Sorting(2/2)

Data Structures
C++ for C Coders

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Merge Sort
Quick Sort

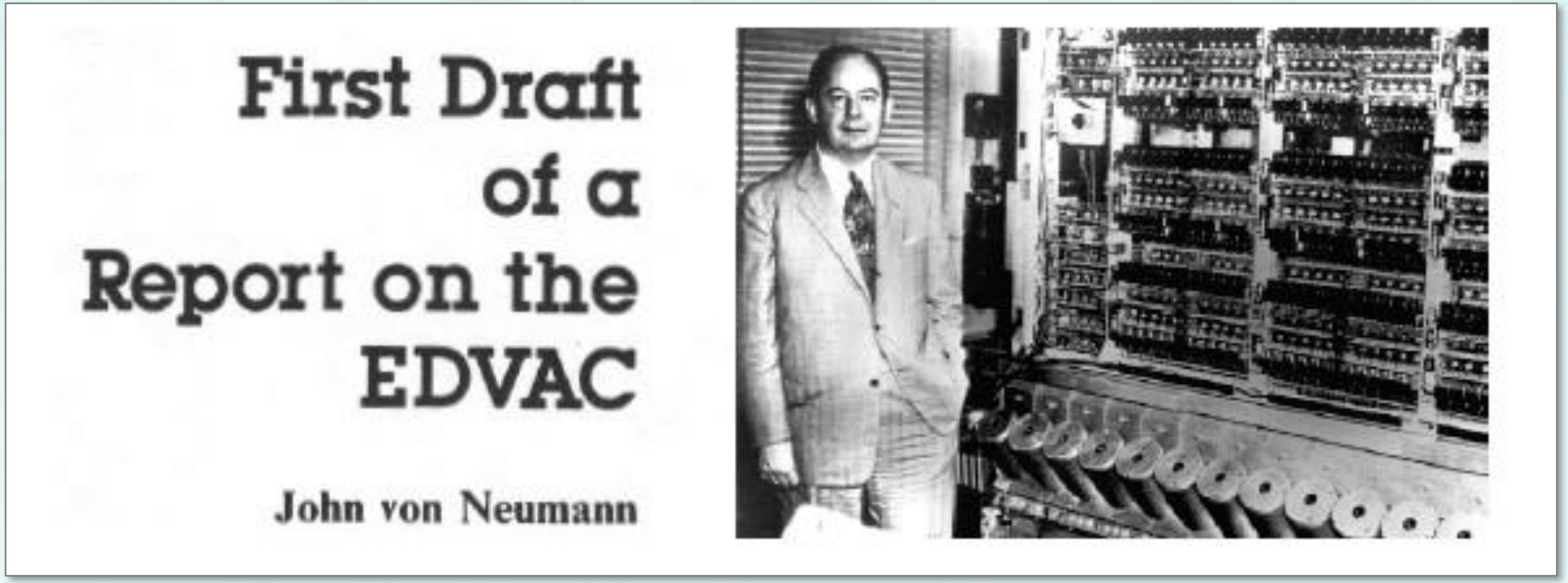
Objectives & Agenda

- **Objectives:**
  - Understand advanced algorithms of sorting.

- **Agenda**
  - Merge Sort
  - Quick Sort

Mergesort

- Divide and conquer algorithm
- Recursive or non-recursive(Iteration) implementation
- It was implemented on the first general purpose computer and is still running.



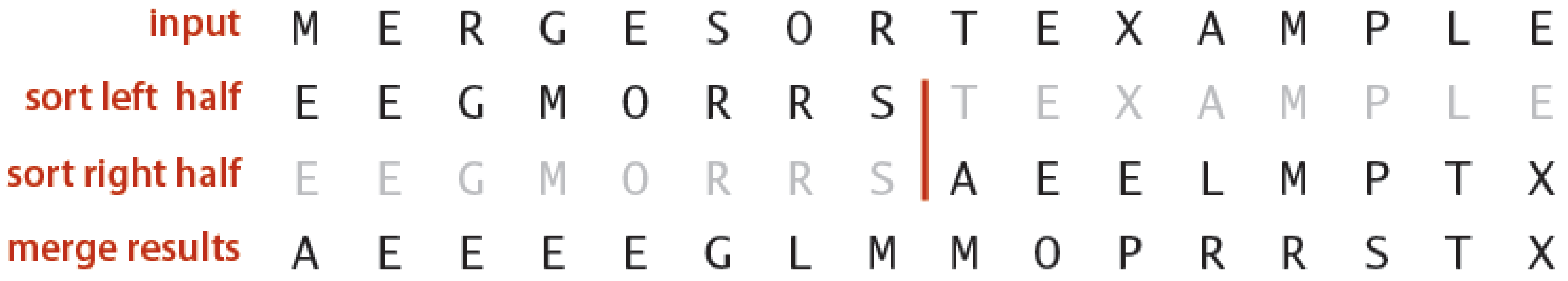
First Draft
of a
Report on the
EDVAC

John von Neumann

the first general
purpose computer
and its inventor,

Mergesort: Algorithm

- Divide array into two halves.
- Recursively sort each half.
- Merge two halves.

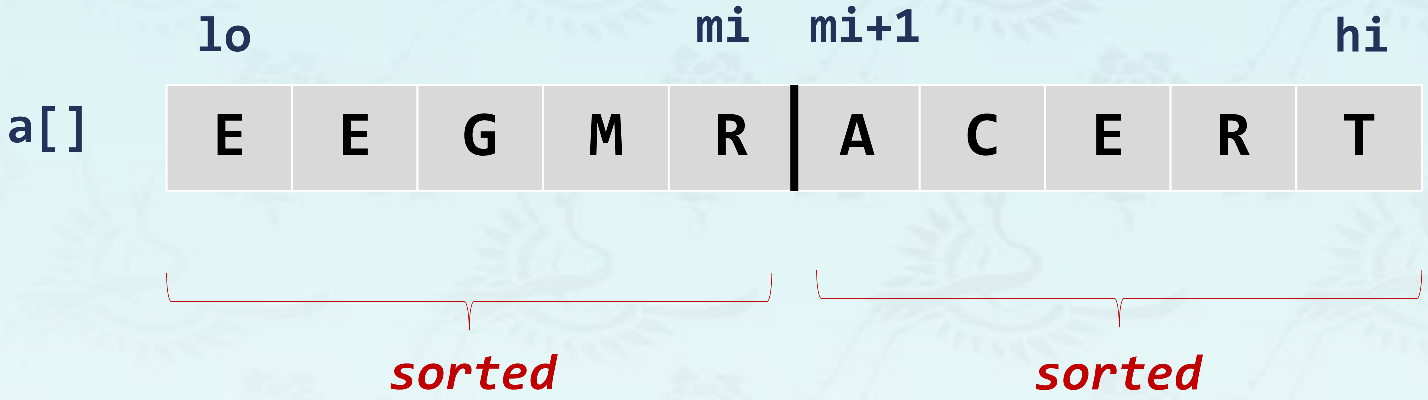


input	M	E	R	G	E	S	O	R	T	E	X	A	M	P	L	E
sort left half	E	E	G	M	O	R	R	S	T	E	X	A	M	P	L	E
sort right half	E	E	G	M	O	R	R	S	A	E	E	L	M	P	T	X
merge results	A	E	E	E	E	G	L	M	M	O	P	R	R	S	T	X

Mergesort overview

Mergesort: merge

- **Goal:** Given two sorted subarrays **`a[lo]`** to **`a[mi]`** and **`a[mi+1]`** to **`a[hi]`**, replace with sorted subarray **`a[lo]`** to **`a[hi]`**.



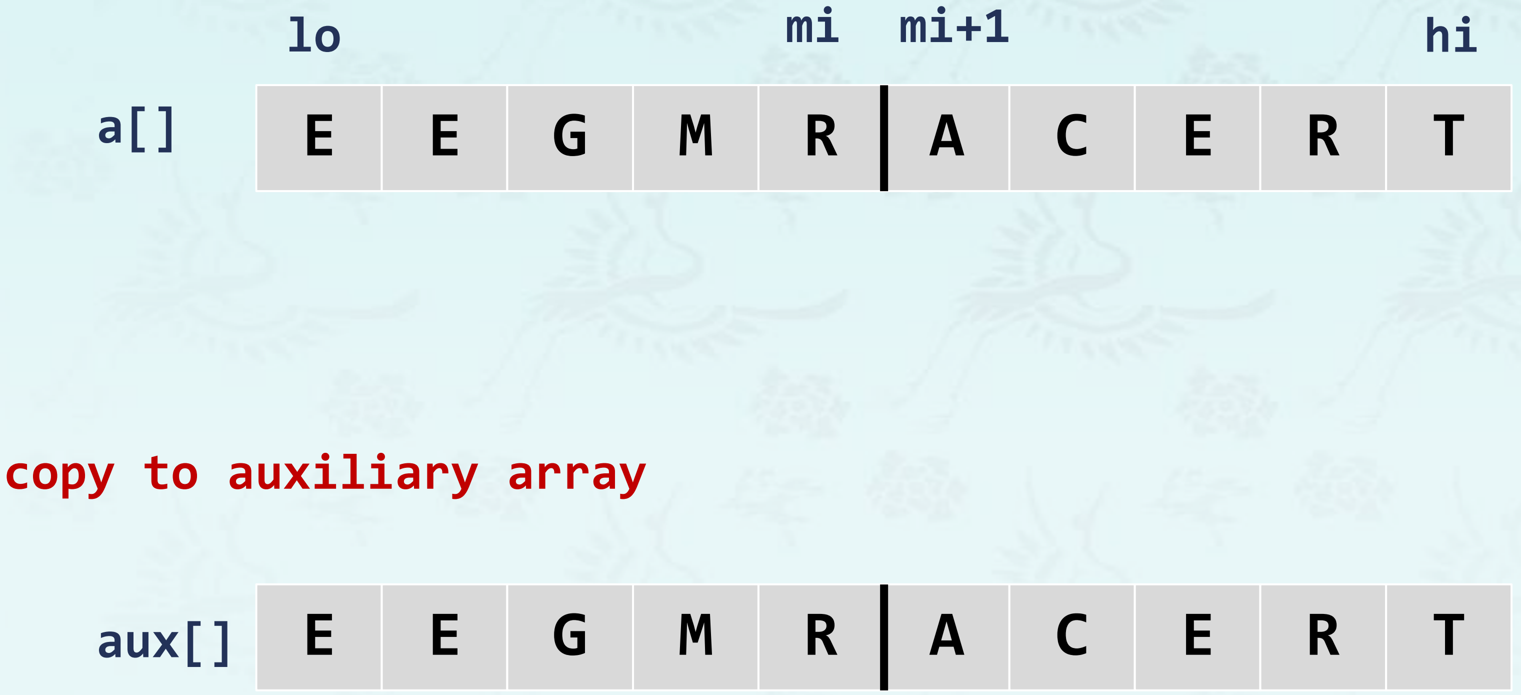
lo mi mi+1 hi

a[] E E G M R | A C E R T

sorted *sorted*

Mergesort: merge

- **Goal:** Given two sorted subarrays **a[lo]** to **a[mi]** and **a[mi+1]** to **a[hi]**, replace with sorted subarray **a[lo]** to **a[hi]**.



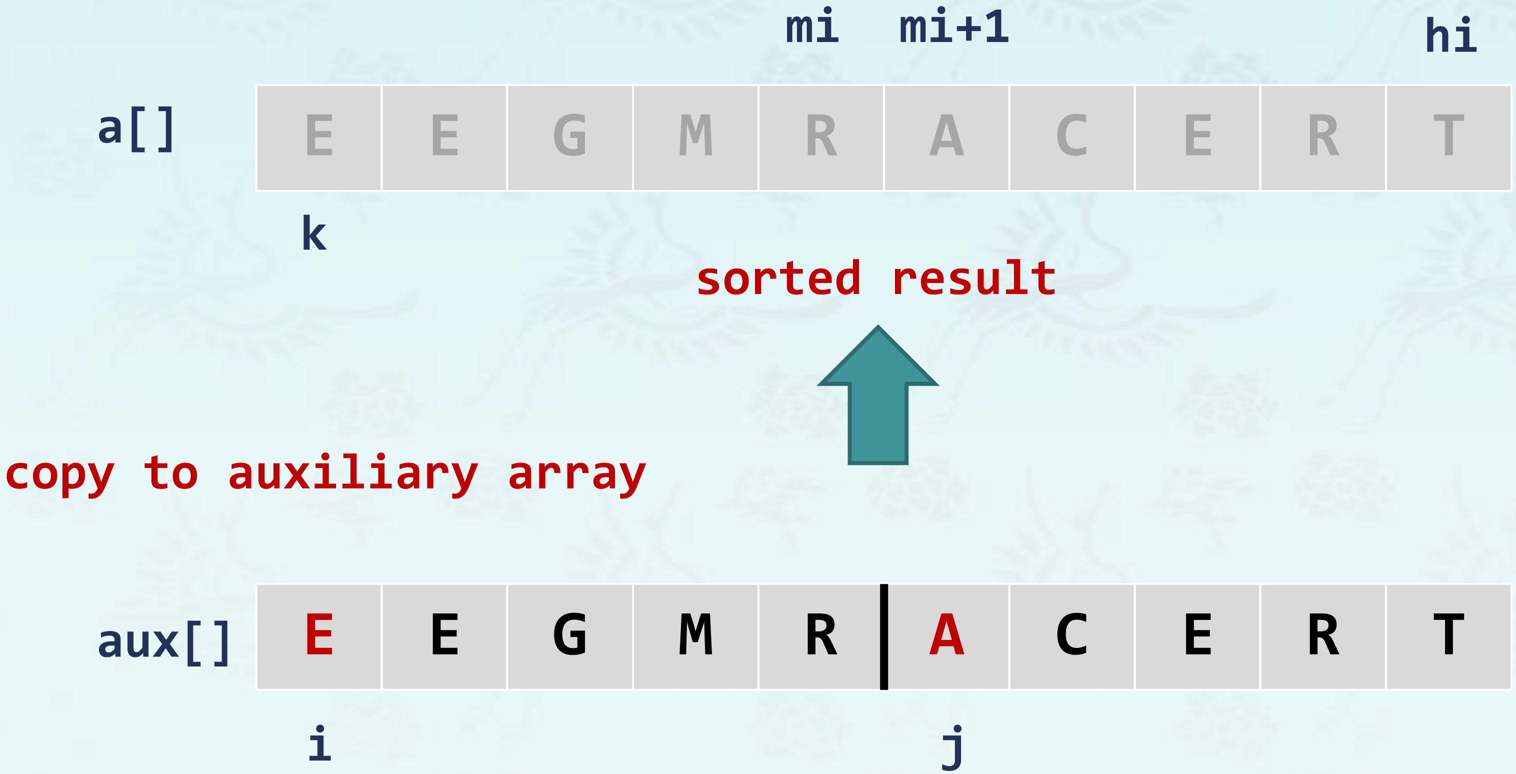
	lo				mi	mi+1				hi
a[]	E	E	G	M	R	A	C	E	R	T

copy to auxiliary array

aux[]	E	E	G	M	R	A	C	E	R	T

Mergesort: merge

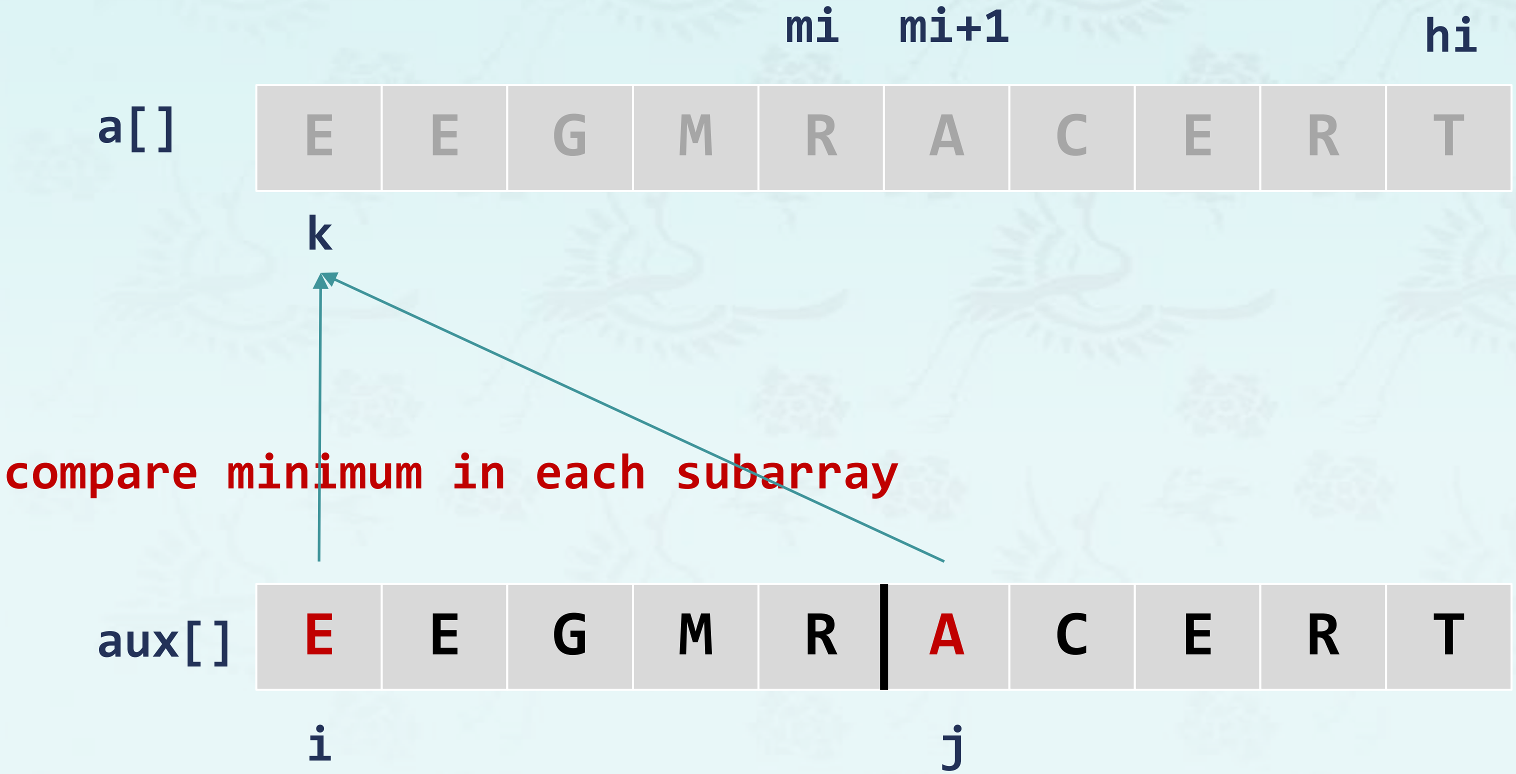
- **Goal:** Given two sorted subarrays **`a[lo]`** to **`a[mi]`** and **`a[mi+1]`** to **`a[hi]`**, replace with sorted subarray **`a[lo]`** to **`a[hi]`**.



a[]	E	E	G	M	R	A	C	E	R	T
aux[]	E	E	G	M	R	A	C	E	R	T

Mergesort: merge

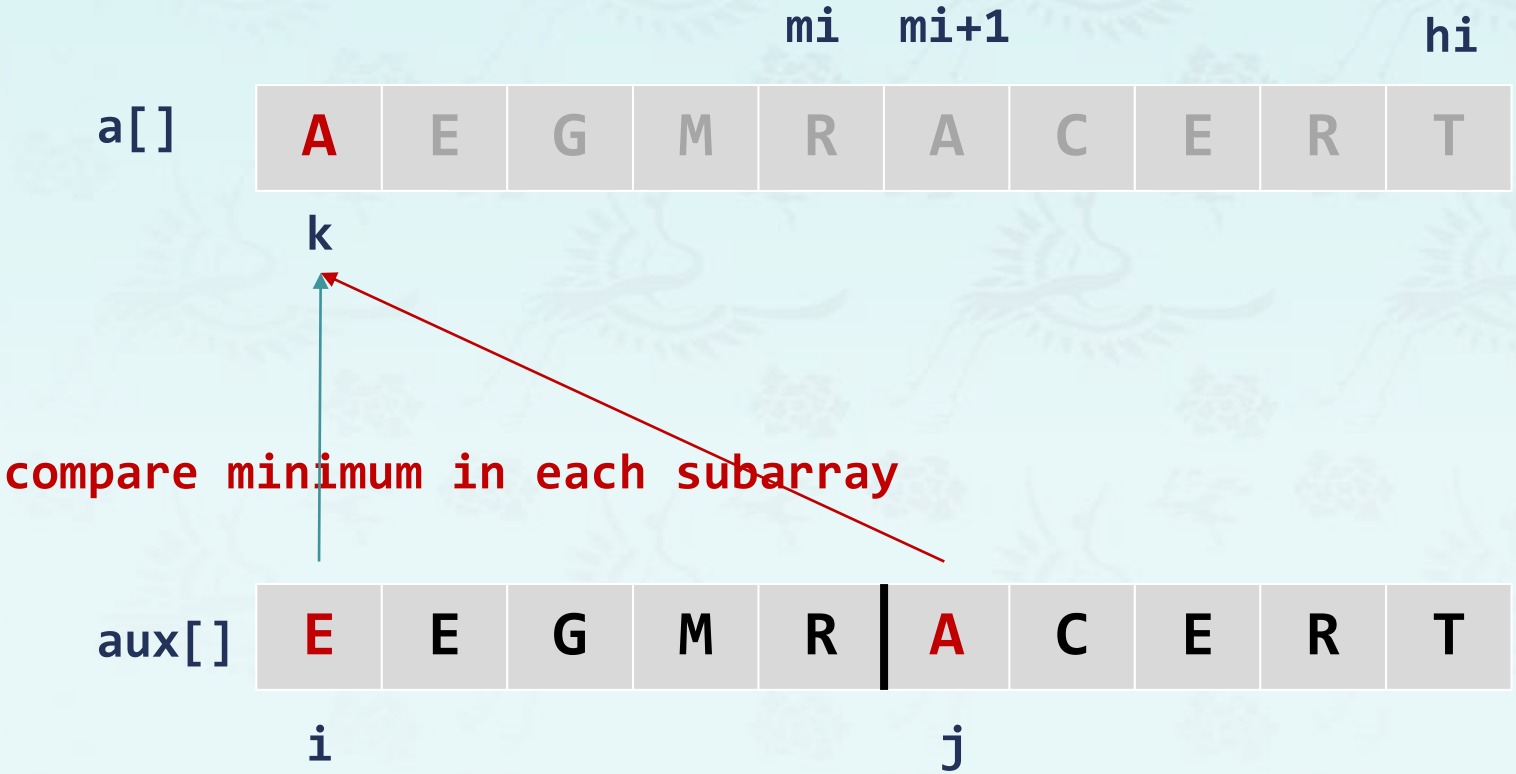
- **Goal:** Given two sorted subarrays **`a[lo]`** to **`a[mi]`** and **`a[mi+1]`** to **`a[hi]`**, replace with sorted subarray **`a[lo]`** to **`a[hi]`**.



compare minimum in each subarray

Mergesort: merge

- **Goal:** Given two sorted subarrays **`a[lo]`** to **`a[mi]`** and **`a[mi+1]`** to **`a[hi]`**, replace with sorted subarray **`a[lo]`** to **`a[hi]`**.



mi mi+1 hi

a[]	A	E	G	M	R	A	C	E	R	T

k

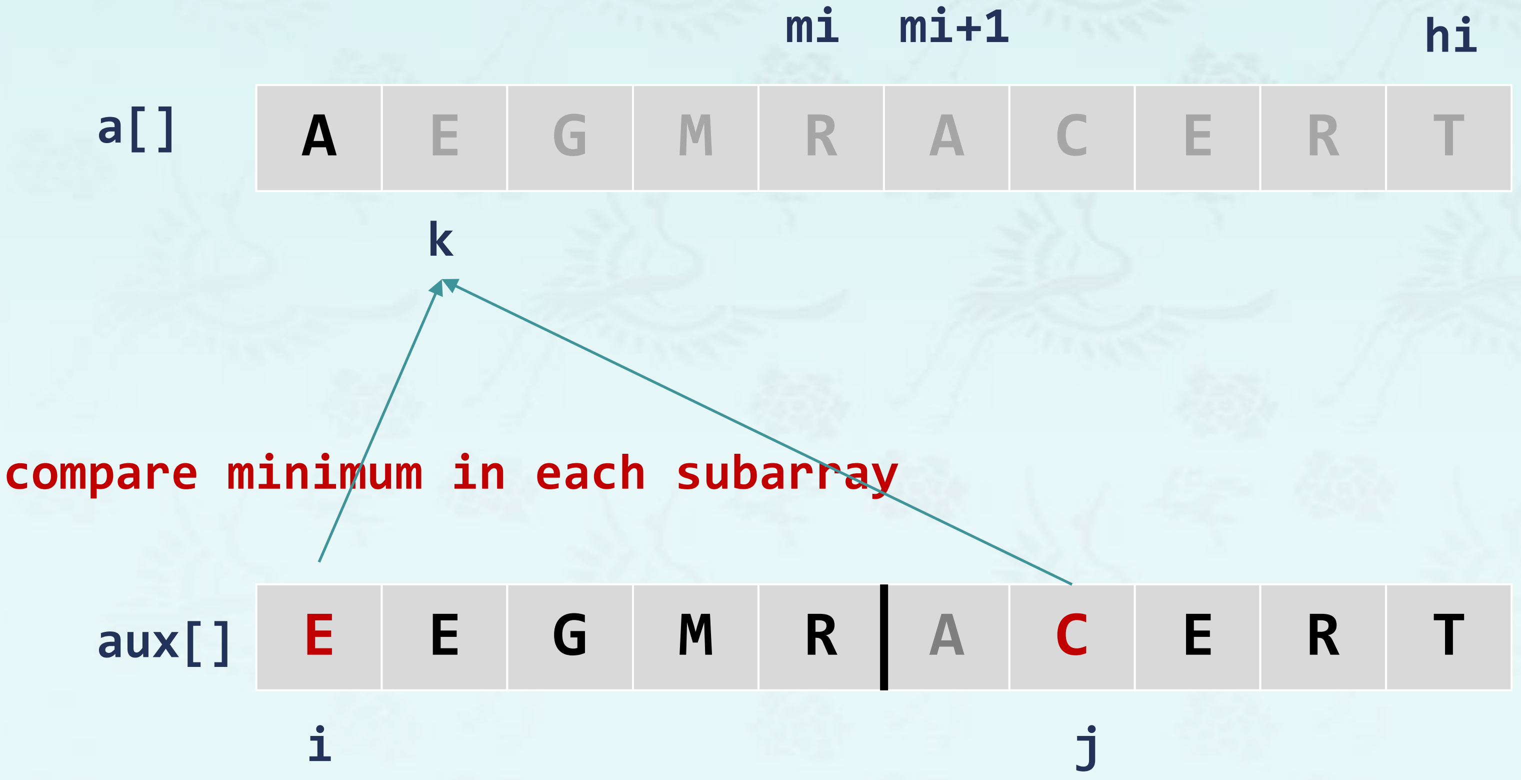
compare minimum in each subarray

aux[]	E	E	G	M	R	A	C	E	R	T

i j

Mergesort: merge

- **Goal:** Given two sorted subarrays **a[lo]** to **a[mi]** and **a[mi+1]** to **a[hi]**, replace with sorted subarray **a[lo]** to **a[hi]**.



