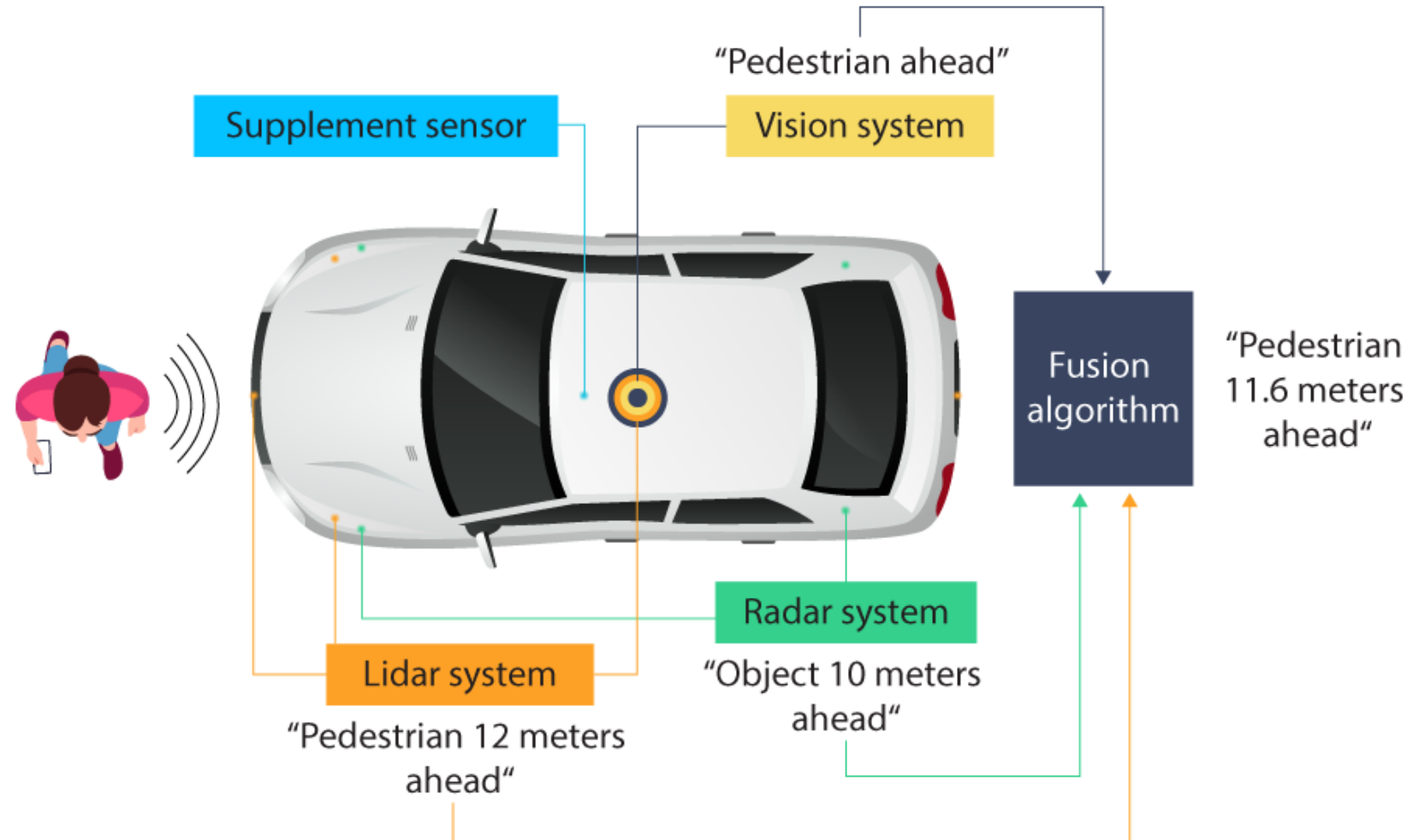


## Sensor Fusion & Robotics

Dr. Karishma Patnaik  
Postdoctoral Researcher  
School of Manufacturing Systems and Networks  
Arizona State University

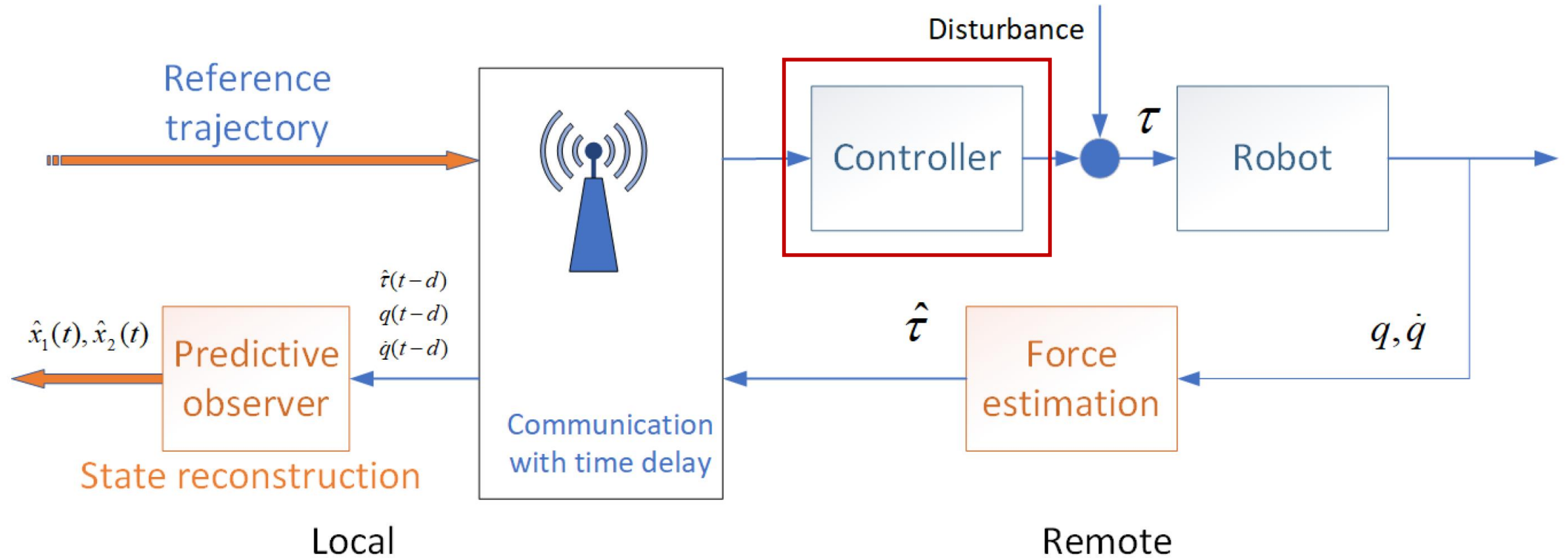
# What is Sensor Fusion?

- Process of combining sensor data or data derived from disparate sources
- For e.g. cameras and lidars have both pros and cons
- With sensor fusion, resulting information has less uncertainty



