Week 4(1/3)

Perceptron

Machine Learning with Python

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Perceptron

- Goals
  - Understanding Perceptron

- Content
  - Perceptron Overview
  - Perceptron Binary Classification
  - Perceptron Learning
  - Overfitting and Underfitting

1. Perceptron History

Frank Rosenblatt(출처: Arvin Calspan Advanced Technology Center; Hecht-Nielsen, R. Neurocomputing)

1. Perceptron History

- Artificial Neuron → Neuron, Node, Perceptron
- Perceptron → The First Artificial Neural Network
  - Frank Rosenblatt, 1957
  - Cornell Aeronautical Laboratory

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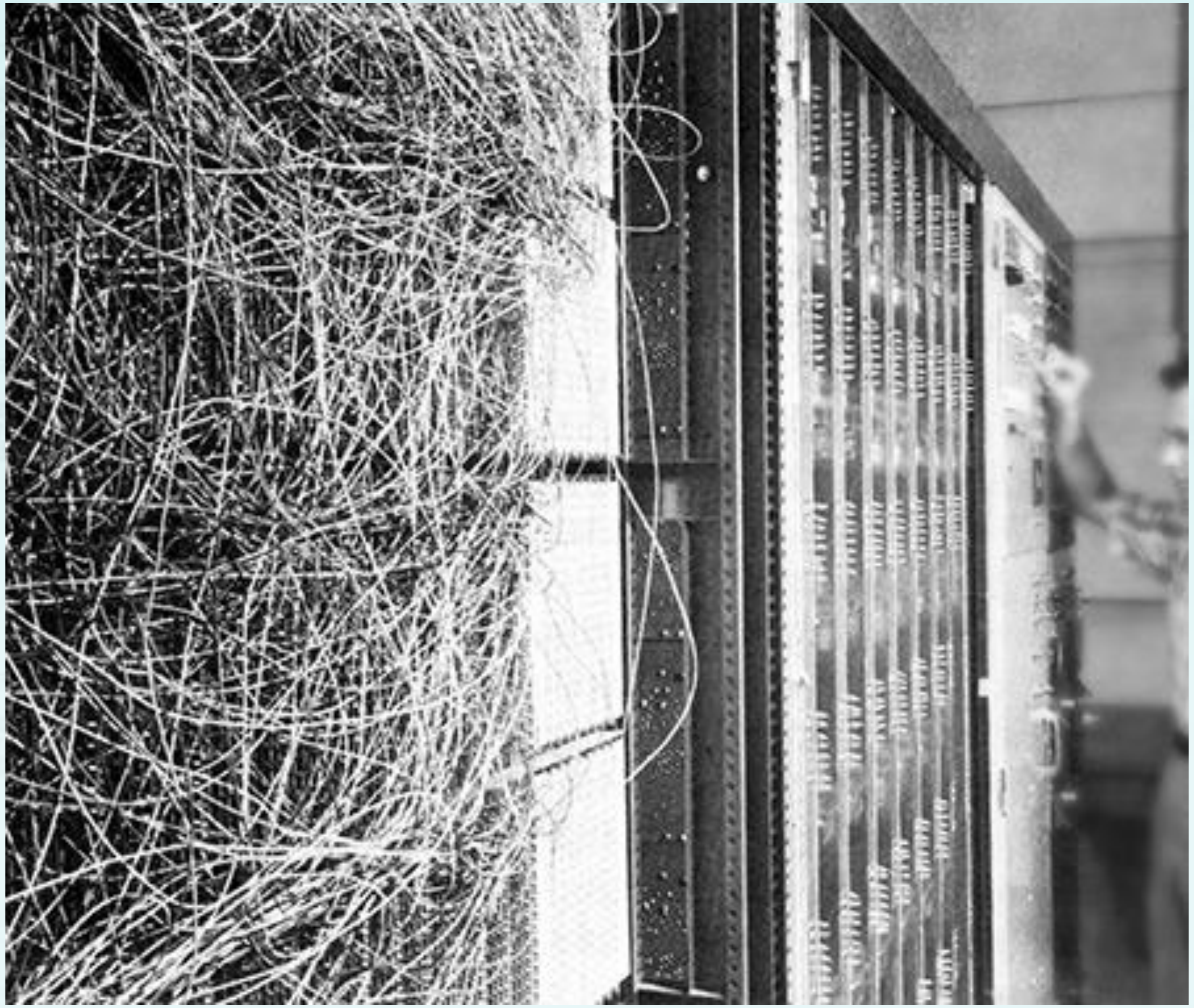

마크 1 퍼셉트론 (출처: Arvin Calspan Advanced Technology Center; Hecht-Nielsen, R. Neurocomputing)

1. Perceptron History

ARCHIVES | 1958

NEW NAVY DEVICE LEARNS BY DOING; Psychologist Shows Embryo of Computer Designed to Read and Grow Wiser

JULY 8, 1958

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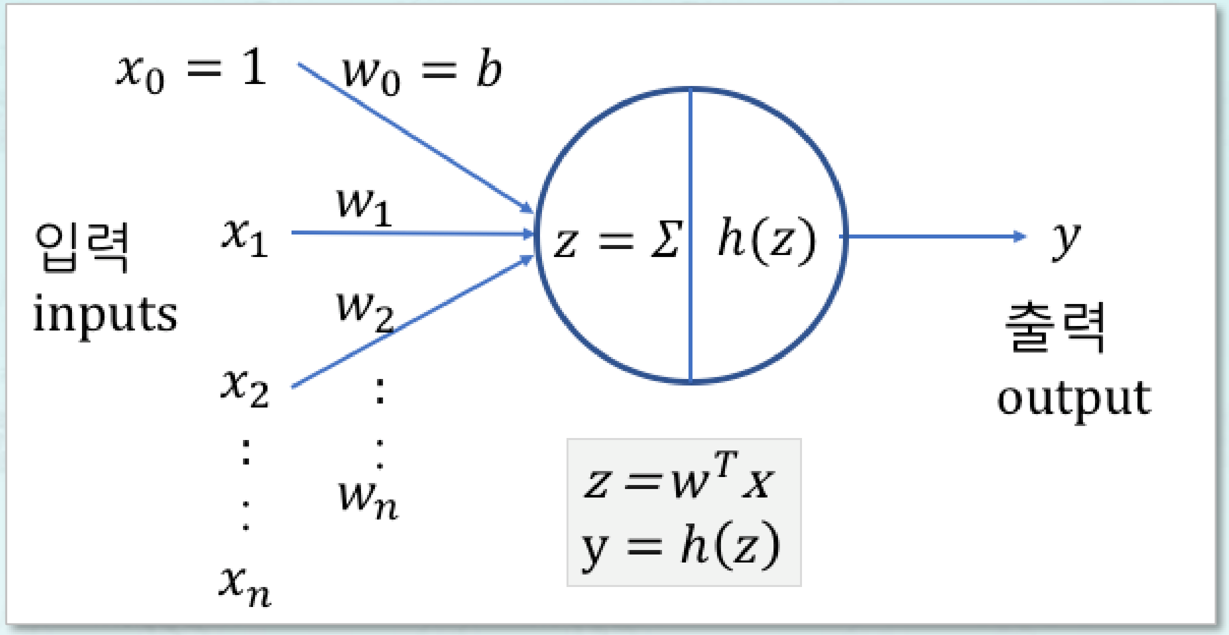
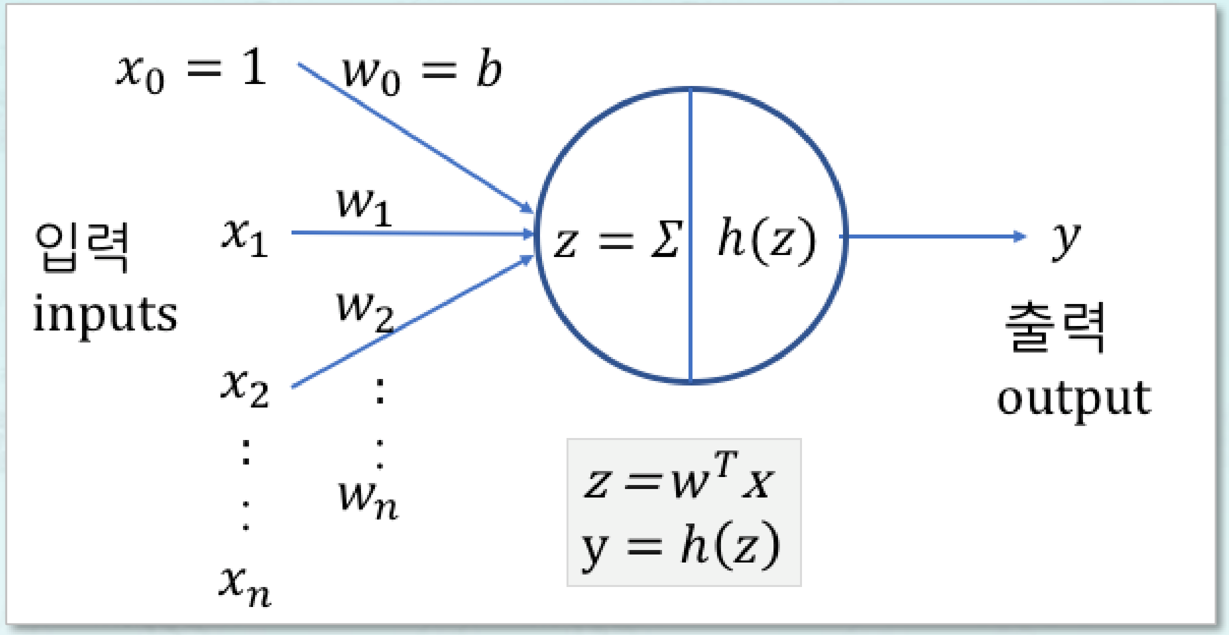
2. Perceptron Structure