Mergesort: merge

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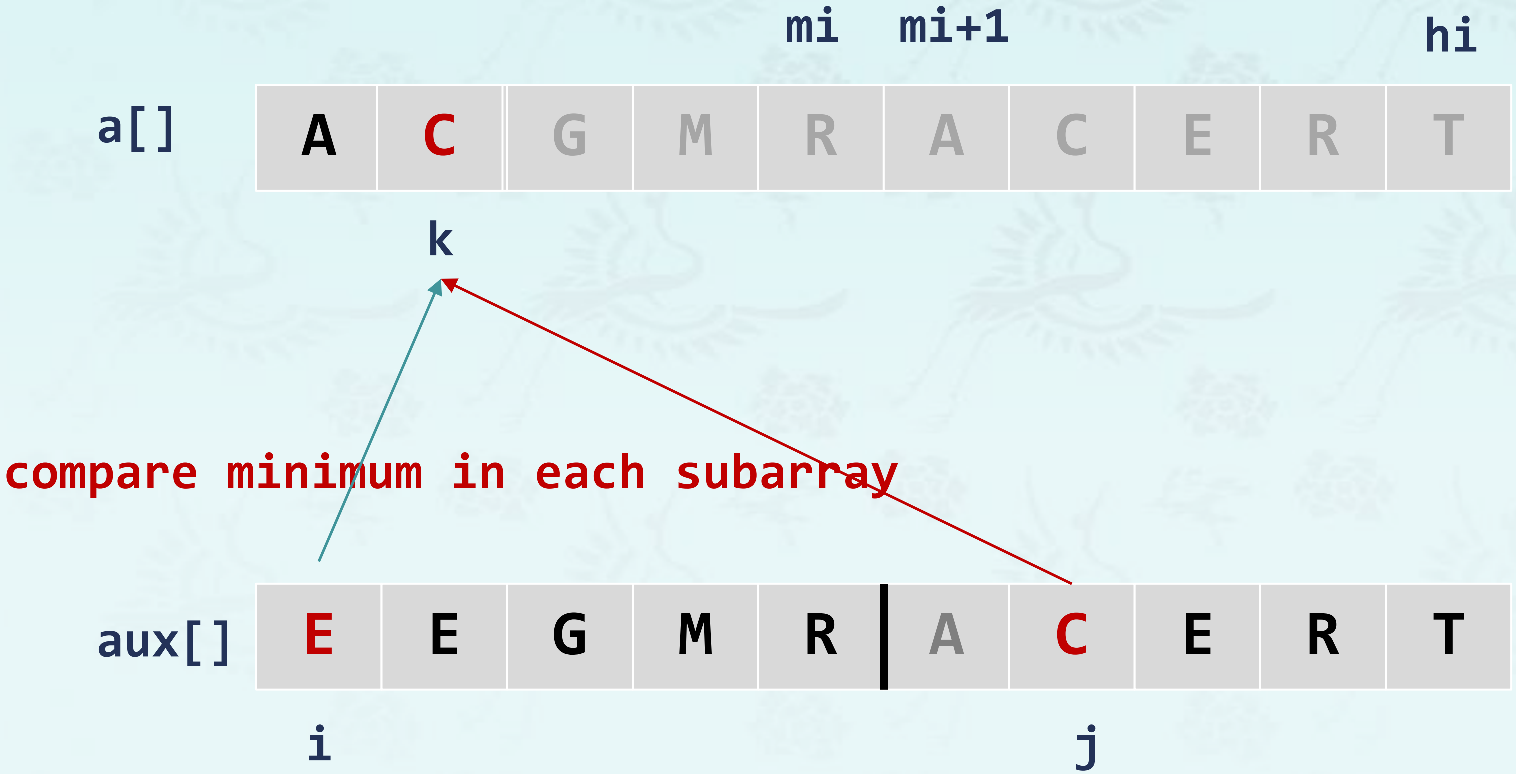

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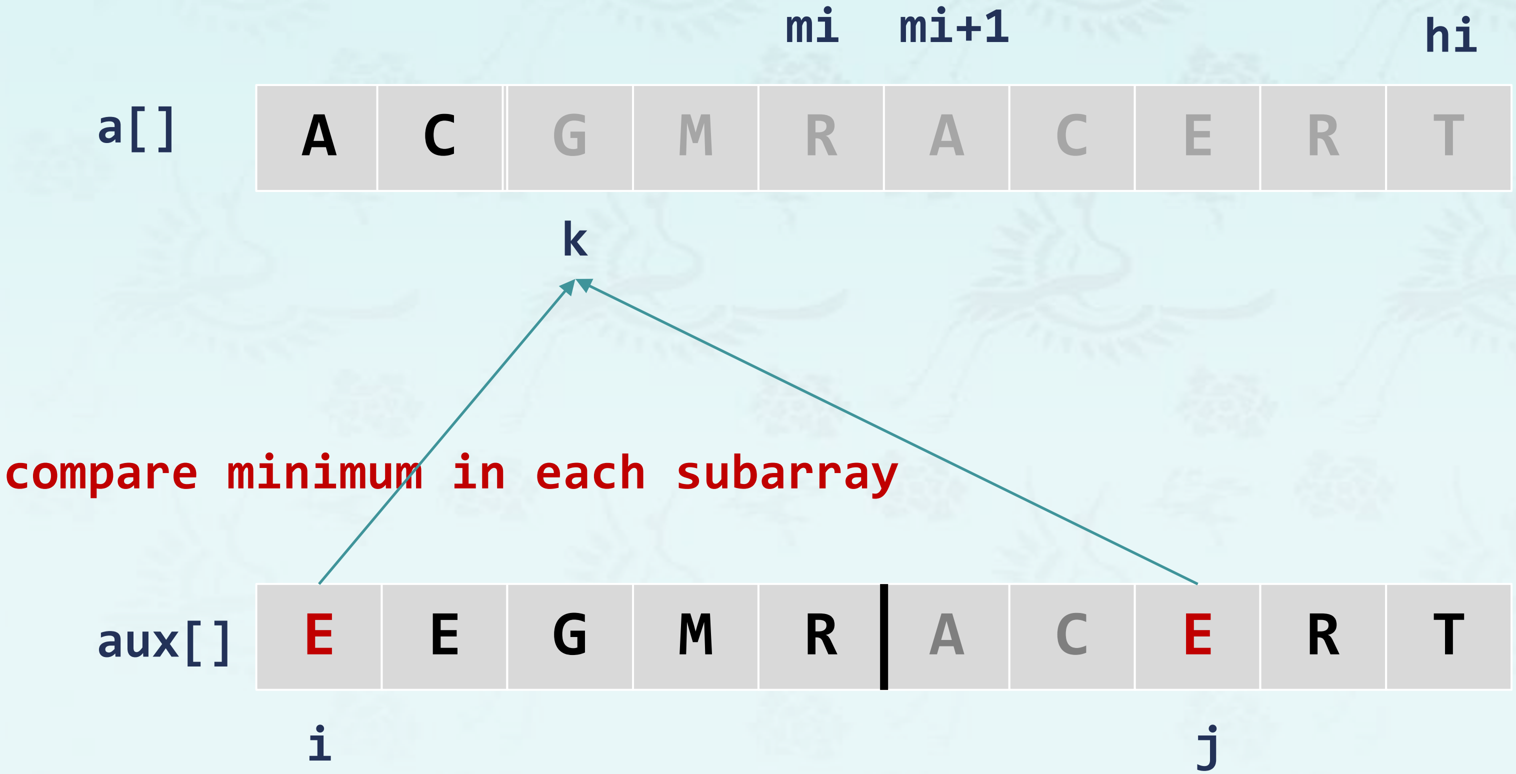
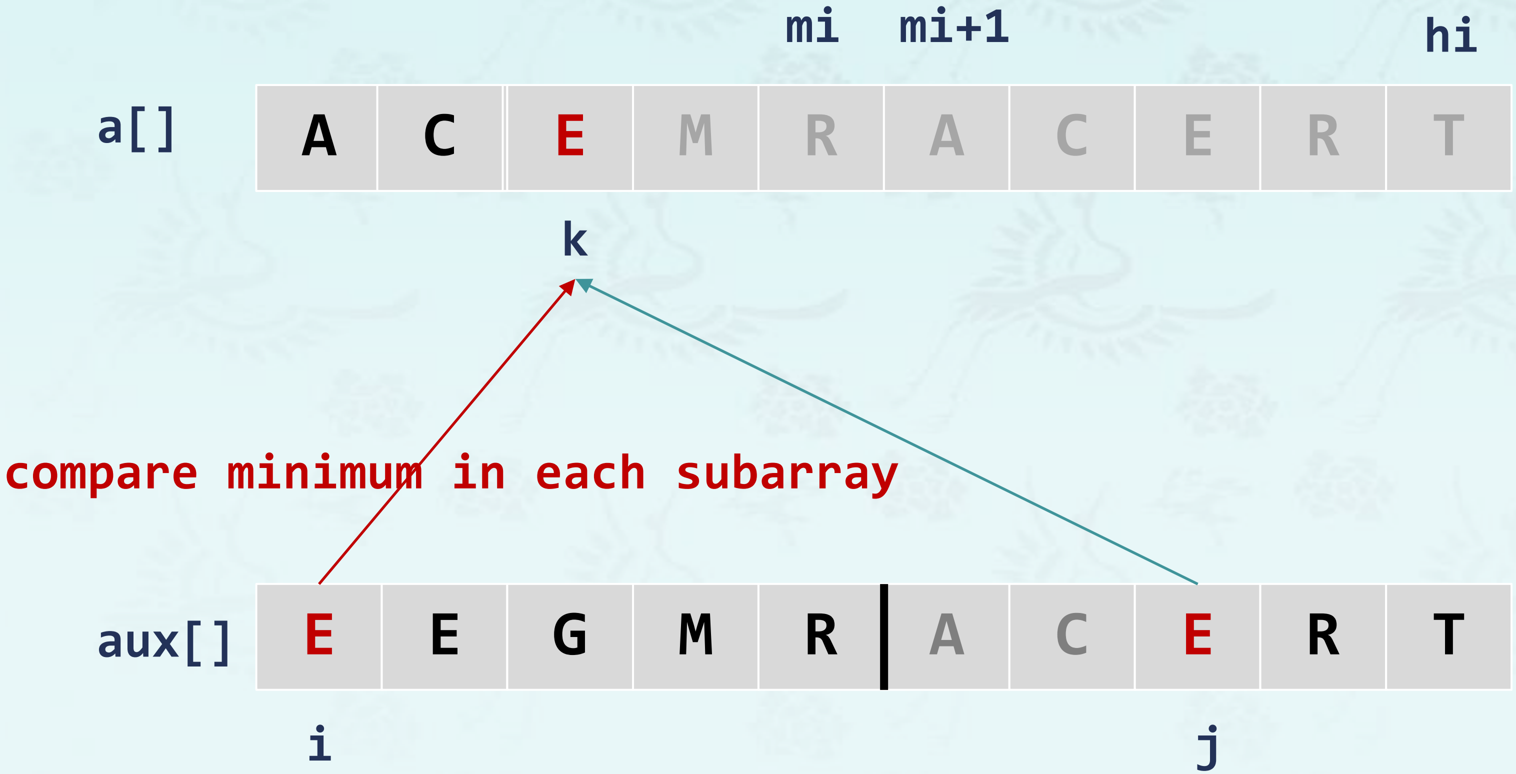


a[] A C G M R A C E R T

aux[] E E G M R A C E R T

Mergesort: merge

- **Goal:** Given two sorted subarrays **`a[lo]`** to **`a[mi]`** and **`a[mi+1]`** to **`a[hi]`**, replace with sorted subarray **`a[lo]`** to **`a[hi]`**.



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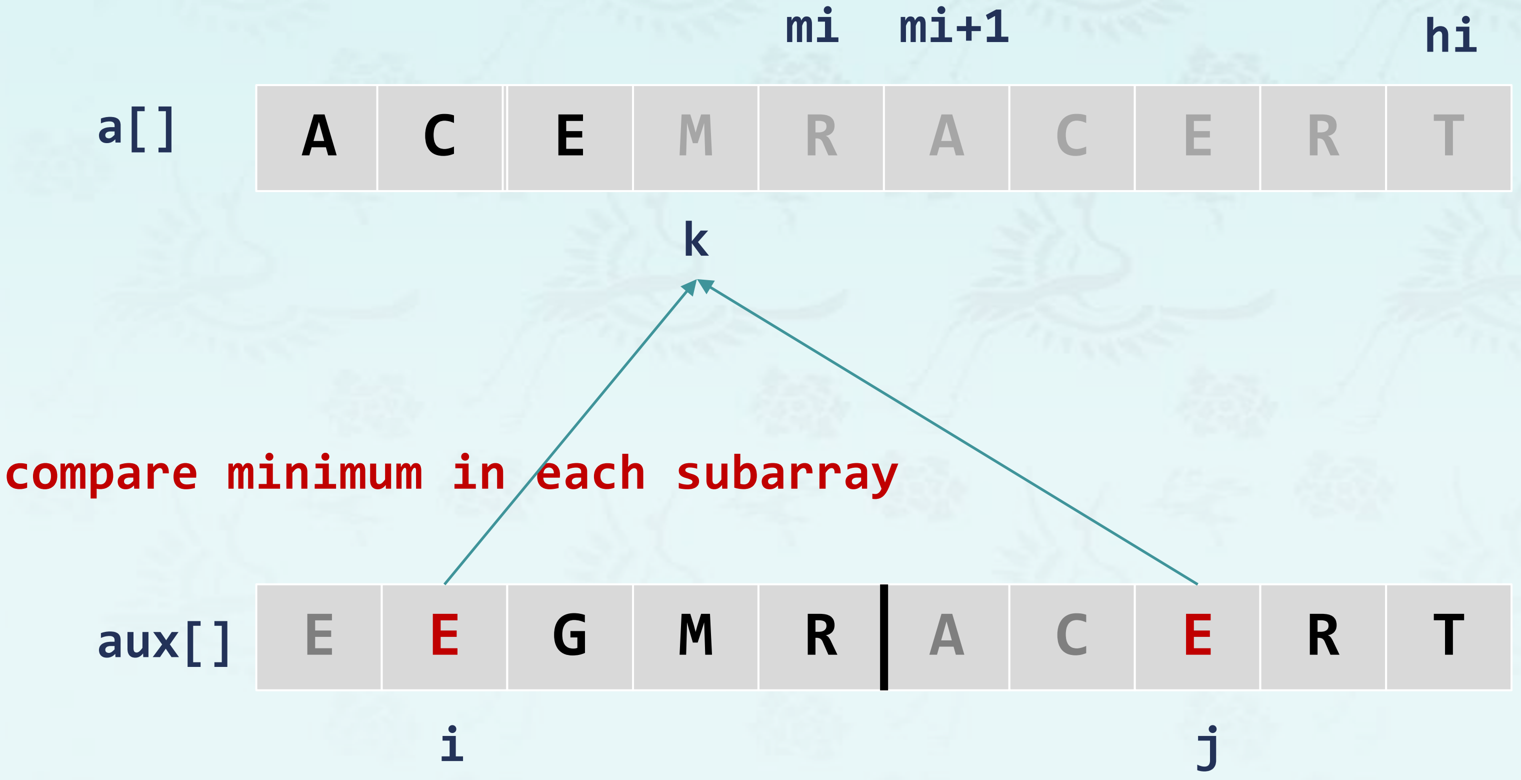

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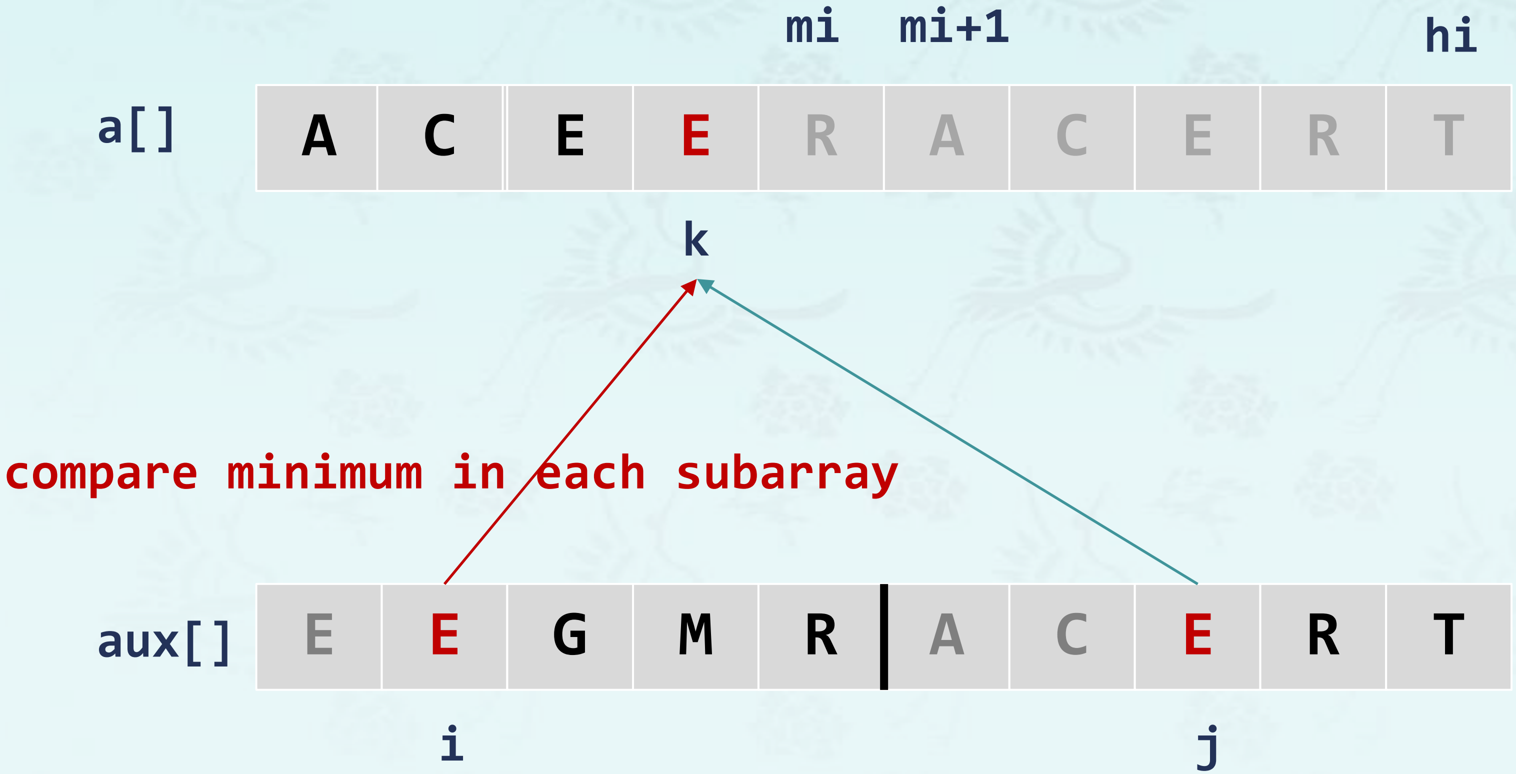

mi mi+1 hi

a[] A C E E R A C E R T

k

compare minimum in each subarray

aux[] E E G M R | A C E R T

i j

Mergesort: merge

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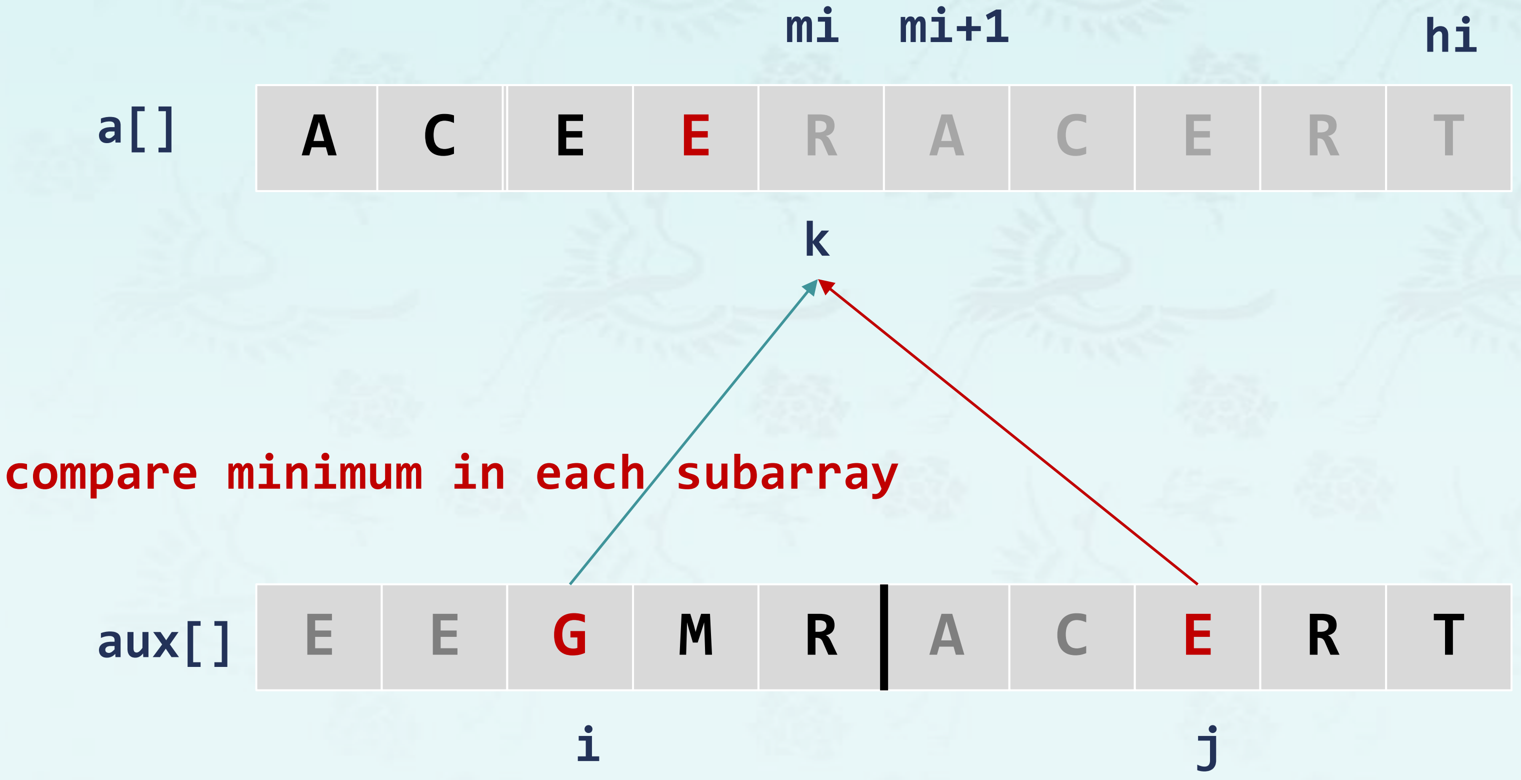

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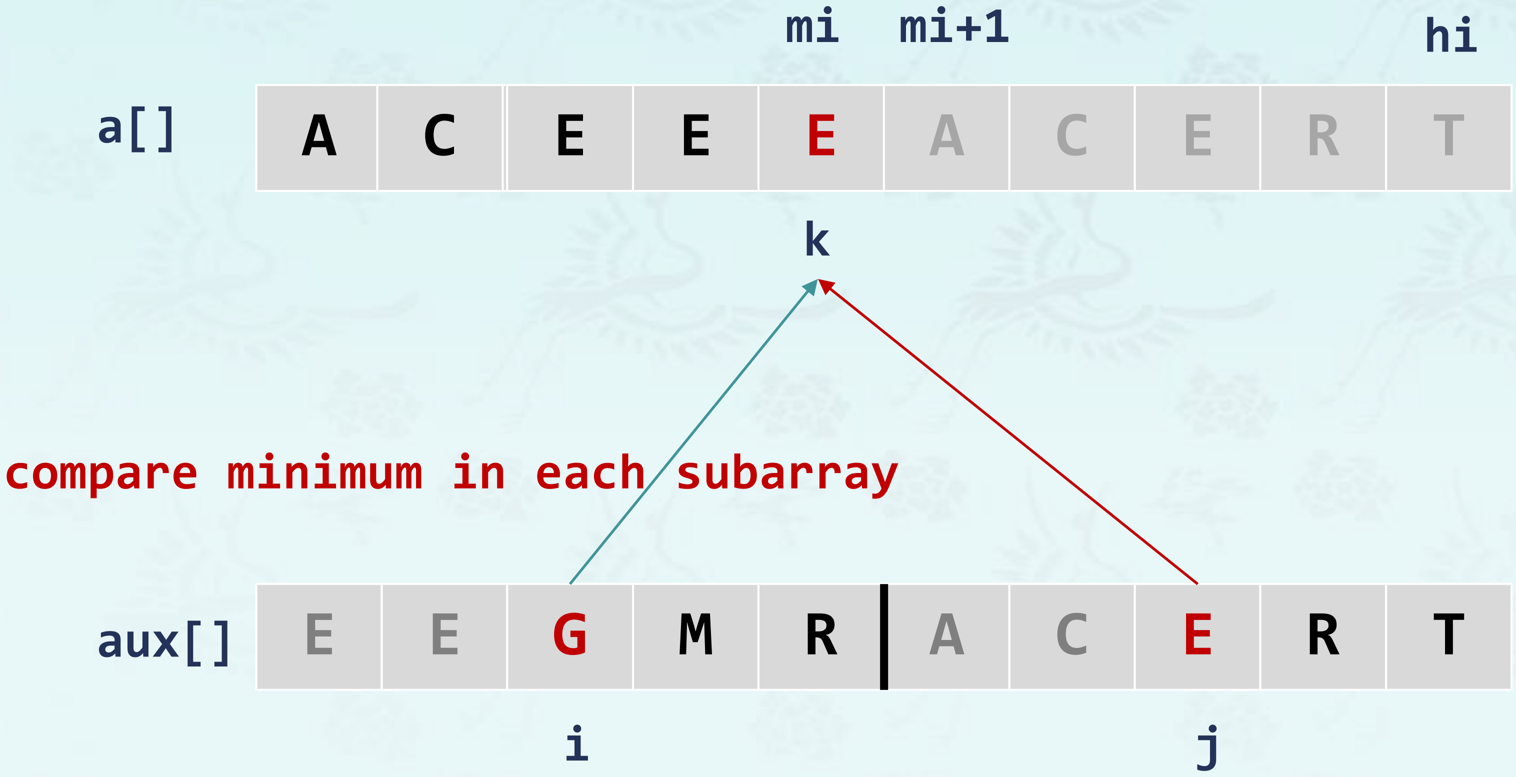

