A Novel Data-Driven Mathematical Framework for Olympic Medal Counting System

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***Abstract*—Current Olympic medal counting systems suffer from significant limitations due to their lack of grounding in rigorous mathematical theory or empirical evidence. Traditional methods, such as the gold-first approach or total medal count, often lead to subjective and potentially flawed assessments of Olympic success. These methods fail to account for the relative difficulty of winning medals across different sports, the variance in competition levels, and the individual performance data of athletes. To address these challenges, we propose a novel data-driven mathematical framework grounded in probability theory and statistical data. This framework aims to provide a more reliable, consistent, and fair measure of athletic achievement, offering a robust tool for ranking countries based on their Olympic performances.**

***Index Terms*—Data-Driven Model, Cumulative Probability Distribution, Non-Cooperative Multi-Player Games, Weighted Medal Count**

I. Introduction

A. Evaluate Olympic Success

In the context of the Olympic Games, ranking countries by their medal counts has long been a subject of debate and controversy. The most commonly used method, favored by the International Olympic Committee (IOC) and the media, ranks countries first by the number of gold medals, followed by silver and bronze. However, this approach is not without its critics. Some argue that it unfairly diminishes the value of silver and bronze medals, suggesting that anything less than gold is of little consequence. This has led to alternative ranking methods, such as the total medal count, which treats all medals as equal, though this too has its drawbacks.

Historically, there have been instances where the country with the most gold medals did not top the overall medal count, such as in the 1896, 1912, and 1964 Summer Olympics. In 2008, for example, the United States led in total medals but was outranked by China due to the latter's higher gold medal count. These anomalies highlight the limitations of the gold-first ranking system. [6]

Various alternative systems have been proposed to address these limitations [7]. These range from weighted point systems, which assign different values to gold, silver, and bronze medals, to demographic-adjusted methods that factor in population size or economic output. Despite the diversity of approaches, there is no consensus on the most fair and accurate method of ranking Olympic success.

This paper seeks to explore these various ranking systems, examining their strengths and weaknesses. By developing a dynamic weighted medal counting system and comparing it to existing methods, this study aims to provide a more nuanced understanding of how Olympic success can be measured and what implications these measurements have for our perception of athletic achievement on the global stage.

B. Overview of Olympic Medal Counting Systems

Several counting systems [8] have been developed to assess the success of countries at the Olympic Games. Among these, weighted systems are often considered the most equitable, although they are more complex to implement compared to the more commonly used methods such as ranking by gold medals or total medal count.

- Gold-First Method: This method prioritizes the number of gold medals won by each country, followed by silver and bronze. However, this raises the question of whether silver and bronze medals should be considered less valuable.
- Total Medals Method: This approach ranks countries based on the total number of medals (gold, silver, and bronze) won, assigning equal value to each. Critics argue that this method may not fairly reflect the different levels of achievement.
- Total Gold Medals: In this system, all gold medals are counted, including those awarded in team sports, where each individual medal within the team contributes to the total.
- Improvement Rating [5]: This method evaluates success based on the percentage improvement in a country's medal tally compared to previous Olympic Games.
- Comparison to Expectations [4]: Several groups use this approach to predict medal outcomes for each country based on factors such as population, GDP, and prior performances at previous Olympics and related competitions.
- Fibonacci Weighted Point System (3:2:1): This system assigns 3 points for a gold medal, 2 points for a silver medal, and 1 point for a bronze medal.

- Exponential Weighted Point System (4:2:1): This variation assigns 4 points for a gold medal, 2 points for a silver medal, and 1 point for a bronze medal.
- London 1908 Weighted Point System (5:3:1): Under this method, gold medals are awarded 5 points, silver 3 points, and bronze 1 point.
- LOF Weighted Point System (5:3:2): This system gives 5 points for a gold medal, 3 points for silver, and 2 points for bronze.
- Topend Sports Weighted Point System (6:2:1): In this method, 6 points are assigned for each gold medal, 2 points for each silver medal, and 1 point for each bronze medal.
- Per-Capita Demographic Ranking: This ranking system divides the number of medals won by a country's population, providing a per-capita analysis of success.
- Per-GDP Demographic Ranking [3]: Similar to the per-capita method, this system divides the number of medals by the country's gross domestic product (GDP), offering a perspective on Olympic success relative to economic output.

