Week 7(1/3)

# Feed-forward Neural Network

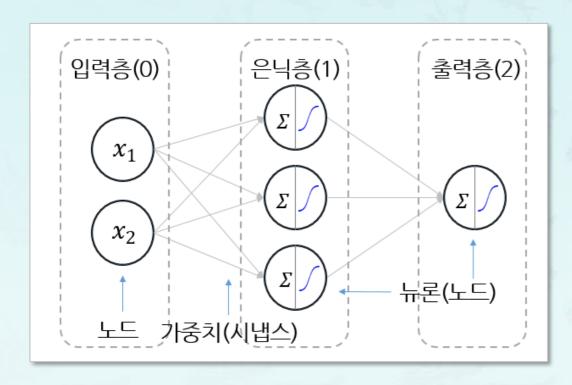
Machine Learning with Python

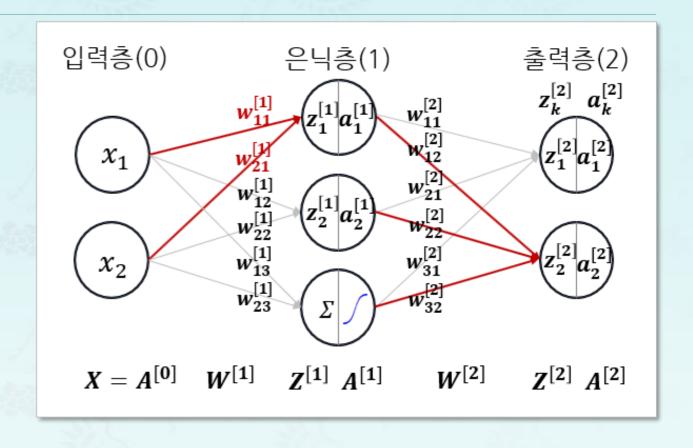
Handong Global University Prof. Youngsup Kim idebtor@gmail.com

#### **Feed-forward Neural Network**

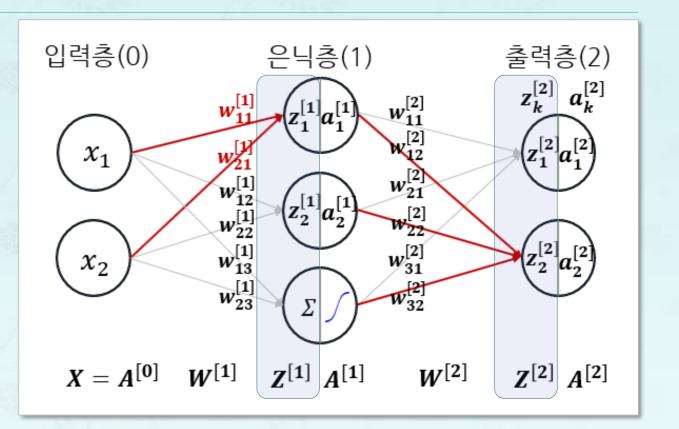
- Objectives
  - Understanding Feed-forward NN Processing.
- Contents
  - Feed-forward NN Notations
  - Feed-forward NN Signal Processing
  - Weights Notation
  - Feed-forward NN Example

### Multi-layer Neural Network

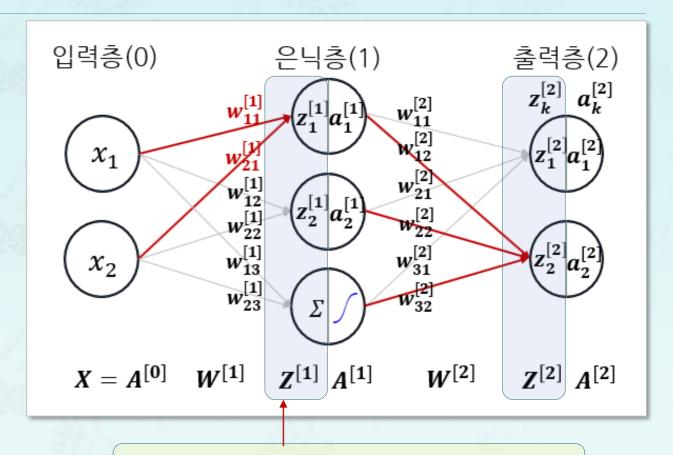




- Z: Input to Neuron
- A: Output from Neuron
- L: Total Number Of Layers
- I: Layer Number(lowercase L)

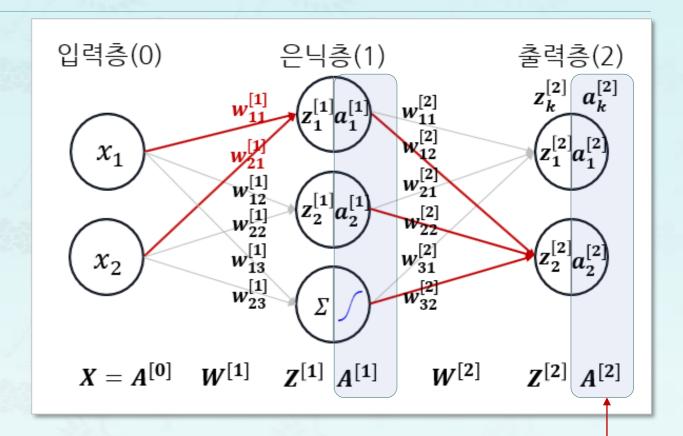


- Z: Input to Neuron
- A: Output from Neuron
- L: Total Number Of Layers
- I: Layer Number(lowercase L)



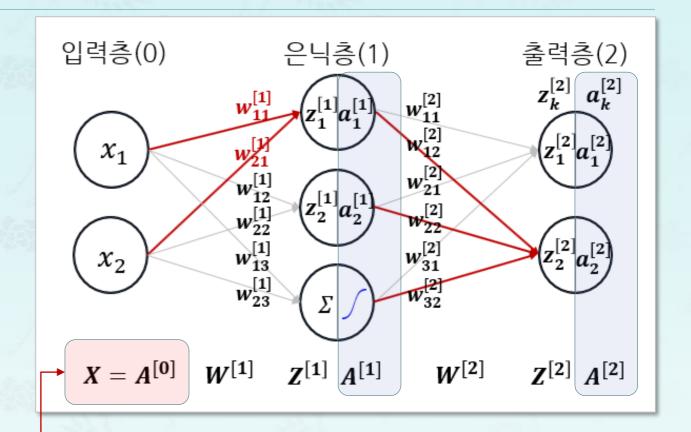
Z<sup>[1]</sup>: Input to Hidden Layer(1)

- Z: Input to Neuron
- A: Output from Neuron
- L: Total Number Of Layers
- I: Layer Number(lowercase L)



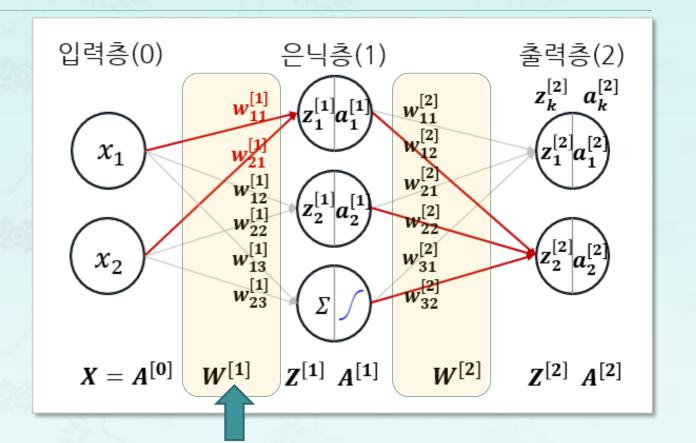
- Z<sup>[1]</sup>: Input to Hidden Layer(1)
- A<sup>[2]</sup>: Output from Output Layer(2)