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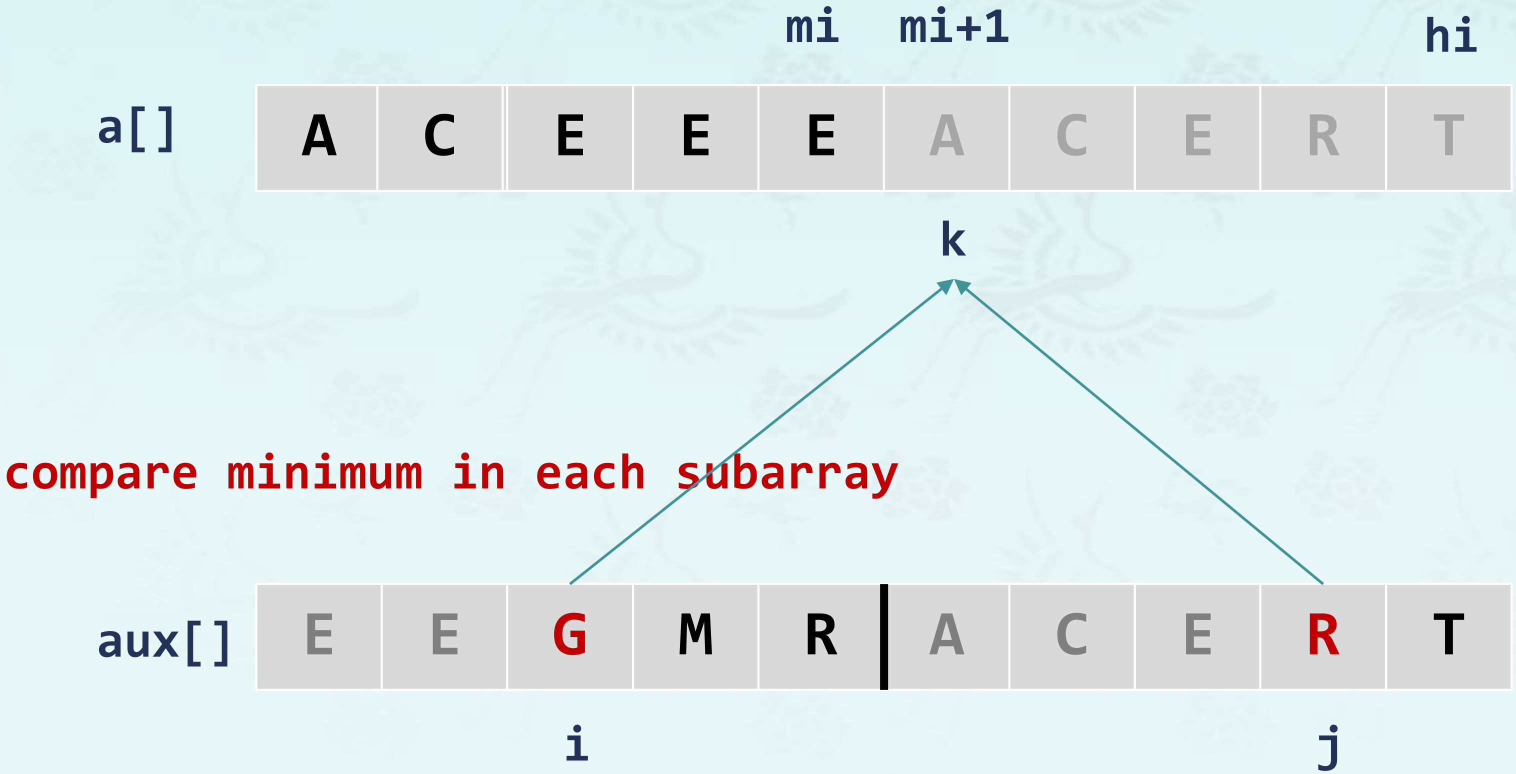


mi mi+1 hi

a[]									
A	C	E	E	E	A	C	E	R	T

k

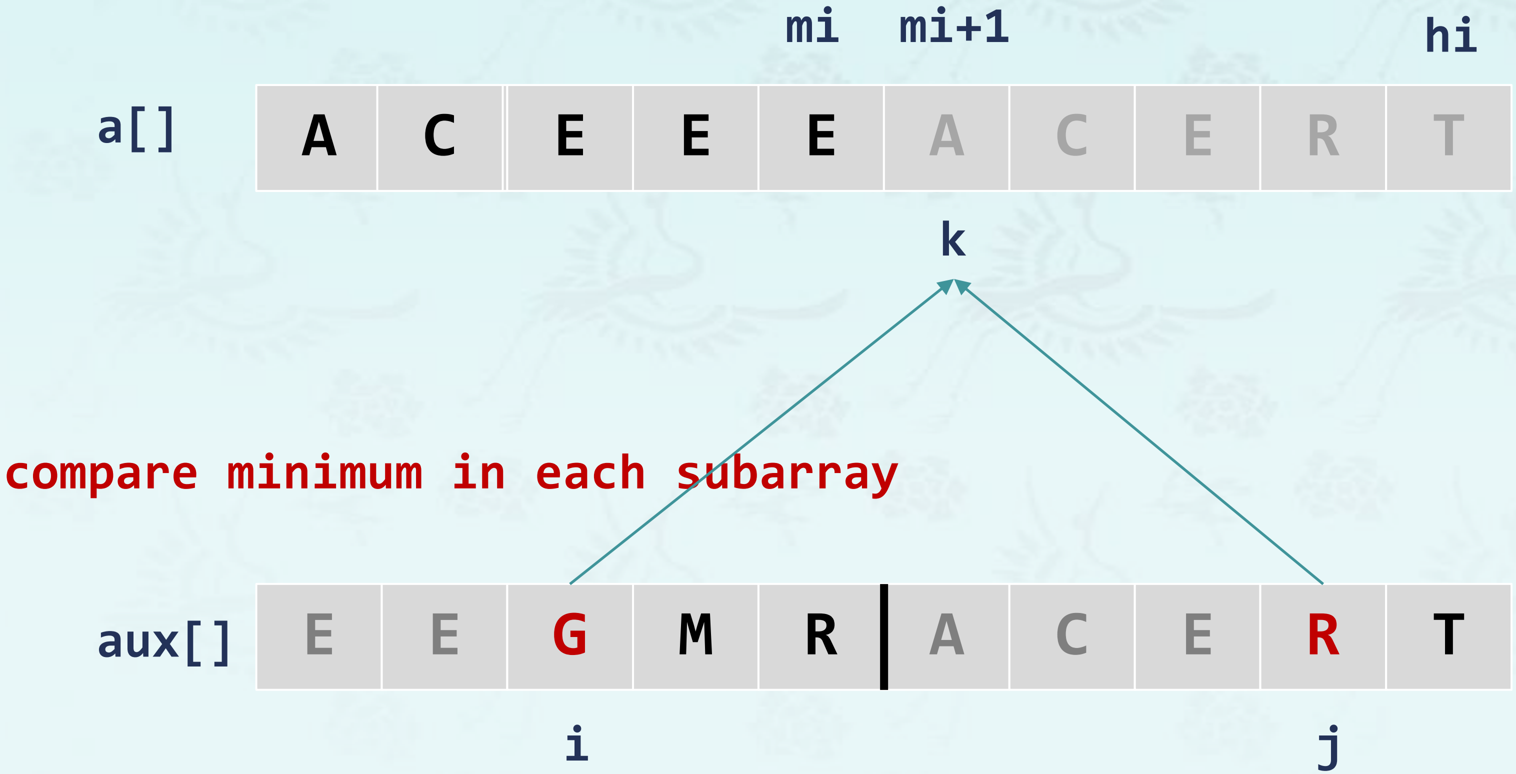
compare minimum in each subarray

aux[]									
E	E	G	M	R	A	C	E	R	T

i j

Mergesort: merge

- **Goal:** Given two sorted subarrays **a[lo]** to **a[mi]** and **a[mi+1]** to **a[hi]**, replace with sorted subarray **a[lo]** to **a[hi]**.



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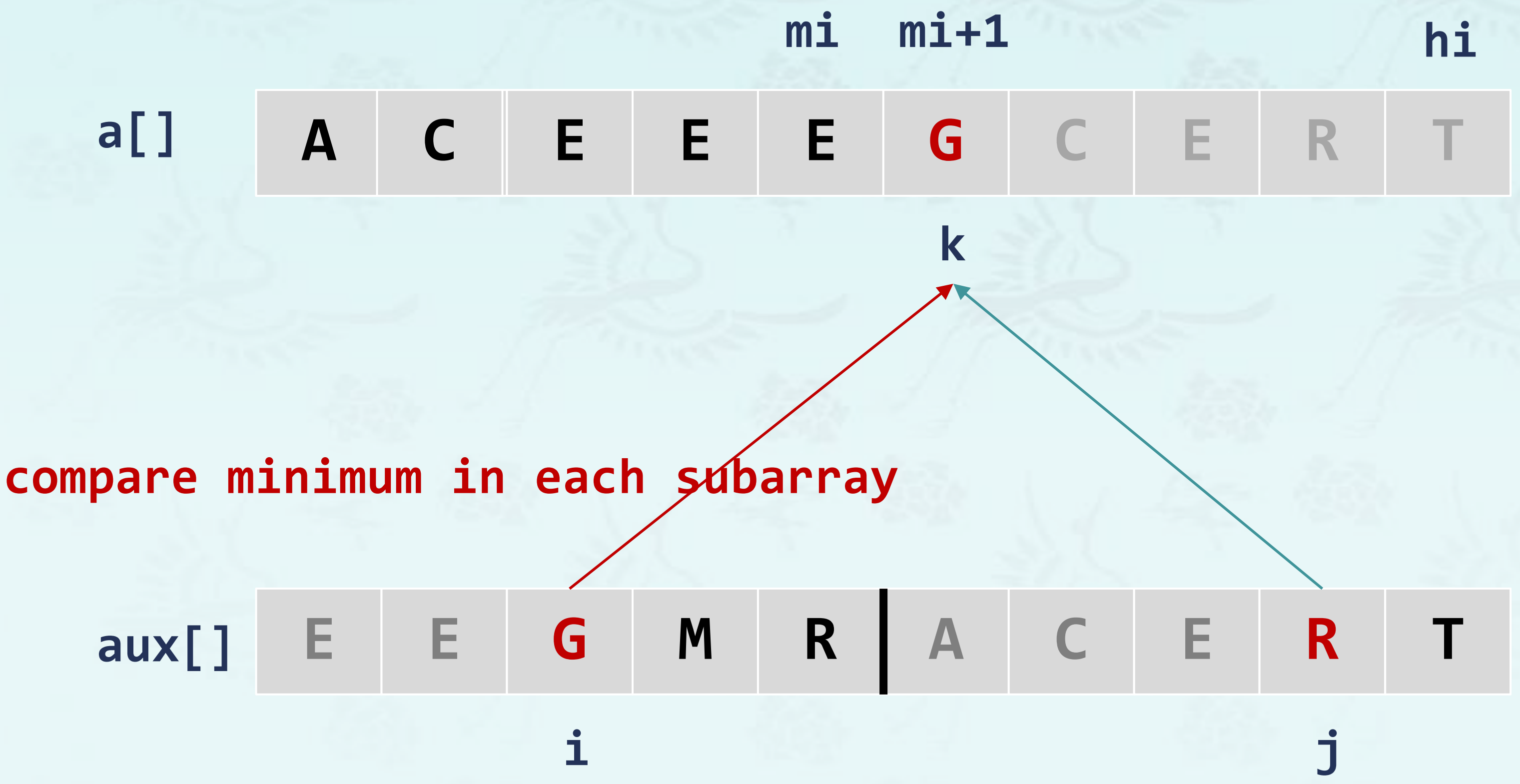

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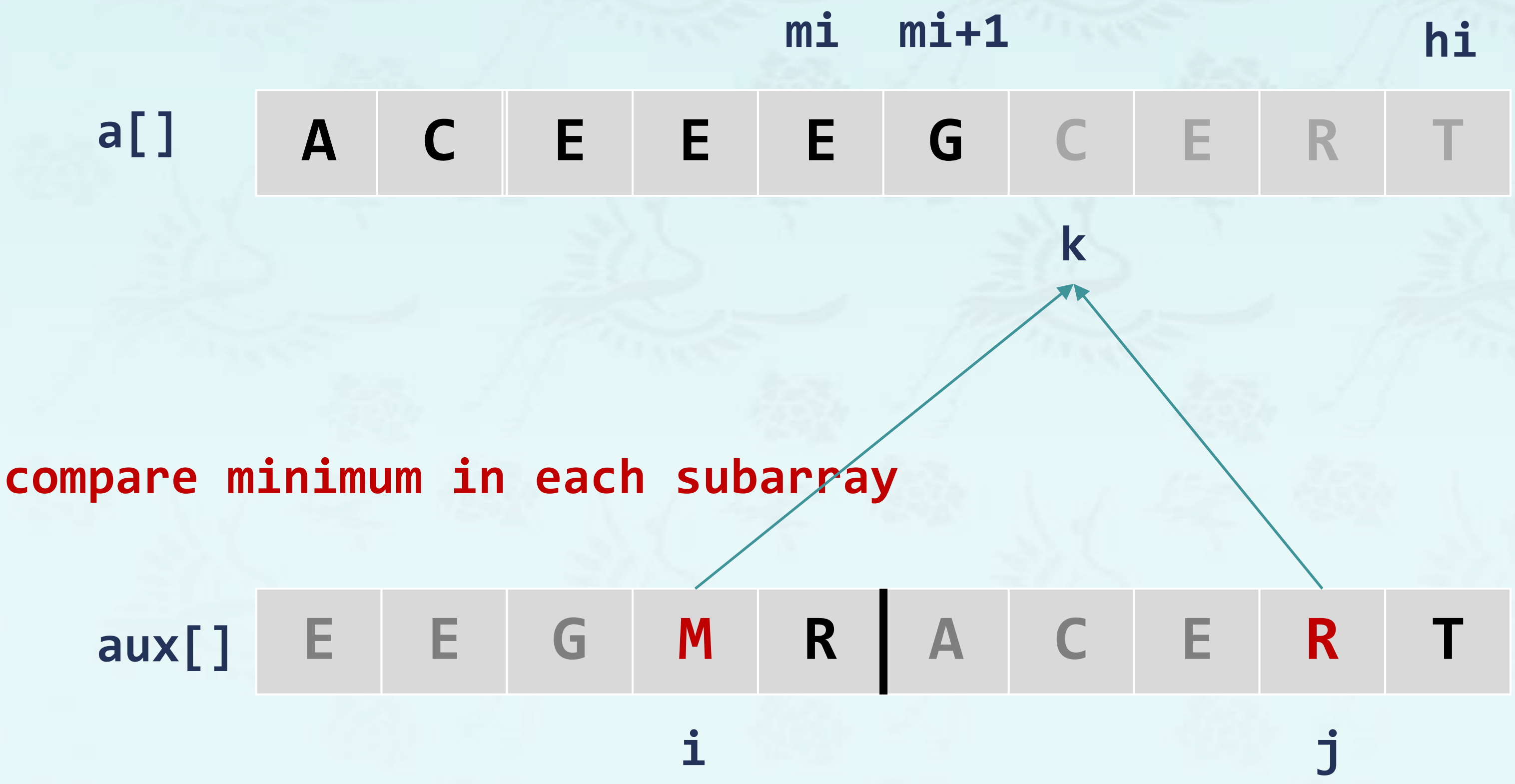
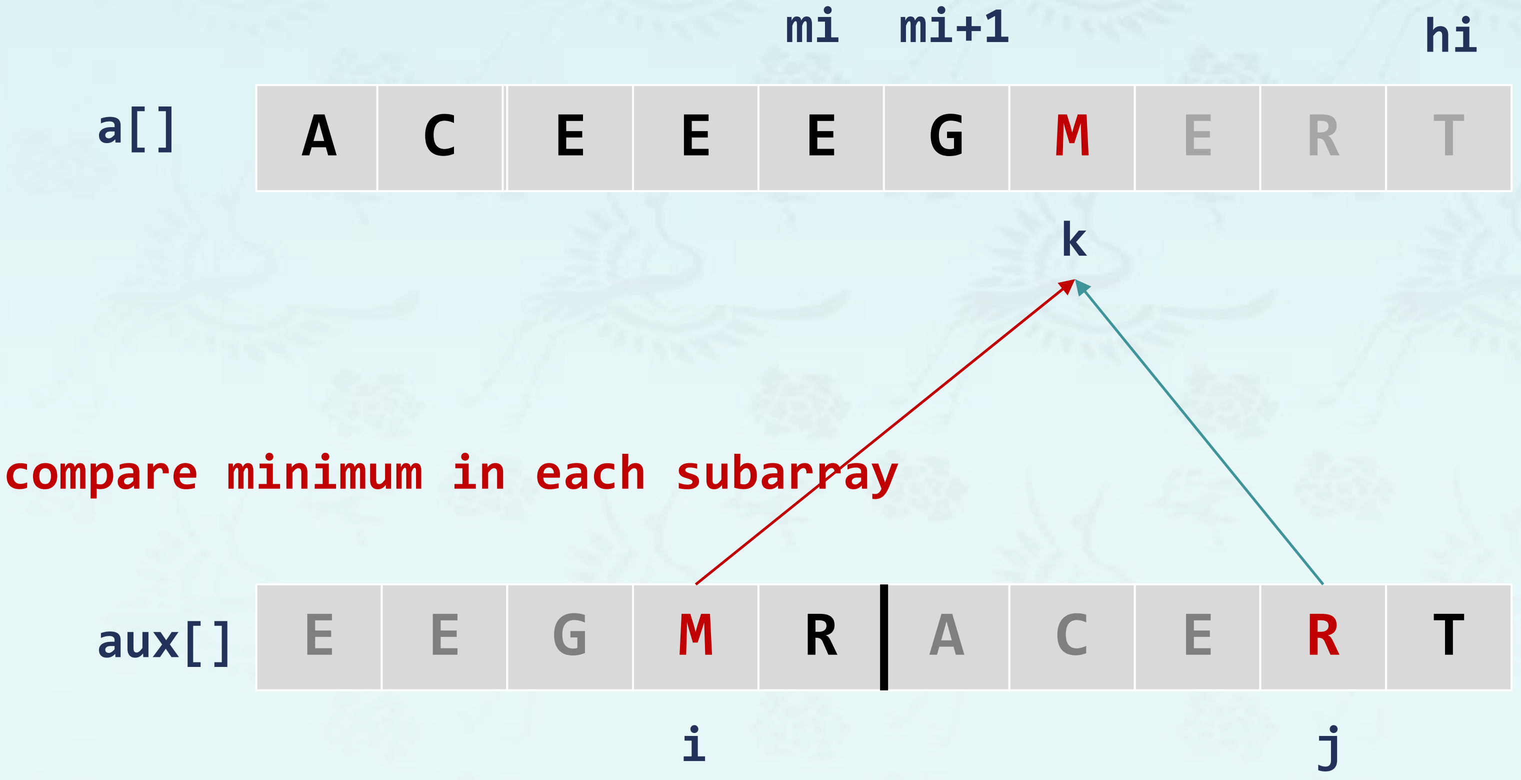


a[]	A	C	E	E	E	G	C	E	R	T
aux[]	E	E	G	M	R	A	C	E	R	T

compare minimum in each subarray

Mergesort: merge

- **Goal:** Given two sorted subarrays **`a[lo]`** to **`a[mi]`** and **`a[mi+1]`** to **`a[hi]`**, replace with sorted subarray **`a[lo]`** to **`a[hi]`**.



Mergesort: merge

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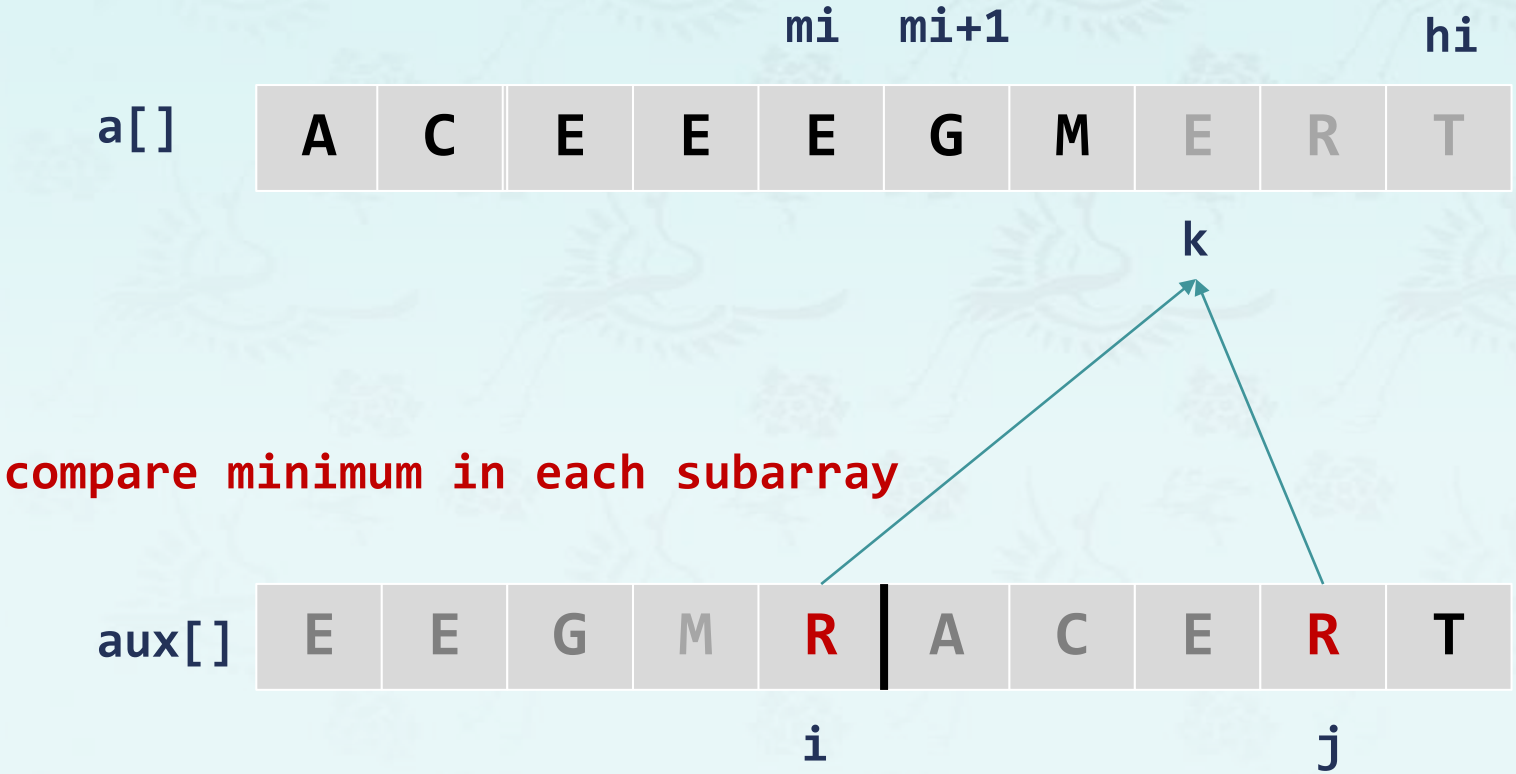

