Fundamental concepts of probability

Recently, you learned that **probability** uses math to quantify uncertainty, or to describe the likelihood of something happening. For example, there might be an 80% chance of rain tomorrow, or a 20% chance that a certain candidate wins an election.

In this reading, you'll learn more about fundamental concepts of probability. We'll discuss the concept of a random experiment, how to represent and calculate the probability of an event, and basic probability notation.

Probability fundamentals

Foundational concepts: Random experiment, outcome, event

Let's begin with three concepts at the foundation of probability theory:

- Random experiment
- Outcome
- Event

Probability deals with what statisticians call random experiments, also known as statistical experiments. A **random experiment** is a process whose outcome cannot be predicted with certainty.

For example, before tossing a coin or rolling a die, you can’t know the result of the toss or the roll. The result of the coin toss might be heads or tails. The result of the die roll might be 3 or 6.

All random experiments have three things in common:

- The experiment can have more than one possible outcome.
- You can represent each possible outcome in advance.
- The outcome of the experiment depends on chance.

In statistics, the result of a random experiment is called an outcome. For example, if you roll a die, there are six possible outcomes: 1, 2, 3, 4, 5, 6.

An event is a set of one or more outcomes. Using the example of rolling a die, an event might be rolling an even number. The event of rolling an even number consists of the outcomes 2, 4, 6. Or, the event of rolling an odd number consists of the outcomes 1, 3, 5.

In a random experiment, an event is assigned a probability. Let's explore how to represent and calculate the probability of a random event.

The probability of an event

The probability that an event will occur is expressed as a number between 0 and 1. Probability can also be expressed as a percent.

- If the probability of an event equals 0, there is a 0% chance that the event will occur.
- If the probability of an event equals 1, there is a 100% chance that the event will occur.

There are different degrees of probability between 0 and 1. If the probability of an event is close to zero, say 0.05 or 5%, there is a small chance that the event will occur. If the probability of an event is close to 1, say 0.95 or 95%, there is a strong chance that the event will occur. If the probability of an event equals 0.5, there is a 50% chance that the event will occur—or not occur.

Knowing the probability of an event can help you make informed decisions in situations of uncertainty. For example, if the chance of rain tomorrow is 0.1 or 10%, you can feel confident about your plans for an outdoor picnic. However, if the chance of rain is 0.9 or 90%, you may want to think about rescheduling your picnic for another day.

Calculate the probability of an event

To calculate the probability of an event in which all possible outcomes are equally likely, you divide the number of desired outcomes by the total number of possible outcomes. You may recall that this is also the formula for classical probability:

# of desired outcomes ÷ total # of possible outcomes

Let's explore the coin toss and die roll examples to get a better idea of how to calculate the probability of a single random event.

Example: Coin toss

Tossing a fair coin is a classic example of a random experiment:

- There is more than one possible outcome.
- You can represent each possible outcome in advance: heads or tails.
- The outcome depends on chance. The toss could turn up heads or tails.

Say you want to calculate the probability of getting heads on a single toss. For any given coin toss, the probability of getting heads is one chance out of two. This is $1 \div 2 = 0.5$, or 50%.

Now imagine that you were to toss a specially designed coin that had heads on both sides. Every time you toss this coin it will turn up heads. In this case, the probability of getting heads is 100%. The probability of getting tails is 0%.

Note that when you say the probability of getting heads is 50%, you aren't claiming that any actual sequence of coin tosses will result in exactly 50% heads. For example, if you toss a fair coin ten times, you may get 4 heads and 6 tails, or 7 heads and 3 tails. However, if you continue to toss the coin, you can expect the long-run frequency of heads to get closer and closer to 50%.

Example: Die roll

Rolling a six-sided die is another classic example of a random experiment:

- There is more than one possible outcome.
- You can represent all possible outcomes in advance: 1, 2, 3, 4, 5, and 6.
- The outcome depends on chance. The roll could turn up

any number 1–6.

Say you want to calculate the probability of rolling a 3. For any given die roll, the probability of rolling a 3 is one chance out of six. This is $1 \div 6 = 0.1666$, or about 16.7%.

Probability notation

It helps to be familiar with probability notation as it's often used to symbolize concepts in educational and technical contexts.

In notation, the letter P indicates the probability of an event. The letters A and B represent individual events.

For example, if you're dealing with two events, you can label one event A and the other event B.

- The probability of event A is written as $P(A)$.
- The probability of event B is written as $P(B)$.
- For any event A, $0 \le P(A) \le 1$. In other words, the probability of any event A is always between 0 and 1.
- If $P(A) > P(B)$, then event A has a higher chance of occurring than event B.
- If $P(A) = P(B)$, then event A and event B are equally likely to occur.

Key takeaways

Data professionals use probability to help stakeholders make informed decisions about uncertain events. Your knowledge of fundamental concepts of probability will be useful as a building block for more complex calculations of probability.

Resources for more information

To learn more about fundamental concepts of probability, refer to the following resources:

- These lecture notes from Richland Community College provide a useful summary of the fundamental concepts and basic rules of probability.