C. Lack of a Solid Mathematical Foundation

These counting systems mentioned are not grounded in rigorous mathematical theory or empirical evidence, leading to subjective and potentially flawed assessments of Olympic success. They fail to provide a reliable, consistent, and fair measure of athletic achievement, which undermines their effectiveness as tools for ranking countries based on their Olympic performances.

- Arbitrary Weight Assignments: Many of the weighted point systems (e.g., Fibonacci, Exponential, London 1908) assign points to gold, silver, and bronze medals based on arbitrary scales without any empirical basis or consistent logic. These scales do not reflect the true value of each medal type, making the choice of weights subjective and ungrounded in any established mathematical or statistical principles.
- Lack of Consistency and Universality: The ranking methods are inconsistent across different systems, with each proposing a unique way to value medals. This lack of a universal standard creates inconsistencies in how Olympic success is measured, leading to potential bias and unfair comparisons. For example, a country ranked highly under one system might rank poorly under another, indicating that the systems do not reliably measure the same underlying concept.
- Ignoring Medal Distribution Variance: Methods like the gold-first or total medals approach do not account for the variance in medal distribution among countries. The gold-first method, for instance, disproportionately values gold medals while ignoring the overall athletic achievement that silver and bronze medals represent. This approach fails to recognize the relative difficulty of winning different medals across various sports and events, leading to a skewed representation of a country's performance.
- Simplification of Athletic Achievement: These systems often oversimplify the complex nature of athletic success by reducing it to a single score or ranking. They do not consider the quality of competition, the number of participants, or the historical and contextual factors that influence outcomes. For example, the "per-capita demographic ranking" and "per-GDP demographic ranking" attempt to adjust for country size or economic power but do so in a way that may not accurately reflect the true difficulty of achieving Olympic success across different contexts.
- Lack of Robustness to Outliers: The systems do not adequately handle outliers, such as countries that win a disproportionate number of medals in a few sports or those that dominate certain events. This lack of robustness can lead to rankings that do not accurately reflect the overall athletic performance of a country across the entire Olympic program.

II. Related work

A. The recent studies

One study [1] examines various methods used to rank countries based on Olympic medals. The lexicographic method is employed by the International Olympic Committee (IOC), which prioritizes gold medals over any number of silver medals, and silver medals over any number of bronze medals. This study explores the challenges of quantifying this ranking system numerically, particularly when using finite positional numeral systems, which fail to uphold the lexicographic order across all possible medal counts.

The solution proposed in [1] is to utilize a new numerical system based on actual infinite numbers, facilitated by a specially designed "Infinity Computer." This system can accurately compute rankings that maintain the lexicographic order, providing a method to rank countries numerically while respecting the relative importance of different medals. The paper also discusses the broader applications of this approach in situations requiring lexicographic ordering.

Another study [2] shows the relationship between a country's socioeconomic factors and its success in the Olympics. This paper uses two models to test these factors: a linear model and a Cobb-Douglas production function. The results indicate that while economic resources and political factors (like being a socialist country) have a strong positive correlation with medal counts, the effect of population size is less clear, showing inconsistencies across the models. The study concludes that economic development and political systems are critical in explaining the uneven distribution of Olympic medals across countries.

