# Data Structures Chapter 3

#### 1. Stack

- Stack Concept
  - STL stack class
- Stack Implementations
  - Using Fixed Array
  - Using Dynamic Array
  - Using Vector
  - Using Template

#### 2. Queue



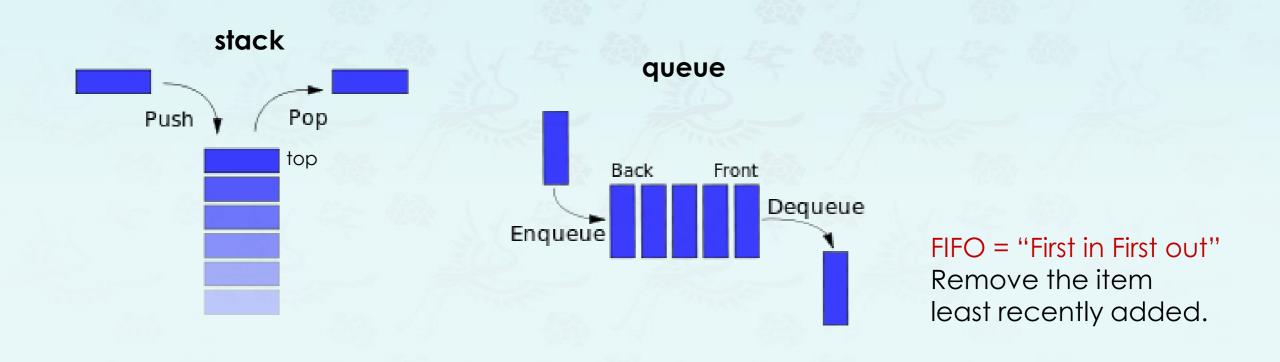
그러므로 예수께서 자기를 믿은 유대인들에게 이르시되 너희가 내 말에 거하면 참으로 내 제자가 되고 진리를 알지니 진리가 너희를 자유롭게 하리라 (요8:31-32)

#### Stack

• **Stack** is a linear data structure represented by a real physical stack or pile, a structure where insertion and deletion of items takes place at one end called top of the stack.

#### Stack

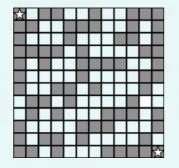
- **Stack** is a linear data structure represented by a real physical stack or pile, a structure where insertion and deletion of items takes place at one end called top of the stack.
- This particular order of the operation is called LIFO(Last In First Out).

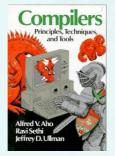


## **Stack Applications**

- Parsing in a compiler.
- Undo in a word processor.
- Back button in a Web browser.
- PostScript language for printers.
- Backtracking as in a maze
- Implementing function calls in a compiler.

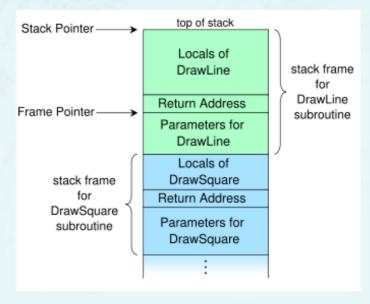
•











# Stack - ADT (Abstract Data Type)

#### **ADT Stack is**

- Objects: a finite ordered list with zero or more elements
- Operations (or Functions):

```
Stack newStack(maxStackSize)
bool empty()
void push(item)
void pop()
int top()
int size;
```

## Stack - Why ADT?

- Separate interface and implementation.
  - Ex: stack, queue, bag, priority queue, symbol table, union-find, ....

#### Benefits.

- Driver (or Client) can't know details of implementation
  - Client has many implementations from which to choose.
  - Program using operations defined in interface.
- Interface is description of data type, basic operations.
- Implementation is actual code implementing operations.
  - Design: creates modular, reusable libraries.
  - Performance: use optimized implementation where it matters.

