파이썬으로 배우는 데이터 구조

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학습 목표

Postfix계산과 Infix to Postfix의

알고리즘을 이해하고 구현할 수 있다

Data Structures in Python Chapter 3 - 1

- Stack Concept and ADT
- Stack Example - Matching
- **Stack Example - Postfix**

Agenda

- Postfix Calculation
- Conversion from Infix to Postfix

Infix, postfix and prefix expressions

Stacks can be used to implement algorithms involving Infix, postfix and prefix expressions.

- Infix:
  - An infix expression is a single letter, or an operator, proceeded by one infix string and followed by another infix string.
  - A, A + B, (A + B) + (C - D)
- Prefix:
  - A prefix expression is a single letter, or an operator, followed by two prefix strings. Every prefix string longer than a single variable contains an operator, first operand and second operand.
  - A, + A B, + + A B - C D
- Postfix:
  - A postfix expression (also called Reverse Polish Notation) is a single letter or an operator, preceded by two postfix strings. Every postfix string longer than a single variable contains first and second operands followed by an operator.
  - A, A B +, A B + C D - +

Infix, postfix and prefix expressions

- Prefix and postfix notations are methods of writing mathematical expressions without parenthesis.
  - **Why**: Time to evaluate a postfix and prefix expression is O(n), where n is the number of elements in the array.

Infix	Prefix	Postfix
A + B	+ A B	A B +
A + B – C	– + A B C	A B + C –
(A + B) * C – D	– * + A B C D	A B + C * D –

Postfix Calculator

- Computation of arithmetic expressions can be efficiently carried out in Postfix notation with the help of stack.

infix		infix		postfix	Result
2 * 3 + 4	$\longrightarrow$	(2 * 3) + 4	$\longrightarrow$	$\underline{\underline{2\ \ 3\ \ *}\ \ 4\ \ +}$	10
		2 * (3 + 4)	$\longrightarrow$	$\underline{2\ \ \underline{3\ \ 4\ \ +}\ \ *}$	14

Postfix Calculator

- Requires you to enter postfix expressions.
    - Example: 2 3 4 + *

Algorithm:

- When an **operand** is entered,
    - the calculator pushes it onto a stack
- When an **operator** is entered,
    - the calculator applies it to the top **two operands** of the stack
        - Pops the top two operands from the stack
        - Pushes the result of the operation on the stack

Postfix Calculator - Algorithm

- Example 1: Evaluating the expression: 2 3 4 + *

Key entered	Calculator action		Stack(bottom to top)
2	push 2		2
3	push 3		2 3
4	push 4		2 3 4
+	operand2 = pop stack	(4)	2 3
	operand1 = pop stack	(3)	2
	result = operand1 + operand2	(7)	2
	push result		2 7
*	operand2 = pop stack	(7)	2
	operand1 = pop stack	(2)	
	result = operand1 * operand2	(14)	
	push result		14

Postfix Calculator

- Example 2: Evaluating the expression: 2 3 * 4 +

```
Key entered   Calculator action                            Stack(bottom to top)

  2           push 2                                       2
  3           push 3                                       2  3

  *           operand2 = pop stack                  (3)    2
              operand1 = pop stack                  (2)

              result = operand1 * operand2          (6)
              push result                                  6
  4           push 4                                       6  4

  +           operand2 = pop stack                  (4)    6
              operand1 = pop stack                  (4)
              result = operand1 + operand2          (10)
              push result                                  10
```

Postfix Calculator

- Example 3: Evaluating the expression: 12 3 - 3 /

```
Key entered   Calculator action                         Stack(bottom to top)

 12           push 12                                   12
 3            push 3                                    12  3

 -            operand2 = pop stack             (3)      12
              operand1 = pop stack             (12)
              result = operand1 + operand2     (9)
              push result                               9
 3            push 3                                    9  3

 /            operand2 = pop stack             (3)      6
              operand1 = pop stack             (9)
              result = operand1 / operand2     (3)
              push result                               3
```

The order of operand1 and operand2 is very important.

Postfix Calculator - Exercise 2

- Evaluate the expression: 10 4 2 - 5 * + 3 -

Key entered	Calculator action	Stack(bottom to top)

Postfix Calculator

- Coding

```
def evaluate_postfixList(postfixList):
    stack = Stack()
    operators = "+-/*"
    for op in postfixList:
        if op in operators:    #operator
            if stack.size() > 1:
                num2 = stack.pop()
                num1 = stack.pop()
                result = compute(int(num1), int(num2), op)
                stack.push(result)
            else:
                return "Failed while parsing postfix expression"
        else: #operand
            stack.push(op)
    return stack.pop()
```

Write your own compute() function
to make this code work properly.

```
#Sample Run:
evaluate_postfixList(['3', '4', '7', '*', '+'])
31
```

Conversion from Infix to Postfix

- Examples:
  - 2 * 3 + 4 → 2 3 * 4 +
  - 2 + 3 * 4 → 2 3 4 * +
  - 1 * 3 + 2 * 4 → 1 3 * 2 4 * +

- Algorithm Concept:
  - Operands always stay in the same order with respect to one another.
  - An operator will move only “to the right” with respect to the operands.
  - All parentheses are removed.

Conversion from Infix to Postfix

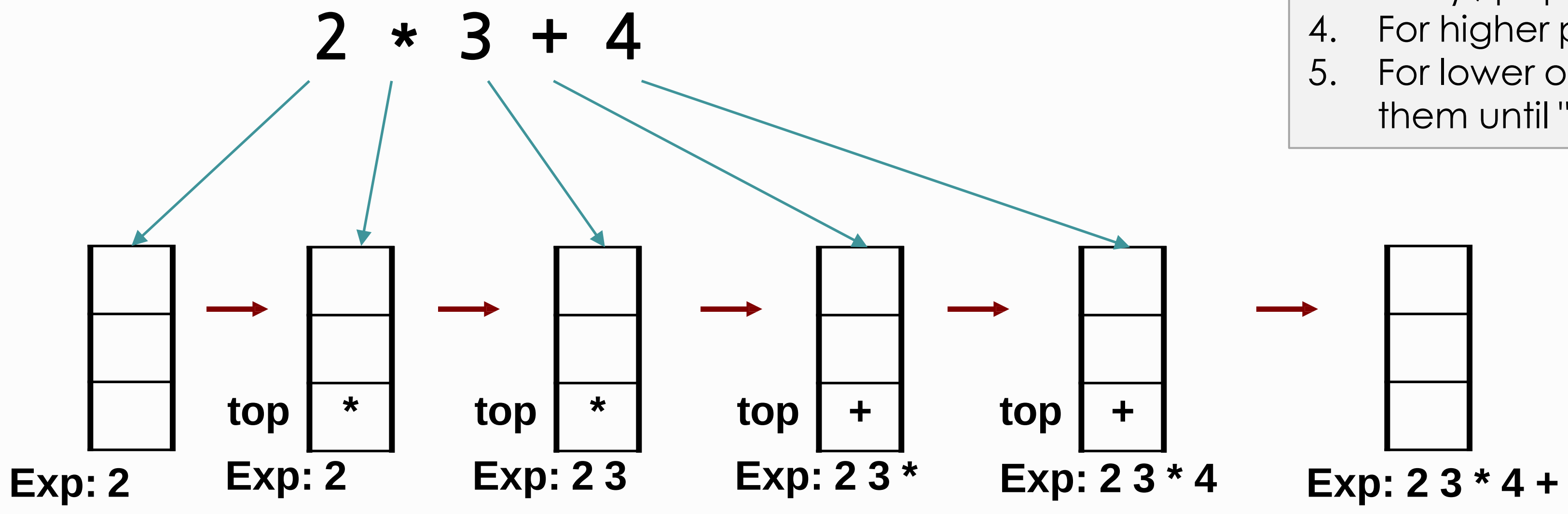
Algorithm:

- operand - output it to postfixExp.
- “(“ - push onto the stack.
- “)” - pop the operators off the stack and append them to the end of postfixExp until encounter the match “(“.
- operator
  - For **higher** precedence operator, push it onto the stack.
  - For **lower or equal** precedence operator, pop them until "(" and push it onto the stack.
- End of the string
  - append the remaining contents of the stack to postfixExp.

Conversion from Infix to Postfix

- Example 1

1. Operands are output immediately
2. Push "(" always and operators in general.
3. For ")", pop until "(". Discard "(" and ")".
4. For higher precedence operator, push it.
5. For lower or equal precedence operator, pop them until "(" and push it.

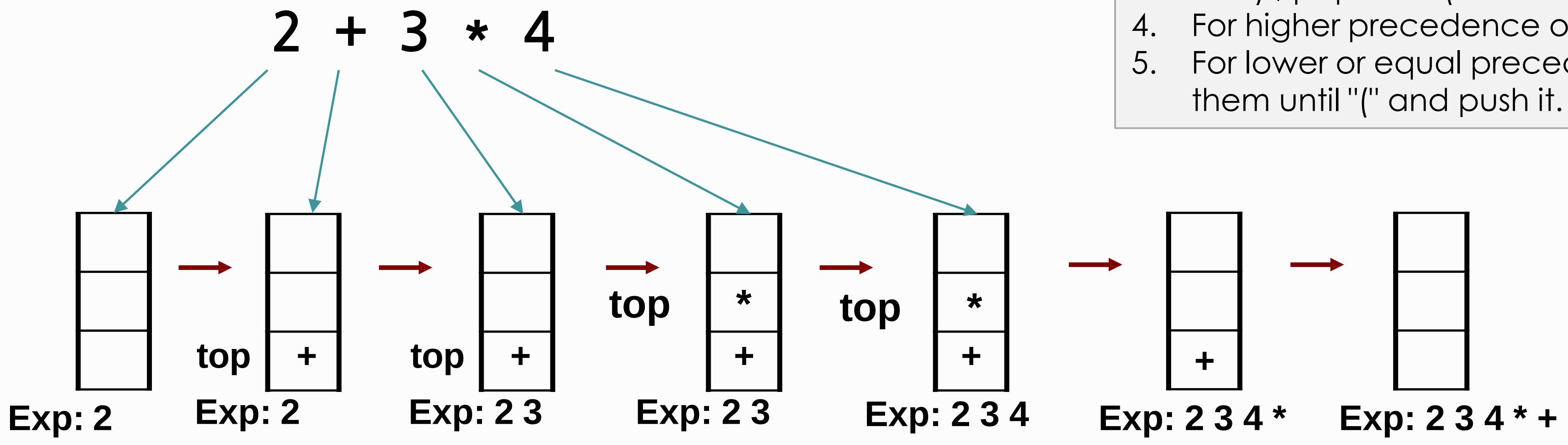


+ has a lower precedence than *
then pop *, push +

Conversion from Infix to Postfix

- Example 2:

1. Operands are output immediately
2. Push "(" always and operators in general.
3. For ")", pop until "(". Discard "(" and ")".
4. For higher precedence operator, push it.
5. For lower or equal precedence operator, pop them until "(" and push it.



top +

Conversion from Infix to Postfix

- Example 3: a - (b + c * d) / e

token	stack	postfix	
a		a	
-	-	a	
(	-(	a	
b	-(	ab	
+	-(+	ab	
c	-(+	abc	
*	-(+*	abcd	
)	-(+	abcd*	get operators
	-(	abcd*+	from stack to
	-	abcd*+	postfix until "("
/	-/	abcd*+	
e	-/	abcd*+e	get operators
	bottom top		from stack to
			postfix until empty

1. Operands are output immediately
2. Push "(" always and operators in general.
3. For ")", pop until "(". Discard "(" and ")".
4. For higher precedence operator, push it.
5. For lower or equal precedence operator, pop them until "(" and push it.

