Week 7(2/3)

# Feed-forward Neural Network

Machine Learning with Python

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#### Feed-forward Neural Network - Example

- Objectives
  - Feed-forward Neural Network Example
- Topics
  - MNIST Dataset
  - Designing Multi-Layer Neural Network
  - Feed-forward Neural Network Signal Processing Example
  - Feed-forward Neural Network Example Implementation

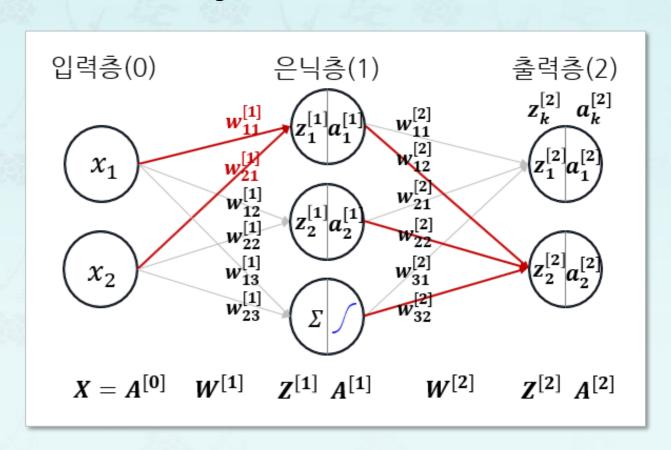
## 1. Multi Layer Neural Network: Input Data

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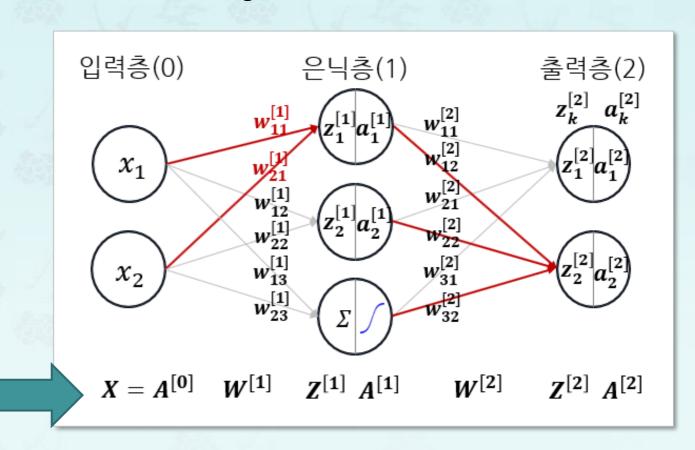
MNIST



### 1. Multi Layer Neural Network: Notation(Review)



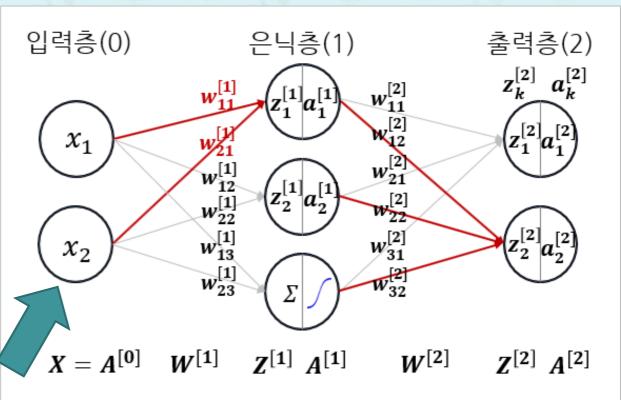
#### 1. Multi Layer Neural Network: Notation(Review)