The study in [3] discusses the intricacies of Olympic medal counts as an indicator of national sporting success. It highlights how a small number of countries dominate the medal counts, though more nations are winning medals now than in the past, partly due to improved athlete support and development programs. The analysis also explores the concept of "medal conversion"—how effectively countries convert top-8 placements into medals—offering insights into the competitiveness of their athlete-support systems. It concludes that while Norway's support system is efficient in converting finalists to medalists, the overall talent pool may be insufficient to consistently meet medal goals

B. The limitations of the recent studies

All of the studies failed to generate a robust quantitative framework for assigning weighted point values to Olympic medals, leaving a critical gap in evaluating the relative worth of different medals. Specifically, they did not address the fundamental question: How many silver medals equate to a single gold medal in terms of value? Furthermore, the studies overlooked the varying levels of competition intensity across different sports, an omission that significantly undermines the validity of their analyses. A gold medal in one sport does not necessarily carry the same value as a gold medal in another due to these differences in competitive intensity. As a result, the methodologies employed in these studies are insufficient for capturing the nuanced dynamics of Olympic success, necessitating a more refined and sport-specific approach to medal valuation.

III. The proposal

A. A Novel Data-driven Mathematical Framework

Ranking countries by Olympic medal counts is a complex issue, with multiple systems employed to measure success as shown above. The novel data-driven mathematical framework proposed in this study addresses the fundamental flaws in existing Olympic medal counting systems. By grounding the medal counting process in probability theory and empirical data, the framework offers a more reliable and objective measure of athletic achievement by a quantitative algorithm to determine the most reasonable weighted point values for each medal in each sport. This approach not only provides a fairer comparison between countries but also enhances the understanding of what constitutes Olympic success, making it a valuable tool for future assessments of global athletic performance. This framework can be extended to every sport in the Olympic Games and any competition-based ranking systems.

B. The proposed methodology

This study adopts a distinct methodology. Rather than finding a universal weighted point value of Olympic medals, the objective of this study is to calculate the probability of winning each type of Olympic medal by analyzing the performance distributions of individual athletes in each sport. Subsequently, these probabilities are transformed into weighted point values corresponding to each medal type.

To illustrate the proposed framework, this study simplifies the final round of competition within a given sport. Specifically, it assumes that each sport culminates in a single final round involving n athletes, where the top three performers are awarded the gold, silver, and bronze medals, respectively. For simplicity, the analysis excludes the possibility of ties, ensuring that each medal is awarded to a unique athlete.

Ultimately, a mathematical framework is derived. This framework also implies that the medals in each sport have different relative value. This paper also includes some examples and simulations based on this framework.

IV. Probability-weighted Medal Count

A. Probability Distribution vs. Medal Count

Consider a straightforward scenario from the perspective of a single athlete in one sport. Assume the probabilities of this athlete winning each medal are as follows:

TABLE I
THE PROBABILITIES OF WINNING MEDALS

Medal Type	m	$P(X=m)$	$C(m)$	$C_w(m)$
Gold Medal	1	1%	1	1
Silver Medal	2	4%	1	1/4
Bronze Medal	3	10%	1	1/10

where

- X is the random variable representing the outcome of this athlete in this sport.
- m represents the medal type of Gold Medal, Silver Medal or Bronze Medal. The numeric code 1, 2 and 3 are for the convenience of the modeling in this paper.
- $P(X=m)$ is the probability of winning each medal type by this athlete in this sport. The derivation of it will be covered later in this paper. For simplicity, numeric values are used here for demonstration purpose.
- $C(m)$ is the raw count of medals. In the total medals method, each medal counts 1.
- $C_w(m)$ is the weighted medal count as explained below.

These probabilities are reasonable because Bronze Medal is considered much easier than Silver Medal, and Silver Medal is much easier than Gold Medal. These probabilities suggest that securing a Gold Medal is four times more challenging than obtaining a Silver Medal and ten times more difficult than winning a Bronze Medal.

From this, it can be inferred that the value of a Gold Medal is approximately four times that of a Silver Medal and ten times that of a Bronze Medal. By normalizing the medal count using gold medal as unit value, the weighted medal counts become 1, $1/4$, and $1/10$ respectively.

In other words, one Silver Medal equates to $1/4$ Gold Medal, and one Bronze Medal equates to $1/10$ Gold Medal.

