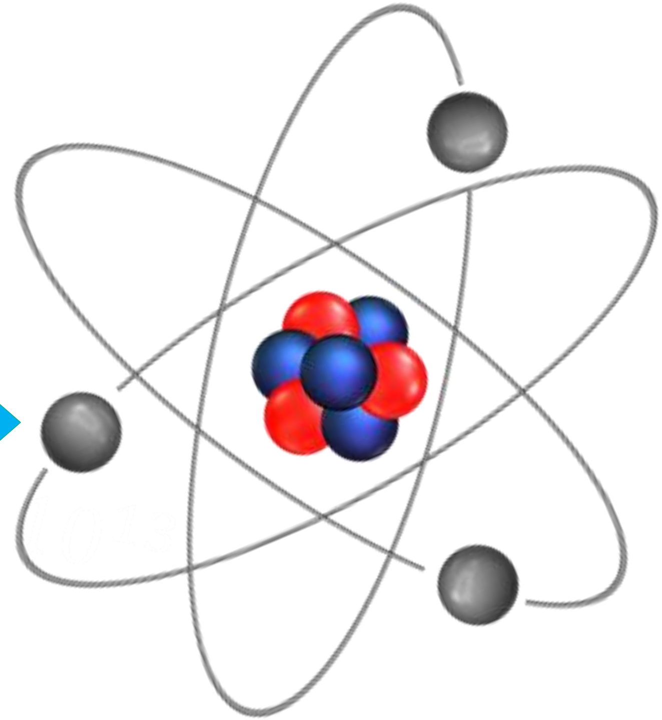
POTENCIACIÓN

¿SABÍAS QUE...?

$$q_{\bar{e}} = -1,6 \, . \, 10^{-19} \, . \, C$$



DEFINICIÓN

EXPONENTE

$$a^n = P$$

BASE

POTENCIA

Talque : $n \in Z \begin{cases} Z^+ \\ \{0\} \\ Z^- \end{cases}$

EXPONENTE ENTERO POSITIVO

$$a^n = \begin{cases} a \quad , & n = 1 \\ \underbrace{a.a.a\dots a}_{n\ veces}, & n > 1 \end{cases}$$

Ejemplo:

$-3^2 = -3.3 = -9$

$(-3)^2 = (-3).(-3) = 9$

EXPONENTE CERO

$a^0 = 1 \ / \ a \neq 0$

Ejemplo:

$$\frac{4^0}{3} = \frac{1}{3}$$

$$\left(\frac{4}{3}\right)^0 = 1$$

Nota:

$\boxed{0^0 = no\ definido}$

EXPONENTE NEGATIVO

$$a^{-n} = \left(\frac{1}{a}\right)^n \ / \ a \neq 0 \wedge n \in Z^+$$

Ejemplo:

$$5^{-3} = \left(\frac{1}{5}\right)^3 = \frac{1}{125}$$

$$\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2 = \left(\frac{4}{3}\right).\left(\frac{4}{3}\right)$$

$$= \frac{16}{9}$$

Nota:

$\boxed{0^{-1} = no\ definido}$

TEOREMAS DE POTENCIACIÓN

I. $a^m . a^n = a^{m+n}$

Ejemplo:

$2^{7-x} . 2^{x-4} = 2^{7-\cancel{x}+\cancel{x}-4}$

$= 2^3 = 8$

II. $\dfrac{a^m}{a^n} = a^{m-n}$

Ejemplo:

$\dfrac{5^{x-8}}{5^{x-12}} = 5^{\cancel{x}-8-\cancel{x}+12}$

$= 5^4 = 625$

III. $a^n . b^n = (a . b)^n$

Ejemplo:

$4^3 . 5^3 = 20^3 = 8000$

IV. $\dfrac{a^n}{b^n} = \left(\dfrac{a}{b}\right)^n$

Ejemplo:

$\dfrac{20^3}{5^3} = 4^3 = 64$

V. $(a^m)^n = a^{m . n}$

Ejemplo:

$$(2^{12})^{5/6} = 2^{12\left(\frac{5}{6}\right)} = 2^{2(5)} = 2^{10} = 1024$$

EJERCICIOS DE APLICACIÓN

1. **Simplificar:** $\mathbf{M = \dfrac{15^2 \cdot 25 \cdot 49}{35^2 \cdot 45^2}}$

