Week 3(2/3)

Derivatives

Machine Learning with Python

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Derivatives

- Goals
  - Understanding Derivatives
  - Finding Derivatives
  - Finding Maxima and Minima using Derivatives

- Content
  - Overview of Derivatives
  - Derivatives of Other Functions
  - Derivative Rules and Finding Min/Max

a good reference: https://www.mathsisfun.com/calculus/derivatives-introduction.html

Derivatives

- **Slope = the rate of change**

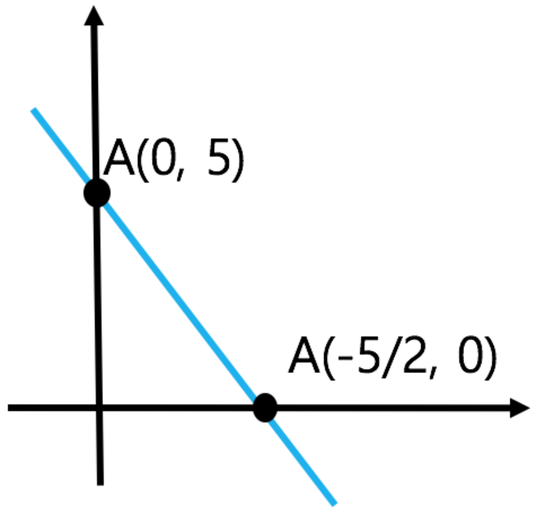
The derivative of a function is the rate of change of the output value with respect to its input value, whereas differential is the actual change of function.

미분**(Differential)**이란 어떤 함수로부터 도함수**(Derivative)**를 구하는 것이며, 미분계수는 어느 한 순간**(**점**)**에서 함수 값의 변화율 즉 순간 변화율을 뜻합니다.

