Data Structures
Chapter 5 Tree

1. Introduction
2. Binary Tree
3. **Binary Search Tree**
    - Introduction
    - Operations
    - **Demo & Coding**
4. Balancing Tree

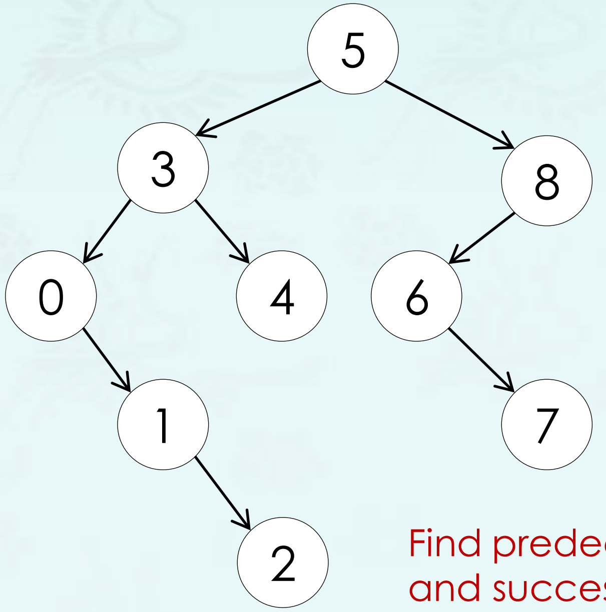
Prof. Youngsup Kim, idebtor@gmail.com, CSEE Dept., Grace School Rm204, Handong Global University

Minimum, Maximum:

- Minimum() and maximum() returns the node with min or max key.
  - Note that the entire tree does not need to be searched.
  - The minimum key is always located at the left most node, the maximum at the right most node.
  - Complexity of algorithm to find the maximum or minimum will be O(log N) in almost balanced binary tree. If tree is skewed, then we have worst case complexity of O(N).

```
tree minimum(tree node) {  // returns left-most node key

}
```

```
tree maximum(tree node) {  // returns right-most node key

}
```



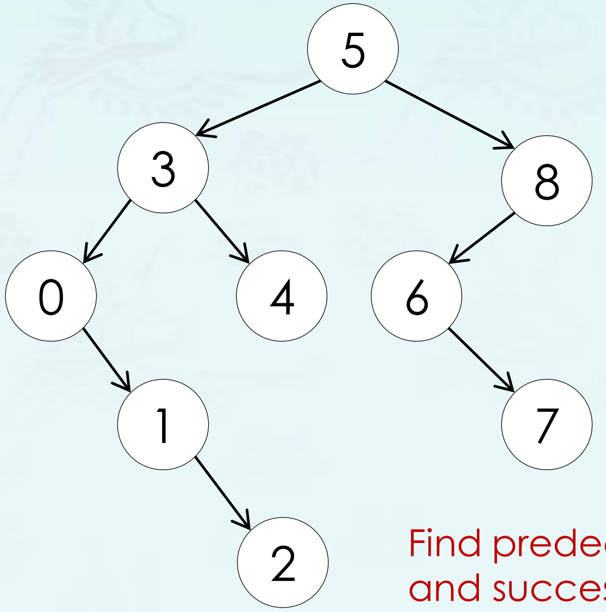
Find predecessors and successors of 5, 3, 8, respectively.

Minimum, Maximum:

- Minimum() and maximum() returns the node with min or max key.
  - Note that the entire tree does not need to be searched.
  - The minimum key is always located at the left most node, the maximum at the right most node.
  - Complexity of algorithm to find the maximum or minimum will be O(log N) in almost balanced binary tree. If tree is skewed, then we have worst case complexity of O(N).

```
tree minimum(tree node) {  // returns left-most node key
  if (node->left == nullptr) return node;
  return minimum(node->left);
}
```

```
tree maximum(tree node) {  // returns right-most node key


}
```

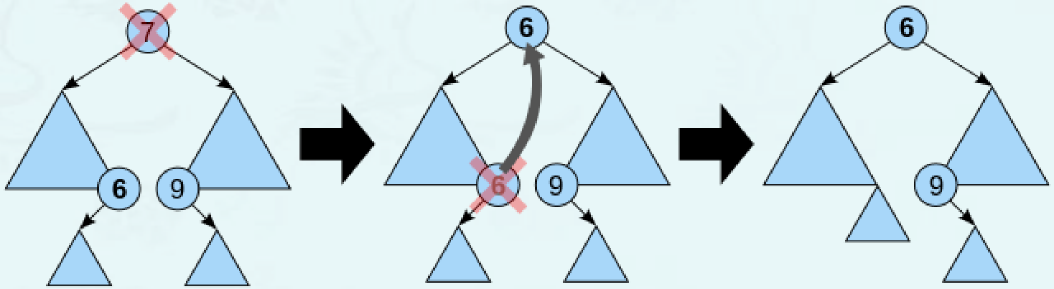


Find predecessors and successors of 5, 3, 8, respectively.

pred(), succ() – predecessor, successor:

- Successor
  - If the given node has a right subtree then by the BST property the next larger key must be in the right subtree. Since all keys in a right subtree are larger than the key of the given node, the successor must be the smallest of all those keys in the right subtree.
- Predecessor
  - If the given node has a left subtree then by the BST property the next smaller key must be in the left subtree. Since all keys in a left subtree are smaller than the key of the given node, the successor must be the largest of all those keys in the left subtree.
- Complexity of algorithm
  - O(log N) in almost balanced binary tree. If tree is skewed, then we have worst case complexity of O(N).