- Z: Input to Neuron
- A: Output from Neuron
- L: Total Number Of Layers
- I: Layer Number(lowercase L)

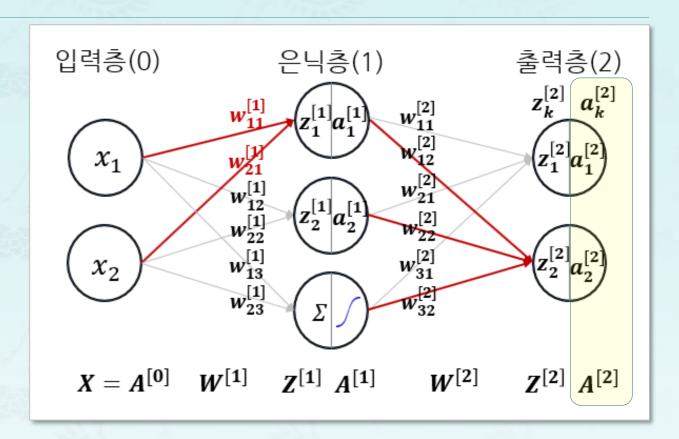


- Z<sup>[1]</sup>: Input to Hidden Layer(1)
- A<sup>[2]</sup>: Output from Output Layer(2)
  - A<sup>[0]</sup>: Output from Input Layer(0)

- Z: Input to Neuron
- A: Output from Neuron
- L: Total Number Of Layers
- I: Layer Number
- W: Weights

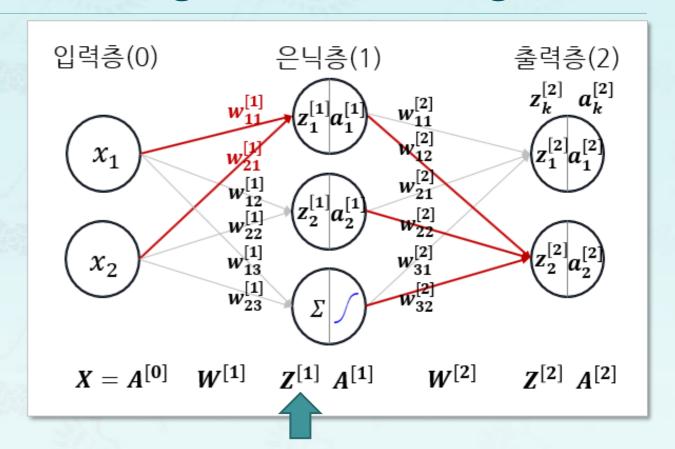


- Z: Input to Neuron
- A: Output from Neuron
- L: Total Number Of Layers
- I: Layer Number
- W: Weights
- $\hat{y}$ : Output predicted

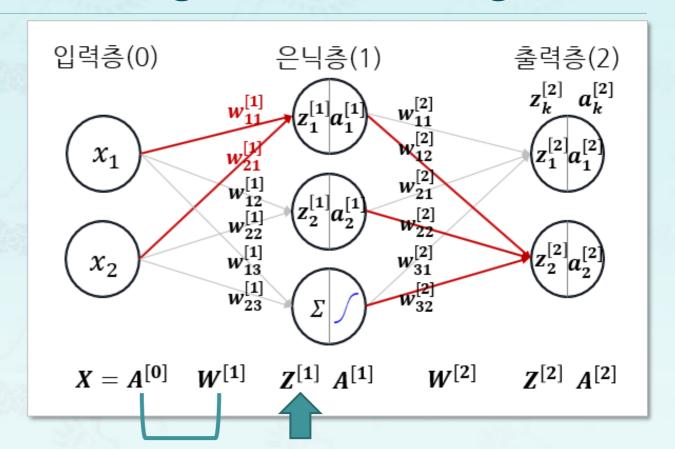


$$\widehat{y} = A^{[2]}$$

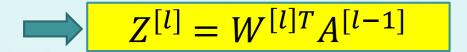
- **Z**:  $\sum$  (Weights \* Input)
  - Net input or Weighted sum

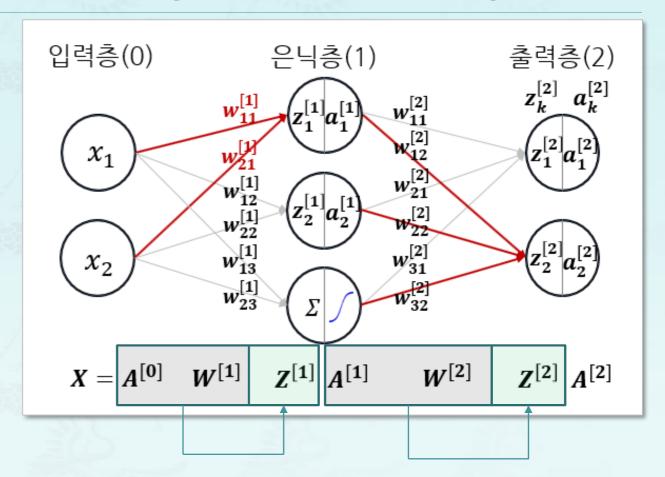


- **Z**:  $\sum$  (Weights \* Input)
  - Net input
  - net input or weighted sum



- **Z**:  $\sum$  (Weights \* Input)
  - Net input
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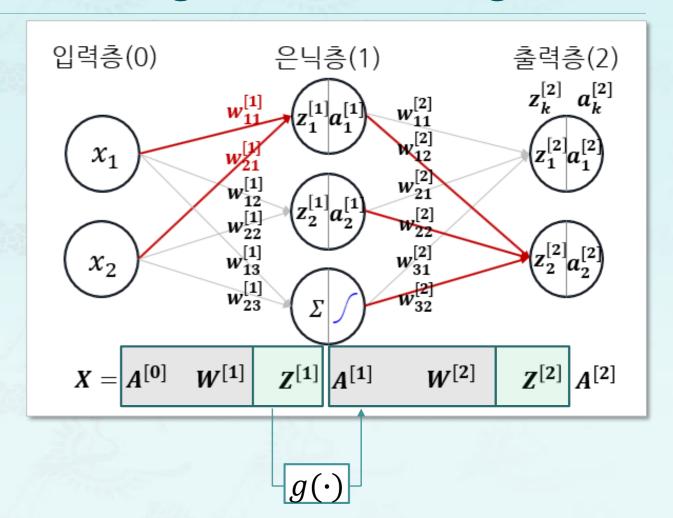