# Stack: Example in C++

| STL         | <pre>#include <stack></stack></pre>         | Stack class in C++ STL                                     |  |  |
|-------------|---|--|--|--|
|             | stack <value_type></value_type>             | creates an empty stack of <value_type></value_type>        |  |  |
| void        | <pre>push(const value_type&amp; item)</pre> | inserts a new item onto stack                              |  |  |
| void        | pop()                                       | removes top item from stack (which is most recently added) |  |  |
| value_type& | top()                                       | returns a reference to the top item                        |  |  |
| bool        | empty()                                     | is the stack empty?  |  |  |
| int         | size()                                      | returns the number of items in the stack                   |  |  |

https://cplusplus.com/reference/stack/stack/

Warm-up client: Reverse sequence of strings using stack.

#### Stack: Driver/Client using stack class in C++ STL

- Read strings from a collection using a range-for loop.
  - If string equals "-", pop string from stack and print.
  - Otherwise, push string onto stack.

```
int main () { // stack initialization using range-based for
 string list[] = {"to", "be", "or", "not", "to", "-", "be",
                    "-", "-", "that", "-", "-", "is"};
 stack<string> s;
 for (auto item : list) { // to be not that or be
    if (item != "-")
      s.push(item);
                                                     void printStack(stack<string> s) {
    else {
                                                       while (!s.empty()) {
      cout << s.top() << ' ';
                                                         cout << s.top() << ' ';
      s.pop();
                                                         s.pop();
                                                       cout << endl; // now, s is empty</pre>
 cout << "\nsize(): " << s.size();</pre>
 cout << "\ntop() : " << s.top();</pre>
```

# **Stack: Implementation**

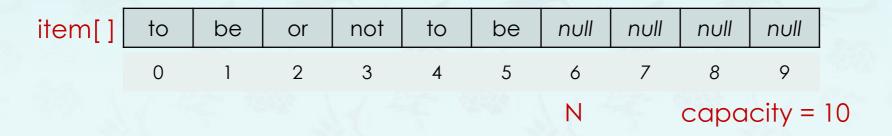
# Let's implement our own stack in several different ways.

- Array implementation
  - fixed size array
  - dynamic array
- Vector implementation
- Using Template
  - Array implementation
  - Vector implementation

# Stack: Array implementation

## Let's implement our own stack in several different ways.

- Array implementation of a stack:
  - Use array item[] to store N items on stack.
  - push(): add new item at item[N].
  - top(): return item from item[N-1].
  - pop(): remove item from item[N-1], it just decrements N by one.



**Defect.** Stack overflows when N exceeds capacity. [stay tuned]

## Stack: Things to consider

- Underflow:
  - Throw exception if pop from an empty stack or return null;

```
string top(stack s) {
  return s->item[s->N - 1];
}
```

```
string top(stack s) {
  if (empty())
    throw std::out_of_range("underflow");
  return s->item[s->N - 1];
}
```

- Overflow:
  - Use resizing array for array implementation. [stay tuned]
  - Use successive doubling method
- Generic programming using Template in C++
  - It makes the stack data(item) type-independent
  - template<typename T>

## Stack: version.2 – using a fixed size array

```
struct Stack {
  string *item;
 int N;
 int capacity;
                                a shortcoming
using stack = Stack *;
                                (stay tuned)
stack newStack(int capacity) {
  stack s = new Stack;
  s->item = new string[capacity];
 s->N=0;
  s->capacity = capacity;
  return s;
void free(stack s) {
  delete[] s->item;
 delete s;
                 item[N] is next to be filled if any.
```

```
int size(stack s) { return s->N; }
bool empty(stack s) { return s->N == 0; }
void pop(stack s) { s->N--; }
string top(stack s) {
 return s->item[s->N - 1]; N is not decremented
void push(stack s, string item) {
  s->item[s->N++] = item; use N and incremented
                            N points an empty slot
void printStack(stack s) {
  while (!empty(s)) {
    cout << top(s) << ' ';
    pop(s);
  cout << endl; // stack is empty now</pre>
```

# TOTAL THE TOTAL



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#### 2. Queue

- Problem:
  - Requiring client to provide capacity (size of stack) is inappropriate.
  - Question: How to grow and shrink array?
- First try.
  - push(): increase size of array item[] by 1.
  - pop(): decrease size of array item[] by 1.
- Too expensive.
  - Need to copy all items to a new array.
  - Inserting first N items takes time proportional to  $1 + 2 + 3 + .... + N \approx N^2/2$ .

infeasible for large N

Challenge: Ensure that array resizing happens infrequently.