#### 1. Multi Layer Neural Network: Input Data

MNIST

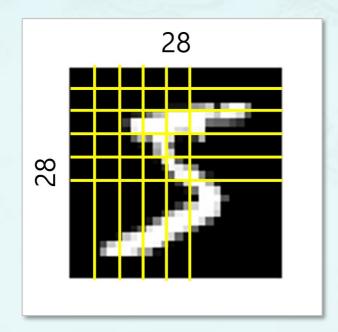




## 1. Multi Layer Neural Network: Input Data

- MNIST
  - 28x28

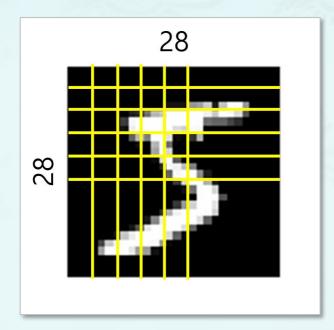




- MNIST
  - 28x28

Input Layer : 784(28x28)

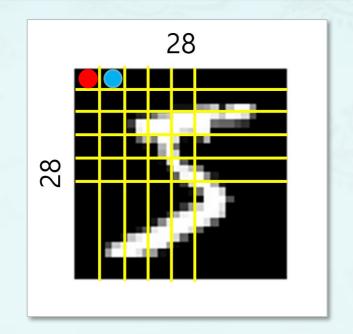


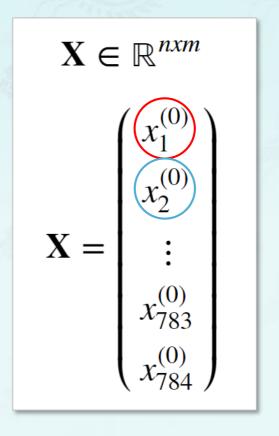


- MNIST
  - 28x28

Input Layer : 784(28x28)

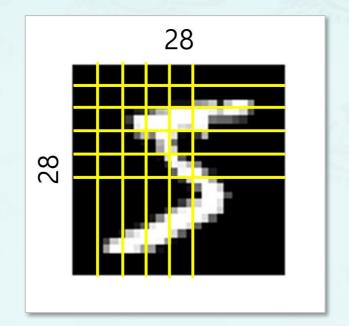






- MNIST
  - 28x28

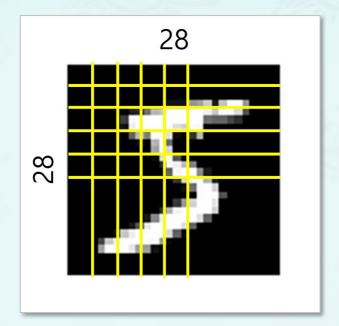




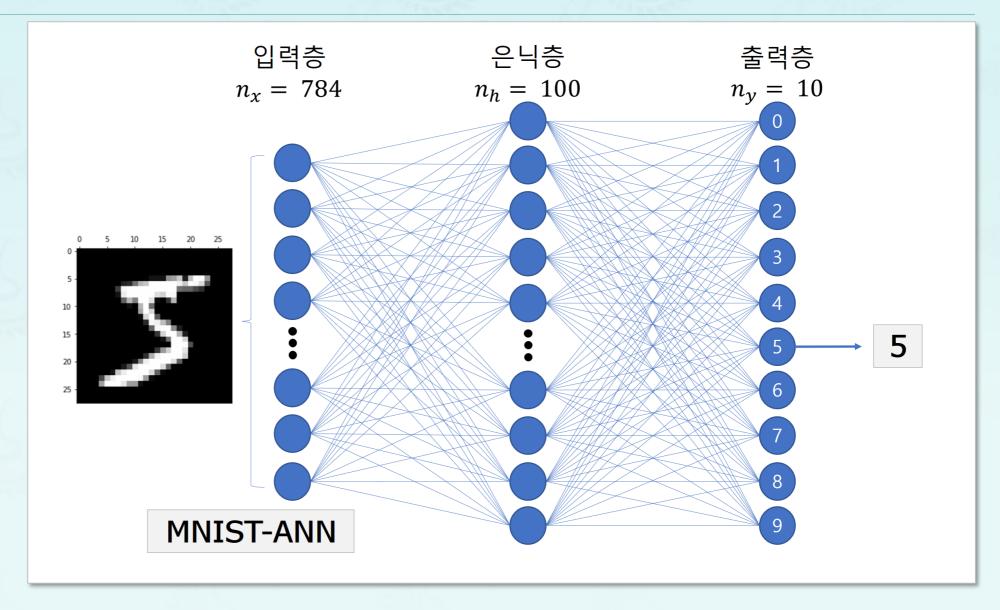
- Input Layer : 784(28x28)
- Hidden Layer: 100