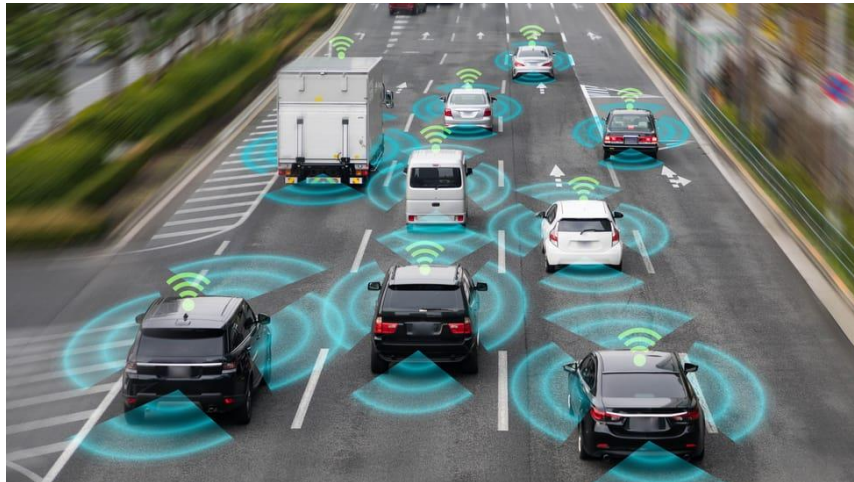


# Sensor Fusion in Autonomy – Crucial for Control

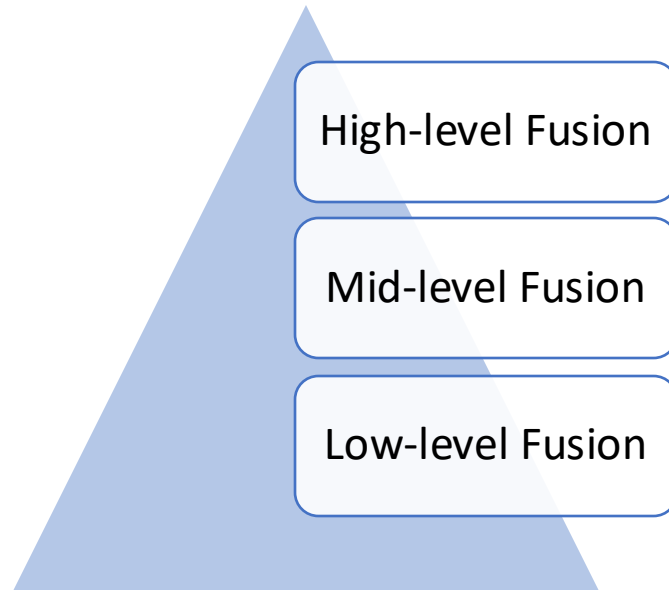


# Applications

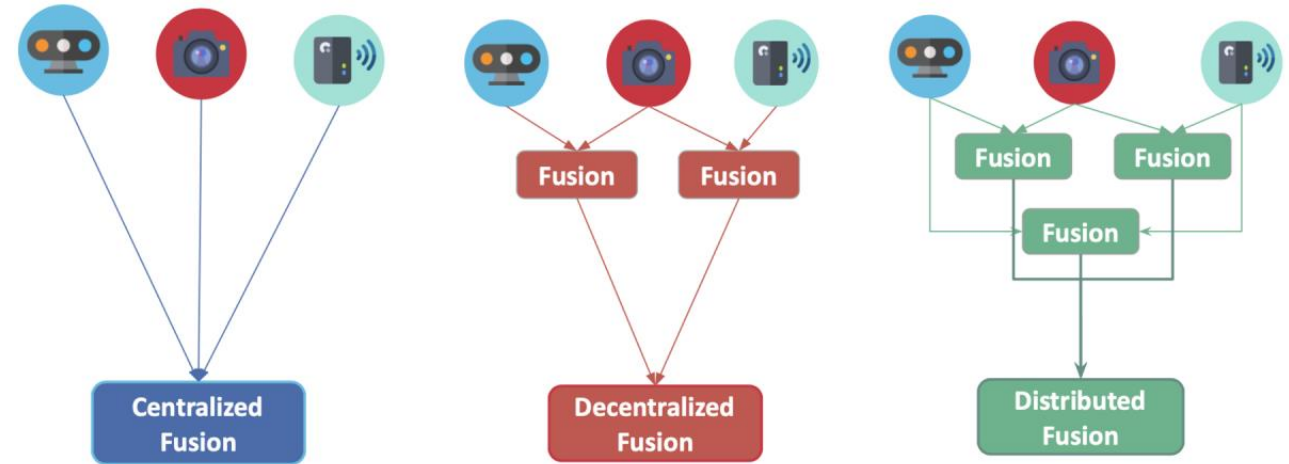
- **Autonomous Vehicles:** Combining data from cameras, LiDAR, and radar to accurately perceive obstacles and navigate safely.
- **Robotics:** Using a combination of cameras, ultrasonic sensors, and inertial measurement units (IMUs) to navigate in complex environments and avoid collisions.
- **Wearable Devices:** Combining accelerometer, gyroscope, and GPS data to accurately track movement and location.
- **Industrial Monitoring:** Using multiple sensors to monitor equipment health and predict potential failures.



# Types of Sensor Fusion



Based on the level of autonomy –  
When?

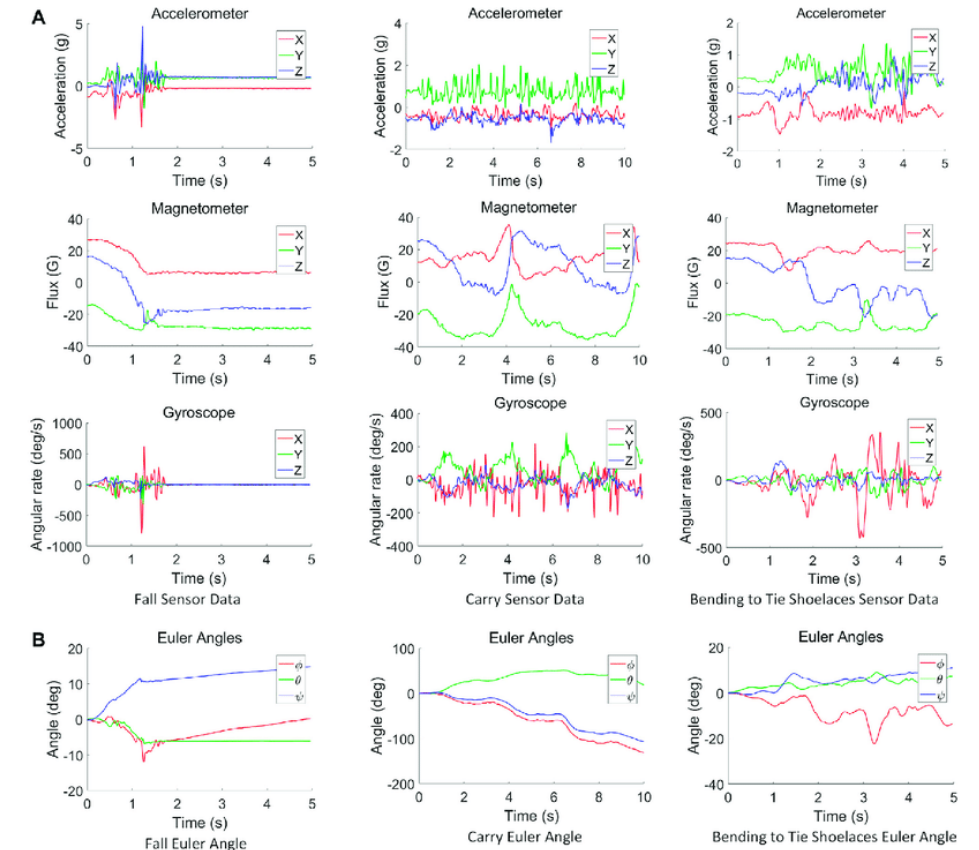


Other types based on Algorithm –  
Where?

# Types of Sensor Fusion – Low-level Example 1

## Low-Level Fusion (Data-Level Fusion)

- Raw data from multiple sensors are combined before any processing.
- Happens early in the pipeline
- Ex: Merging **accelerometer, magnetometers & gyroscope** in (IMUs).
  - Accelerometer:
    - Linear accelerations
    - Effected by gravity
  - Magnetometer
    - Accurate yaw
    - Takes measurements over long time to converge
  - Gyro
    - Gives angular velocities
    - Drifts over time due to integration under noise, bias



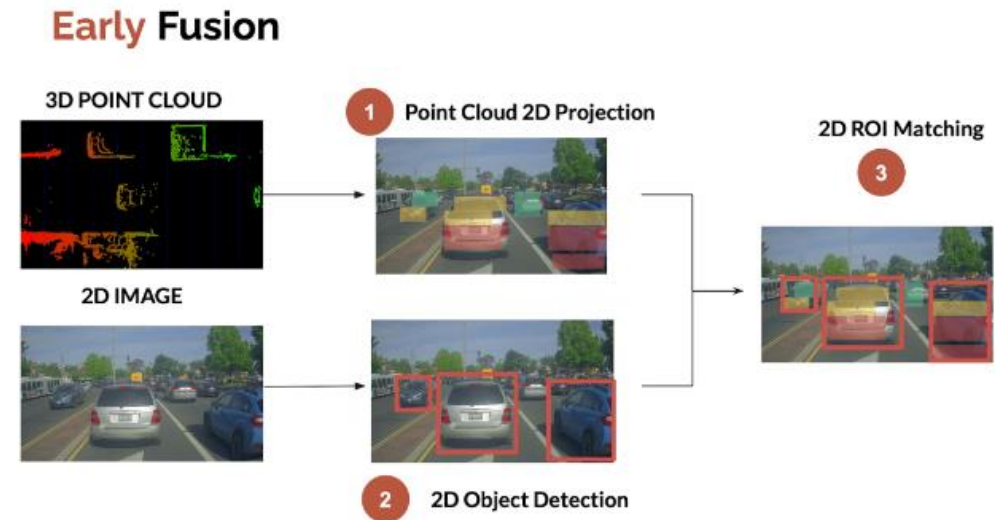
IMU-based Sensing [1]



# Types of Sensor Fusion – Low-level Example 2

Combining LiDAR point clouds and camera pixel data for enhanced perception

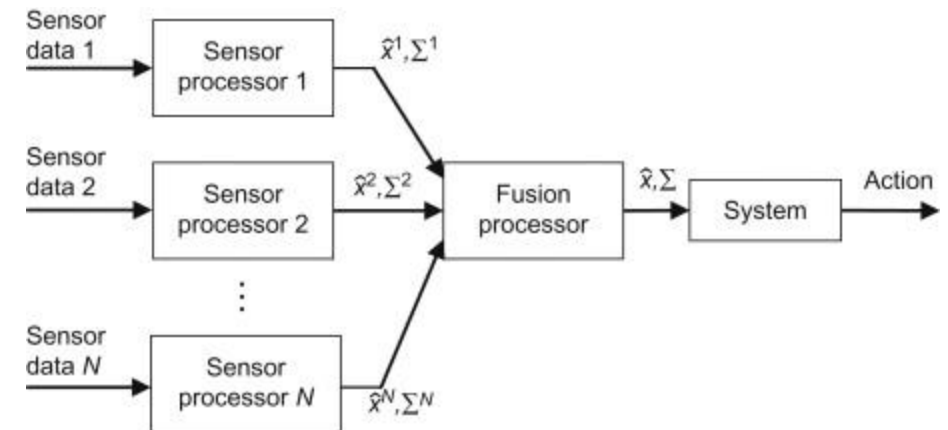
- Project the 3D PCL into 2D projection – using geometric principles
- Use YOLO to detect objects from the camera image
- Region Of Interest (ROI) Identification
  - For each bounding box, the camera gives us the classification
  - For each LiDAR projected point, we have a very accurate distance.
  - Can use clustering / thresholding
- Feature Extractions – Extract edge, color information,
- Data Fusion – Combine using feature concatenation



Fusing PCL data with camera image [2]

## Other Low-Level Fusion Techniques

- **Weighted Averaging:**
  - Combines raw data by assigning weights based on sensor confidence.
  - Example: Fusing temperature readings from multiple sensors.
- **Bayesian Networks:**
  - Probabilistic models that combine raw data with prior knowledge.
  - Example: Fusing radar and LiDAR point clouds for obstacle detection.
- **Direct Concatenation:**
  - Combines raw data vectors directly (e.g., concatenating LiDAR point clouds with camera RGB data).
- **Wavelet Transform:**
  - Used for combining sensor signals by decomposing them into frequency bands.
  - Example: Fusing audio signals with vibration sensor data.