Q. How to grow and shrink array?

"successive doubling"

A. If array is full, create a new array of twice the size, and copy items.

```
stack newStack(int capacity = 1) {
                                                  void push(stack s, string item) {
 stack s = new Stack;
 s->item = new string[capacity];
 s->capacity = capacity;
 s \rightarrow N = 0;
                                                  void pop(stack s) {
 return s;
void resize(stack s, int new_capacity) {
  string *copied = new string[new_capacity];
 for (int i = 0; i < s->N; i++)
                                       copy(s->item; s->item + s->N, copied);
    copied[i] = s->item[i];
 delete[] s->item;
 s->item = copied;
 s->capacity = new capacity;
```

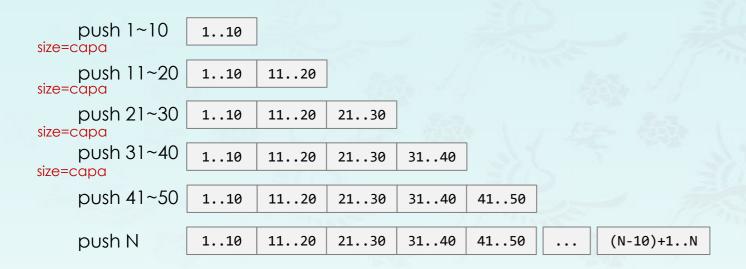
Q. Cost of inserting first N items by resize(s.length + 10)?

• A. 
$$T(N) = 10N + (10 + 20 + 30 + ... + N)$$

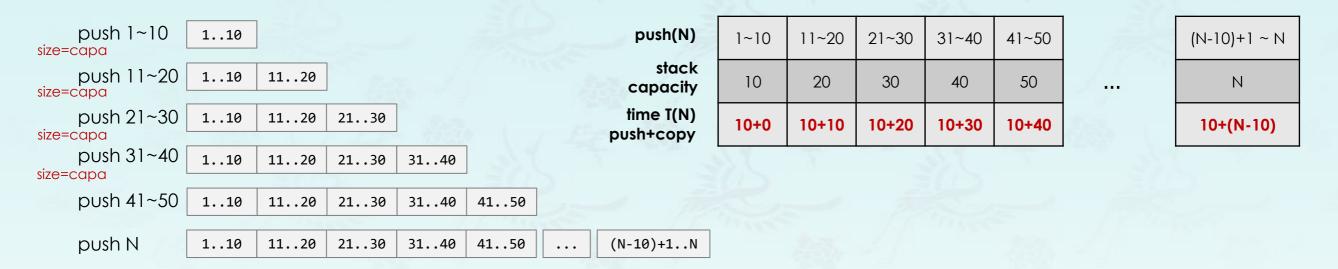
1 array access per push

k array accesses when memory is resized by increment of 10 (ignoring cost to create new array) (assuming new() costs copying each item one by one)

- Q. Cost of inserting first N items by resize(s.length + 10)?
- Let us consider an example of a simple array stack pushes.

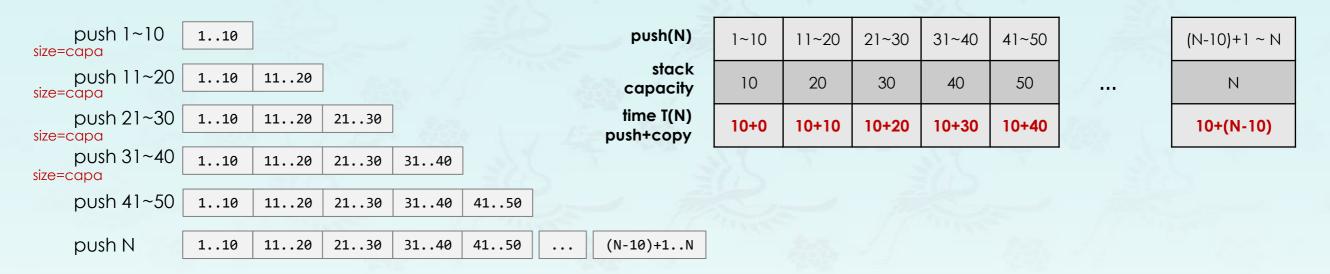


- Q. Cost of inserting first N items by resize(s.length + 10)?
- Let us consider an example of a simple array stack pushes.