Conversion from Infix to Postfix - Exercise 3

- Debug the following program.

```
def get_postfix(infixList):
    precedence = {"*":3, "/":3, "+":2, "-":2, "(":1 }
    operators = "+-/*"
    op_stack = Stack()
    postfixList = []
    for op in infixList:
        if op in operators:
            while (not op_stack.is_empty()) and (precedence[op_stack.peek()] >= precedence[op]):
                postfixList.append(op_stack.pop())
                op_stack.push(op)
        elif op == "(":
            op_stack.push(op)
        elif op == ")":
            op = op_stack.pop()
            while not op == "(":
                postfixList.append(op)
            op = op_stack.pop()
        else: #operand
            postfixList.append(op)
    while not op_stack.is_empty():
        postfixList.append(op_stack.pop())
    return " ".join(postfixList), postfix
```

```
#Sample Run:
a, b = get_postfixList(['3', '+', '4', '*', '7'])
print(a)
print(b)
```

```
3 4 7 * +
['3', '4', '7', '*', '+']
```

Conversion from Infix to Postfix - Exercise 4

- Converting the infix expression to postfix: (B - C) * (D - E)

token	stack	postfix

1. Operands are output immediately
2. Push "(" always and operators in general.
3. For ")", pop until "(". Discard "(" and ")".
4. For higher precedence operator, push it.
5. For lower or equal precedence operator, pop them until "(" and push it.

Conversion from Infix to Postfix - Exercise 4 solution

- Converting the infix expression to postfix: (B - C) * (D - E)

token	stack	postfix
(	(	
B		B
-	(-	B
C	(-	BC
)	(	BC-
	)	BC-
*	*	BC-
(	*(	BC-
D	*(	BC-D
-	*(-	BC-D
E	*(-	BC-DE
)	*(	BC-DE-
	*	BC-DE-
		BC-DE-*

```
a, b = get_postfix(['(', 'B', '-', 'C', ')', '*', '(', 'D', '-', 'E', ')'])
print(a)
print(b)
```

```
3 4 7 * +
['3', '4', '7', '*', '+']
```

1. Operands are output immediately
2. Push "(" always and operators in general.
3. For ")", pop until "(". Discard "(" and ")".
4. For higher precedence operator, push it.
5. For lower or equal precedence operator, pop them until "(" and push it.

Summary

- Stacks are used in applications that manage data items in LIFO manner, such as:
  - Checking for Balanced Braces
  - Matching bracket symbols in expressions
  - Evaluating postfix expressions
  - Conversion from Infix to Postfix

학습 정리

1) 전위(Prefix) 및 후위(Postfix) 표현식에는 괄호 연산이 없기 때문에 Infix(중위) 표현식보다 효율적으로 계산할 수 있다

2) Infix to Postfix 표현식 변환 알고리즘의 핵심은 괄호를 모두 제거하고 연산자(operator)에 우선순위를 부여하는 것이다

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수고했습니다
곧 다음 시간에
다시 뵙겠습니다

Postfix Evaluation

◈ **Goal**: Evaluate postfix expressions.

infix	postfix
2 + 3 * 4	
a * b + 5	
(1 + 2) * 7	
a * b / c	
(a / (b – c + d)) * (e – a) * c	
a / b – c + d * e – a * c	

Postfix Evaluation

- **Goal**: Evaluate postfix expressions.

infix	postfix
2 + 3 * 4	2 3 4 * +
a * b + 5	a b * 5 +
(1 + 2) * 7	1 2 + 7 *
a * b / c	a b * c /
(a / (b - c + d)) * (e - a) * c	a b c - d + / e a - * c *
a / b - c + d * e - a * c	a b / c - d e * + a c * -

Postfix Evaluation

◈ **Goal**: Evaluate postfix expressions.

```
a b c - d + / e a - * c *
```

⬇

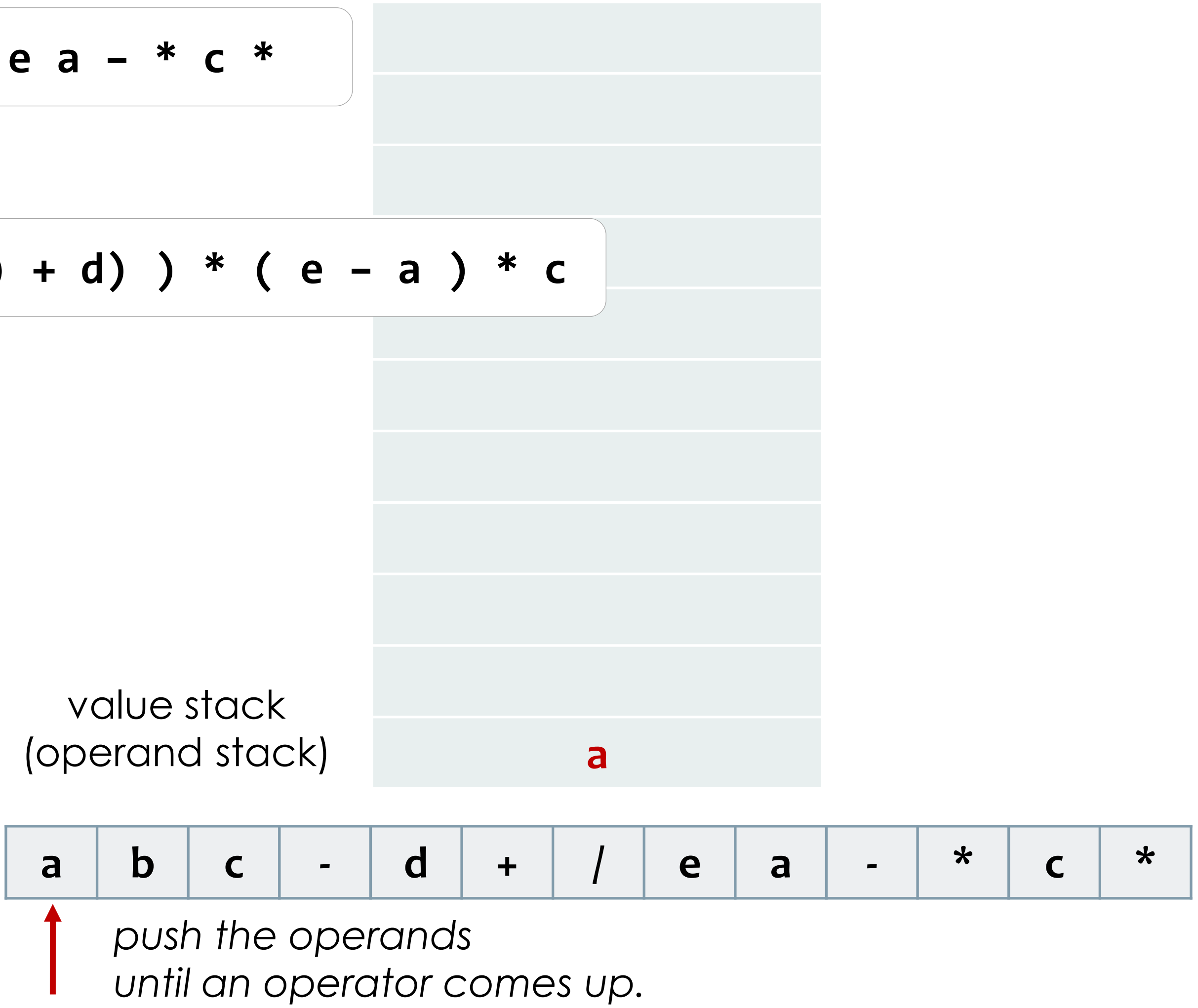
```
( a / ((b - c) + d) ) * ( e - a ) * c
```

Postfix Evaluation

◈ **Goal**: Evaluate postfix expressions.

```
a b c – d + / e a – * c *
```

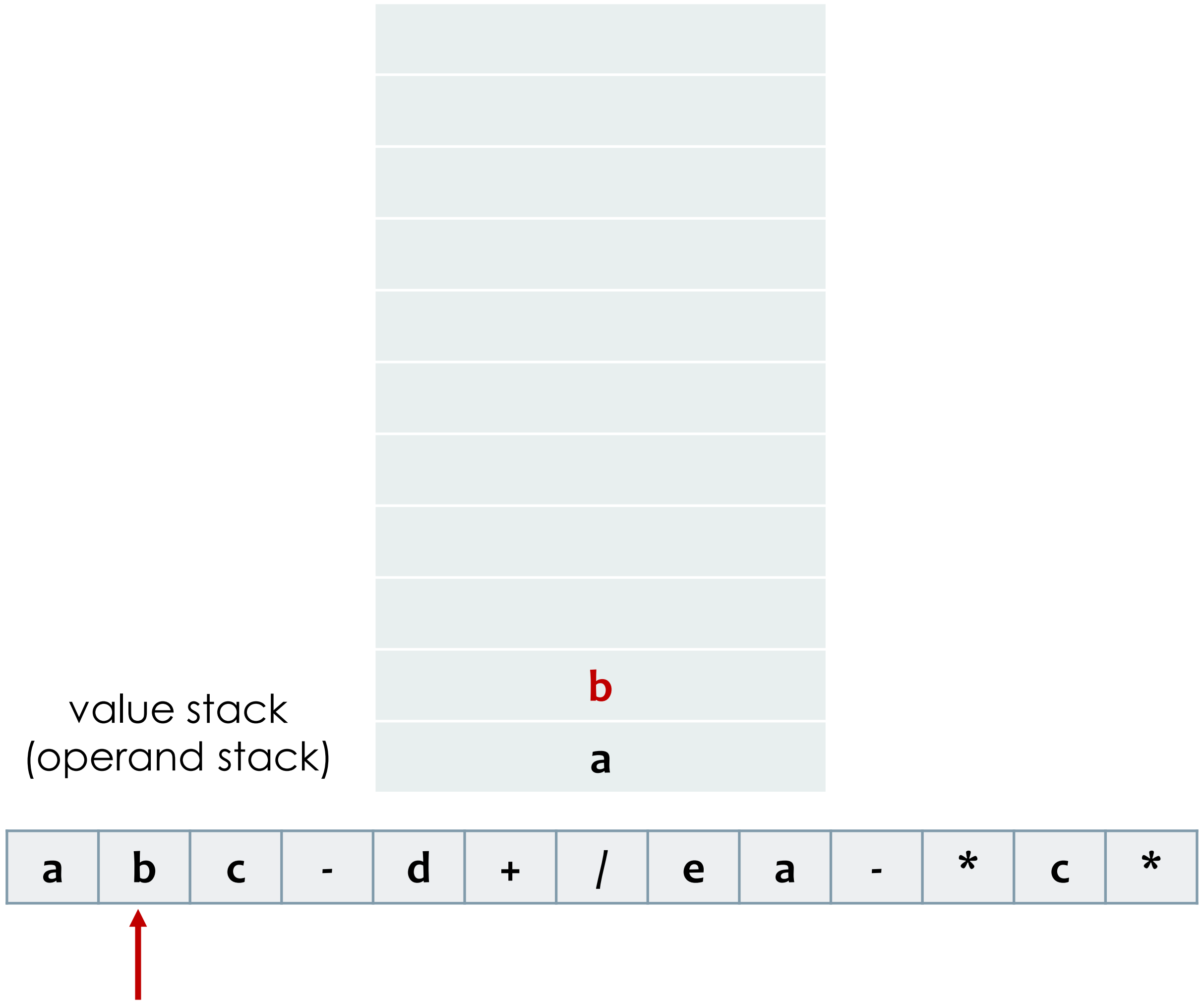
```
( a / ((b – c) + d) ) * ( e – a ) * c
```



push the operands
until an operator comes up.