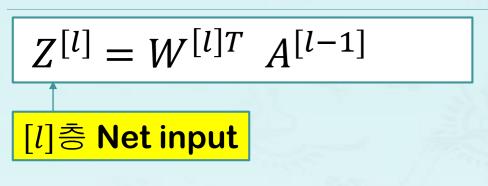


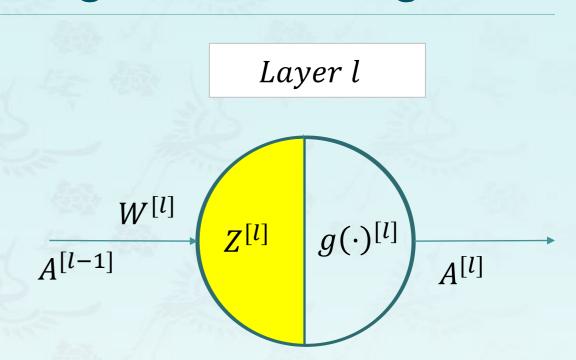
- **Z**:  $\sum$  (Weights \* Input)
  - Net input
  - net input or weighted sum

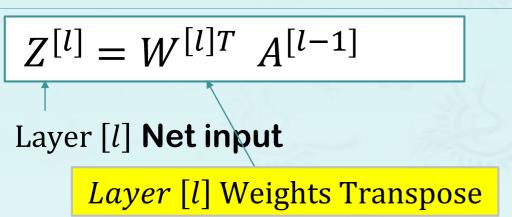
$$Z^{[l]} = W^{[l]T}A^{[l-1]}$$

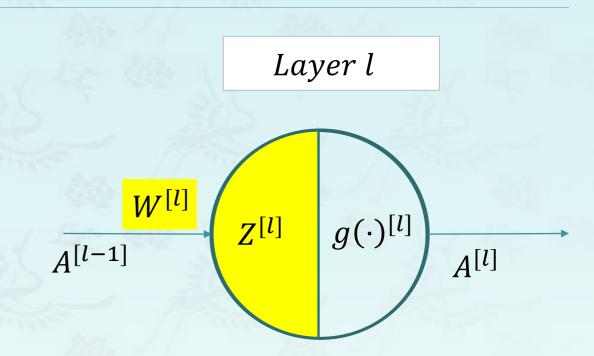
$$A^{[l]} = g(Z^{[l]})$$
Activation Function

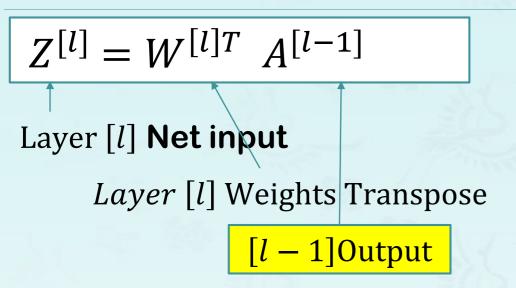


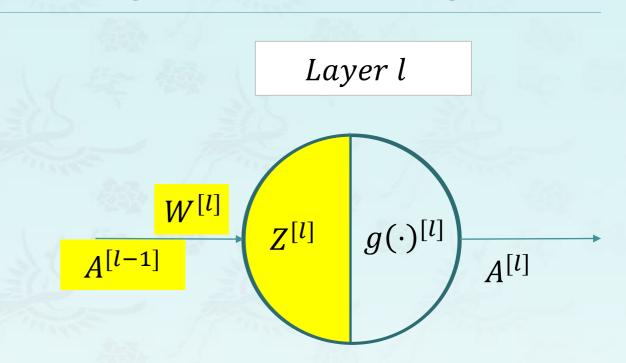


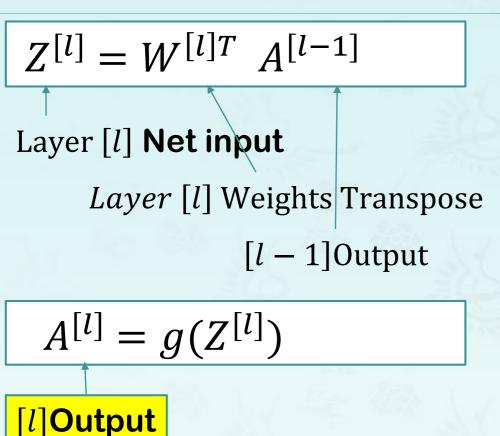


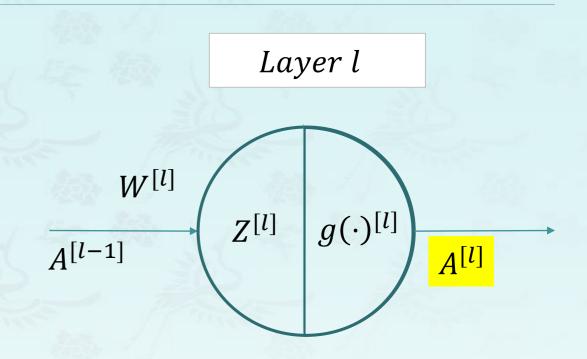


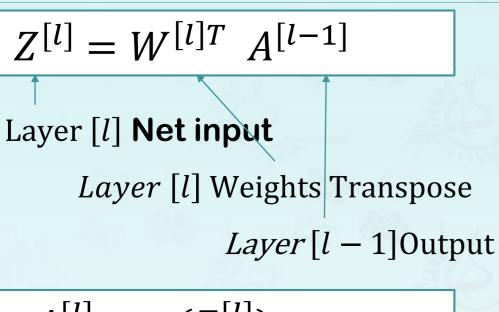


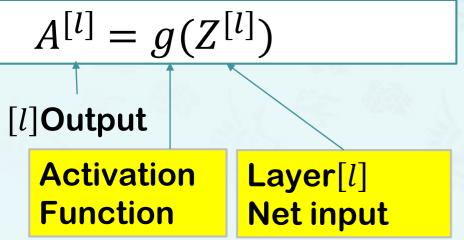


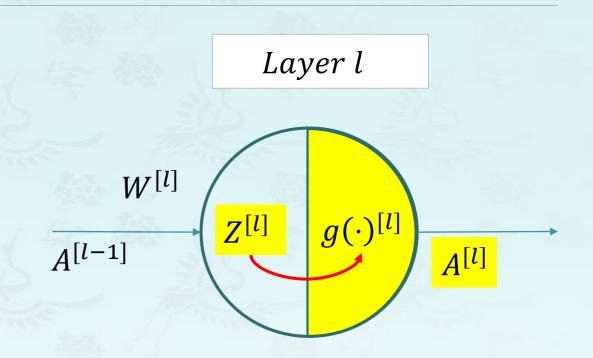




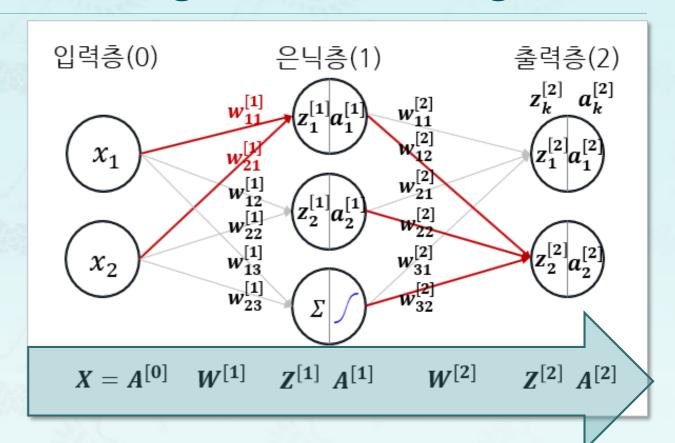








- **Z**:  $\sum$  (Weights \* Input)
  - Net input
  - net input or weighted sum



$$\mathbf{Z}^{[l]} = W^{[l]T} A^{[l-1]}$$

$$\mathbf{Z}^{[l]} = W^{[l]T} A^{[l-1]}$$
 
$$\mathbf{W}^{(l)} = \begin{pmatrix} w_{11}^{(l)} & w_{12}^{(l)} & w_{13}^{(l)} \\ w_{21}^{(l)} & w_{22}^{(l)} & w_{23}^{(l)} \end{pmatrix}$$
 Layer I Weights  $w_{21}^{(l)} = w_{22}^{(l)} + w_{23}^{(l)} + w_{23}^{(l)}$ 

