

# RADICACIÓN EN R

## DEFINICIÓN

ÍNDICE                      RAÍZ

$\downarrow$                                        $\downarrow$

$$\sqrt[n]{a} = b \iff b^n = a$$

RADICANDO

$\nearrow$

Donde:  $n \in \mathbb{N} \wedge n \geq 2$

**Ejemplo:**

$$\sqrt[5]{32} = 2 \text{ pues } 2^5 = 32$$

$$\sqrt{100} = \sqrt[2]{100} = 10 \text{ pues } 10^2 = 100$$

## EXPONENTE FRACCIONARIO

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \sqrt[n]{a}^m$$

Donde  $\frac{m}{n}$  es irreducible

**Ejemplo:**

$$16^{3/4} = \sqrt[4]{16^3} = 2^3 = 8$$

## TEOREMAS

I)  $\sqrt[n]{a^n} = a$

**Ejemplo:**

$$\sqrt[6]{4^{12}} = \sqrt[6]{4^{(6)(2)}} = 4^2 = 16$$

$$\text{II)} \quad \sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$$

**Ejemplo:**

$$\sqrt{8} \cdot \sqrt{2} = \sqrt{16} = 4$$

$$\text{III)} \quad \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

**Ejemplo:**

$$\frac{\sqrt[3]{56}}{\sqrt[3]{7}} = \sqrt[3]{8} = 2$$

$$\text{IV)} \quad \sqrt[m]{\sqrt[n]{a}} = \sqrt[m \cdot n]{a}$$

**Ejemplo:**

$$\left( \sqrt[4]{\sqrt[6]{4}} \right)^{48} = \sqrt[4]{\sqrt[6]{4}}^{(4)(6)} = \sqrt[4]{4^{(24)(2)}} = 4^2 = 16$$

$$\text{V)} \quad \sqrt[n]{a^m} = \sqrt[n \cdot k]{a^{m \cdot k}}$$

**Ejemplo:**

$$\begin{aligned} \sqrt[3]{a^2} \cdot \sqrt[4]{a^3} &= \sqrt[3 \cdot 4]{a^{2 \cdot 4}} \cdot \sqrt[4 \cdot 3]{a^{3 \cdot 3}} = \sqrt[12]{a^8 \cdot a^9} \\ &= \sqrt[12]{a^{17}} \end{aligned}$$

**MCM(3:4)= 12**

$$\text{VI)} \quad \sqrt[m]{a^\alpha \cdot \sqrt[n]{a^\beta \cdot \sqrt[p]{a^\theta}}} = \sqrt[m \cdot n \cdot p]{a^{(\alpha \cdot n + \beta)p + \theta}}$$

## Ejemplo:

$$\sqrt[3]{x^2} \sqrt[4]{x^3} \sqrt[5]{x^4} = \sqrt[60]{x^{59}}$$

## EJERCICIOS DE APLICACIÓN

1. Efectuar:

$$M = \frac{\overbrace{^3\sqrt{x} \cdot ^3\sqrt{x} \cdot ^3\sqrt{x} \dots ^3\sqrt{x}}^{45 \text{ factores}}}{\sqrt{\underbrace{\sqrt{x} \cdot \sqrt{x} \cdot \sqrt{x} \dots \sqrt{x}}_{44 \text{ factores}}} \div \frac{x^{-3}}{x^{-1}}}$$

Resolución

$$M = \frac{(^3\sqrt{x})^{45}}{\sqrt{(\sqrt{x})^{44}} \left( \frac{x^{-1}}{x^{-3}} \right)}$$

$$M = \frac{x^{15}}{\sqrt{x^{22}}} (x^{-1+3}) = \frac{x^{15+2}}{x^{11}}$$

$$\therefore M = x^6$$

2. Reducir:  $E = \frac{\sqrt[6]{2} \cdot \sqrt[3]{2} \cdot \sqrt[4]{2}}{\sqrt[5]{2} \cdot \sqrt[20]{2}}$

Resolución

Calculamos:  $\text{MCM}(6;3;4;5;20) = 60$

$$E = \frac{6^{(10)}\sqrt[6]{2^{(10)}} \cdot 3^{(20)}\sqrt[3]{2^{(20)}} \cdot 4^{(15)}\sqrt[4]{2^{(15)}}}{5^{(12)}\sqrt[5]{2^{(12)}} \cdot 20^{(3)}\sqrt[20]{2^{(3)}}}$$

$$E = \sqrt[60]{\frac{2^{10} \cdot 2^{20} \cdot 2^{15}}{2^{12} \cdot 2^3}} = \sqrt[60]{\frac{2^{35}}{2^{15}}} = \sqrt[60]{2^{20}} = \sqrt[3]{2^4} = \sqrt{2}$$

3. Reducir:

$$M = \frac{\sqrt[5]{x^2} \sqrt[3]{x^4} \sqrt{x^7}}{\sqrt[3]{\sqrt[4]{\sqrt[5]{\frac{1}{x^6}}}} \sqrt{x^2}}$$

Resolución

$$M = \frac{\sqrt[5]{x^2} \sqrt[3]{x^4} \sqrt{x^7}}{\sqrt[3]{x^{-2}} \sqrt[4]{x^{-1}} \sqrt[5]{x^{-6}}} = \frac{(2)^{30} \sqrt{x^{27(2)}}}{60 \sqrt{x^{-51}}}$$

$$M = \sqrt[60]{x^{54+51}} = \sqrt[60]{x^{105}} = \sqrt[4]{x^7}$$

$$\therefore M = \sqrt[4]{x^7}$$

Calcular : "A + B + C"

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Si:  $A = \sqrt[3]{9 \cdot \sqrt[3]{9 \cdot \sqrt[3]{9} \dots}}$

$$B = \sqrt{132 + \sqrt{132 + \sqrt{132 + \dots}}}$$

$$C = \sqrt[5]{64 \sqrt[5]{64 \sqrt[5]{64 \dots}}}$$

## RESOLUCIÓN

$$A = \sqrt[3]{9} = \sqrt{9}$$



$$A = 3$$

$$132 = (11) \cdot (12)_{\text{mayor}}$$



$$B = 12$$

$$C = \sqrt[5]{64} = \sqrt[6]{64}$$



$$C = 2$$

$$\therefore A + B + C = 17$$