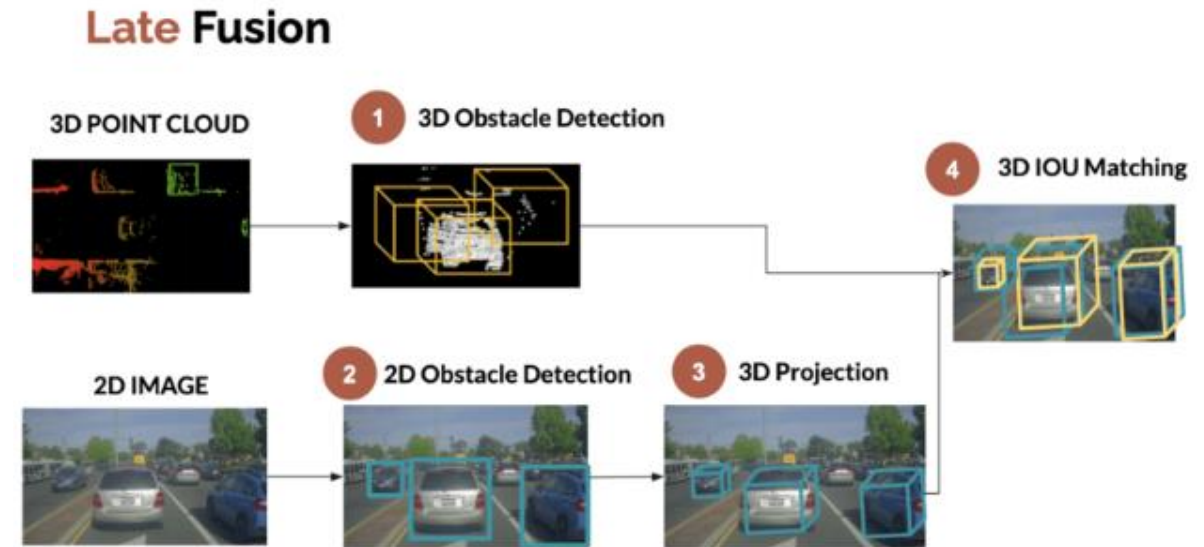




# Types of Sensor Fusion – Mid-level

## Mid-Level Fusion (Feature-Level Fusion)

- Extracted features from each sensor (e.g., **edges in an image or clusters in LiDAR data**) are combined.
- Fusion occurs at the level of processed data representations.
- Examples:
  - Combining object detections from cameras and depth information from LiDAR to **classify and localize objects**.
  - Using radar speed readings and image-based tracking for better object trajectory prediction.
- Pros:
  - Reduces data size compared to low-level fusion.
  - Easier to handle than raw data.
- Cons:
  - Loss of detailed information.
  - Requires effective feature extraction techniques.

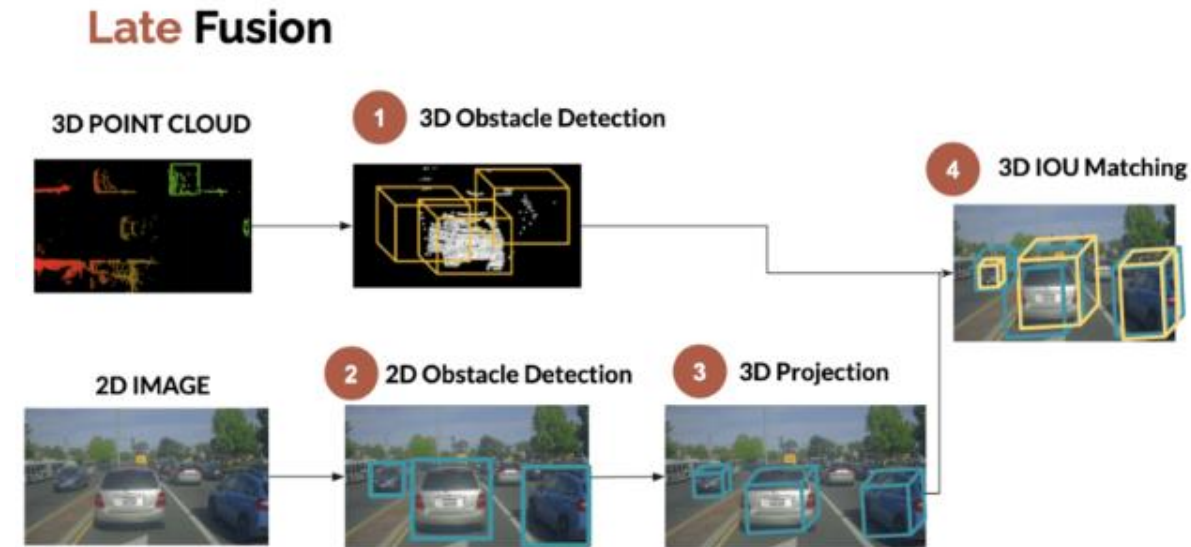


Fusing PCL data with camera image [2]

# Types of Sensor Fusion – Example Mid-level

Combining 3D Bounding boxes from LiDAR point clouds and camera pixel data

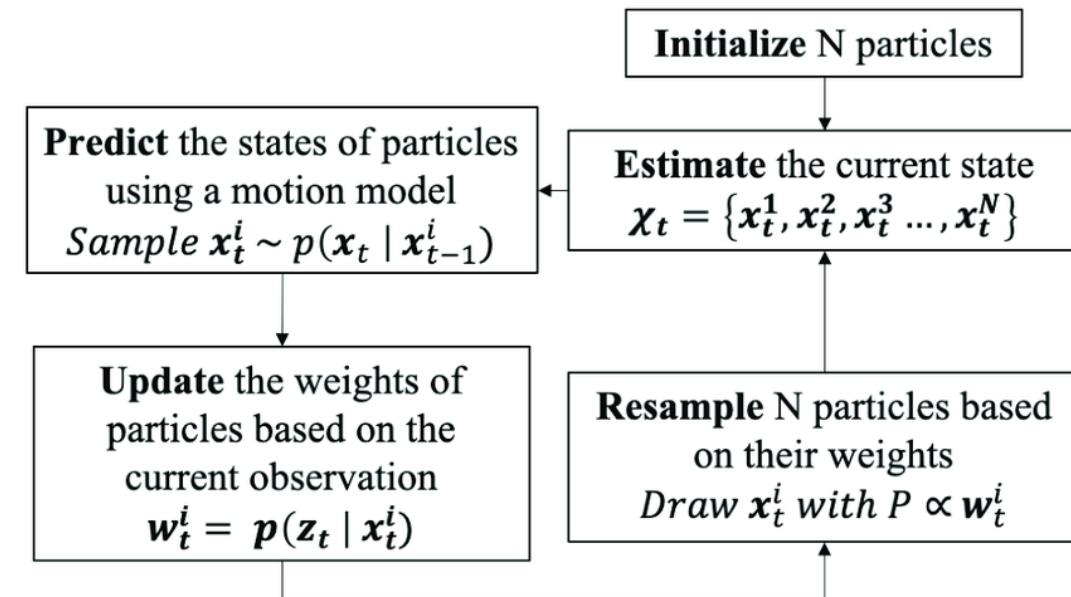
- 3D bounding boxes in LiDAR
  - Unsupervised 3D machine learning
  - Deep learning
- 3D obstacle detection via camera
  - Camera calibration
  - Depth map
- Intersection over Union (IoU) Matching
  - If the bounding boxes from camera and LiDAR overlap, in 2D or 3D, we consider that obstacle to be the same



Fusing PCL data with camera image [2]

## Other Mid-Level Fusion Techniques

- Kalman Filters:
  - Recursive estimation for fusing features like position & velocity
- Particle filters:
  - Non-linear non-Gaussian approach for tracking and estimation
  - Example: fusing radar and visual features for object tracking
- Principal Component Analysis:
  - Reduces dimensionality of fused features while preserving variance
  - Example: Fusing multiple-camera views for 3D reconstruction
- Neural Networks:
  - CNNs for images, LSTMs for temporal features
  - Example: fusing image and LIDAR features for object detection

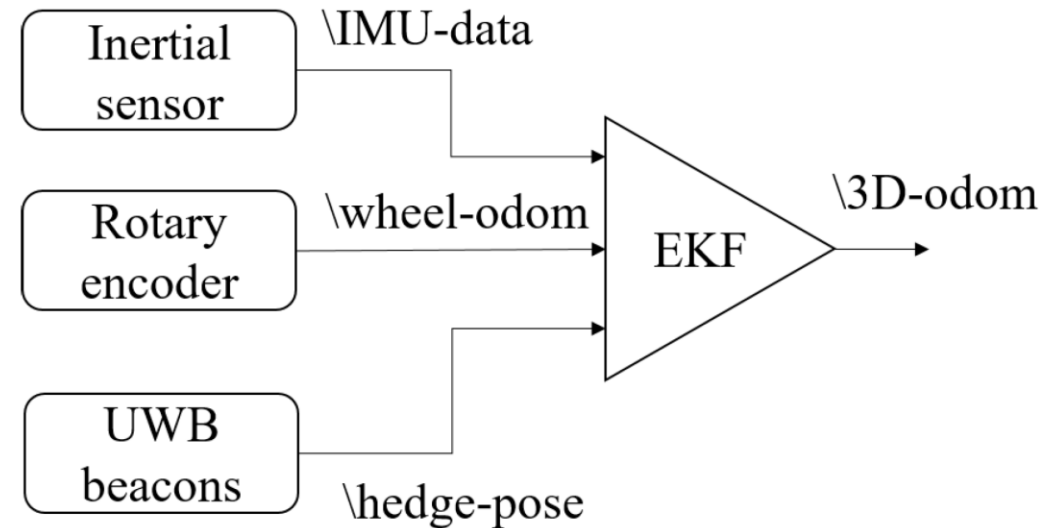




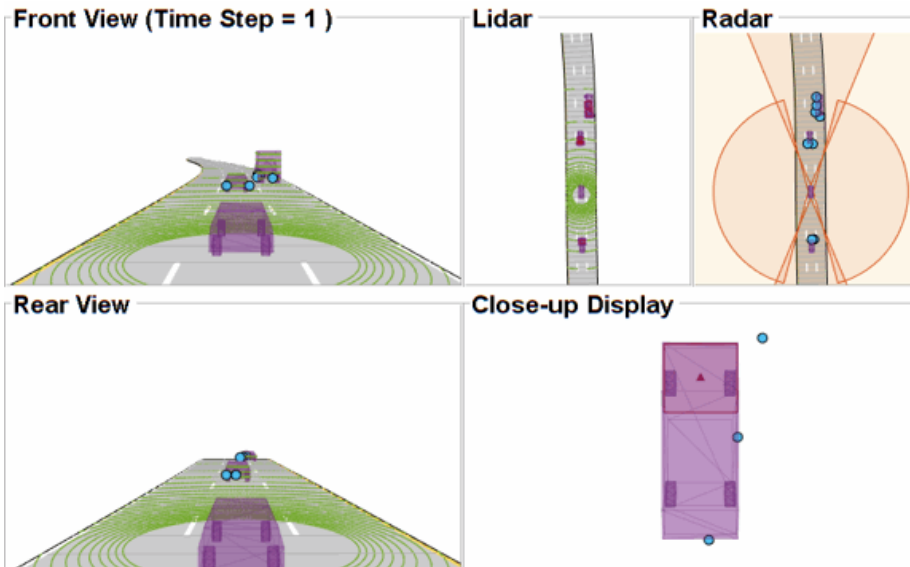
# Sensors: Measurement, Process and Noise

Goal: You are a robot

1. Need to know your own position, velocity etc. – **state (6D pose)**
2. Need to know your surroundings, e.g. **other cars state**



Position, velocity and uncertainty!



