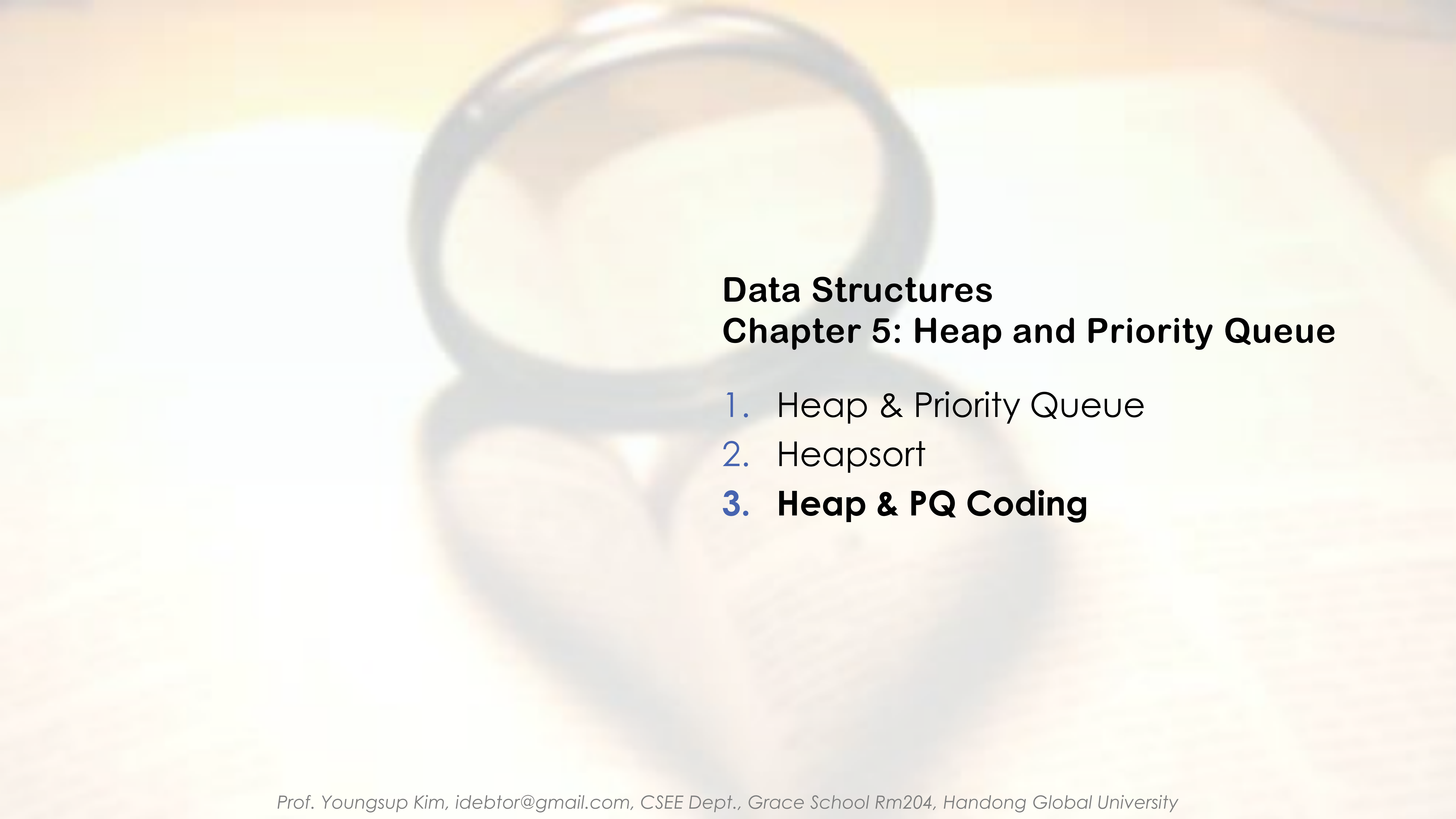

Data Structures
Chapter 5: Heap and Priority Queue

1. Heap & Priority Queue
2. Heapsort
3. **Heap & PQ Coding**

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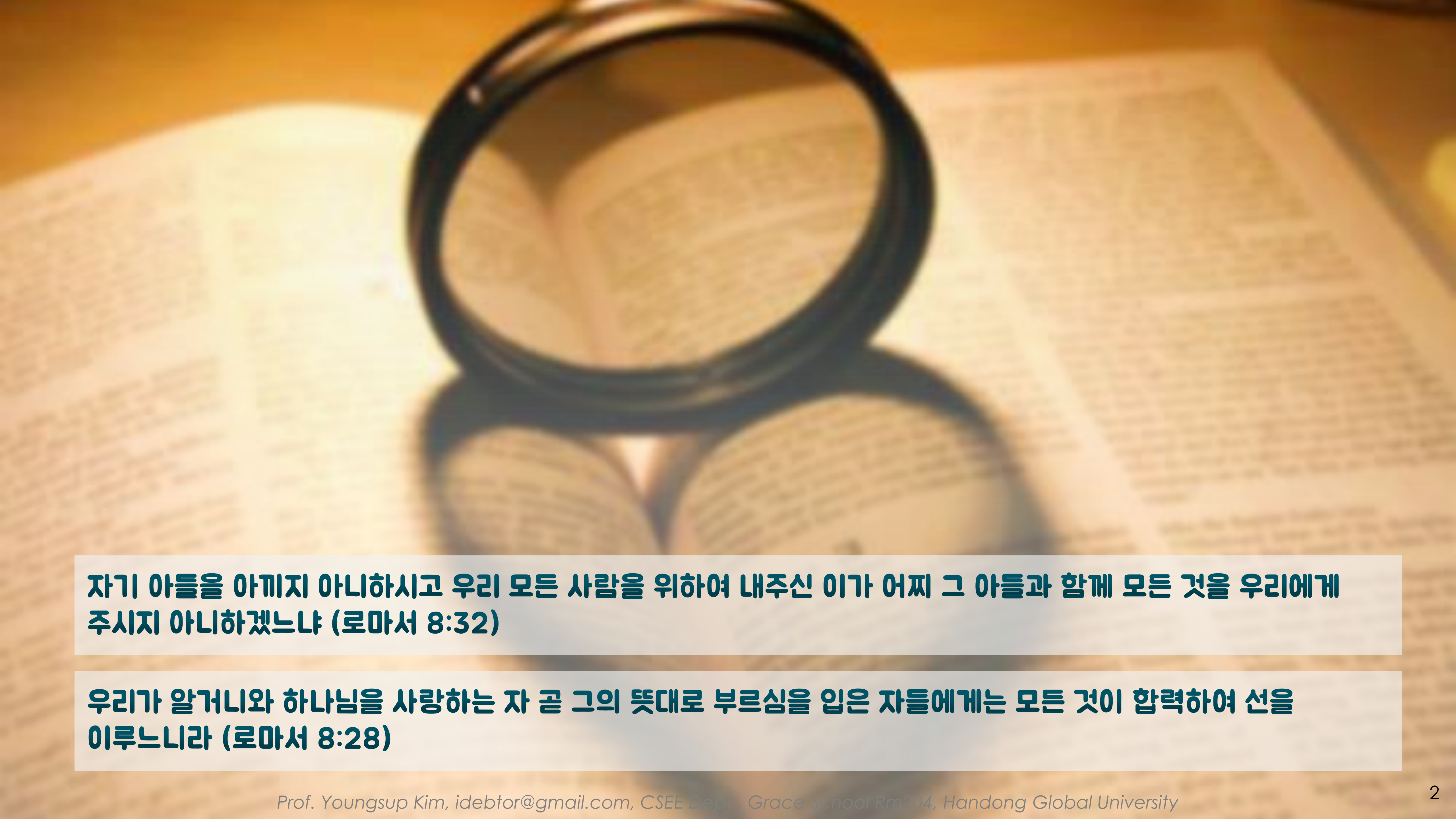


자기 아들을 아끼지 아니하시고 우리 모든 사람을 위하여 내주신 이가 어찌 그 아들과 함께 모든 것을 우리에게 주시지 아니하겠느냐 (로마서 8:32)

우리가 알거니와 하나님을 사랑하는 자 곧 그의 뜻대로 부르심을 입은 자들에게는 모든 것이 합력하여 선을 이루느니라 (로마서 8:28)

Heap ADT - heap.h

- Heap ADT: A **one - based** and **one dimensional array** is used to simplify parent and child calculations.
- heap.h

```
struct Heap {
  int *nodes;          // an array of nodes
  int capacity;        // array size of node or key, item
  int N;               // the number of nodes in the heap
  bool (*comp)(Heap*, int, int);
  Heap(int capa = 2) {
    capacity = capa;
    nodes = new int[capacity];
    N = 0;
    comp = nullptr;
  };
  ~Heap() {};
};
using heap = Heap*;
```