Derivatives

- **Slope of a straight line**

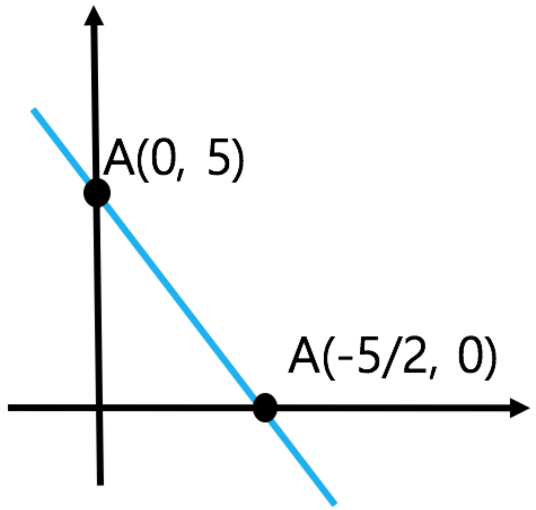
$$f(x) = \boxed{-2x} + 5$$



Derivatives

- **Slope of a straight line**

$$f(x) = -2x + 5$$



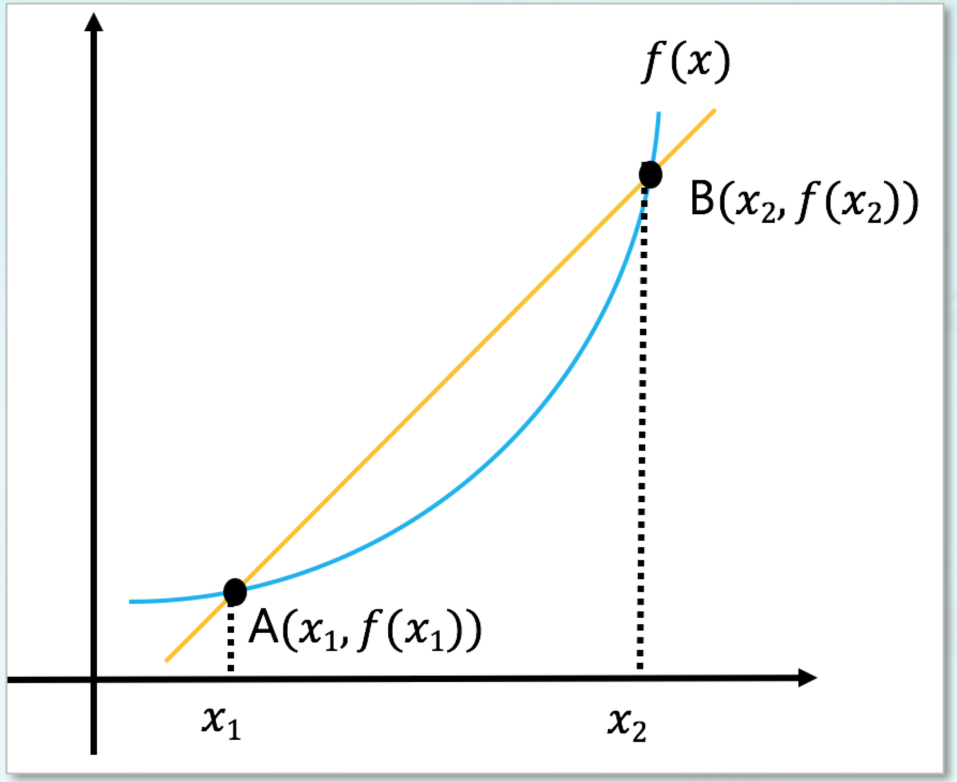
$$slope(d) = \frac{5-0}{0-\frac{5}{2}} = -2$$