2. Perceptron Structure

- **input x**

$$\mathbf{x} = \begin{bmatrix} x_0 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$



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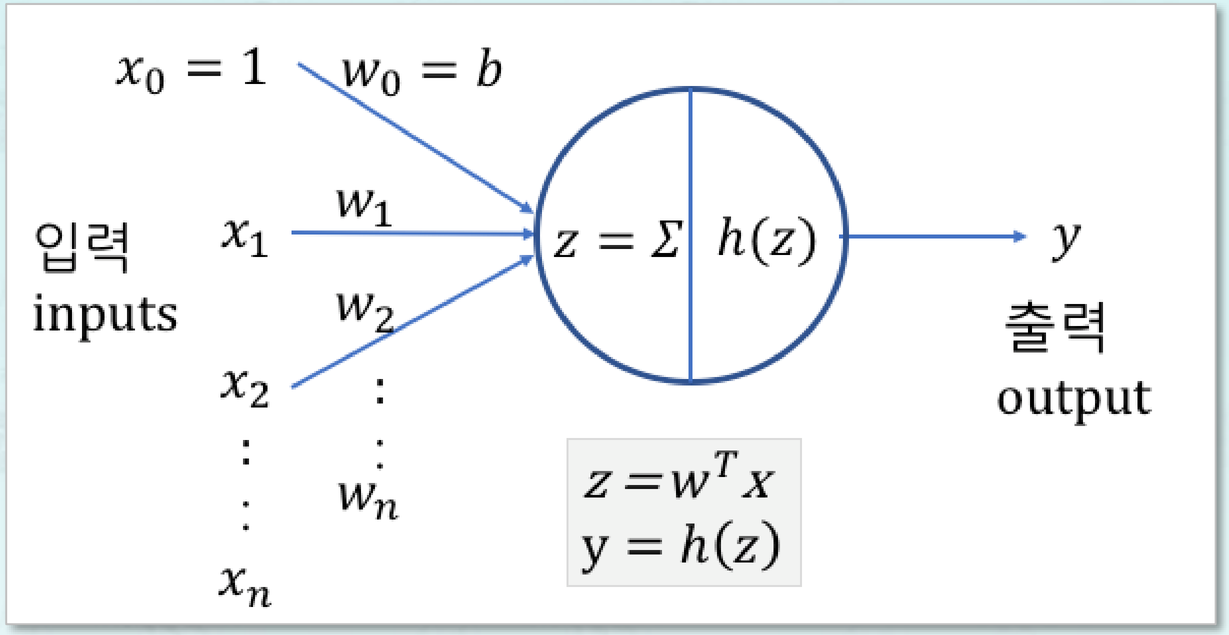

- **weight w**

$$\mathbf{w} = \begin{bmatrix} w_0 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

2. Perceptron Structure

- **input x**

$$\mathbf{x} = \begin{bmatrix} x_0 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

bias
b

- **weight w**

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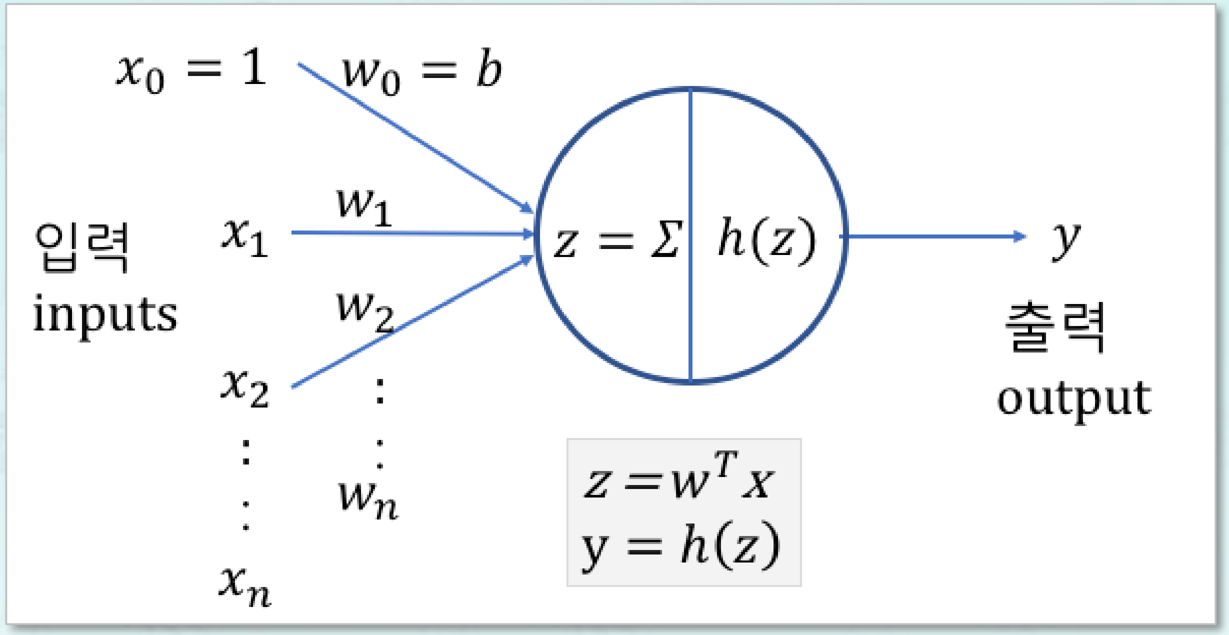
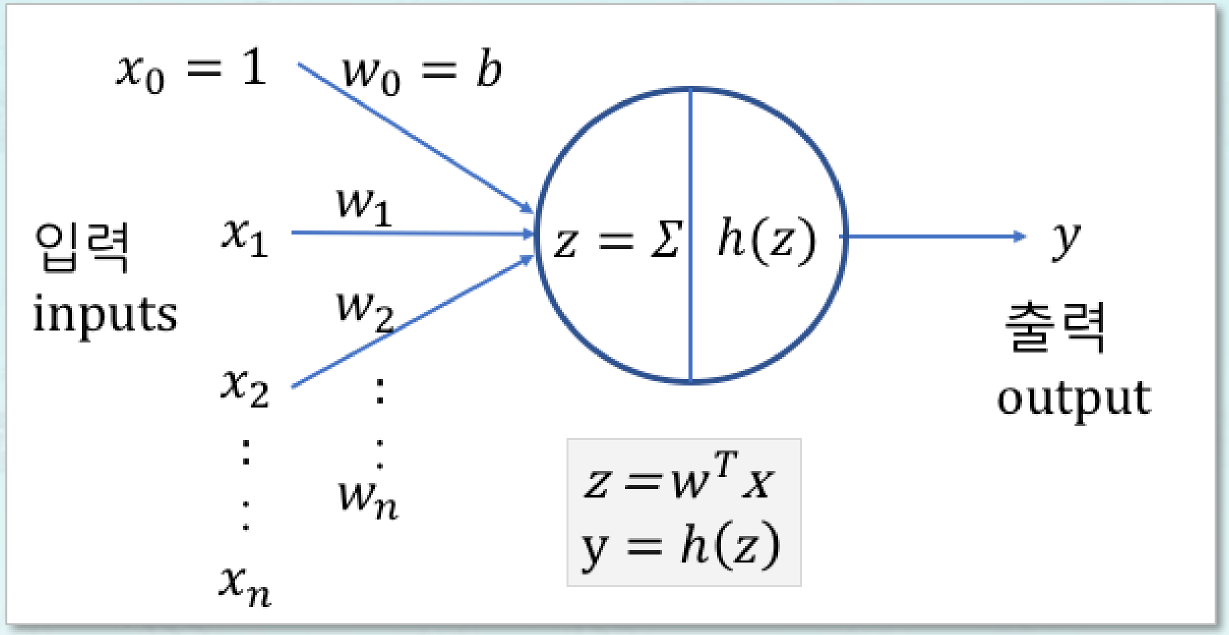