$$\mathbf{Z}^{[l]} = W^{[l]T} A^{[l-1]}$$

$$\mathbf{W}^{(l)} = \begin{pmatrix} w_{11}^{(l)} & w_{12}^{(l)} & w_{13}^{(l)} \\ w_{21}^{(l)} & w_{22}^{(l)} & w_{23}^{(l)} \end{pmatrix}$$
Layer I Weights  $w_{21}^{(l)}$   $w_{22}^{(l)}$   $w_{23}^{(l)}$ 

$$\mathbf{W}^{(1)} = \begin{pmatrix} w_{11}^{(1)} & w_{12}^{(1)} & w_{13}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} & w_{23}^{(1)} \end{pmatrix}$$

## 2. Weights Notation: Wij Style

$$\mathbf{Z}^{[l]} = W^{[l]T} A^{[l-1]}$$

$$\mathbf{W}^{(l)} = \begin{pmatrix} w_{11}^{(l)} & w_{12}^{(l)} & w_{13}^{(l)} \\ w_{21}^{(l)} & w_{22}^{(l)} & w_{23}^{(l)} \end{pmatrix}$$
Layer I Weights  $w_{21}^{(l)}$   $w_{22}^{(l)}$   $w_{23}^{(l)}$ 

$$\mathbf{W}^{(1)} = \begin{pmatrix} w_{11}^{(1)} & w_{12}^{(1)} & w_{13}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} & w_{23}^{(1)} \end{pmatrix}$$



$$\mathbf{Z}^{[1]} = W^{[1]T} A^{[0]}$$

## 2. Weights Notation: Wii Style

$$\mathbf{Z}^{[l]} = W^{[l]T} A^{[l-1]}$$

$$\mathbf{W}^{(l)} = \begin{pmatrix} w_{11}^{(l)} & w_{12}^{(l)} & w_{13}^{(l)} \\ w_{21}^{(l)} & w_{22}^{(l)} & w_{23}^{(l)} \end{pmatrix}$$
 Layer I Weights  $w_{21}^{(l)}$   $w_{22}^{(l)}$   $w_{23}^{(l)}$ 

$$\mathbf{W}^{(1)} = \begin{pmatrix} w_{11}^{(1)} & w_{12}^{(1)} & w_{13}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} & w_{23}^{(1)} \end{pmatrix}$$



$$\mathbf{Z}^{[1]} = W^{[1]T} A^{[0]}$$

$$= \begin{pmatrix} w_{11}^{(1)} & w_{12}^{(1)} & w_{13}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} & w_{23}^{(1)} \end{pmatrix}^{T} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\mathbf{Z}^{[l]} = W^{[l]T} A^{[l-1]}$$

$$\mathbf{W}^{(l)} = \begin{pmatrix} w_{11}^{(l)} & w_{12}^{(l)} & w_{13}^{(l)} \\ w_{21}^{(l)} & w_{22}^{(l)} & w_{23}^{(l)} \end{pmatrix}$$
Layer I Weights  $w_{21}^{(l)}$   $w_{22}^{(l)}$   $w_{23}^{(l)}$ 

$$\mathbf{W}^{(1)} = \begin{pmatrix} w_{11}^{(1)} & w_{12}^{(1)} & w_{13}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} & w_{23}^{(1)} \end{pmatrix}$$
Hidden Layer Weights

$$\mathbf{Z}^{[1]} = W^{[1]T} A^{[0]}$$

$$= \begin{pmatrix} w_{11}^{(1)} & w_{12}^{(1)} & w_{13}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} & w_{23}^{(1)} \end{pmatrix}^{T} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= \begin{pmatrix} w_{11}^{(1)} & w_{21}^{(1)} \\ w_{11}^{(1)} & w_{21}^{(1)} \\ w_{12}^{(1)} & w_{22}^{(1)} \\ w_{13}^{(1)} & w_{23}^{(1)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

## 2. Weights Notation: Wij Style

$$\mathbf{Z}^{[l]} = W^{[l]T} A^{[l-1]}$$

$$\mathbf{W}^{(l)} = \begin{pmatrix} w_{11}^{(l)} & w_{12}^{(l)} & w_{13}^{(l)} \\ w_{21}^{(l)} & w_{22}^{(l)} & w_{23}^{(l)} \end{pmatrix}$$
Layer I Weights  $w_{21}^{(l)}$   $w_{22}^{(l)}$   $w_{23}^{(l)}$ 

$$\mathbf{W}^{(1)} = \begin{pmatrix} w_{11}^{(1)} & w_{12}^{(1)} & w_{13}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} & w_{23}^{(1)} \end{pmatrix}$$

$$\mathbf{Z}^{[1]} = W^{[1]T} A^{[0]}$$

$$= \begin{pmatrix} w_{11}^{(1)} & w_{12}^{(1)} & w_{13}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} & w_{23}^{(1)} \end{pmatrix}^{T} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= \begin{pmatrix} w_{11}^{(1)} & w_{21}^{(1)} \\ w_{12}^{(1)} & w_{22}^{(1)} \\ w_{13}^{(1)} & w_{23}^{(1)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= \begin{pmatrix} w_{11}^{(1)} x_1 + w_{21}^{(1)} x_2 \\ w_{12}^{(1)} x_1 + w_{22}^{(1)} x_2 \\ w_{13}^{(1)} x_1 + w_{23}^{(1)} x_2 \end{pmatrix} = \begin{pmatrix} z_{1}^{(1)} \\ z_{2}^{(1)} \\ z_{3}^{(1)} \end{pmatrix}$$

$$\mathbf{Z}^{[l]} = W^{[l]T} A^{[l-1]}$$

$$\mathbf{W}^{(l)} = \begin{pmatrix} w_{11}^{(l)} & w_{12}^{(l)} & w_{13}^{(l)} \\ w_{21}^{(l)} & w_{22}^{(l)} & w_{23}^{(l)} \end{pmatrix}$$

$$\mathbf{W}^{(1)} = \begin{pmatrix} w_{11}^{(1)} & w_{12}^{(1)} & w_{13}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} & w_{23}^{(1)} \end{pmatrix}$$