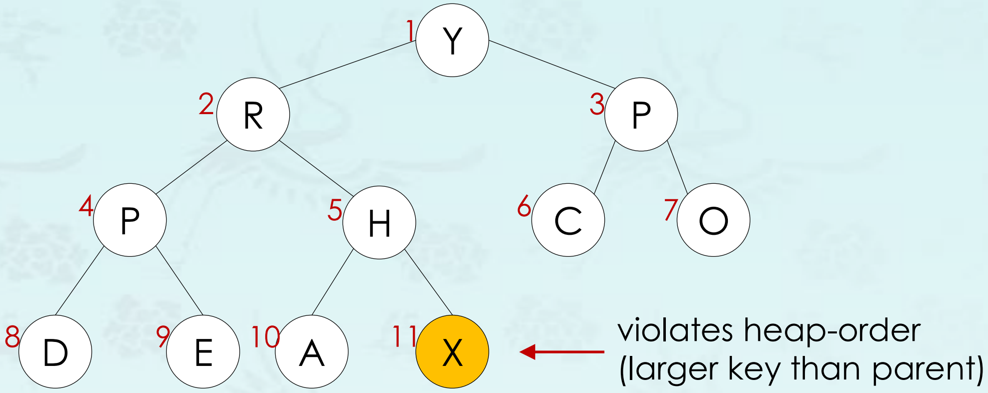
Heap ADT - heap.h

```
void clear(heap hp);                // deallocate heap
int size(heap hp);                  // return nodes in heap currently
int level(int n);                   // return level based on num of nodes
int capacity(heap hp);              // return its capacity (array size)
int reserve(heap hp, int capa);     // reserve the array size (= capacity)
int full(heap hp);                  // return true/false
int empty(heap hp);                 // return true/false
void grow(heap hp, int key);        // add a new key
void trim(heap hp);                 // delete a queue
int heapify(heap hp);               // convert a complete BT into a heap
// helper functions to support grow/trim functions
int less(heap hp, int i, int j);    // used in max heap
int more(heap hp, int i, int j);    // used in min heap
void swim(heap hp, int k);          // bubble up
void sink(heap hp, int k);          // tickle down
// helper functions to check heap invariant
int heapOrdered(heap hp);           // is heap[1..N] a heap?
```

Promotion in a heap: swim

- To eliminate the violation:
  - Swap key in child with key in parent.
  - Repeat until **heap order** restored.

This is a maxheap example.



swim up or sink down ?

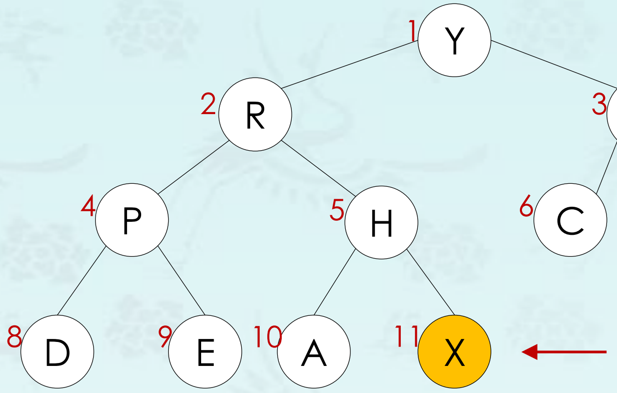
Promotion in a heap: swim

- To eliminate the violation:
  - Swap key in child with key in parent.
  - Repeat until **heap order** restored.

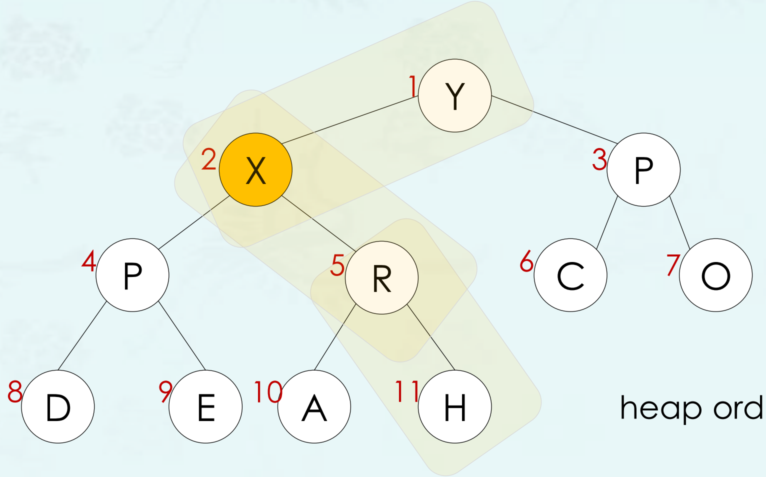
This is a maxheap example.

```
bool less (heap h, int p, int c) {
    return h->nodes[p] < h->nodes[c];
}
```

```
void swap (heap h, int p, int c) {
    int item = h->nodes[p];
    h->nodes[p] = h->nodes[c];
    h->nodes[c] = item;
}  // Inside this swap(), we may use swap() in c++
```



violates heap-order
(larger key than parent)



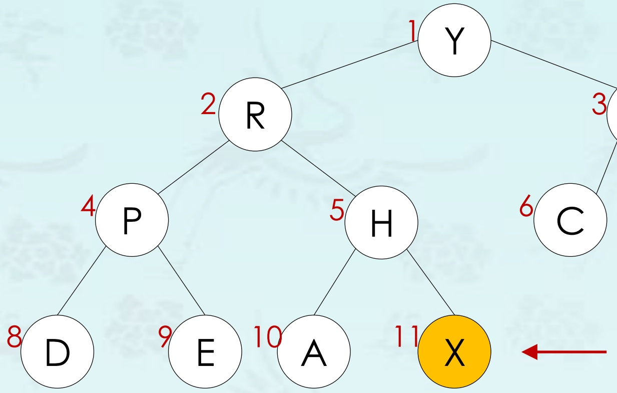
heap ordered

Promotion in a heap: swim

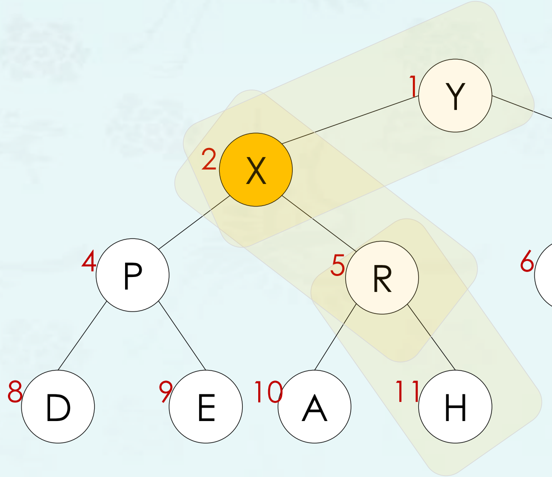
- To eliminate the violation:
  - Swap key in child with key in parent.
  - Repeat until **heap order** restored.

This is a maxheap example.

```
void swim (heap h, int k)
{
  while (not reached at root &&
          k's parent key < k's key)
  {
        swap k and its parent
        go for the next
  }
}
```



violates heap-order
(larger key than parent)



heap ordered

Promotion in a heap: swim

- To eliminate the violation:
  - Swap key in child with key in parent.
  - Repeat until **heap order** restored.

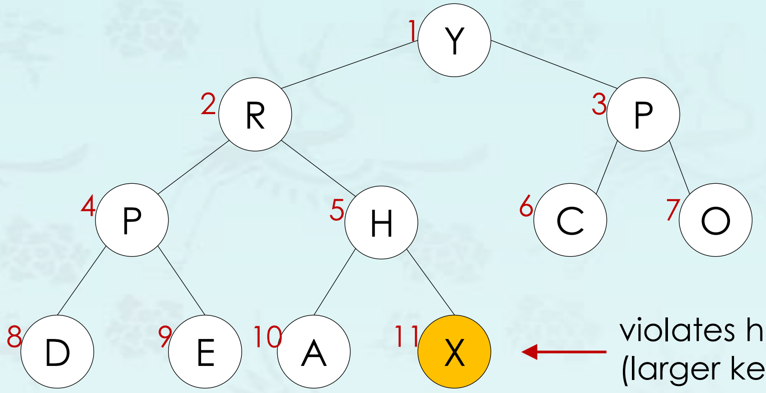
This is a maxheap example.

not reached at root , parent is less than its child(k)

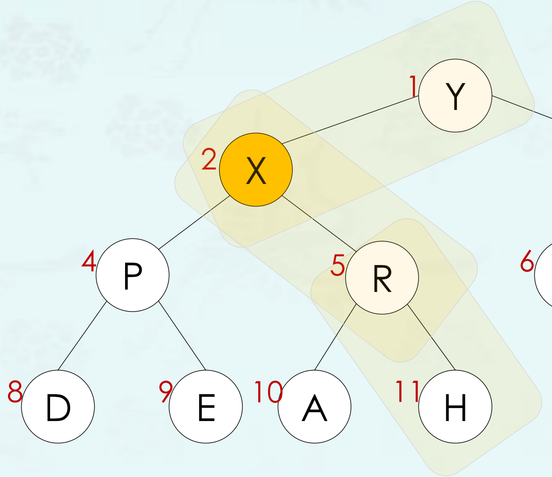
```
{
  while (k > 1 && less(h, k / 2, k))
  {
    swap(h, k / 2, k);
    k = k / 2;
  }
}
```

swap parent(k/2) and its child(k)

move up one level



violates heap-order (larger key than parent)