2. Perceptron Structure: net input

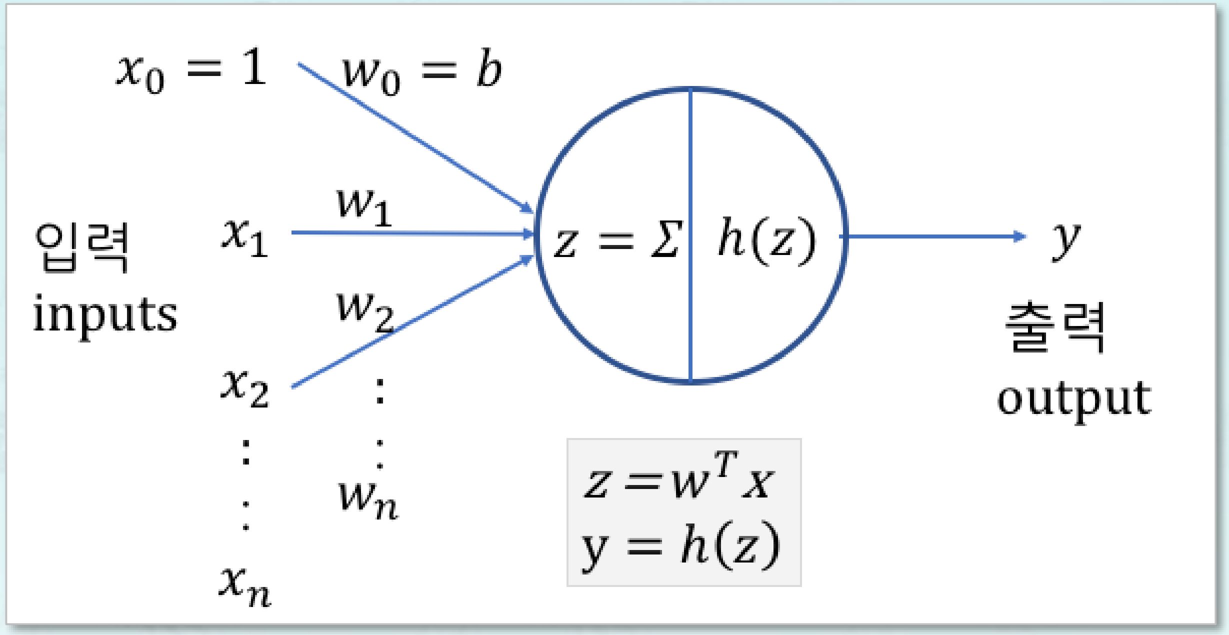
- **net input z**



2. Perceptron Structure: net input

- **net input z**

  $$z = w_0 x_0 + w_1 w_1 + \ \ldots + w_n x_n$$



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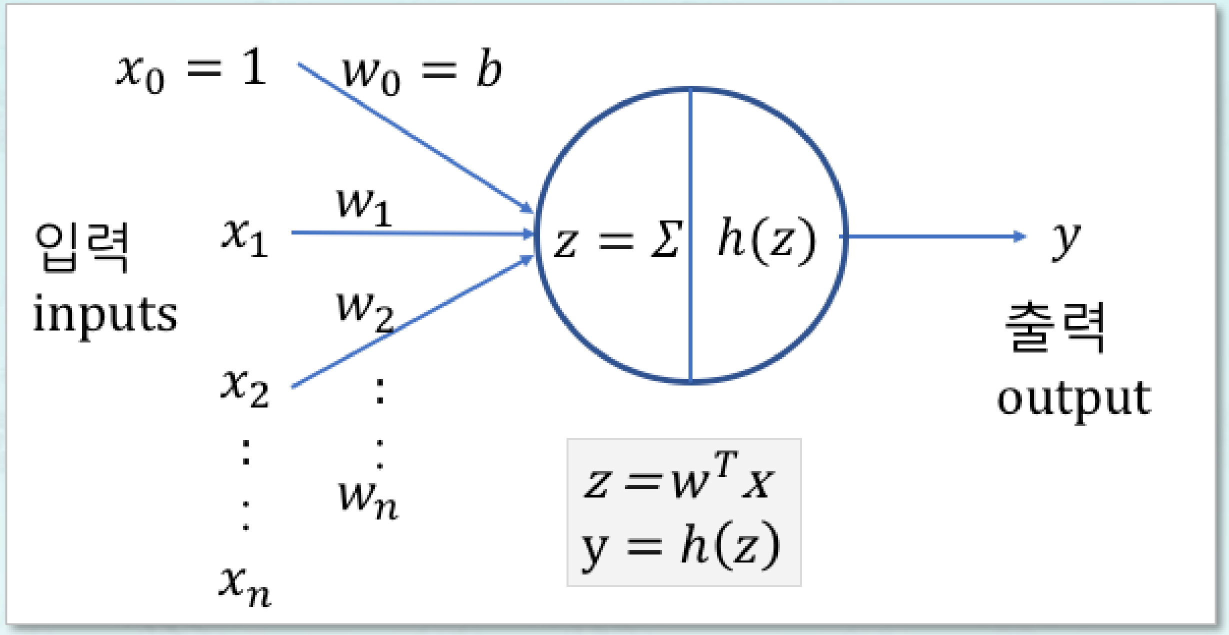
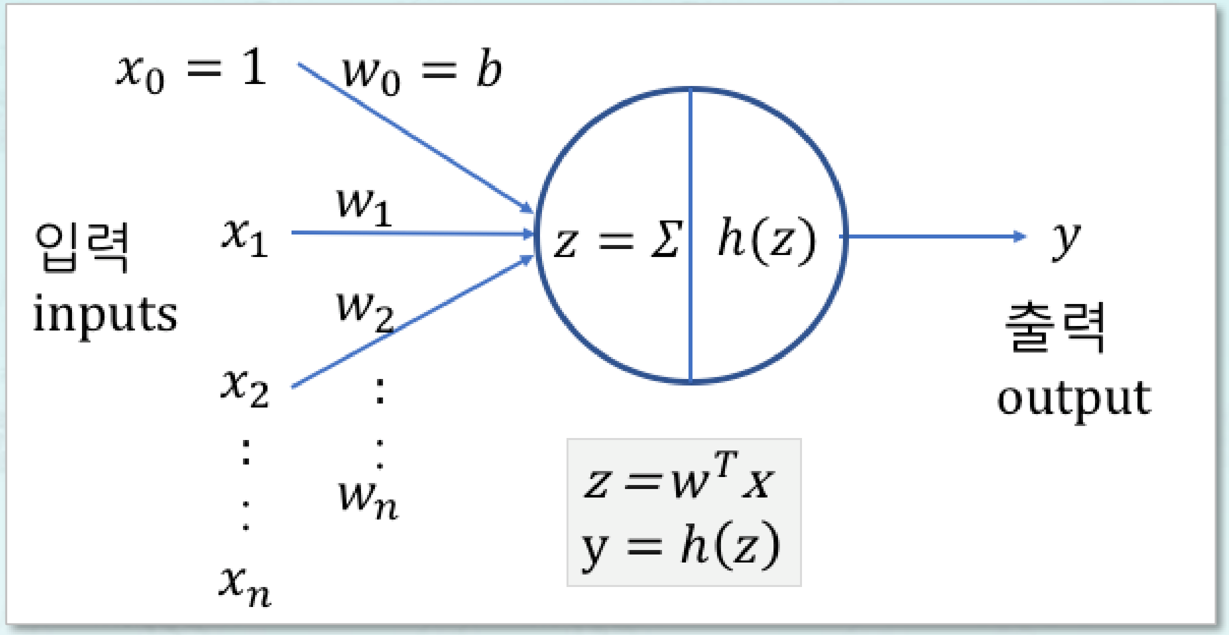


2. Perceptron Structure: net input

- **net input z**

$$z = w_0 x_0 + w_1 w_1 + \ \dots + w_n x_n$$
$$= \sum_{j=0}^{n} x_j w_j$$
$$= \mathbf{w}^{\mathrm{T}}\mathbf{x}$$



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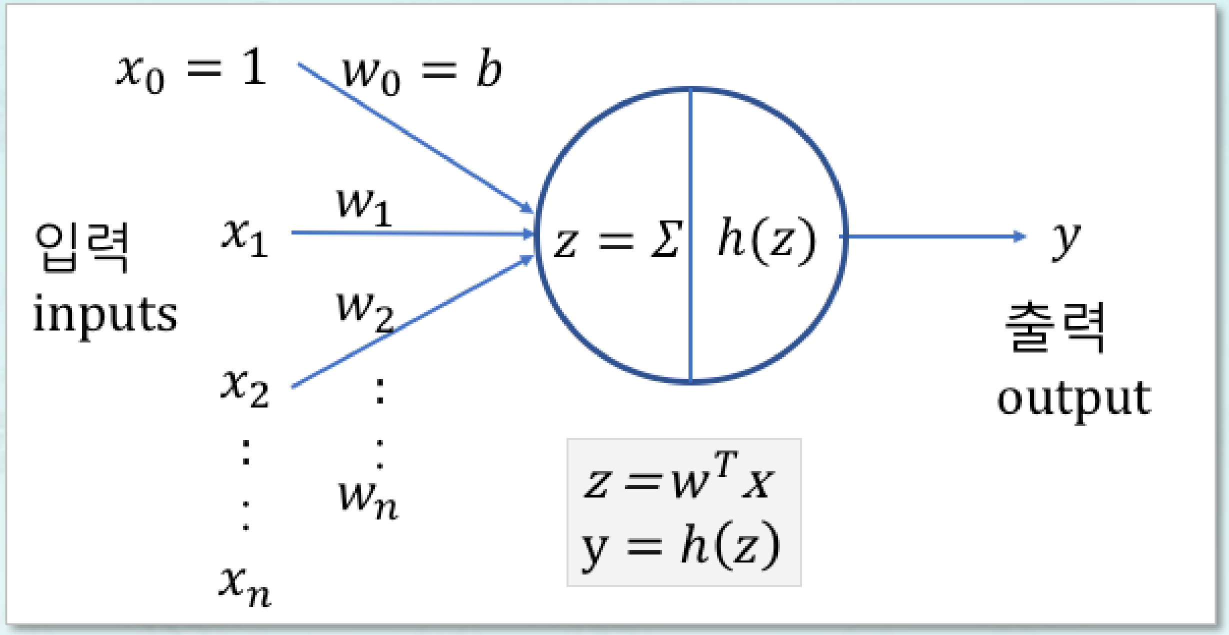

$$\mathbf{w} = \begin{bmatrix} w_0 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} x_0 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

2. Perceptron Structure: net input computation

- **net input z example:**
  1. input x = [0, 1, 2, 3]
  2. weight w = [0..1]
  3. Compute net input z.