In summary, one can conclude,

$$C_w(m) = \frac{P(X=1)}{P(X=m)} C(m) \tag{1}$$

B. Cumulative Probability Distribution vs. Medal Count

Consider a slightly different scenario from the perspective of a single athlete in one sport. Assume the probabilities of this athlete winning each medal are as follows:

In a revised scenario where the probability of winning a Bronze Medal is 1%—in contrast to the 10% probability

TABLE II
THE PROBABILITIES OF WINNING MEDALS

Medal Type	m	$P(X=m)$	$C(m)$	$C_w(m)$
Gold Medal	1	1%	1	1
Silver Medal	2	4%	1	1/4
Bronze Medal	3	1%	1	1 ?

in the previous example—the probabilities reflect a different competitive landscape. For this highly competitive athlete, the probability of securing a Silver Medal is higher than that of the Bronze Medal. Applying the same methodology as before, these probabilities imply that winning a Gold Medal is four times more challenging than obtaining a Silver Medal and equally challenging as winning a Bronze Medal. Consequently, the value of a Gold Medal would be approximately four times that of a Silver Medal and equal to that of a Bronze Medal.

The result appears counterintuitive when the probability of winning the Bronze Medal is low, not due to the inherent difficulty of winning the Bronze Medal, but rather because the athlete's performance is exceptionally strong. In such cases, a more appropriate approach would be to utilize the cumulative probability distribution rather than the individual probability distribution. This adjustment is necessary because the probability of winning a Gold Medal is inherently included in the probability of securing a Silver Medal or higher, from the perspective of a single athlete competing in a given sport.

The probabilities table will become,

TABLE III
THE CUMULATIVE PROBABILITIES OF WINNING MEDALS

Medal Type	m	$P(X=m)$	$P(X \leq m)$	$C(m)$	$C_w(m)$
Gold Medal	1	1%	1%	1	1
Silver Medal	2	4%	5%	1	1/5
Bronze Medal	3	1%	6%	1	1/6

Therefore, the equation (1) can be modified to,

$$C_w(m) = \frac{P(X \leq 1)}{P(X \leq m)} C(m) \tag{2}$$

Notice that $P(X \leq 1) = P(X = 1)$ in all cases.

In this revised scenario, Silver Medal and Bronze Medal have the similar difficulty level, so their similar medal counts are reasonable.

Using the cumulative probability methodology, Table I can be re-computed as,

TABLE IV
THE CUMULATIVE PROBABILITIES OF WINNING MEDALS

Medal Type	m	$P(X=m)$	$P(X \leq m)$	$C(m)$	$C_w(m)$
Gold Medal	1	1%	1%	1	1
Silver Medal	2	4%	5%	1	1/5
Bronze Medal	3	10%	15%	1	1/15

V. NON-COOPERATIVE MULTI-PLAYER GAMES

A. Individual Player's Statistical Model

The preceding section derived the equation for weighted medal counting from the perspective of an individual athlete within a single sport. This framework can be extended to competitions involving n athletes in the same sport. For simplicity, and without loss of generality, we assume that each athlete competes against every other athlete in the sport.

Furthermore, it is assumed that each athlete's performance follows a normal distribution. Statistical data for each athlete can be obtained from recent sporting events.

$$p_i(x_i) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{(x_i - \mu_i)^2}{2\sigma_i^2}\right) \tag{3}$$

where

- i represents the ith athlete in this sport. $1 \leq i \leq n$.
- x_i represents the score of one competition performed by athlete i in this sport.
- p_i is the probability density function of a normal distribution for athlete i in this sport, assuming the x value follows the normal distribution.
- σ_i represents the standard deviation of athlete i's performance in this sport.
- μ_i represents the mean of athlete i's performance in this sport.

B. Non-Cooperative Multi-Player Competitions Model

Under the assumption that each athlete competes against every other athlete in the sport, the probability density of the outcome between the athlete i and the athlete j is computed as,

$$p_{ij}(x_i - x_j) = \frac{1}{\sqrt{2\pi(\sigma_i^2 + \sigma_j^2)}} \exp\left(-\frac{((x_i - x_j) - (\mu_i - \mu_j))^2}{2(\sigma_i^2 + \sigma_j^2)}\right) \tag{4}$$

Here it assumes that there is no correlation between the two athletes i and j.