						mi	mi+1				hi
a[]	A	C	E	E	E	G	M	E	R	T	

aux[]	E	E	G	M	R	A	C	E	R	T

compare minimum in each subarray

Mergesort: merge

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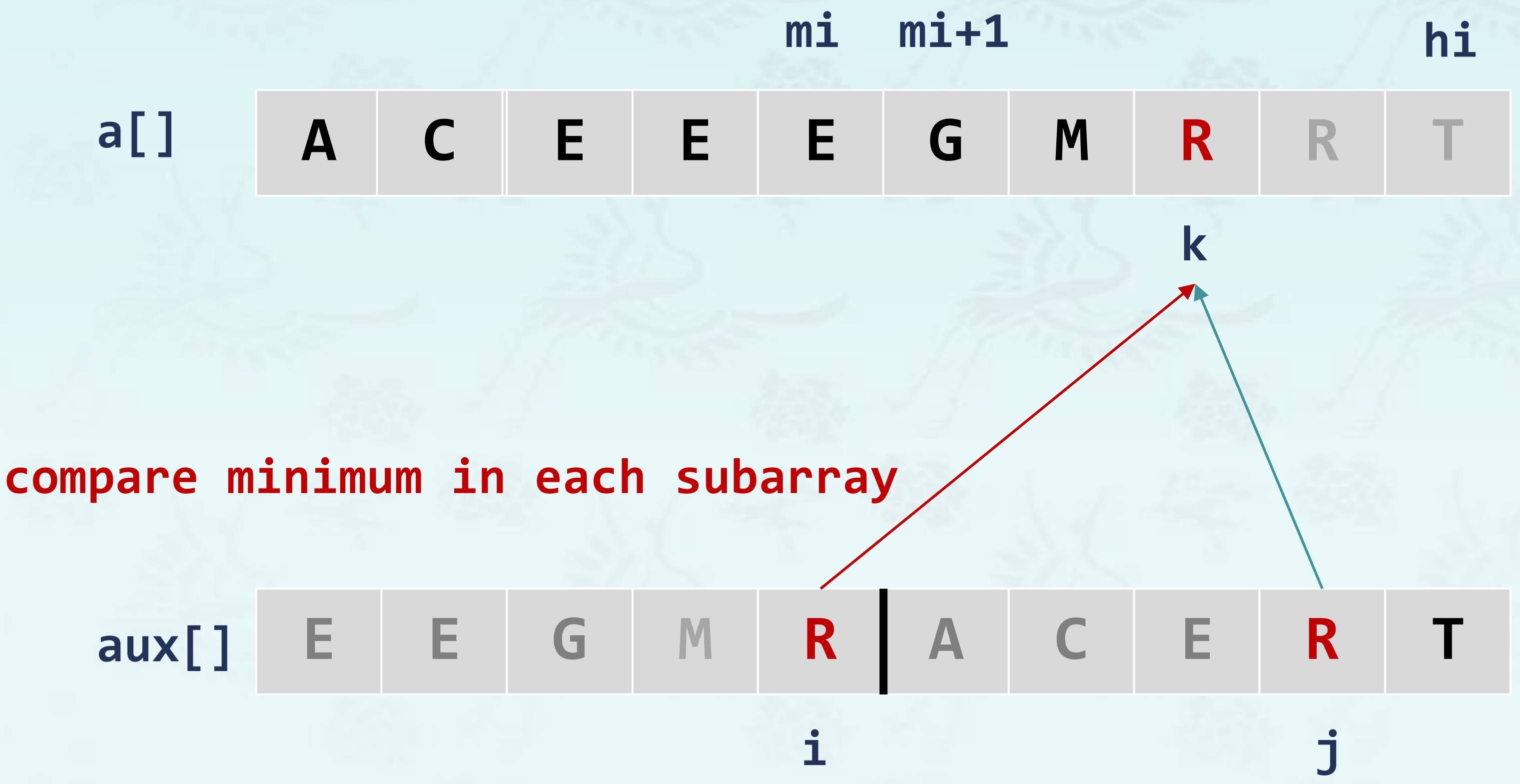


mi mi+1 hi

a[] A C E E E G M R R T

k

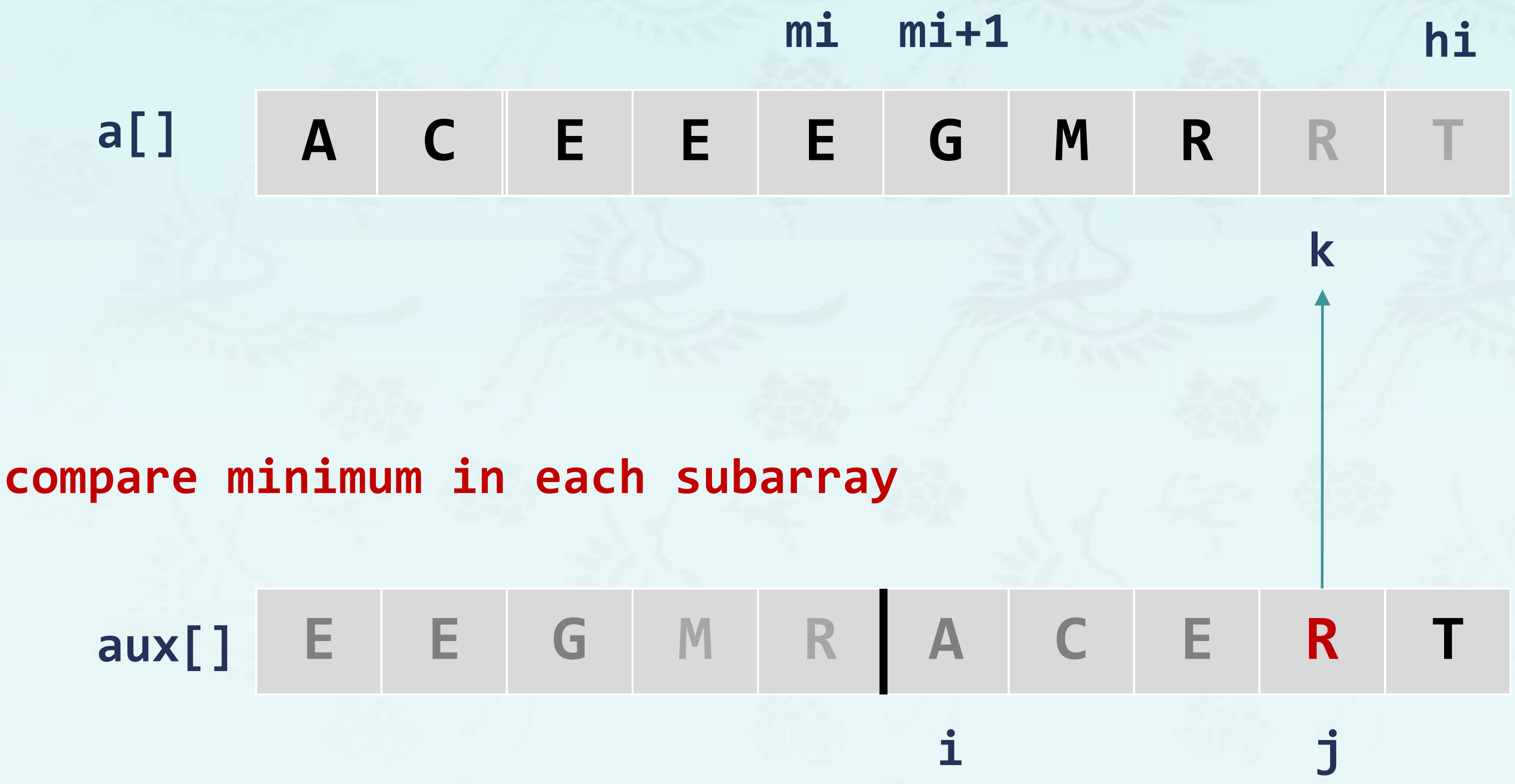
compare minimum in each subarray

aux[] E E G M R | A C E R T

i j

Mergesort: merge

- **Goal:** Given two sorted subarrays **`a[lo]`** to **`a[mi]`** and **`a[mi+1]`** to **`a[hi]`**, replace with sorted subarray **`a[lo]`** to **`a[hi]`**.



mi mi+1 hi

a[] A C E E E G M R R T

k

compare minimum in each subarray

aux[] E E G M R | A C E R T

i j

Mergesort: merge

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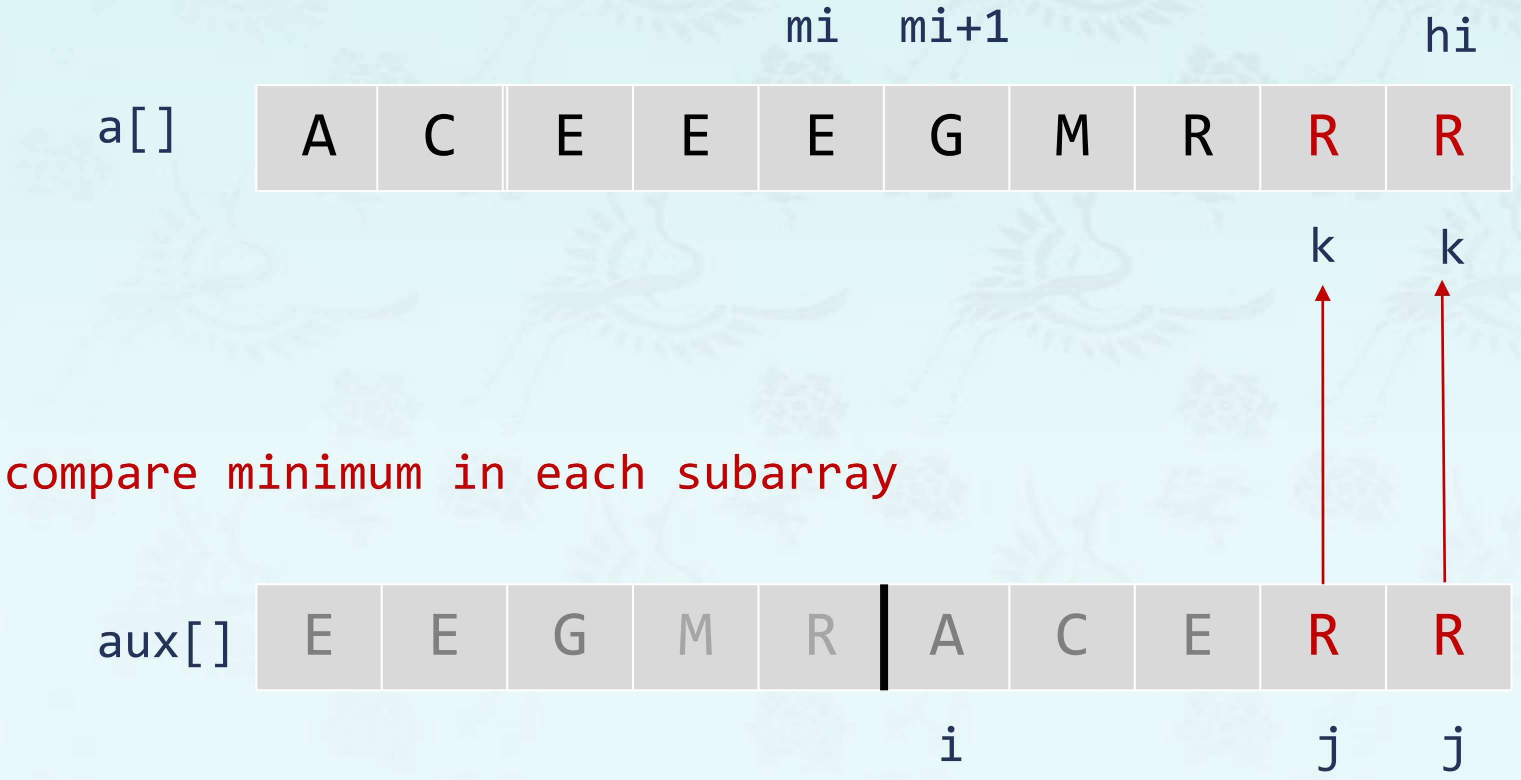

Mergesort: merge

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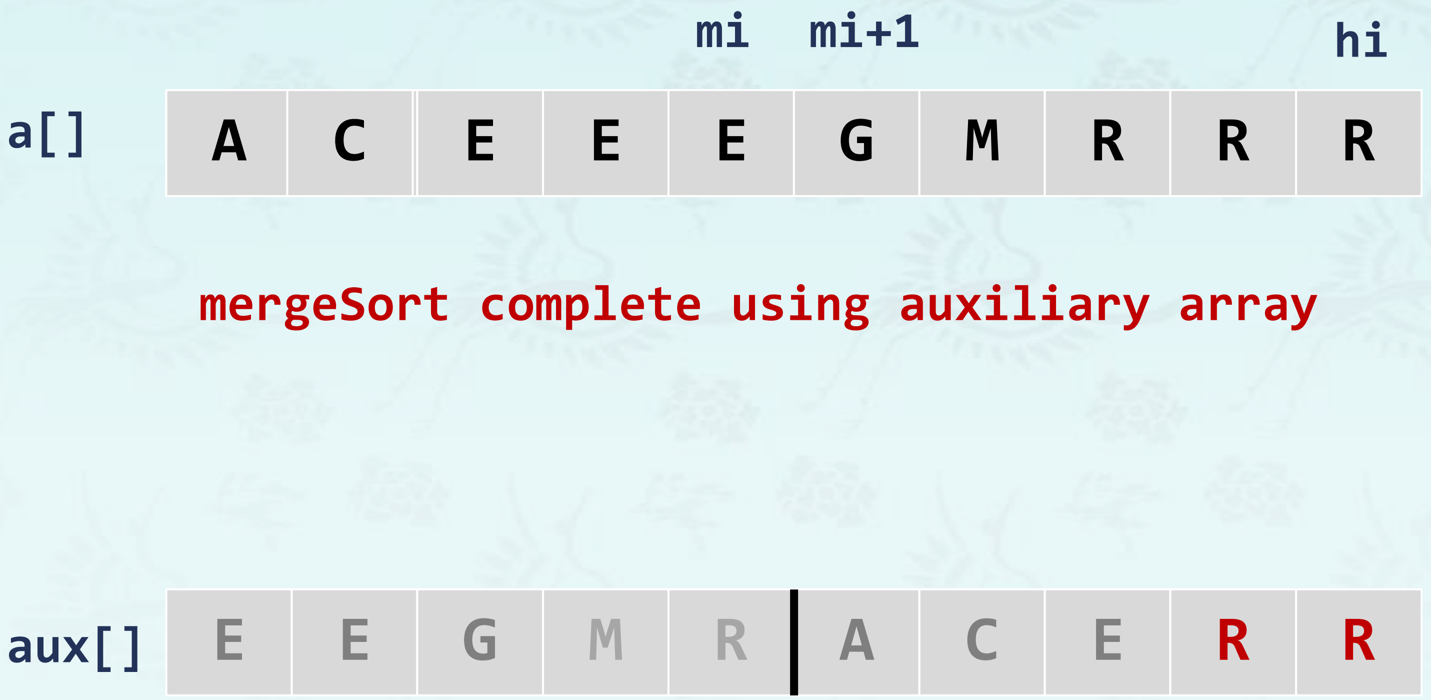


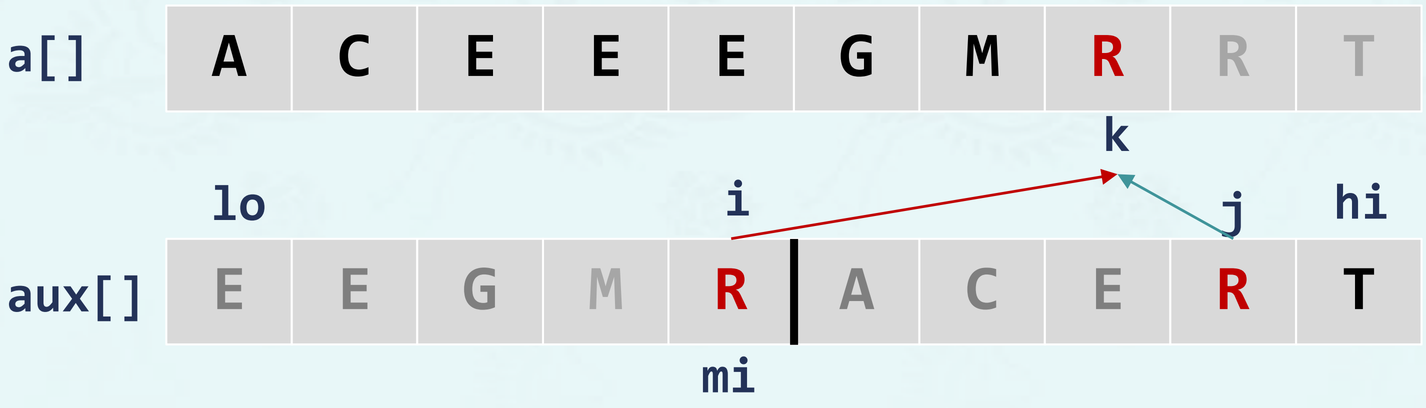
						mi	mi+1				hi
a[]		A	C	E	E	E	G	M	R	R	R

mergeSort complete using auxiliary array

aux[]	E	E	G	M	R	A	C	E	R	R

Mergesort: merge

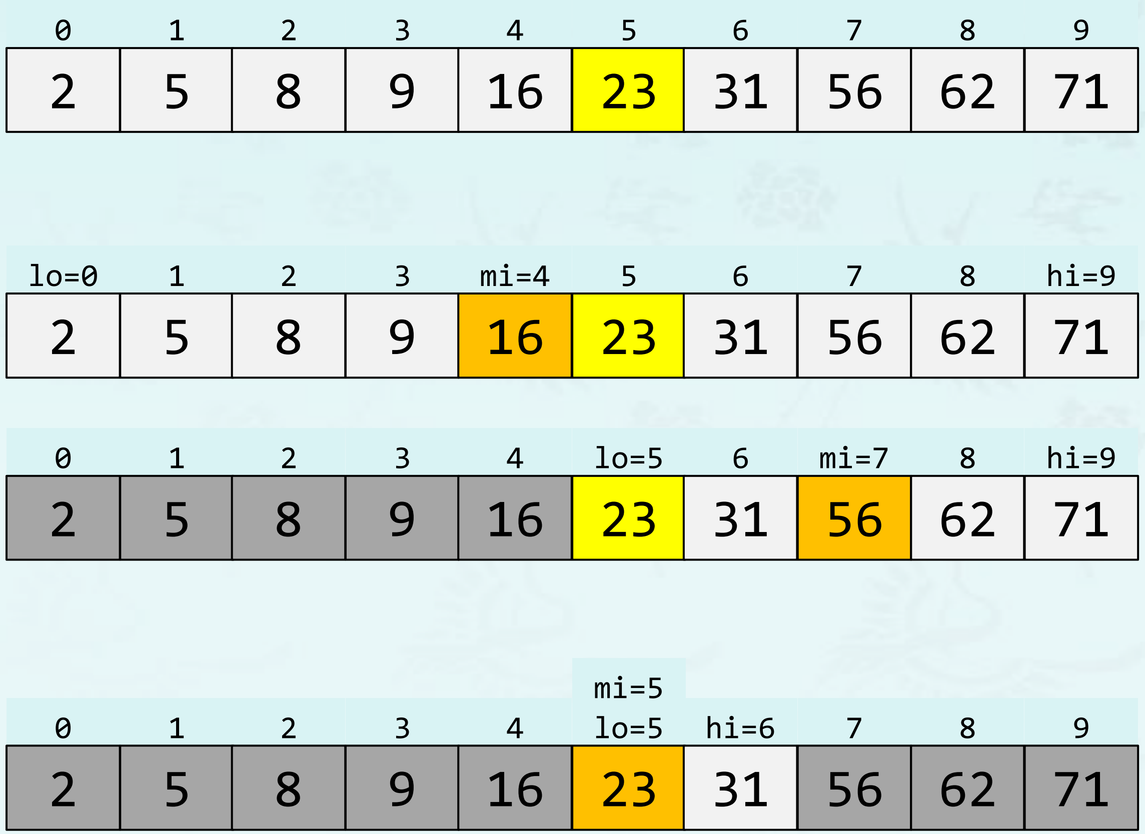
- If your array is empty or has one element, it is sorted.
- If it has two elements, sort it by swapping as appropriate.
- If it has more than two elements, do this:
    - **split** the array in half at the midpoint **mi**;
    - **merge** on the left half and **merge** on the right half;
        - **merge** the arrays by picking the smallest head element from the two sub-arrays until they are exhausted.



a[]	A	C	E	E	E	G	M	R	R	T
aux[]	E	E	G	M	R	A	C	E	R	T

Example 5: Recursive binary search

- For instance, we want to search "23" from the array. If we find it, we return its array index; otherwise, -1 or something else.



0	1	2	3	4	5	6	7	8	9
2	5	8	9	16	23	31	56	62	71

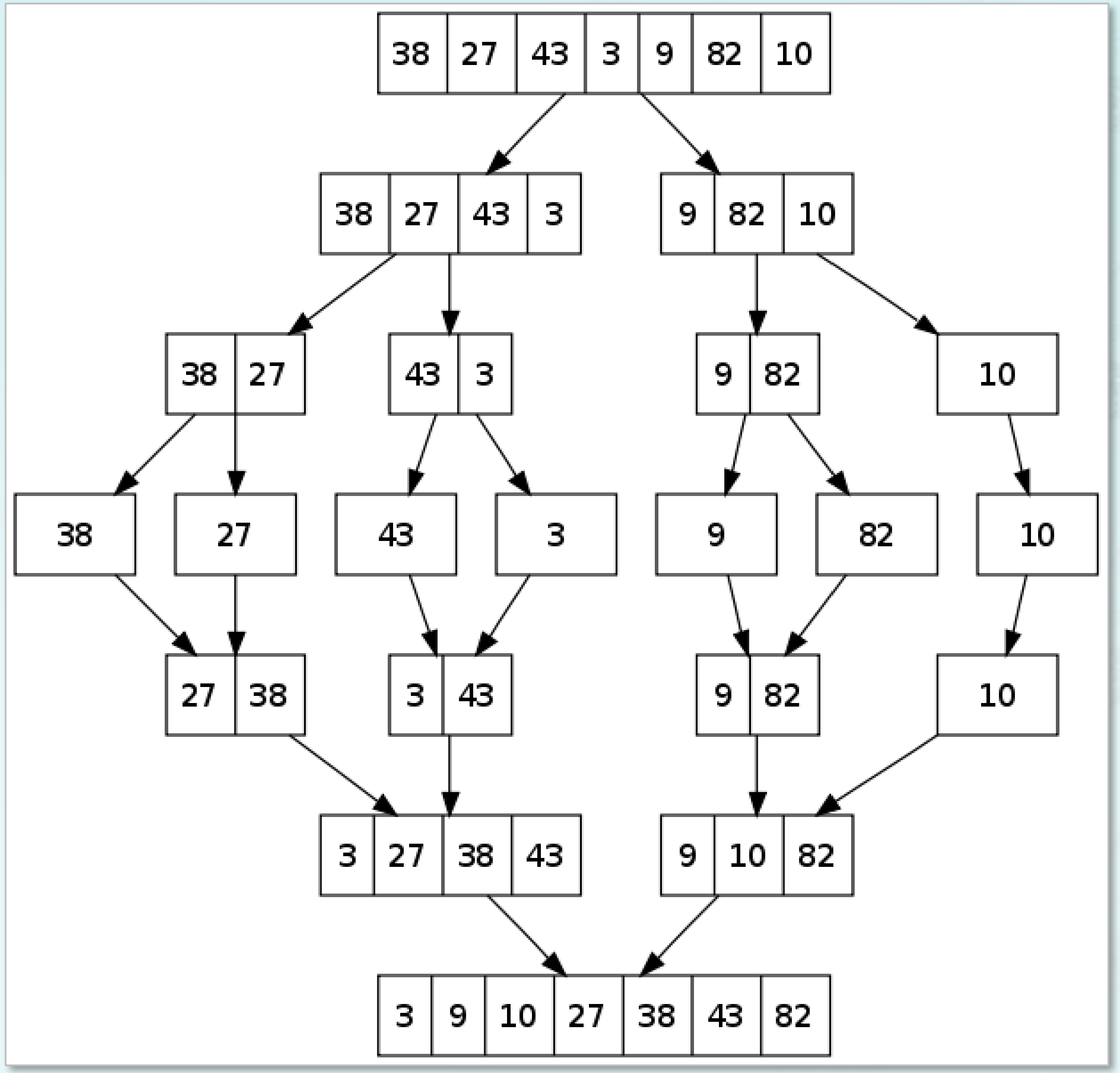
lo=0	1	2	3	mi=4	5	6	7	8	hi=9
2	5	8	9	16	23	31	56	62	71

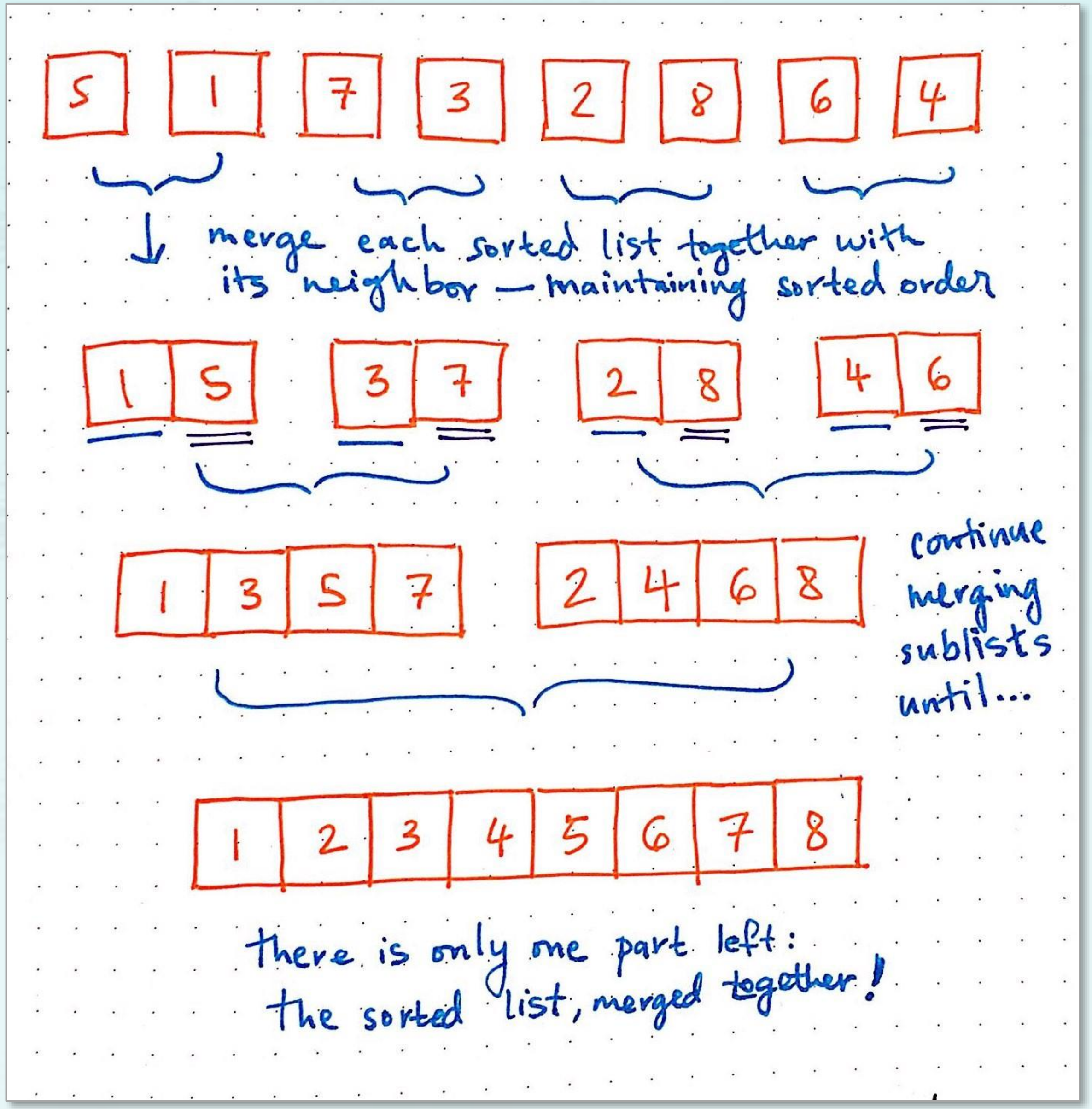
0	1	2	3	4	lo=5	6	mi=7	8	hi=9
2	5	8	9	16	23	31	56	62	71