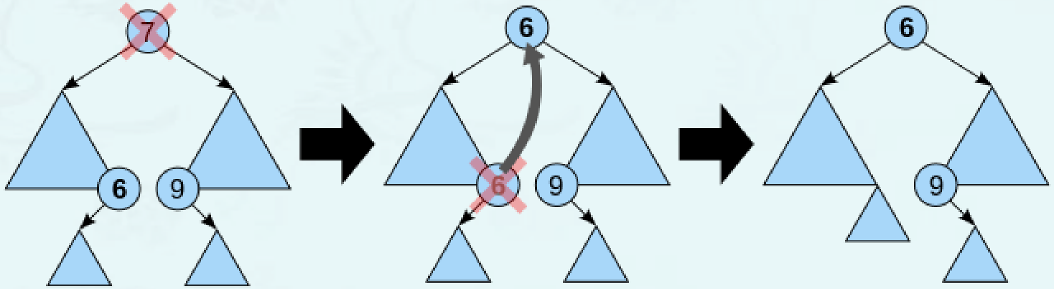
```
tree successor(tree node) {
  if (node != nullptr && node->right != nullptr)

  return nullptr;
}
```



pred(), succ() – predecessor, successor:

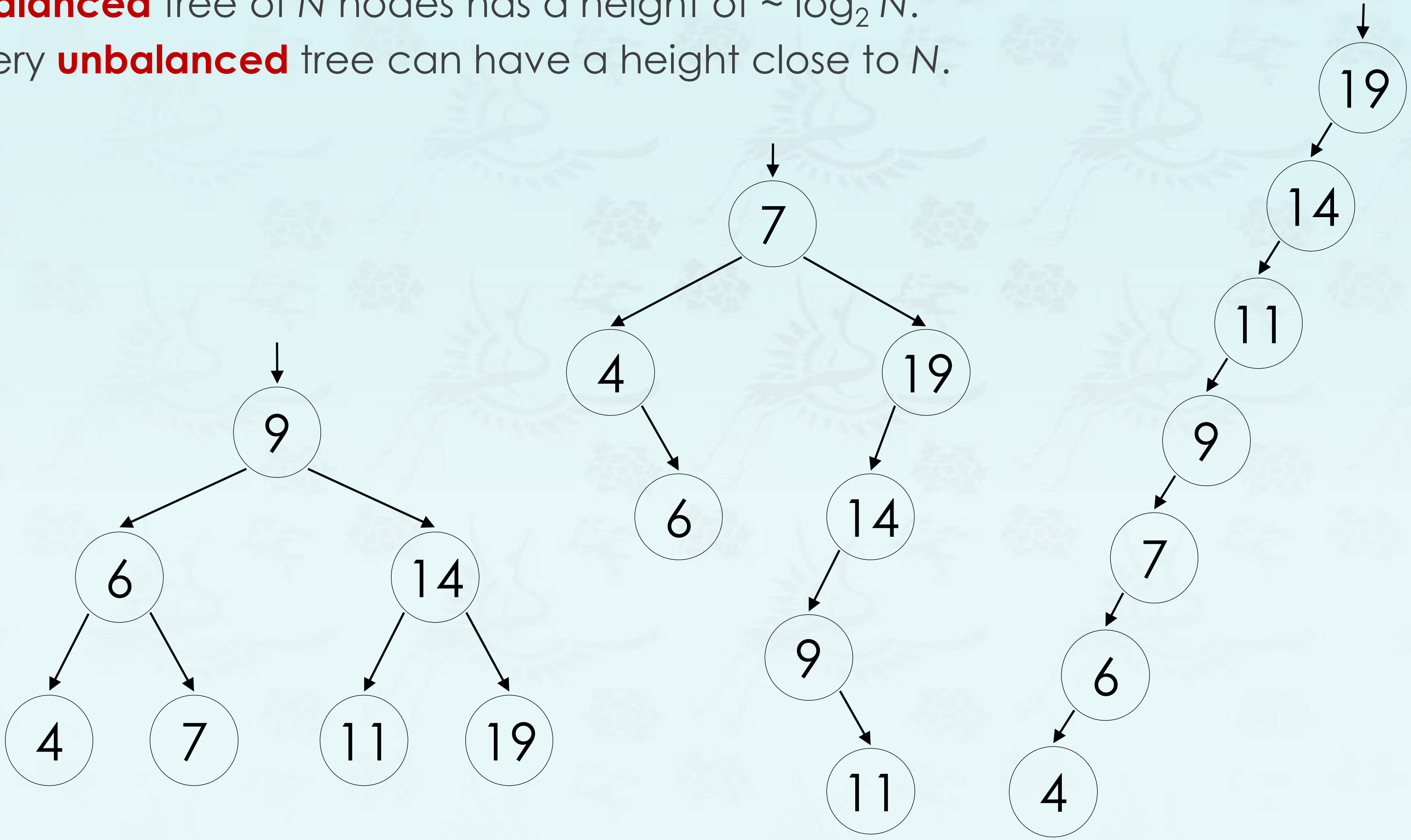
- Successor
  - If the given node has a right subtree then by the BST property the next larger key must be in the right subtree. Since all keys in a right subtree are larger than the key of the given node, the successor must be the smallest of all those keys in the right subtree.
- Predecessor
  - If the given node has a left subtree then by the BST property the next smaller key must be in the left subtree. Since all keys in a left subtree are smaller than the key of the given node, the successor must be the largest of all those keys in the left subtree.
- Complexity of algorithm
  - O(log N) in almost balanced binary tree. If tree is skewed, then we have worst case complexity of O(N).

```
tree successor(tree node) {
  if (node != nullptr && node->right != nullptr)
    return minimum(node->right);
  return nullptr;
}
```