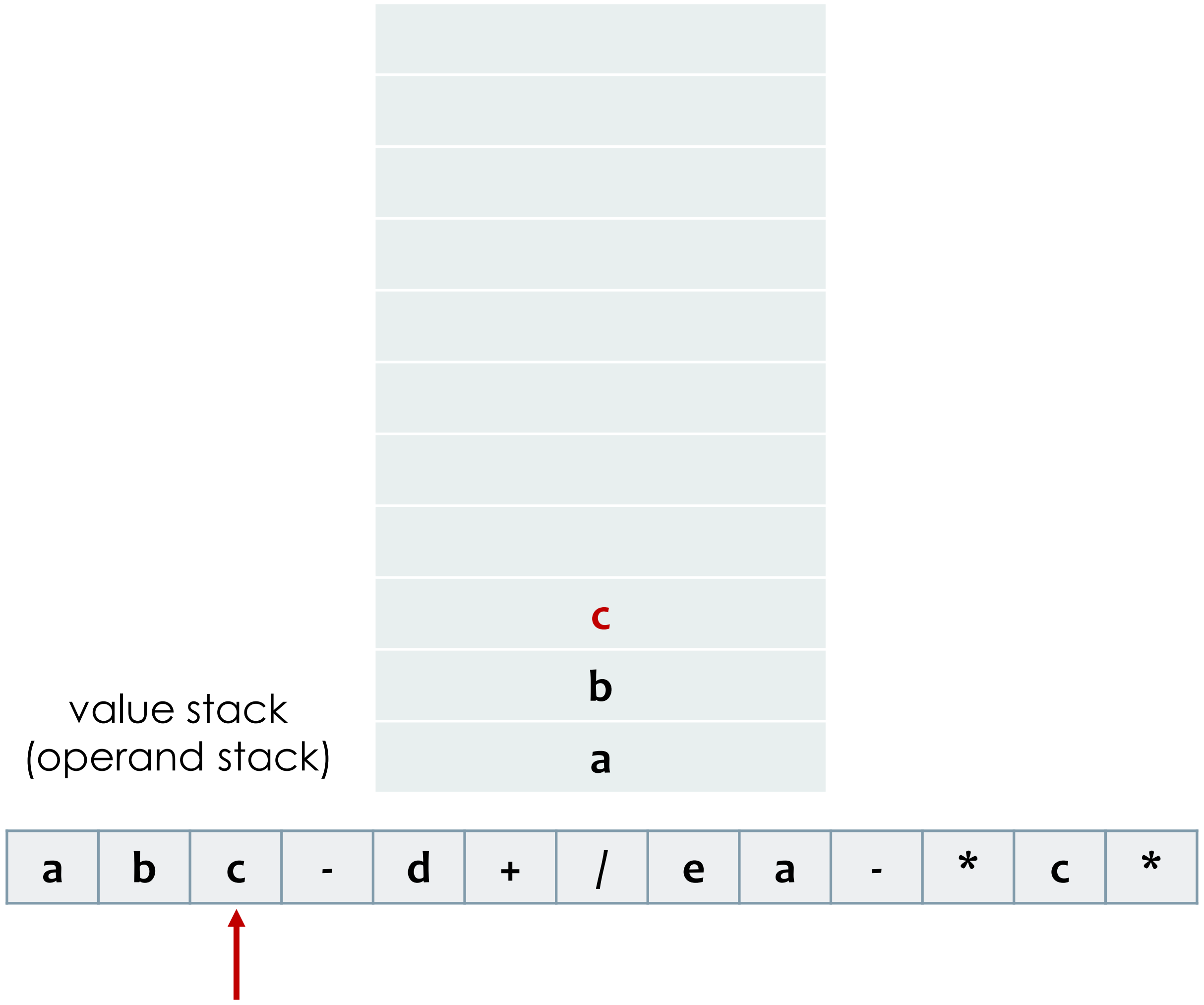
Postfix Evaluation

- **Goal**: Evaluate postfix expressions.



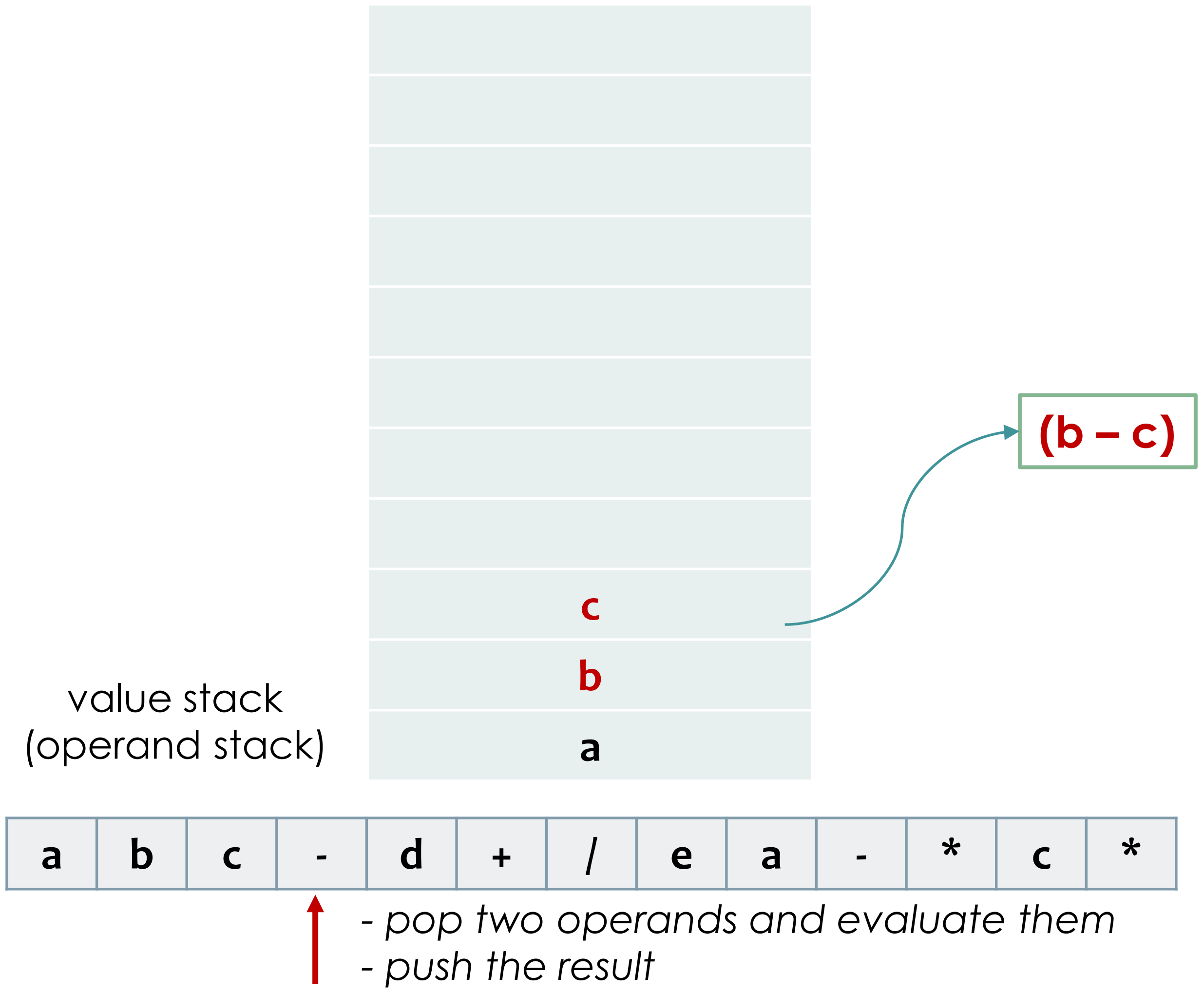
Postfix Evaluation

- **Goal**: Evaluate postfix expressions.



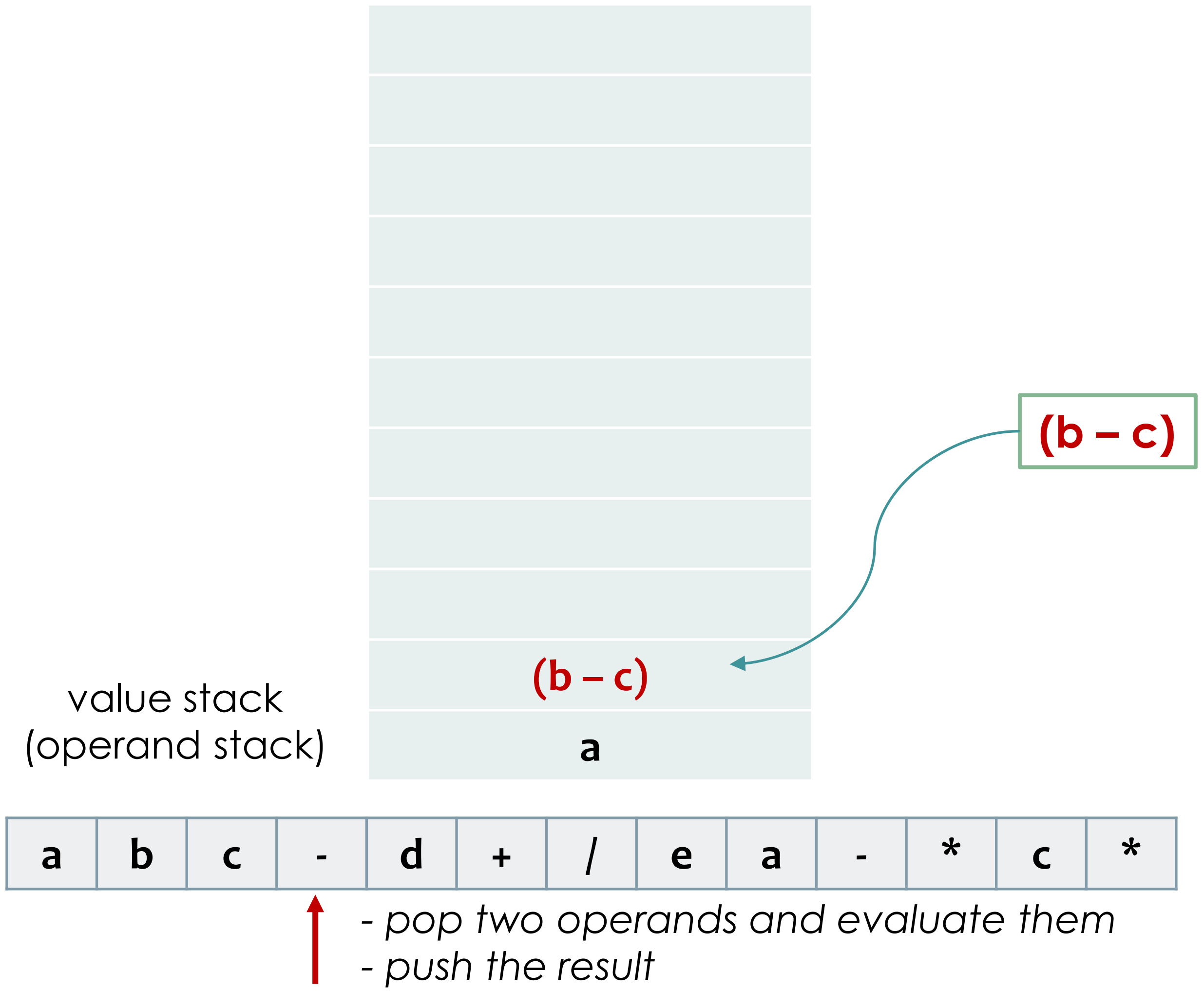
Postfix Evaluation

◈ **Goal**: Evaluate postfix expressions.