0	1	2	3	4	mi=5 lo=5	hi=6	7	8	9
2	5	8	9	16	23	31	56	62	71

```
int binarySearch(int list[], int key,
                             int lo, int hi) {
  if (lo > hi) return -1;

  mi = (lo + hi)/2;
  if (key == list[mi]) return mi;
  if (key <  list[mi])
    return binarySearch(list, key, lo, mi - 1);
  else
    return binarySearch(list, key, mi + 1, hi);
}
```

Mergesort: Coding



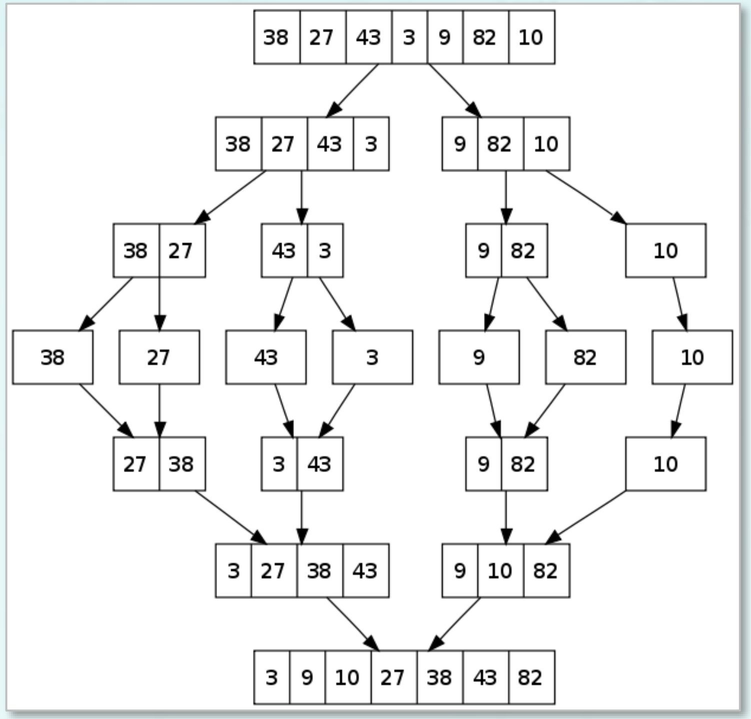


5 1 7 3 2 8 6 4
merge each sorted list together with its neighbor — maintaining sorted order
1 5 3 7 2 8 4 6
continue merging sublists until...
1 3 5 7 2 4 6 8
1 2 3 4 5 6 7 8
there is only one part left: the sorted list, merged together!

https://en.wikipedia.org/wiki/File:Merge_sort_algorithm_diagram.svg

Mergesort: Coding

```
mergeSort(a[], aux[], N, lo,  hi)
If hi > lo
  Find the middle to divide the array into two: mi = (lo+hi)/2
  Split 1st half: mergeSort(a, aux, N, lo,   mi)
  Split 2nd half: mergeSort(a, aux, N, mi+1, hi)
  Merge the two halves sorted: merge(a, aux, lo, mi, hi)
```



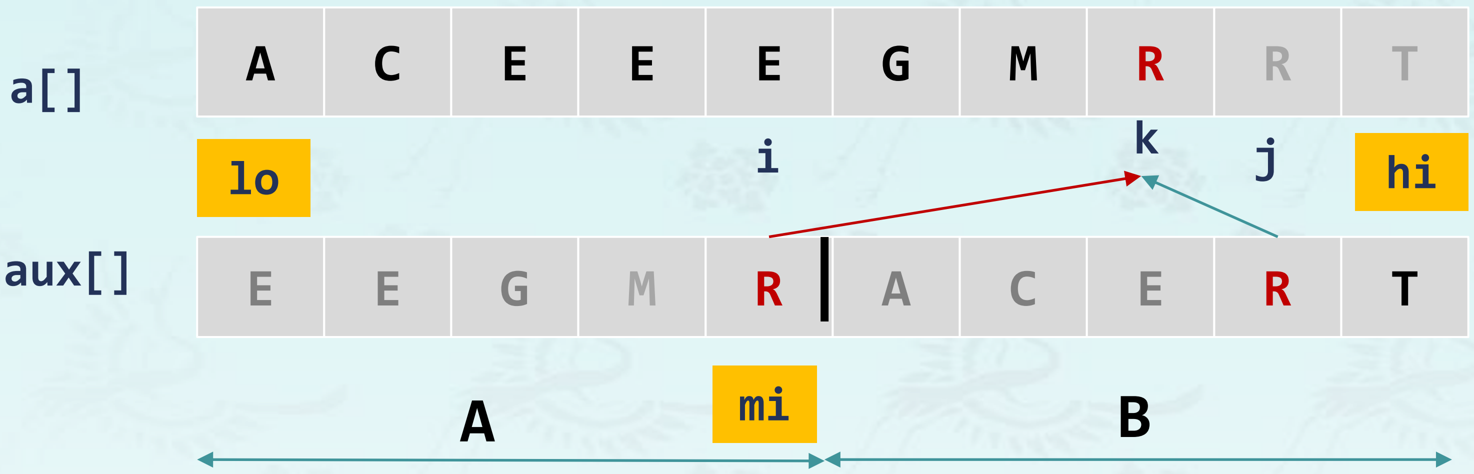
Mergesort: Coding

```
void mergeSort(char *a, char *aux, int N, int lo, int hi) {
  if (hi <= lo) return;
  int mi = lo + (hi - lo) / 2;                   // mi=(lo+hi)/2
  mergeSort (a, aux, N, lo,     mi);
  mergeSort (a, aux, N, mi + 1, hi);
  merge(a, aux, lo, mi, hi);
}

int main() {
  char a[]={'M','E','R','G','E','S','O','R','T','E','X','A','M','P','L','E'};
  cout << "UNSORTED: "; for (auto x: a) cout << x; cout << endl;
  int N = sizeof(a) / sizeof(a[0]);
  char *aux = new char[N];
  mergeSort(a, aux, N, 0, N - 1);
  cout << "  SORTED: "; for (auto x: a) cout << x; cout << endl;
}
```

Mergesort: Coding

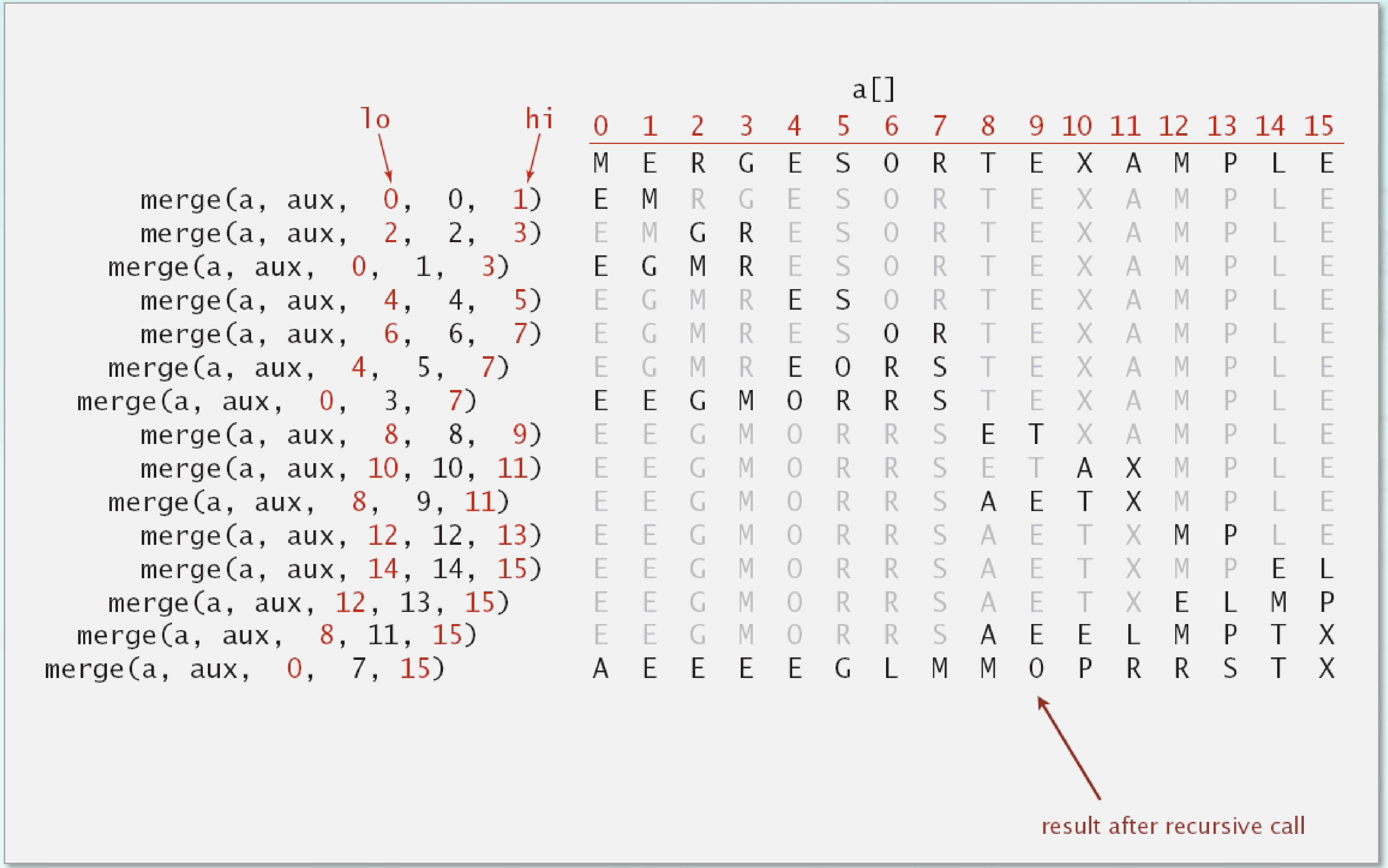
```
int isSorted(char *a, int i, int j){return a[i] <= a[j];}

void merge(char *a, char *aux, int lo, int mi, int hi) {
    assert(isSorted(a, lo,   mi));     // precondition: a[lo..mi]   sorted
    assert(isSorted(a, mi+1, hi));     // precondition: a[mi+1..hi] sorted
    for (int k = lo; k <= hi; k++) aux[k] = a[k];

    int i = lo;
    int j = mi + 1;
    for (int k = lo; k <= hi; k++) {
        if      (i > mi)          a[k] = aux[j++];     // A is exhausted, take B[j]
        else if (j > hi)          a[k] = aux[i++];     // B is exhausted, take A[i]
        else if (aux[j] < aux[i]) a[k] = aux[j++];     // B[j] <  A[i], take B[j]
        else                      a[k] = aux[i++];     // A[i] <= B[j], take A[i]
    }
    assert(isSorted(a, lo, hi));       // postcondition: a[lo..hi] sorted
}
```

Mergesort: Coding



a[]	A	C	E	E	E	G	M	R	R	T
aux[]	E	E	G	M	R	A	C	E	R	T

Mergesort: Coding

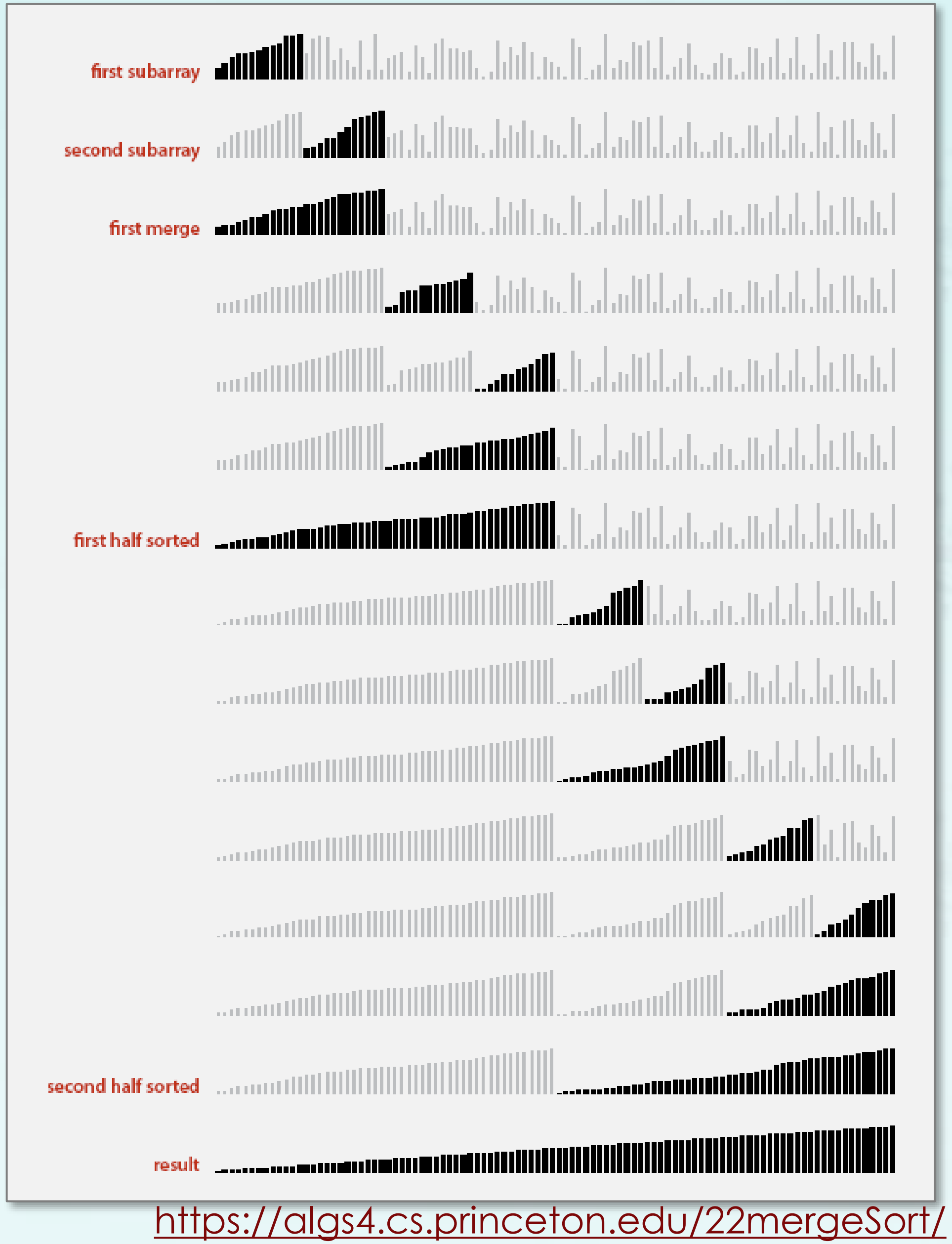


```
                                                         a[]
                      lo         hi       0  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15
                                          M  E  R  G  E  S  O  R  T  E  X  A  M  P  L  E
      merge(a, aux,  0,  0,  1)           E  M  R  G  E  S  O  R  T  E  X  A  M  P  L  E
      merge(a, aux,  2,  2,  3)           E  M  G  R  E  S  O  R  T  E  X  A  M  P  L  E
    merge(a, aux,  0,  1,  3)             E  G  M  R  E  S  O  R  T  E  X  A  M  P  L  E
      merge(a, aux,  4,  4,  5)           E  G  M  R  E  S  O  R  T  E  X  A  M  P  L  E
      merge(a, aux,  6,  6,  7)           E  G  M  R  E  S  O  R  T  E  X  A  M  P  L  E
    merge(a, aux,  4,  5,  7)             E  G  M  R  E  O  R  S  T  E  X  A  M  P  L  E
  merge(a, aux,  0,  3,  7)               E  E  G  M  O  R  R  S  T  E  X  A  M  P  L  E
      merge(a, aux,  8,  8,  9)           E  E  G  M  O  R  R  S  E  T  X  A  M  P  L  E
      merge(a, aux, 10, 10, 11)           E  E  G  M  O  R  R  S  E  T  A  X  M  P  L  E
    merge(a, aux,  8,  9, 11)             E  E  G  M  O  R  R  S  A  E  T  X  M  P  L  E
      merge(a, aux, 12, 12, 13)           E  E  G  M  O  R  R  S  A  E  T  X  M  P  L  E
      merge(a, aux, 14, 14, 15)           E  E  G  M  O  R  R  S  A  E  T  X  M  P  E  L
    merge(a, aux, 12, 13, 15)             E  E  G  M  O  R  R  S  A  E  T  X  E  L  M  P
  merge(a, aux,  8, 11, 15)               E  E  G  M  O  R  R  S  A  E  E  L  M  P  T  X
merge(a, aux,  0,  7, 15)                 A  E  E  E  E  G  L  M  M  O  P  R  R  S  T  X
```

result after recursive call

https://algs4.cs.princeton.edu/22mergeSort/

Mergesort: Coding



https://algs4.cs.princeton.edu/22mergeSort/

Assertion in C/C++

- **Assertion:** Statement to test assumptions about your program in Java.
  - Helps detect logic bugs.
  - Documents code.
- **Assert statement:** abort the program and print an error message (the function name and a line number) unless Boolean condition is true.

```
#include <cassert>
assert( isSorted(a, lo, hi) );
```

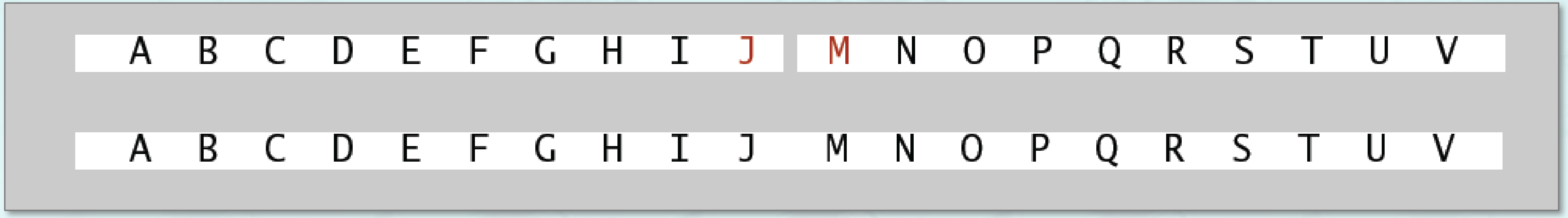
- **Best practices:** Use assertions to check internal invariants;
  - Assume assertions will be disabled in production code.
  - Do not use for external argument checking.

Mergesort: Quiz 1

1. Improvement by reducing the number of `merge()` function call.
   Some hints for this problem are provided in the following pages.
2. How many times did you spare `merge()` calls for "`MERGESORTEXAMPLE`" case?
   - Total number of **merge()** calls without your improvement: _______________
   - The number of **merge()** calls spared with your improvement: _________________
3. Identify those sets of char array groups that `merge()` call was unnecessary.

Mergesort: Quiz 1

- **Hint:** Do not invoke "merge()" function **if two halves are already sorted..**
  - Is the biggest item in first half ≤ the smallest item in second half?
  - For example, the following case should not call merge() since `J <= M`.

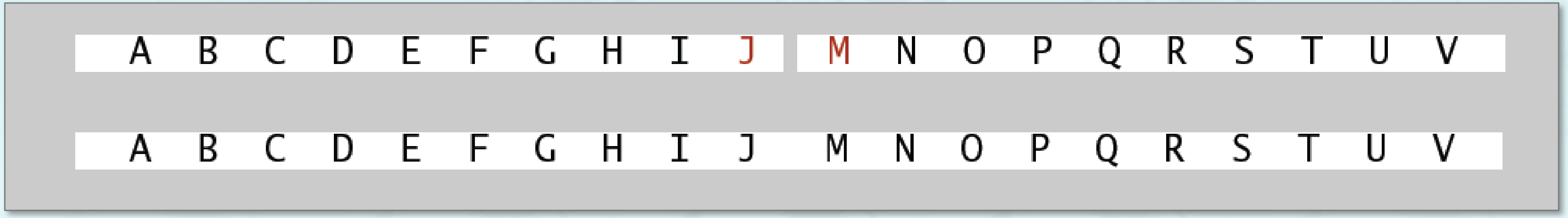


```
A B C D E F G H I J   M N O P Q R S T U V
A B C D E F G H I J M N O P Q R S T U V
```

정렬들에 관한 좋은 자료를 읽어 보길 적극 추천합니다.
영어: https://medium.com/basecs/making-sense-of-merge-sort-part-1-49649a143478
한글: https://gmlwjd9405.github.io/2018/05/08/algorithm-merge-sort.html

Mergesort: Quiz 1

- **Hint:** Do not invoke "merge()" function **if two halves are already sorted..**
  - Is the biggest item in first half ≤ the smallest item in second half?
  - For example, the following case should not call merge() since `J <= M`.



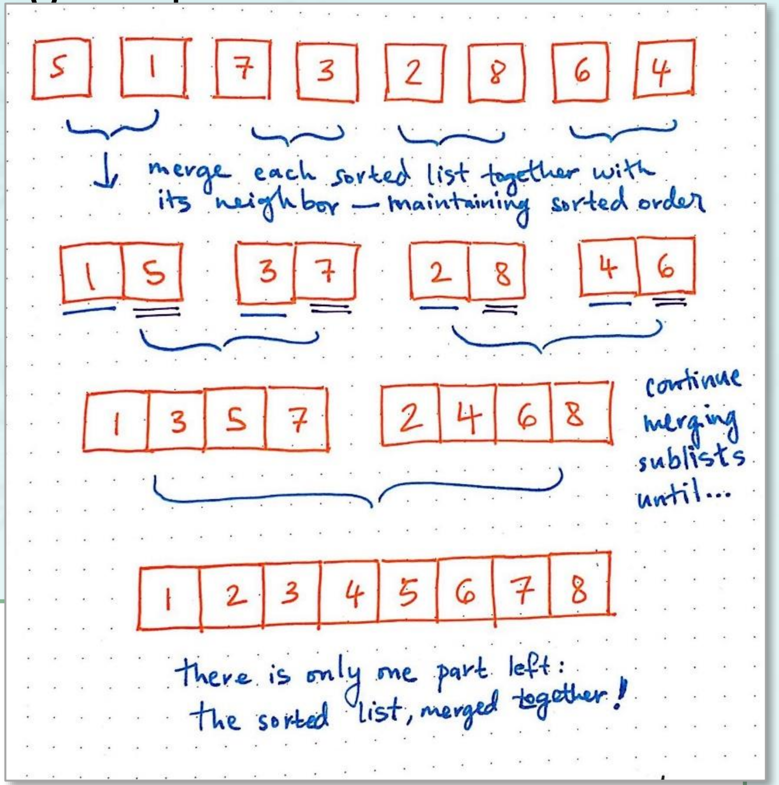
A B C D E F G H I J | M N O P Q R S T U V

A B C D E F G H I J M N O P Q R S T U V

```
void mergeSort(char *a, char *aux, int N, int lo, int hi) {
    if (hi <= lo)  return;
    int mi = lo + (hi - lo) / 2;
    mergeSort(a, aux, N, lo,     mi);
    mergeSort(a, aux, N, mi + 1, hi);
    // your code here                       // already sorted
    merge(a, aux, lo, mi, hi);
}
```

Mergesort: Quiz 2

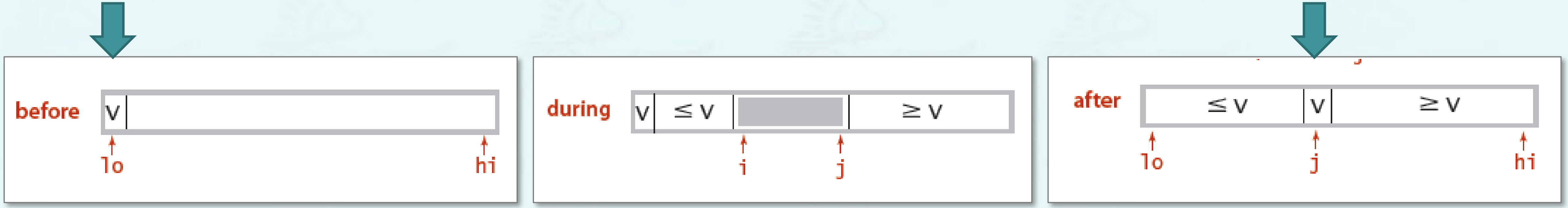
- In the figure, which elements are compared in isSorted() at postcondition?
- Why isSorted() checks only two elements?
  Is this enough?