- MNIST
  - 28x28

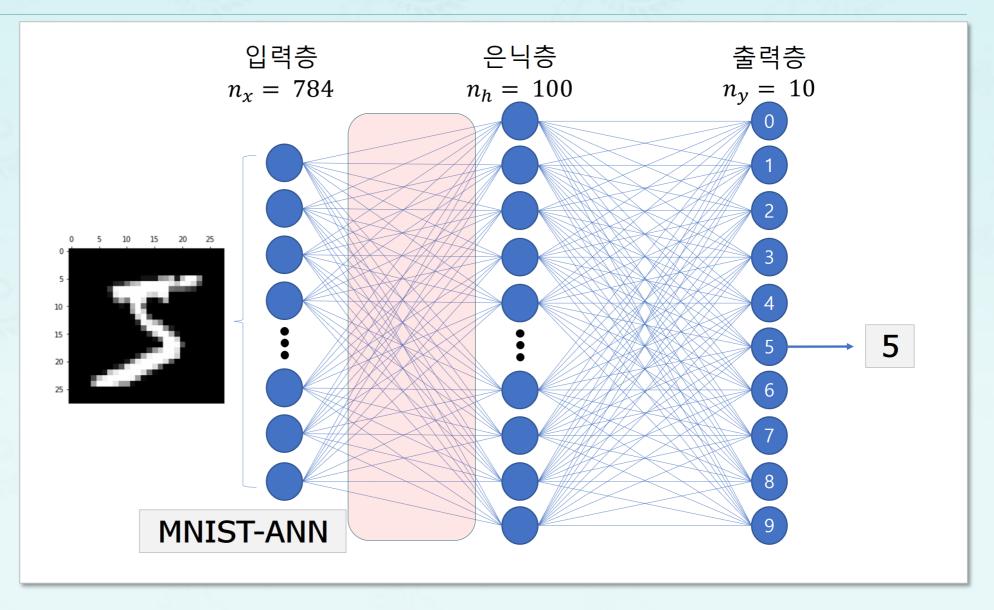




- Input Layer : 784(28x28)
- Hidden Layer: 100
- Output Layer Computation: 10

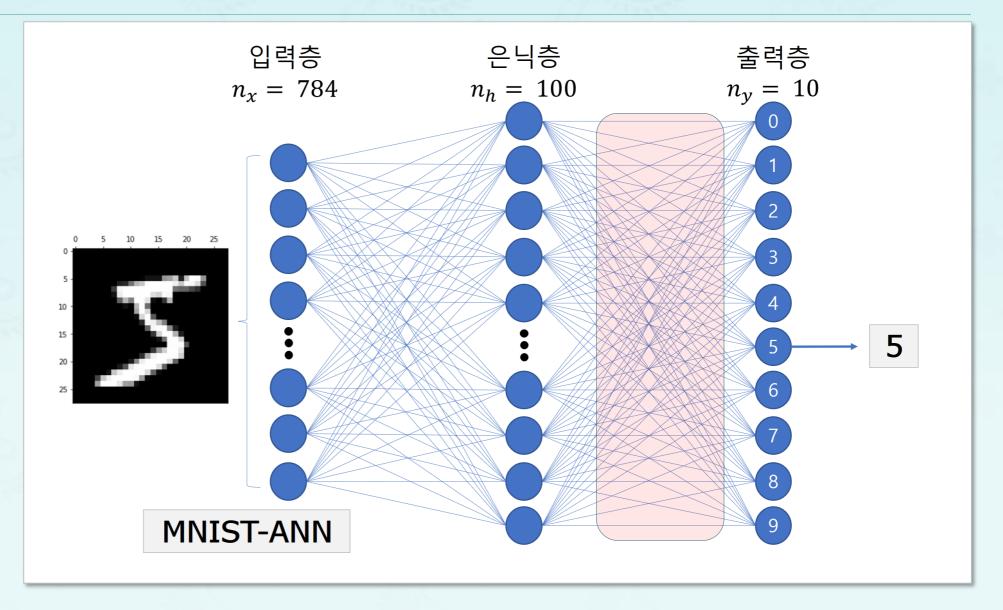


- Weights
  - **784x100**



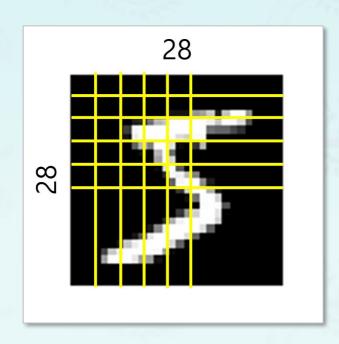
#### Weights

- **784x100**
- 100x10



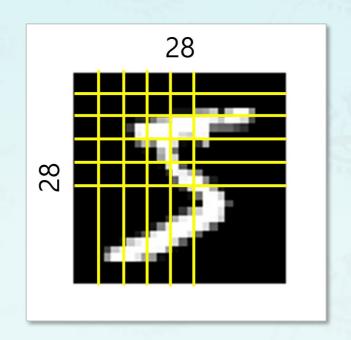
## 2. ANN Implementation: Input Dataset

- Xnxm
- n = 784, m = 1



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- Xnxm
- n = 784, m = 1

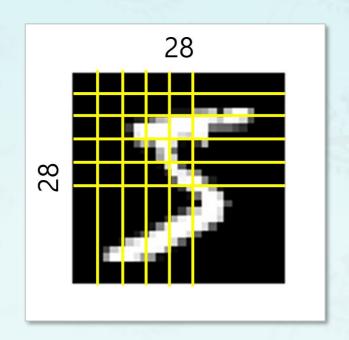


$$\mathbf{X} \in \mathbb{R}^{nxm}$$

$$\mathbf{X} = \begin{pmatrix} x_1^{0} \\ x_2^{0} \\ \vdots \\ x_{783}^{0} \\ x_{784}^{0} \end{pmatrix}$$

## 2. ANN Implementation: Input Dataset

- Xnxm
- n = 784, m = 1



$$\mathbf{X} \in \mathbb{R}^{n \times m}$$

$$\mathbf{X} = \begin{pmatrix} x_{1}^{(0)} \\ x_{2}^{(0)} \\ \vdots \\ x_{783}^{(0)} \\ x_{784}^{(0)} \end{pmatrix}$$

## 2. ANN Implementation: Reusing Weights

- Preprocessed Weights
- 96% Accuracy

#### 2. ANN Implementation: Reusing Weights

•  $W^{[1]} = 100x784$ 

$$W^{[1]} = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & \cdots & W_1^{(783)} & w_1^{(784)} \\ w_2^{(1)} & w_2^{(2)} & \cdots & W_2^{(783)} & w_2^{(784)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{100}^{(1)} & w_{100}^{(2)} & \cdots & w_{100}^{(783)} & w_{100}^{(784)} \end{pmatrix}$$

### 2. ANN Implementation: Reusing Weights

•  $W^{[1]} = 100x784$ 

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!type data/w\_xh.txt

## 2. ANN Implementation: Hidden Layer Computation

### 2. ANN Implementation: Hidden Layer Computation

$$\mathbf{Z}^{[1]} = W^{[1]} A^{[0]}$$

$$= \begin{pmatrix} w_{11}^{(1)} & w_{21}^{(1)} \\ w_{12}^{(1)} & w_{22}^{(1)} \\ w_{13}^{(1)} & w_{23}^{(1)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= \begin{pmatrix} z_1^{(1)} \\ z_2^{(1)} \\ z_3^{(1)} \end{pmatrix}$$