Postfix Evaluation

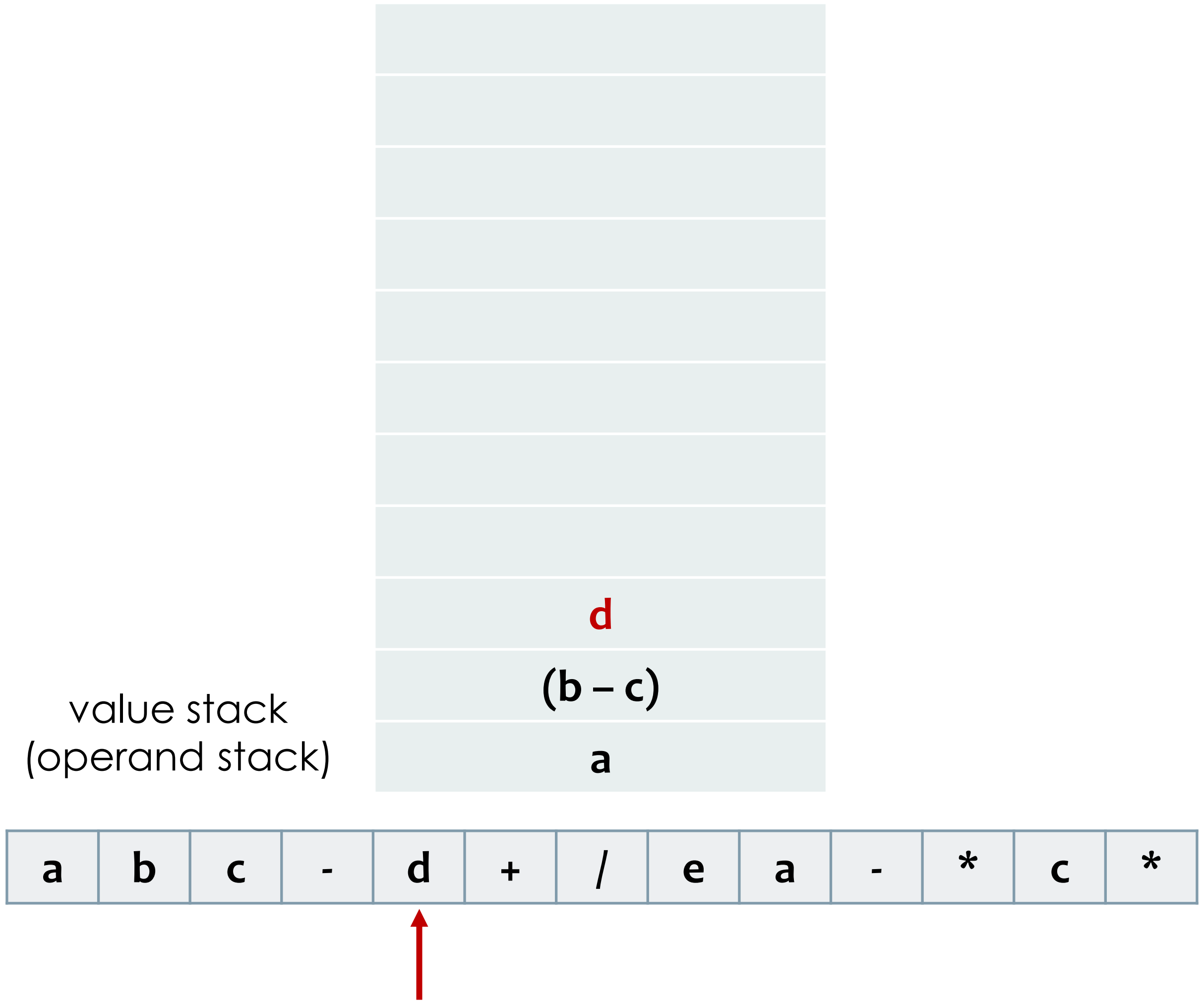
◈ **Goal**: Evaluate postfix expressions.



- *pop two operands and evaluate them*
- *push the result*

Postfix Evaluation

- **Goal**: Evaluate postfix expressions.



Postfix Evaluation

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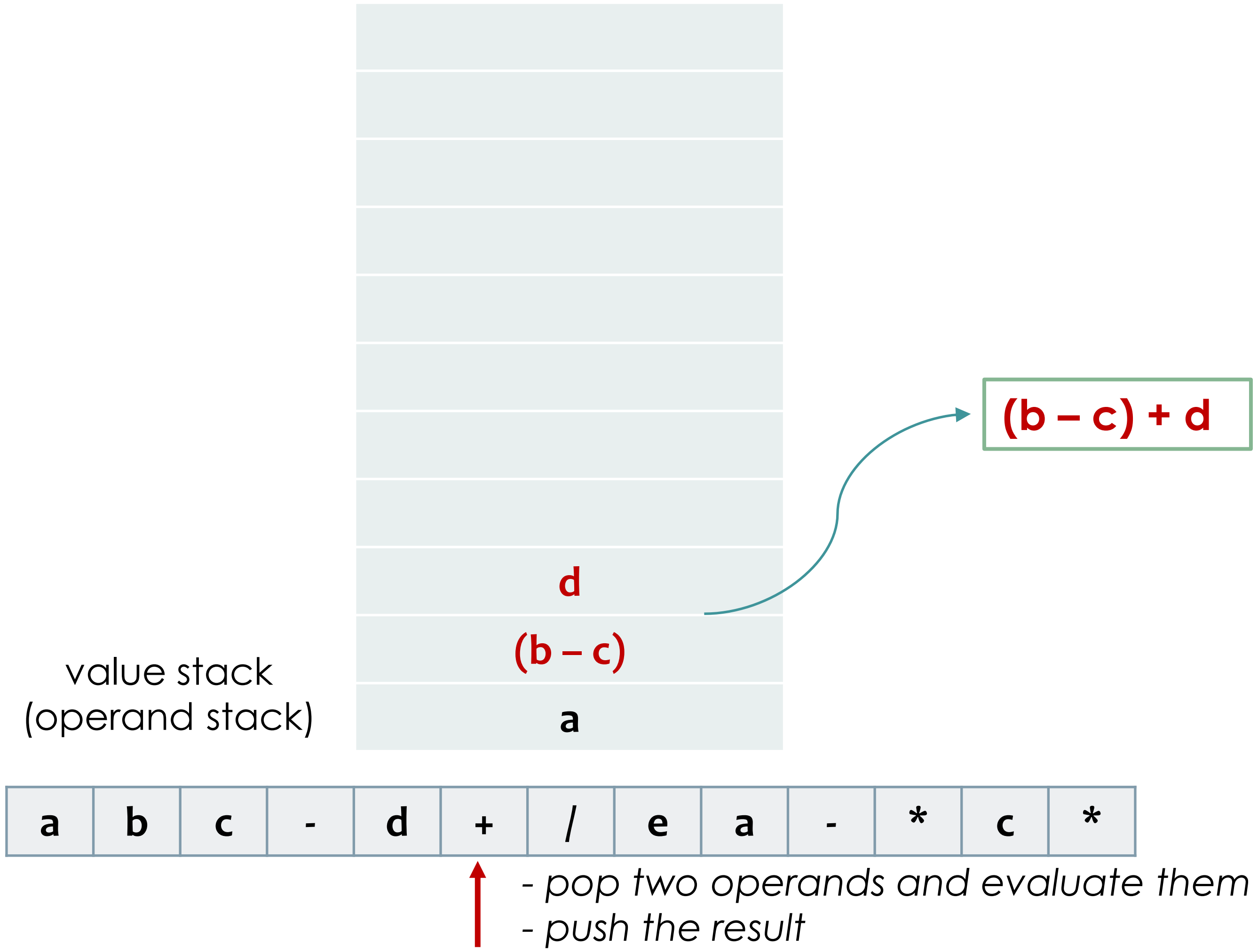
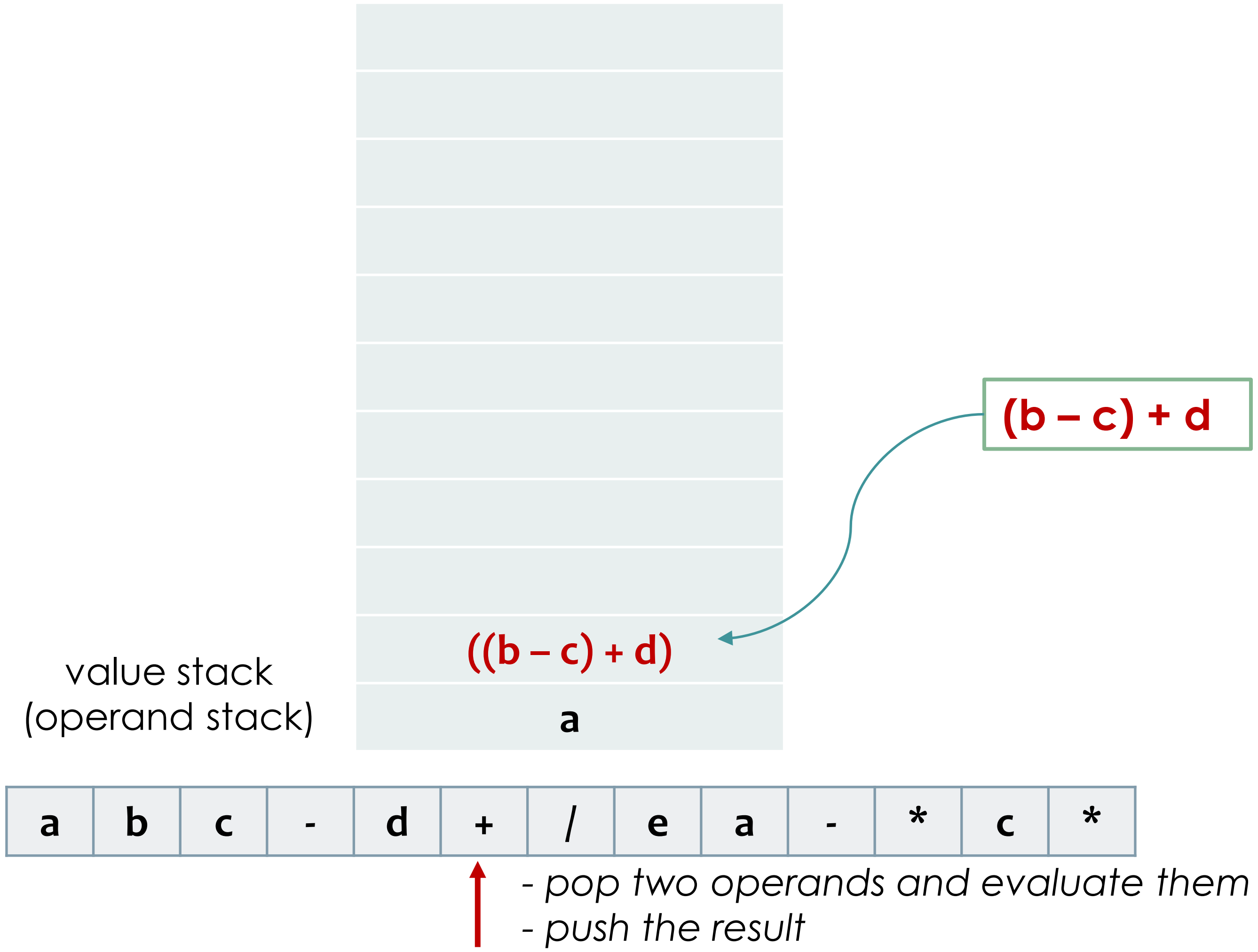