2. Perceptron Structure: net input computation

```
x = np.array([0, 1, 2, 3])
w = np.array([0, 0.1, 0.2, 0.3])
z = np.dot(x, w)
print(z)
```

```
1.4
```

Solution(1)

- **net input z example:**
  1. input x = [0, 1, 2, 3]
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```

```
1.4
```

Solution(1)

```
import numpy as np
x = np.array(np.arange(4))
w = np.array(np.random.random(4))
z = np.dot(w, x)
print(z)
```

```
1.329056653793057
```

Solution(2)

- **net input z example:**
  1. input x = [0, 1, 2, 3]
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  3. Compute net input z.

2. Perceptron Structure: net input computation

- net input z

$$z = w_0x_0 + w_1w_1 + \ \dots + w_nx_n$$
$$= \sum_{j=0}^{n} x_jw_j$$
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- **net input z example:**
  1. input x = [0, 1, 2, 3]
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- Thoughts on Solution(2):

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Solution(2)

2. Perceptron Structure: net input computation

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2. Perceptron Structure: net input computation

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```
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w = np.array(np.random.random(4))
z = np.dot(w.T, x)
print(z)
```

```
1.4781818847304011
```

Solution(3)

2. Perceptron Structure: net input computation

- net input z

$$z = w_0x_0 + w_1w_1 + \ ... + w_nx_n$$
$$= \sum_{j=0}^{n} x_j w_j$$
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- **net input z example:**
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Solution(3)

2. Perceptron Structure: net input computation

- net input z

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import numpy as np
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```

```
3.553656675063983
```

Solution(2A)

```
import numpy as np
np.random.seed(0)
x = np.array(np.arange(4))
w = np.array(np.random.random(4))
z = np.dot(w.T, x)
print(z)
```

```
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```

Solution(3A)

2. Perceptron Structure: net input computation

- net input z

$$z = w_0 x_0 + w_1 w_1 + \ \ldots + w_n x_n$$
$$= \sum_{j=0}^{n} x_j w_j$$
$$= \mathbf{w}^{\mathrm{T}}\mathbf{x}$$

```
print('x.shape={}, w.shape{}, w.T.shape{}'.
      format(x.shape, w.shape, w.T.shape))
```

```
x.shape=(4,), w.shape(4,), w.T.shape(4,)
```

- :
- :
-

- **net input z example:**
  1. input x = [0, 1, 2, 3]
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2. Perceptron Structure: net input computation

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- .
- .
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- Thoughts on Solution(2):

2. Perceptron Structure: net input computation

- input x, w:
  - row vector(행 벡터)
  - shape n x 1 or (n, 1)

$$\mathbf{x} = \begin{bmatrix} x_0 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad \mathbf{w} = \begin{bmatrix} w_0 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

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  3. Compute net input z.
- Thoughts on Solution(2):

```
np.random.seed(0)
x = np.array(np.arange(4)).reshape(4,1)
w = np.array(np.random.random(4)).reshape(4,1)
z = np.dot(w.T, x)    #.random((4,1)) is OK!
print(z)
```

```
[[3.55536567]]
```

2. Perceptron Structure: net input computation

```
print(x)
print(w.T)
print(z)
print('shapes: x{}, w{}, w.T{}, z{}'.
  format(x.shape, w.shape, w.T.shape, z.shape))
```

```
[[0]
 [1]
 [2]
 [3]]
[[0.5488135  0.71518937 0.60276338 0.54488318]]
[[3.55536567]]
shapes: x(4, 1), w(4, 1), w.T(1, 4), z(1, 1)
```

2. Perceptron Structure: net input computation

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print(x)
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```

```
z = np.dot(w.T, x).squeeze()
print(z)
```

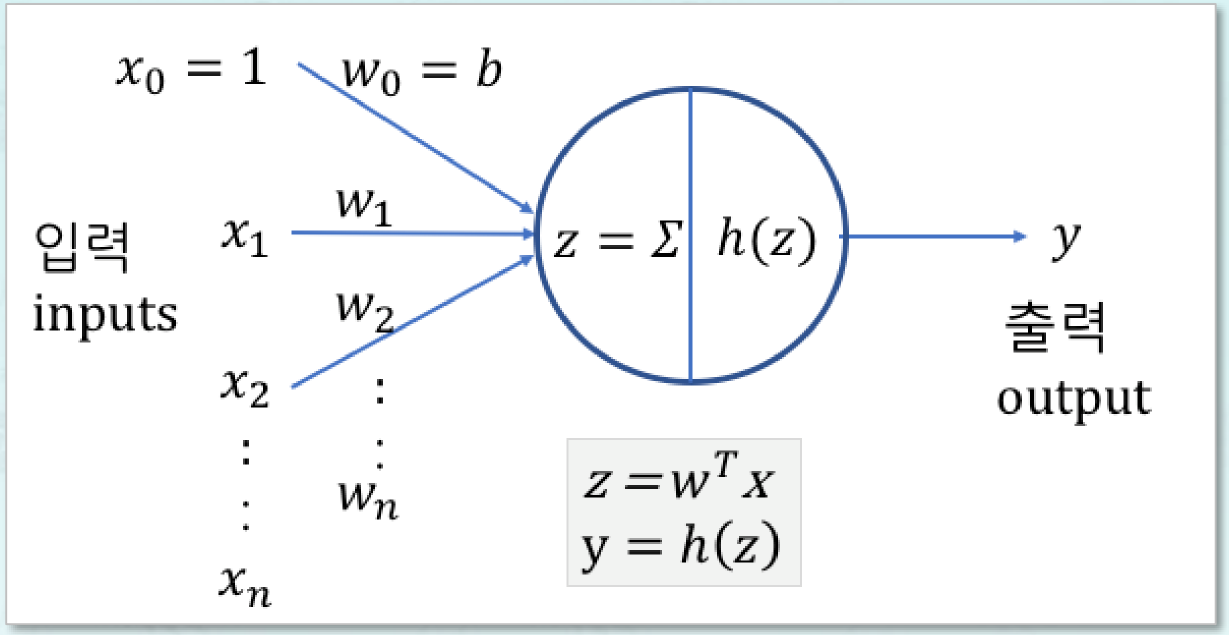
```
3.555365667506398
```

3. Perceptron Binary Classification

- Binary classification
- Linear binary classifier

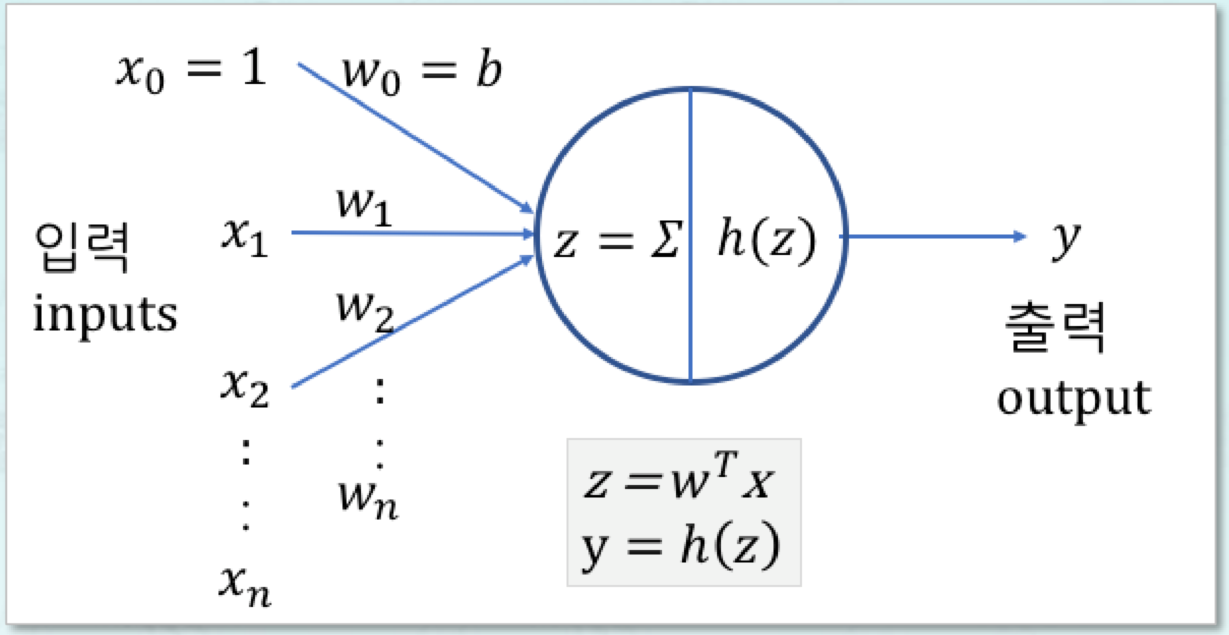
3. Perceptron Binary Classification

- **input**
- **weight**
- ?