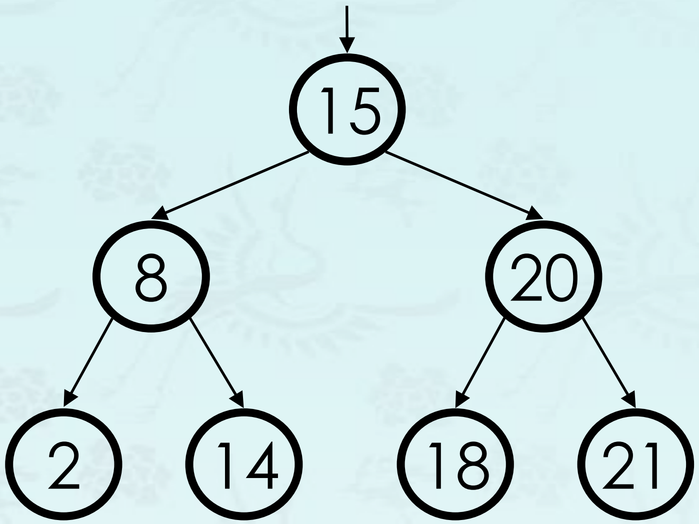
Binary Search Trees: Observations

- What do you see in the following BSTs?
  - A **balanced** tree of *N* nodes has a height of ~ $\log_2 N$.
  - A very **unbalanced** tree can have a height close to *N*.



Binary Search Trees: Observations

- For binary tree of height h:
  - max # of leaves: 2^h
  - max # of nodes: $2^{h+1} - 1$
  - min # of leaves: 1
  - min # of nodes: $h + 1$
- The shallower the BST the better.
  - Average case height is O(log *N*)
  - Worst case height is O(*N*)
  - Simple cases such as adding (1, 2, 3, ..., *N*), or the opposite order, lead to the worst case scenario: height O(*N*).



Binary Search Trees: Observations

- Q: If you have a sorted sequence, and we want to design a data structure for it. Which one are you going to use an array or BST? and why?

Time Complexity	
BST	$O(h)$
Array	$O(\log n)$

- Q: When searching, we're traversing a path (since we're always moving to one of the children); since the length of the longest path is the height h of the binary search tree, then finding an element takes $O(h)$.
  - Since $h = \log n$ (where n is the number of elements), then it's good! – right?
  - No, of course, it is wrong! Why?

  A: The nodes could be arranged in linear sequence in BST, so the $height$ h could be n. In worst case, it is $O(n)$ instead of $O(h)$.

Operations: growN() & trimN() for testing

- It performs a user specified number of insertion(or grow) or deletion(or trim) of nodes in the tree.
- The function **`growN()`** inserts a user specified number N of nodes in the tree.
  - If it is an empty tree, the value of keys to add ranges from 0 to N-1.
  - If there are some existing nodes in the tree, the value of keys to add ranges from max + 1 to max + 1 + N, where max is the maximum value of keys in the tree.
- This function growN() is provided for your reference^^.

Operations: growN() & trimN() for testing

- It performs a user specified number of insertion(or grow) or deletion(or trim) of nodes in the tree.
- The function **`growN()`** inserts a user specified number N of nodes in the tree.
  - If it is an empty tree, the value of keys to add ranges from 0 to N-1.
  - If there are some existing nodes in the tree, the value of keys to add ranges from max + 1 to max + 1 + N, where max is the maximum value of keys in the tree.
- This function growN() is provided for your reference^^.

- If the function is called with AVLtree = true, nodes are added using BST grow() function first. Then reconstruct the BST tree into an AVL tree using reconstruct() function which is much faster.

Operations: growN() & trimN() for testing

- The function **`trimN()`** deletes N number of nodes in the tree.
  - The nodes to trim are **randomly** selected from the tree.
  - If N is less than the tree size (which is not N), you just trim N nodes.
  - If the N is larger than the tree size, set it to the tree size.
  - At any case, you should trim all nodes one by one, but randomly.
  - With an AVL tree, reconstruct it **after** trimming N nodes from BST.

