


Data Structures

Chapter 4

1. Singly Linked List
2. Doubly Linked List
 - **Revisit – Singly Linked List**
 - **Sentinel Nodes & Basic Operations**
 - Two Key Operations: erase, insert
 - Advanced Operations



**우리가 알거니와 하나님을 사랑하는 자 곧 그의 뜻대로 부르심을 입은 자들에게엔 모든 것이
합력하여 선을 이루느니라 (롬8:28)**

And we know that in all things God works for the good of those who love him, who have been called according to his purpose. (Rom8:28)

**하나님이 우리를 구원하사 거룩하신 소명으로 부르심은 우리의 행위대로 하심이 아니요 오직
자기의 뜻과 영원 전부터 그리스도 예수 안에서 우리에게 주신 은혜대로 하심이라 (딤후1:9)**

A Node and List Data Structure with Constructor and Destructor

```
struct Node {
    int    data;
    Node*  prev;
    Node*  next;
};

struct List {
    Node*  head;
    Node*  tail;
    int    size; //optional
};

using pNode = Node*;
using pList = List*;
```

← unused in singly linked list

```
struct Node {
    int    data;
    Node*  prev;
    Node*  next;
    Node(int d = 0, Node* p = nullptr,
           Node* x = nullptr)
        { data = d; prev = p; next = x; }
    ~Node() {}
};

struct List {
    Node*  head;
    Node*  tail;
    int    size; // optional
    List() { head = new Node{}; tail = new Node{};
            head->next = tail; tail->prev = head;
            size = 0;
        }
    ~List() {}
};

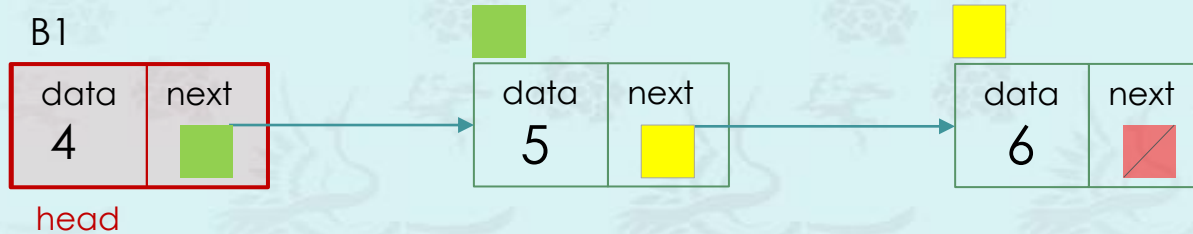
using pNode = Node*;
using pList = List*;
```

push a node – three different cases

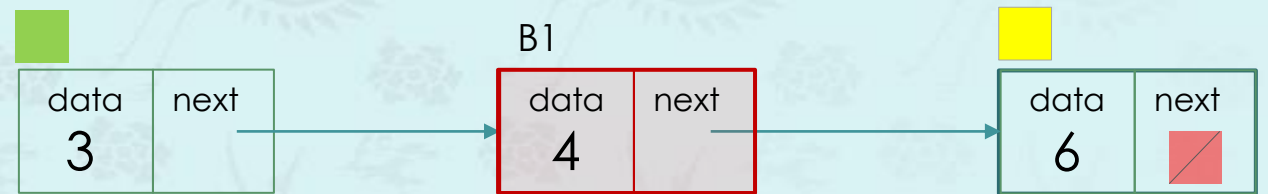
Given: an data(4) **to insert as sorted** – What was the most difficult part of this coding?



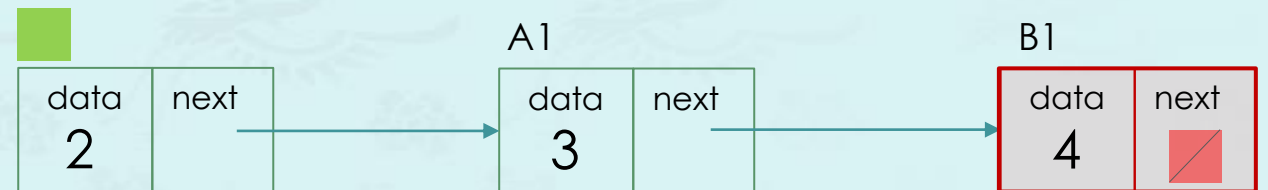
Case 1: in front



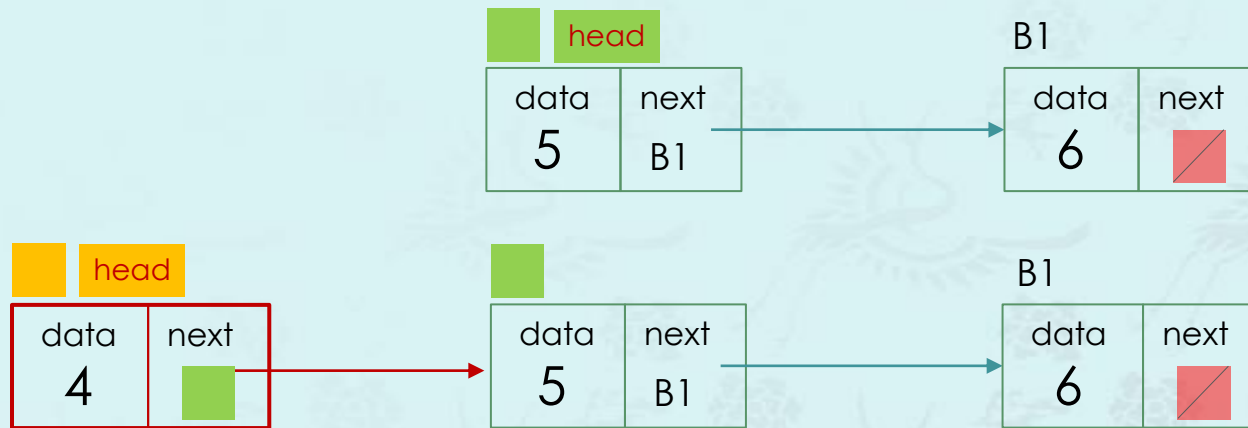
Case 2: in middle



Case 3: at end



push a node – **Case 1**: insert in front, head given



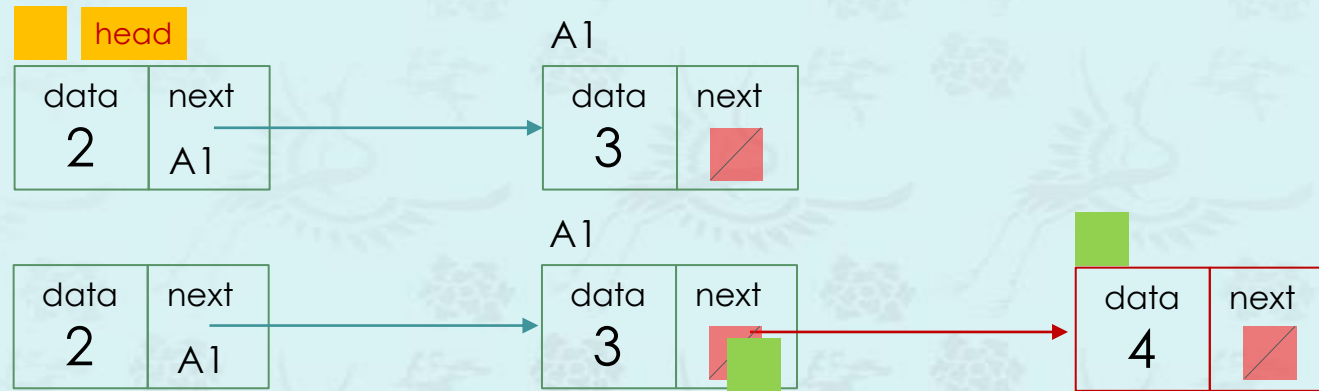
```
void push_front(pList p, int value) {  
    p->head = new Node{value, p->head};  
}
```

```
struct Node{  
    int data;  
    Node* next;  
    Node(const int d = 0, pNode x = nullptr){  
        data = d;        next = x;  
    }  
    ~Node() {}  
};
```

```
struct List {  
    Node* head;  
    Node* tail;  
    int size; //optional  
};  
using pList = List*;
```

push a node – **Case 3**; insert at end, head given

Append a new Node{4}; then find the last node first to append.

