PROFESOR:

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TEMA: **LEYES DE EXPONENTES**

(EJERCICIOS)

NIVEL BÁSICO

①

Reducir: $M = \dfrac{2^{16}.16^{2}}{8^{8}}$

RESOLUCIÓN

Descomponer en forma canónica las bases

$$M = \frac{2^{16}.\ 2^{4(2)}}{2^{3(8)}} = \frac{2^{16+8}}{2^{24}}$$

$$\therefore\ M = 1$$

2

Simplificar: $J = \sqrt[x-2]{\dfrac{5^{x-2}+3^{x-2}}{5^{2-x}+3^{2-x}}}$

RESOLUCIÓN

Aplicamos el siguiente teorema.

$$\frac{a^{x}+b^{x}}{a^{-x}+b^{-x}} = (a.b)^{x}$$

En el ejercicio

$$J = \sqrt[\cancel{x-2}]{(5\,.\,3)^{\cancel{x-2}}} \Longrightarrow J = 15$$

Calcular : “A + B + C”

3

Si: $A = \sqrt[3]{9.\sqrt[3]{9.\sqrt[3]{9\ldots}}}$ $\quad B = \sqrt{132+\sqrt{132+\sqrt{132+\ldots}}}$ $\quad C = \sqrt[5]{\dfrac{64}{\sqrt[5]{\dfrac{64}{\sqrt[5]{\dfrac{64}{\ddots}}}}}}$

RESOLUCIÓN

$$A = \sqrt[3-1]{9} = \sqrt{9} \longrightarrow A = 3$$

$$132 = (11).\underset{\text{mayor}}{(12)} \longrightarrow B = 12$$

$$C = \sqrt[5+1]{64} = \sqrt[6]{64} \longrightarrow C = 2$$

$$\therefore\ A + B + C = 17$$

NIVEL INTERMEDIO

1

Reducir: $E=\dfrac{15^{20}.35^{10}.10^{30}}{12^{20}.25^{15}.49^{5}.5^{30}}$

RESOLUCIÓN

$$E=\frac{15^{20}.35^{10}.10^{30}}{12^{20}.25^{15}.49^{5}.5^{30}}$$

Descomponer en forma canónica las bases

$$E=\frac{3^{20}.5^{20}.5^{10}.7^{10}.2^{30}.5^{30}}{2^{2(20)}.3^{20}.5^{2(15)}.7^{2(5)}.5^{30}}=\frac{2^{30}}{2^{40}}$$

$$\therefore\ E=2^{-10}$$

NIVEL INTERMEDIO

②

Si: $b^a = 5 \quad \wedge \quad a^{-b} = \frac{1}{2}$ ➡ $a^b = 2$

Calcular: $E = a^{b^{a+1}}$

RESOLUCIÓN

$$E = a^{b^{a+1}} = a^{b^a \cdot b^1}$$

$$E = \left(a^b\right)^{b^a} = (2)^5$$ ➡ $E = 32$

EJERCICIOS DE APLICACIÓN

1. Simplificar: $\mathbf{M = \dfrac{15^2 \, . \, 25 \, . \, 49}{35^2 \, . \, 45^2}}$

Resolución

Descomponer en forma canónica las bases

$$M = \frac{15^2 \, . \, 25 \, . \, 49}{35^2 \, . \, 45^2} = \frac{3^2 \, . \, \cancel{5^2} \, . \, \cancel{5^2} \, . \, \cancel{7^2}}{\cancel{5^2} \, . \, \cancel{7^2} \, . \, 3^{2(2)} \, . \, \cancel{5^2}}$$

$$M = \frac{3^2}{3^4} = \frac{9}{81} \quad \Rightarrow \quad \therefore \; M = \frac{1}{9}$$

2. $Simplificar: \quad G = \dfrac{x^2.x^4.x^6 \ldots n\, factores}{x^3.x^5.x^7 \ldots n\, factores} \quad ; \quad x \neq 0$

Resolución

Seleccionamos

$$G = \frac{x^2.x^4.x^6 \ldots n\, factores}{x^3.x^5.x^7 \ldots n\, factores}$$

$$G = \underbrace{x^{-1}.\,x^{-1}.\,x^{-1} \ldots x^{-1}}_{\text{n veces}}$$

$$G = (x^{-1})^n \qquad \longrightarrow \qquad G = x^{-n}$$

$$3. Simplificar: E = \frac{2^{n+1} + 2^{n+2} + 2^{n+3} + 2^{n+4}}{2^{n-1} + 2^{n-2} + 2^{n-3} + 2^{n-4}}$$

Resolución

Factorizamos:

$$E = \frac{2^{n+1}\left(2^{0} + 2^{1} + 2^{2} + 2^{3}\right)}{2^{n-4}\left(2^{3} + 2^{2} + 2^{1} + 2^{0}\right)}$$

$$E = 2^{n+1-n+4} = 2^{5} = 32$$

3

$$\text{Reducir:} \quad M = \left(\sqrt[128^{5^n}]{\sqrt[4]{7^{2^{5^n+3}}}} \right)^{64^{5^n}}$$

a) 7 b) 1 c) 49 d) 343 e) 2401

RESOLUCIÓN

Si: $n = 0 \quad \rightarrow \quad 5^n = 1$ Reemplazamos:

$$M = \left(\sqrt[128^{1}]{\sqrt[4]{7^{2^{1+3}}}} \right)^{64^{1}} = \sqrt[128.(4)]{7^{(2^3.2).(64)}}$$

$$M = 7^{\frac{8}{4}} = 7^{2}$$

$$M = 49$$

NIVEL AVANZADO

①

Si: $x^{x^x} = 2$ Calcular: $M = x^{2x^x + x^{x+x^x+x^{x+x^x}}}$

RESOLUCIÓN

$$M = x^{\boxed{2x^x} + \boxed{x^{x+x^x+x^{x+x^x}}}}$$

$$M = x^{2 \cdot x^x} \cdot x^{x^{x} + x^x + x^{x+x^x}}$$

$$M = \left(x^{x^x}\right)^2 \cdot x^{x^x \cdot x^{x^x} \cdot x^{x^{x+x^x}}}$$

$$M = (2)^2 \cdot 2^{2 \cdot x^{x^x} \cdot x^{x^x}}$$

$$M = 4 \cdot 4^{2^2} = 4 \cdot 4^4$$

$$M = 1024$$