5 1 7 3 2 8 6 4
merge each sorted list together with its neighbor — maintaining sorted order
1 5 3 7 2 8 4 6
1 3 5 7 2 4 6 8
continue merging sublists until...
1 2 3 4 5 6 7 8
there is only one part left: the sorted list, merged together!

```
int isSorted(int *a, int i, int j){return a[i] <= a[j];}

void merge(int *a, char *aux, int lo, int mi, int hi) {
    assert(isSorted(a, lo,   mi));    // precondition: a[lo..mi]   sorted
    assert(isSorted(a, mi+1, hi));    // precondition: a[mi+1..hi] sorted
    for (int k = lo; k <= hi; k++) aux[k] = a[k];
     ……
    assert(isSorted(a, lo, hi));      // postcondition: a[lo..hi] sorted
}
```

Quick Sort(퀵 정렬)

- Quick Sort is a divide-and-conquer algorithm.
- It works by selecting a '**pivot**' element from the array and partitioning the other elements into two sub-arrays, according to whether they are less than or greater than the pivot.
- The sub-arrays are then sorted recursively.



https://en.wikipedia.org/wiki/quicksort

Quick Sort(퀵 정렬)

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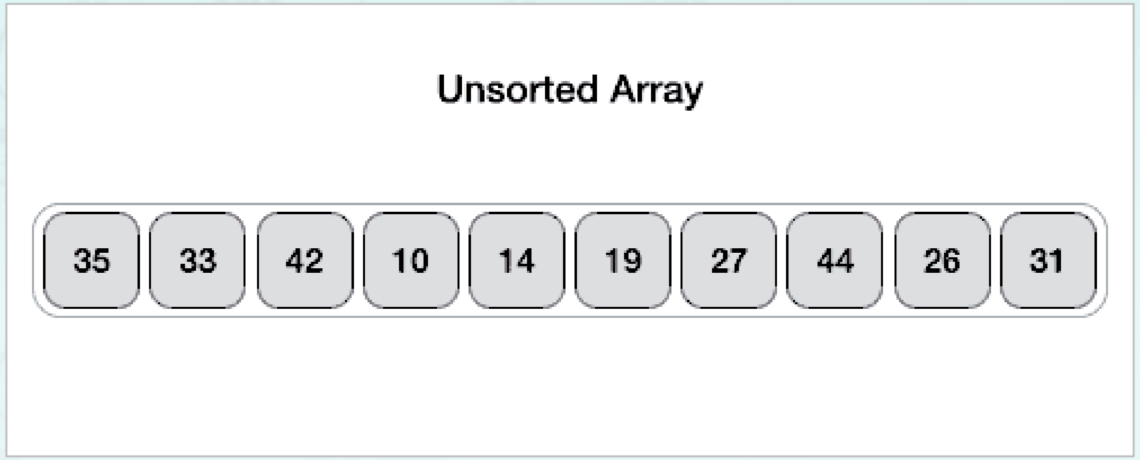
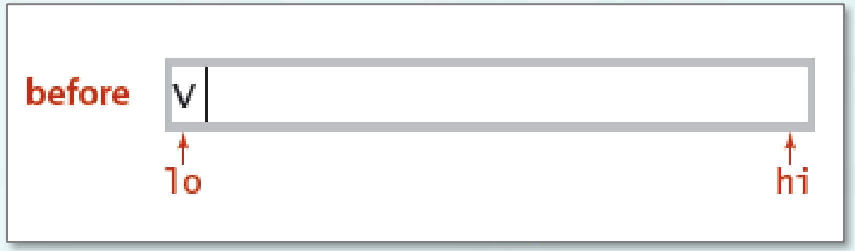
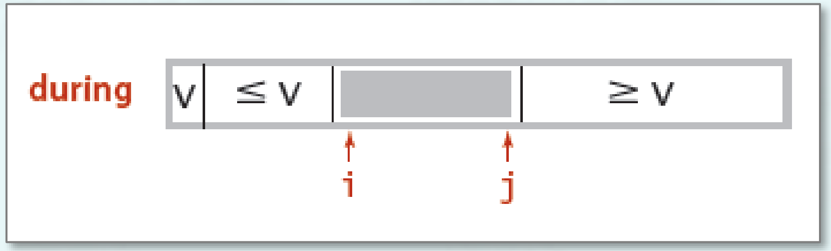
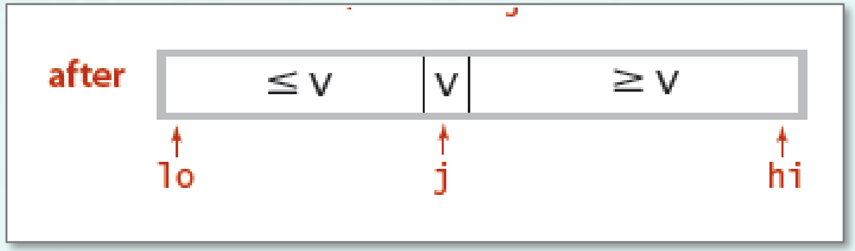




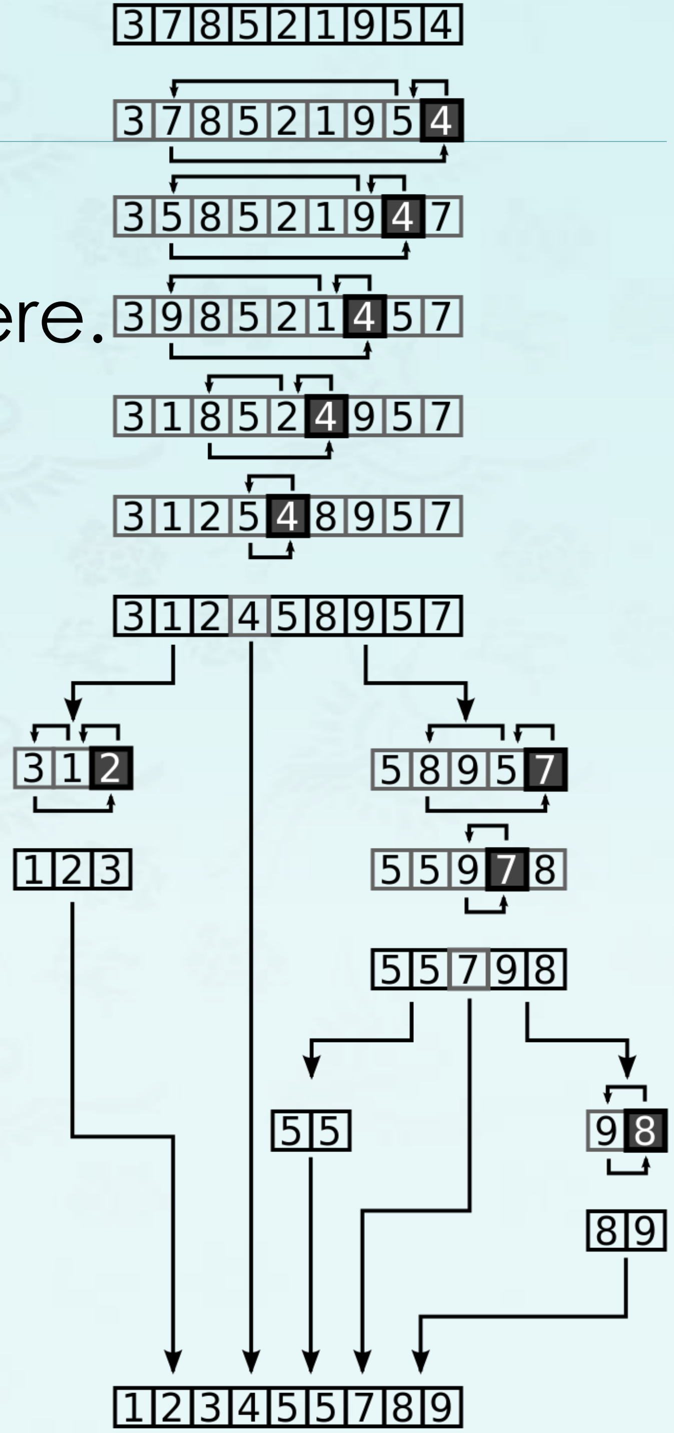




https://en.wikipedia.org/wiki/quicksort

Quick Sort(퀵 정렬)

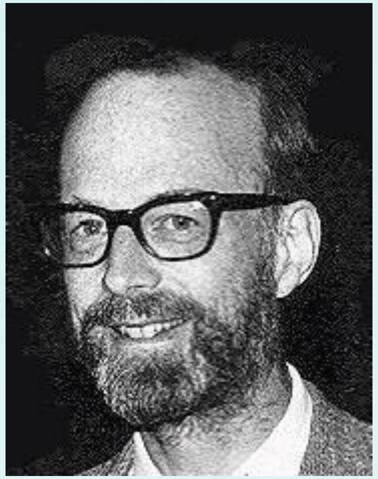
- The shaded element is the pivot.
- It is chosen as the last element of the partition here.



https://en.wikipedia.org/wiki/quicksort

4. Quick Sort - by Hoare in 1961

- Algorithm:
  - Shuffle the array.
  - Partition so that, for some j
    - entry a[j] is in place
    - no larger entry to the left of j
    - no smaller entry to the right of j
  - Sort each piece recursively.



Sir Charles Antony Richard Hoare
1980 Turing award

4. Quick Sort - by Hoare in 1961

- Algorithm:
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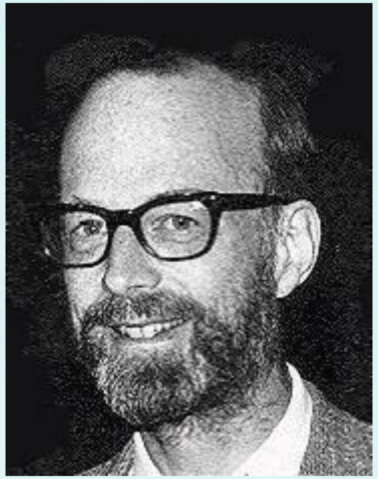

Sir Charles Antony Richard Hoare
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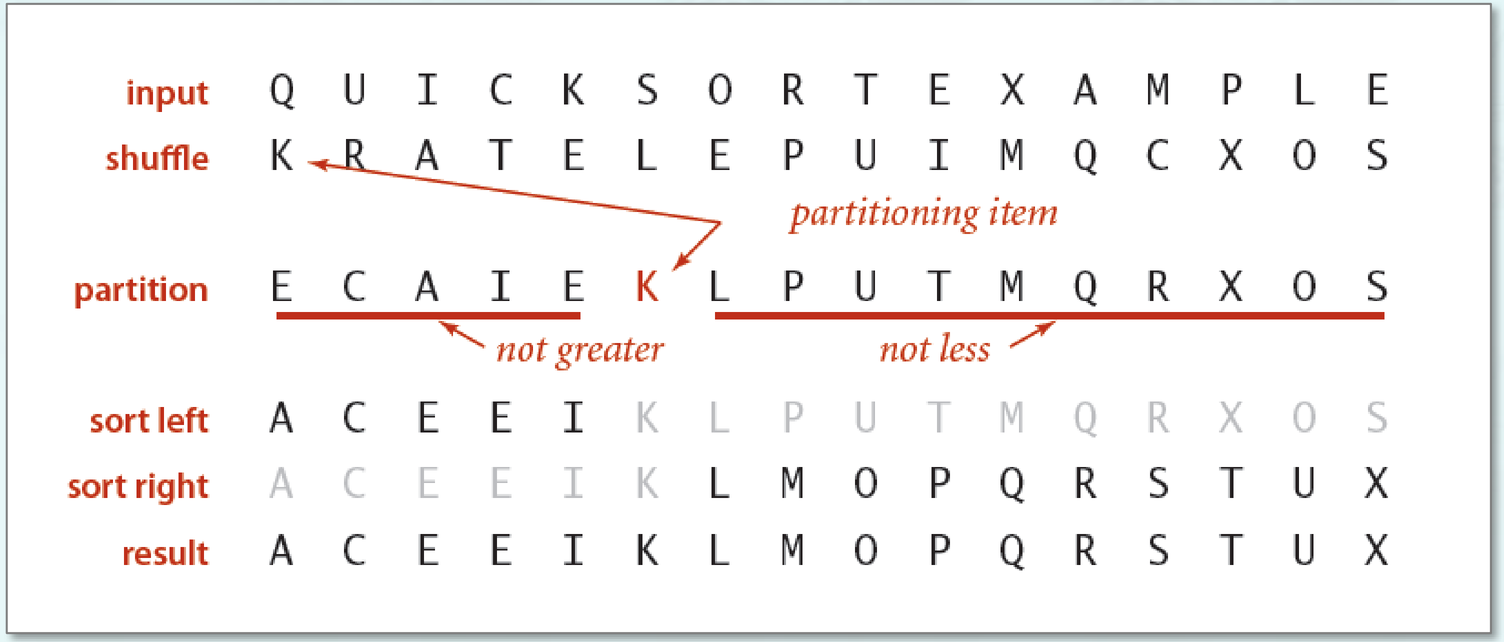


input	Q	U	I	C	K	S	O	R	T	E	X	A	M	P	L	E
shuffle	K	R	A	T	E	L	E	P	U	I	M	Q	C	X	O	S
partition	E	C	A	I	E	K	L	P	U	T	M	Q	R	X	O	S
sort left	A	C	E	E	I	K	L	P	U	T	M	Q	R	X	O	S
sort right	A	C	E	E	I	K	L	M	O	P	Q	R	S	T	U	X
result	A	C	E	E	I	K	L	M	O	P	Q	R	S	T	U	X

partitioning item

not greater

not less

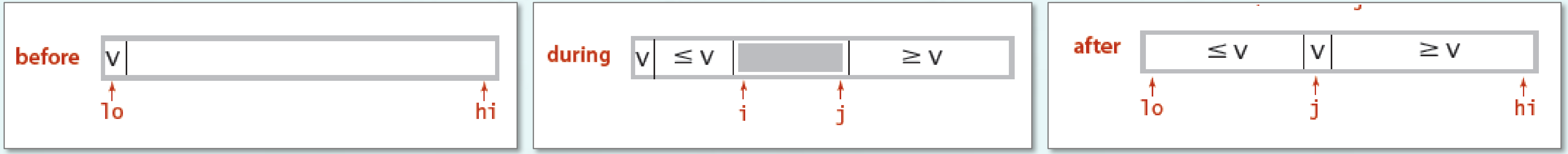
https://algs4.cs.princeton.edu/23quicksort/

Quick Sort partitioning demo

- Phase I. Repeat until ***i*** and ***j*** pointers cross:
  - Scan i from left to right so long as $(a[i] < a[lo])$
  - Scan j from right to left so long as $(a[j] > a[lo])$.
  - Exchange $a[i]$ with $a[j]$.

pivot partitioning element

K	R	A	T	E	L	E	P	U	I	M	Q	C	X	O	S
lo	i														j



Quick Sort partitioning demo

- Phase I. Repeat until $\boldsymbol{i}$ and $\boldsymbol{j}$ pointers cross:
  - Scan i from left to right so long as $(a[i] < a[lo])$
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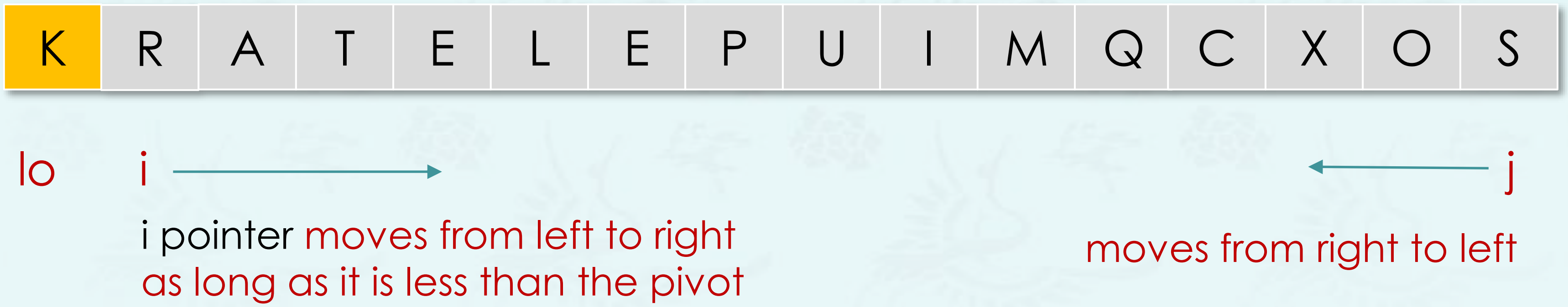
  - Exchange $a[i]$ with $a[j]$.



K	R	A	T	E	L	E	P	U	I	M	Q	C	X	O	S

i pointer moves from left to right as long as it is less than the pivot

moves from right to left

Quick Sort partitioning demo

- Phase I. Repeat until $\boldsymbol{i}$ and $\boldsymbol{j}$ pointers cross:
  - Scan i from left to right so long as $(a[i] < a[lo])$
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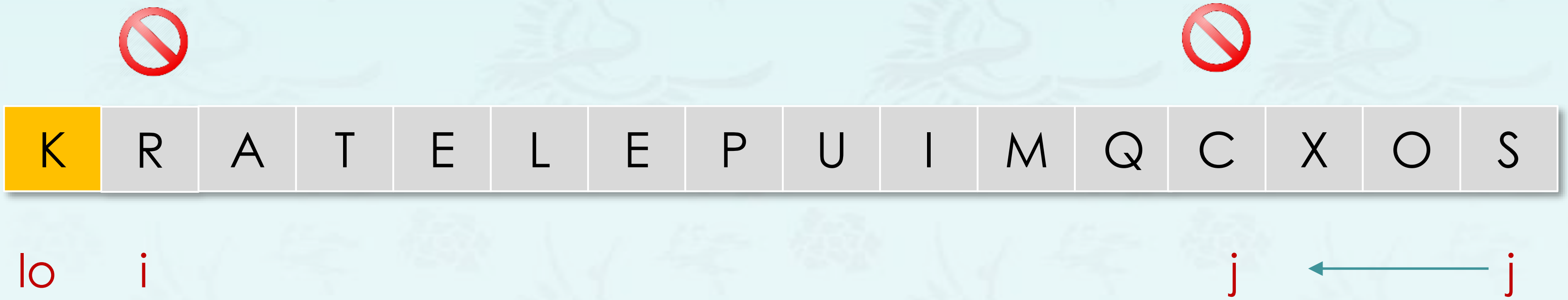

K	R	A	T	E	L	E	P	U	I	M	Q	C	X	O	S

i pointer moves from left to right
as long as it is less than the pivot

i pointer stops immediately since ….

moves from right to left

now decrement j until ….

j pointer moves and stops at "C"

Quick Sort partitioning demo

- Phase I. Repeat until $\boldsymbol{i}$ and $\boldsymbol{j}$ pointers cross:
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  - Scan j from right to left so long as $(a[j] > a[lo])$.
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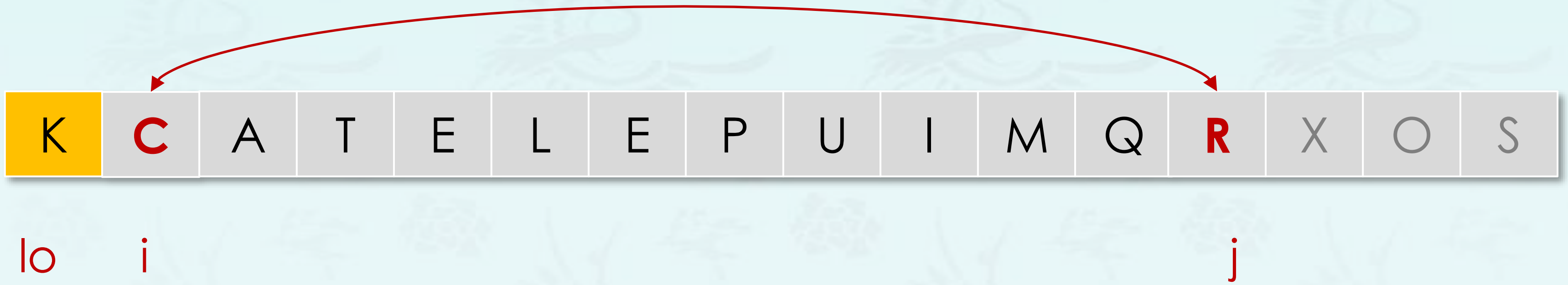
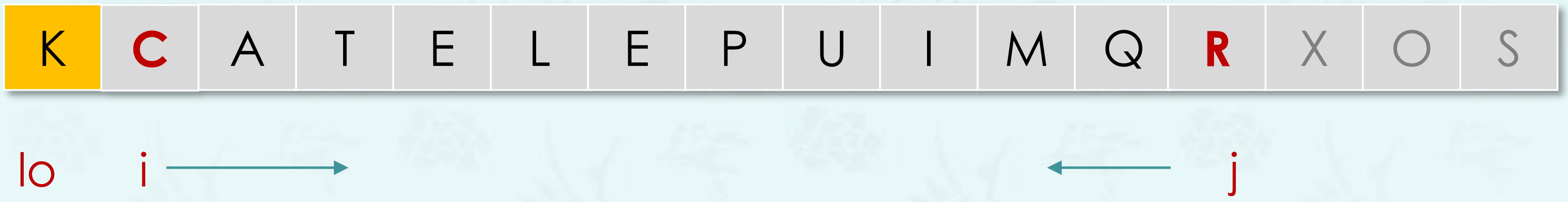


K	C	A	T	E	L	E	P	U	I	M	Q	R	X	O	S
lo	i											j			

stop scan and exchange a[i] with a[j]

Quick Sort partitioning demo

- Phase I. Repeat until $\boldsymbol{i}$ and $\boldsymbol{j}$ pointers cross:
  - Scan i from left to right so long as $(a[i] < a[lo])$
  - Scan j from right to left so long as $(a[j] > a[lo])$.
  - Exchange $a[i]$ with $a[j]$.



K	**C**	A	T	E	L	E	P	U	I	M	Q	**R**	X	O	S

now increment i until ....

stop i scan because a[i] >= a[lo]

pivot

now decrement j until ....

Quick Sort partitioning demo

- Phase I. Repeat until $\boldsymbol{i}$ and $\boldsymbol{j}$ pointers cross:
  - Scan i from left to right so long as $(a[i] < a[lo])$
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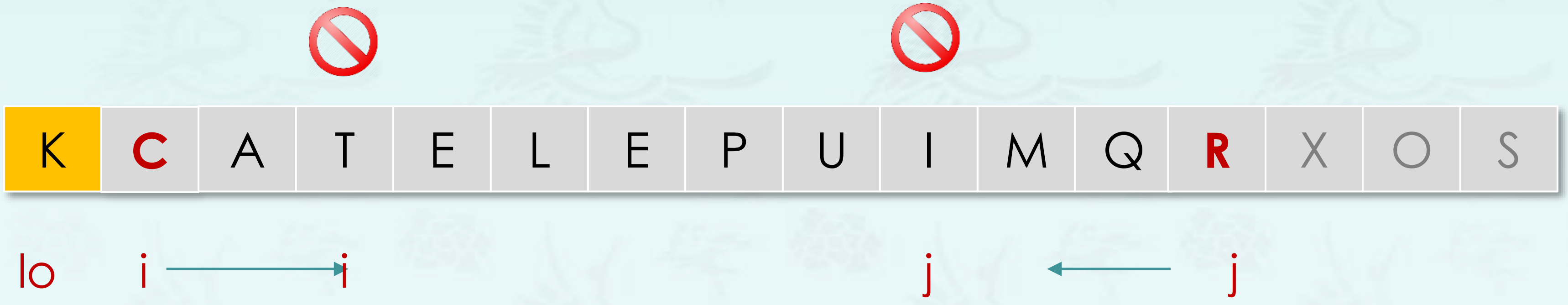

K	**C**	A	T	E	L	E	P	U	I	M	Q	**R**	X	O	S

now increment i until ....

stop i scan because a[i] >= a[lo]

pivot

now decrement j until ....

Quick Sort partitioning demo

- Phase I. Repeat until $\boldsymbol{i}$ and $\boldsymbol{j}$ pointers cross:
  - Scan i from left to right so long as $(a[i] < a[lo])$
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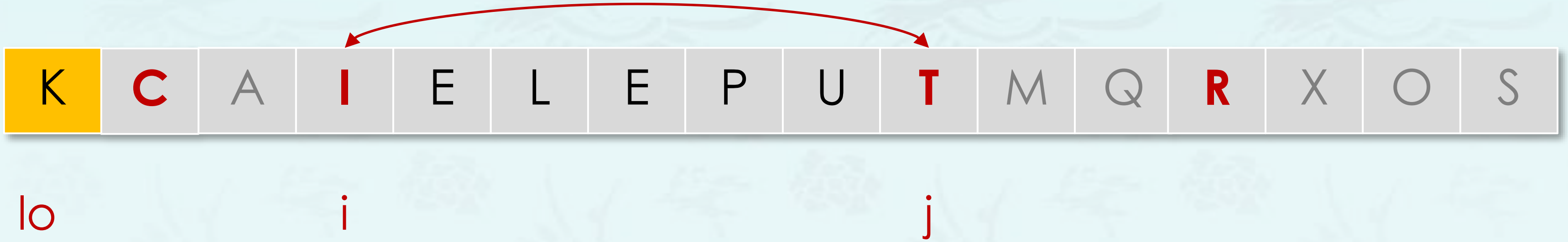

K	**C**	A	**I**	E	L	E	P	U	**T**	M	Q	**R**	X	O	S
lo			i						j						

stop scan and exchange a[i] with a[j]

Quick Sort partitioning demo

- Phase I. Repeat until ***i*** and ***j*** pointers cross:
  - Scan i from left to right so long as $(a[i] < a[lo])$
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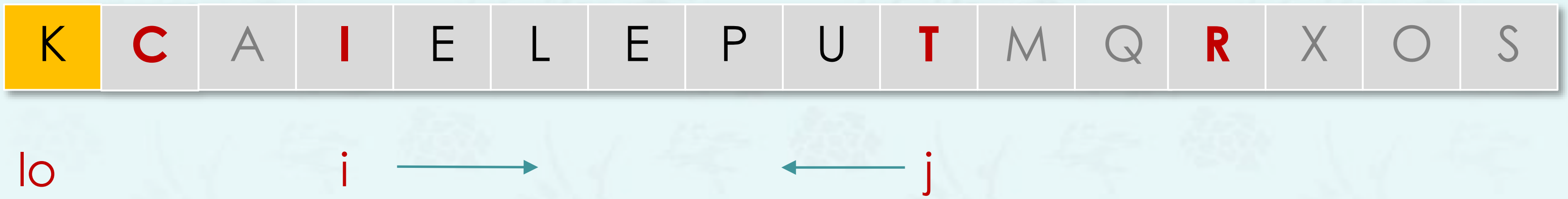

K	C	A	I	E	L	E	P	U	T	M	Q	R	X	O	S

now increment i until ….

now decrement j until ….