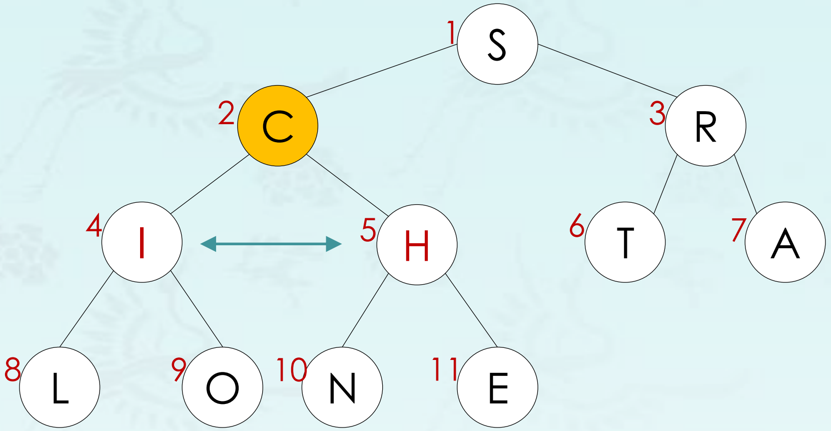


heap ordered

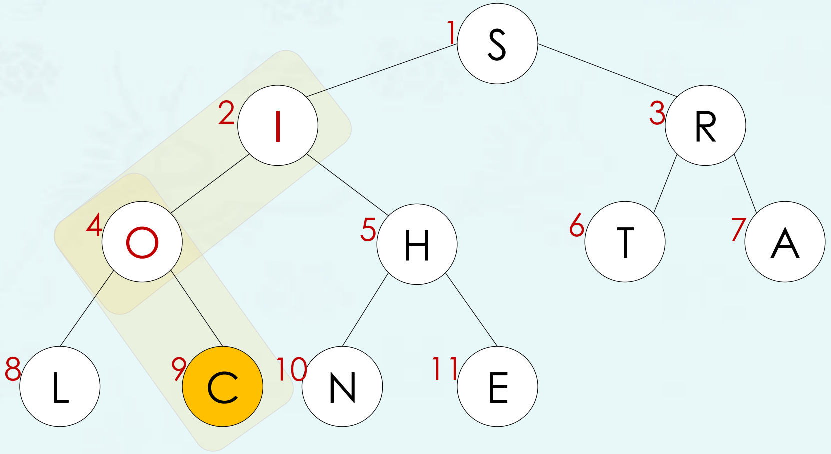
Demotion in a heap: sink

- Parent's key becomes smaller than one (or both) of its children's.
- To eliminate the violation:
  - Swap key in parent with key in larger child (of two)
  - Repeat until heap order restored.

Top-down reheapify (sink)



swim up or sink down ?

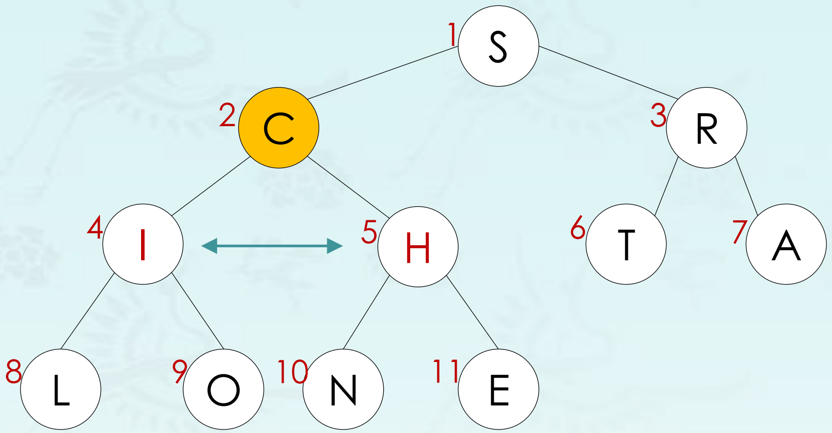


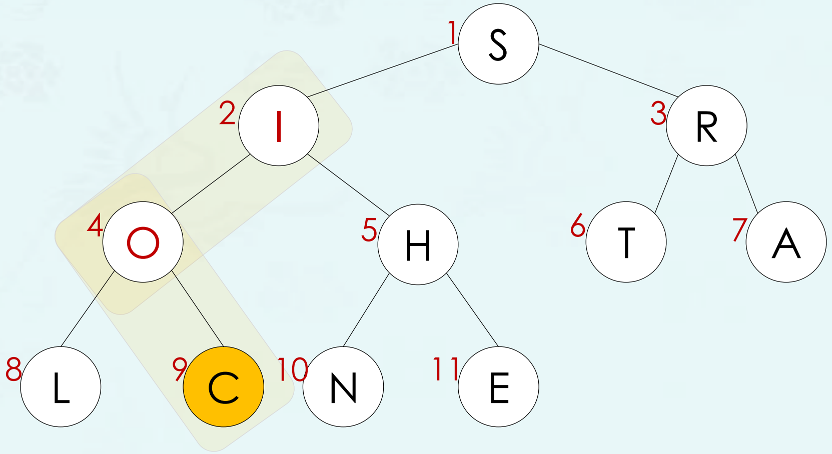
Demotion in a heap: sink

- Parent's key becomes smaller than one (or both) of its children's.
- To eliminate the violation:
  - Swap key in parent with key in **larger** child (of two)
  - Repeat until heap order restored.

Why not smaller child?

Top-down reheapify (sink)





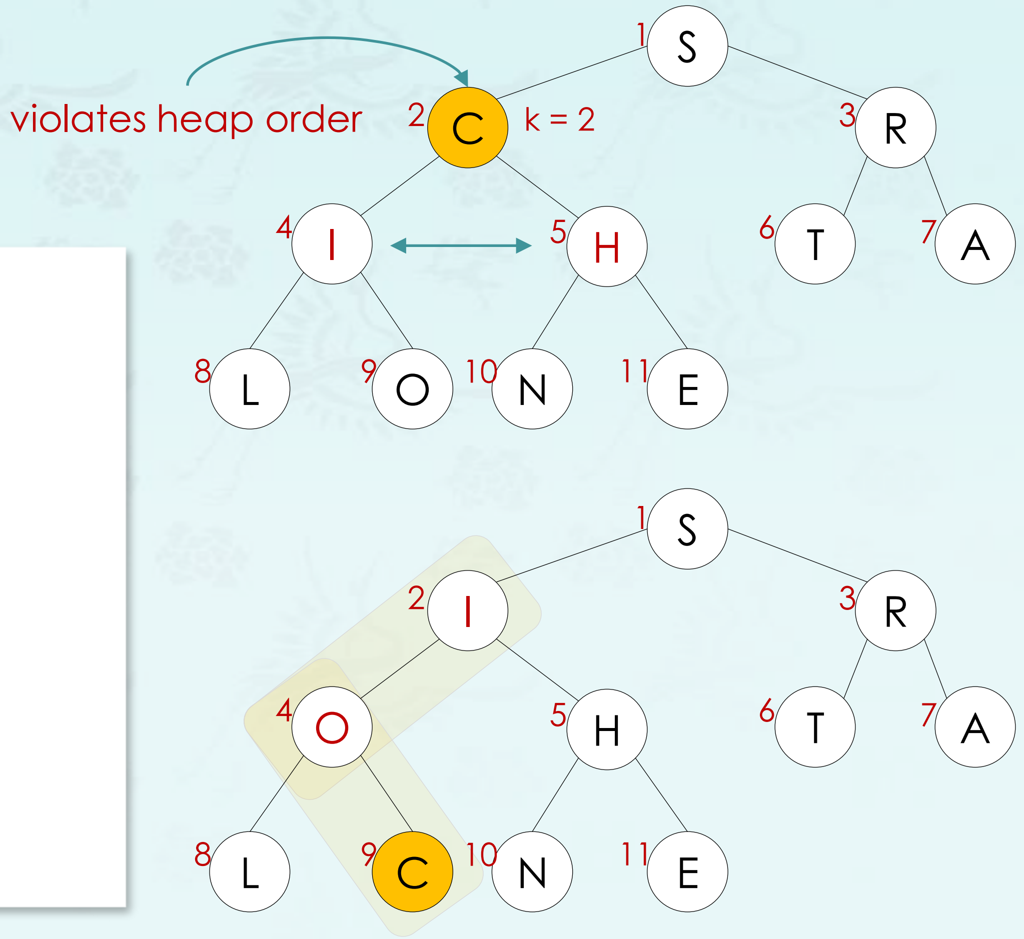
```
void sink(heap h, int k)
{
   while (k's child not reached the last)
   {
      find the larger child of k, let it be j. (j = 5)

      if k's key is not less than j's key, break;
      swap k and j since k's key < j's key
      set k to the next node which is j.
   }
}
```

Demotion in a heap: sink

- Parent’s key becomes smaller than one (or both) of its children’s.
- To eliminate the violation:
  - Swap key in parent with key in **larger** child (of two)
  - Repeat until heap order restored.

Why not smaller child?