The probability of the athlete i winning the athlete j is,

$$P_{ij}(X_{ij} > 0) = 1 - \Phi\left(\frac{0 - \mu_{ij}}{\sigma_{ij}}\right) \tag{5}$$

where

- X_{ij} is a random variable representing $x_i - x_j$.
- $\mu_{ij} = \mu_i - \mu_j$
- $\sigma_{ij} = \sqrt{\sigma_i^2 + \sigma_j^2}$
- Φ is the cumulative distribution function (CDF) of the standard normal distribution. $\Phi(z) = \frac{1}{2}\left[1 + \text{erf}\left(\frac{z}{\sqrt{2}}\right)\right]$ where erf is the error function.

The equation (5) also implies the probability of the athlete i losing to the athlete j,

$$P_{ij}(X_{ij} < 0) = \Phi\left(\frac{0 - \mu_{ij}}{\sigma_{ij}}\right) \tag{6}$$

Here we can assume there is no tie, so it is safe to ignore $P_{ij}(X_{ij}=0)$.

In a sport with n athletes, for one athlete to win Gold Medal, this athlete i must win all other athletes in this sport. The probability is,

$$\begin{aligned} P_i(X=1) &= \prod_{\substack{j=1\\ j\neq i}}^{n} P_{ij}(X_{ij}>0) \\ &= \prod_{\substack{j=1\\ j\neq i}}^{n} \left(1-\Phi\left(\frac{0-\mu_{ij}}{\sigma_{ij}}\right)\right) \end{aligned} \tag{7}$$

Similarly, for one athlete to win Silver Medal, this athlete i must win all but one other athlete k in this sport. The probability of winning Silver Medal is,

$$\begin{aligned} P_i(X=2) &= \sum_{\substack{k=1\\ k\neq i}}^{n} \left[P_{ik}(X_{ik}<0) \prod_{\substack{j=1\\ j\neq i\\ j\neq k}}^{n} P_{ij}(X_{ij}>0) \right] \\ &= \sum_{\substack{k=1\\ k\neq i}}^{n} \left[\Phi\left(\frac{0-\mu_{ik}}{\sigma_{ik}}\right) \prod_{\substack{j=1\\ j\neq i\\ j\neq k}}^{n} \left(1-\Phi\left(\frac{0-\mu_{ij}}{\sigma_{ij}}\right)\right) \right] \end{aligned} \tag{8}$$

The probability of winning Bronze Medal is more complicated but following the same methodology, where this athlete i must win all but two other athletes k and l in this sport.

$$\begin{aligned} &P_i(X=3) \\ &= \sum_{\substack{k=1\\ l=1\\ k\neq i\\ l\neq i\\ k<l}}^{n} \left[P_{ik}(X_{ik}<0) P_{il}(X_{il}<0) \prod_{\substack{j=1\\ j\neq i\\ j\neq k\\ j\neq l}}^{n} P_{ij}(X_{ij}>0) \right] \\ &= \sum_{\substack{k=1\\ l=1\\ k\neq i\\ l\neq i\\ k<l}}^{n} \left[\Phi\left(\frac{0-\mu_{ik}}{\sigma_{ik}}\right) \Phi\left(\frac{0-\mu_{il}}{\sigma_{il}}\right) \prod_{\substack{j=1\\ j\neq i\\ j\neq k\\ j\neq l}}^{n} \left(1-\Phi\left(\frac{0-\mu_{ij}}{\sigma_{ij}}\right)\right) \right] \end{aligned} \tag{9}$$

C. Evaluate the Difficulty Levels of Medals

The previous subsection outlined the methodology for calculating the probability of athlete i winning each type of medal in a given sport. To quantify the relative weight of each medal, this methodology employs the average probability as an indicator of the difficulty level associated with obtaining each medal.