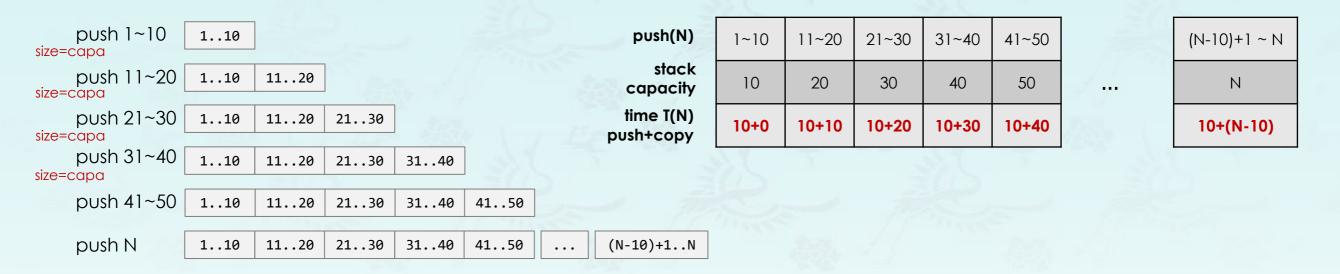
```
T(N) = (10) + (10+10) + (10+20) + (10+30) + \dots + (10+N-10)
= (10+10) + (10+20) + (10+30) + \dots + (10+N)
= (10 + 10 + 10 + \dots) + (10 + 20 + 30 + \dots + N)
```

- Q. Cost of inserting first N items by resize(s.length + 10)?
- Let us consider an example of a simple array stack pushes.



```
T(N) = (10) + (10+10) + (10+20) + (10+30) + \dots + (10+N-20) + (10+N-10)
= (10+10) + (10+20) + (10+30) + \dots + (10+N-20) + (10+N-10) + (10+N) - N
(Move 10 from the front to the end, then add N and subtract N)
= (10 + 10 + 10 + \dots + 10) + (10 + 20 + 30 + \dots + N) - N
= (10 * N/10) + (10 + 20 + 30 + \dots + N) - N
T(N) = (10 + 20 + 30 + \dots + N)
```

- Q. Cost of inserting first N items by resize(s.length + 10)?
- Let us consider an example of a simple array stack pushes.



```
T(N) = (10) + (10+10) + (10+20) + (10+30) + \dots + (10+N-20) + (10+N-10)
= (10+10) + (10+20) + (10+30) + \dots + (10+N-20) + (10+N-10) + (10+N-N)
(Move 10 \text{ from the front to the end, then add N and subtract N})
= (10 + 10 + 10 + \dots + 10) + (10 + 20 + 30 + \dots + N) - N
= (10 * N/10) + (10 + 20 + 30 + \dots + N) - N
T(N) = (10 + 20 + 30 + \dots + N)
Let N = 10k, then T(N) = 10(1 + 2 + 3 + \dots + k) = 10 \text{ k(k + 1)/2}
Let k = N/10, then T(N) = 10 * N/10 * (N/10 + 1)/2 = N/2(N/10 + 1)
```

- Q. Cost of inserting first N items by resize(s.length + 10)?
- A. T(N) = 10N + (10 + 20 + 30 + ... + N)

1 array access per push

k array accesses when memory is resized by increment of 10 (ignoring cost to create new array) (assuming new() costs copying each item one by one)

- Q. Cost of inserting first N items by resize(s.length + 10)?
- A. T(N) = 10N + (10 + 20 + 30 + ... + N)