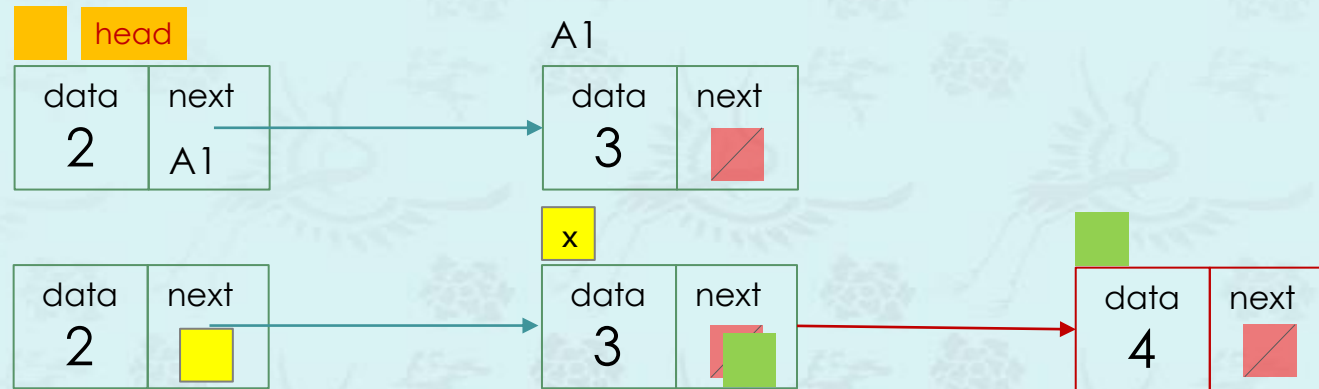


```
void push_back(pList p, int value) {  
      
}  

```


push a node – **Case 3**; insert at end, head given

Append a new Node{4}; then find the last node first to append.



$O(n)$

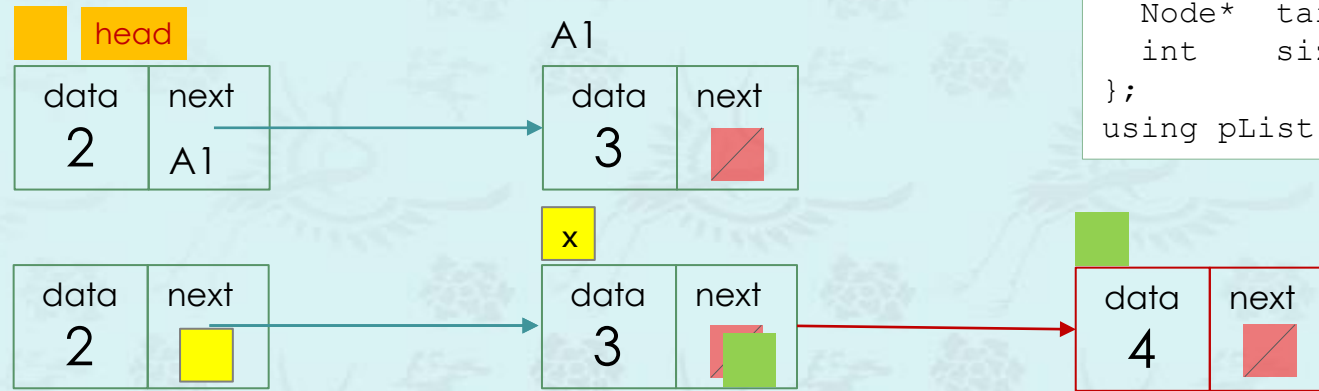
```
void push_back(pList p, int value) {  
    pNode x = last(p->head);  
    x->next = new Node{ value, nullptr };  
}
```

```
pNode last(pNode head) {  
    pNode x = head; ← need this or not?  
    while (x->next != nullptr)  
        x = x->next;  
    return x;  
}
```

```
while (head->next != nullptr)  
    head = head->next;  
return head; ← OK
```

push a node – **Case 3**; insert at end, head given

Append a new Node{4}; then find the last node first to append.

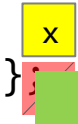


```
struct List {  
    Node* head;  
    Node* tail;  
    int size; //optional  
};  
using pList = List*;
```

$O(n)$



```
void push_back(pList p, int value) {  
    pNode x = last(p->head);  
    x->next = new Node{ value, nullptr };  
}
```



$O(1)$



```
void push_back(pList p, int value) {  
    p->tail->next = new Node{ val };  
    p->tail = p->tail->next;  
}
```

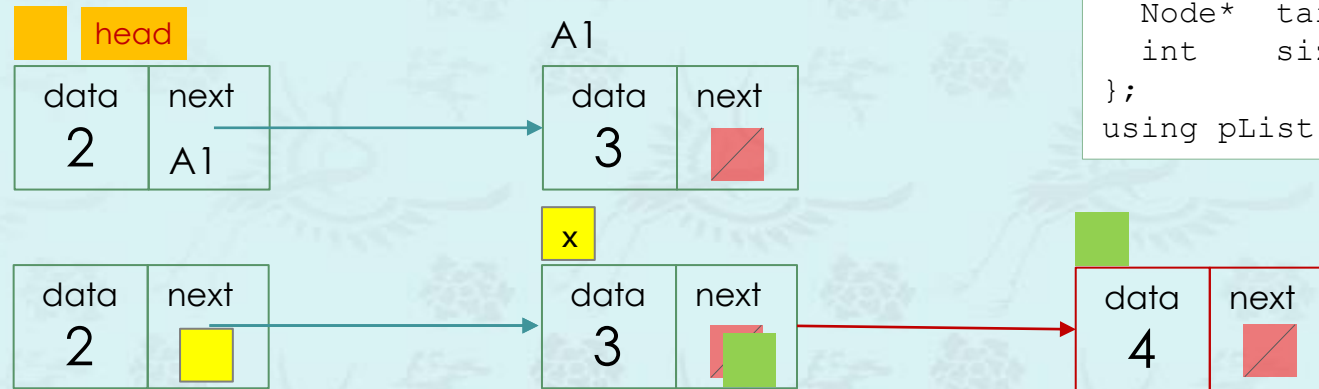
```
void push_back(pList p, int value) {  
    p->tail = p->tail->next = new Node{ val };  
}
```

```
pNode last(pNode head) {  
    while (head->next != nullptr)  
        head = head->next;  
    return head;  
}
```

it can be shortened?

push a node – **Case 3**; insert at end, head given

Append a new Node{4}; then find the last node first to append.