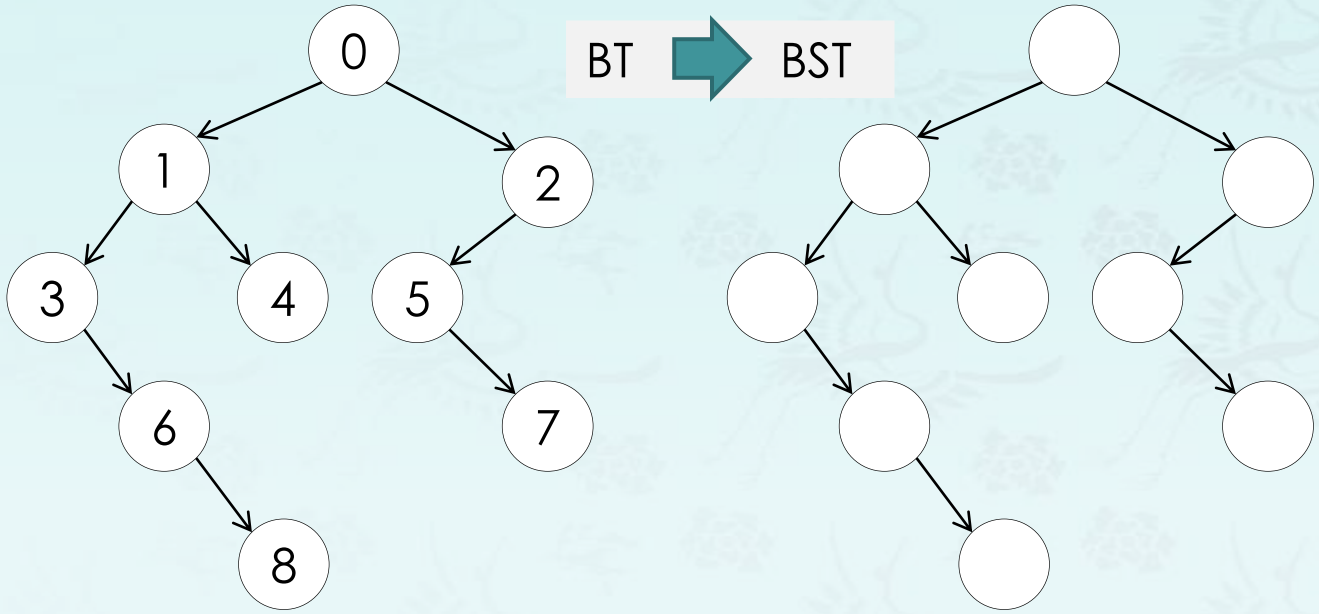
Operations: growN() & trimN() for testing

- The function **`trimN()`** deletes N number of nodes in the tree.
  - The nodes to trim are **randomly** selected from the tree.
  - If N is less than the tree size (which is not N), you just trim N nodes.
  - If the N is larger than the tree size, set it to the tree size.
  - At any case, you should trim all nodes one by one, but randomly.
  - With an AVL tree, reconstruct it **after** trimming N nodes from BST.

Step 1: Get a list (vector) of all keys from the tree first.
Get the size of the tree using the size().
Use assert to check two sizes;
Step 2: Shuffle the vector with keys. – shuffle()
Step 3: Invoke trim() N times with a key from the vector in sequence.
Inside a for loop, trim() may return a new root of the tree.
Step 4: The function is called with AVLtree = true, then reconstruct the tree.

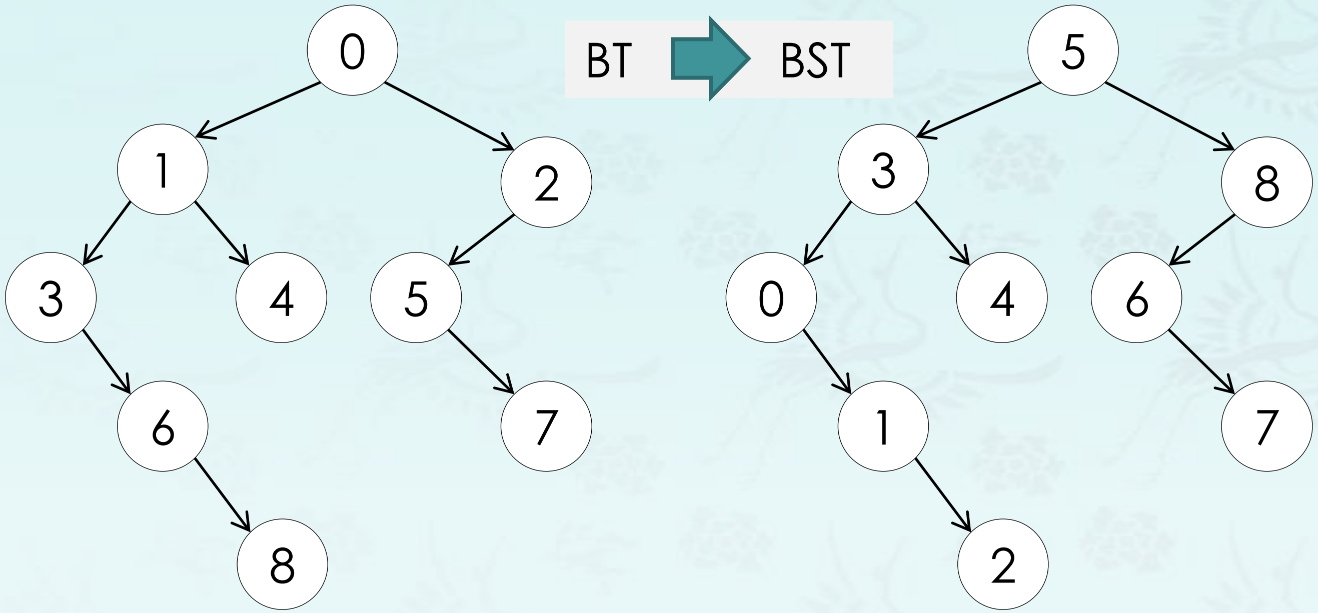
Convert BT to BST in-place

- Convert a binary tree to a binary search tree while keeping its tree structure as it is.
- For example:



Convert BT to BST in-place

- Convert a binary tree to a binary search tree while keeping its tree structure as it is.
- For example:



Convert BT to BST in-place

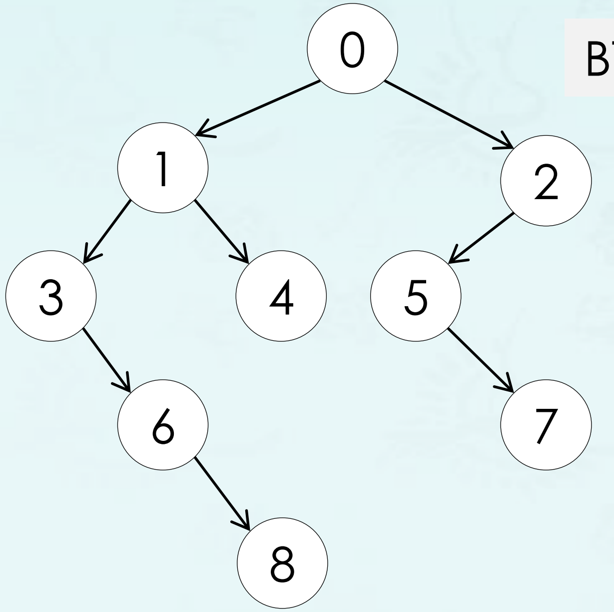
- Convert a binary tree to a binary search tree while keeping its tree structure as it is.
- Algorithm:

  only for pedagogical reason

  - **Step 1** – store keys of a binary tree into a container like **vector** or **set**. (**Do not use an array**.)
  - **Step 2** – sort the keys in **vector**. Skip this step if **set** is used since it is already sorted.
  - **Step 3** – Now, do the **inorder** traversal of the tree and copy back the elements of the container into the nodes of the tree one by one.

Convert BT to BST in-place

- Convert a binary tree to a binary search tree while keeping its tree structure as it is.
- Algorithm:
  - **Step 1** – store keys of a binary tree into a container like **vector** or **set**. (**Do not use an array**.)
  - **Step 2** – sort the keys in **vector**. Skip this step if **set** is used since it is already sorted.
  - **Step 3** – Now, do the **inorder** traversal of the tree and copy back the elements of the container into the nodes of the tree one by one.

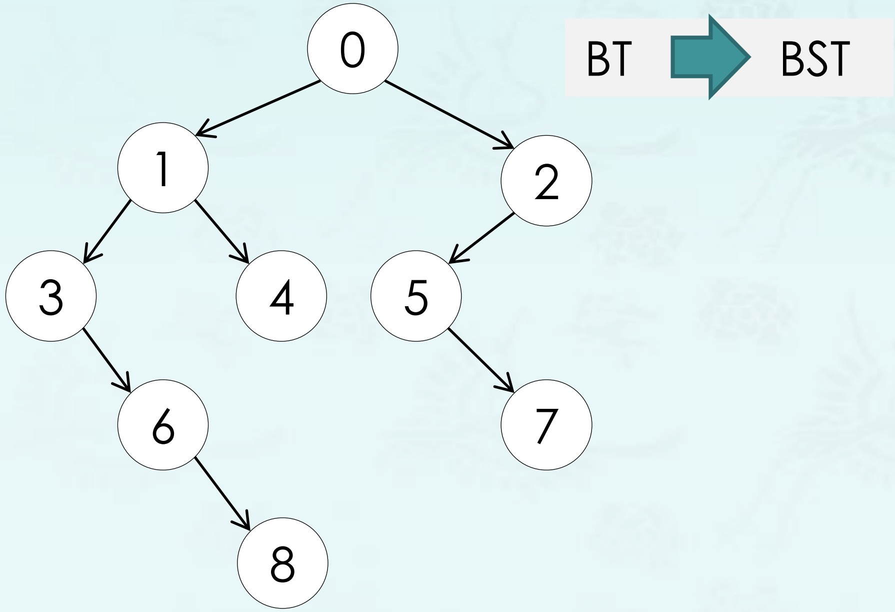


(1) Retrieve the keys from BT:
3 6 8 1 4 0 5 7 2 // if in-order used

```
void inorder(tree root) {
   if (root == nullptr) return;

   inorder(root->left);         L
   cout << root->key;           V
   inorder(root->right);        R
}
```

Convert BT to BST in-place

- Convert a binary tree to a binary search tree while keeping its tree structure as it is.
- Algorithm:
  - **Step 1** – store keys of a binary tree into a container like **vector** or **set**. (**Do not use an array**.)
  - **Step 2** – sort the keys in vector. Skip this step if set is used since it is already sorted.
  - **Step 3** – Now, do the **inorder** traversal of the tree and copy back the elements of the container into the nodes of the tree one by one.