Top-down reheapify (sink)



```
void
{
    while (2 * k <= h->N)
    {
        int j = 2 * k;
                                   j++;
    }
}
```

while (k's child not reached the last)

find the larger child of k

if k is not less than j, break;
swap k and j since k < j
set k to the next node (which is j)

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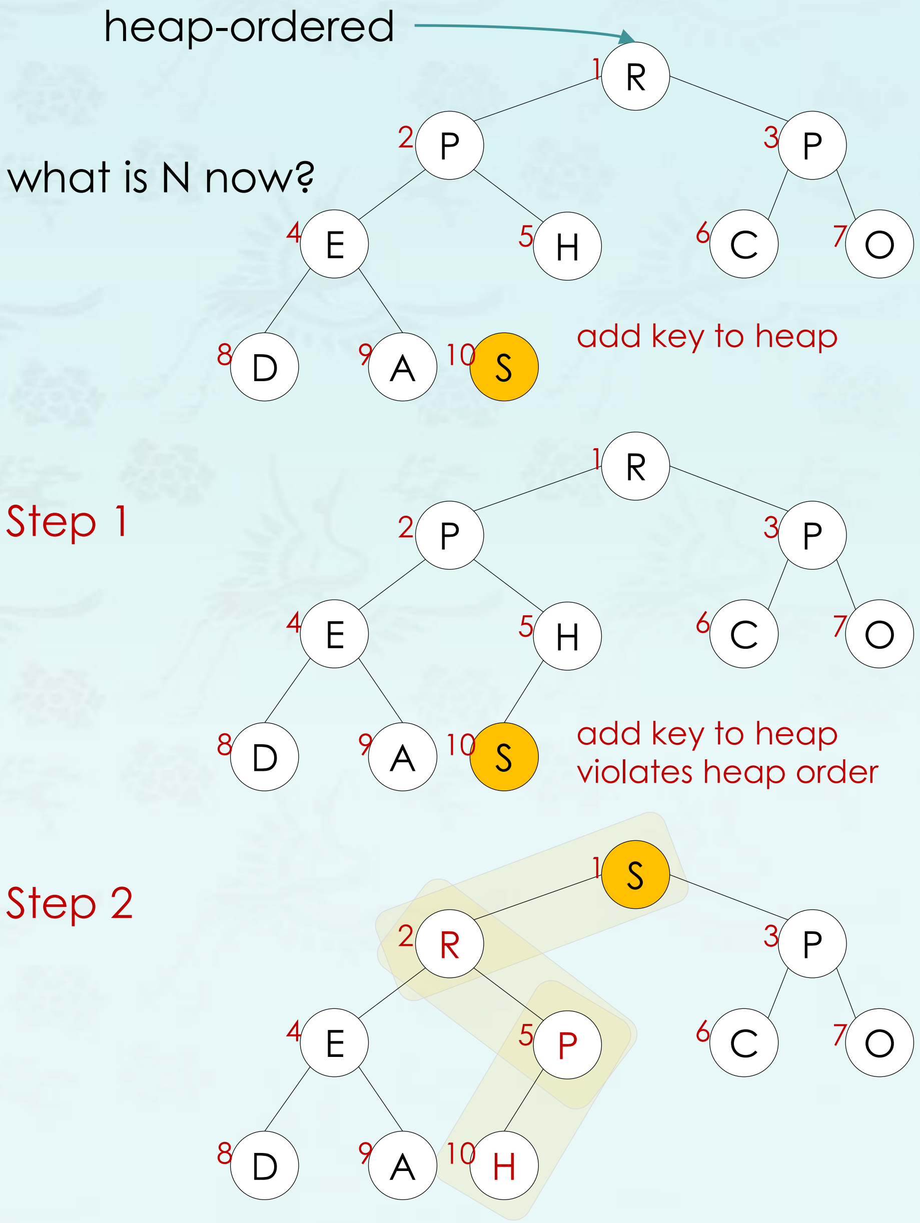
Insert in a heap

- Insert: Add node at end, then **swim** it up.
  - Cost: At most 1 + log N compares.

```
void insert(heap h, int key)
{
    h->nodes[++h->N] = key;

}
```

```
struct Heap {
  int *nodes;
  int capacity;
  int N;
};
using heap = *Heap;
```



heap-ordered

what is N now?

add key to heap

Step 1

add key to heap
violates heap order

Step 2

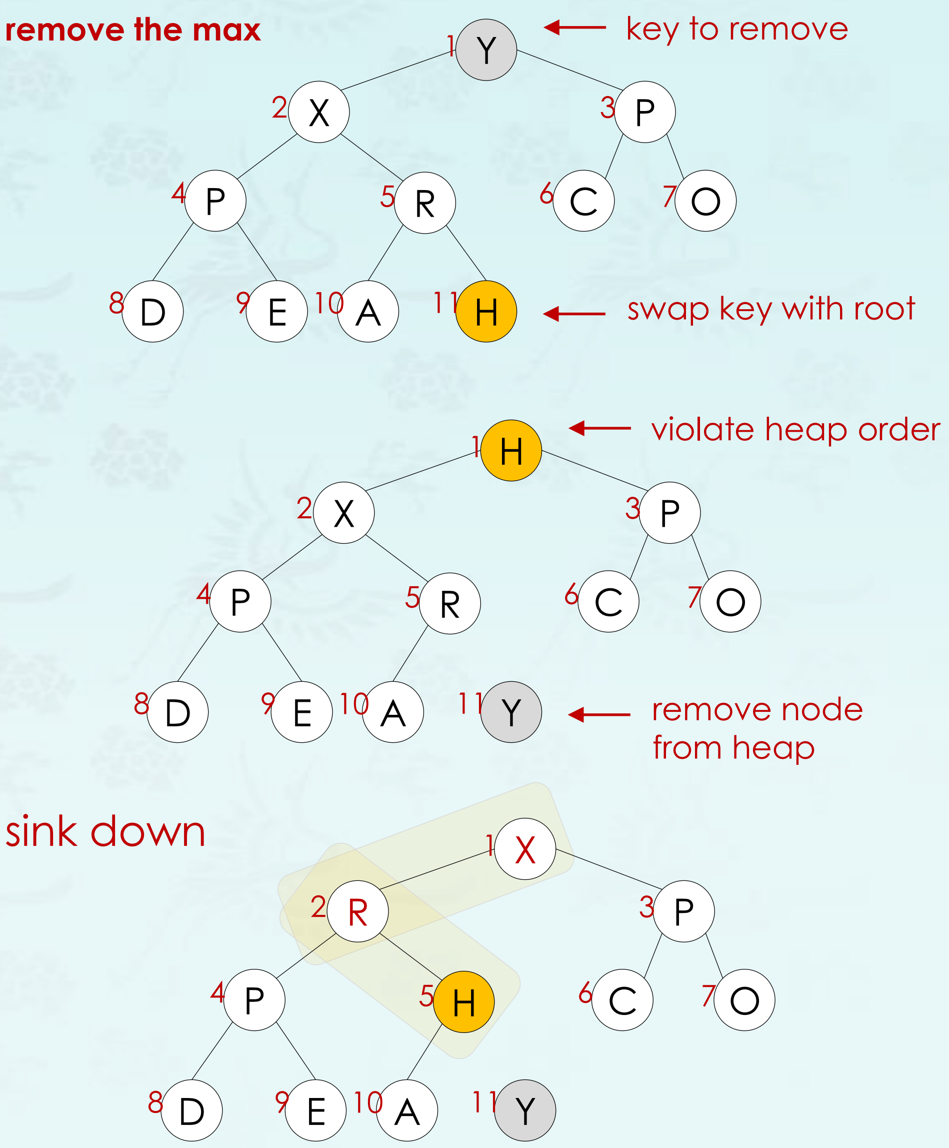
Delete in a heap

Delete the root (max or min) in a heap:

- **Delete root:** Swap root with node at end, then sink it down.
- **Cost:** At most 2 log N compares.

```
void delete(heap h)
{
   swap(h, 1, h->N--);

}
```

```
void swim(heap h, int k)
void sink(heap h, int k)
bool less(heap h, int p, int c)
void swap(heap h, int p, int c)
```



remove the max

key to remove

swap key with root

violate heap order

remove node from heap

sink down

Heap ADT

```
void clear(heap hp);                     // deallocate heap
int size(heap hp);                       // return nodes in heap currently
int level(int n);                        // return level based on num of nodes
int capacity(heap hp);                   // return its capacity (array size)
int reserve(heap hp, int capa);          // reserve the array size (= capacity)
int full(heap hp);                       // return true/false
int empty(heap hp);                      // return true/false
void grow(heap hp, int key);             // add a new key
void trim(heap hp);                      // delete a queue
int heapify(heap hp);                    // convert a complete BT into a heap

// helper functions to support grow/trim functions
int less(heap hp, int i, int j);         // used in max heap
int more(heap hp, int i, int j);         // used in min heap
void swim(heap hp, int k);               // bubble up
void sink(heap hp, int k);               // tickle down
// helper functions to check heap invariant
int heapOrdered(heap hp);                // is it heap-ordered?
```

Data Structures
Chapter 5: Heap and Priority Queue

1. Heap & Priority Queue
2. Heapsort
3. **Heap & PQ Coding**

Heap Coding

```
// return the number of items in heap
int size(heap hp) {
    return hp->N;
}
```

```
// Is this heap empty?
int empty(heap hp) {
    return (hp->N == 0) ? true : false;
}
```

```
// Is this heap full?
int full(heap hp) {
    return (hp->N == hp->capacity - 1) ? true : false;
}
```

Heap Coding

```
int less(heap hp, int i, int j) {
    return hp->nodes[i] < hp->nodes[j];
}
```

```
void swap(heap hp, int i, int j) {
    int t = hp->nodes[i];
    hp->nodes[i] = hp->nodes[j];
    hp->nodes[j] = t;
}
```

```
void swim(heap hp, int k) {

}
```

```
void sink(heap hp, int k) {

}
```

Heap Coding

```
void grow(heap hp, int key) {

    cout << "YOUR CODE HERE\n";

    // add key @ ++heap->N

    // swim up @ heap->N

}
```

```
void trim(heap hp) {
    if (empty(hp)) return;

    cout << "YOUR CODE HERE\n";

}
```