### Hidden Layer Net input ${}^{|T}A^{[0]}$

$$= \begin{pmatrix} w_{11}^{(1)} & w_{12}^{(1)} & w_{13}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} & w_{23}^{(1)} \end{pmatrix}^{T} \begin{pmatrix} x_{1} \\ x_{2} \end{pmatrix}$$

$$= \begin{pmatrix} w_{11}^{(1)} & w_{21}^{(1)} \\ w_{12}^{(1)} & w_{22}^{(1)} \\ w_{13}^{(1)} & w_{23}^{(1)} \end{pmatrix} \begin{pmatrix} x_{1} \\ x_{2} \end{pmatrix}$$

$$= \begin{pmatrix} w_{11}^{(1)} x_{1} + w_{21}^{(1)} x_{2} \\ w_{13}^{(1)} x_{1} + w_{22}^{(1)} x_{2} \\ w_{13}^{(1)} x_{1} + w_{23}^{(1)} x_{2} \end{pmatrix} = \begin{pmatrix} z_{1}^{(1)} \\ z_{2}^{(1)} \\ z_{3}^{(1)} \end{pmatrix}$$

**Hidden Layer Net inp** 

## 2. Weights Notation: Wii Style

#### **Hidden Layer Net input**

$$A^{[1]} = g(Z^{[1]}) = \begin{pmatrix} a_1^{[1]} \\ a_2^{[1]} \\ a_3^{[1]} \end{pmatrix}$$

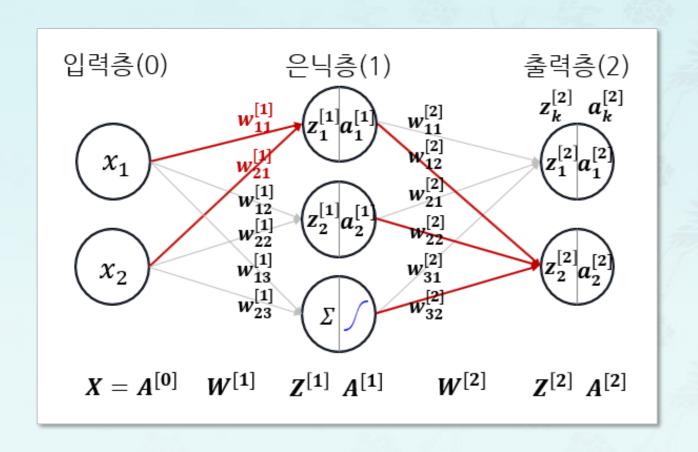
**Activation Function** 

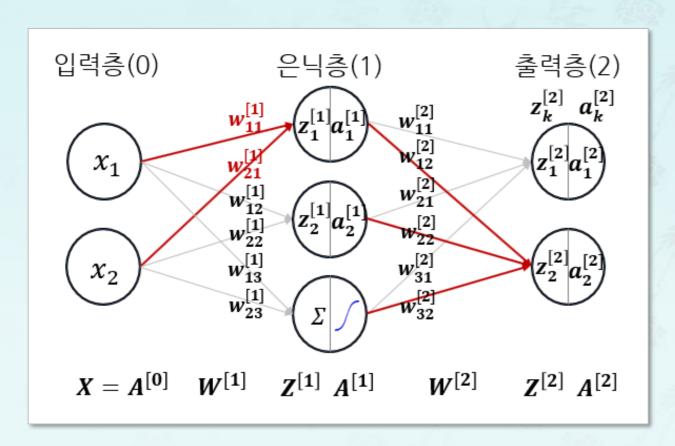
$$\mathbf{Z}^{[1]} = W^{[1]T} A^{[0]}$$

$$= \begin{pmatrix} w_{11}^{(1)} & w_{12}^{(1)} & w_{13}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} & w_{23}^{(1)} \end{pmatrix}^T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= \begin{pmatrix} w_{11}^{(1)} & w_{21}^{(1)} \\ w_{12}^{(1)} & w_{22}^{(1)} \\ w_{13}^{(1)} & w_{23}^{(1)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

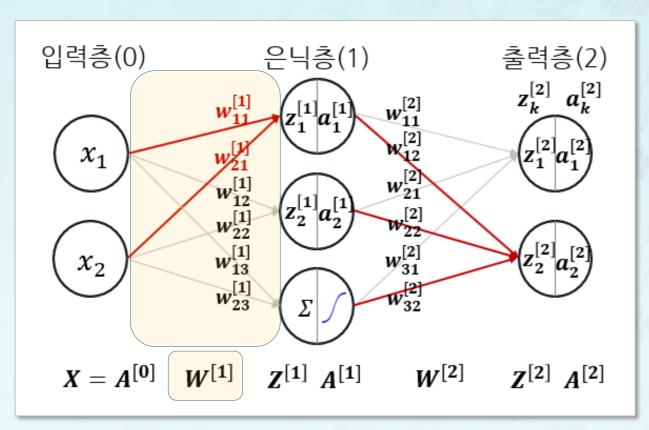
$$= \begin{pmatrix} w_{11}^{(1)} x_1 + w_{21}^{(1)} x_2 \\ w_{12}^{(1)} x_1 + w_{21}^{(1)} x_2 \\ w_{13}^{(1)} x_1 + w_{23}^{(1)} x_2 \end{pmatrix} = \begin{pmatrix} z_1^{(1)} \\ z_2^{(1)} \\ z_3^{(1)} \end{pmatrix}$$





## • $W_{ij}^{T}$ Shape

Layer l Node Nums **x** Layer (l-1) Node Nums

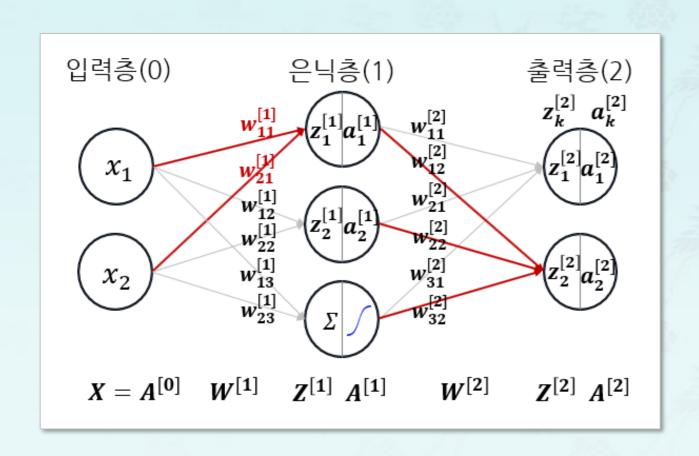


## • $W_{ij}^{T}$ Shape

- Layer l Node Nums x Layer (l − 1)Node Nums
- $W^1$ .shape = (3,2)