Quick Sort partitioning demo

- Phase I. Repeat until $\boldsymbol{i}$ and $\boldsymbol{j}$ pointers cross:
  - Scan i from left to right so long as $(a[i] < a[lo])$
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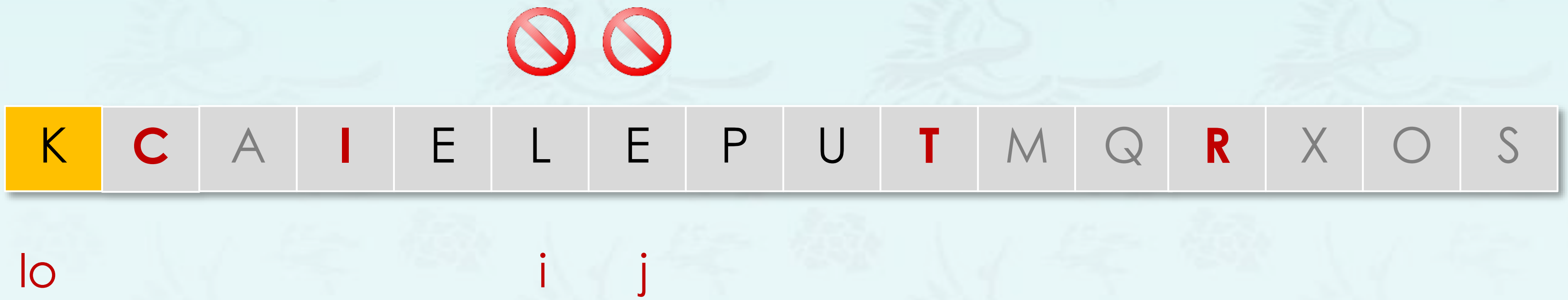

					🚫	🚫									
K	**C**	A	**I**	E	L	E	P	U	**T**	M	Q	**R**	X	O	S
lo					i	j									

Quick Sort partitioning demo

- Phase I. Repeat until $\boldsymbol{i}$ and $\boldsymbol{j}$ pointers cross:
  - Scan i from left to right so long as $(a[i] < a[lo])$
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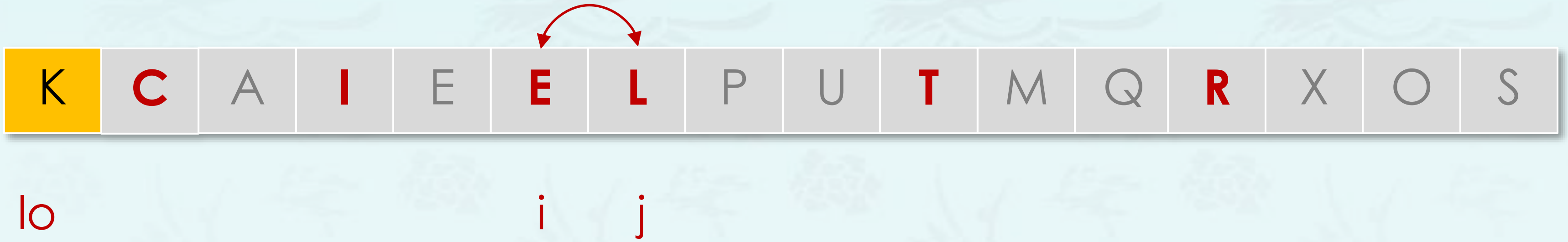
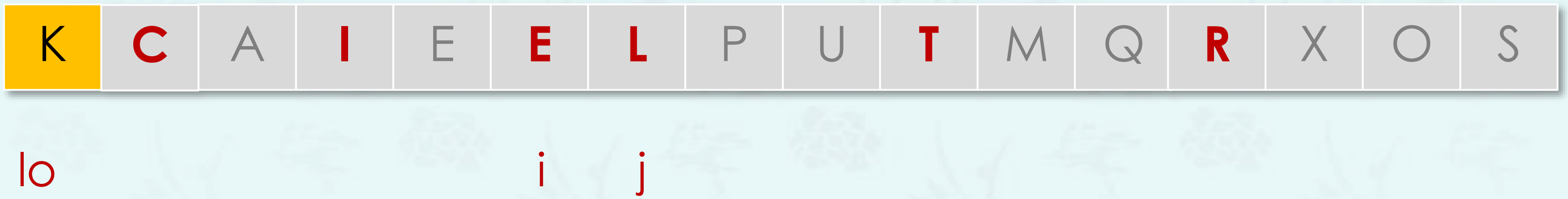


K	**C**	A	**I**	E	**E**	**L**	P	U	**T**	M	Q	**R**	X	O	S
lo					i	j									

stop scan and exchange a[i] with a[j]

Quick Sort partitioning demo

- Phase I. Repeat until ***i*** and ***j*** pointers cross:
  - Scan i from left to right so long as $(a[i] < a[lo])$
  - Scan j from right to left so long as $(a[j] > a[lo])$.
  - Exchange $a[i]$ with $a[j]$.



K	**C**	A	**I**	E	**E**	**L**	P	U	**T**	M	Q	**R**	X	O	S
lo					i	j									

now increment i until ….

now decrement j until ….

Quick Sort partitioning demo

- Phase I. Repeat until ***i*** and ***j*** pointers cross:
  - Scan i from left to right so long as $(a[i] < a[lo])$
  - Scan j from right to left so long as $(a[j] > a[lo])$.
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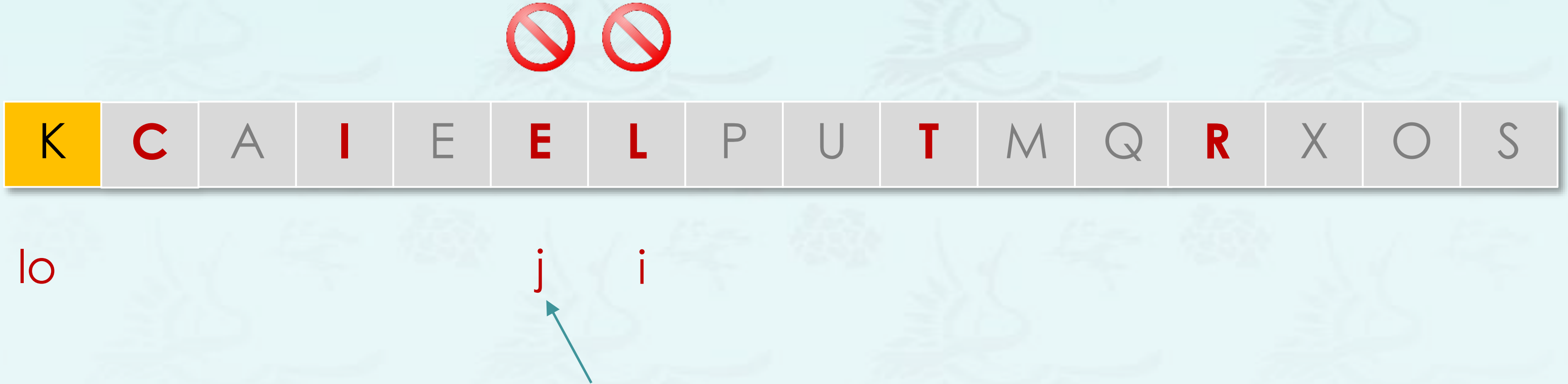


K	**C**	A	**I**	E	**E**	**L**	P	U	**T**	M	Q	**R**	X	O	S
lo					j	i									

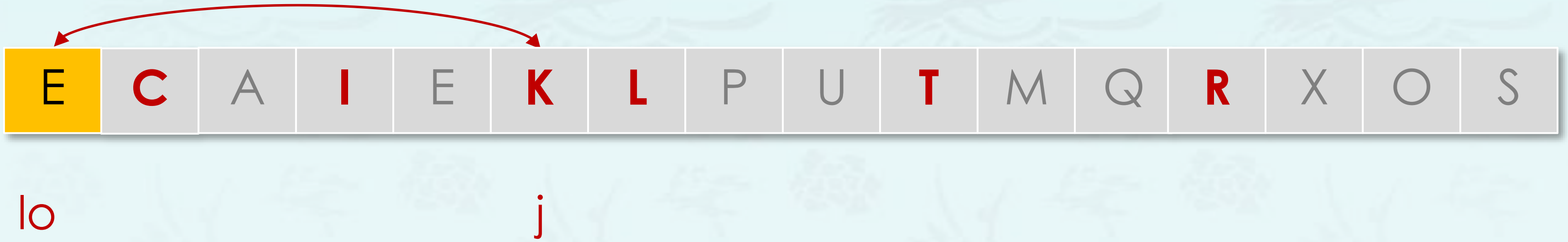
stop j scan because a[j] <= a[lo]

now j points the last element of left subarray

at this point, partitioning process is complete!

Quick Sort partitioning demo

- Phase I. Repeat until $\boldsymbol{i}$ and $\boldsymbol{j}$ pointers cross:
  - Scan i from left to right so long as ($a[i] < a[lo]$)
  - Scan j from right to left so long as ($a[j] > a[lo]$).
  - Exchange $a[i]$ with $a[j]$.
- **Phase II. When pointers cross.**
  - Exchange a[lo] with a[j].

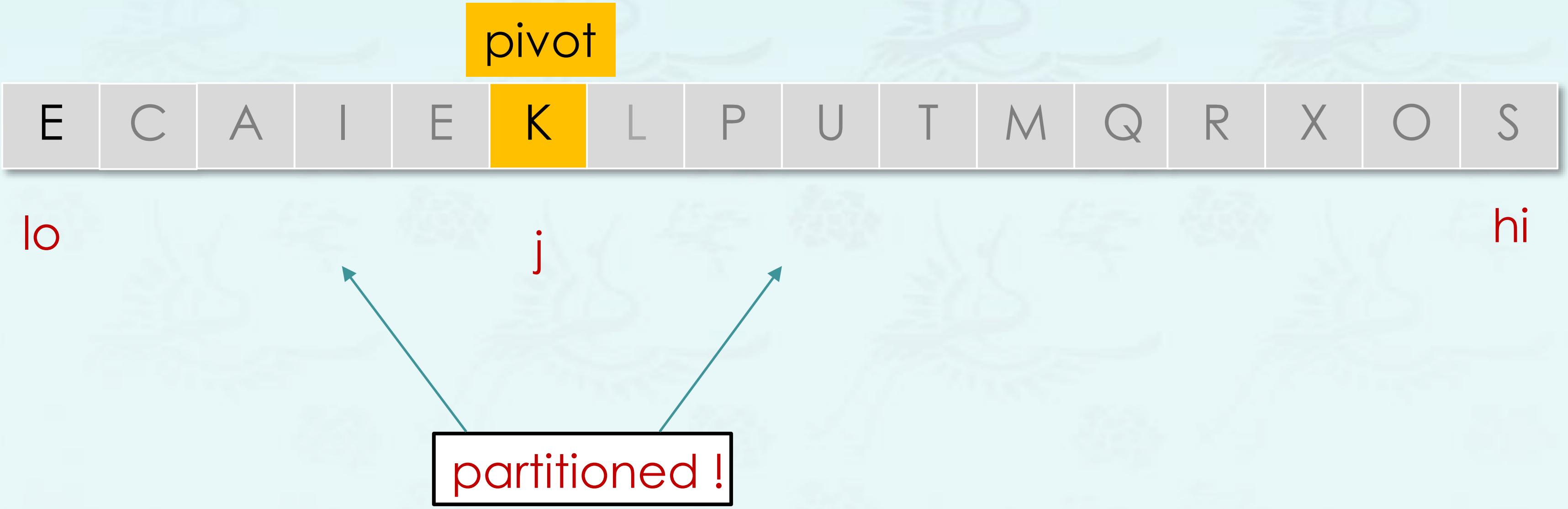


E	C	A	I	E	K	L	P	U	T	M	Q	R	X	O	S

at this point, partitioning process is complete!

Quick Sort partitioning demo

- Phase I. Repeat until $\boldsymbol{i}$ and $\boldsymbol{j}$ pointers cross:
  - Scan i from left to right so long as ($a[i] < a[lo]$)
  - Scan j from right to left so long as ($a[j] > a[lo]$).
  - Exchange $a[i]$ with $a[j]$.
- **Phase II. When pointers cross.**
  - Exchange a[lo] with a[j].



pivot

E	C	A	I	E	K	L	P	U	T	M	Q	R	X	O	S

lo

j

hi

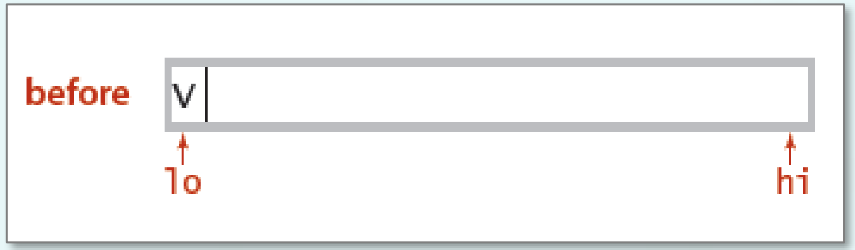
partitioned !

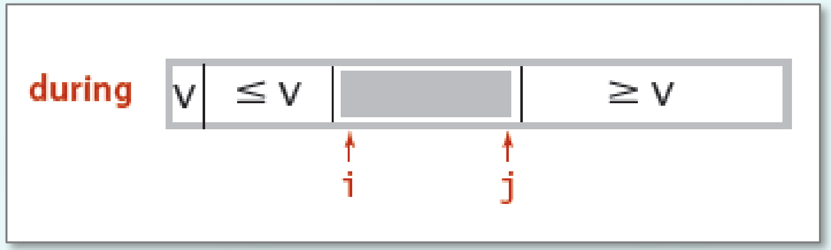
at this point, partitioning process is complete!

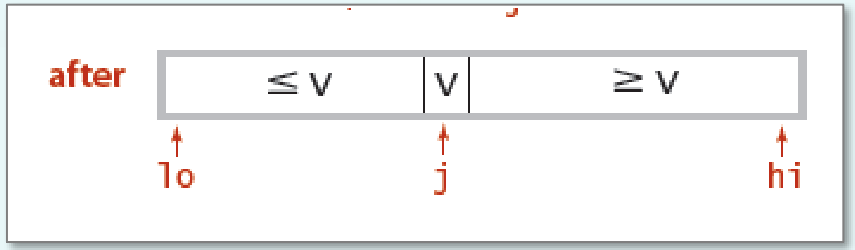
Quick Sort implementation

```
bool less(char a, char b)  { return a < b; }
void swap(char *a, int i, int j)  { char t = a[i];  a[i] = a[j];   a[j] = t; }

int partition(char *a, int lo, int hi) {
    int i = lo; int j = hi + 1;
    while (1) {
        while (less(a[++i], a[lo]))          find item on left to swap
            if (i == hi) break;

        while (less(a[lo], a[--j]))          find item on right to swap
            if (j == lo) break;

        if (i >= j) break;                   check if pointers cross
        swap(a, i, j);                       swap
    }
    swap(a, lo, j);                          swap with pivot
    return j;                                return index of item now sorted
}
```

pivot







Quick Sort implementation

```
void quicksort(char *a, int lo, int hi) {
    if (hi <= lo) return;

    int j = partition(a, lo, hi);

    quicksort(a,
    quicksort(a,
}

void main() {
  char a[] = {'Q', 'U', 'I', 'C', 'K', 'S', 'O', 'R', 'T', 'E', 'X', 'A', 'M', 'P', 'L', 'E'};
  int N = sizeof(a) / sizeof(a[0]);

  cout << "UNSORTED: \n";
  for (int i = 0; i < N; i++) cout << a[i]; cout << endl;
  //  shuffle(a, N);

  quicksort(a, 0, N-1);
  cout << "SORTED: \n";
  for (int i = 0; i < N; i++) cout << a[i]; cout << endl;
}
```

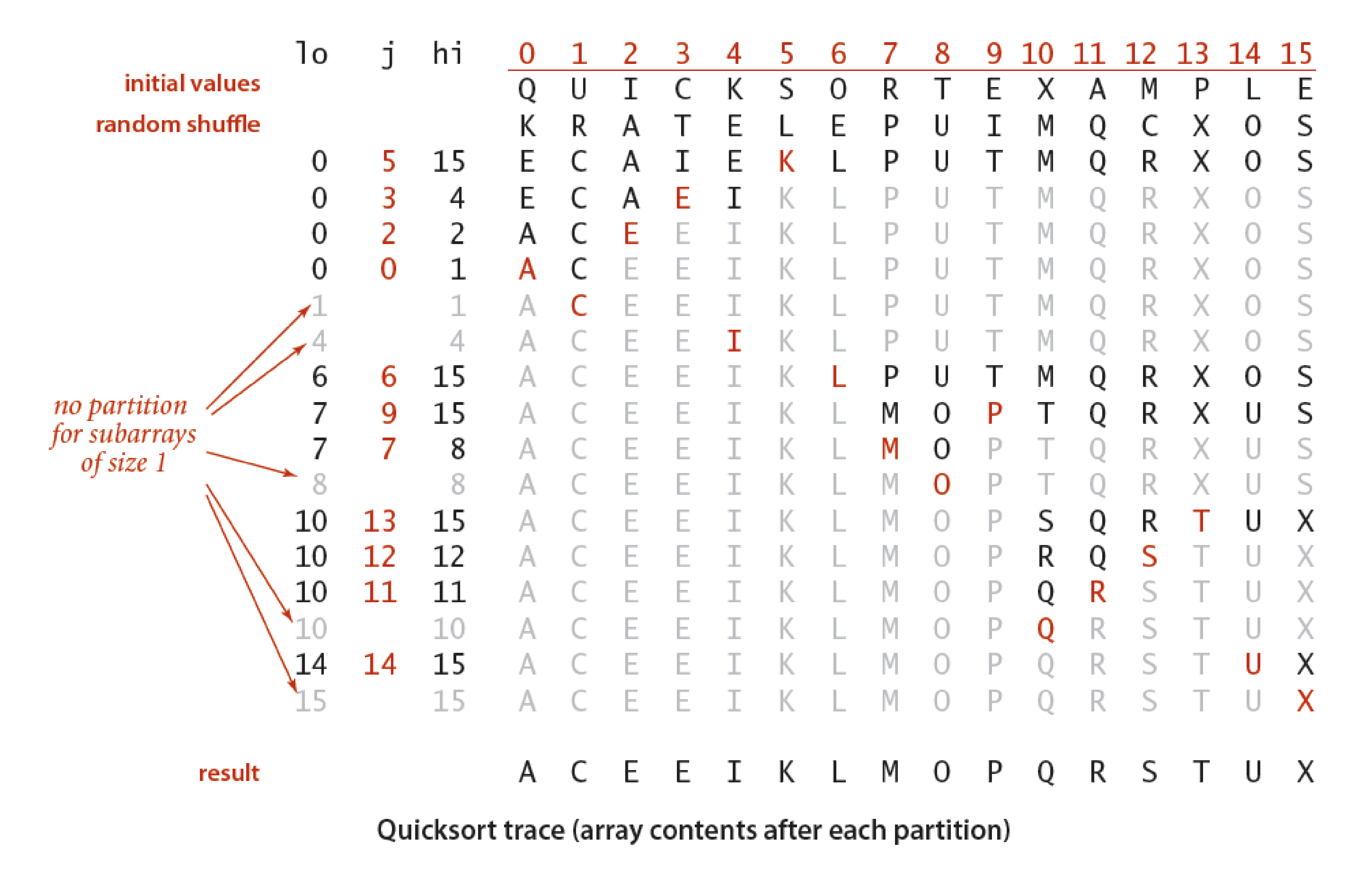
Quick Sort implementation

```
void quicksort(char *a, int lo, int hi) {
    if (hi <= lo) return;

    int j = partition(a, lo, hi);

    quicksort(a, lo,    j - 1);
    quicksort(a, 
}

void main() {
  char a[] = {'Q', 'U', 'I', 'C', 'K', 'S', 'O', 'R', 'T', 'E', 'X', 'A', 'M', 'P', 'L', 'E'};
  int N = sizeof(a) / sizeof(a[0]);

  cout << "UNSORTED: \n";
  for (int i = 0; i < N; i++) cout << a[i]; cout << endl;
  //  shuffle(a, N);

  quicksort(a, 0, N-1);
  cout << "SORTED: \n";
  for (int i = 0; i < N; i++) cout << a[i]; cout << endl;
}
```

Quick Sort implementation

```
void quicksort(char *a, int lo, int hi) {
    if (hi <= lo) return;

    int j = partition(a, lo, hi);

    quicksort(a, lo,    j - 1);
    quicksort(a, j + 1, hi   );
}

void main() {
  char a[] = {'Q', 'U', 'I', 'C', 'K', 'S', 'O', 'R', 'T', 'E', 'X', 'A', 'M', 'P', 'L', 'E'};
  int N = sizeof(a) / sizeof(a[0]);

  cout << "UNSORTED: \n";
  for (int i = 0; i < N; i++) cout << a[i]; cout << endl;
  //  shuffle(a, N);

  quicksort(a, 0, N-1);
  cout << "SORTED: \n";
  for (int i = 0; i < N; i++) cout << a[i]; cout << endl;
}
```

Quick Sort implementation



	lo	j	hi	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
initial values				Q	U	I	C	K	S	O	R	T	E	X	A	M	P	L	E
random shuffle				K	R	A	T	E	L	E	P	U	I	M	Q	C	X	O	S
	0	5	15	E	C	A	I	E	K	L	P	U	T	M	Q	R	X	O	S
	0	3	4	E	C	A	E	I	K	L	P	U	T	M	Q	R	X	O	S
	0	2	2	A	C	E	E	I	K	L	P	U	T	M	Q	R	X	O	S
	0	0	1	A	C	E	E	I	K	L	P	U	T	M	Q	R	X	O	S
	1		1	A	C	E	E	I	K	L	P	U	T	M	Q	R	X	O	S
	4		4	A	C	E	E	I	K	L	P	U	T	M	Q	R	X	O	S
	6	6	15	A	C	E	E	I	K	L	P	U	T	M	Q	R	X	O	S
	7	9	15	A	C	E	E	I	K	L	M	O	P	T	Q	R	X	U	S
	7	7	8	A	C	E	E	I	K	L	M	O	P	T	Q	R	X	U	S
	8		8	A	C	E	E	I	K	L	M	O	P	T	Q	R	X	U	S
	10	13	15	A	C	E	E	I	K	L	M	O	P	S	Q	R	T	U	X
	10	12	12	A	C	E	E	I	K	L	M	O	P	R	Q	S	T	U	X
	10	11	11	A	C	E	E	I	K	L	M	O	P	Q	R	S	T	U	X
	10		10	A	C	E	E	I	K	L	M	O	P	Q	R	S	T	U	X
	14	14	15	A	C	E	E	I	K	L	M	O	P	Q	R	S	T	U	X
	15		15	A	C	E	E	I	K	L	M	O	P	Q	R	S	T	U	X
result				A	C	E	E	I	K	L	M	O	P	Q	R	S	T	U	X