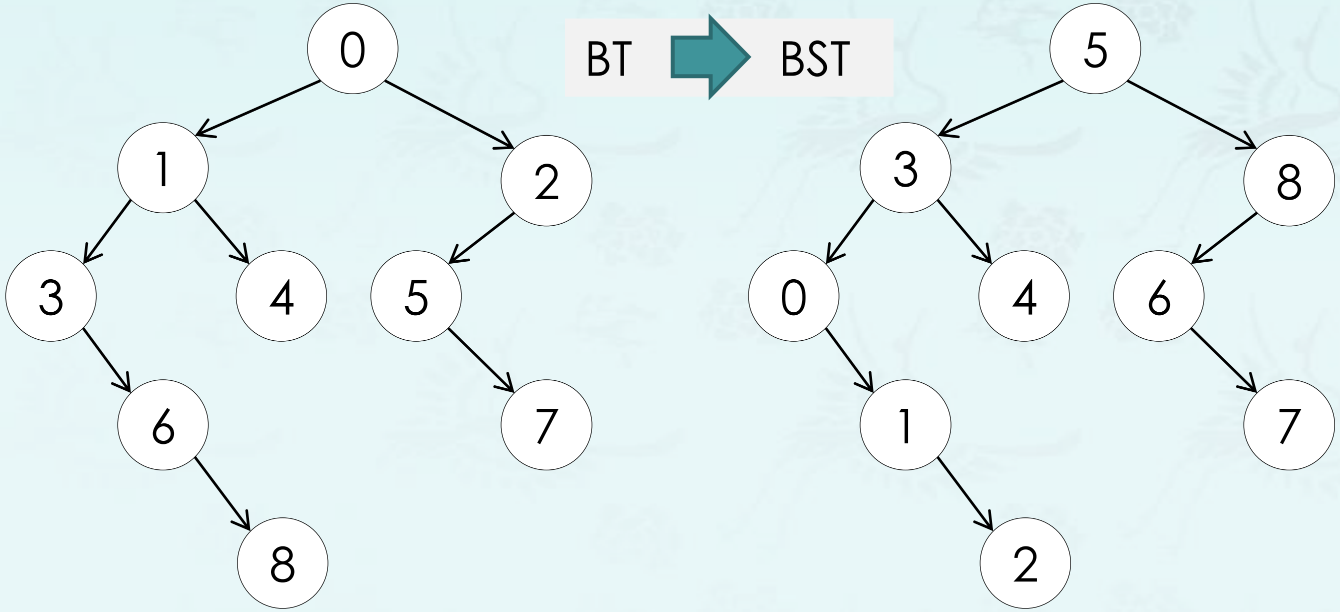


(1) Retrieve the keys from BT:
3 6 8 1 4 0 5 7 2   // if in-order used
(2) Sort keys in the container:
0 1 2 3 4 5 6 7 8

```
void inorder(tree root) {
   if (root == nullptr) return;

   inorder(root->left);          L
   cout << root->key;            V
   inorder(root->right);         R
}
```

Convert BT to BST in-place

- Convert a binary tree to a binary search tree while keeping its tree structure as it is.
- Algorithm:
  - **Step 1** – store keys of a binary tree into a container like **vector** or **set**. (**Do not use an array**.)
  - **Step 2** – sort the keys in vector. Skip this step if set is used since it is already sorted.
  - **Step 3** – Now, do the **inorder** traversal of the tree and copy back the elements of the container into the nodes of the tree one by one.



(1) Retrieve the keys from BT:
    3 6 8 1 4 0 5 7 2   // if in-order used
(2) Sort keys in the container:
    0 1 2 3 4 5 6 7 8
(3) Replace keys in BT with sorted keys while in-order traversal.

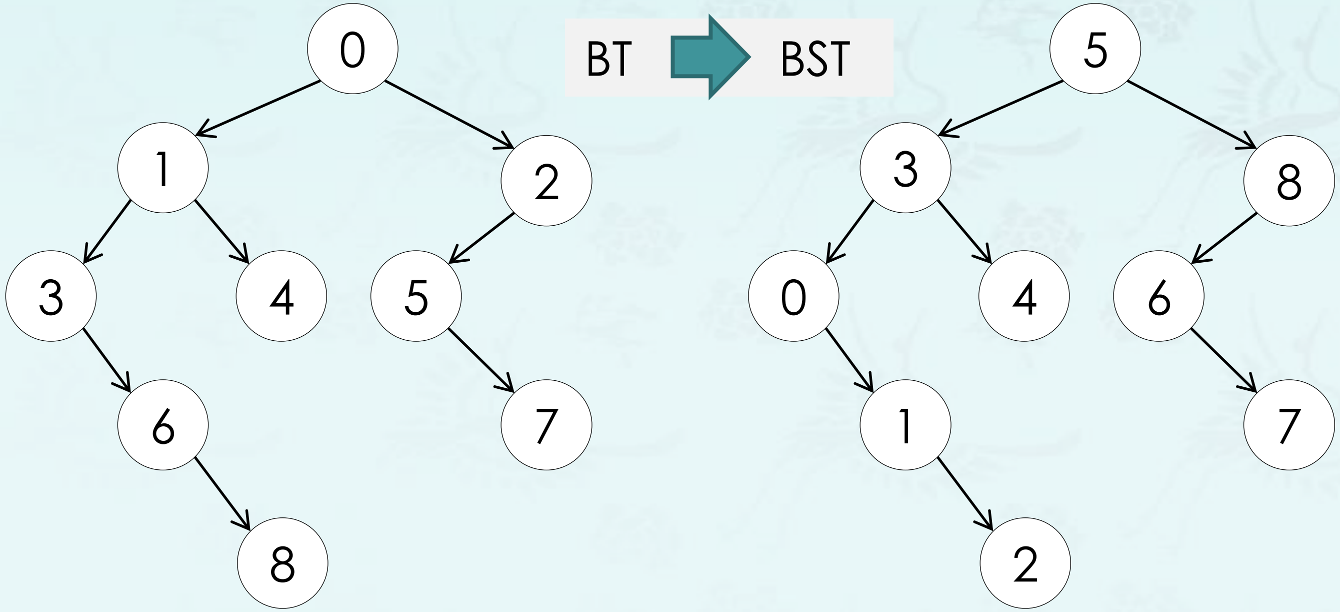
```
void inorder(tree root) {
   if (root == nullptr) return;

   inorder(root->left);         L
   cout << root->key;           V
   inorder(root->right);        R
}
```

Convert BT to BST in-place

- Convert a binary tree to a binary search tree while keeping its tree structure as it is.