How many terms? k terms, then N = 10k

$$T(N) = 10N + (10 + 20 + 30 + \dots + N)$$

$$Let N = 10k, then it becomes$$

$$T(N) = 10N + (10 + 20 + 30 + \dots + 10k)$$

$$= 10N + 10(1 + 2 + 3 + \dots + k)$$

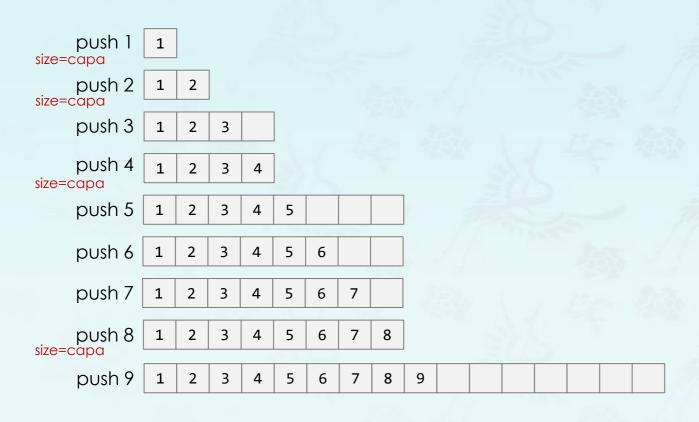
$$= 10N + 10 \frac{k(k+1)}{2}$$

$$= 10N + 10 \frac{\frac{N}{10}(\frac{N}{10} + 1)}{2}$$

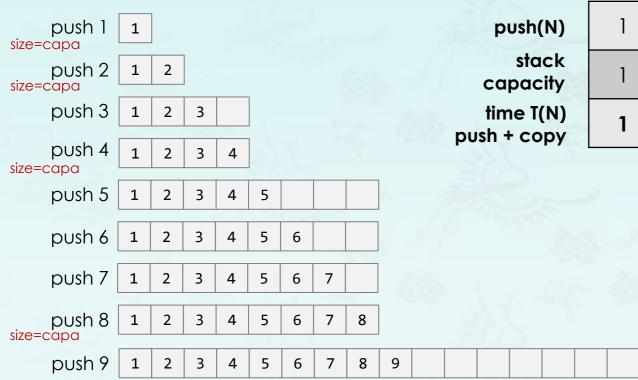
$$Therefore, T(N) = 10N + \frac{N}{2}(\frac{N}{10} + 1)$$
The time complexity of the algorithm is  $O(n^2)$ .

- Q. Cost of inserting first N items by resize(capacity \* 2)?
- A. T(N) = N + (1 + 2 + 4 + 8 + ... + N)

- Q. Cost of inserting first N items by resize(capacity \* 2)?
- Let us consider an example of a simple array stack pushes.



- Q. Cost of inserting first N items by resize(capacity \* 2)?
- Let us consider an example of a simple array stack pushes.



| 1 | 1+1 | 1+2 | 1 | 1+4 | 1 | 1 | 1 | 1+8 | 1  |
|---|-----|-----|---|-----|---|---|---|-----|----|
| 1 | 2   | 4   | 4 | 8   | 8 | 8 | 8 | 16  | 16 |
| 1 | 2   | 3   | 4 | 5   | 6 | 7 | 8 | 9   | 10 |

```
(1+1) + (1+2) + (1+4) + (1+8) + \dots + (1+2^k)
= (1+1+1+\dots) + (1+2+4+8+\dots+2^k)
= (k+1) + (1+2+4+8+\dots+2^k)
```

```
For example, N = 9 (or push 9, k = 3):

T'(9) = (1+1) + (1+2) + (1+4) + (1+8)

= (1+1+1+1) + (1+2+4+8)

= (3+1) + (1+2+4+8)

= 19  // resizing steps only
```

```
How many 1's in the time for N pushes? Then T(N) can be expressed T(N) = x + (k + 1) + (1 + 2 + 4 + 8 + ... + 2^k)= N + (1 + 2 + 4 + 8 + ... + 2^k)= N + (1 + 2 + 4 + 8 + ... + N)
```

- Q. Cost of inserting first N items by resize (capacity \* 2)?
- Let us consider an example of a simple array stack pushes.