Heap Coding

```
newCBT()       with a given array, instantiate a new complete binary tree
               its result is neither maxheap nor minheap.

heapify()      make a complete binary tree into a max/minheap
heapsort()     use max/min-heap to sort elements in heap
heapprint()    build a binary tree from heap/CBT for display purpose only
```

newCBT() – convert an int array to CBT

```
// instantiates a CBT with given data and its size.
heap newCBT(int *a, int n) {
    int capa = ?

    heap p = new Heap{ capa };

    p->N = n;
    for (int i = 0; i < n; i++)
        p->nodes[i + 1] = a[i];
    return p;
}
```

```
struct Heap {
  int *nodes;
  int capacity;
  int N;
  bool (*comp)(Heap*, int, int);
  Heap(int capa = 2) {
    capacity = capa;
    nodes = new int[capacity];
    N = 0;
    comp = nullptr;
  };
  ~Heap() {};
};
using heap = Heap*;
```

heapify – convert an int array to max/minheap

```
// start sink() at the last internal node(or parent of the last node)
// since leaf nodes already satisfy the max/min priority property
// This is O(n) algorithm.
void heapify(heap p) {
  for (int k = p->N; k >= 1; k--)
    sink(p, k);
} // this works but inefficiently. Fix it if you can.
```

Convert maxheap to minheap and vice versa

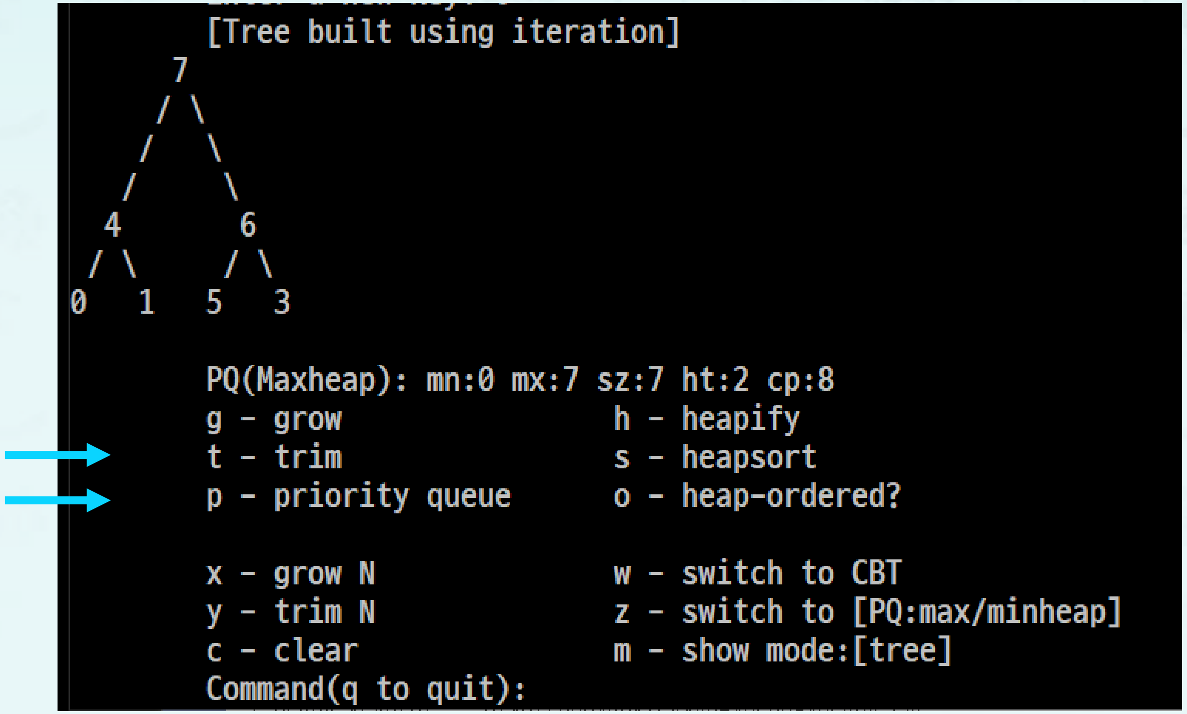
```
        ...
        case 'z':   // turn into max-heap or min-heap
          if (ordered)
            maxType = maxType ? false : true;
          else
            maxType = true;

          setType(hp, maxType);
          heapify(hp);
          ordered = true;
          break;
        ...

// driver.cpp
```

```
// sets the compare function less() for maxheap, more() for minheap.

void setType(heap p, bool maxType) {
  p->comp = maxType ? _________ : ____________;  // comparator fp
} // heap.cpp
```

Priority Queue

It is like a regular queue or stack data structure, but where additionally **each element has a "priority" associated with it.** In a priority queue, an element with high priority is served before an element with low priority.

- **"trim"** removes the root which has the highest priority
- **"priority queue"** lets user modify the priority (or value) of an element) and move it to the position based on its new priority in the queue.

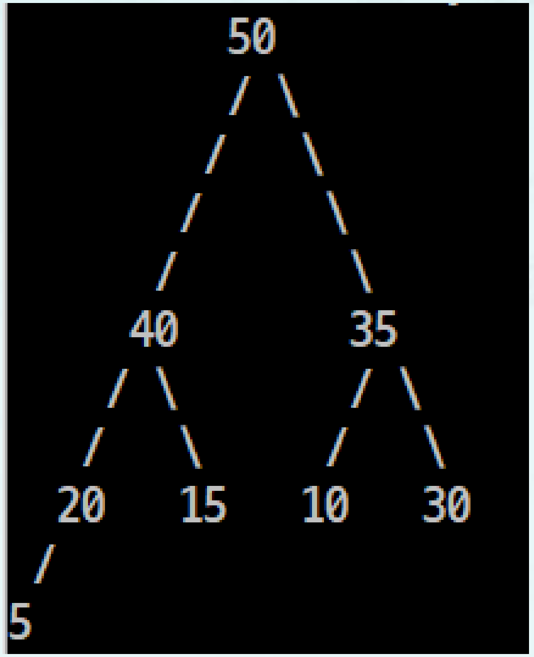


```
        [Tree built using iteration]
      7
     / \
    /   \
   /     \
  4       6
 / \     / \
0   1   5   3

        PQ(Maxheap): mn:0 mx:7 sz:7 ht:2 cp:8
        g - grow                h - heapify
        t - trim                s - heapsort
        p - priority queue      o - heap-ordered?

        x - grow N              w - switch to CBT
        y - trim N              z - switch to [PQ:max/minheap]
        c - clear               m - show mode:[tree]
        Command(q to quit):
```

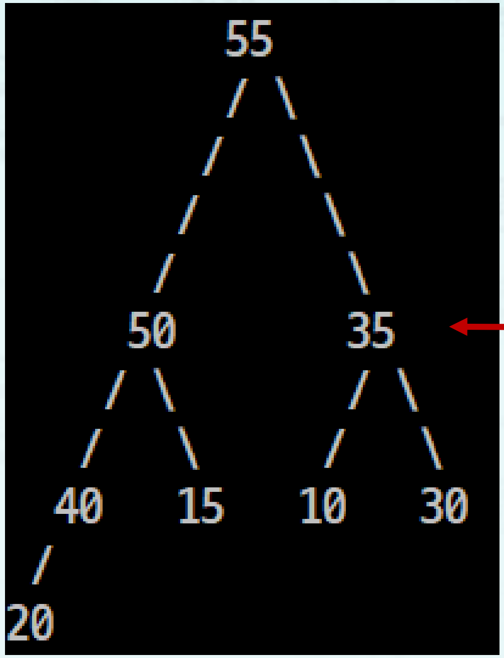
Priority Queue

For example:

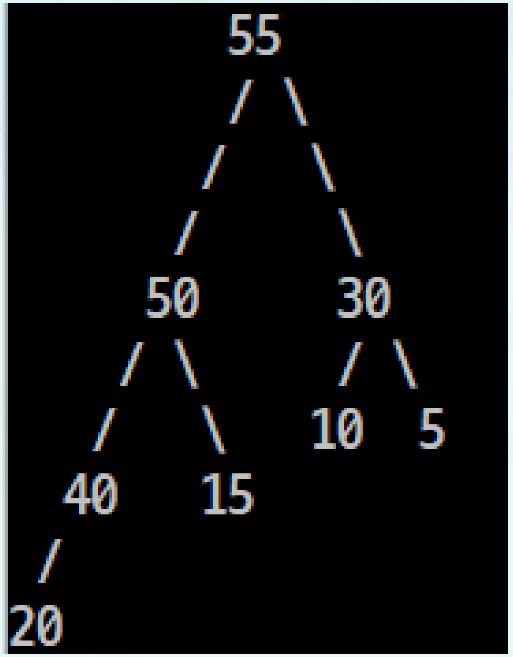
- If you **change 5 to 55**, it will go up to the root and 20 is placed at the bottom.
- If you **change 35 to 5**, 30 will go up where 35 is, then 5 goes down to the right corner.



```
         50
         / \
        /   \
       /     \
      /       \
     40        35
    / \       / \
   /   \     /   \
  20   15   10   30
 /
5
```



```
         55
         / \
        /   \
       /     \
      /       \
     50        35
    / \       / \
   /   \     /   \
  40   15   10   30
 /
20
```



```
         55
         / \
        /   \
       /     \
      50      30
     / \     / \
   /    \   10  5
  40    15
 /
20
```

grow() - inserts a new key to the max-heap or min-heap.

grow(heap p, int key)