(1) Retrieve the keys from BT:
    3 6 8 1 4 0 5 7 2   // if in-order used
(2) Sort keys in the container:
    0 1 2 3 4 5 6 7 8
(3) Replace keys in BT with sorted keys
    while in-order traversal.

```
void get_keys(tree root, set<int> &keys) {
   if (root == nullptr) return;
   keys.insert(root->key);
   get_keys(root->left, keys);
   get_keys(root->right, keys);
}
```




```
void put_keys(tree root,
              set<int>::iterator& it) {



}
```

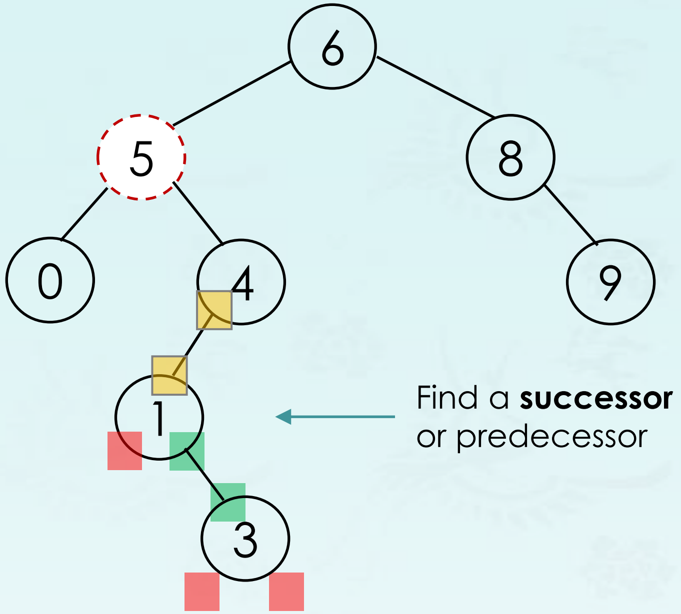
```
void BTtoBST(tree root) {
   set<int> keys;
   get_keys(root, keys);
   assert(                         );
   set<int>::iterator it = keys.begin();
   put_keys(root, it);
}
```

Data Structures
Chapter 5 Tree

1. Introduction
2. Binary Tree
3. **Binary Search Tree**
   - Introduction
   - Operations
   - Demo & Coding
4. Balancing Tree

Operations: delete (or trim)

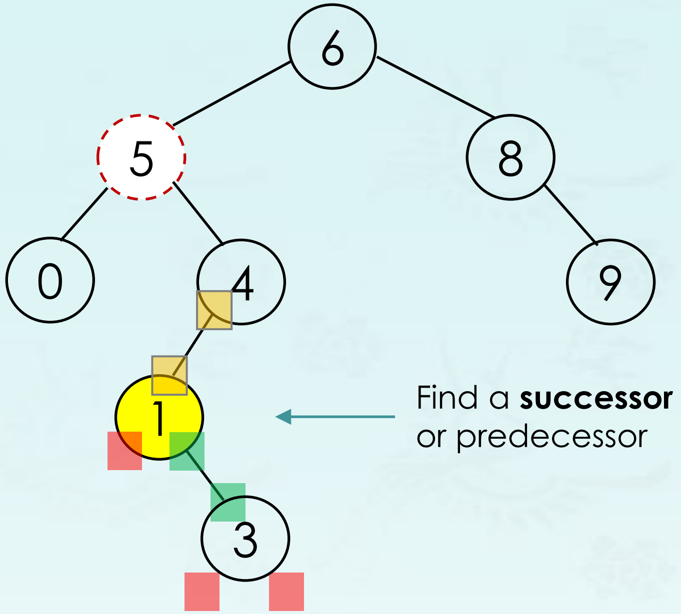
- **Example:** Case 3: Two children




```
1. find the node 5 to delete
```

Operations: delete (or trim)

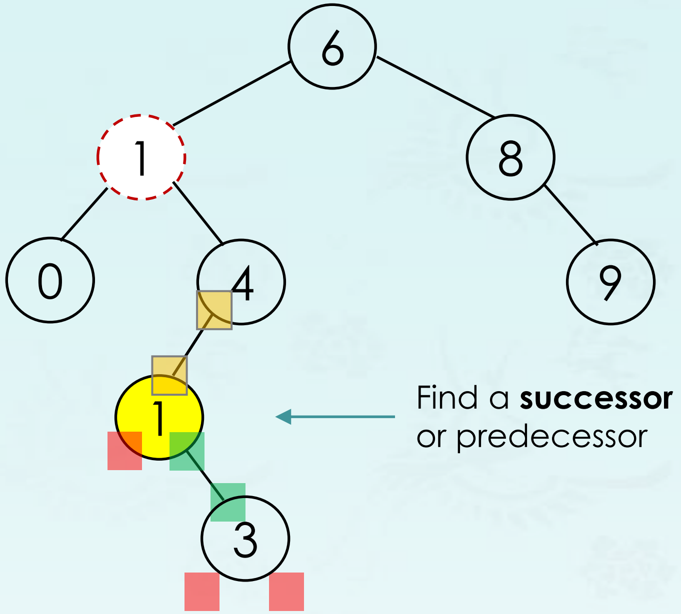
- **Example:** Case 3: Two children




```
1. find the node 5 to delete
2. if (two children case),
   find 5's successor's key = 1
```

Operations: delete (or trim)