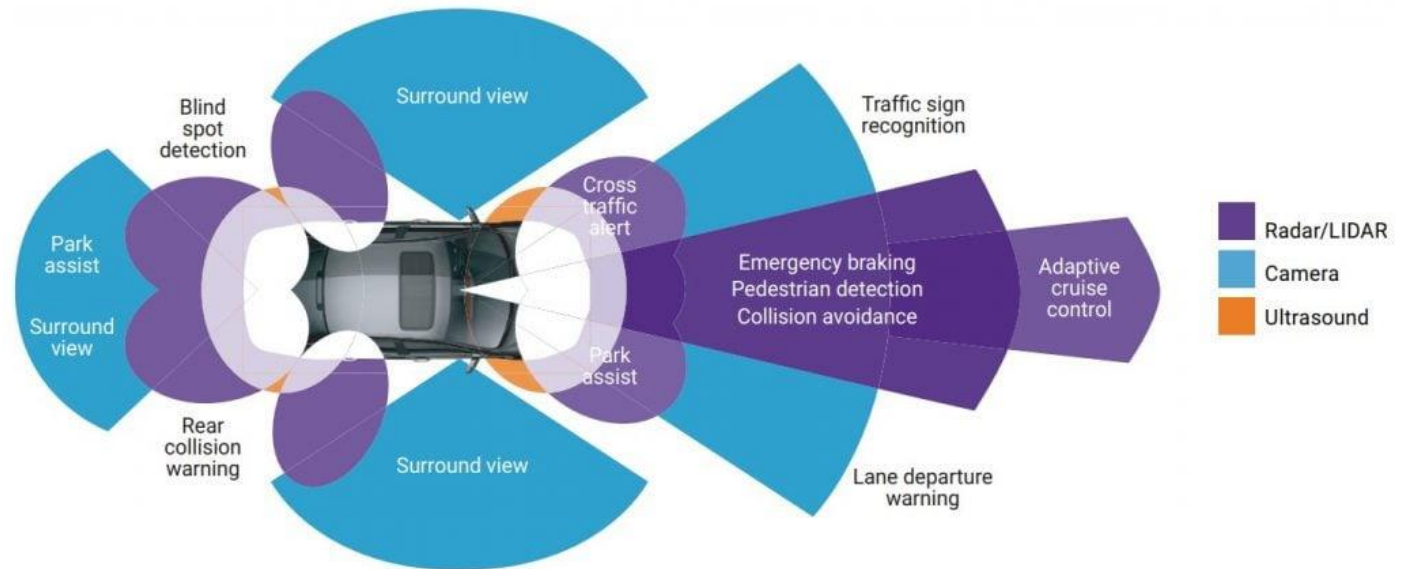
# Sensors: Measurement, Process and Noise

Have a sensor :

1. Raw data is available, or some kind of filter has been applied
2. You want accuracy

Have multiple sensors:

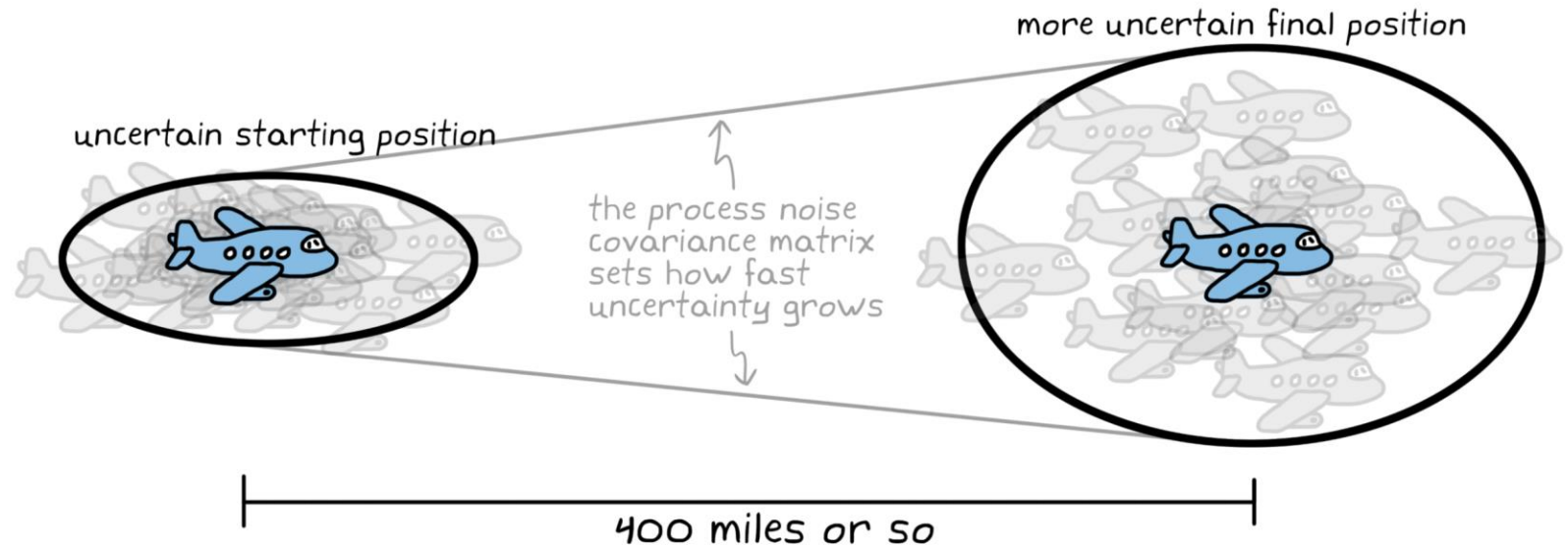
1. One sensor gives one kind of information only for example, radar give speed
2. Want additional information fused to get a full estimate



# Sensors: Measurement, Process and Noise

Have problems:

1. Less information
2. Sensor inefficient

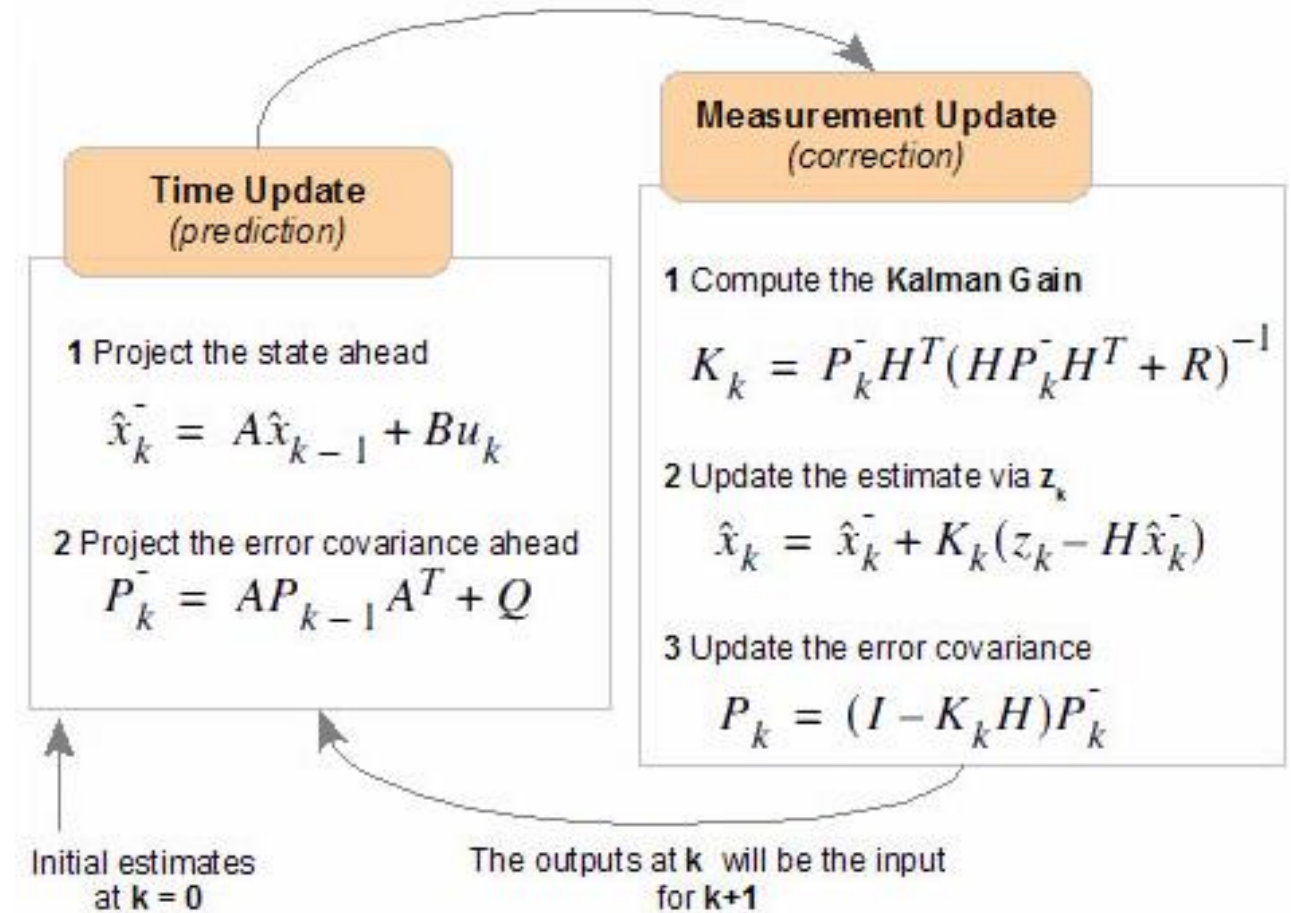
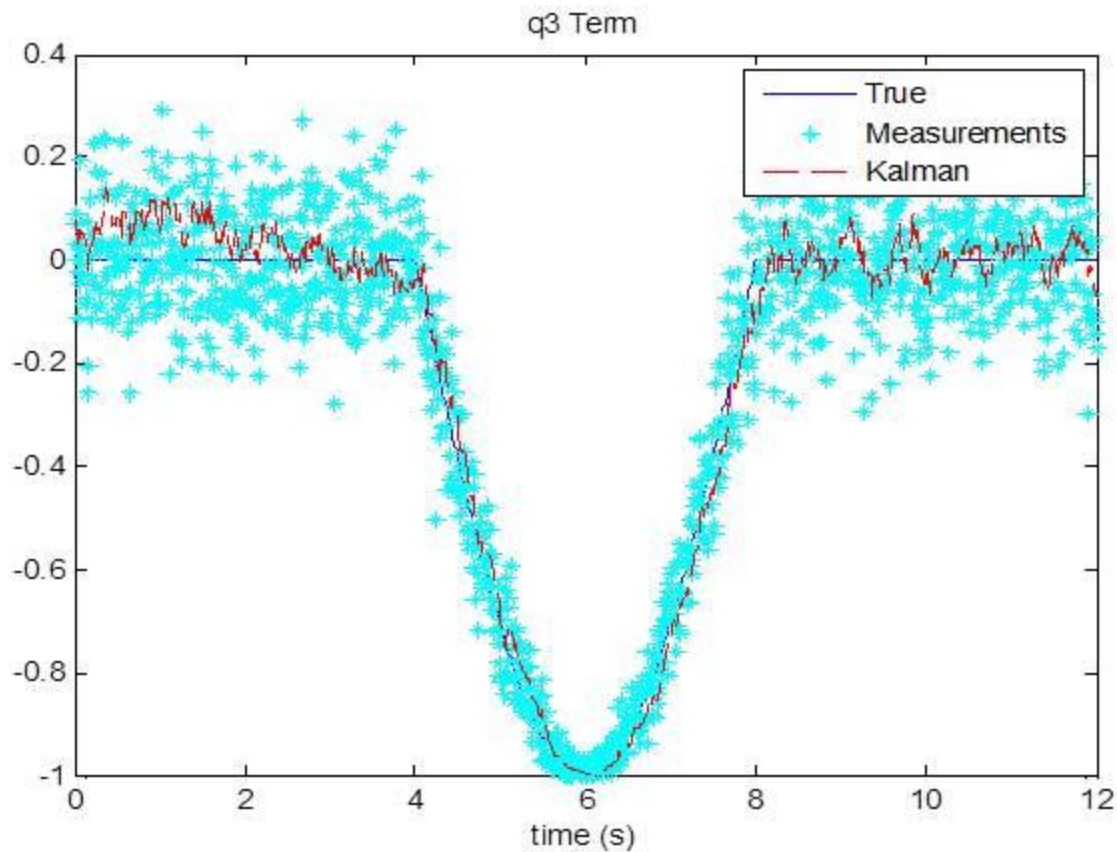