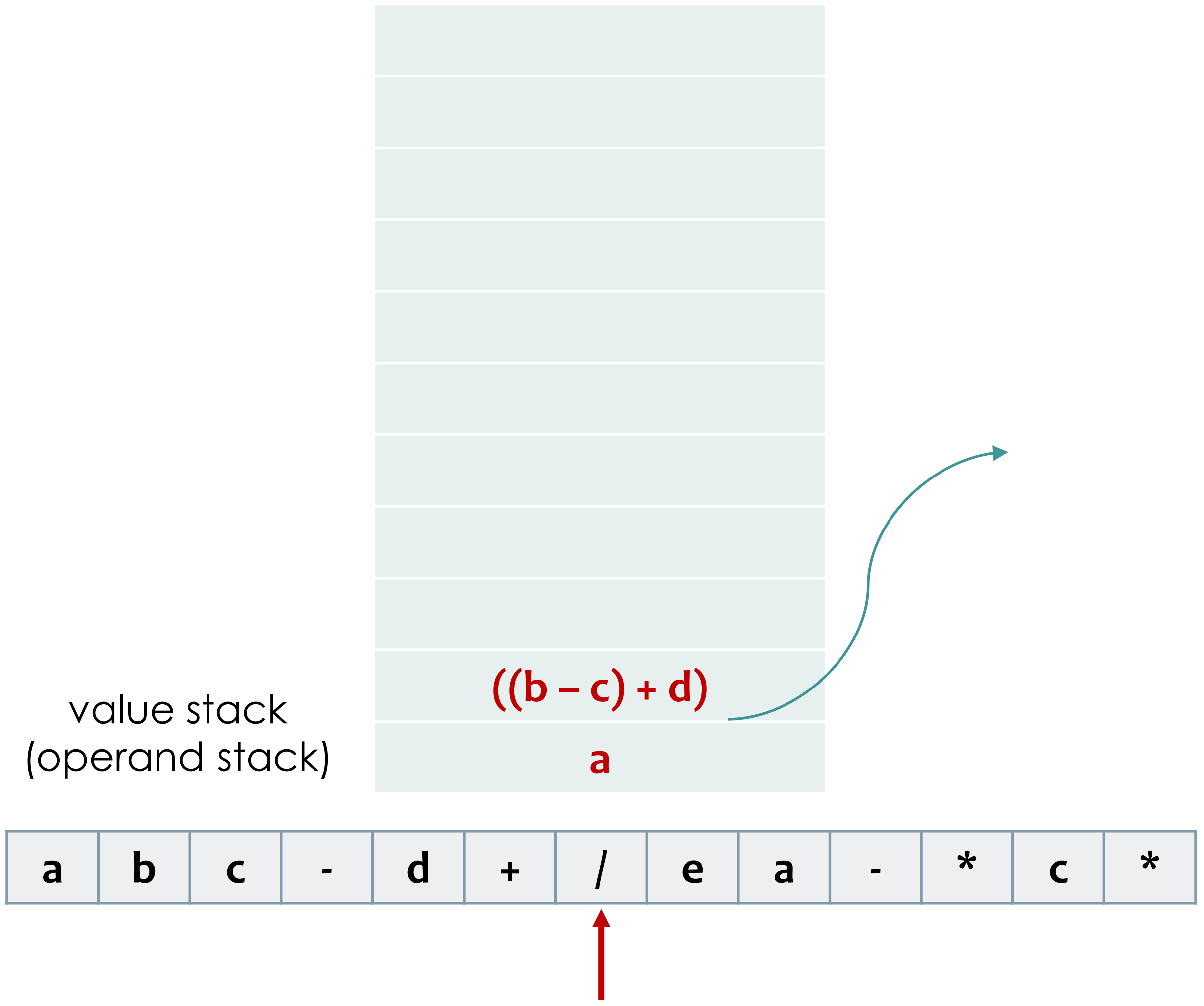
Postfix Evaluation

◈ **Goal**: Evaluate postfix expressions.



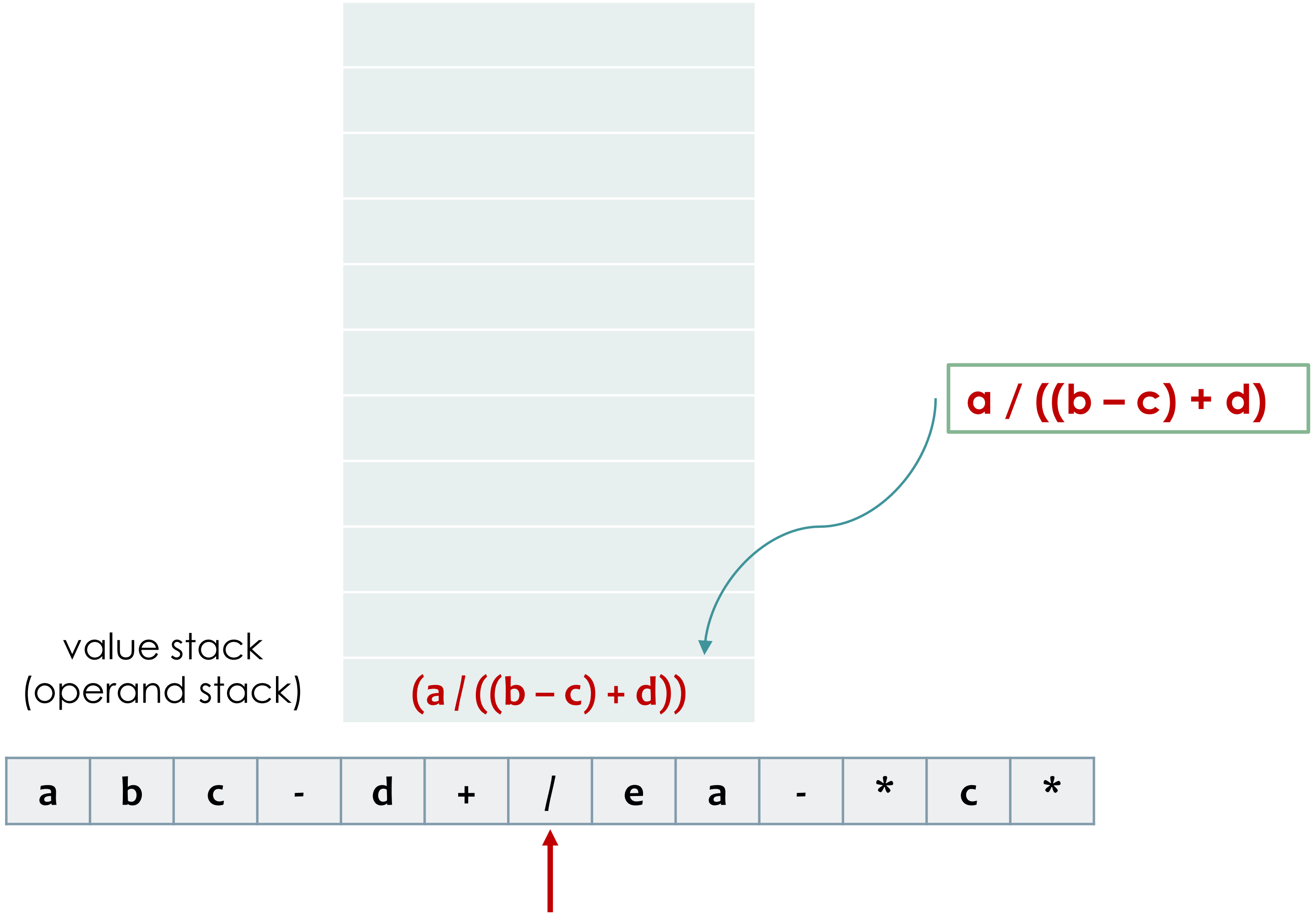
Postfix Evaluation

- **Goal**: Evaluate postfix expressions.



Postfix Evaluation

- **Goal**: Evaluate postfix expressions.