Derivatives

- An average rate of change

Derivatives

- **An average rate of change**



$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

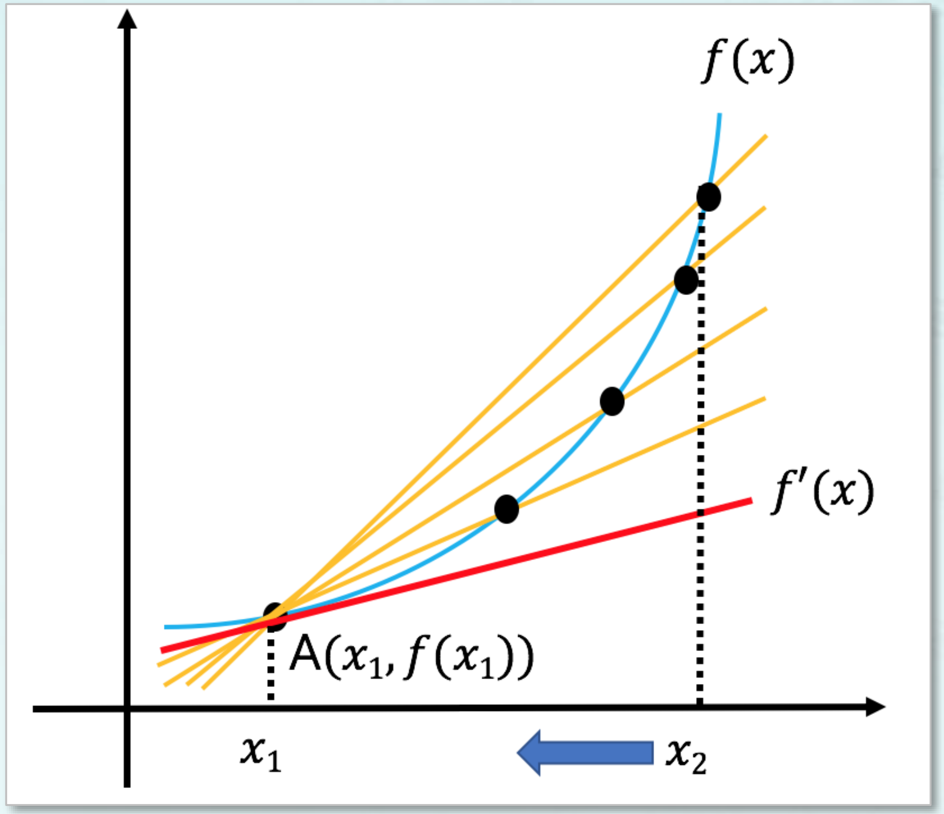
$average\ rate\ of\ \ change$

Derivatives

- **The instantaneous rate of change**

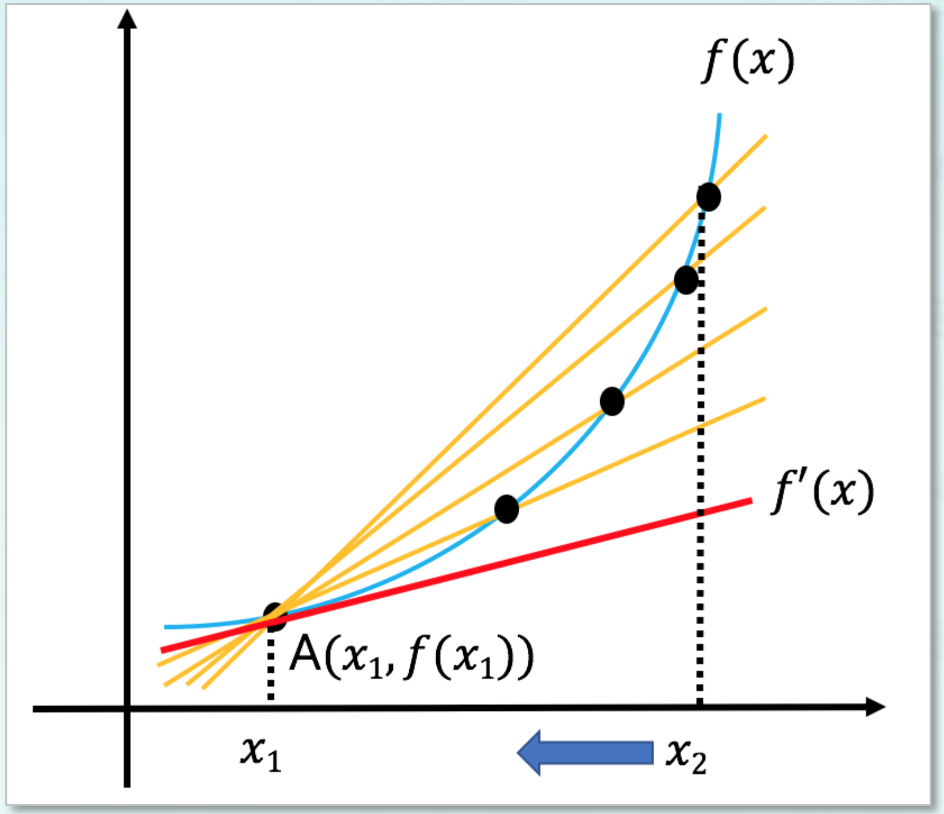
Derivatives

- **The instantaneous rate of change**



Derivatives

- **The instantaneous rate of change**

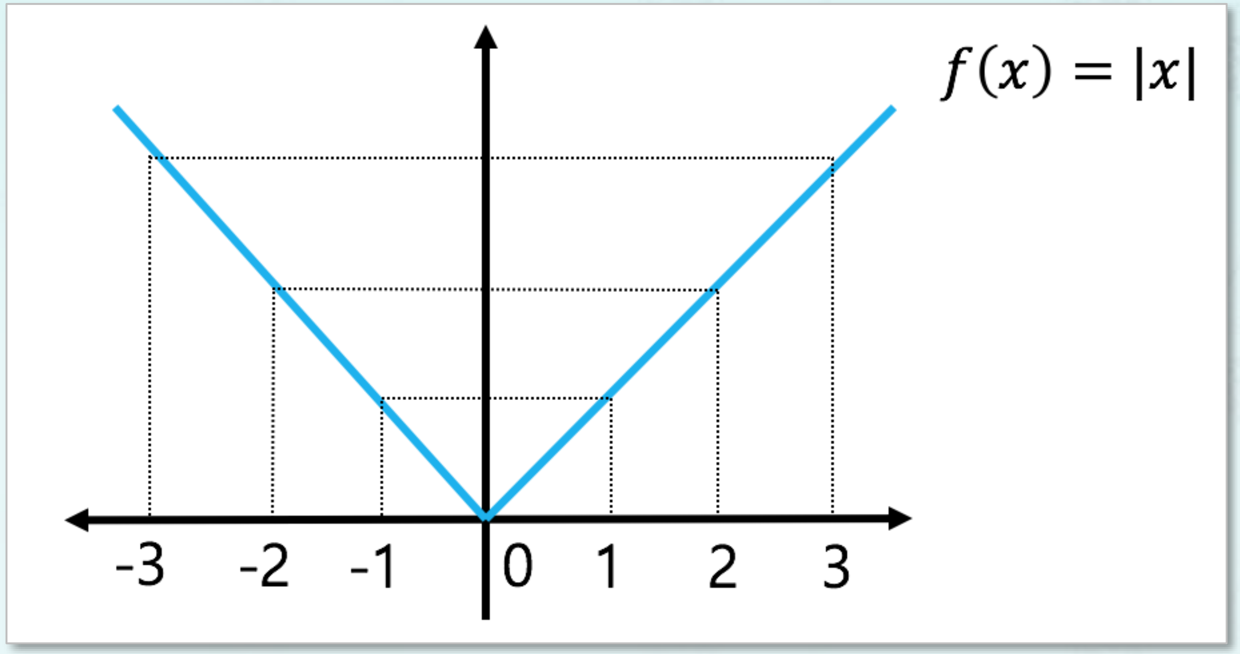


- **Derivatives**

$$\lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1} = f'(x_1)$$

Derivatives

- **The instantaneous rate of change**
  - **Differentiable**



$$\lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1} = f'(x_1)$$

Derivative Rules

- **General Rules**

Derivative Rules

- $f(x) = c$ 이면, $f'(x) = 0$
- $f(x) = cg(x)$ 이면, $f'(x) = cg'(x)$
- $f(x) = g(x) \pm t(x)$이면, $f'(x) = g'(x) \pm t'(x)$
- $f(x) = g(x)t(x)$ 이면, $f'(x) = g'(x)t(x) + g(x)t'(x)$
- $f(x) = \frac{t(x)}{g(x)}$ 이면, $f'(x) = \frac{t'(x)g(x) - t(x)g'(x)}{g^2(x)}$
- $f(x) = x^n$ 이면, $f'(x) = nx^{n-1}$

Derivative Rules

- $f(x) = \frac{t(x)}{g(x)}\ given, then$