| push(N)     |
|-------------|
| stack       |
| capacity    |
| time T(N)   |
| push + copy |

| 1 | 2   | 3   | 4 | 5   | 6 | 7 | 8 | 9   | 10 |
|---|-----|-----|---|-----|---|---|---|-----|----|
| 1 | 2   | 4   | 4 | 8   | 8 | 8 | 8 | 16  | 16 |
| 1 | 1+1 | 1+2 | 1 | 1+4 | 1 | 1 | 1 | 1+8 | 1  |

$$(1+1) + (1+2) + (1+4) + (1+8) + \dots + (1 + 2^k)$$
  
=  $(1 + 1 + 1 + \dots) + (1 + 2 + 4 + 8 + \dots + 2^k)$   
=  $(k + 1) + (1 + 2 + 4 + 8 + \dots + 2^k)$ 

```
For example, N = 9 (or push 9, k = 3):

T'(9) = (1+1) + (1+2) + (1+4) + (1+8)

= (1+1+1+1) + (1+2+4+8)

= (3+1) + (1+2+4+8)

= 19  // resizing steps only
```



```
How many 1's in the time for N pushes?

Then T(N) can be expressed

T(N) = (k + 1) + (1 + 2 + 4 + 8 + ... + 2^k)

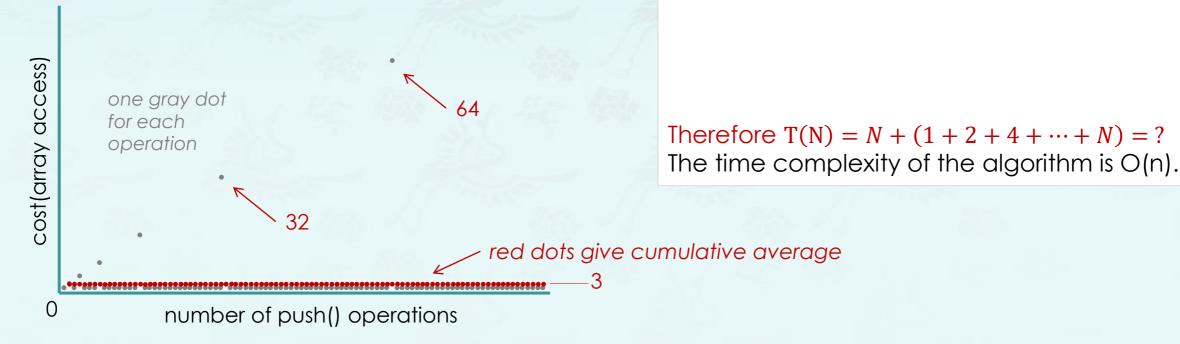
= N + (1 + 2 + 4 + 8 + ... + 2^k)

= N + (1 + 2 + 4 + 8 + ... + N)
```

- Q. Cost of inserting first N items by resize(capacity \* 2)?
- A. T(N) = N + (1 + 2 + 4 + 8 + ... + N)

$$1 + a + a^2 + a^3 + \dots + a^n = \frac{a^{n+1} - 1}{a - 1}$$

$$1 + 2 + 4 + \dots + 2^n = \frac{2^{n+1} - 1}{2 - 1} = 2^{n+1} - 1$$



Q. How to grow and shrink array?

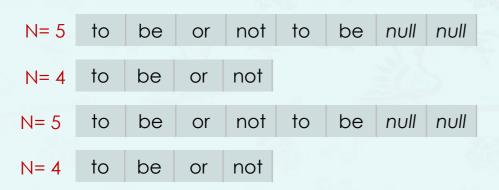
"successive doubling"

A. If array is full, create a new array of twice the size, and copy items.

```
stack newStack(int capacity = 1) {
  stack s = new Stack;
 s->item = new string[capacity];
 s->capacity = capacity;
 s \rightarrow N = 0;
 return s;
void resize(stack s, int new_capacity) {
  string *copied = new string[new_capacity];
  for (int i = 0; i < s->N; i++)
    copied[i] = s->item[i];
 delete[] s->item;
 s->item = copied;
  s->capacity = new_capacity;
```

```
void push(stack s, string item) {
   // your code here
   s->item[s->N++] = item;
}
```

- Q: How to shrink array?
- First try.
  - push(): double size of array item[] when array is full
  - pop(): halve size of array item[] when array is one-half full.
- Too expensive in worst case.
  - Consider push-pop-push-pop- ... sequence when array is full
  - Each operation takes time proportional to N.