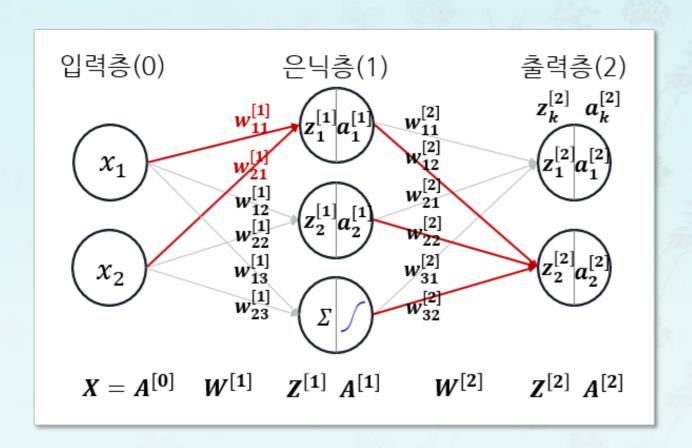
$$W^{[1]} = \begin{pmatrix} w_{11}^{[1]} & w_{21}^{[1]} \\ w_{12}^{[1]} & w_{22}^{[1]} \\ w_{13}^{[1]} & w_{23}^{[1]} \end{pmatrix}$$

$$W^{[2]} = \begin{pmatrix} w_{11}^{[2]} & w_{21}^{[2]} & w_{31}^{[2]} \\ w_{12}^{[2]} & w_{22}^{[2]} & w_{32}^{[2]} \end{pmatrix}$$



$$\mathbf{Z}^{[l]} = W^{[l]} A^{[l-1]}$$

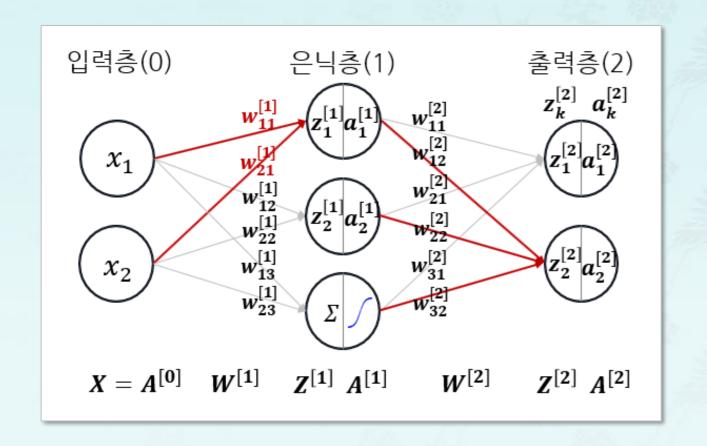
$$\mathbf{Z}^{[l]} = W^{[l]} A^{[l-1]}$$



$$\mathbf{Z}^{[l]} = W^{[l]} A^{[l-1]}$$

$$\mathbf{Z}^{[l]} = W^{[l]}A^{[l-1]}$$

$$\mathbf{Z}^{[1]} = W^{[1]}A^{[0]}$$

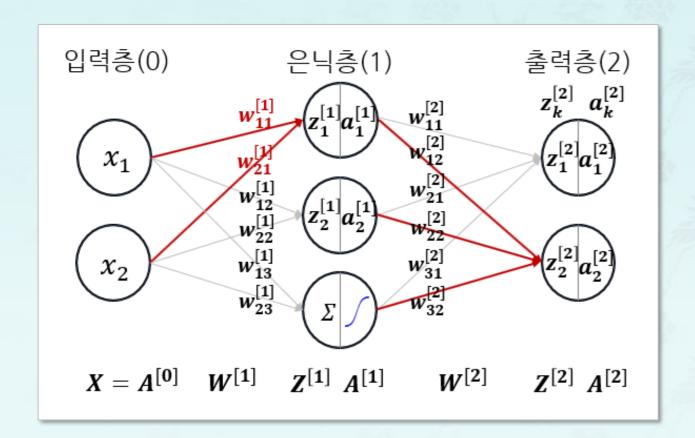




$$\mathbf{Z}^{[1]} = W^{[1]} A^{[0]}$$

$$= \begin{pmatrix} w_{11}^{(1)} & w_{21}^{(1)} \\ w_{12}^{(1)} & w_{22}^{(1)} \\ w_{13}^{(1)} & w_{23}^{(1)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\mathbf{Z}^{[l]} = W^{[l]} A^{[l-1]}$$





$$\mathbf{Z}^{[1]} = W^{[1]} A^{[0]}$$

$$= \begin{pmatrix} w_{11}^{(1)} & w_{21}^{(1)} \\ w_{12}^{(1)} & w_{22}^{(1)} \\ w_{13}^{(1)} & w_{23}^{(1)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

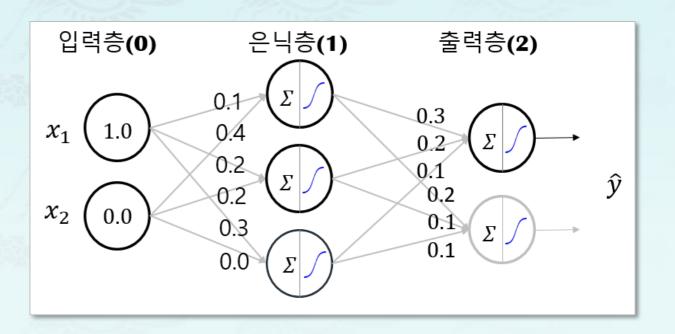
$$= \begin{pmatrix} z_{1}^{(1)} \\ z_{2}^{(1)} \\ z_{3}^{(1)} \end{pmatrix}$$

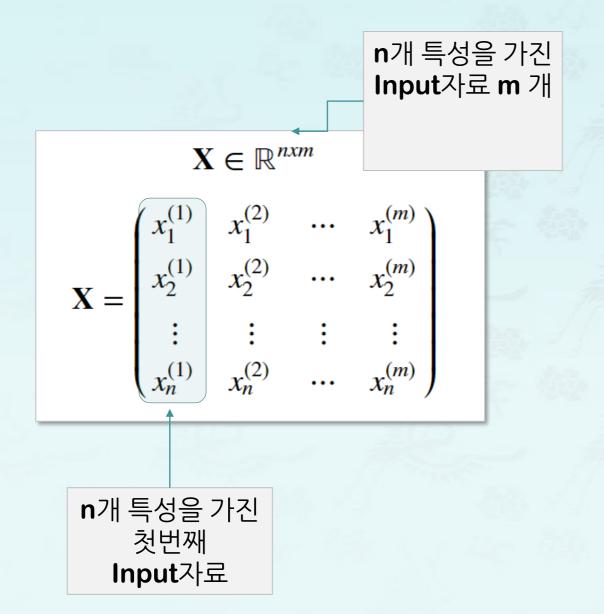
# 2. Weights Notation: $W_{ij}^T$ Style(or $W_{ji}$ Style)

•  $W_{ij}$  와  $W_{ij}^T$  Notations

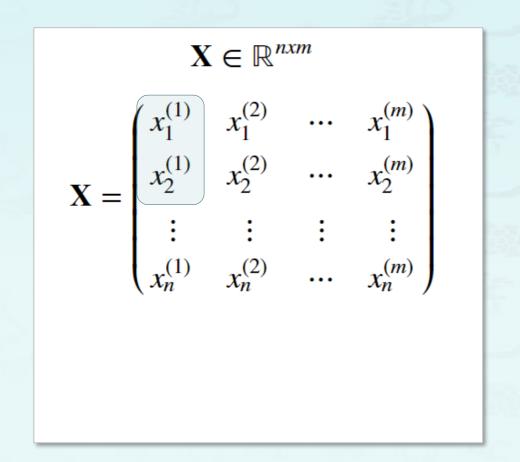
# 3. Feed-forward NN Example: $W_{ij}^T$ Notations