That's why Kalman Filters!

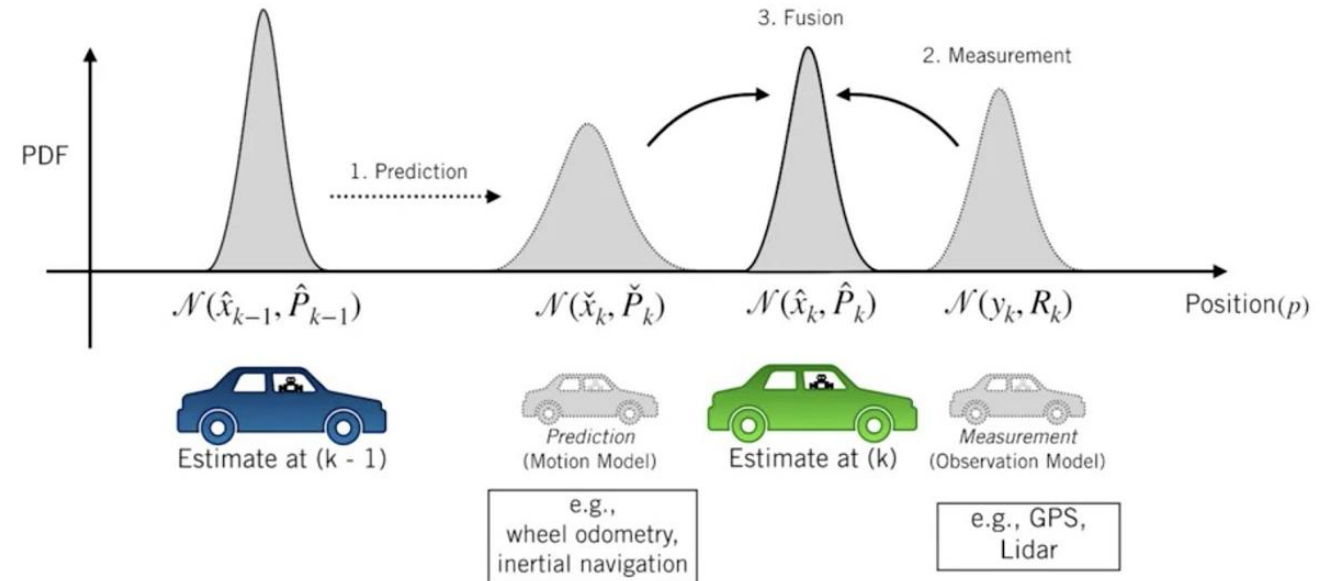
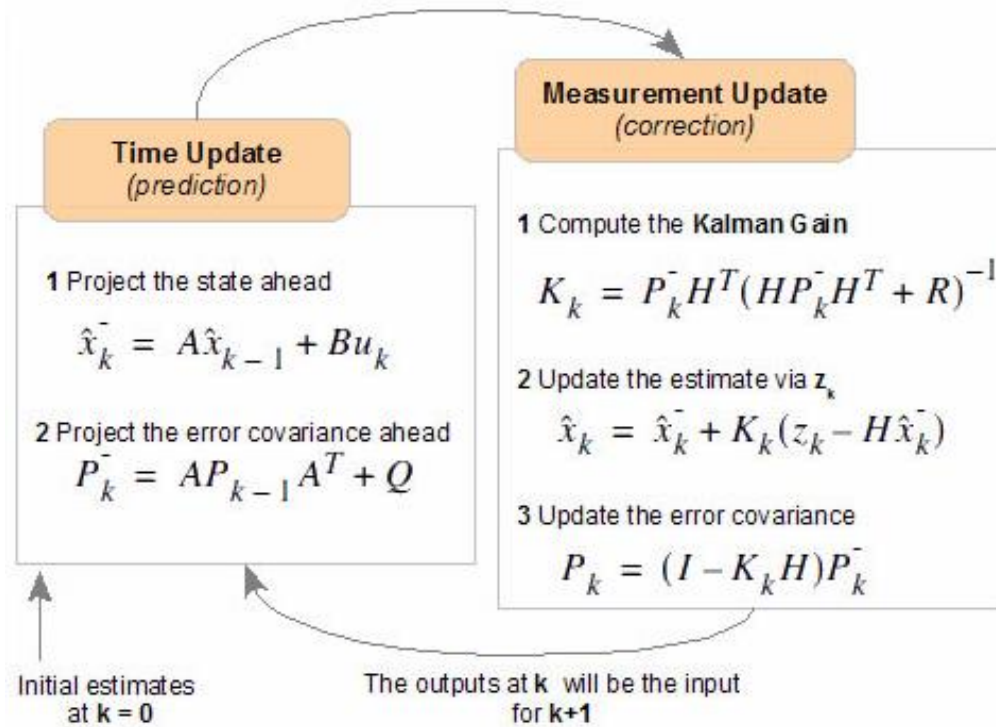
Position, velocity and uncertainty!