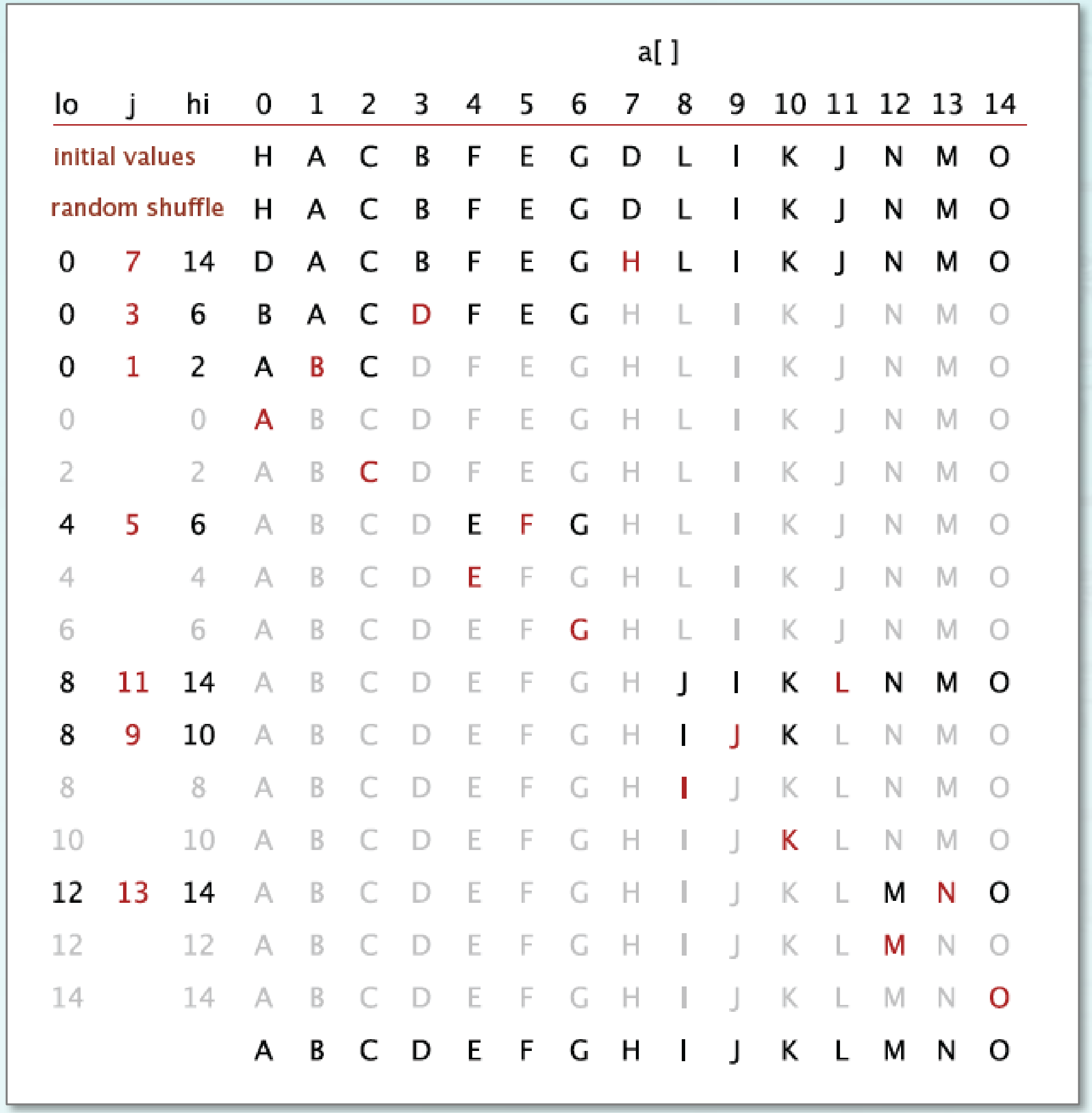
no partition for subarrays of size 1

Quicksort trace (array contents after each partition)

https://algs4.cs.princeton.edu/23quicksort/

Quick Sort: best-case analysis

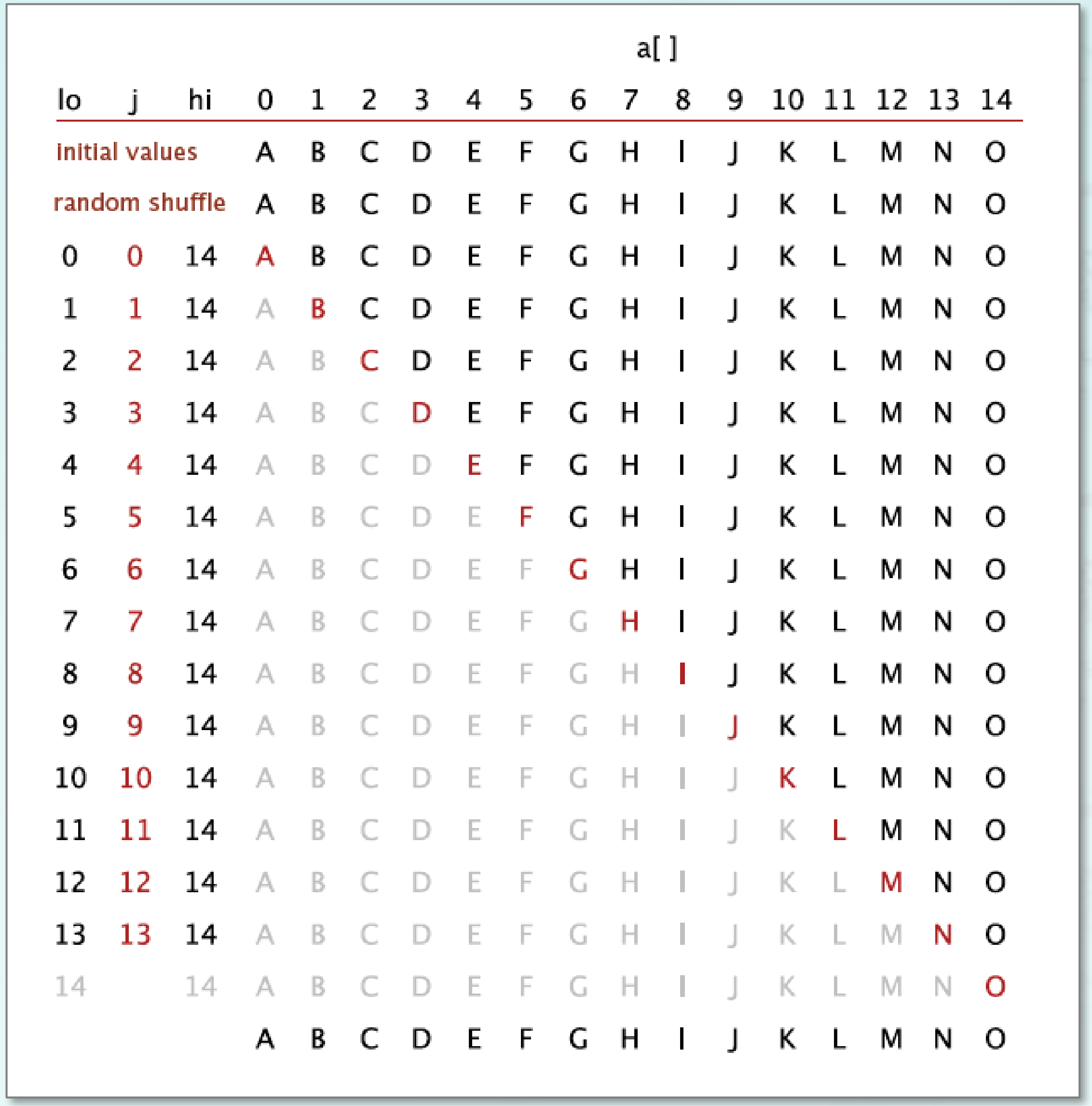
- **Best case:** Number of compares is ~ *N* lg *N*.



			a[ ]														
lo	j	hi	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
initial values			H	A	C	B	F	E	G	D	L	I	K	J	N	M	O
random shuffle			H	A	C	B	F	E	G	D	L	I	K	J	N	M	O
0	7	14	D	A	C	B	F	E	G	H	L	I	K	J	N	M	O
0	3	6	B	A	C	D	F	E	G	H	L	I	K	J	N	M	O
0	1	2	A	B	C	D	F	E	G	H	L	I	K	J	N	M	O
0		0	A	B	C	D	F	E	G	H	L	I	K	J	N	M	O
2		2	A	B	C	D	F	E	G	H	L	I	K	J	N	M	O
4	5	6	A	B	C	D	E	F	G	H	L	I	K	J	N	M	O
4		4	A	B	C	D	E	F	G	H	L	I	K	J	N	M	O
6		6	A	B	C	D	E	F	G	H	L	I	K	J	N	M	O
8	11	14	A	B	C	D	E	F	G	H	J	I	K	L	N	M	O
8	9	10	A	B	C	D	E	F	G	H	I	J	K	L	N	M	O
8		8	A	B	C	D	E	F	G	H	I	J	K	L	N	M	O
10		10	A	B	C	D	E	F	G	H	I	J	K	L	N	M	O
12	13	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
12		12	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
14		14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
			A	B	C	D	E	F	G	H	I	J	K	L	M	N	O

Quick Sort: worst-case analysis

- **Worst case:** Number of compares is ~ ½ N^2 .

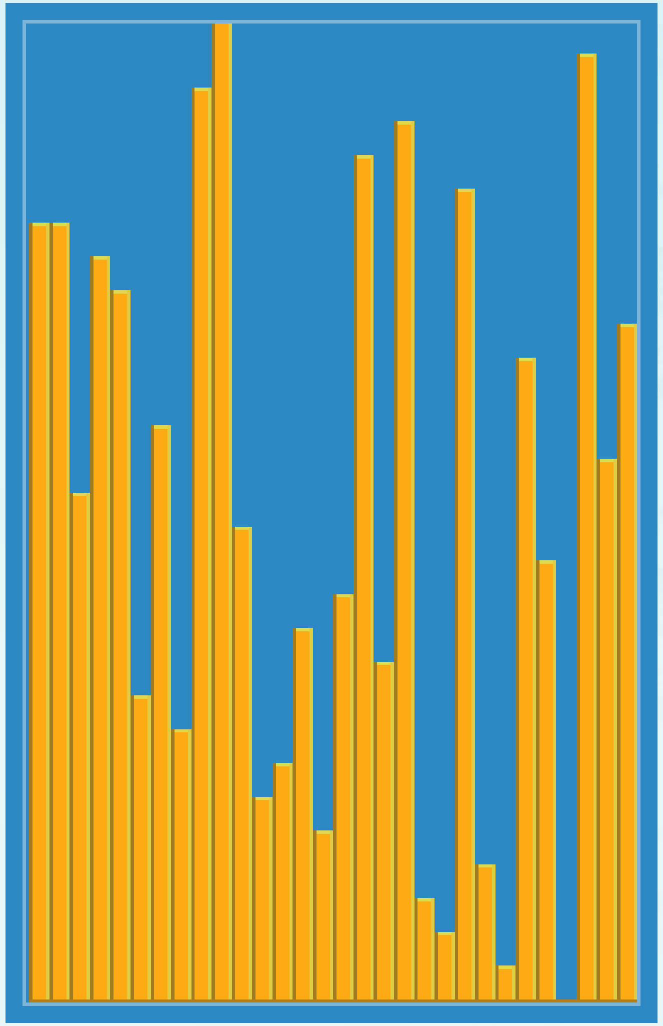


			a[ ]														
lo	j	hi	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
initial values			A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
random shuffle			A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
0	0	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	1	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
2	2	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
3	3	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
4	4	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
5	5	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
6	6	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
7	7	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
8	8	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
9	9	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
10	10	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
11	11	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
12	12	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
13	13	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
14		14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
			A	B	C	D	E	F	G	H	I	J	K	L	M	N	O

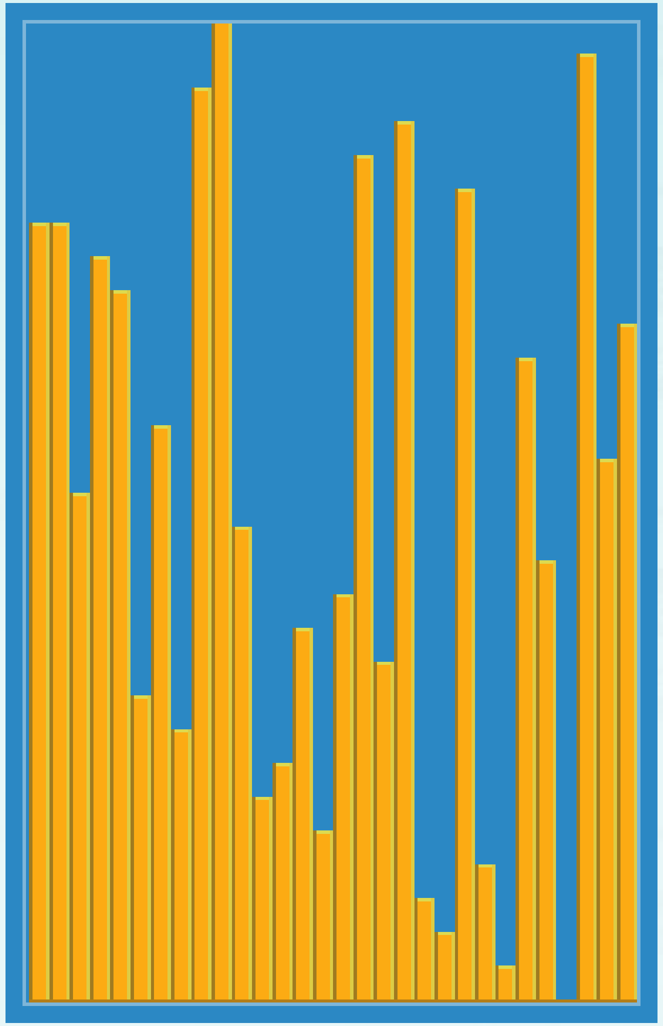
Quick Sort: average-case analysis

- Worst case: Number of compares is quadratic.
  - $N + (N - 1) + (N - 2) + \ldots + 1 \sim ½ N^2$.
  - More likely that your computer is struck by lightning bolt
- **Average case:** Number of compares is ~ 1.39 N lg N.
  - 39% more compares than mergesort.
  - But faster than mergesort in practice because of less data movement.
- Random shuffle:
  - Probabilistic guarantee against worst case.
  - Basis for math model that can be validated with experiments.

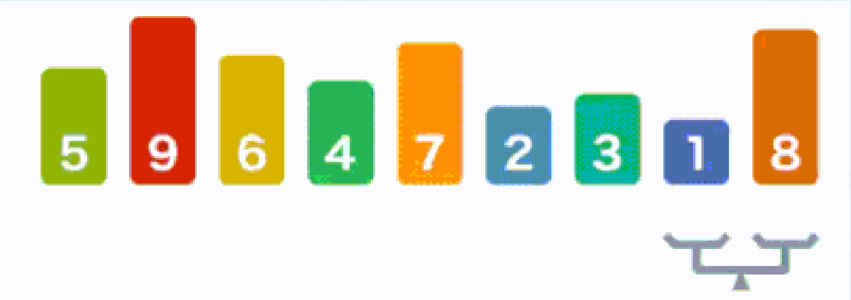
Sorting Algorithm Animation



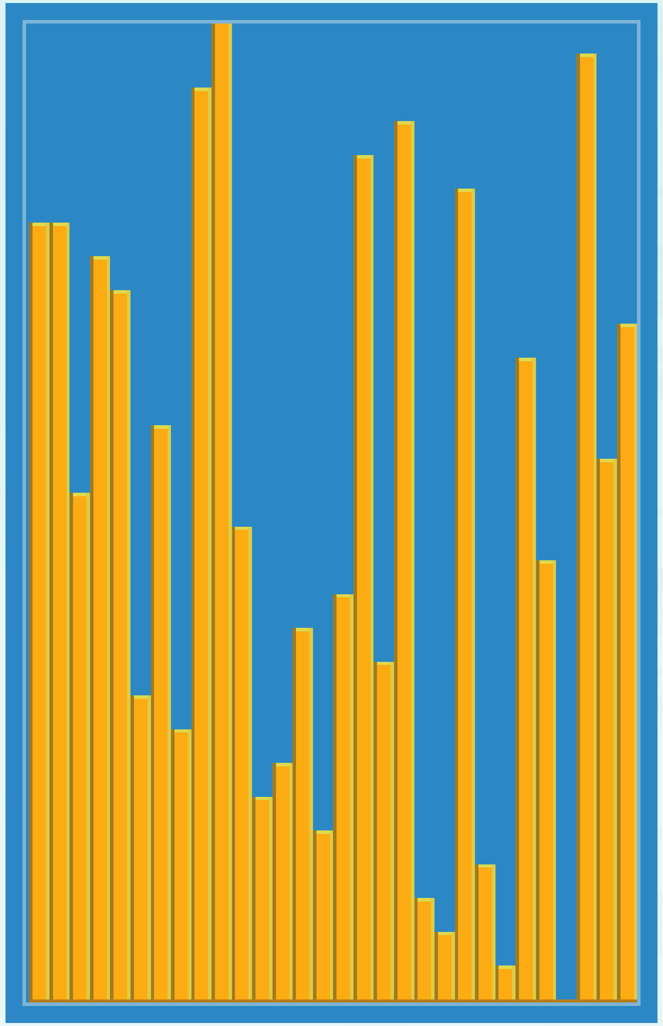
Sorting Algorithm Animation



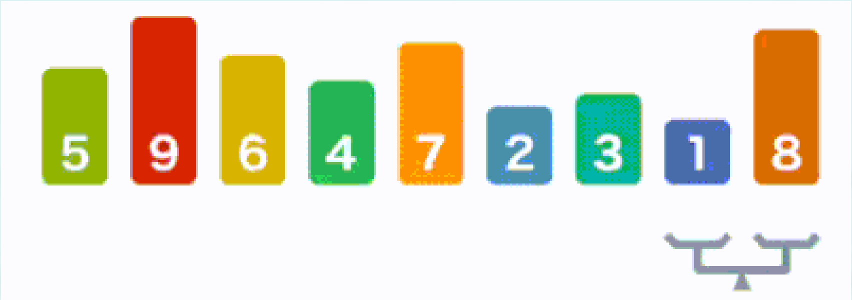
Insertion Sort



Sorting Algorithm Animation

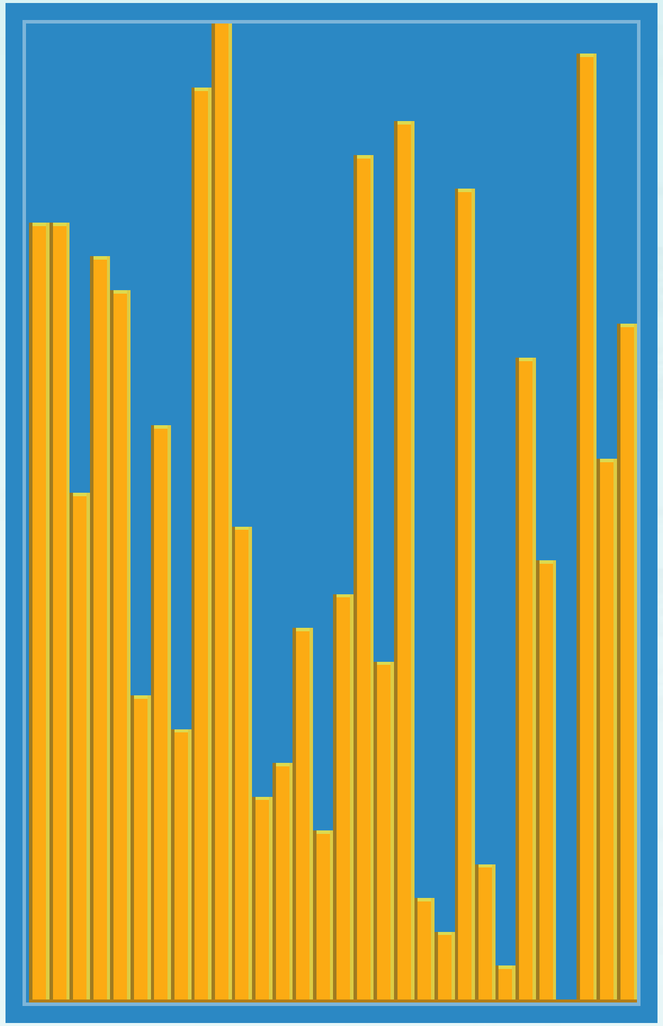


Insertion Sort

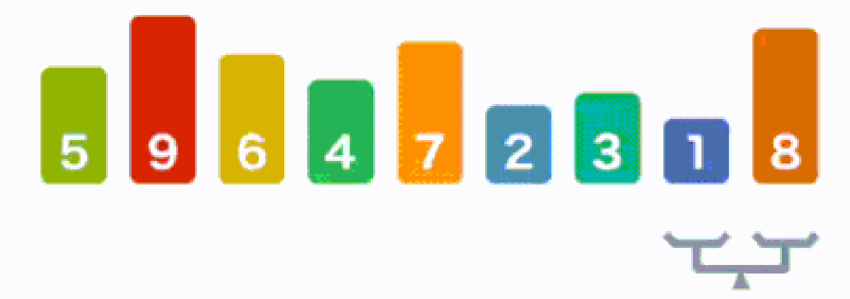


Bubble Sort

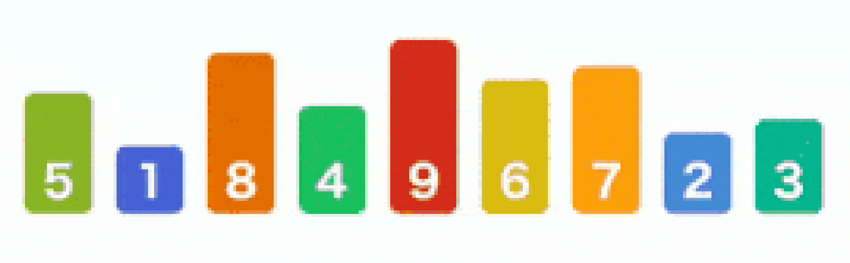
Sorting Algorithm Animation



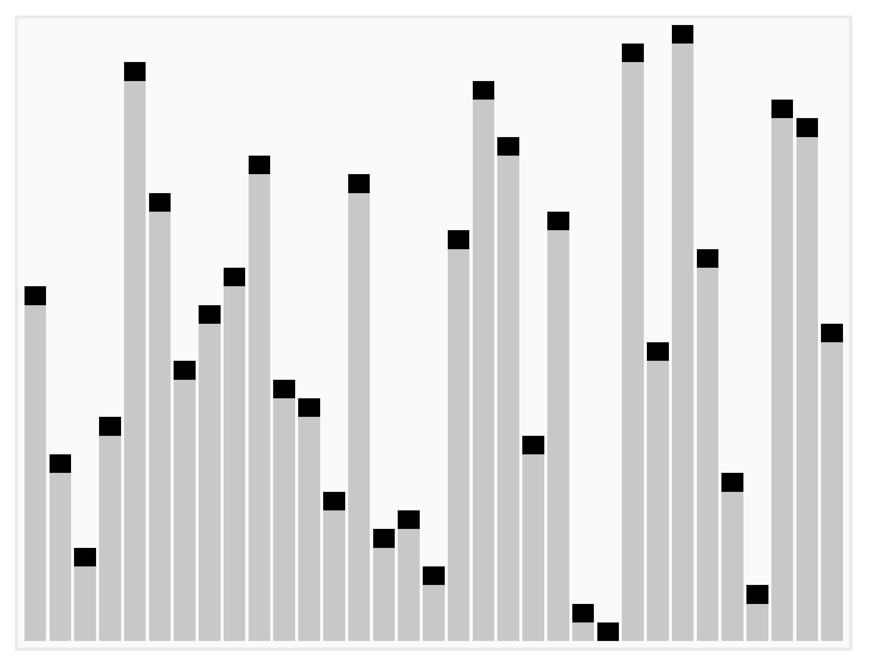
Insertion Sort



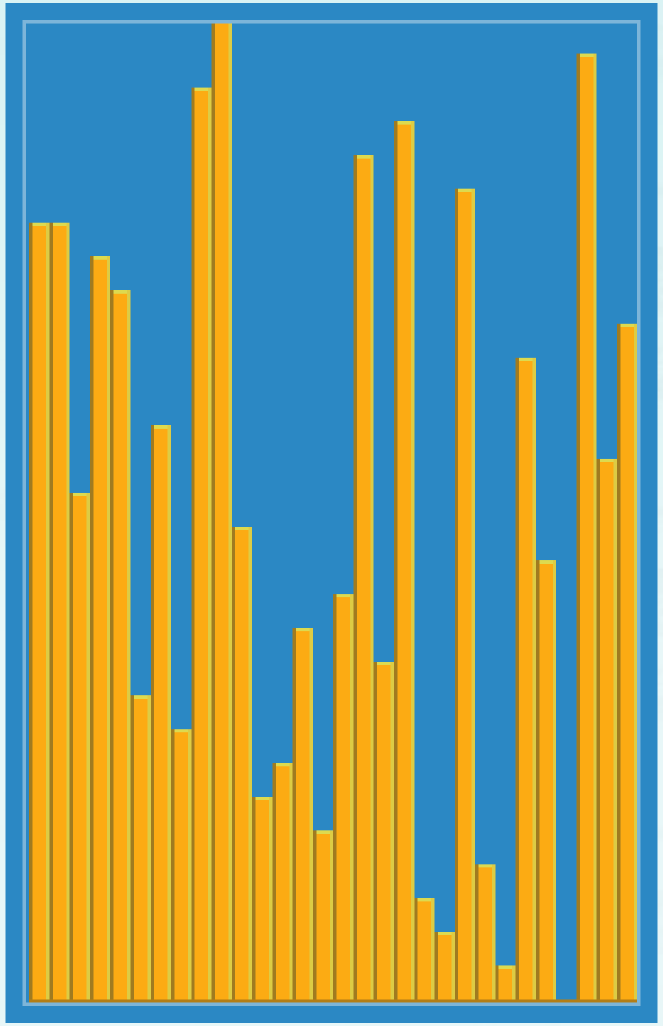
Bubble Sort



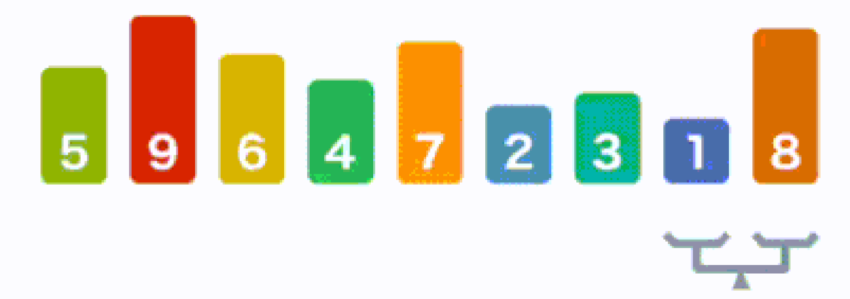
Selection Sort



Sorting Algorithm Animation

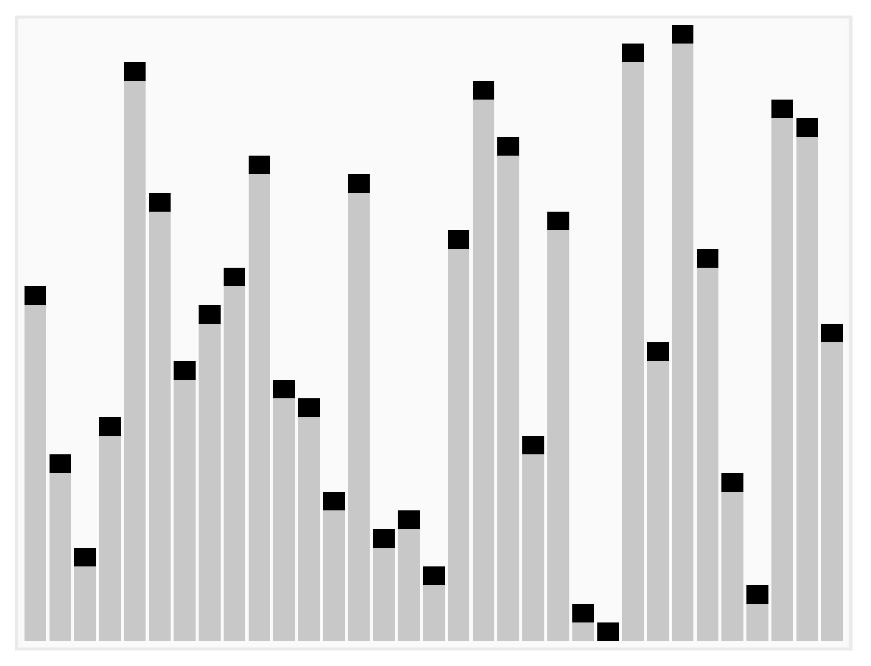


Insertion Sort



Bubble Sort

Selection Sort



quicksort

https://commons.wikimedia.org/wiki/File:Insertion-sort-example.gif
https://en.wikipedia.org/wiki/Insertion_sort

Sorting(2/2)

Data Structures
C++ for C Coders

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Merge Sort
Quick Sort