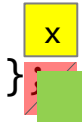


```
struct List {  
    Node* head;  
    Node* tail;  
    int size; //optional  
};  
using pList = List*;
```

$O(n)$



```
void push_back(pList p, int value) {  
    pNode x = last(p->head);  
    x->next = new Node{ value, nullptr };  
}
```



$O(1)$



```
void push_back(pList p, int value) {  
    p->tail->next = new Node{ val };  
    p->tail = p->tail->next;  
}
```

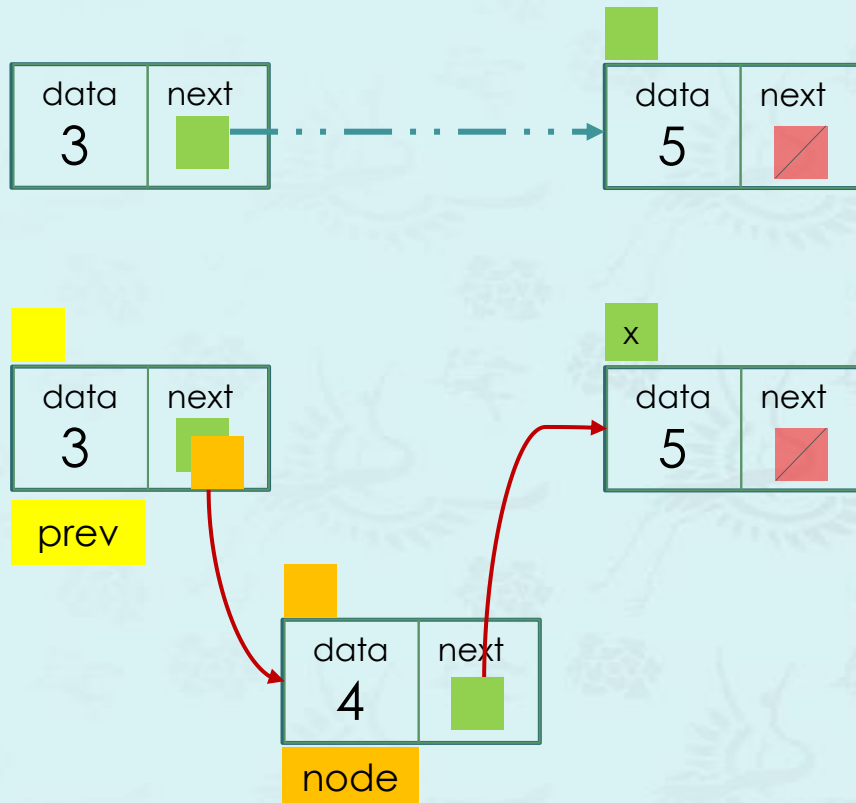
```
void push_back(pList p, int value) {  
    p->tail = p->tail->next = new Node{ val };  
}
```

```
pNode last(pNode head) {  
    while (head->next != nullptr)  
        head = head->next;  
    return head;  
}
```

it can be shortened, but not recommended

It is a matter of associativity!
Then, what is the associativity of '='?
right to left

push a node – **Case 2**; insert in middle, head given



```
// inserts a node value at node z
void push_at(pList p, int value, int z) {
    if (empty(p) || (p->head->data == z) )
        return push_front(p, value);
```

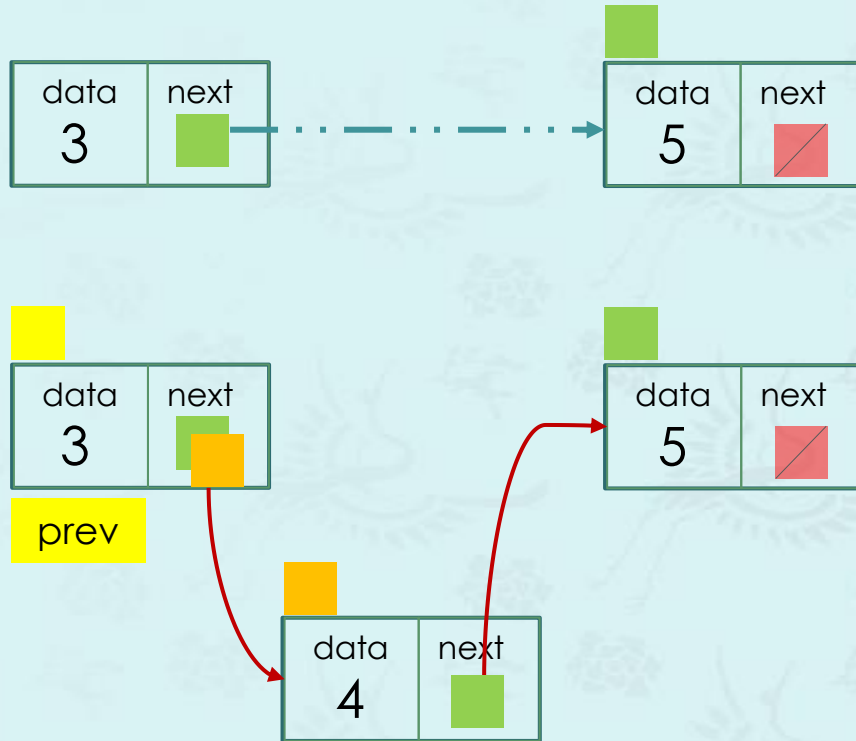
```
pNode x = p->head;
pNode prev = nullptr;
while (x != nullptr) {
```

```
    prev = x;
    x = x->next;
```

```
}
```

```
}
```

push a node – **Case 2**; insert in middle, head given



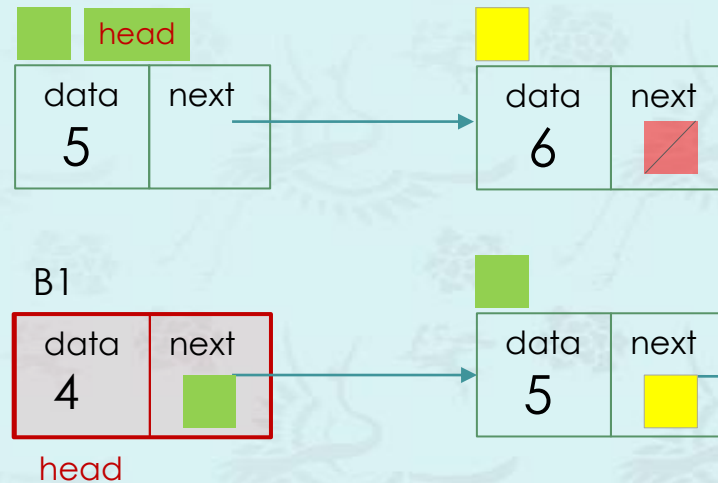
```
// inserts a node value at node z
void push_at(pList p, int value, int z) {
    if (empty(p) || (p->head->data == z) )
        return push_front(p, value);

    pNode x = p->head;
    pNode prev = nullptr;
    while (x != nullptr) {
        if (x->data == z) {
            prev->next = new Node{value, prev->next};
            return;
        }
        prev = x;
        x = x->next;
    }
}
```

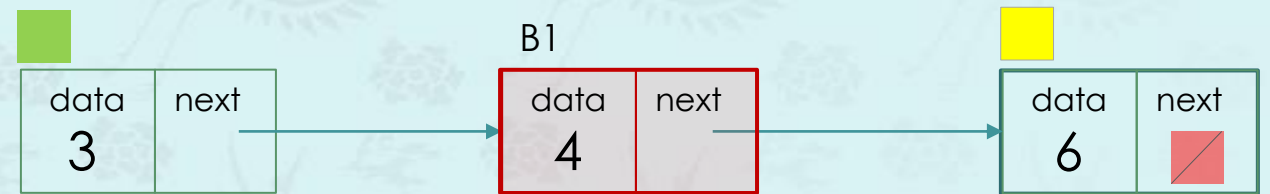
push a node – Three different cases

Given: an data(4) to insert – **What was the most difficult part of this coding?**