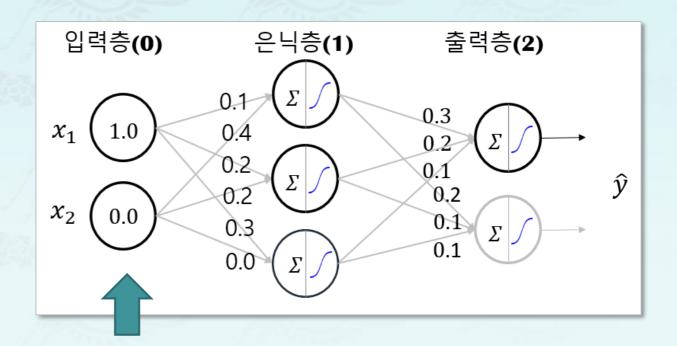
•  $W_{ij} \hookrightarrow W_{ij}^T$  Notations





• Input X: m = 1, n = 2





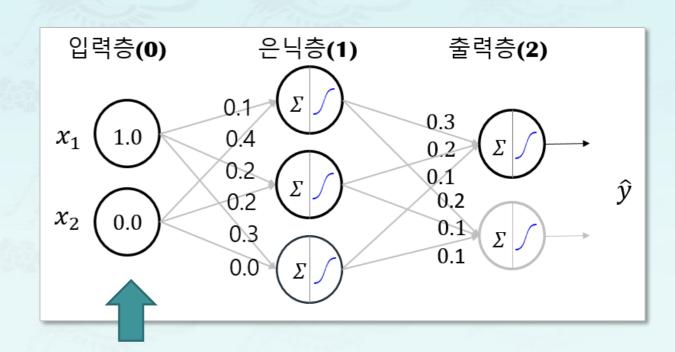
• Input X: m = 1, n = 2

$$\mathbf{X} \in \mathbb{R}^{n \times m}$$

$$\mathbf{X} = \begin{bmatrix} x_1^{(1)} & x_1^{(2)} & \cdots & x_1^{(m)} \\ x_2^{(1)} & x_2^{(2)} & \cdots & x_2^{(m)} \\ \vdots & \vdots & \vdots & \vdots \\ x_n^{(1)} & x_n^{(2)} & \cdots & x_n^{(m)} \end{bmatrix}$$

$$\mathbf{x}^{(1)} = \begin{pmatrix} x_1^{(1)} \\ x_2^{(1)} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Weights Initialization



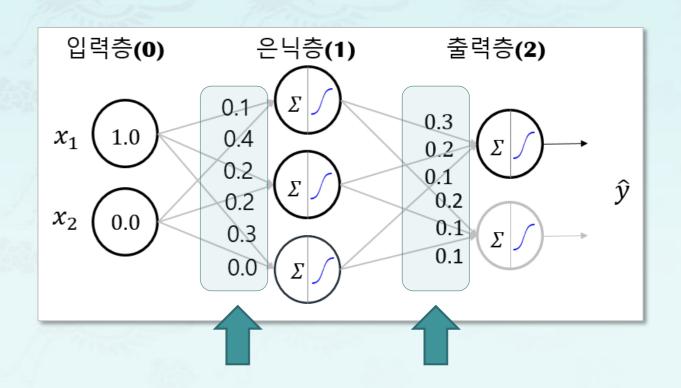
• Input X: m = 1, n = 2

$$\mathbf{X} \in \mathbb{R}^{n \times m}$$

$$\mathbf{X} = \begin{pmatrix} x_1^{(1)} & x_1^{(2)} & \cdots & x_1^{(m)} \\ x_2^{(1)} & x_2^{(2)} & \cdots & x_2^{(m)} \\ \vdots & \vdots & \vdots & \vdots \\ x_n^{(1)} & x_n^{(2)} & \cdots & x_n^{(m)} \end{pmatrix}$$

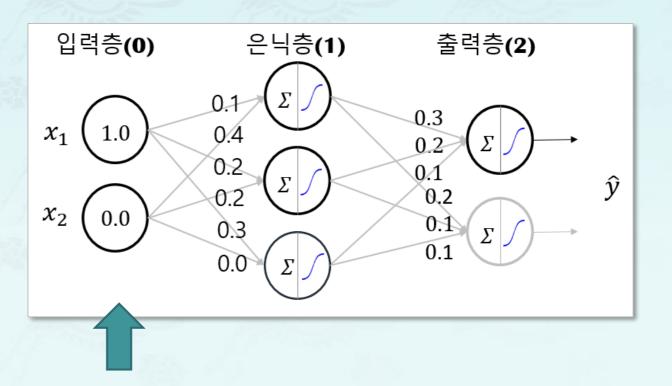
$$\mathbf{x}^{(1)} = \begin{pmatrix} x_1^{(1)} \\ x_2^{(1)} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

#### Weights Initialization



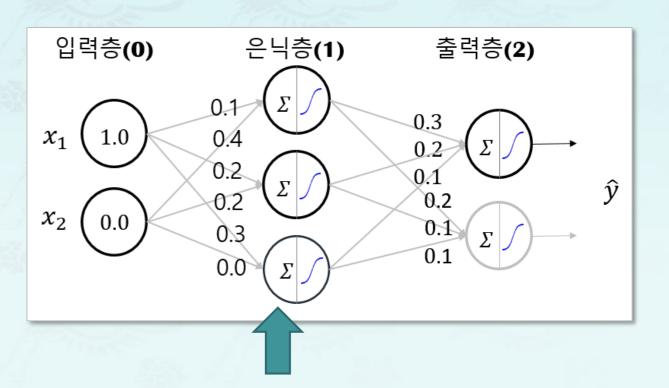
## 4. Feed-forward NN Computation: Input층

•  $A^{[0]} = X$ 

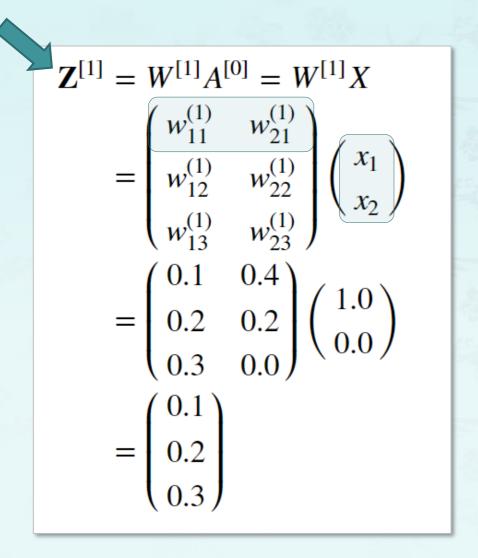


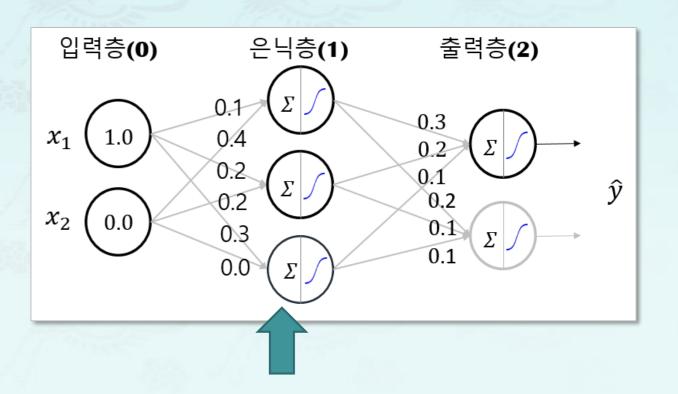
### 4. Feed-forward NN Computation: Hidden Layer