3. Perceptron Binary Classification

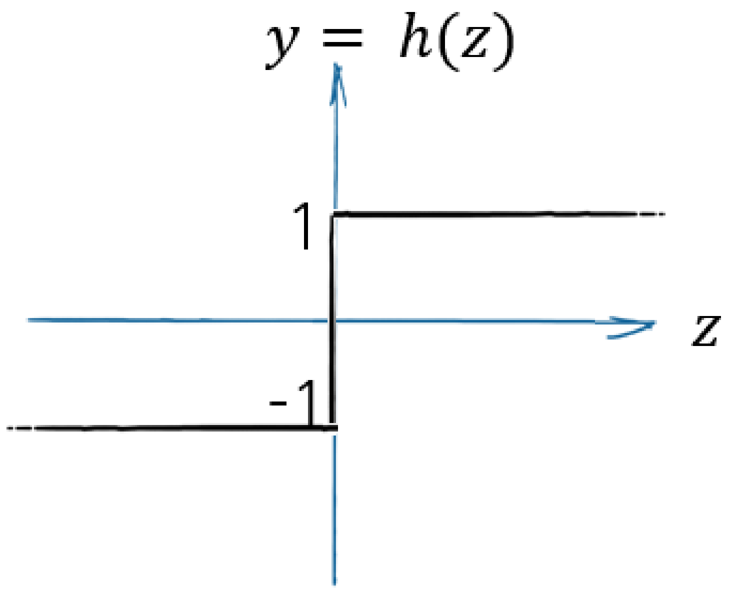
- input
- weight
- Activation Function
  - Sigmoid Function
  - Step Function
  - tanh Function
  - ReLU Function



3. Perceptron Binary Classification

- **Activation Function for Binary Classification**

$$h(z) = \begin{cases} +1 & \textit{if } z > 0 \\ -1 & \textit{otherwise.} \end{cases}$$



계단함수(양극성)

bipolar step function

3. Perceptron Binary Classification

- **Activation Function for Binary Classification**

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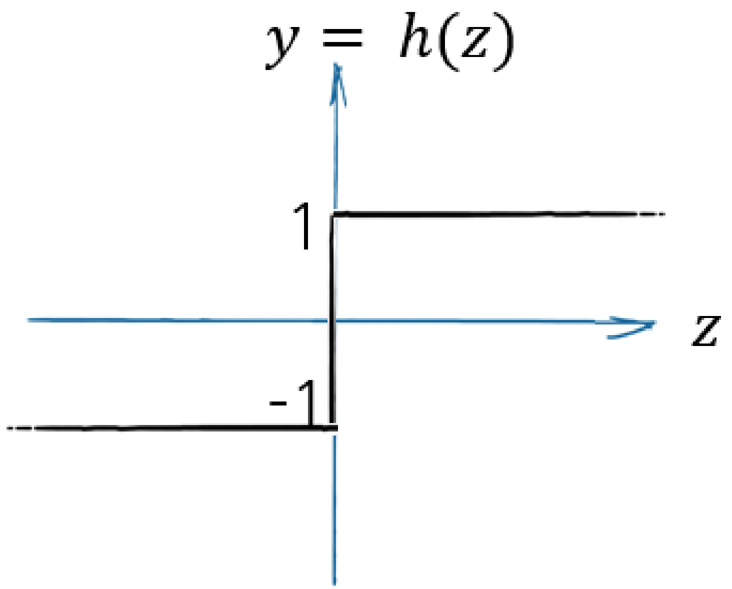
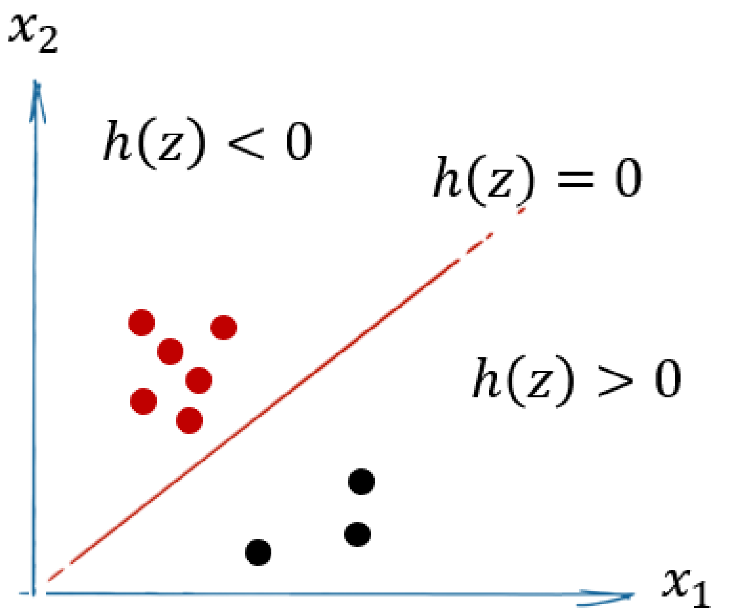


계단함수(양극성)
bipolar step function



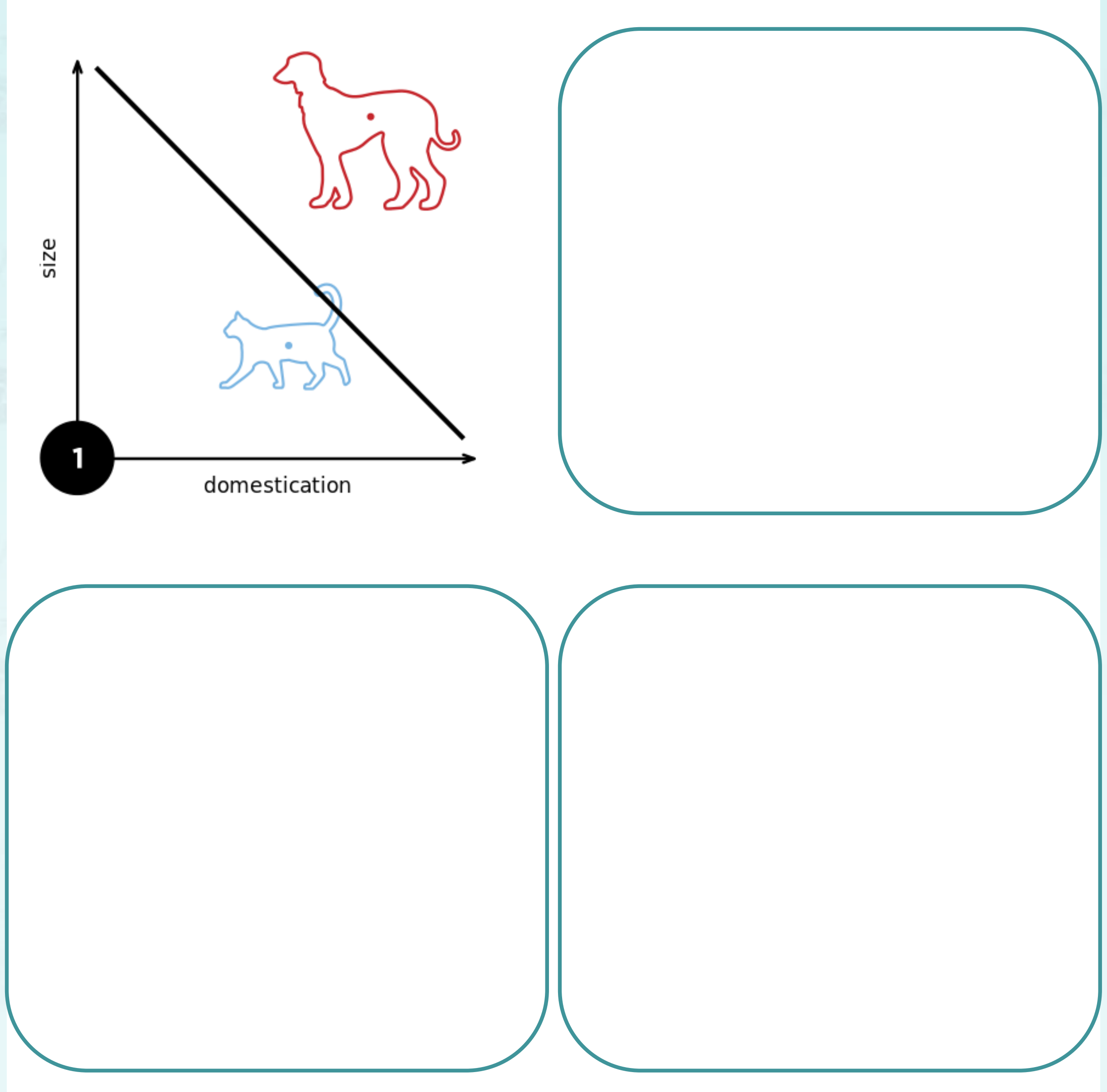
선형 이진 분류기($z = w^T x$)
linear binary classifier