### 2. ANN Implementation: Hidden Layer Computation

$$\mathbf{Z}^{[1]} = W^{[1]} A^{[0]}$$

$$= \begin{pmatrix} w_{11}^{(1)} & w_{21}^{(1)} \\ w_{12}^{(1)} & w_{22}^{(1)} \\ w_{13}^{(1)} & w_{23}^{(1)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= \begin{pmatrix} z_1^{(1)} \\ z_2^{(1)} \\ z_3^{(1)} \end{pmatrix}$$

$$= \begin{bmatrix} w_{11}^{(1)} & w_{21}^{(1)} \\ w_{11}^{(1)} & w_{21}^{(1)} \\ w_{12}^{(1)} & w_{23}^{(1)} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= \begin{bmatrix} w_{11}^{(1)} & w_{1}^{(2)} & \cdots & w_{1}^{(783)} & w_{1}^{(784)} \\ w_{21}^{(1)} & w_{22}^{(2)} & \cdots & w_{2}^{(783)} & w_{2}^{(784)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{100}^{(1)} & w_{100}^{(2)} & \cdots & w_{100}^{(783)} & w_{100}^{(784)} \end{bmatrix} \begin{pmatrix} x_{1}^{(0)} \\ x_{2}^{(0)} \\ \vdots \\ x_{783}^{(0)} \\ x_{784}^{(0)} \end{pmatrix}$$

$$\mathbf{A}^{[1]} = sigmoid(\mathbf{Z}^{[1]})$$

### 2. ANN Implementation: Output Layer Computation

A<sup>[1]</sup>: 100x1

$$W^{[2]}A^{[1]} = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & \cdots & w_1^{(99)} & w_1^{(100)} \\ w_2^{(1)} & w_2^{(2)} & \cdots & w_2^{(99)} & w_2^{(100)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{10}^{(1)} & w_{10}^{(2)} & \cdots & w_{10}^{(99)} & w_{10}^{(100)} \end{pmatrix} \begin{pmatrix} a_1^{(1)} \\ a_2^{(1)} \\ \vdots \\ a_{99}^{(1)} \\ a_{100}^{(1)} \end{pmatrix}$$

### 2. ANN Implementation: Output Layer Computation

A<sup>[1]</sup>: 100x1

W<sup>[2]</sup>: 10x100

$$W^{[2]}A^{[1]} = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & \cdots & w_1^{(99)} & w_1^{(100)} \\ w_2^{(1)} & w_2^{(2)} & \cdots & w_2^{(99)} & w_2^{(100)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{10}^{(1)} & w_{10}^{(2)} & \cdots & w_{10}^{(99)} & w_{10}^{(100)} \end{pmatrix} \begin{pmatrix} a_1^{(1)} \\ a_2^{(1)} \\ \vdots \\ a_{99}^{(1)} \\ a_{100}^{(1)} \end{pmatrix}$$