Postfix Evaluation

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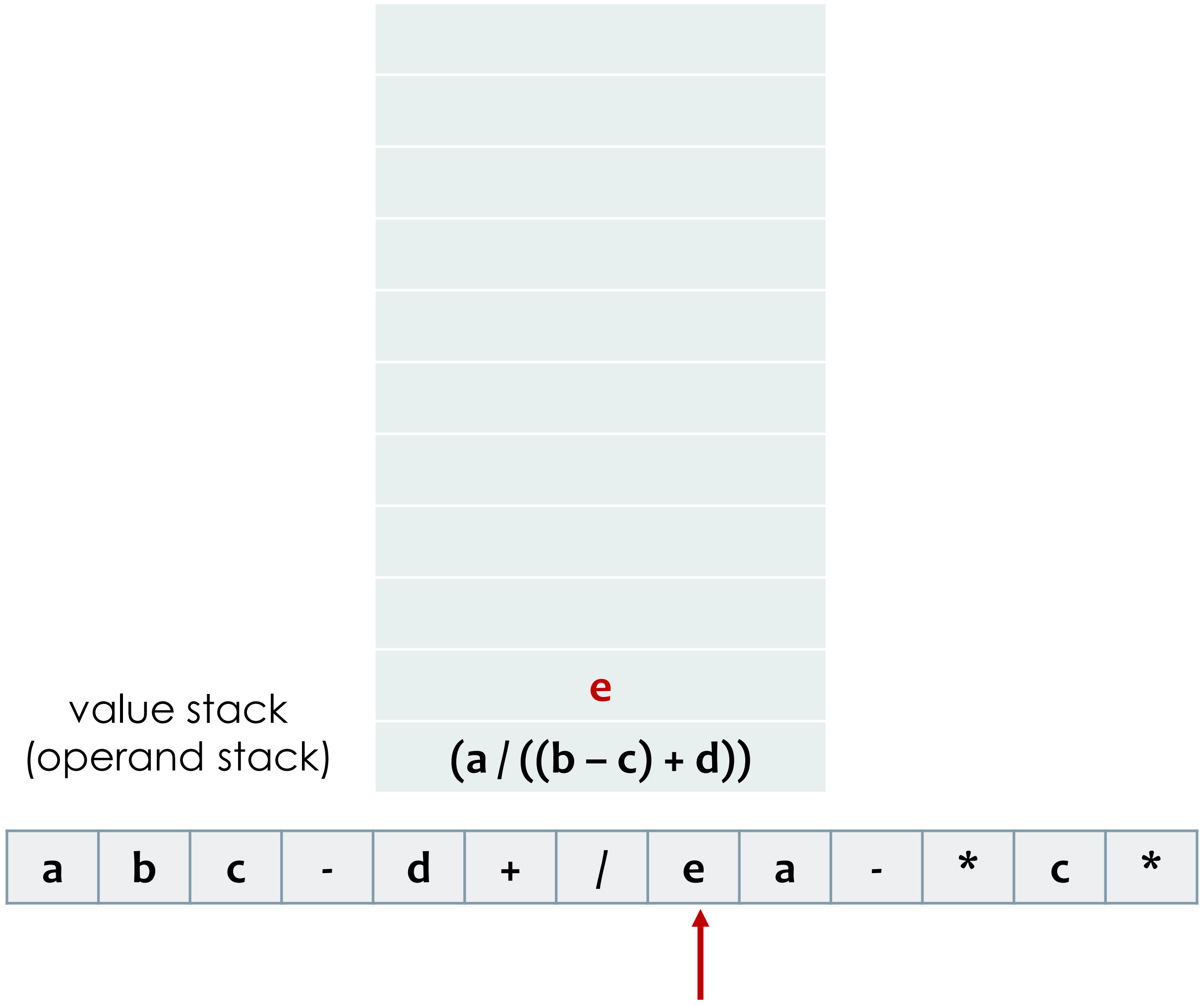

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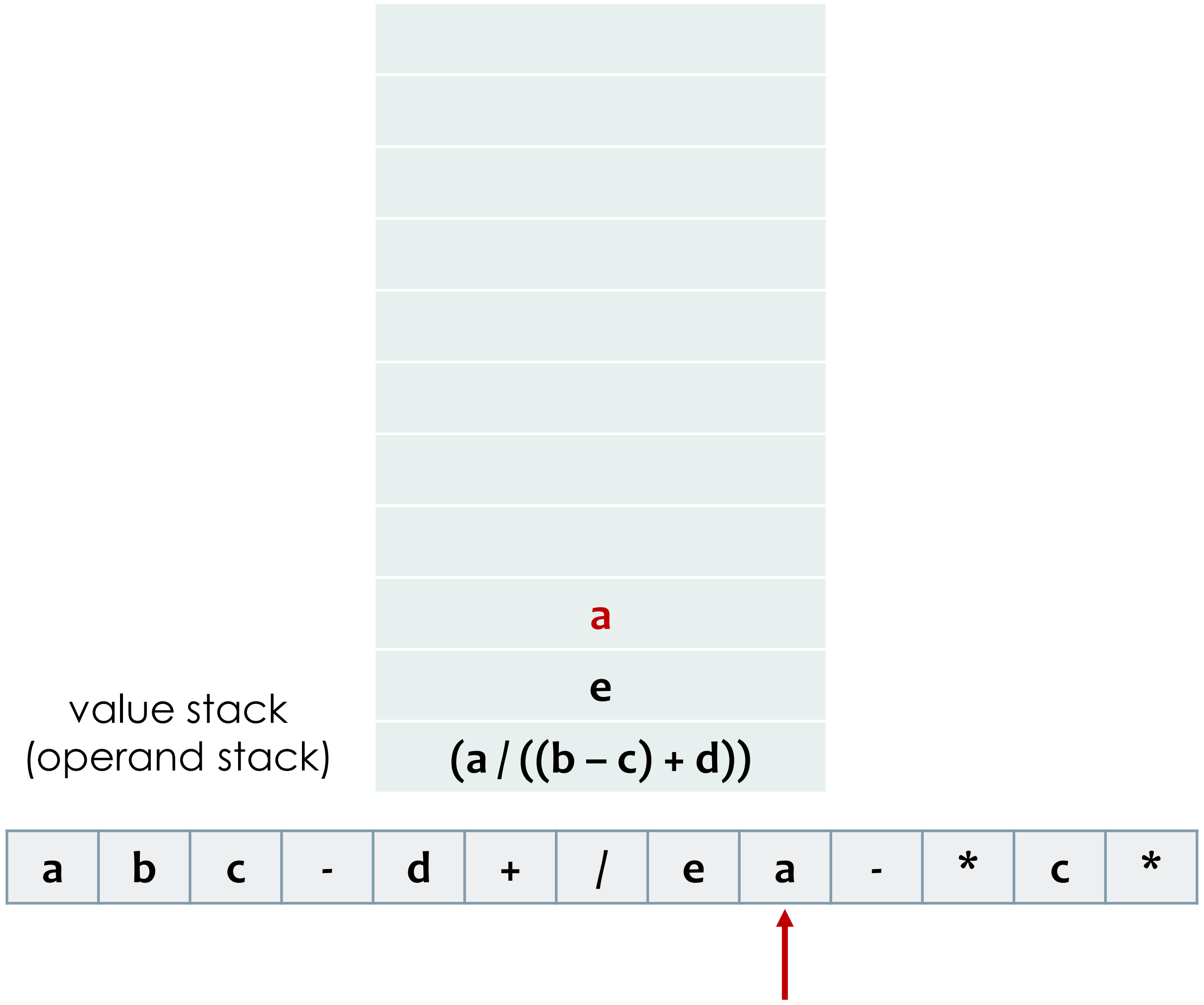

Postfix Evaluation

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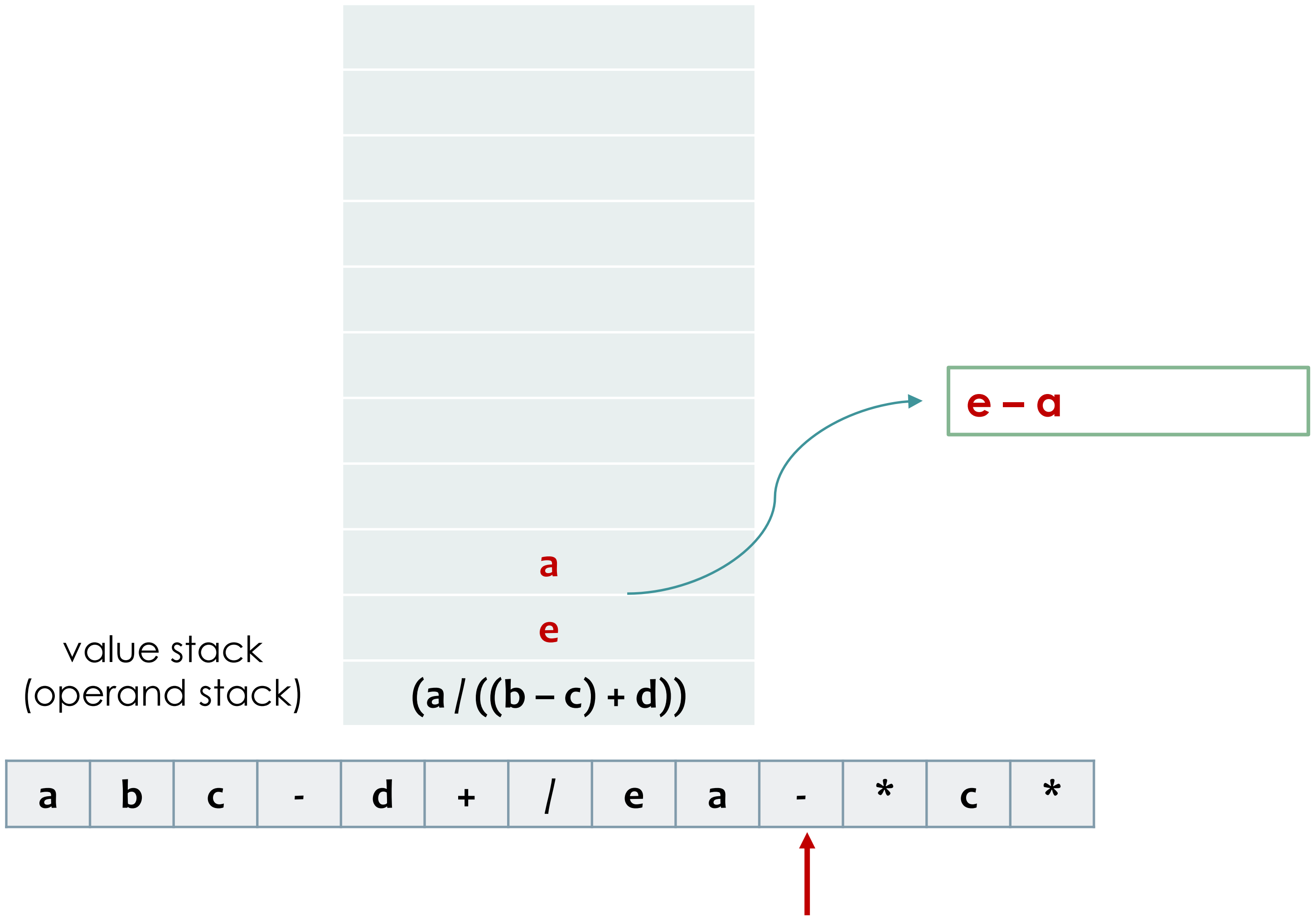
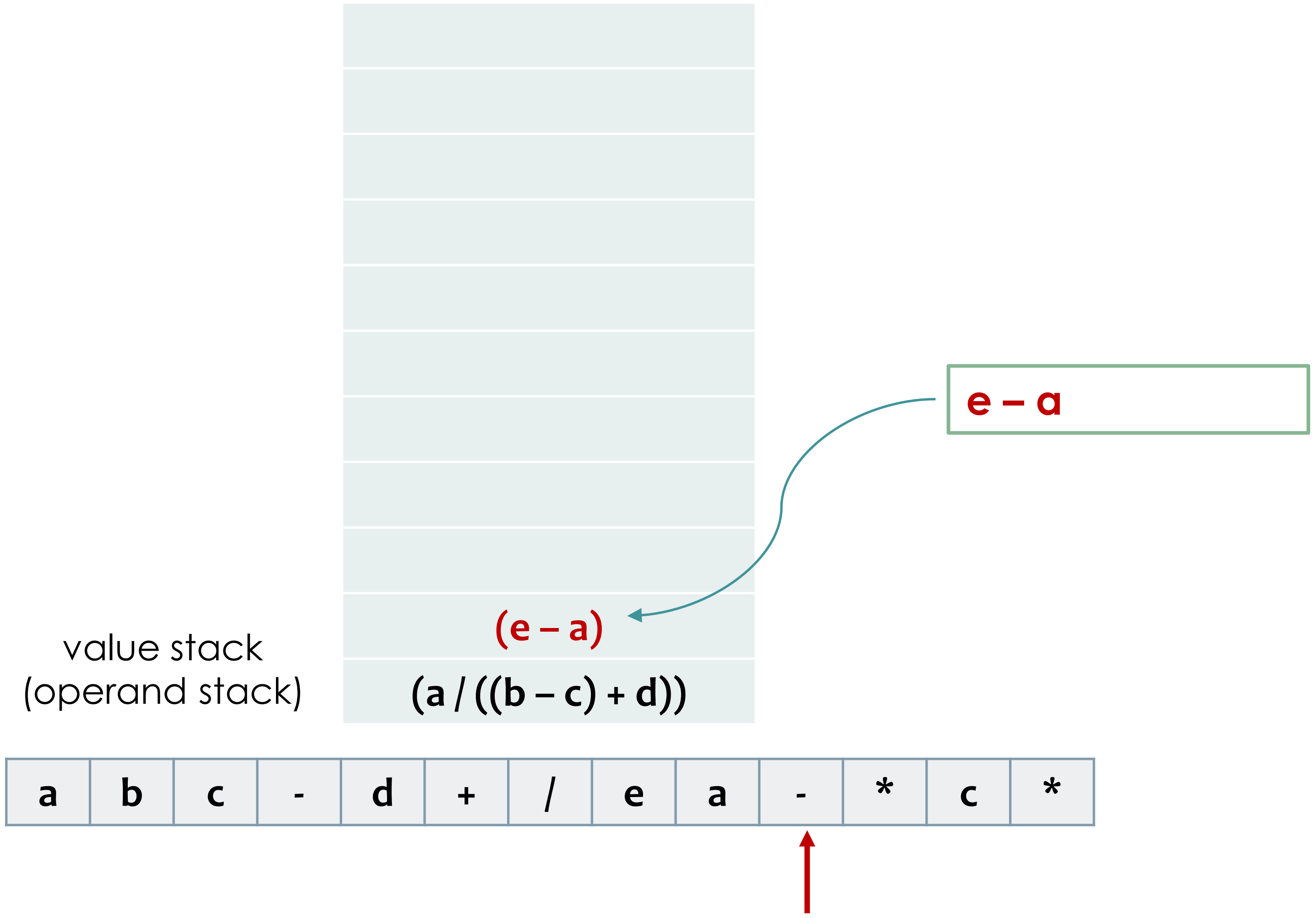