4. Perceptron Learning Method

- **Learning – Change in Weights**

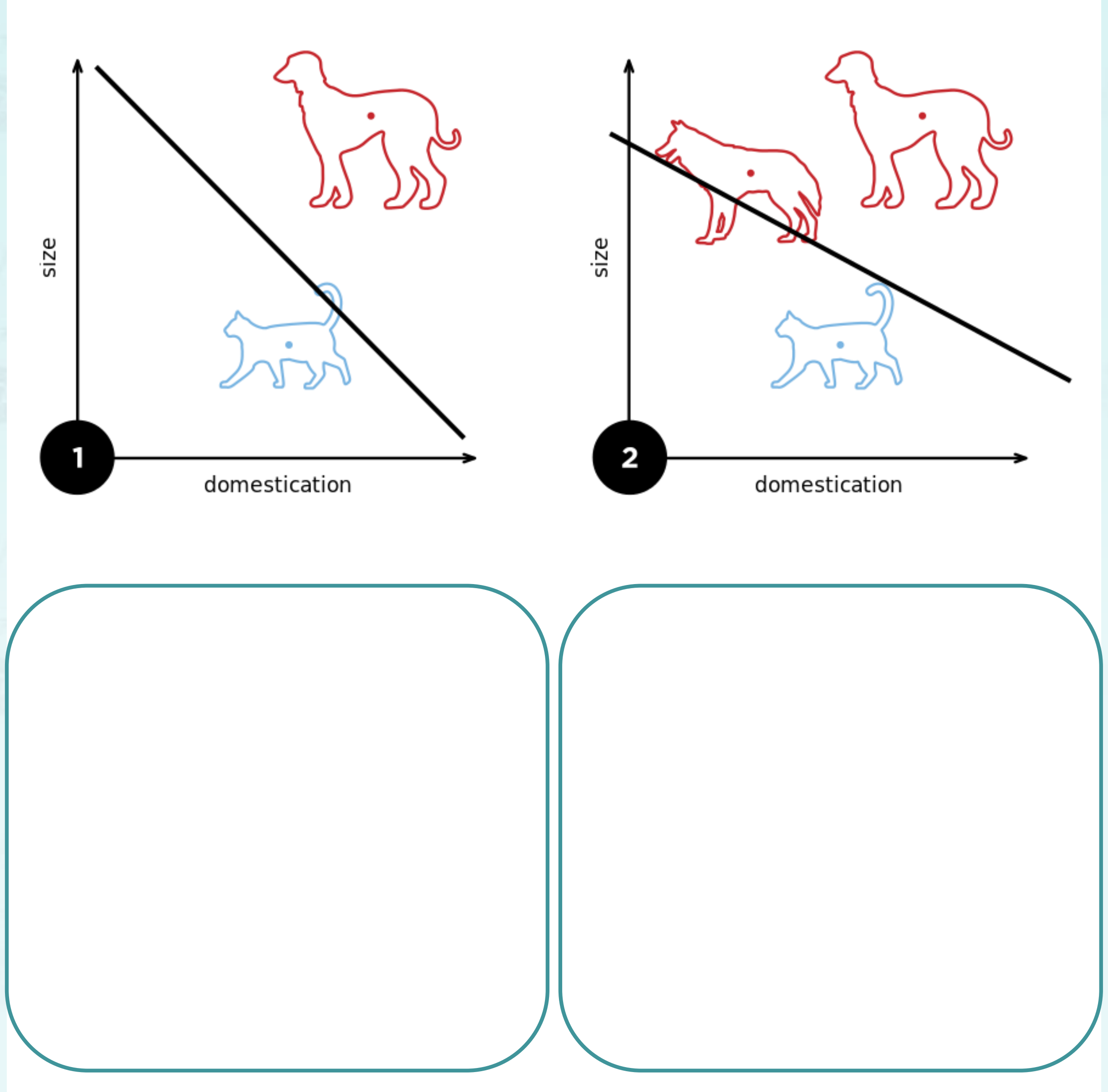
4. Perceptron Learning Method

- **Learning – Change in Weights**



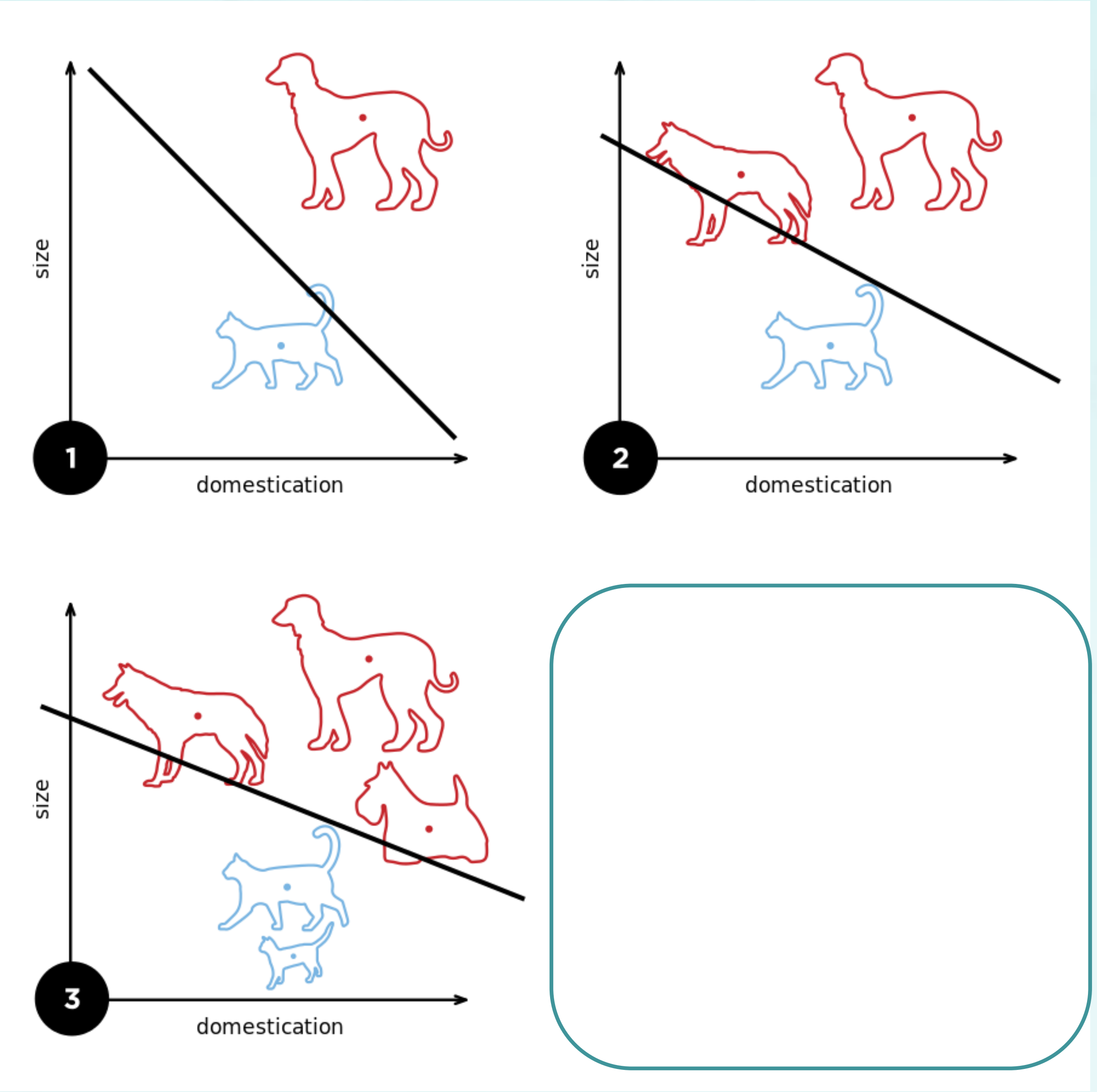
4. Perceptron Learning Method

- **Learning – Change in Weights**



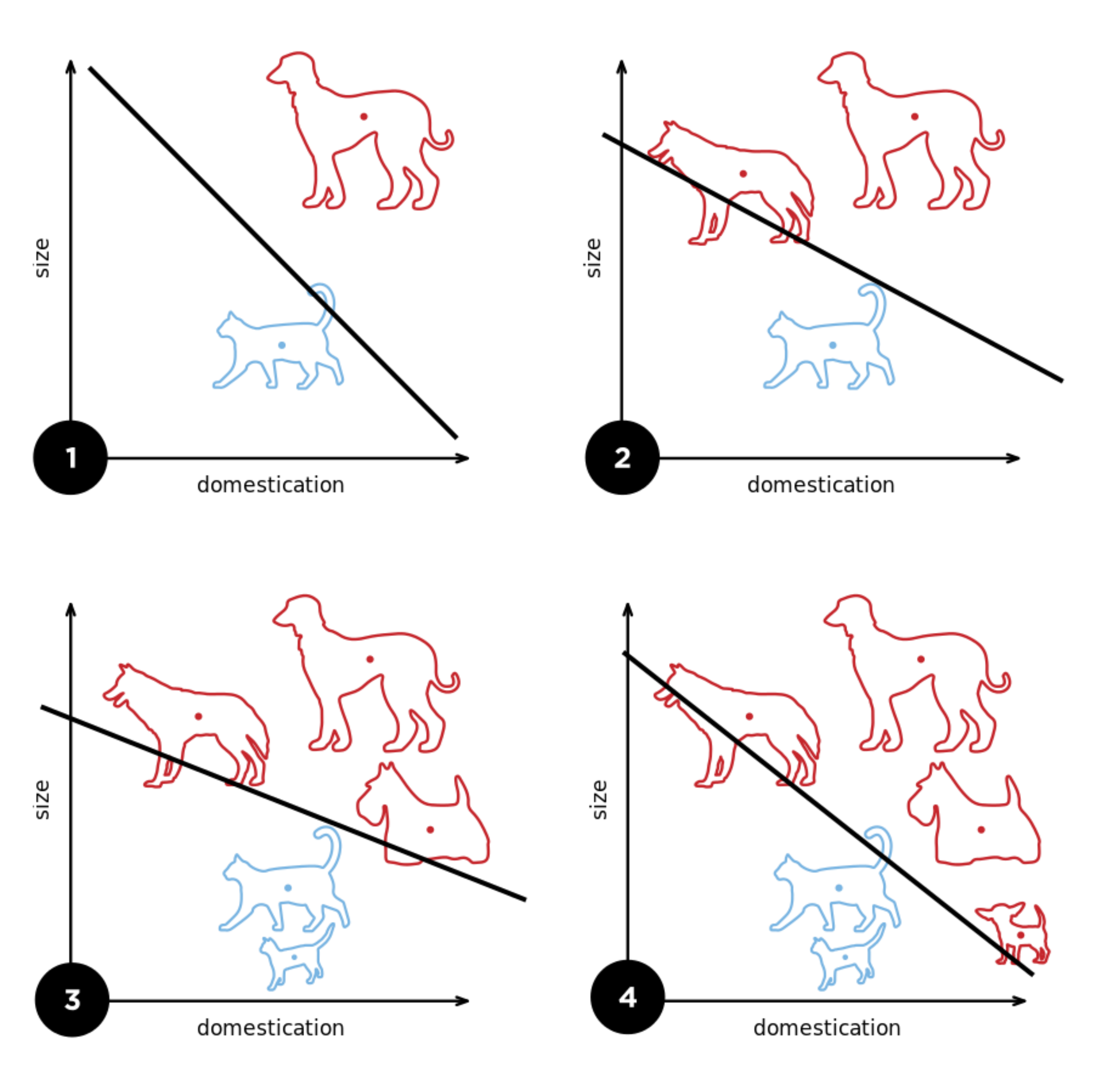
4. Perceptron Learning Method

- **Learning – Change in Weights**



4. Perceptron Learning Method