### 2. ANN Implementation: Output Layer Computation

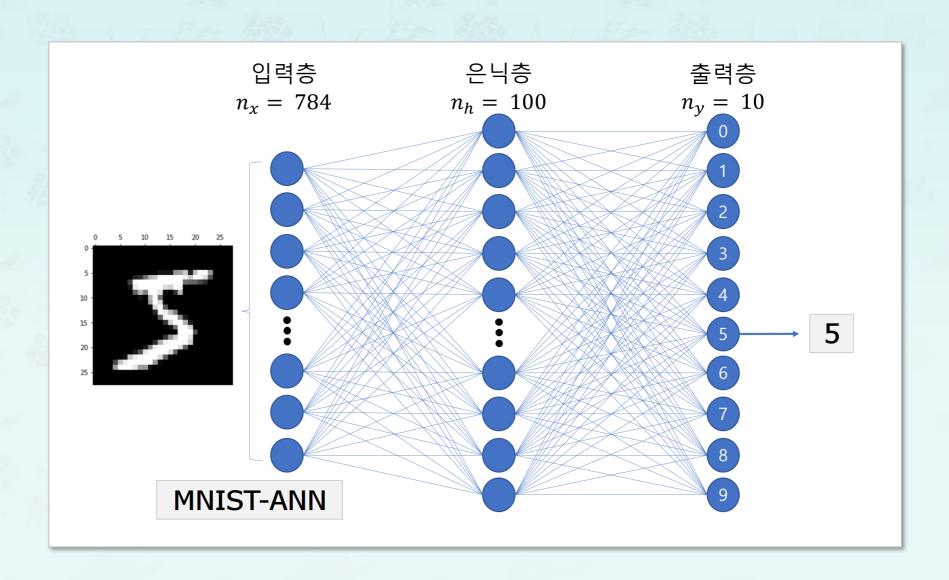
A<sup>[1]</sup>: 100x1

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$$\mathbf{A}^{[2]} = sigmoid(\mathbf{Z}^{[2]})$$

## 2. ANN Implementation: Structure



```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```

```
(X, y) = joy.load_mnist_num(7)
W1 = joy.load_mnist_weight('data/w_xh.weights')
Z1 = np.dot(W1, X)
A1 = g(Z1)

W2 = joy.load_mnist_weight('data/w_hy.weights')
Z2 = np.dot(W2, A1)
yhat = g(Z2)

print('image:', y)
print('predict:', np.round_(yhat, 3))
```

#### Library

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```

```
(X, y) = joy.load_mnist_num(7)
W1 = joy.load_mnist_weight('data/w_xh.weights')
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print('image:', y)
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#### Activation function

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
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print('image:', y)
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```

#### Input Dataset Loading

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```



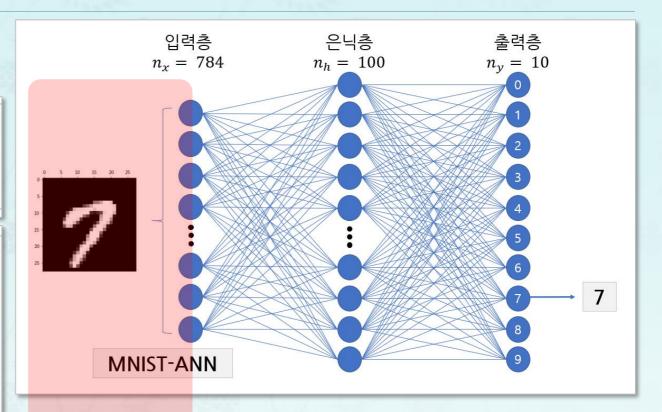
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#### Reusing Weights