1. if full(p), invoke **reserve()** to double the size of nodes[]. Use p->capacity * 2.
2. add the key to nodes[]. The index of nodes[] must be ++p->N.
3. swim up to maintain heap invariant.

```
void grow(heap p, int key) {

  if (full(p))
    ...
  p->nodes ...
  swim...

  return;
}
```

growN()

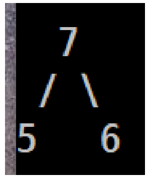
1. Find the max key(max) in heap or CBT.
2. Set a function pointer to the function to insert a node.
3. Allocate a Key type array such as keys to store random keys.
4. Invoke randomN() function to generate keys in the range [(max + 1)..(max + count)]
5. Invoke the function to insert keys in keys[], but one key at a time.
6. Print the heap if DEBUG is defined whenever a node is inserted.
7. Don't forget freeing the array of keys you allocated in Step 3.

```
void growN(heap p, int count, bool heapOrdered) {
  int max = empty(p) ? 1 : maximum(p) + 1;
  void(*insertFunc)(heap h, int key) = heapOrdered ? grow : growCBT;




                                               // insertFunc(p, keys[i]);




}
```

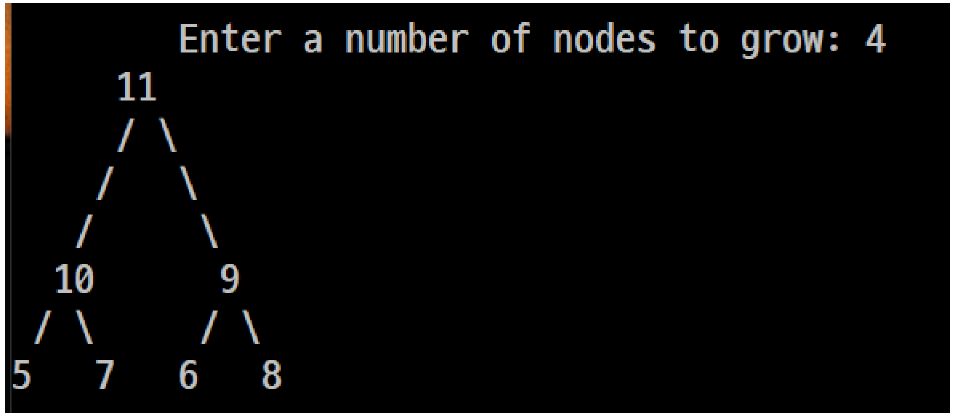
growN()

1. Find the max key(max) in heap or CBT.
2. Set a function pointer to the function to insert a node.
3. Allocate a Key type array such as keys to store random keys.
4. Invoke randomN() function to generate keys in the range [(max + 1)..(max + count)]
5. Invoke the function to insert keys in keys[], but one key at a time.
6. Print the heap if DEBUG is defined whenever a node is inserted.
7. Don't forget freeing the array of keys you allocated in Step 3.

```
void growN(heap p, int count, bool heapOrdered) {
  int max = empty(p) ? 1 : maximum(p) + 1;
  void(*insertFunc)(heap h, int key) = heapOrdered ? grow : growCBT;



}
```

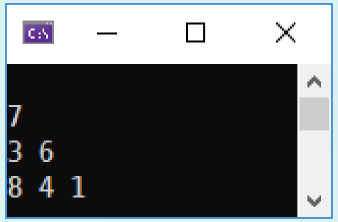


```
   7
  / \
 5   6
```

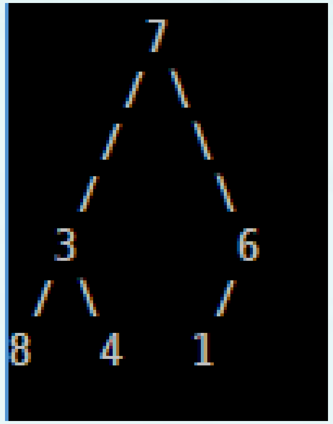


```
        Enter a number of nodes to grow: 4
     11
     / \
    /   \
   /     \
  10      9
 / \     / \
5   7   6   8
```

Heapprint(): build a tree from CBT - heapprint.cpp



```
7
3 6
8 4 1
```



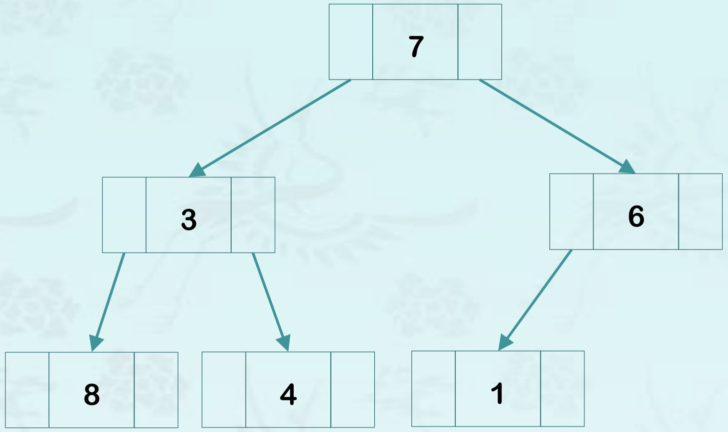
```
      7
     / \
    /   \
   /     \
  3       6
 / \     /
8   4   1
```

```
// print a heap using treeprint() –
// build a tree to call treeprint()

void heapprint(heap p, int mode) {
  if (empty(p)) return;
  if (size(p) % 2 == 0) {
    cout << "\t[Tree built using recursion]\n";
    root = buildBT(p->nodes, 1, size(p));  // using recursion
  }
  else {
    cout << "\t[Tree built using iteration]\n";
    root = buildBT(p);                      // using iteration
  }
  ...
  ... treeprint(root);
  ...
  clear(root);
}
```

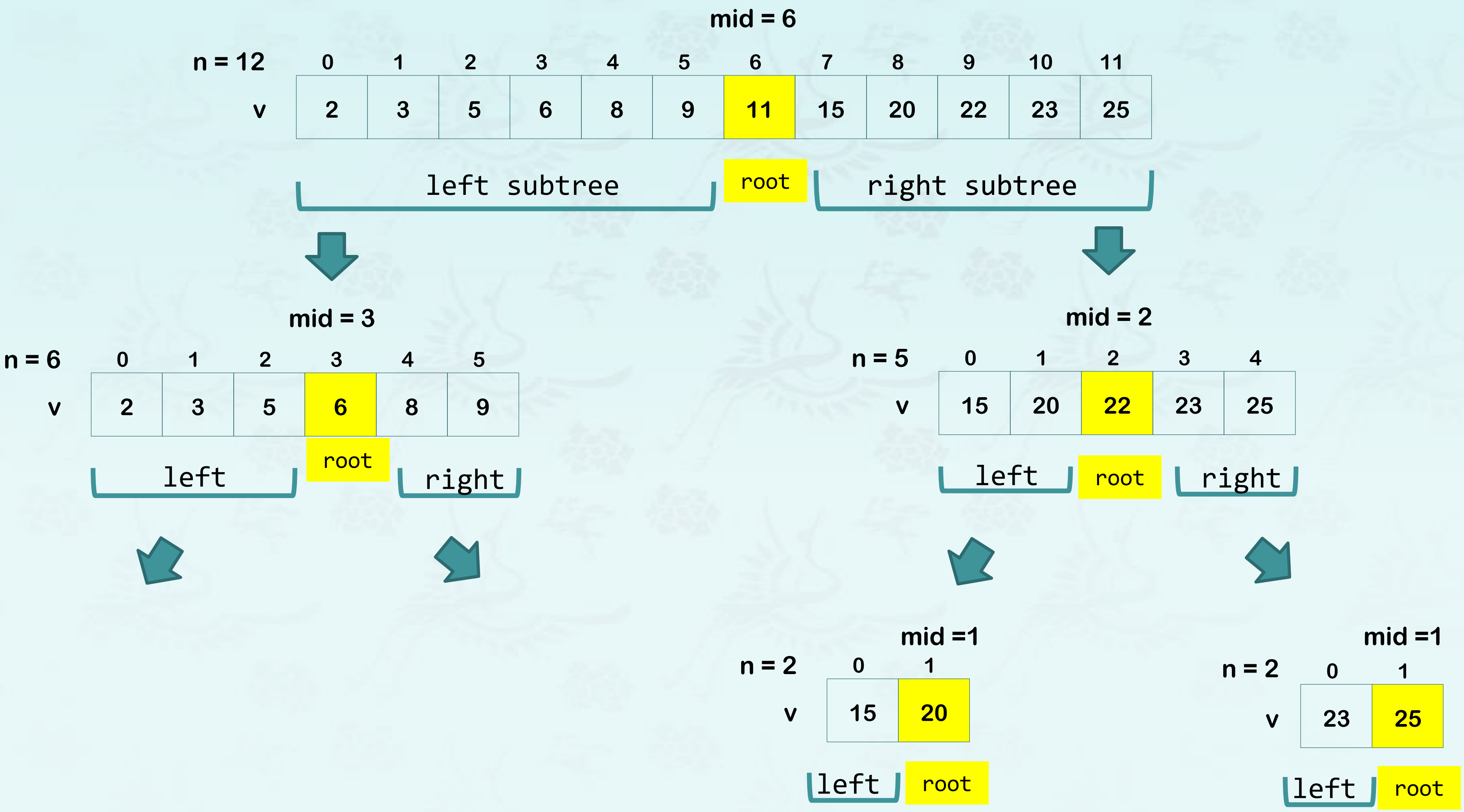
heapprint() – build a binary tree from heap/CBT using recursion