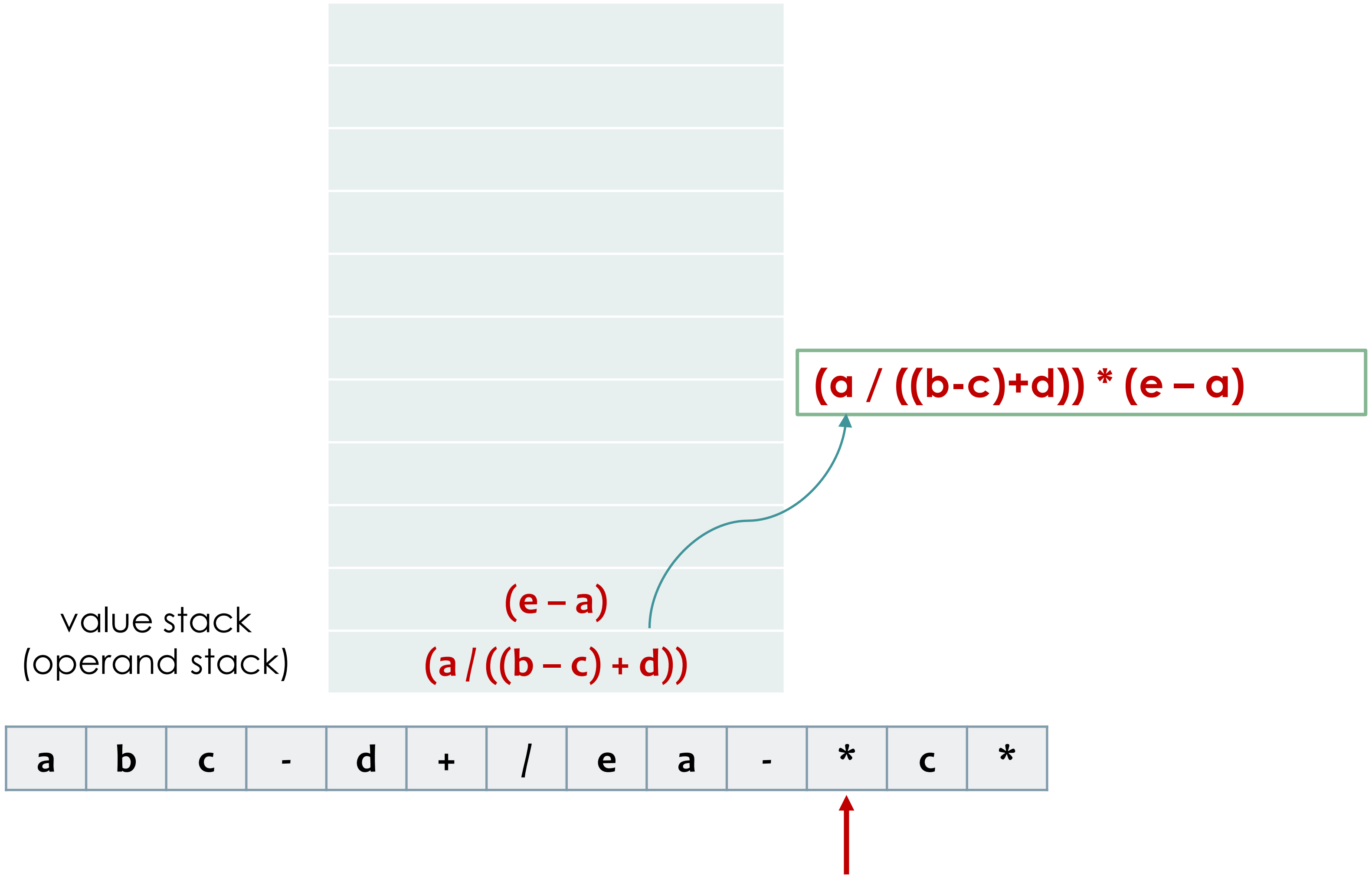
Postfix Evaluation

- **Goal**: Evaluate postfix expressions.



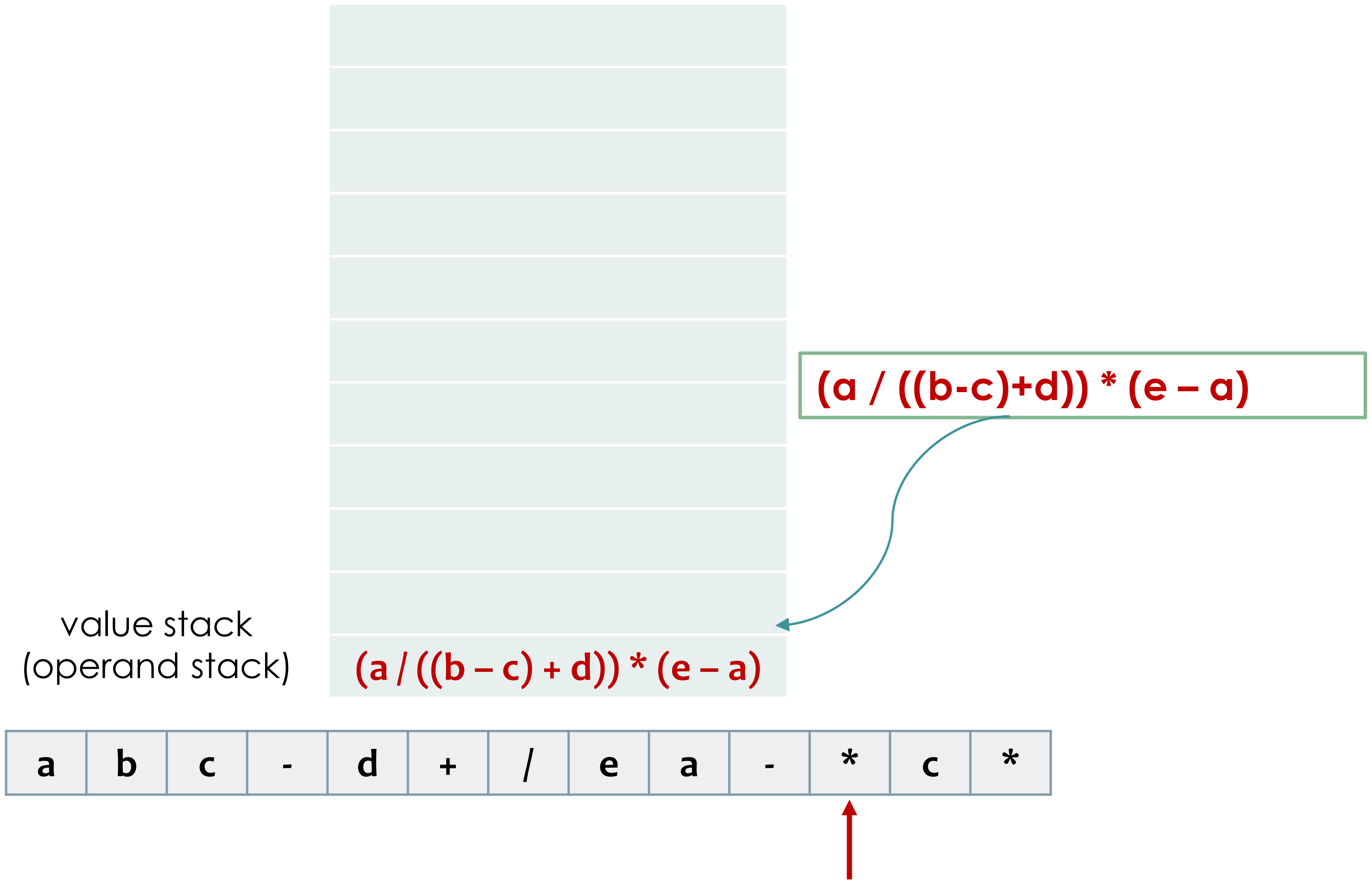
Postfix Evaluation

- **Goal**: Evaluate postfix expressions.



Postfix Evaluation

◈ **Goal**: Evaluate postfix expressions.