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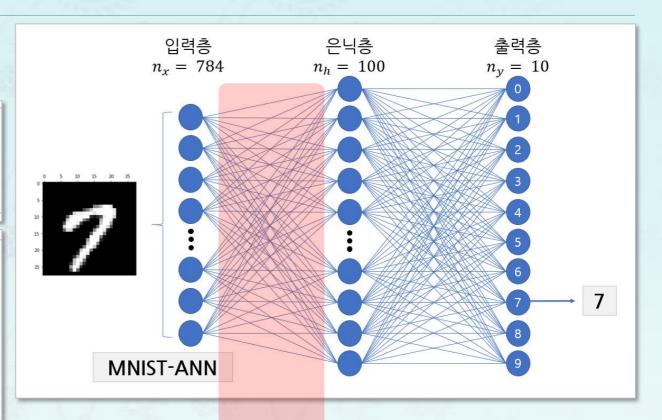
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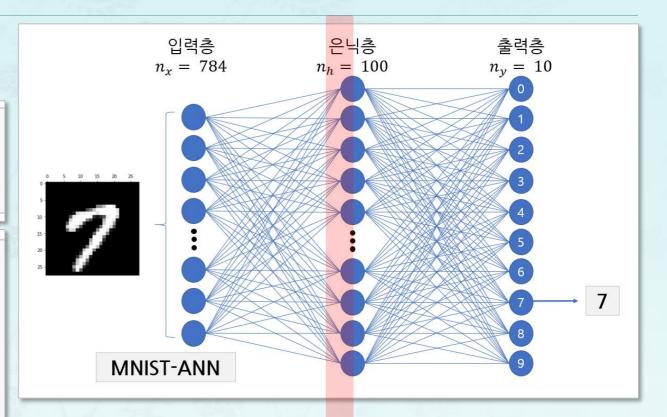
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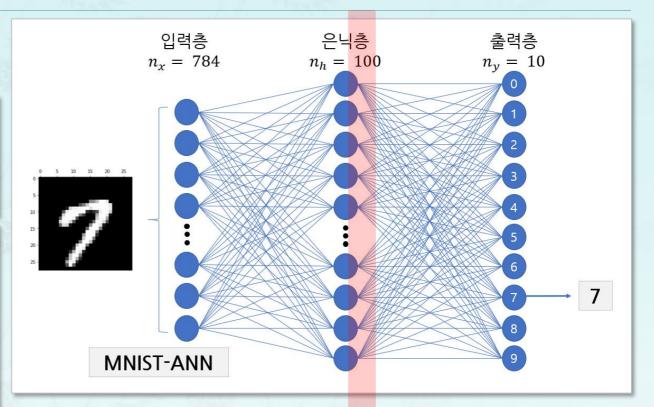
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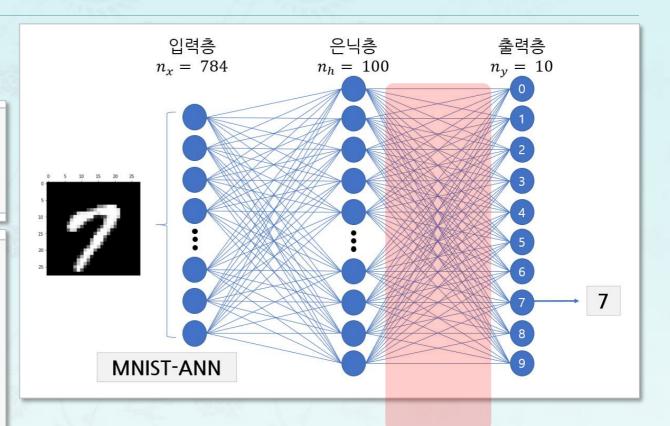
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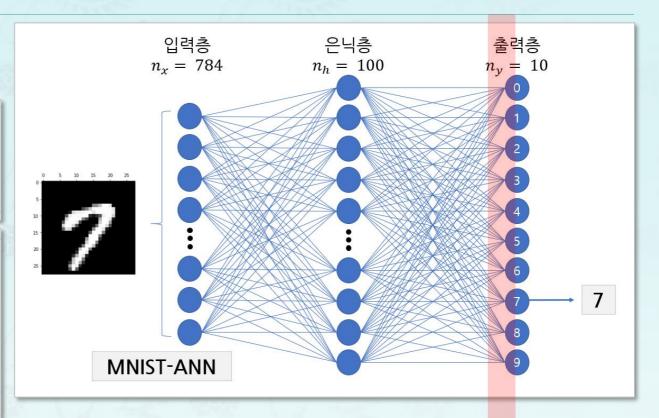
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print('image:', y)
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```