- **Learning – Change in Weights**



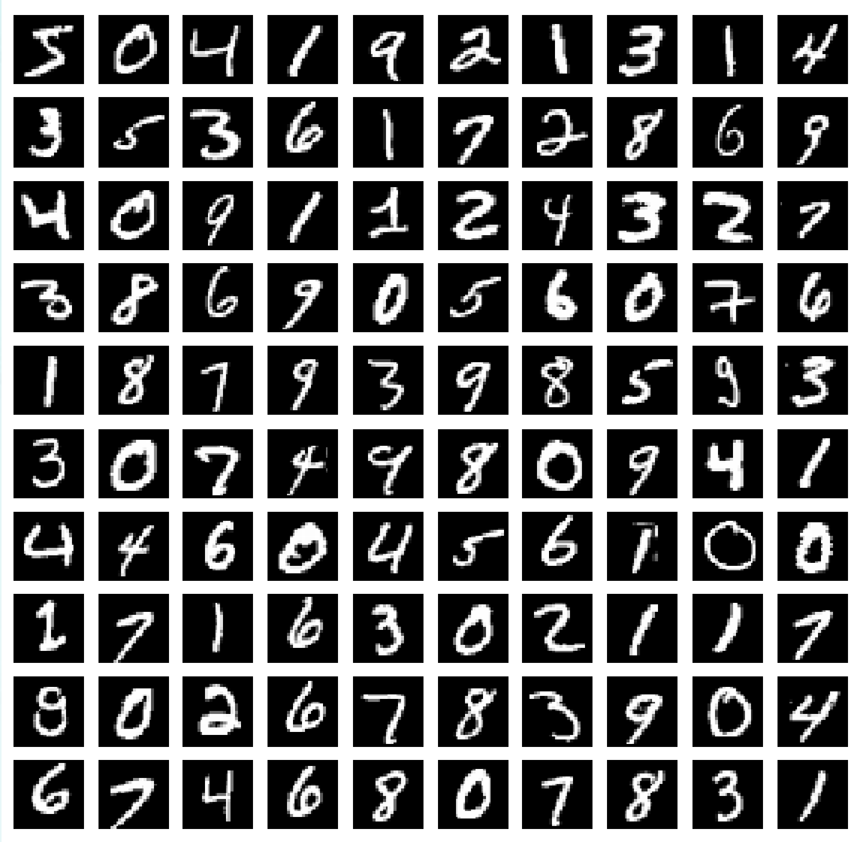
5. Overfitting

- **Perfect Perceptron?**

5. Overfitting

- Perfect Perceptron?

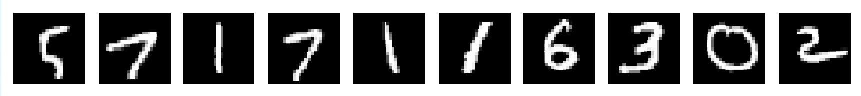
Training Data:



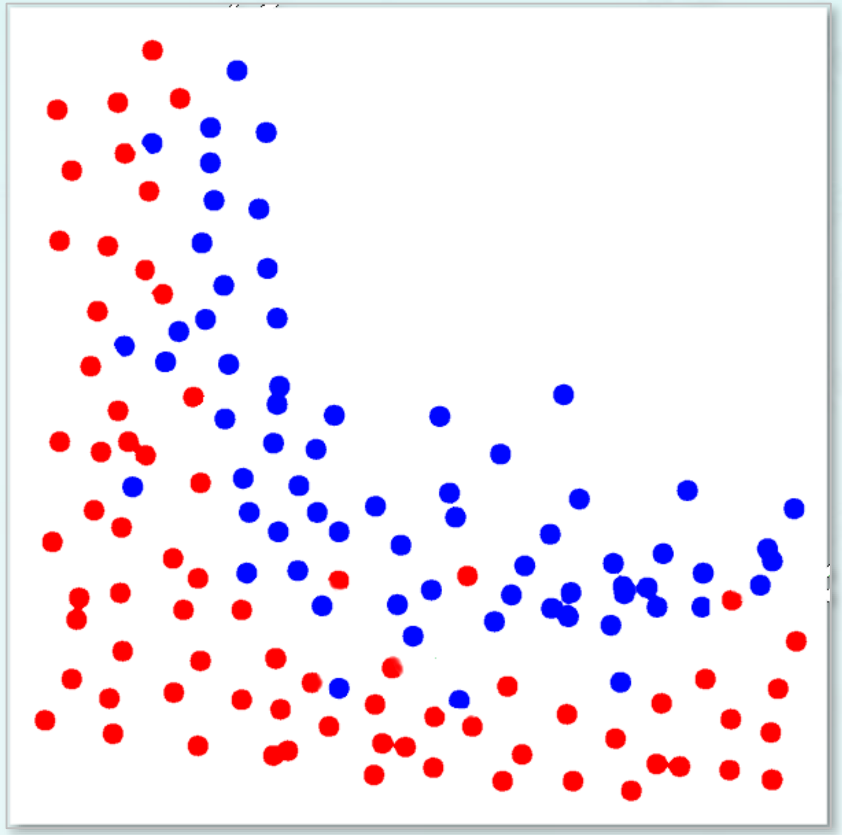
Label:

5 0 4 1 9 2 1 3 1 4
3 5 3 6 1 7 2 8 6 9
4 0 9 1 …
…

Test Data:



5. Overfitting



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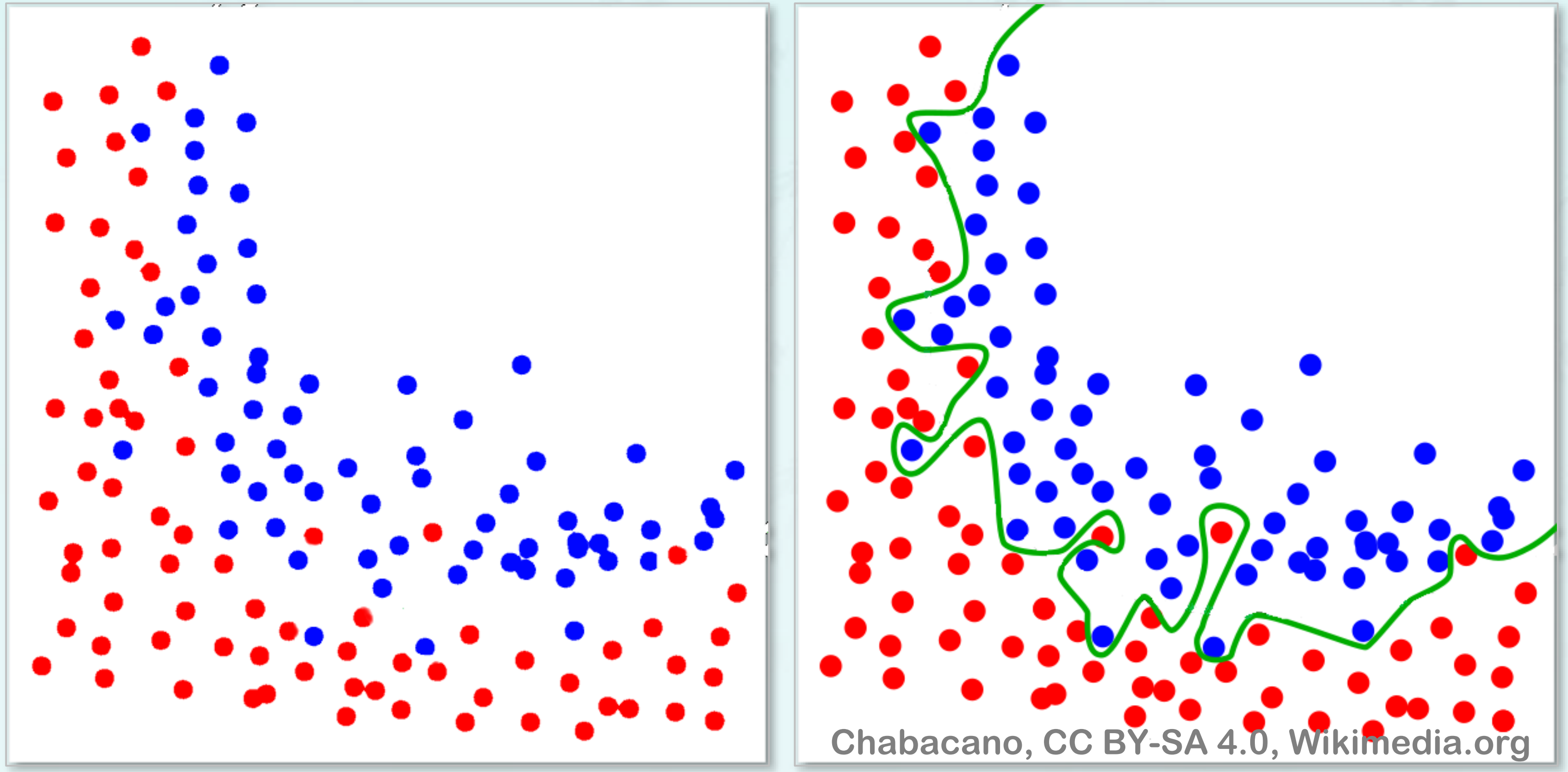