Resolución

Descomponer en forma canónica las bases

$$M = \frac{15^2 \cdot 25 \cdot 49}{35^2 \cdot 45^2} = \frac{3^2 \cdot \cancel{5^2} \cdot \cancel{5^2} \cdot \cancel{7^2}}{\cancel{5^2} \cdot \cancel{7^2} \cdot 3^{2(2)} \cdot \cancel{5^2}}$$

$$M = \frac{3^2}{3^4} = \frac{9}{81} \qquad \therefore\ M = \frac{1}{9}$$

$$2.\ \ Simplificar:\quad G=\frac{x^2.x^4.x^6\ldots n\,factores}{x^3.x^5.x^7\ldots n\,factores}\quad ;\quad x\neq 0$$

Resolución

Seleccionamos

$$G=\frac{x^2.x^4.x^6\ldots n\,factores}{x^3.x^5.x^7\ldots n\,factores}$$

$$G=\underbrace{x^{-1}.x^{-1}.x^{-1}\ldots x^{-1}}_{\textbf{n veces}}$$

$$G=(x^{-1})^n \longrightarrow G=x^{-n}$$

$$3. Simplificar: E = \frac{2^{n+1} + 2^{n+2} + 2^{n+3} + 2^{n+4}}{2^{n-1} + 2^{n-2} + 2^{n-3} + 2^{n-4}}$$

Resolución

Factorizamos:

$$E = \frac{2^{n+1}\left(2^{0} + 2^{1} + 2^{2} + 2^{3}\right)}{2^{n-4}\left(2^{3} + 2^{2} + 2^{1} + 2^{0}\right)}$$

$$E = 2^{n+1-n+4} = 2^{5} = 32$$

14. Efectuar:

$$(a^2b^3c^4)(a^{-5}b^{-6}c^{-7})(a^8b^9c^{10})$$

RESOLUCIÓN

Aplicamos la propiedad asociativa de la multiplicaión

$$= \left(\boldsymbol{a^2}.\, b^3.\, \boldsymbol{c^4}\right)\left(\boldsymbol{a^{-5}}.\, b^{-6}.\, \boldsymbol{c^{-7}}\right)\left(\boldsymbol{a^8}.\, b^9.\, \boldsymbol{c^{10}}\right)$$

$$= \left(\boldsymbol{a^2}.\, \boldsymbol{a^{-5}}.\, \boldsymbol{a^8}\right)(b^3.\, b^{-6}.\, b^9)\left(\boldsymbol{c^4}.\, \boldsymbol{c^{-7}}.\, \boldsymbol{c^{10}}\right)$$

$$= \boldsymbol{a^{2-5+8}}.\, b^{3-6+9}.\, \boldsymbol{c^{4-7+10}}$$

$$= \boldsymbol{a}^{\boldsymbol{5}}.\, b^{\boldsymbol{6}}.\, \boldsymbol{c}^{\boldsymbol{7}}$$

NIVEL INTERMEDIO

Si: $b^a = 5 \quad \wedge \quad a^{-b} = \frac{1}{2}$ ⟹ $a^b = 2$

Calcular: $E = a^{b^{a+1}}$

RESOLUCIÓN

$$E = a^{b^{a+1}} = a^{b^a \cdot b^1}$$

$$E = \left(a^b\right)^{b^a} = (2)^5 \Rightarrow E = 32$$

<u>**NIVEL INTERMEDIO**</u>

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Reducir: $$E = \frac{15^{20}.35^{10}.10^{30}}{12^{20}.25^{15}.49^{5}.5^{30}}$$

RESOLUCIÓN

$$E = \frac{15^{20}.35^{10}.10^{30}}{12^{20}.25^{15}.49^{5}.5^{30}}$$

Descomponer en forma canónica las bases

$$E = \frac{3^{20}.5^{20}.5^{10}.7^{10}.2^{30}.5^{30}}{2^{2(20)}.3^{20}.5^{2(15)}.7^{2(5)}.5^{30}} = \frac{2^{30}}{2^{40}}$$

$$\therefore\ E = 2^{-10}$$