$$f'(x) = \frac{t'(x)g(x)\ - t(x)g'(x)}{g^2(x)}$$

- **Example**
- $f(x) = \frac{1}{x},\quad f'(x) =?$

1. $t(x) = 1,\ \ g(x) = x$

Derivative Rules

- $f(x) = \frac{t(x)}{g(x)}$ $given, then$

$$f'(x) = \frac{t'(x)g(x) \ - t(x)g'(x)}{g^2(x)}$$

- **Example**
- $f(x) = \frac{1}{x}, \quad f'(x) = ?$

1. $t(x) = 1, \ \ g(x) = x$
2. $t'(x) = 0, \ g'(x) = 1$

Derivative Rules

- $f(x) = \frac{t(x)}{g(x)}\ given, then$

$$f'(x) = \frac{t'(x)g(x)\ - t(x)g'(x)}{g^2(x)}$$

- **Example**
- $f(x) = \frac{1}{x},\quad f'(x) = ?$

1. $t(x) = 1,\ \ g(x) = x$
2. $t'(x) = 0,\ g'(x) = 1$
3. $t'(x)g(x) - t(x)g'(x) = -1$

Derivative Rules

- $f(x) = \frac{t(x)}{g(x)}$ $given, then$

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- **Example**
- $f(x) = \frac{1}{x}, \quad f'(x) = ?$

1. $t(x) = 1, \;\; g(x) = x$
2. $t'(x) = 0, \; g'(x) = 1$
3. $t'(x)g(x) - t(x)g'(x) = -1$
4. $f'(x) = \frac{t'(x)g(x) - t(x)g'(x)}{g^2(x)} = \frac{-1}{x^2}$

Derivative Rules

- **Trigonometric functions**
  - $f(x) = \sin x$ 이면, $f'(x) = \cos x$
  - $f(x) = \cos x$ 이면, $f'(x) = -\sin x$
  - $f(x) = \tan x$ 이면, $f'(x) = \left(\frac{1}{\cos(x)}\right)^2 = \sec^2 x$