- Q: How to shrink array?
- Efficient solution
  - push(): double size of array item[] when array is full
  - pop(): halve size of array item[] when array is one-quarter full.

```
void pop(stack s) {
  s->N--;
  // your code here
}
```

❖ Invariant. Array is between 25% and 100% full.

#### Amortized analysis:

Average running time per operation over a worst-case sequence of operations.

#### Proposition:

 Starting from an empty stack, any sequence of N push and pop operations takes time proportional to N.

|           | best | worst  | amortized |                                    |
|-----------|------|--------|-----------|------------------------------------|
| construct | O(1) | O(1)   | O(1)      | 300                                |
| push      | O(1) | O(n) ← | O(1)      |                                    |
| рор       | O(1) | O(n) ← | O(1)      | doubling and<br>halving operations |
| size      | O(1) | O(1)   | O(1)      |                                    |

order of growth of running time for resizing stack with N items

Q. How to grow and shrink array?

"successive doubling"

A. If array is full, create a new array of twice the size, and copy items.

```
stack newStack(int capacity = 1) {
  stack s = new Stack;
 s->item = new string[capacity];
 s->capacity = capacity;
 s \rightarrow N = 0;
 return s;
void resize(stack s, int new_capacity) {
  string *copied = new string[new_capacity];
  for (int i = 0; i < s->N; i++)
    copied[i] = s->item[i];
 delete[] s->item;
 s->item = copied;
  s->capacity = new capacity;
```

```
bool empty(stack s) { return s->N == 0; }
string top(stack s) {
  return s->item[s->N - 1];
void push(stack s, string item) {
  // your code here
  s \rightarrow item[s \rightarrow N++] = item;
void pop(stack s) {
  s->N--;
  // your code here
```

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## **Stack: Vector implementation**

# Let's implement our own stack in several different ways.

- Array implementation
  - fixed size array
  - dynamic array
- Vector implementation
  - A good tutorial in Korean at <a href="https://codechacha.com/ko/cpp-stl-vector/">https://codechacha.com/ko/cpp-stl-vector/</a>
- Using Template
  - Array implementation
  - Vector implementation

#### **Vector Container**

- An alternative to the built-in array.
- A vector is self-grown.
  - No allocation/free is necessary!
- Template implementation advantage!!
- For example:
   vector<int> vector of integers.
   vector<string> vector of strings.
   vector<int \*> vector of pointers to integers.
   vector<Shape> vector of Shape objects. Shape is a user defined class.

```
iterator begin();
 iterator end();
 bool empty();
void push_back(const T& x);
void pop_back();
value_type& back();
void clear();
size_type size();
size_type capacity();
```

# **Vector Container Example**

```
#include<iostream>
#include<vector>
using namespace std;
int main(){
  vector<int> v(5);
  for(int i=0; i < v.size(); i++)</pre>
    cin >> v[i];
  for(int i=0; i < v.size(); i++)</pre>
    cout << v[i] << ' ';
  cout << endl;</pre>
```