- **Example:** Case 3: Two children

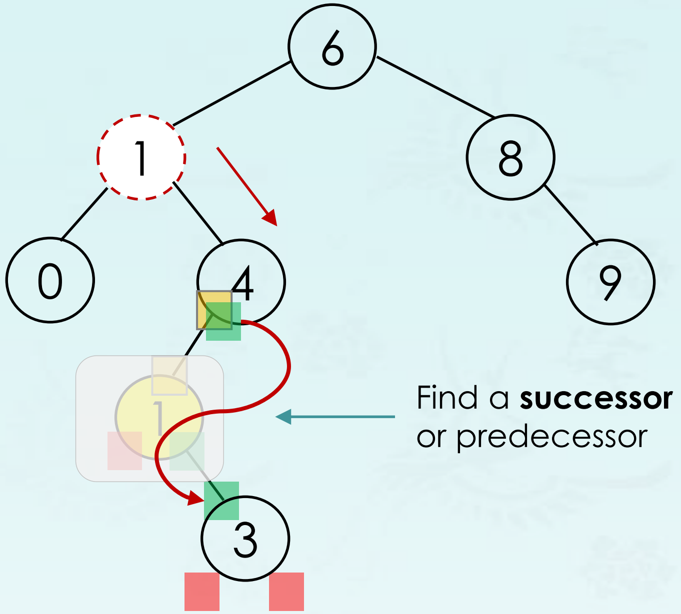



```
1. find the node 5 to delete
2. if (two children case),
   find 5's successor's key = 1
3. replace 5 with 1
```


Prof. Youngsup Kim, idebtor@gmail.com, CSEE Dept., Grace School Rm204, Handong Global University

Operations: delete (or trim)

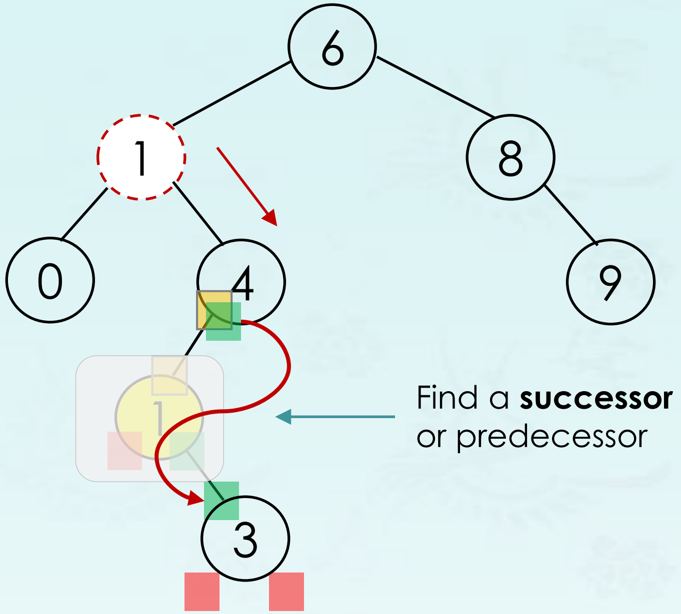
- **Example:** Case 3: Two children




```
1. find the node 5 to delete
2. if (two children case),
   find 5's successor's key = 1
3. replace 5 with 1
4. invoke
   node->right = trim(node->right, 1)
```

Operations: delete (or trim)

- **Example:** Case 3: Two children



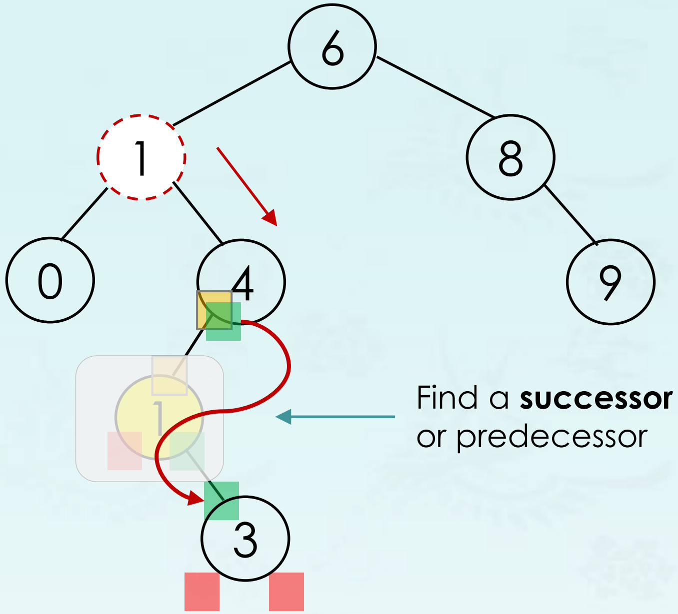

```
1. find the node 5 to delete
2. if (two children case),
   find 5's successor's key = 1
3. replace 5 with 1
4. invoke
   node->right = trim(node->right, 1)
```

Some thoughts:

- Step 2 Get the heights of two subtree first.
  - If right subtree height is larger, then use the successor. Otherwise use the predecessor to shorten the tree height.
- Step 4 simply uses the code for one-child case deletion.

Operations: delete (or trim)

- **Example:** Case 3: Two children




```
1. find the node 5 to delete
2. if (two children case),
   find 5's successor's key = 1
3. replace 5 with 1
4. invoke
   node->right = trim(node->right, 1)
```

Some thoughts:

- Step 2 Get the heights of two subtree first.
  - If right subtree height is larger, then use the successor. Otherwise use the predecessor to shorten the tree height.
- Step 4 simply uses the code for one-child case deletion.

Some questions:

- What if successor has **two** children?
  - **Not possible !**
  - Because if it has two nodes, at least one of them is less than it, then in the process of finding successor, we won't pick it !