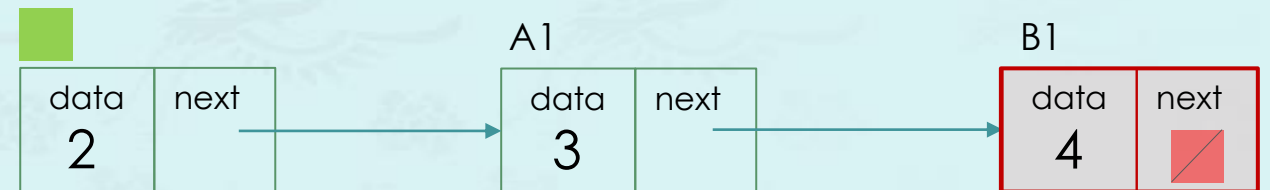
Case 1: in front



Case 2: in middle

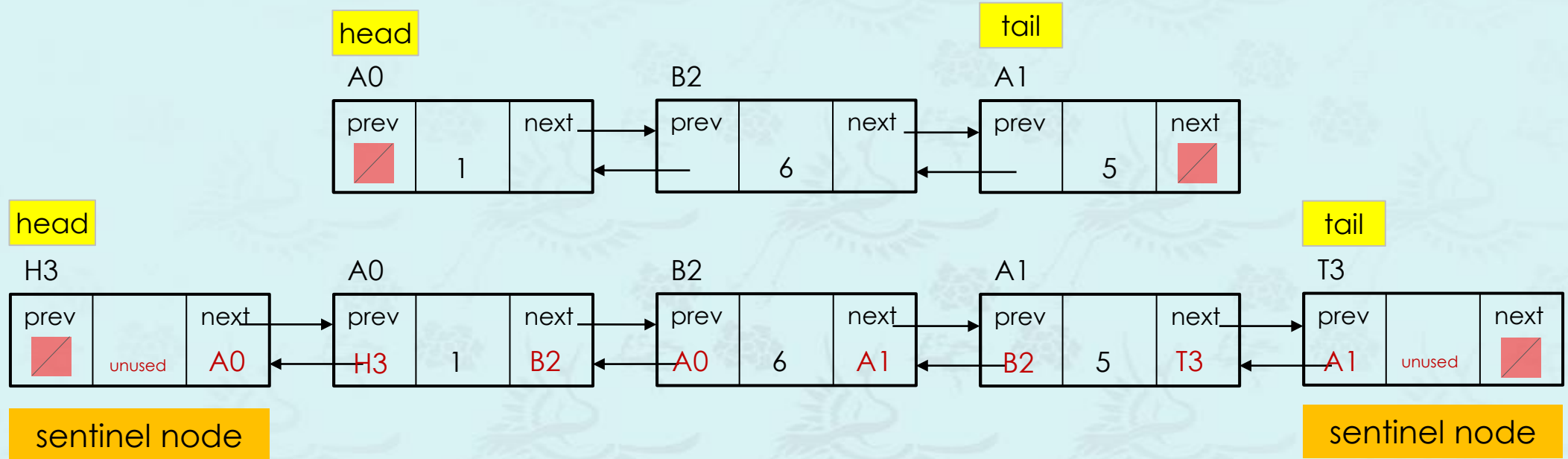


Case 3: at end



- Coding case by case
 - front, back, or middle
 - not generalized
- Lack of information about
 - the previous node
 - the tail/last node
- **Solution?**
 - have previous node information
 - have sentinel nodes

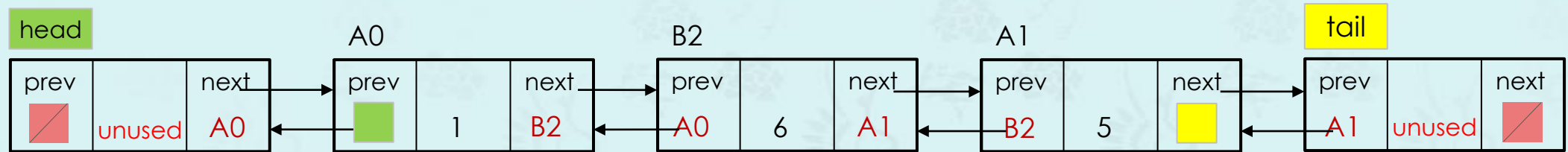
doubly linked list with sentinel nodes



- **Solution**

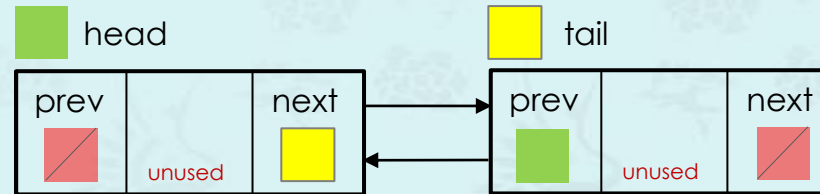
- **doubly linked list with two sentinel nodes**
- Each node carries the pointer to the previous node.
- There is only one case (middle) with two sentinel nodes.

doubly linked list with sentinel nodes



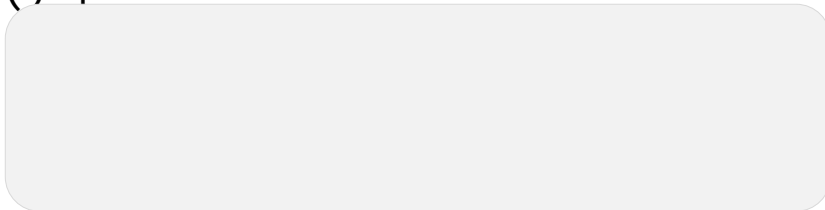
- These extra nodes are known as **sentinel nodes**. The node at the front is known as **head node**, and the node at the end as a **tail node**. The head and tail nodes are created when the doubly linked list is initialized. The purpose of these nodes is to simply the insert, push/pop front and back, remove methods by eliminating all need for special-case code when the list empty, or when we insert at the head or tail of the list. **This would greatly simplify the coding unbelievably.**
- For instance, if we do not use a head node, then removing the first node becomes a special case, because we must reset the list's link to **the first node** during the remove and because the remove algorithm in general needs to access the node prior to the node being removed (and without a head node, the first node does not have a node prior to it).


doubly linked list with sentinel nodes



An **empty** doubly linked list with sentinel nodes

Initializing doubly linked list with sentinel nodes

```
struct Node {  
    int    data;  
    Node*  prev;  
    Node*  next;  
};  
  
struct List {  
    Node*  head; //sentinel  
    Node*  tail; //sentinel  
    int    size; //size of list, optional  
    List() {  
          
    }  
    ~List() {}  
};  
using pNode = Node*;  
using pList = List*;
```