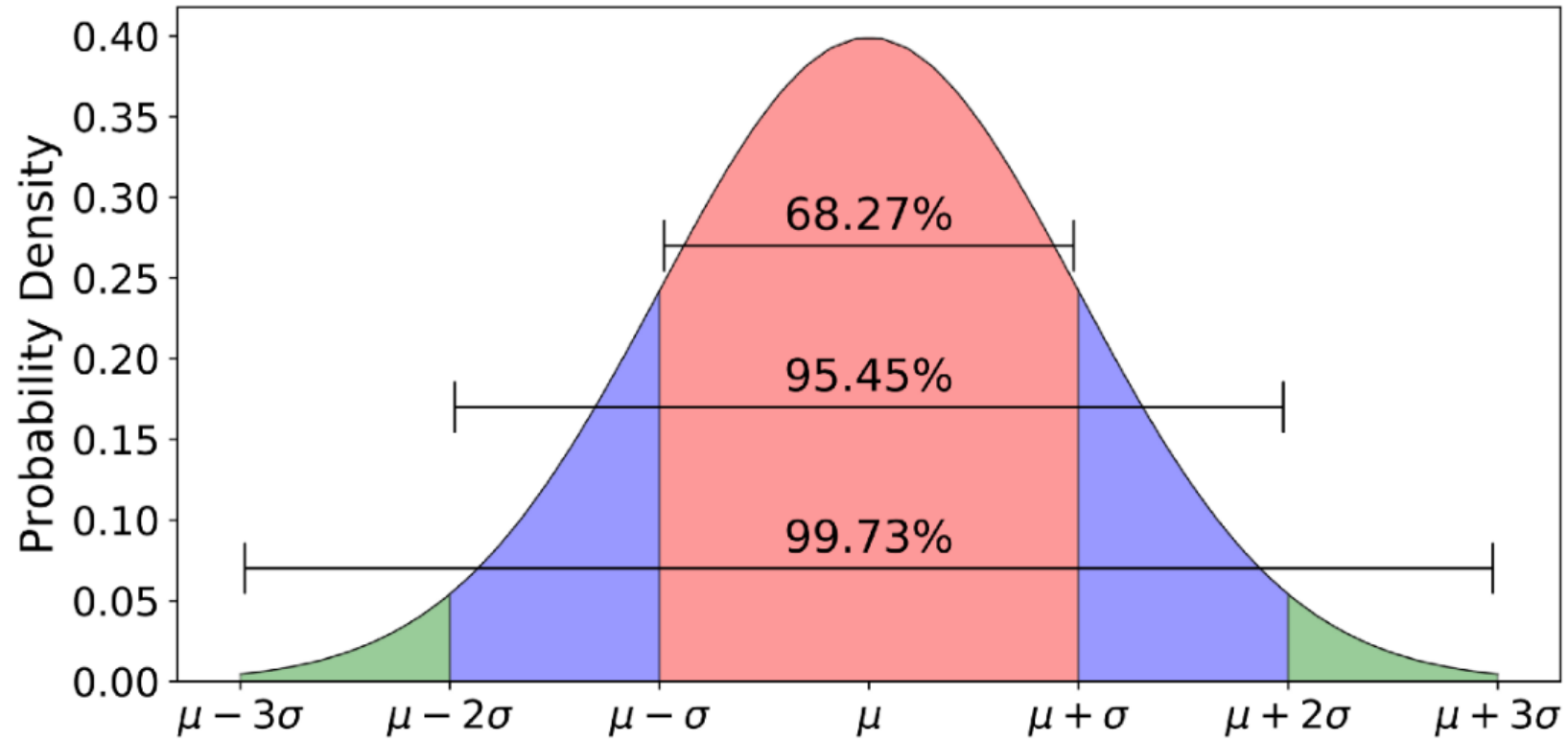
# Discussion: Kalman Filters



# Discussion: Kalman Filters

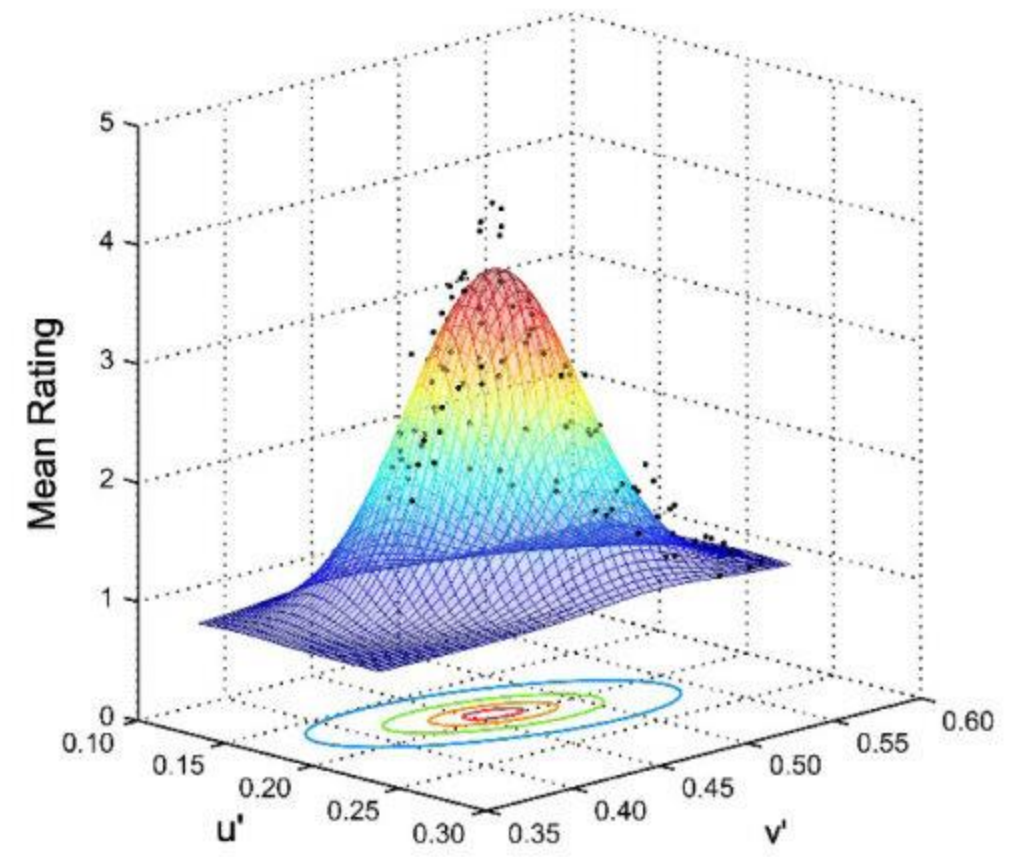
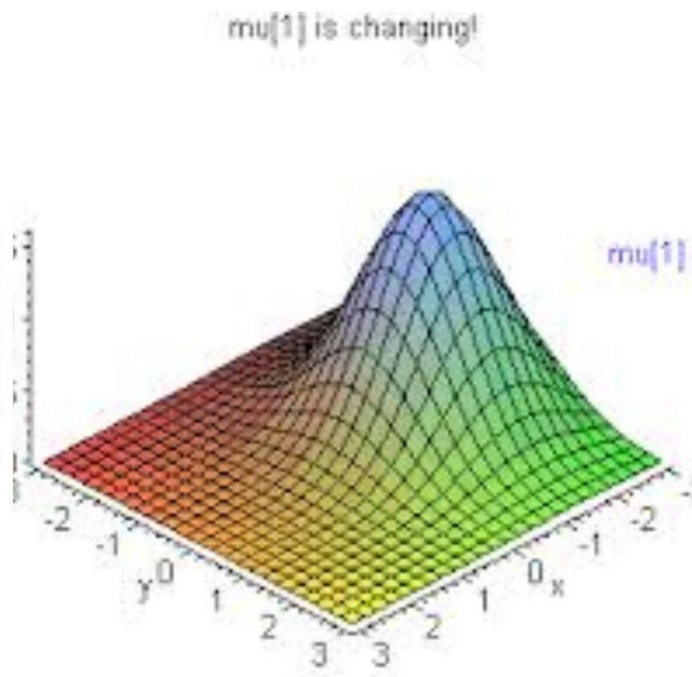
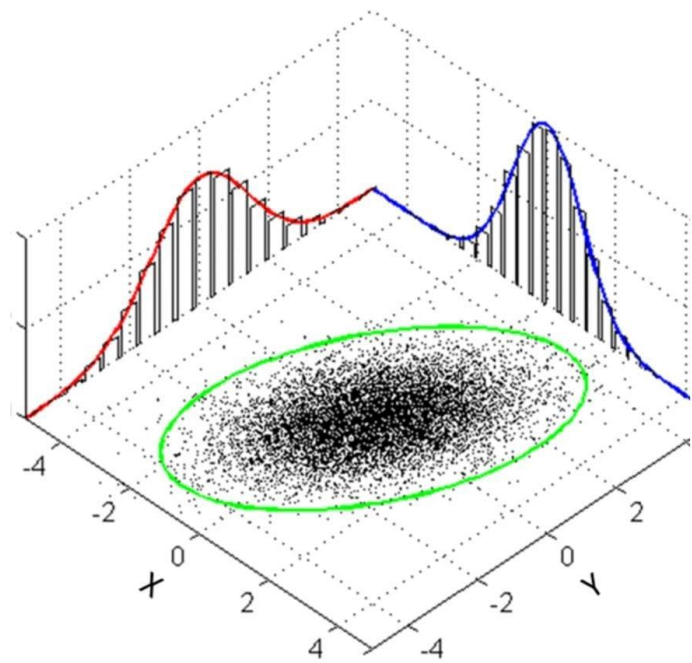


## Discussion: Kalman Filters (Gaussian Noise)





# Discussion: Kalman Filters



## KFs: Measurement, Process and Noise

- **Linear system model:** The system dynamics (how the state evolves over time) and the measurement model (how the state is observed) must be represented by linear equations with matrices  $F$  (state transition),  $H$  (observation), and  $B$  (control input).
- **Gaussian noise assumptions:** Both the process noise (system uncertainty) and measurement noise (sensor uncertainty) must be assumed to be white Gaussian noise with known covariance matrices ( $Q$  and  $R$  respectively).
- **Known initial conditions:** The initial state estimate ( $\hat{x}$ ) and its corresponding error covariance matrix ( $P$ ) must be provided to start the filtering process.

# KFs: Measurement, Process and Noise

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}u_k + \mathbf{w}_k$$

$\mathbf{x}_k = [x_k; \dot{x}_k]$ : State vector at time  $k$  (position and velocity).

$\mathbf{A}$ : State transition matrix.

$\mathbf{B}$ : Control input matrix.

$u_k$ : Control input (constant here).

$\mathbf{w}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{Q})$ : Process noise with covariance  $\mathbf{Q}$ .

$$\mathbf{z}_k = \mathbf{H}\mathbf{x}_k + \mathbf{v}_k$$

$\mathbf{z}_k = [z_{k,1}; z_{k,2}]$ : Measurement vector (noisy position and velocity).

$\mathbf{H}$ : Measurement matrix.

$\mathbf{v}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{R})$ : Measurement noise with covariance  $\mathbf{R}$ .

% state equations, discrete system

muQ = 0;

Q = [0.01 0; 0 0.03]; % we need to know this: noise covariance of state

A = [1.0000 0.0010; 0 1.0000];

B = 1.0e-03 \* [0.0005; 1.0000];

wk = normrnd(muQ,Q);

% output equations

H = [1 0; 0 1];

muR = 0;

R = [0.5 0; 0 0.5]; % we need to know this: noise covariance of output

vk = normrnd(muR, R);

% simulation parameters:

t0 = 0;

dt = 0.001;

tend = 10;

x = [0 0]'; % initial state

z = x; % initial measurement;

len = length(t0:dt:tend);

xf = zeros(len, length(x));

xf(1,:) = x';

zf(1,:) = z';

u = 1;

% apriori x

xhat\_minus = [0 0]';

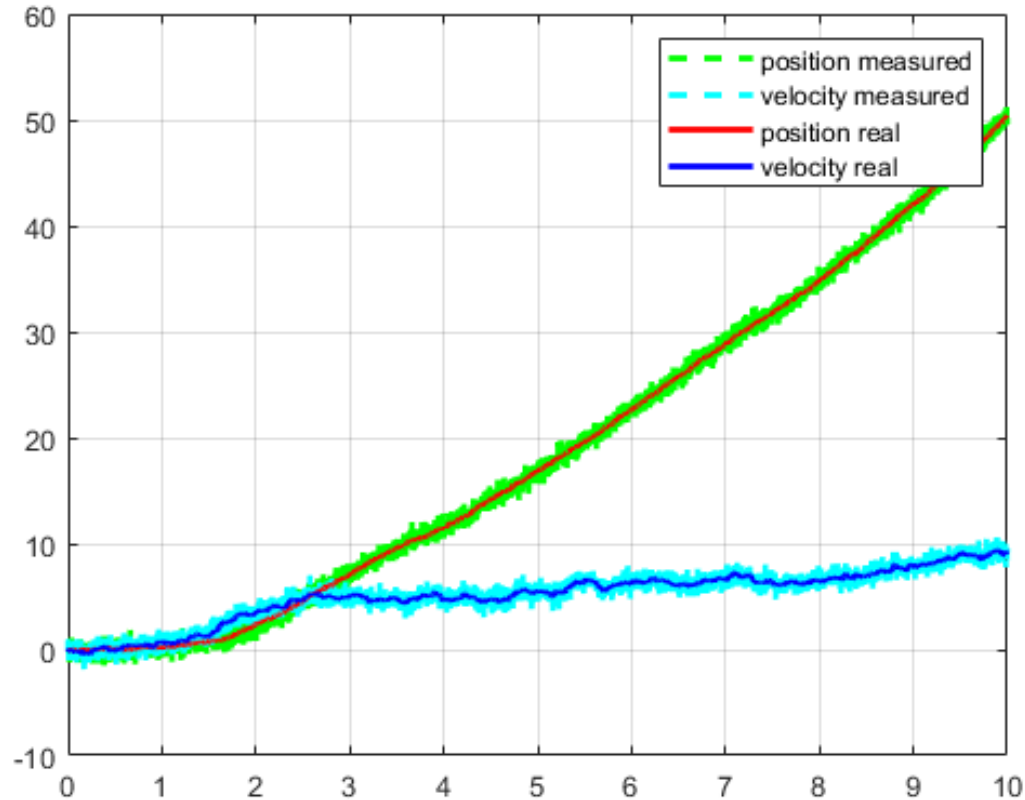
xhat = [0 0]';

xhatf = xf;