Postfix Evaluation

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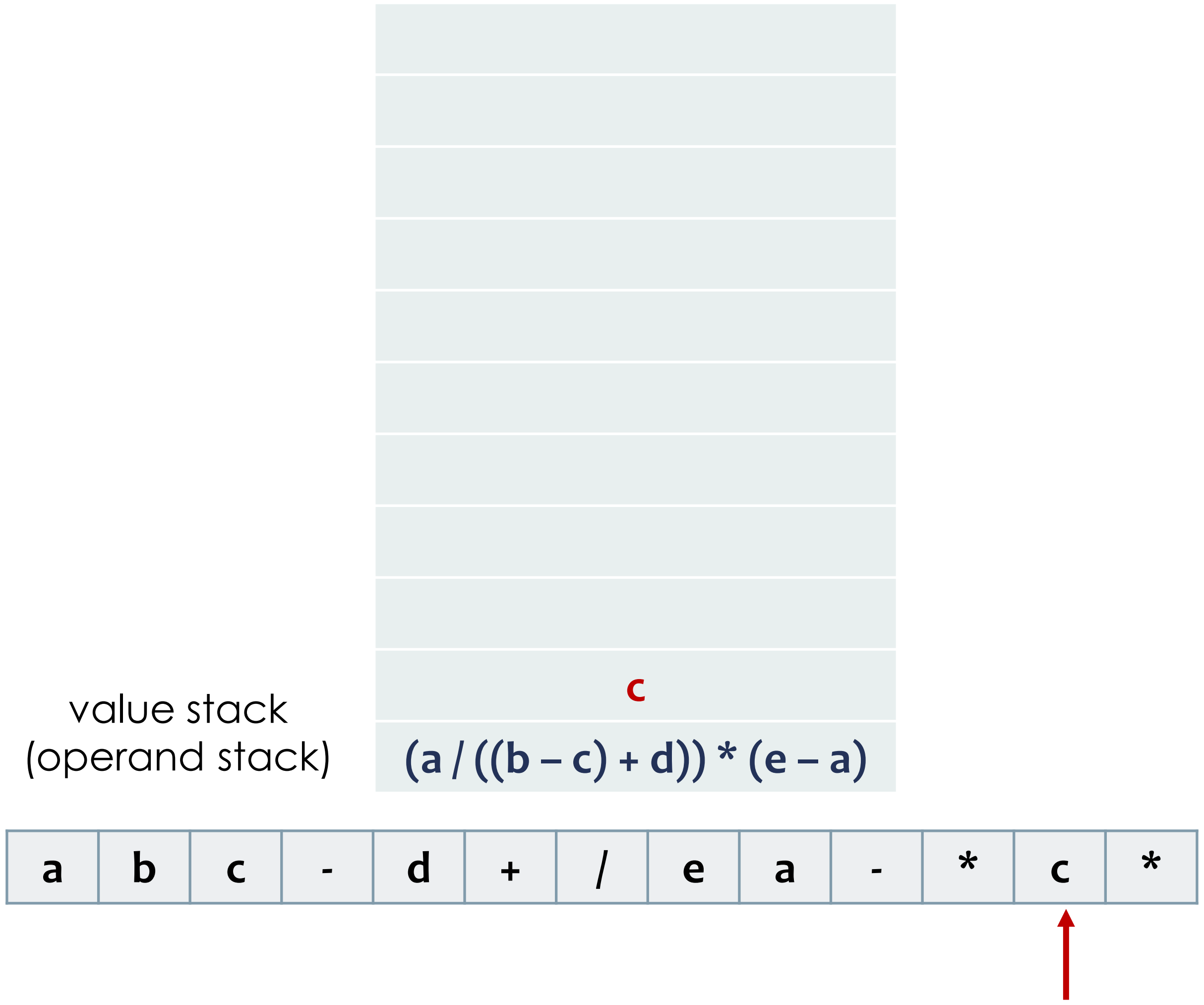
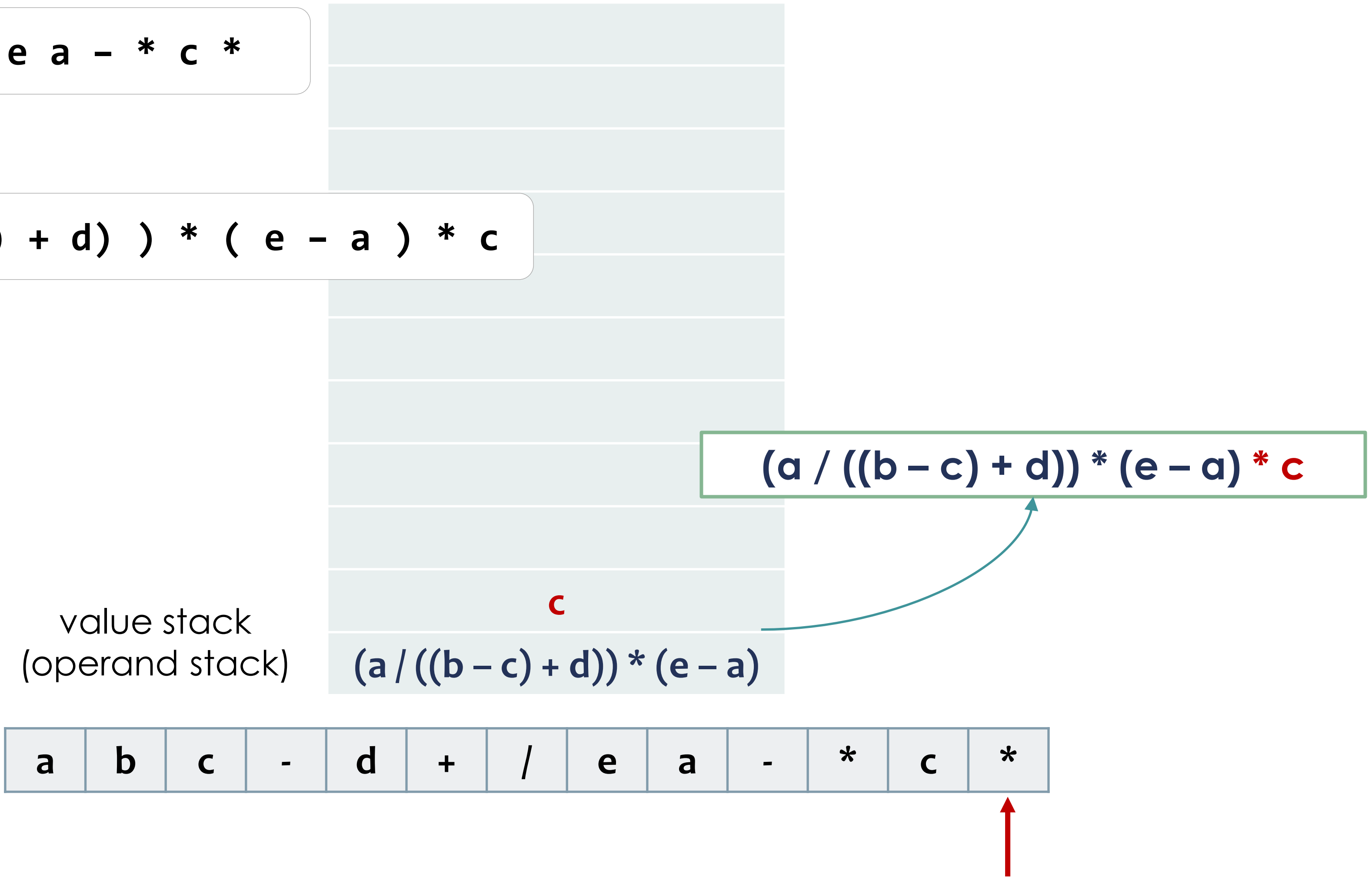


Postfix Evaluation

◈ **Goal**: Evaluate postfix expressions.

```
a b c – d + / e a – * c *
```

```
( a / ((b – c) + d) ) * ( e – a ) * c
```



value stack
(operand stack)
c
(a / ((b – c) + d)) * (e – a)
(a / ((b – c) + d)) * (e – a) * c
a
b
c
-
d
+
/
e
a
-
*
c
*

Extra Examples - Infix to Postfix Conversion

◈ **Example 1:** 3+4*5/6

in	stack(bottom to top)	postfix
3		3
+	+	
4		3 4
*	+ *	
5		3 4 5
/	+ /	3 4 5 *
6		3 4 5 * 6
		3 4 5 * 6 / +

1. Operands are output immediately
2. Push "(" always and operators in general.
3. For ")", pop until "(". Discard "(" and ")".
4. For higher precedence operator, push it.
5. For lower or equal precedence operator, pop them until "(" and push it.

Extra Examples - Infix to Postfix Conversion

◈ **Example 2:** (1+3)*(4-2)/(5+7)

in	stack (bottom to top)	postfix
(	(	
1		1
+	(+	
3		1 3
)		1 3 +
*	*	
(	* (	
4		1 3 + 4
-	* (-	
2		1 3 + 4 2
)	*	1 3 + 4 2 -
/	/	1 3 + 4 2 - *

in	stack	postfix
(	/ (	1 3 + 4 2 - *
5		1 3 + 4 2 - * 5
+	/ (+	
7		1 3 + 4 2 - * 5 7
)		1 3 + 4 2 - * 5 7 +
		1 3 + 4 2 - * 5 7 + /

1. Operands are output immediately
2. Push "(" always and operators in general.
3. For ")", pop until "(". Discard "(" and ")".
4. For higher precedence operator, push it.
5. For lower or equal precedence operator, pop them until "(" and push it.

Extra Examples - Infix to Postfix Conversion

◈ **Example 3:** a – (b + c * d) / e

in	stack(bottom to top)	postfix
a		
-		
(		
b		
+		
c		
*		
d		
)		
/		
e		

1. Operands are output immediately
2. Push "(" always and operators in general.
3. For ")", pop until "(". Discard "(" and ")".
4. For higher precedence operator, push it.
5. For lower or equal precedence operator, pop them until "(" and push it.

Extra Examples - Infix to Postfix Conversion

◈ **Example 4:** A * (B + C * D) + E

	in	stack(bottom to top)	postfix
1	A		
2	*		
3	(		
4	B		
5	+		
6	C		
7	*		
8	D		
9	)		
10	+		
11	E		
12			

1. Operands are output immediately
2. Push "(" always and operators in general.
3. For ")", pop until "(". Discard "(" and ")".
4. For higher precedence operator, push it.
5. For lower or equal precedence operator, pop them until "(" and push it.

Extra Examples - Infix to Postfix

- **Example 5:**
  - A + (B * C - (D/E^F) * G) * H
  - where ^ is an exponential operator.

1. Operands are output immediately
2. Push "(" always and operators in general.
3. For ")", pop until "(". Discard "(" and ")".
4. For higher precedence operator, push it.
5. For lower or equal precedence operator, pop them until "(" and push it.

	in	stack	postfix	
1	A		A	
2	+	+		
3	(	+ (		
4	B		A B	
5	*	+ (*		
6	C		A B C	
7	-	+ (-	A B C *	(5)
8	(	+ (- (	A B C *	
9	D		A B C * D	
10	/	+ (- (/		
11	E		A B C * D E	
12	^	+ (- (/ ^		(4)
13	F		A B C * D E F	
14	)	+ (-	A B C * D E F ^ /	(3)
15	*	+ (- *		
16	G		A B C * D E F ^ / G	
17	)	+	A B C * D E F ^ / G * -	(3)
18	*	+ *		
19	H		A B C * D E F ^ / G * - H	
20			A B C * D E F ^ / G * - H * +	