true when values are the same **Dual**, the dual is found by replacing all OR's with AND's and all AND's with OR's, and all 1's with 0's and all 0's with 1's

Complement, found by negating each individual value and replaving all OR's with AND's and all AND's with OR's and all 1's with 0's and all 0's with 1's

Memorizing boolean algebra laws is extremely beneficial to being able to solve problems quickly and efficiently. Here is a link to page with almost every law. Most are derivable, but should still be memorized. <http://www.uilTEXAS.org/files/academics/UI LCS-BooleanIdentities.pdf>

Demorgan's Rule is crucial to simplifying boolean algebra problems. It states:

$$\bar{A} + \bar{B} = \overline{AB}$$

or

$$\bar{A} * \bar{B} = \overline{A + B}$$

With these basic rules memorized, all boolean algebra problems should be simple to work through

4 Exercises

1. Simplify completely:(ACSL 2001-2002) $(A + B) \oplus AB$
2. Simplify the following expression: $F = BC + \overline{BC} + BA$
3. Simplify the Boolean expression $(A+B+C)\overline{(D+E)} + (A+B+C)(D+E)$:

For more practice resources on boolean algebra, read through [the ACSL Wiki page](#).