Derivative Rules

- **For Exponentials**
  - **For** a^x $where$ $a > 0$ and $a \neq 1$

$$f(x) = a^x, \qquad f'(x) = a^x \ln a$$
$$f(x) = e^x, \qquad f'(x) = e^x$$

Derivative Rules

- **Composition of Functions**

  When $t = f(x), \;\; u = g(x),$ **if** $both\; f(x)\; and\; g(x)$ **differentialable.**

  Then $f\big(g(x)\big)' = f'\big(g(x)\big) \times g'(x)$

Derivative Rules

- **Composition of Functions**
  - $f\big(g(x)\big)' = f'\big(g(x)\big) \times g'(x)$
- **Example**
- $f(g(x)) = \frac{1}{1+e^{-x}}, \quad f\big(g(x)\big)' = ?$

Derivative Rules

- **Composition of Functions**
  - $f\big(g(x)\big)' = f'\big(g(x)\big) \times g'(x)$

- **Example**
- $f(g(x)) = \frac{1}{1+e^{-x}}, \quad f\big(g(x)\big)' = ?$

1. $f(x) = \frac{1}{x}, \quad g(x) = 1 + e^{-x}$

Derivative Rules

- **Composition of Functions**
  - $f\big(g(x)\big)' = f'\big(g(x)\big) \times g'(x)$

- **Example**
- $f(g(x)) = \frac{1}{1+e^{-x}}, \quad f\big(g(x)\big)' = ?$

1. $f(x) = \frac{1}{x}, \quad g(x) = 1 + e^{-x}$
2. $f'(x) = \frac{-1}{x^2}$

Derivative Rules

- **Composition of Functions**
  - $f(g(x))' = f'(g(x)) \times g'(x)$

- **Example**
- $f(g(x)) = \frac{1}{1+e^{-x}}$, $f(g(x))' = ?$

1. $f(x) = \frac{1}{x}$, $g(x) = 1 + e^{-x}$
2. $f'(x) = \frac{-1}{x^2}$
3. $g'(x) = -e^{-x}$
4. $f'(g(x)) = \frac{1}{(1+e^{-x})^2}$

Derivative Rules

- **Composition of Functions**
  - $f\big(g(x)\big)' = f'\big(g(x)\big) \times g'(x)$
- **Example**
- $f(g(x)) = \frac{1}{1+e^{-x}},\quad f\big(g(x)\big)' = ?$

1. $f(x) = \frac{1}{x},\quad g(x) = 1 + e^{-x}$
2. $f'(x) = \frac{-1}{x^2}$
3. $g'(x) = -e^{-x}$
4. $f'\big(g(x)\big) = \frac{1}{(1+e^{-x})^2}$
5. $f\big(g(x)\big)' = f'\big(g(x)\big) \times g'(x)$

   $= \frac{1}{(1+e^{-x})^2}\, e^{-x}$

Derivative Rules

- **Composition of Functions**
  - $f(g(x))' = f'(g(x)) \times g'(x)$

- **Example**
- $f(g(x)) = \frac{1}{1+e^{-x}}, \quad f(g(x))' = ?$

1. $f(x) = \frac{1}{x}, \quad g(x) = 1 + e^{-x}$
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3. $g'(x) = -e^{-x}$
4. $f'(g(x)) = \frac{1}{(1+e^{-x})^2}$
5. $f(g(x))' = f'(g(x)) \times g'(x)$

$$= \frac{1}{(1+e^{-x})^2} e^{-x}$$

Derivative Rules

- **What if the function has more than one variable?**
  - $f(x) \rightarrow f(x, y)$
- **Partial Derivative!**
  - **Compute derivative of a variable given and hold the rest as constants**
- **Ex.** $f(x, y) = x^2 + xy + y^2$
  - $partial\ derivative\ with\ respect\ to\ x$:
  - $partial\ derivative\ with\ respect\ to\ y$:

Derivative Rules

- **What if the function has more than one variable?**
  - $f(x) \to f(x,y)$
- **Partial Derivative!**
  - **Compute derivative of a variable given and hold the rest as constants**
- **Ex.** $f(x,y) = x^2 + xy + y^2$
  - $partial\ derivative\ with\ respect\ to\ x$: $f_x(x,y) = 2x + y$
  - $partial\ derivative\ with\ respect\ to\ y$:

Derivative Rules

- **What if the function has more than one variable?**
  - $f(x) \rightarrow f(x, y)$
- **Partial Derivative!**
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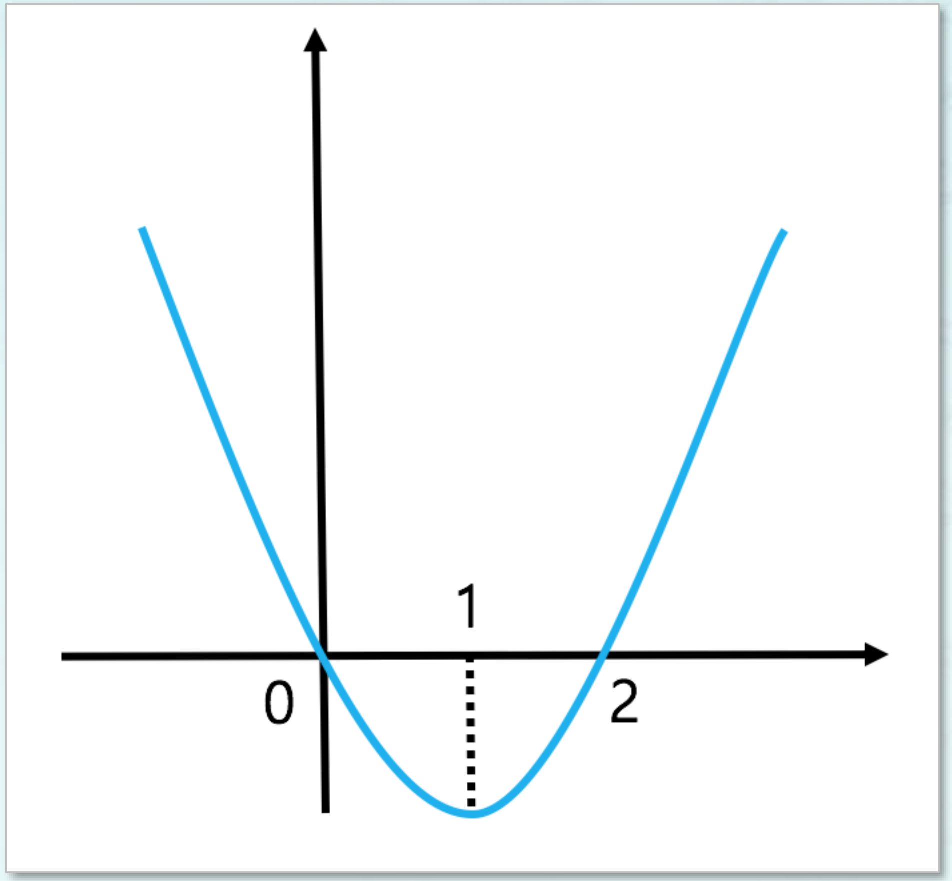
- **Ex.** $f(x, y) = x^2 + xy + y^2$
  - $partial\ derivative\ with\ respect\ to\ x$: $f_x(x, y) = 2x + y$
  - $partial\ derivative\ with\ respect\ to\ y$: $f_y(x, y) = 2y + x$