P = [0.1 0.2; 0.3 0.1];



# KFs: Measurement, Process and Noise



% state equations, discrete system

muQ = 0;

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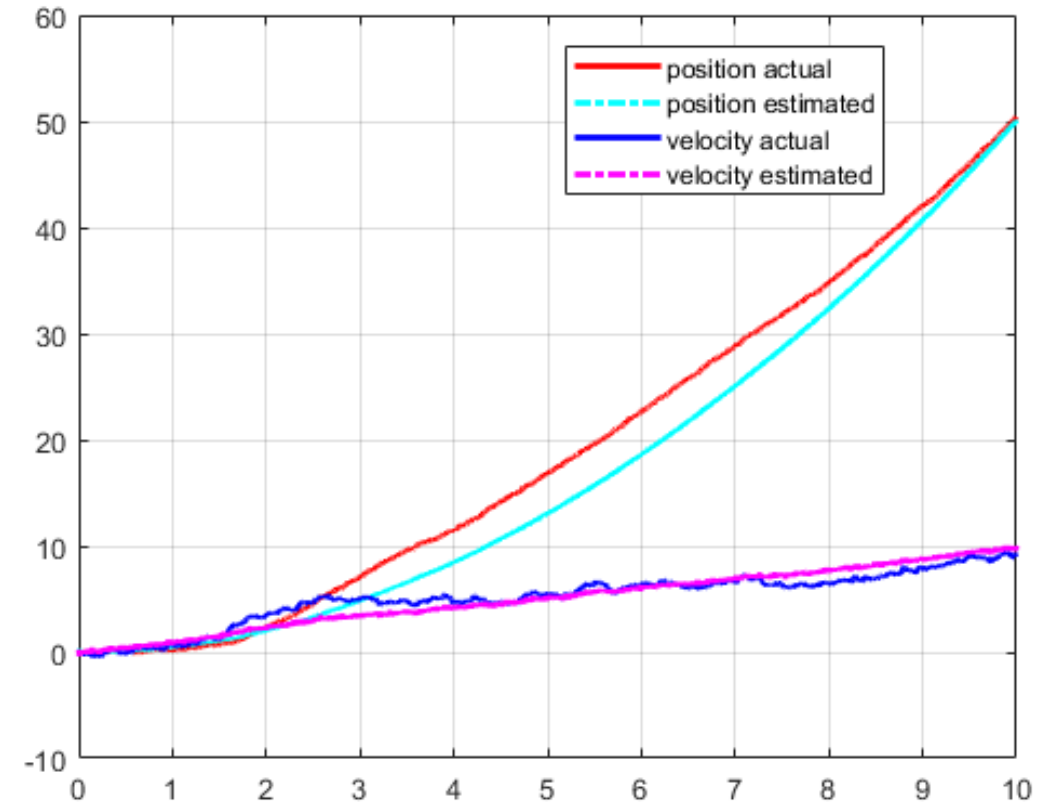
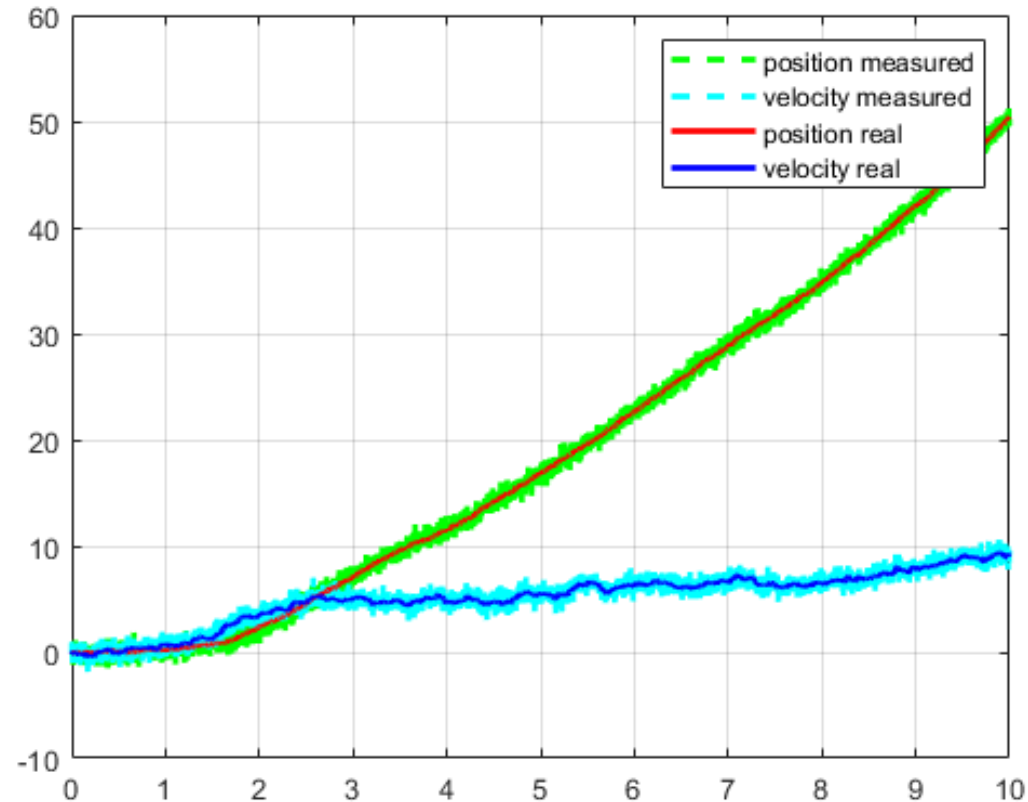
xhat\_minus = [0 0]';

xhat = [0 0]';

xhatf = xf;

P = [0.1 0.2; 0.3 0.1];

# KFs: Measurement, Process and Noise



# Sliding Window State Estimation using Optimization

State estimation can also be formulated as a simple linear/quadratic optimization problem! Here, we know the system dynamics, so we try to find the best state estimate that the minimizes some error metric over a window.

$$y = \underbrace{x_1 \sin(t) + x_2 \sin(2t) + x_3 \sin(3t) + x_4 \sin(4t) + x_5 \sin(5t) + x_6 \sin(6t)}_{\text{known}} + \text{noise}$$

i.e.,  $y = Ax + \text{noise}$ , with  $x = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6]$  ← unknown

Let us predict  $x$ , so denote prediction by  $\hat{x}$

$$e = y - A\hat{x}$$

known

$$\|e\|_1 := \sum_{i=1}^n |e_i|$$

derived from  $L_1$

$$\|e\|_2 := \sum_{i=1}^n \|e_i^2\|$$

derived from  $L_2$

$$\|e\|_\infty := \max_i |e_i|$$

derived from  $L_\infty$

# Sliding Window State Estimation using Optimization

Depending on the nature of the objective function i.e, whether it is L1, L2 or  $L_\infty$  norm on  $x$ , we can use LP, QP or LP respectively.

$$\|e\|_1 := \sum_{i=1}^n |e_i|$$

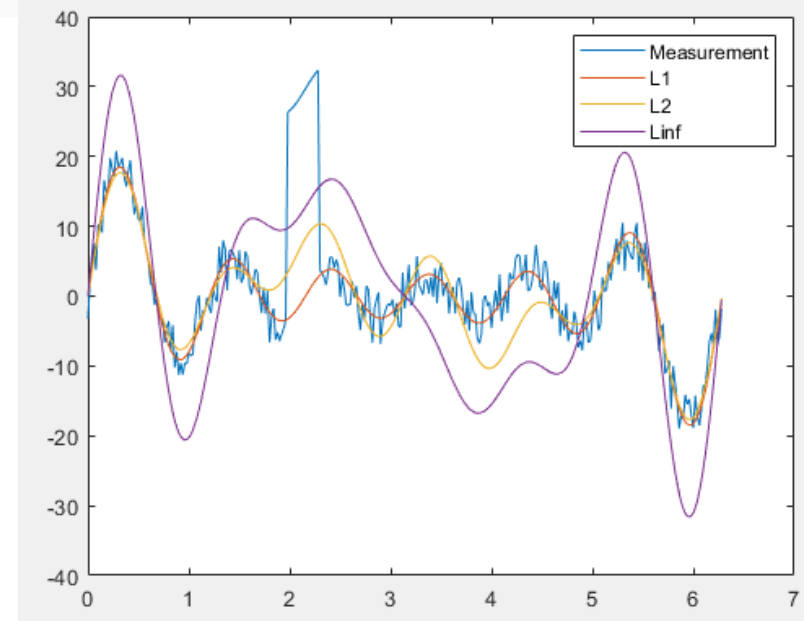
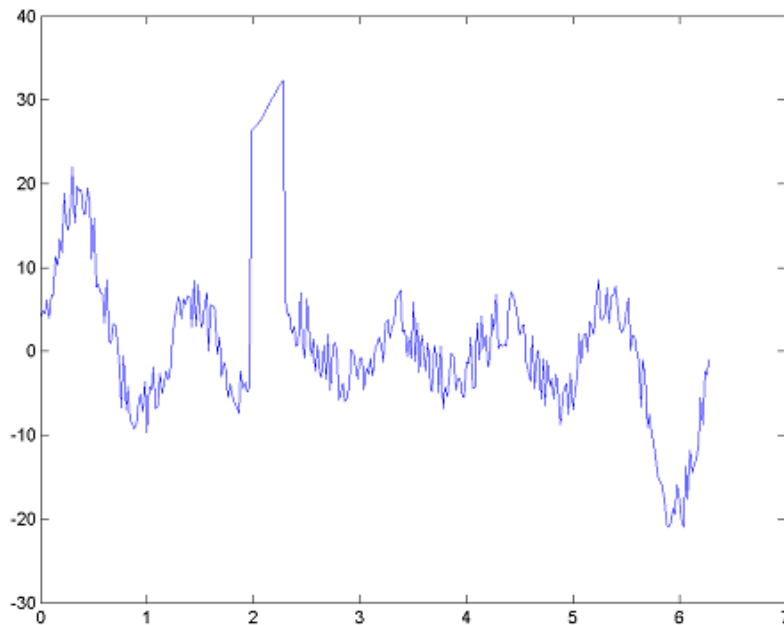
derived from  $L_1$