0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

hp->nodes[] hp->N



Building AVL tree from BST in O(n) – Review



mid = 6

n = 12	0	1	2	3	4	5	6	7	8	9	10	11
v	2	3	5	6	8	9	11	15	20	22	23	25

left subtree root right subtree

mid = 3

n = 6	0	1	2	3	4	5
v	2	3	5	6	8	9

left root right

mid = 2

n = 5	0	1	2	3	4
v	15	20	22	23	25

left root right

mid =1

n = 2	0	1
v	15	20

left root

mid =1

n = 2	0	1
v	23	25

left root

Building AVL tree from BST in O(n) – Review

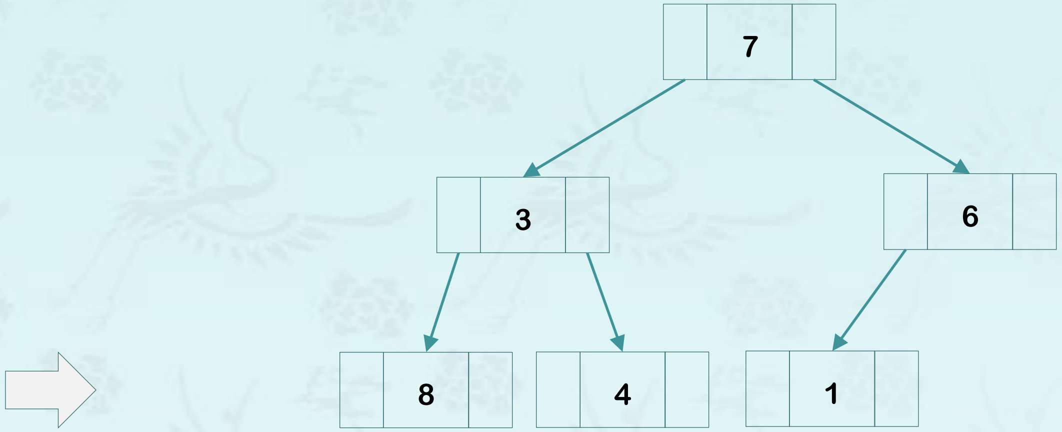
```
// rebuilds an AVL tree with a list of keys sorted.
// v - an array of keys sorted, n - the array size

tree buildAVL(int* v, int n) {
  if (n <= 0) return nullptr;

  int mid = n / 2;
  ...
  return root;
}
```

```
tree reconstruct(tree root) {
  if (root == nullptr) return nullptr;
  if (size(root) > 10) { // recycling method
    vector<tree> v;      // get nodes sorted
    ...
    root = buildAVL(...);
  }
  else {                 // recreation method
    vector<int> v;       // get keys sorted
    ...
    ...
    root = buildAVL(...);
  }
  return root;
}
```

heapprint() – build a binary tree from heap/CBT using recursion

0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

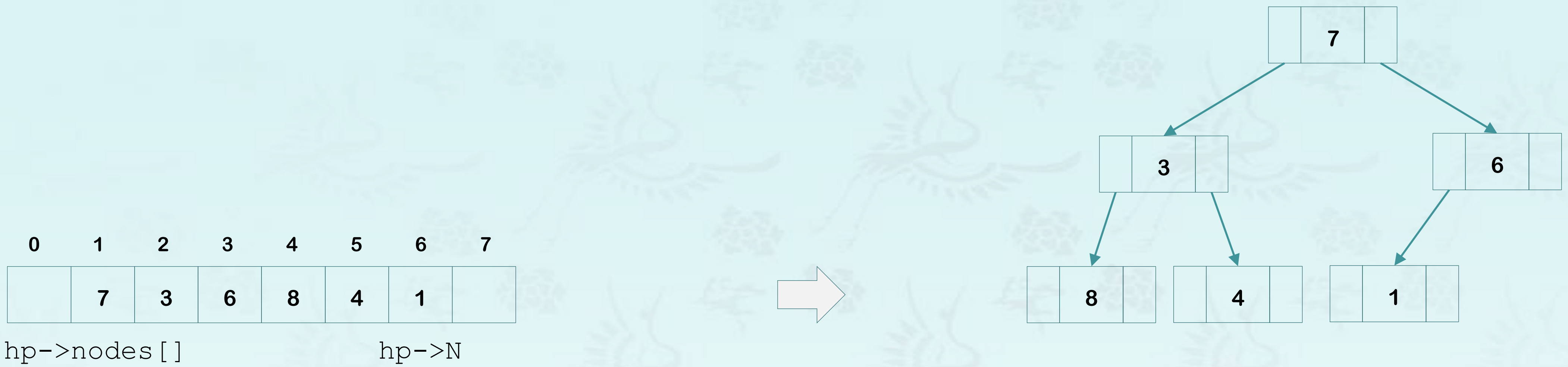
`hp->nodes[]` `hp->N`



Create a recursive function that creates a binary tree from an int array. This function takes an int array, starting index, and size of the array and returns the root as shown below:

```
tree buildBT(int *nodes, int i, int n) {
        1.  If i > n, return nulltptr – terminate condition
        2.  Create the tree (root) node with nodes[i]).
              A.  Invoke buildBT() for all its left children (or i * 2).
                  Set its return to the left child of the root.
              B.  Invoke buildBT() for all its right children (or i * 2 + 1).
                  Set its return to the right child of the root.
        3.  return root
}
```

heapprint() – build a binary tree from heap/CBT using recursion



0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

hp->nodes[] hp->N

Create a recursive function that creates a binary tree from an int array. This function takes an int array, starting index, and size of the array and returns the root as shown below:

tree buildBT(int *nodes, int i, int n) {

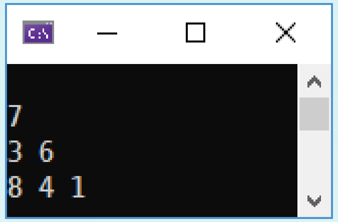
1. If **i > n**, return nulltptr – terminate condition
2. Create the **tree (root) node** with **nodes[i]**).
   A. Invoke buildBT() for all its left children (or **i * 2**). Set its return to the left child of the root.
   B. Invoke buildBT() for all its right children (or **i * 2 + 1**). Set its return to the right child of the root.
3. return root

}

```
tree buildBT(int *nodes, int i, int n) {
  if (i > n) return nullptr;
  tree root = new TreeNode{ nodes[i] };
  root->left  = ...
  root->right = ...
  return  root;
}
```

```
void heapprint(heap p) {
  if (empty(p)) return;
  tree root = buildBT(p->nodes, 1, size(p));
  treeprint(root);
}
```

heapprint() – build a binary tree from heap/CBT using recursion



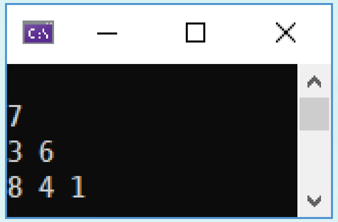
```
7
3 6
8 4 1
```

0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

hp->nodes[] hp->N

```
tree buildBT(*nodes, i=1, n=6) { }
```

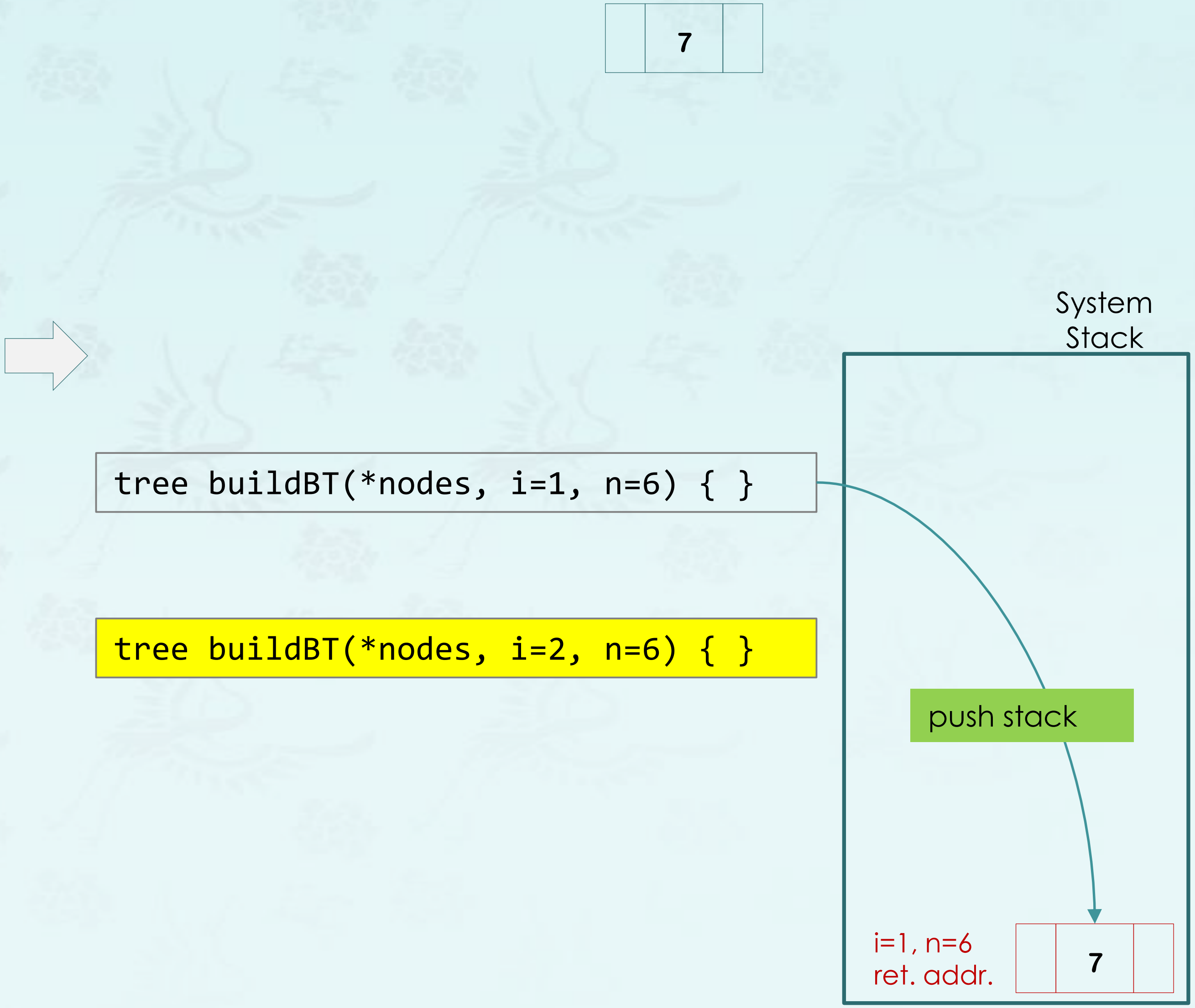
heapprint() – build a binary tree from heap/CBT using recursion



```
7
3 6
8 4 1
```