### **Vector Container Example**

```
#include<iostream>
#include<vector>
using namespace std;
int main(){
  vector<int> v(5);
  for(int i=0; i < v.size(); i++)</pre>
    cin >> v[i];
  for(int i=0; i < v.size(); i++)</pre>
    cout << v[i] << ' ';
  cout << endl;</pre>
```

```
for(int x: v)
  cout << x << ' ';
cout << endl;</pre>
for(auto x: v)
  cout << x << ' ';
cout << endl;</pre>
vector<int>::iterator it;
for(it = v.begin(); it!=v.end(); it++)
    cout << *it << ' ';
cout << endl;</pre>
```

```
iterator begin();
 iterator end();
bool empty();
void push_back(const T& x);
void pop_back();
value_type& back();
void clear();
size_type size();
size_type capacity();
```

```
int main() {
  int count = 0;
  vector<int> vec;
  vec.push_back(1);
  vec.push_back(2);
  vec.push_back(3);
  while (!vec.empty()) {
    count++;
    vec.pop back();
  cout << count;</pre>
  return 0;
```

```
iterator begin();
 iterator end();
bool empty();
void push back(const T& x);
void pop_back();
value_type& back();
void clear();
size_type size();
size_type capacity();
```

```
int main () {
  vector<int> vec;
  vec.push_back(10);
  while (vec.back() != 0) {
    vec.push_back ( vec.back() - 1 );
  cout << "vec contains: ";</pre>
  for (auto x: vec)
    cout << x << ' '; cout << endl;</pre>
  return 0;
```

```
iterator begin();
 iterator end();
bool empty();
void push back(const T& x);
void pop_back();
value_type& back();
void clear();
size_type size();
size_type capacity();
```

```
int main () {
  vector<int> vec;
  vec.push_back(10);
  while (vec.back() != 0) {
    vec.push_back ( vec.back() - 1 );
  cout << "vec contains: ";</pre>
  for (auto x: vec)
    cout << x << ' ';
  cout << endl;</pre>
  return 0;
```

vec contains: 10 9 8 7 6 5 4 3 2 1 0

```
iterator begin();
iterator end();
bool empty();
void push_back(const T& x);
void pop_back();
value_type& back();
void clear();
size_type size();
```

size\_type capacity();

```
int main () {
   vector<int> vec;
   for (int i=0; i<100; i++) vec.push_back(i);
   cout << "size: " << vec.size() << endl;
   cout << "capa: " << vec.capacity() << endl;
   return 0;
}</pre>
```

size: 100 capa: 128

### Stack: version.4 – using a vector in C++ STL

```
struct Stack {
 vector<string> item;
using stack = Stack *;
void free(stack s) {
 delete s;
int size(stack s) {
 return s->item.size();
bool empty(stack s) {
 return s->item.empty();
```

```
void pop(stack s) {
  // your code here
string top(stack s) {
 // your code here
void push(stack s, string item) {
  // your code here
void printStack(stack s) {
  while (!empty(s)) {
    cout << top(s) << ' ';
    pop(s);
  cout << endl; // stack is empty now</pre>
```

### Stack: Using template

- A template is a mechanism that allows a programmer to use types as
  parameters for a class or a function. The compiler then generates a specific
  class or function when we later provide specific types as arguments.
- A function/class defined using template is called a generic function/class.
   This is one of the key features of C++.
- Use templates when we need functions/classes that apply the same algorithm to a several types. So we can use the same function/class regardless of the types of the argument or result.
- The syntax is:
  - template <class T> function\_declaration;
    or
  - template <typename T> function\_declaration;

### Stack: Using template

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### Stack: Using template

#### Pros and Cons of Templates

#### Pros:

- It provides us type-safe, efficient generic containers and generic algorithms
- The main reason for using C++ and templates is the trade-offs in performance and maintainability outweigh the bigger size of the resulting code and longer compile times.
- The drawbacks of not using them are likely to be much greater.

#### Cons:

- Templates can lead to slower compile-times and possibly larger executable.
- Compilers often produce incomprehensible poor error diagnostics and poor error messages.

# Stack: version.4T – using a vector<> in C++ STL

```
struct Stack {
                       stack4_vec.cpp
 vector<string> item;
};
using stack = Stack *;
void free(stack s) {
 delete s;
string top(stack s) {
 return s->item.back();
```

Compare these two program segments and see how to use **Templates** in C++ for generic programming.

```
template<typename T>
struct Stack {
 vector<T> item;
template<typename T>
using stack = Stack<T> *;
template<typename T>
void free(stack<T> s) {
  delete s;
template<typename T>
T top(stack<T> s) {
  return s->item.back();
```

# Data Structures Chapter 3

#### 1. Stack

- Stack Concept
  - STL stack class
- Stack Implementations
  - Using Fixed Array
  - Using Dynamic Array
  - Using Vector
  - Using Template

#### 2. Queue