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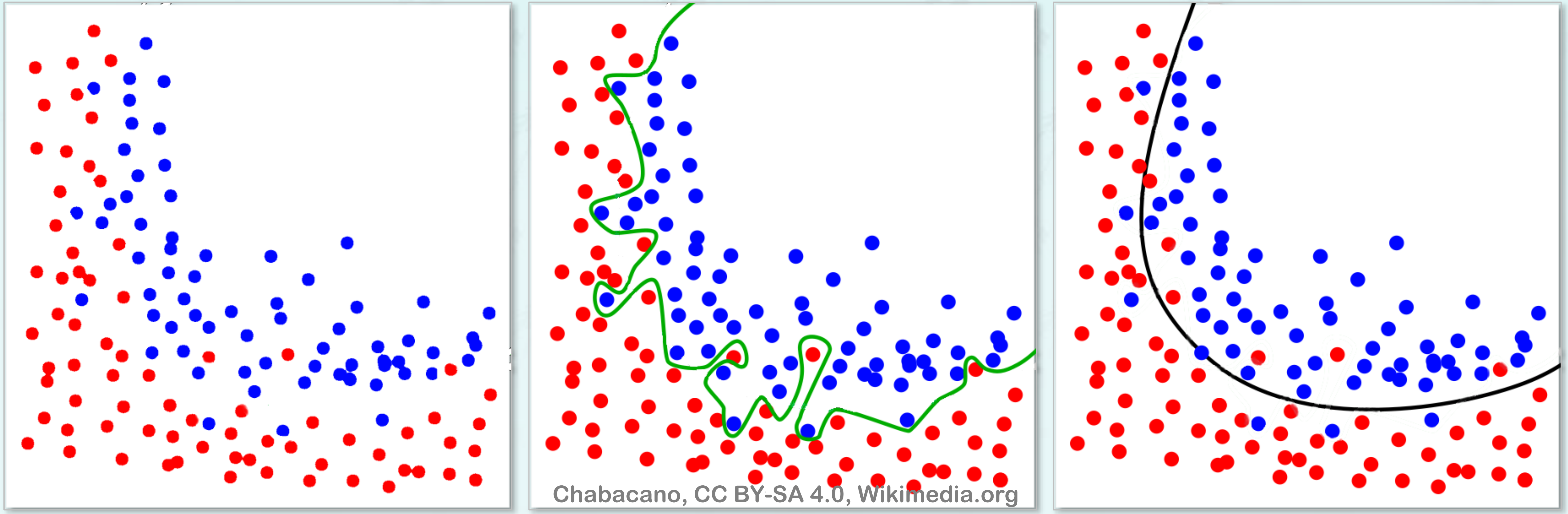
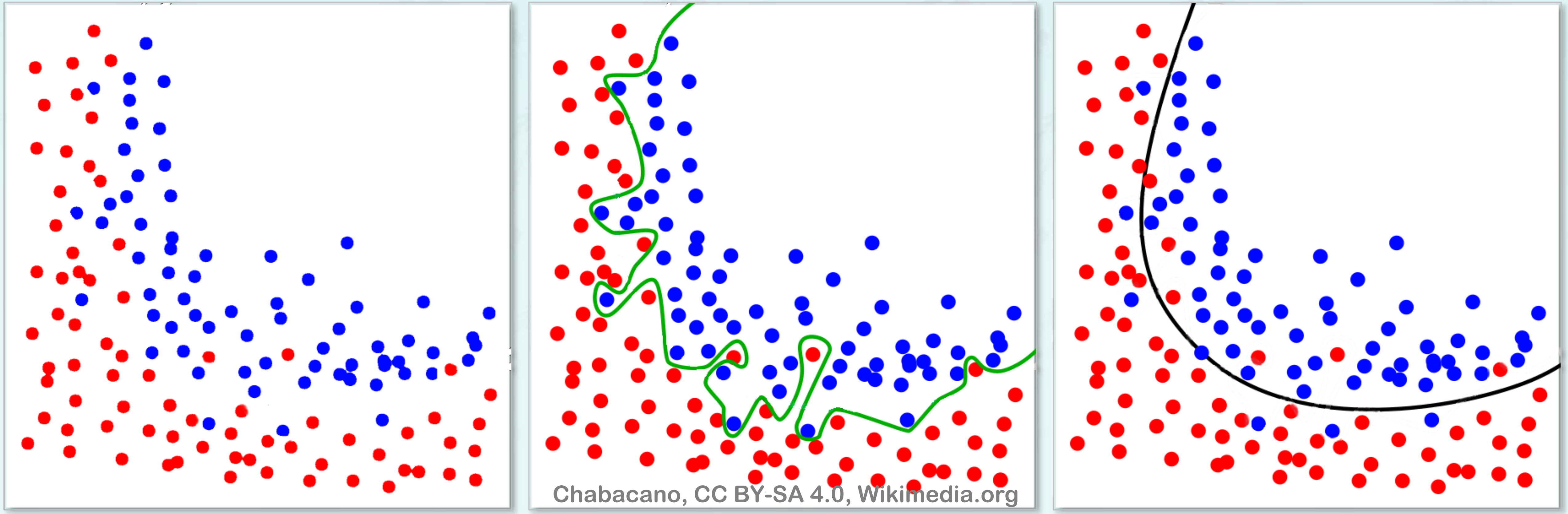


5. Overfitting

- **A Better Classifier?**
  1. Green line
  2. Black line



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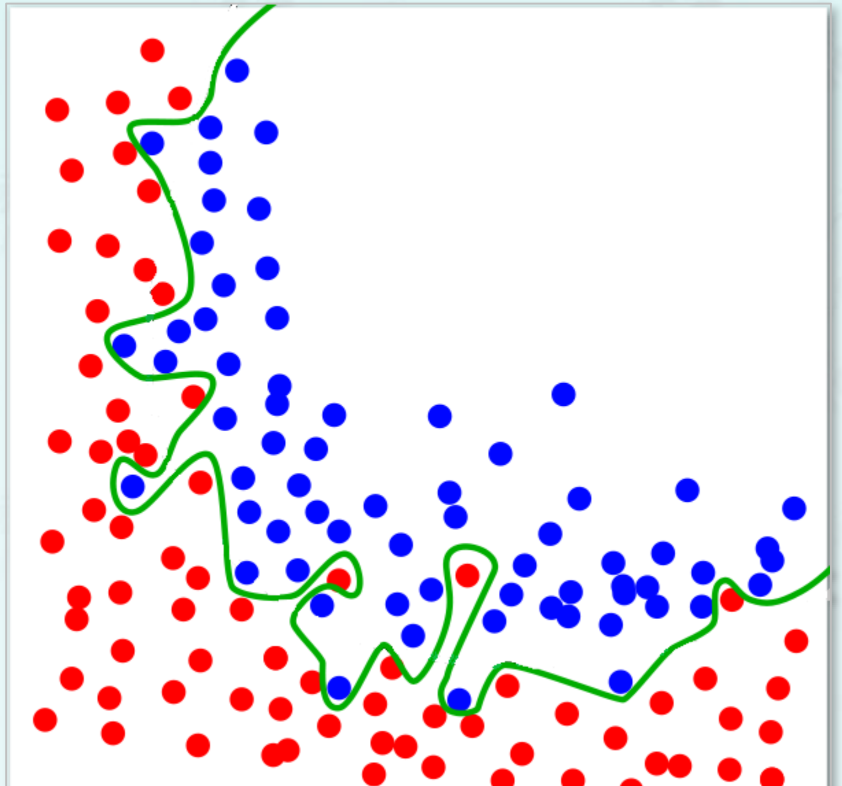
5. Overfitting

- Overfitting
- Underfitting



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Perceptron

- Summary
  - Perceptron History
  - Perceptron Structure
  - Perceptron Learning
  - Binary Classifier and Activation Function
  - Overfitting and Underfitting

- Next
  - 4-2 Perceptron Algorithm

3주차(3/3)

Activation Function

Machine Learning with Python

Handong Global University
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여러분 곁에 항상 열려 있는 K-MOOC 강의실에서 만나 뵙기를 바랍니다.