Finding Maxima and Minima

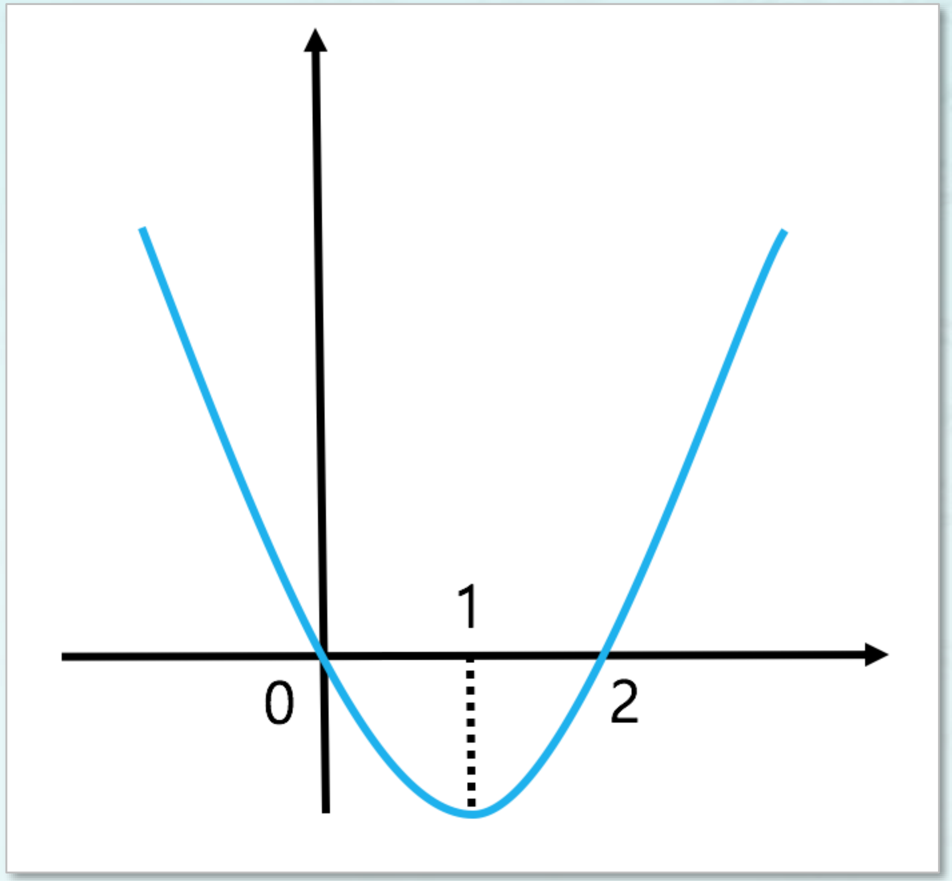
Finding Maxima and Minima

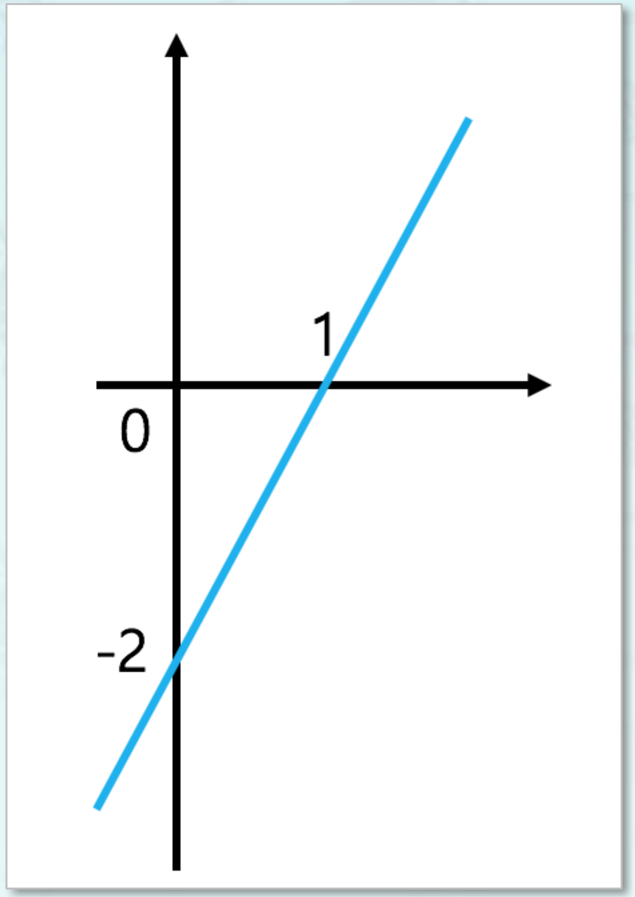
- **Example**
  - $f(x) = x^2 - 2x$



Finding Maxima and Minima

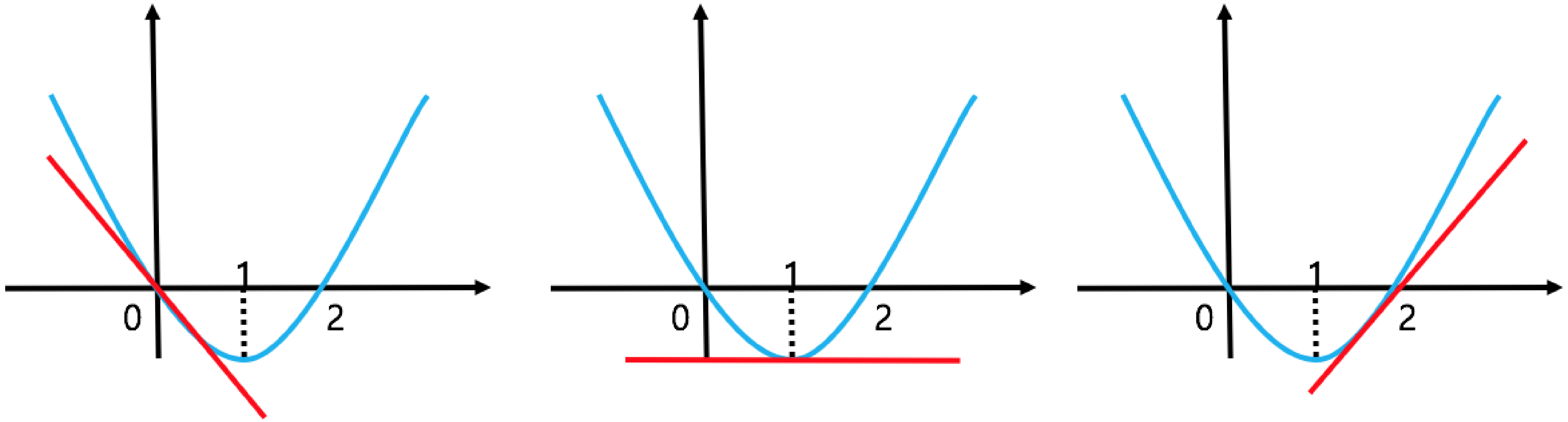
- **Example**
  - $f(x) = x^2 - 2x$
  - $f'(x) = 2x - 2$





Finding Maxima and Minima

- **Meaning of $f'(x)$**



$f'(x_1) < 0$ $f'(x_2) = 0$ $f'(x_3) > 0$

Derivatives

- summary
  - Understanding Derivatives Concept
  - Using Rules of Derivatives
  - Finding Minima and Maxima Using Derivatives

- Next
  - 3-3 Activation Function

Week3(2/3)

Derivatives

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미분에 나오는 수학적 기호들

- 수학적 기호
  - Δ $(Delta,$ 델타)

$$\lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1} = f'(x_1) \quad \Rightarrow \quad \lim_{\Delta h \to 0} \frac{f(x + \Delta h) - f(x)}{\Delta h} = f'(x)$$

미분에 나오는 수학적 기호들

- 수학적 기호
  - d (d, 디)

$$f(x) = g(x) \pm t(x)\text{이면}, f'(x) = g'(x) \pm t'(x)$$

$$\Downarrow$$

$$f(x) = g(x) \pm t(x)\text{이면}, \frac{d}{dx}f(x) = \frac{d}{dx}g(x) \pm \frac{d}{dx}t(x)$$

미분에 나오는 수학적 기호들

- 수학적 기호
  - ∂ ($round\ d$, 라운드 디)

$$f(x) = g(x) \pm t(x) \text{이면}, f'(x) = g'(x) \pm t'(x)$$

$$\Downarrow$$

$$f(x,y) = g(x,y) \pm t(x,y) \text{이면}, \frac{\partial}{\partial x} f(x,y) = \frac{\partial}{\partial x} g(x,y) \pm \frac{\partial}{\partial x} t(x,y)$$