The average probability of winning Gold Medal is,

$$\begin{aligned} P(X=1) &= \frac{1}{n}\sum_{i=1}^{n} P_i(X=1) \\ &= \frac{1}{n}\sum_{i=1}^{n} \prod_{\substack{j=1\\ j\neq i}}^{n} P_{ij}(X_{ij}>0) \\ &= \frac{1}{n}\sum_{i=1}^{n} \prod_{\substack{j=1\\ j\neq i}}^{n} \left(1-\Phi\left(\frac{0-\mu_{ij}}{\sigma_{ij}}\right)\right) \end{aligned} \tag{10}$$

The average probability of winning Silver Medal is,

$$\begin{aligned} &P(X=2) = \frac{1}{n}\sum_{i=1}^{n} P_i(X=2) \\ &= \frac{1}{n}\sum_{i=1}^{n}\sum_{\substack{k=1\\ k\neq i}}^{n} \left[P_{ik}(X_{ik}<0) \prod_{\substack{j=1\\ j\neq i\\ j\neq k}}^{n} P_{ij}(X_{ij}>0) \right] \\ &= \frac{1}{n}\sum_{i=1}^{n}\sum_{\substack{k=1\\ k\neq i}}^{n} \left[\Phi\left(\frac{0-\mu_{ik}}{\sigma_{ik}}\right) \prod_{\substack{j=1\\ j\neq i\\ j\neq k}}^{n} \left(1-\Phi\left(\frac{0-\mu_{ij}}{\sigma_{ij}}\right)\right) \right] \end{aligned} \tag{11}$$

The average probability of winning Bronze Medal is,

$$\begin{aligned} &P(X=3) = \frac{1}{n}\sum_{i=1}^{n} P_i(X=3) \\ &= \frac{1}{n}\sum_{i=1}^{n}\sum_{\substack{k=1\\ l=1\\ k\neq i\\ l\neq i\\ k<l}}^{n} \left[P_{ik}(X_{ik}<0) P_{il}(X_{il}<0) \prod_{\substack{j=1\\ j\neq i\\ j\neq k\\ j\neq l}}^{n} P_{ij}(X_{ij}>0) \right] \\ &= \frac{1}{n}\sum_{i=1}^{n}\sum_{\substack{k=1\\ l=1\\ k\neq i\\ l\neq i\\ k<l}}^{n} \left[\Phi\left(\frac{0-\mu_{ik}}{\sigma_{ik}}\right) \Phi\left(\frac{0-\mu_{il}}{\sigma_{il}}\right) \prod_{\substack{j=1\\ j\neq i\\ j\neq k\\ j\neq l}}^{n} \left(1-\Phi\left(\frac{0-\mu_{ij}}{\sigma_{ij}}\right)\right) \right] \end{aligned} \tag{12}$$

D. Compute the Weighted Medal Count

Given the equations (10), (11), and (12), one can apply them to the equation (2).

The weighted Gold Medal count is always 1.

$$\begin{aligned} C_w(1) &= \frac{P(X\leq 1)}{P(X\leq 1)} C(m) \\ &= C(m) \\ &= 1 \end{aligned} \tag{13}$$

The weighted Silver Medal count is,

$$
\begin{aligned}
C_w(2) &= \frac{P(X \le 1)}{P(X \le 2)} C(m) \\
&= \frac{P(X=1)}{P(X=1)+P(X=2)}
\end{aligned} \tag{14}
$$

The weighted Bronze Medal count is,

$$
\begin{aligned}
C_w(3) &= \frac{P(X \le 1)}{P(X \le 3)} C(m) \\
&= \frac{P(X=1)}{P(X=1)+P(X=2)+P(X=3)}
\end{aligned} \tag{15}
$$

Given the quantitative formulas, one can compute the weighted medal count in every sport.

VI. Simulations

Based on the mathematical framework derived above, numeric simulations are demonstrated in this section.