- $Z^{[l]} = W^{[l]}A^{[l-1]}$
- $A^{[i]} = g(Z^{[i]})$

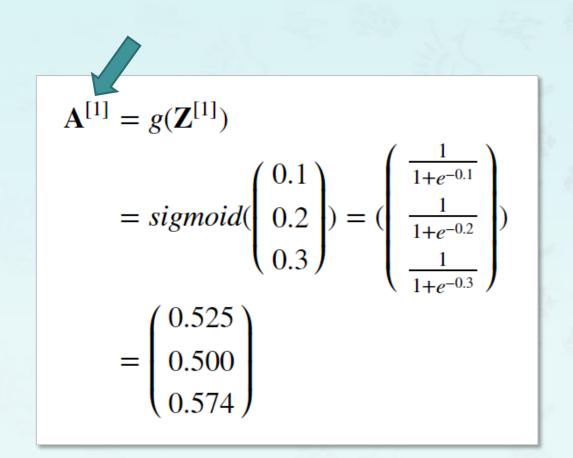


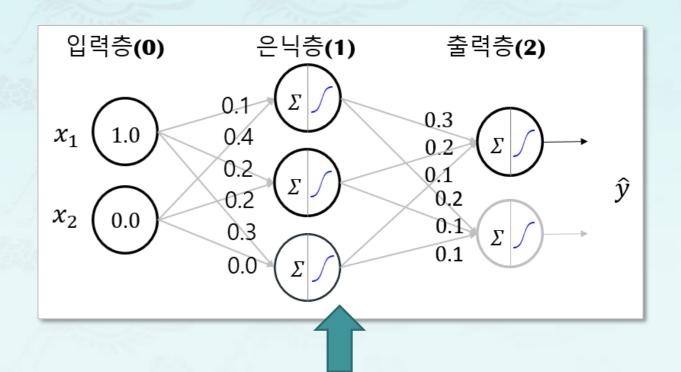
### 4. Feed-forward NN Computation: Hidden Layer



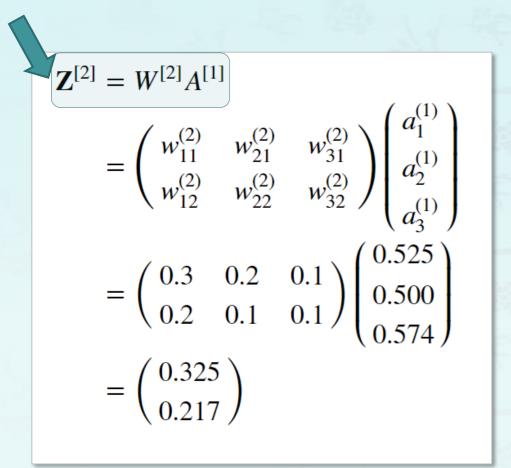


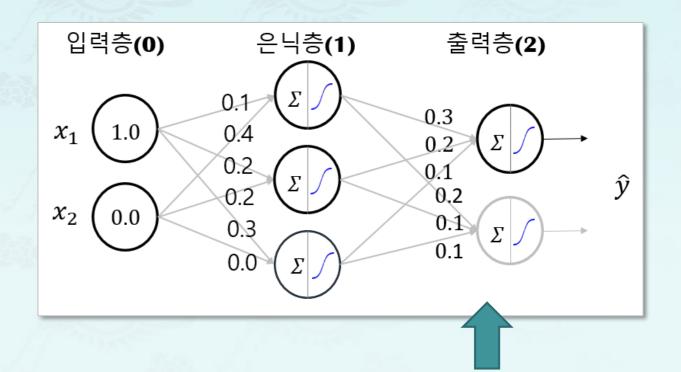
### 4. Feed-forward NN Computation: Hidden Layer



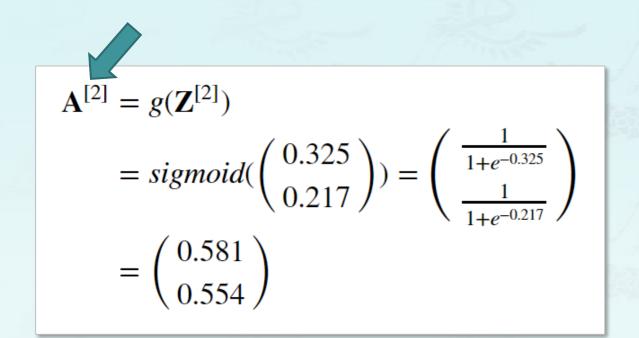


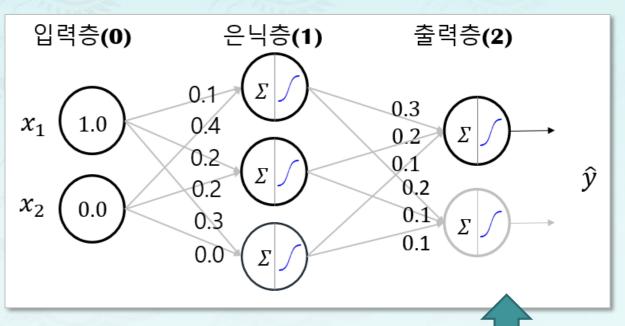
#### 4. Feed-forward NN Computation: Output Layer





## 4. Feed-forward NN Computation: Output Layer





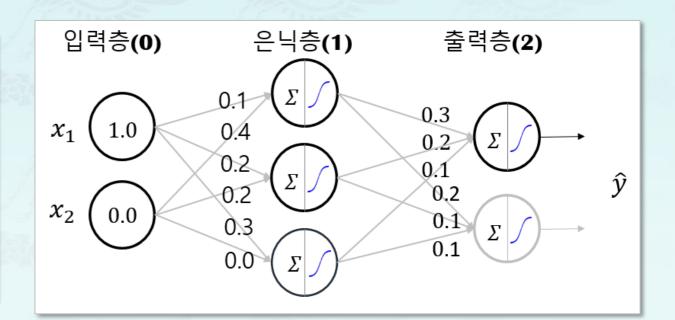
#### 4. Feed-forward NN Computation: Output Layer

#### Output Layer(2)

$$\mathbf{A}^{[2]} = g(\mathbf{Z}^{[2]})$$

$$= sigmoid(\begin{pmatrix} 0.325 \\ 0.217 \end{pmatrix}) = \begin{pmatrix} \frac{1}{1+e^{-0.325}} \\ \frac{1}{1+e^{-0.217}} \end{pmatrix}$$

$$= \begin{pmatrix} 0.581 \\ 0.554 \end{pmatrix}$$





$$\hat{y} = \begin{pmatrix} \hat{y_1} \\ \hat{y_2} \end{pmatrix} = \begin{pmatrix} 0.581 \\ 0.554 \end{pmatrix}$$

#### **Feed-forward Neural Network**

- Summary
  - Feed-forward NN Notation
  - Feed-forward NN Processing
  - Weights  $W_{iJ}$  and  $W_{ij}^{T}$  Notation Styles
  - Feed-forward NN Example

7-2 Feed-forward NN Example

Week 7(1/3)

# Feed-forward NN

Machine Learning with Python

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여러분 곁에 항상 열려 있는 K-MOOC 강의실에서 만나 뵙기를 바랍니다.