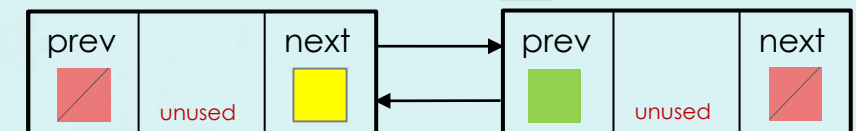
 head

 tail

An **empty** doubly-linked list with sentinel nodes

 head

 tail

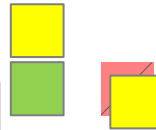


Initializing doubly linked list with sentinel nodes

```
struct Node {
    int    data;
    Node*  prev;
    Node*  next;
};

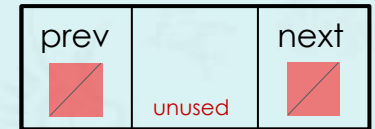
struct List {
    Node*  head; //sentinel
    Node*  tail; //sentinel
    int    size; //size of list, optional
    List() {
        tail = new Node{};
    }
    ~List() {}
};

using pNode = Node*;
using pList = List*;
```



head

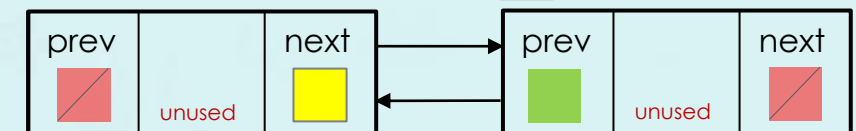
tail



An **empty** doubly-linked list with sentinel nodes

head

tail

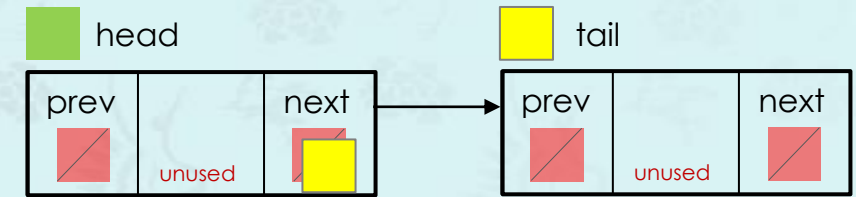


Initializing doubly linked list with sentinel nodes

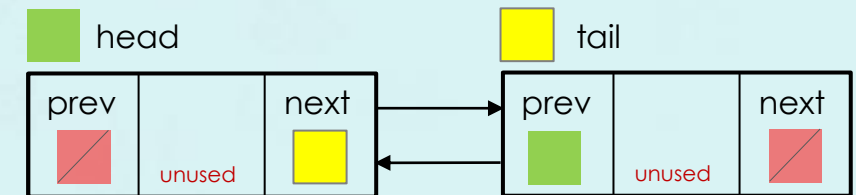
```
struct Node {
    int    data;
    Node*  prev;
    Node*  next;
};

struct List {
    Node*  head; //sentinel
    Node*  tail; //sentinel
    int    size; //size of list, optional
    List() {
        tail = new Node{};
        head = new Node{0, nullptr, tail};
    }
    ~List() {}
};

using pNode = Node*;
using pList = List*;
```



An **empty** doubly-linked list with sentinel nodes

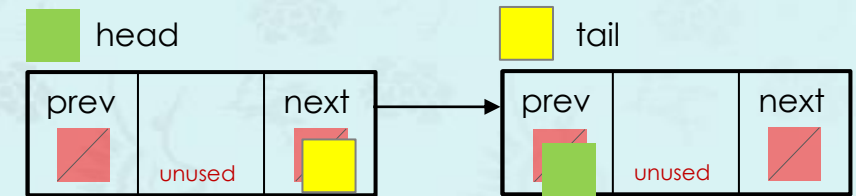


Initializing doubly linked list with sentinel nodes

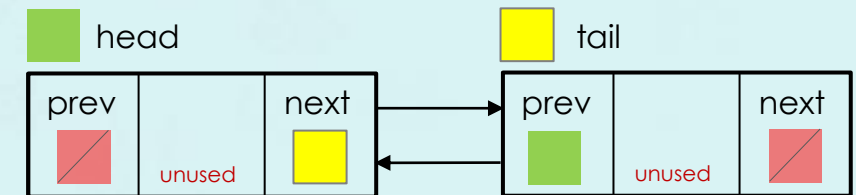
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        tail = new Node{};
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    }
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An **empty** doubly-linked list with sentinel nodes

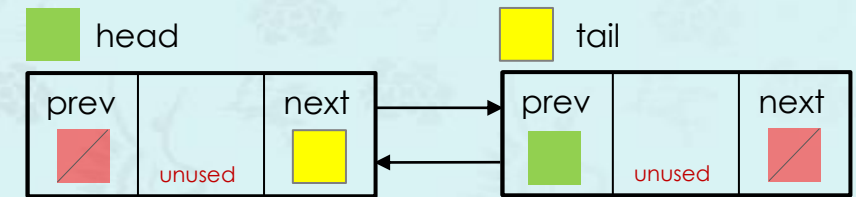
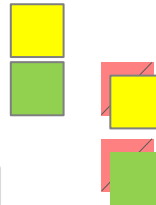


Initializing doubly linked list with sentinel nodes

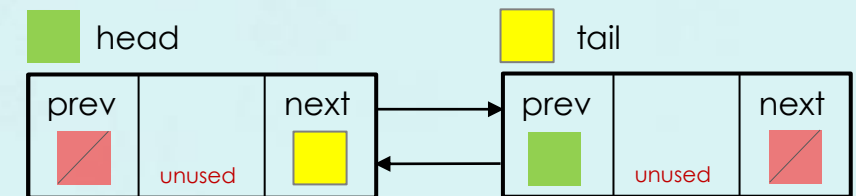
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    Node*  prev;
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};

struct List {
    Node*  head; //sentinel
    Node*  tail; //sentinel
    int    size; //size of list, optional
    List() {
        tail = new Node{};
        head = new Node{0, nullptr, tail};
        tail->prev = head;
    }
    ~List() {}
};

using pNode = Node*;
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```



An **empty** doubly-linked list with sentinel nodes

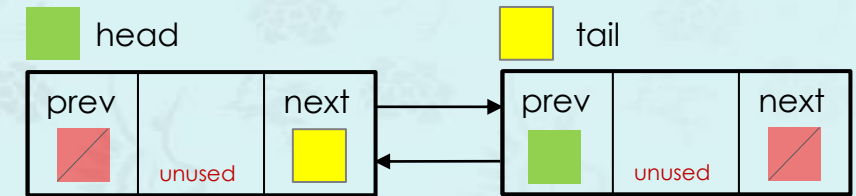
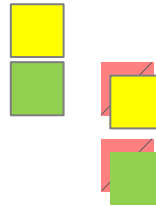


Initializing doubly linked list with sentinel nodes

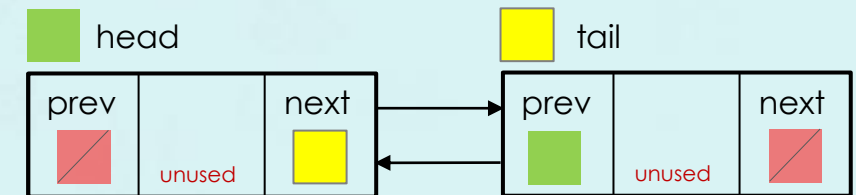
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};

struct List {
    Node*  head; //sentinel
    Node*  tail; //sentinel
    int    size; //size of list, optional
    List() {
        tail = new Node{};
        head = new Node{0, nullptr, tail};
        tail->prev = head;
        size = 0;
    }
    ~List() {}
};

using pNode = Node*;
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```



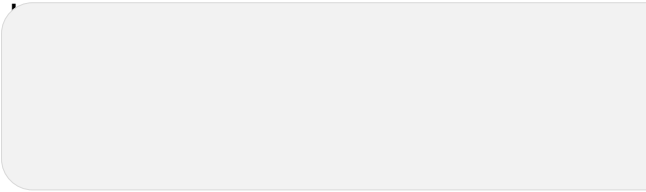
An **empty** doubly-linked list with sentinel nodes