### Output Layer Computation

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```

```
(X, y) = joy.load_mnist_num(7)
W1 = joy.load_mnist_weight('data/w_xh.weights')
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A1 = g(Z1)

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Z2 = np.dot(W2, A1)
yhat = g(Z2)

print('image:', y)
print('predict:', np.round_(yhat, 3))
```

$$W^{[2]}A^{[1]} = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & \cdots & w_1^{(99)} & w_1^{(100)} \\ w_2^{(1)} & w_2^{(2)} & \cdots & w_2^{(99)} & w_2^{(100)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{10}^{(1)} & w_{10}^{(2)} & \cdots & w_{10}^{(99)} & w_{10}^{(100)} \end{pmatrix} \begin{pmatrix} a_1^{(1)} \\ a_2^{(1)} \\ \vdots \\ a_{99}^{(1)} \\ a_{100}^{(1)} \end{pmatrix}$$

$$\mathbf{A}^{[2]} = sigmoid(\mathbf{Z}^{[2]})$$

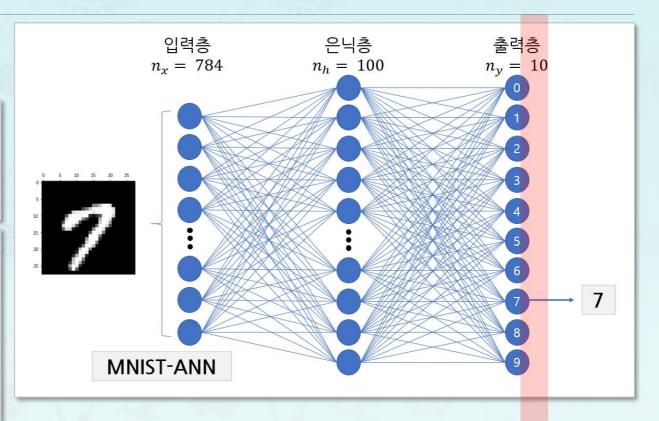
### Output Layer Computation

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```

```
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Z2 = np.dot(W2, A1)
yhat = g(Z2)

print('image:', y)
print('predict:', np.round_(yhat, 3))
```



#### Prediction

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```

```
(X, y) = joy.load_mnist_num(7)
W1 = joy.load_mnist_weight('data/w_xh.weights')
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Z2 = np.dot(W2, A1)
yhat = g(Z2)

print('image:', y)
print('predict:', np.round_(yhat, 3))
```



```
image: 7
predict: [0.
           0.002
           0.001
           0.
           0.
           0.001
           0.
           0.979
           0.006
           0.003]
```

#### Prediction

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```

```
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Z1 = np.dot(W1, X)
A1 = g(Z1)

W2 = joy.load_mnist_weight('data/w_hy.weights')
Z2 = np.dot(W2, A1)
yhat = g(Z2)

print('image:', y)
print('predict:', np.round_(yhat, 3))
```

```
image: 7
predict: [0.
           0.002
           0.001
           0.
           0.
           0.001
           0.
           0.979
           0.006
           0.003]
```

#### Prediction

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```

```
(X, y) = joy.load_mnist_num(7)
W1 = joy.load_mnist_weight('data/w_xh.weights')
Z1 = np.dot(W1, X)
A1 = g(Z1)

W2 = joy.load_mnist_weight('data/w_hy.weights')
Z2 = np.dot(W2, A1)
yhat = g(Z2)

print('image:', y)
print('predict:', np.round_(yhat, 3))
```

```
image: 7
predict: [0.
           0.002
           0.001
           0.
           0.
                        5
           0.001
          0.979
           0.006
           0.003]
                        9
```

### Feed-forward Neural Network - Example

#### Summary

- Understanding Feed-forward Neural Network Example
- Understanding Features from data
- Feed-forward Neural Network Example Modeling
- Reusing Weights

#### Next

7-3 Adaline Gradient Descent

7주차(2/3)

# Feed-forward Neural Network

Machine Learning with Python

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여러분 곁에 항상 열려 있는 K-MOOC 강의실에서 만나 뵙기를 바랍니다.