$$\|e\|_2 := \sum_{i=1}^n \|e_i^2\|$$

derived from  $L_2$

$$\|e\|_\infty := \max_i |e_i|$$

derived from  $L_\infty$





# Implementing KFs in Python and ROS2

```
import rclpy
from rclpy.node import Node
from std_msgs.msg import Float32, Float32MultiArray
import numpy as np
from filterpy.kalman import KalmanFilter
```

```
class KalmanFilterNode(Node):
    def __init__(self):
        super().__init__('kalman_filter_node')

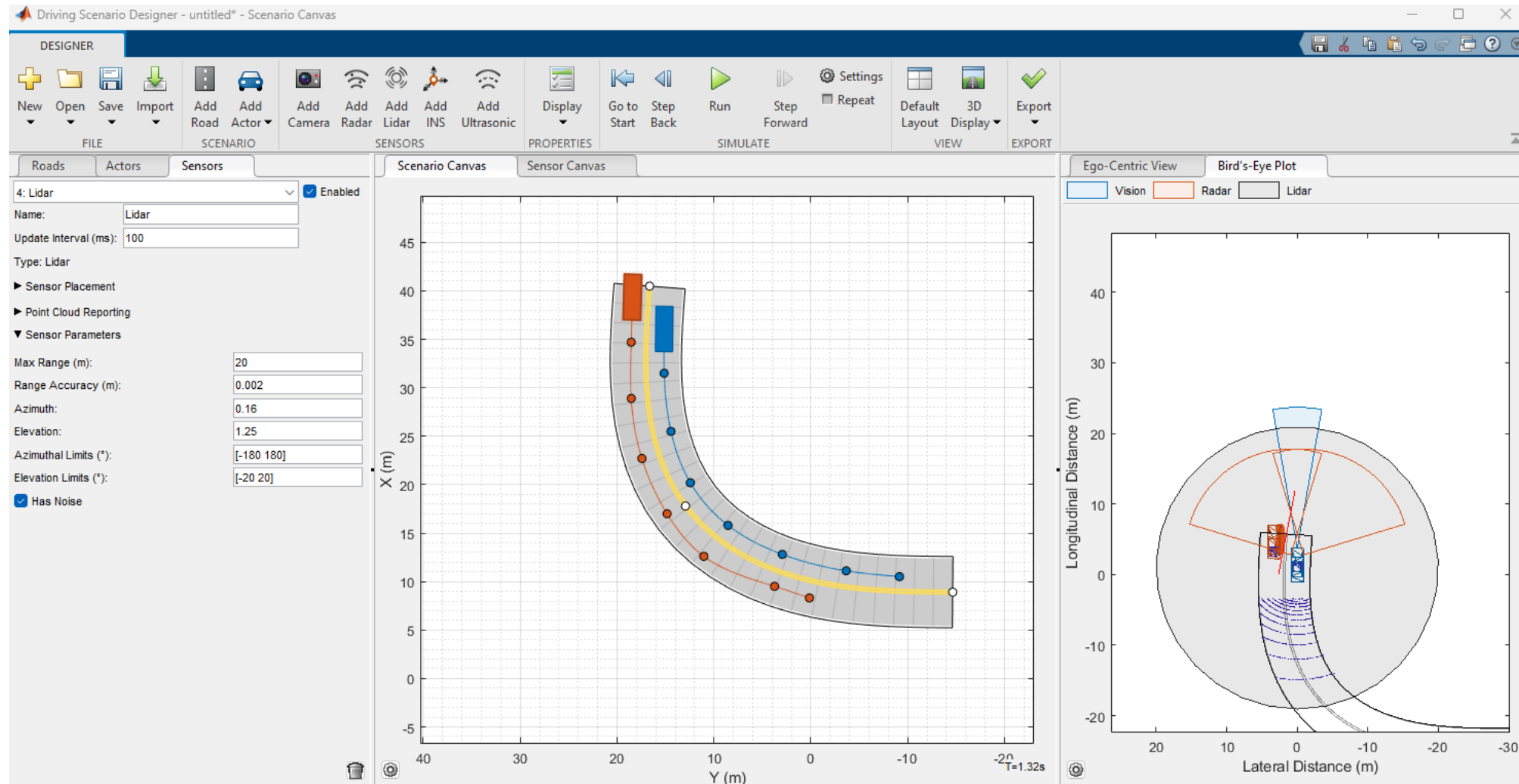
        # Subscriber for measurements
        self.subscription = self.create_subscription(
            Float32, 'measurement', self.measurement_callback, 10)

        # Publisher for estimated state
        self.state_publisher = self.create_publisher(Float32MultiArray, 'estimated_state', 10)

        # Kalman filter initialization
        dt = 0.1 # Time step
        self.kf = KalmanFilter(dim_x=2, dim_z=1)
        self.kf.x = np.array([0, 0]) # Initial state: [position, velocity]
        self.kf.F = np.array([[1, dt], [0, 1]]) # State transition matrix
        self.kf.H = np.array([[1, 0]]) # Measurement matrix
        self.kf.P = np.eye(2) * 500 # Covariance matrix (large initial uncertainty)
        self.kf.R = 1 # Measurement noise covariance
        self.kf.Q = np.array([[0.1, 0], [0, 0.1]]) # Process noise covariance
```

# Radars, Cameras and Lidars

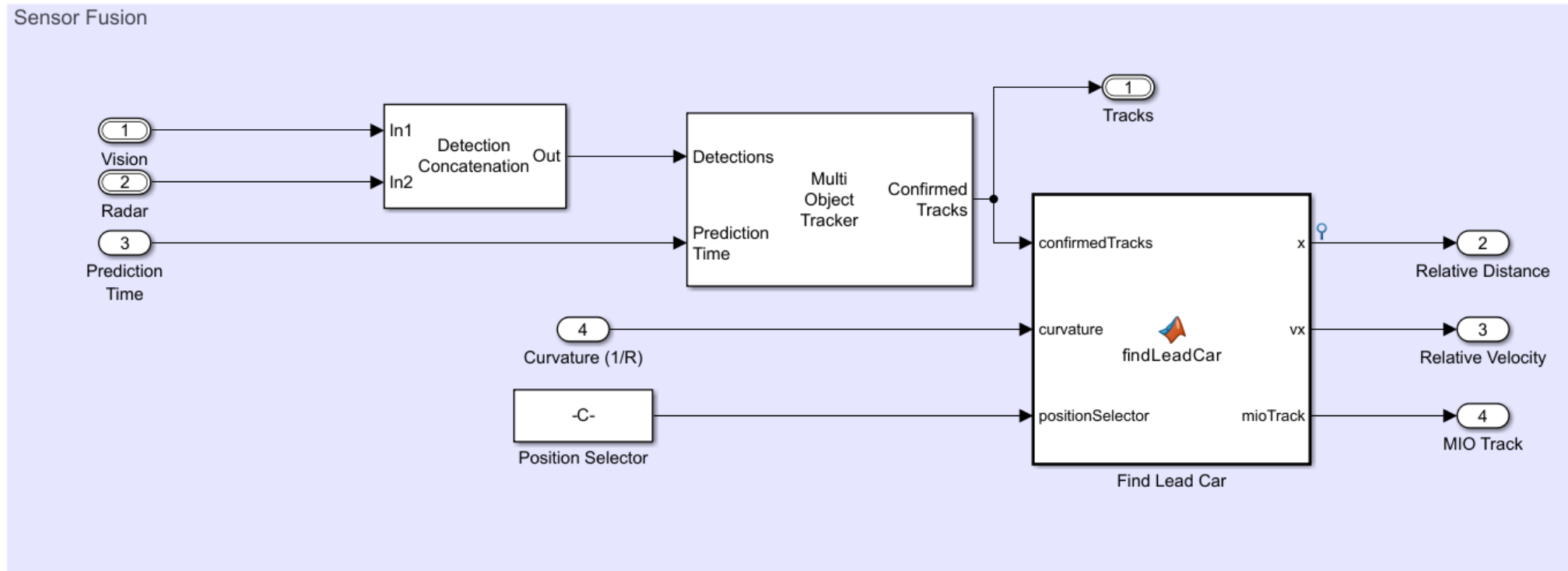
## Creating Synthetic Data in MATLAB



[3] Codes available at [https://github.com/karishmapatnaik/sensor\\_fusion/tree/main](https://github.com/karishmapatnaik/sensor_fusion/tree/main)

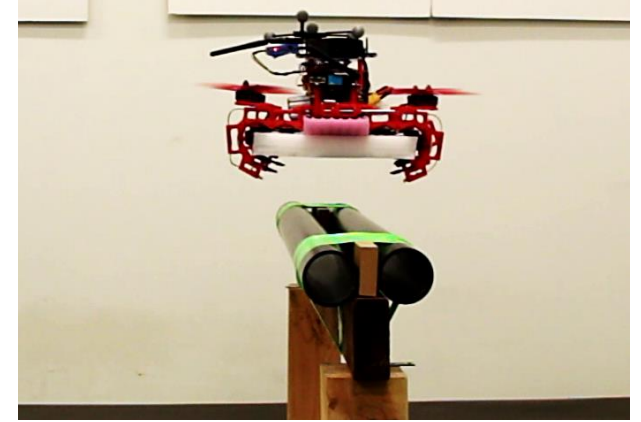
[4] <https://www.mathworks.com/help/driving/ug/create-driving-scenario-interactively-and-generate-synthetic-detections.html>

# Creating Synthetic Data in MATLAB Visualization and Fusing



[3] Codes available at [https://github.com/karishmapatnaik/sensor\\_fusion/tree/main](https://github.com/karishmapatnaik/sensor_fusion/tree/main)

[4] <https://www.mathworks.com/help/driving/ug/create-driving-scenario-interactively-and-generate-synthetic-detections.html>



# Questions