Initializing doubly linked list with sentinel nodes

```
struct Node {  
    int    data;  
    Node*  prev;  
    Node*  next;  
    Node(const int d = 0, Node* p = nullptr, Node* x = nullptr) {  
        data = d; prev = p; next = x;  
    }  
    ~Node() {}  
};
```

struct List { another way of doubly linked list initialization

```
    Node* head;  
    Node* tail;  
    int    size; //size of list, optional  
    List() {  
          
    }  
    ~List() {}  
};
```

```
using pNode = Node*;  
using pList = List*;
```



An **empty** doubly-linked list with sentinel nodes

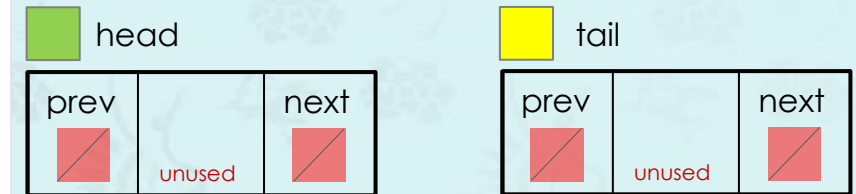
Initializing doubly linked list with sentinel nodes

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struct Node {  
    int    data;  
    Node*  prev;  
    Node*  next;  
    Node(const int d = 0, Node* p = nullptr, Node* x = nullptr) {  
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    }  
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};
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struct List { another way of doubly linked list initialization

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    Node* head;  
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    int    size; //size of list, optional  
    List() {  
  
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    ~List() {}  
};
```

```
using pNode = Node*;  
using pList = List*;
```



An **empty** doubly-linked list with sentinel nodes

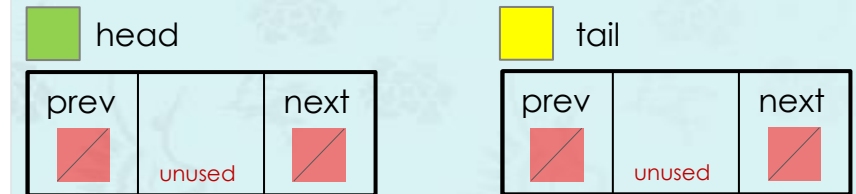
Initializing doubly linked list with sentinel nodes

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    Node* head;  
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    List() { head = new Node{};    tail = new Node{};  
  
    }  
    ~List() {}  
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```

```
using pNode = Node*;  
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```



An **empty** doubly-linked list with sentinel nodes

Initializing doubly linked list with sentinel nodes

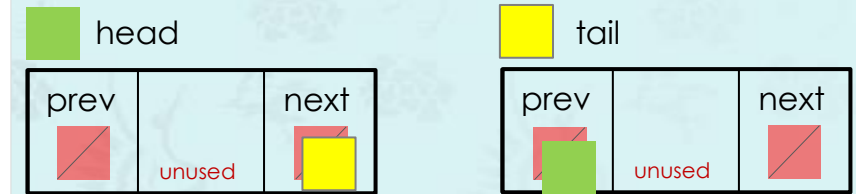
```
struct Node {  
    int    data;  
    Node*  prev;  
    Node*  next;  
    Node(const int d = 0, Node* p = nullptr, Node* x = nullptr) {  
        data = d; prev = p; next = x;  
    }  
    ~Node() {}  
};
```

struct List { another way of doubly linked list initialization

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    Node* head;  
    Node* tail;  
    int    size; //size of list, optional  
    List() { head = new Node{};    tail = new Node{};
```

```
    }  
    ~List() {}  
};
```

```
using pNode = Node*;  
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```



An **empty** doubly-linked list with sentinel nodes

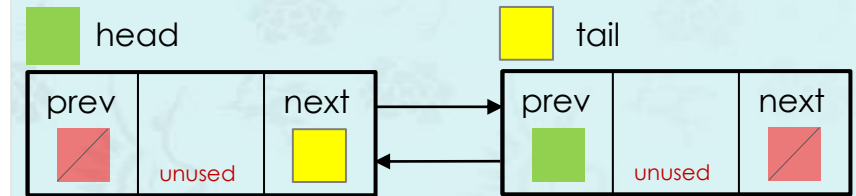
Initializing doubly linked list with sentinel nodes

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    Node* head;  
    Node* tail;  
    int    size; //size of list, optional  
    List() { head = new Node{};    tail = new Node{};  
            head->next = tail;    tail->prev = head;  
    }  
    ~List() {}  
};
```

```
using pNode = Node*;  
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An **empty** doubly-linked list with sentinel nodes

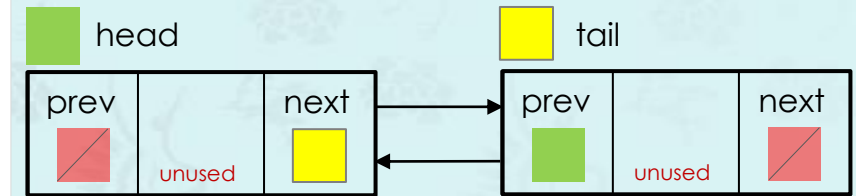
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struct List { another way of doubly linked list initialization

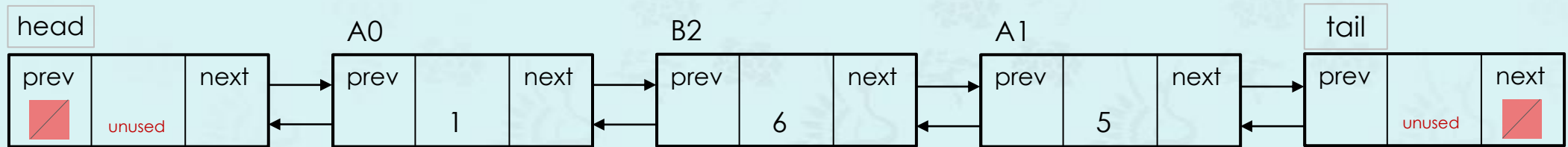
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    Node* tail;  
    int    size; //size of list, optional  
    List() { head = new Node{};    tail = new Node{};  
            head->next = tail;    tail->prev = head;  
            size = 0;  
    }  
    ~List() {}  
};
```

```
using pNode = Node*;  
using pList = List*;
```



An **empty** doubly-linked list with sentinel nodes

Basic Operations:



```
pNode begin(pList p) {  
    return p->head->next;  
}
```

← What is the name of this function?