A. Three-Player Games

Assume there are in total 3 athletes in a sport. They have identical skill level, i.e., they have the same μ_i and σ_i. From the equation (5) and (6), one can simply estimate,

$$
\begin{aligned}
P_{ij}(X_{ij} > 0) &= \frac{1}{2} \\
P_{ij}(X_{ij} < 0) &= \frac{1}{2}
\end{aligned} \tag{16}
$$

Using equations (7), (8) and (9), the probabilities of winning each medal by the athlete i are,

$$
\begin{aligned}
P_i(X=1) &= \frac{1}{4} \\
P_i(X=2) &= \frac{1}{2} \\
P_i(X=3) &= \frac{1}{4}
\end{aligned} \tag{17}
$$

Since the 3 athletes have identical skill level, the above probabilities are also the average probabilities.

$$
\begin{aligned}
P(X=1) &= \frac{1}{4} \\
P(X=2) &= \frac{1}{2} \\
P(X=3) &= \frac{1}{4}
\end{aligned} \tag{18}
$$

Therefore, the weighted medal counts are,

$$
\begin{aligned}
C_w(1) &= \frac{P(X \le 1)}{P(X \le 1)} C(m) \\
&= 1 \\
C_w(2) &= \frac{P(X \le 1)}{P(X \le 2)} C(m) \\
&= \frac{P(X=1)}{P(X=1)+P(X=2)} \\
&= \frac{1/4}{1/4+1/2} \\
&= \frac{1}{3} \\
C_w(3) &= \frac{P(X \le 1)}{P(X \le 3)} C(m) \\
&= \frac{P(X=1)}{P(X=1)+P(X=2)+P(X=3)} \\
&= \frac{1/4}{1/4+1/2+1/4} \\
&= \frac{1}{4}
\end{aligned} \tag{19}
$$

B. Ten-Player Games

Assume there are in total 10 athletes in a sport. They have identical skill level, i.e., they have the same μ_i and σ_i. Similarly the average probabilities of winning each medal are,

$$
\begin{aligned}
P(X=1) &= \frac{1}{2^9} = \frac{1}{512} \\
P(X=2) &= \binom{9}{1} \frac{1}{512} = \frac{9}{512} \\
P(X=3) &= \binom{9}{2} \frac{1}{512} = \frac{36}{512}
\end{aligned} \tag{20}
$$

Then the weighted medal counts are,

$$
\begin{aligned}
C_w(1) &= \frac{P(X \le 1)}{P(X \le 1)} C(m) \\
&= 1 \\
C_w(2) &= \frac{P(X \le 1)}{P(X \le 2)} C(m) \\
&= \frac{P(X=1)}{P(X=1)+P(X=2)} \\
&= \frac{1}{1+9} \\
&= \frac{1}{10} \\
C_w(3) &= \frac{P(X \le 1)}{P(X \le 3)} C(m) \\
&= \frac{P(X=1)}{P(X=1)+P(X=2)+P(X=3)} \\
&= \frac{1}{1+9+36} \\
&= \frac{1}{46}
\end{aligned} \tag{21}
$$

VII. Conclusions and Discussions

This study presents a mathematical framework rooted in data-driven analysis that leverages cumulative probability distributions to estimate the weighted medal count, based on statistical data of each athlete's recent performance metrics.

While the theory is derived from a simplified scenario in Olympic games, it remains applicable to more complex cases where athletes compete in subgroups, with winners advancing to subsequent rounds, and then ultimately, only two athletes contend for the Gold Medal. In this case, this framework can also be applied to estimate the weighted medal count by evaluating the winning probability at each round, where each athlete has a certain probability to compete with a subset of other athletes within each round, and then eventually the gold medalist triumphs over a subset of athletes through a sequential rounds of competition.

This study further demonstrates that, in highly competitive sports, the Gold Medal represents a substantially greater value compared to the Silver or Bronze Medals, particularly when the athletes exhibit similar skill levels. This finding aligns with the real-world value system observed in such sports, contrasting with less competitive events.

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