```
pNode end(pList p) {  
    return p->tail;  
}
```

← What is the name of this function?

Basic Operations: begin() and end()

```
// returns the first node which list::head points to in the container.  
pNode begin(pList p) {  
    return p->head->next;  
}
```

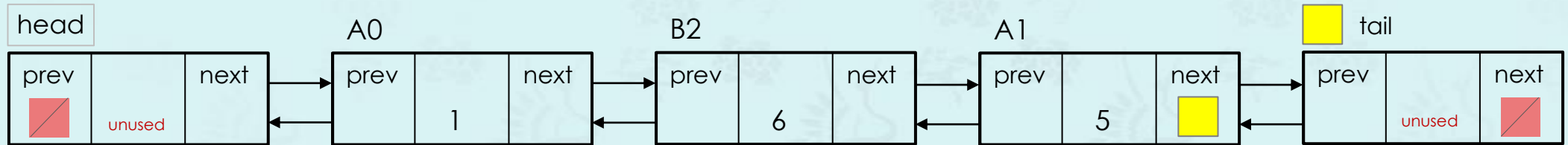
```
// returns the tail node referring to the past -the last- node in the list.  
// The past -the last- node is the sentinel node which is used only as a sentinel that would  
// follow the last node. It does not point to any node next, and thus shall not be dereferenced.  
// Because the way we are going use during the iteration, we don't want to include the node  
// pointed by this. This function is often used in combination with List::begin to specify a  
// range including all the nodes in the list. This is a kind of simulated used in STL. If the  
// container is empty, this function returns the same as List::begin.
```

```
pNode end(pList p) {  
    return p->tail;  
}
```

With the container given below,
what does begin(p) and end(p) return
in mnemonic code, respectively?



Basic Operations:



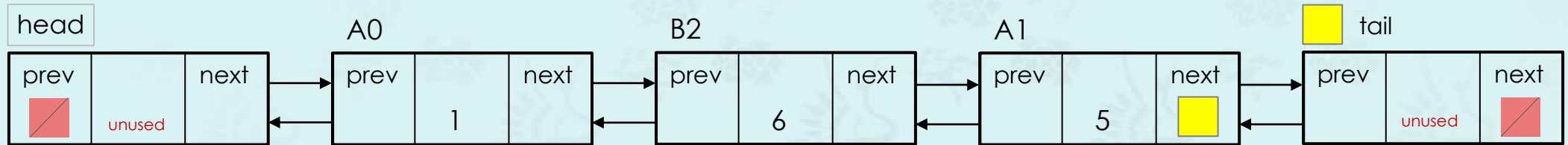
```
pNode begin(pList p) {  
    return p->head->next;  
}
```

← What is the return value of begin() in mnemonic shown above?

```
pNode end(pList p) {  
    return p->tail;  
}
```

← What is the return value of begin() in mnemonic shown above?

Basic Operations:

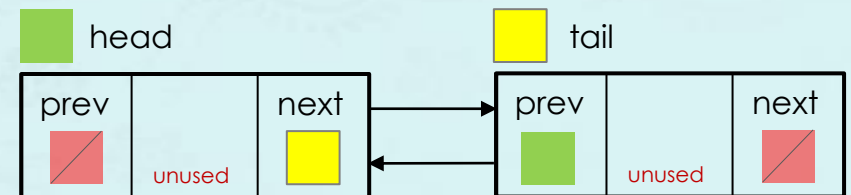


```
pNode begin(pList p) {  
    return p->head->next;  
}
```

```
bool empty(pList p) {  
    return   
}
```

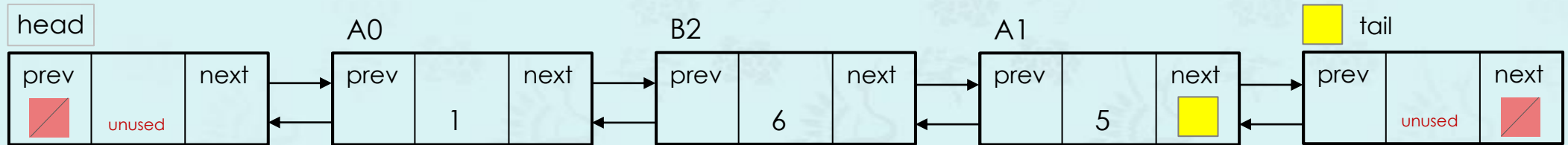
```
pNode end(pList p) {  
    return p->tail;  
}
```

```
pNode last(pList p) {  
    return   
}
```



An **empty** doubly-linked list with sentinel nodes

Basic Operations:

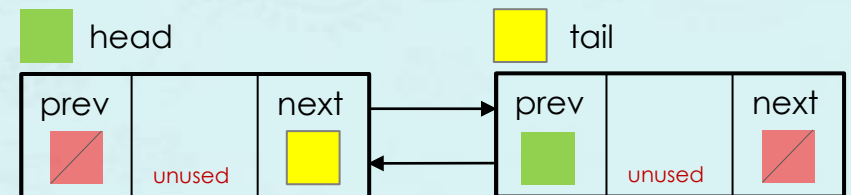


```
pNode begin(pList p) {  
    return p->head->next;  
}
```

```
bool empty(pList p) {  
    return begin(p) == end(p);  
}
```

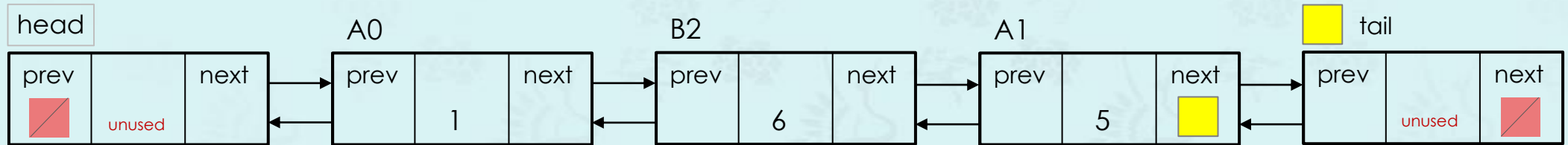
```
pNode end(pList p) {  
    return p->tail;  
}
```

```
pNode last(pList p) {  
    return end(p)->prev;  
}
```



An **empty** doubly-linked list with sentinel nodes

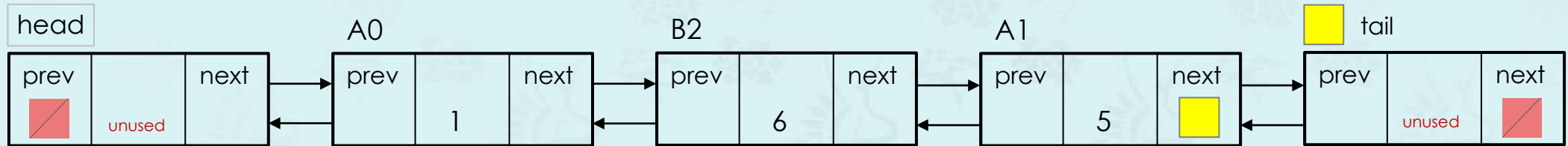
Basic Operations:



```
int size(pList p) {  
    int count = 0;  
      
    return count;  
}
```

```
int size(pList p) {  
    int count = 0;  
    pNode x = begin(p);  
    while(x != end(p)) {  
        count++;  
        x = x->next;  
    }  
    return count;  
}
```

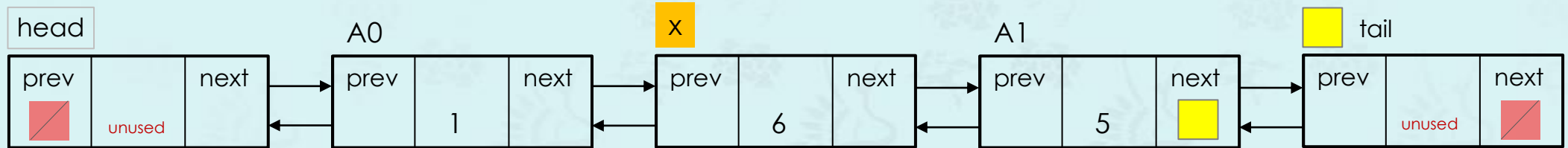
Basic Operations:



```
int size(pList p) {  
    int count = 0;  
    for (pNode x = begin(p); x != end(p); x = x->next)  
        count++;  
    return count;  
}
```

```
int size(pList p) {  
    int count = 0;  
    pNode x = begin(p);  
    while(x != end(p)) {  
        count++;  
        x = x->next;  
    }  
    return count;  
}
```

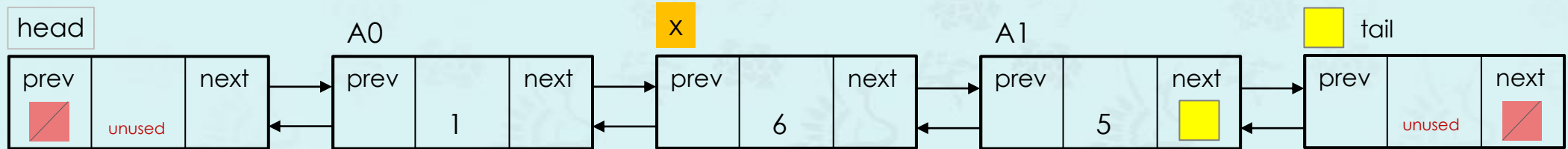
Exercise 1: What does it return if not found?



- What does it return if not found? →
(1) A1, (2) tail, (3) nullptr.

```
pNode find(pList p, int value){  
    pNode x = begin(p);  
    while(x != end(p)) {  
        if (x->data == value) return x;  
        x = x->next;  
    }  
    return x;  
}
```

Exercise 2: Rewrite find() using a for-loop



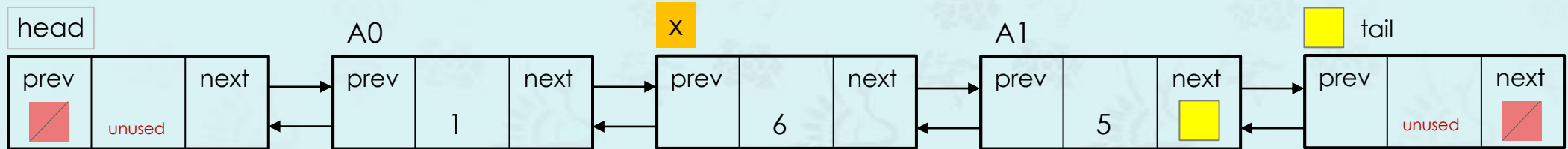
```
int find(pList p, int value) {  
    for (pNode x = begin(p); x != end(p); x = x->next)  
        if (x->data == value) return x;  
    return x;  
} // there is a bug.
```



```
int find(pList p, int value) {  
  
    return x;  
}
```

```
pNode find(pList p, int value){  
    pNode x = begin(p);  
    while(x != end(p)) {  
        if (x->data == value) return x;  
        x = x->next;  
    }  
    return x;  
}
```

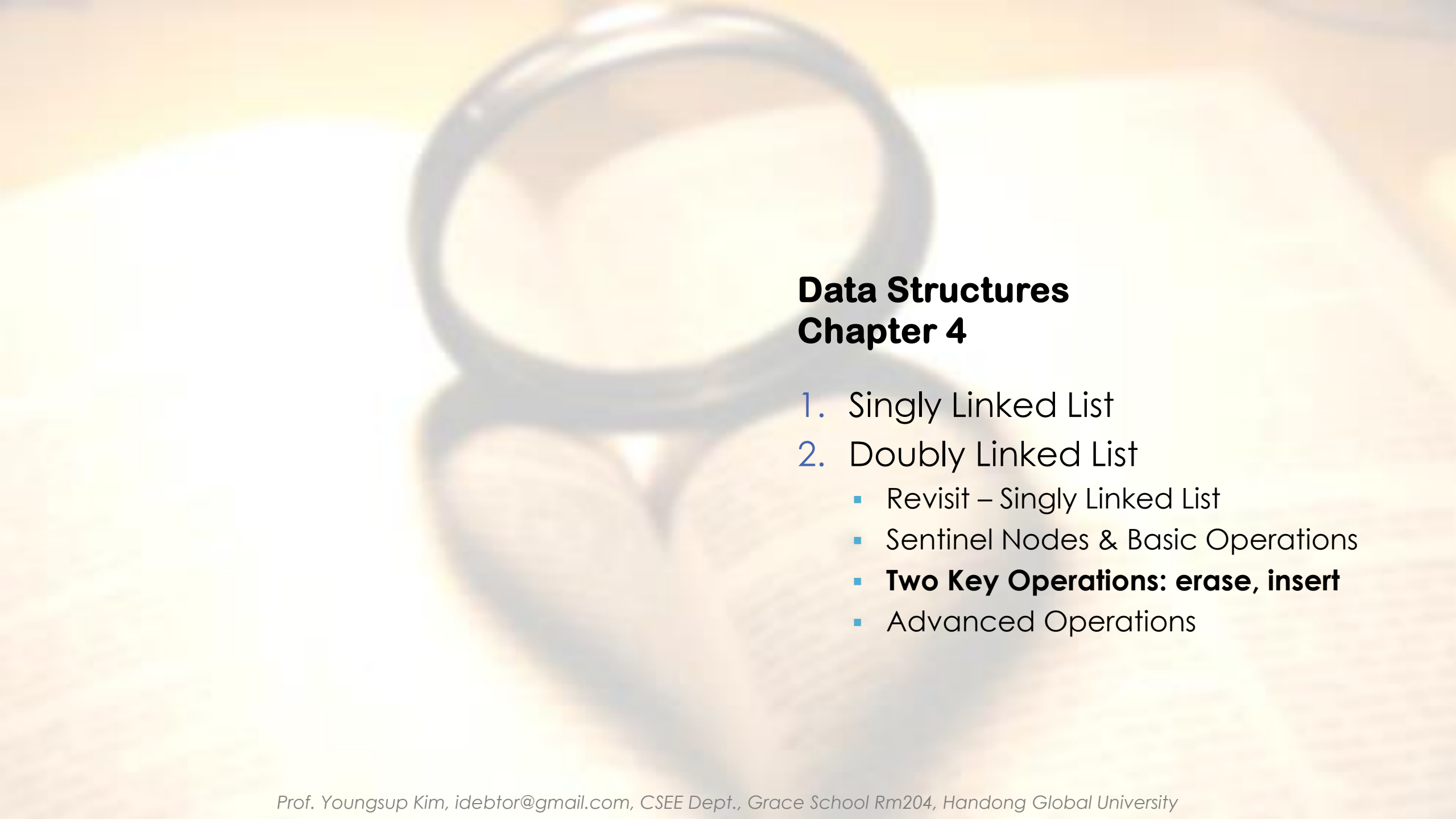

Exercise 3: Reduce the while-loop in two lines



```
pNode find(pList p, int value){  
    pNode x = begin(p);  
  
    return x;  
}
```

```
pNode find(pList p, int value){  
    pNode x = begin(p);  
    while(x != end(p)) {  
        if (x->data == value) return x;  
        x = x->next;  
    }  
    return x;  
}
```

- Can we reduce the number of lines by two? →



Data Structures

Chapter 4

1. Singly Linked List
2. Doubly Linked List
 - Revisit – Singly Linked List
 - Sentinel Nodes & Basic Operations
 - **Two Key Operations: erase, insert**
 - Advanced Operations