0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

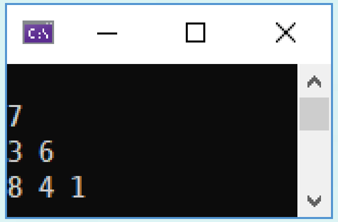
`hp->nodes[]` `hp->N`



```
tree buildBT(*nodes, i=1, n=6) { }
```

```
tree buildBT(*nodes, i=2, n=6) { }
```

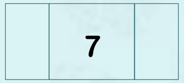
heapprint() – build a binary tree from heap/CBT using recursion



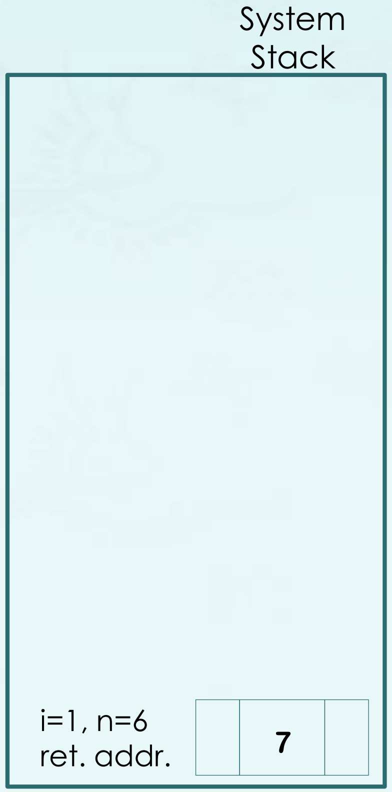
```
7
3 6
8 4 1
```

0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

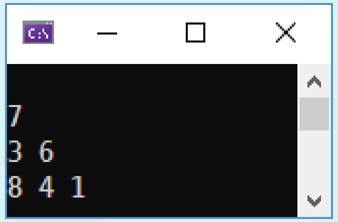
hp->nodes[]   hp->N



```
tree buildBT(*nodes, i=2, n=6) { }
```



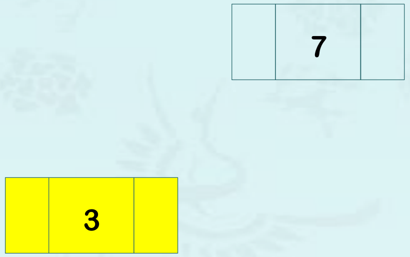
heapprint() – build a binary tree from heap/CBT using recursion



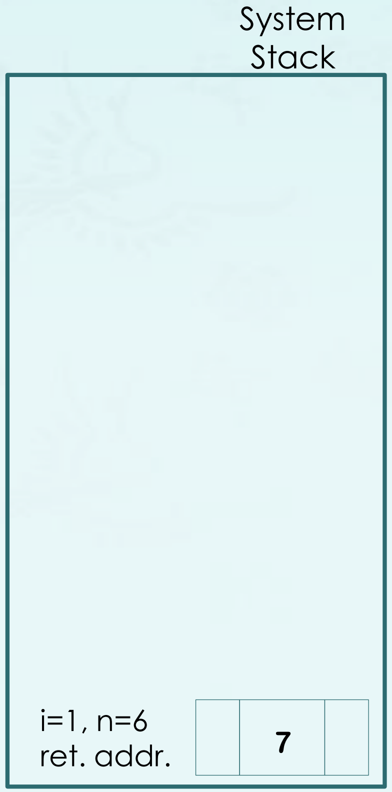
```
7
3 6
8 4 1
```

0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

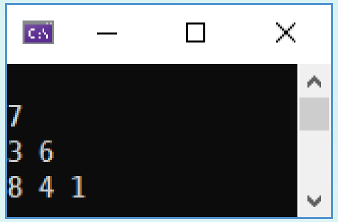
hp->nodes[] hp->N



```
tree buildBT(*nodes, i=2, n=6) { }
```



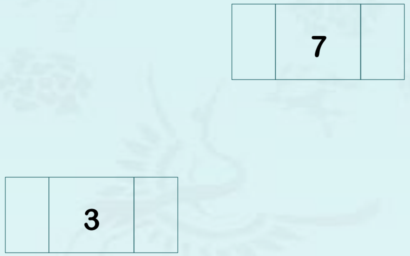
heapprint() – build a binary tree from heap/CBT using recursion



```
7
3 6
8 4 1
```

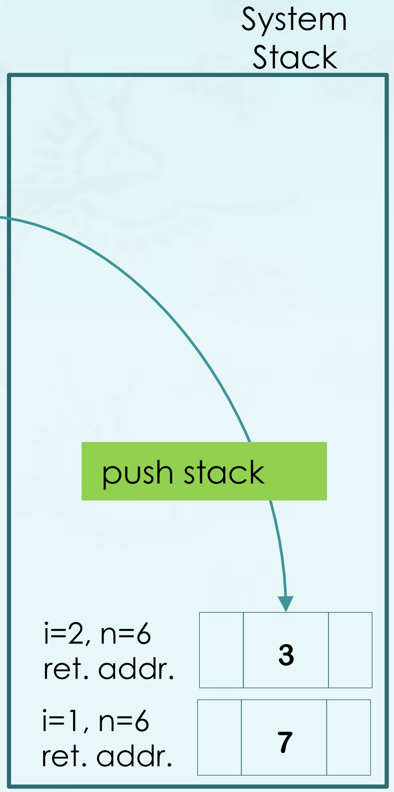
0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

`hp->nodes[]` `hp->N`

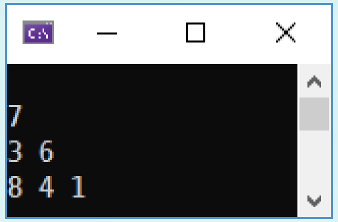


```
tree buildBT(*nodes, i=2, n=6) { }
```

```
tree buildBT(*nodes, i=4, n=6) { }
```



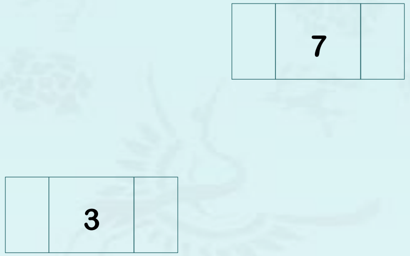
heapprint() – build a binary tree from heap/CBT using recursion



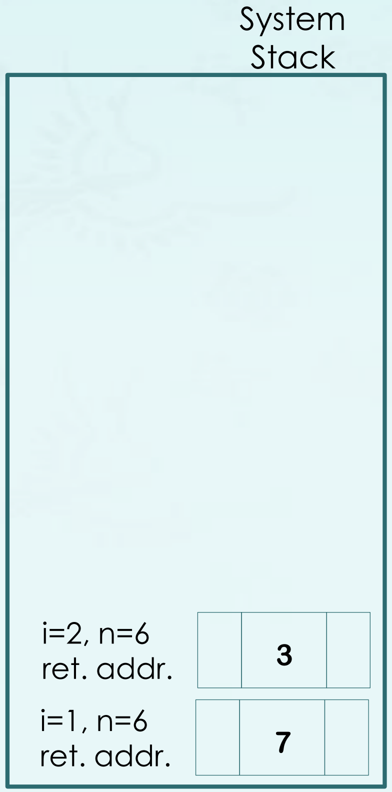
```
7
3 6
8 4 1
```

0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

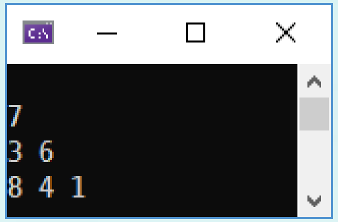
hp->nodes[] hp->N



```
tree buildBT(*nodes, i=4, n=6) { }
```



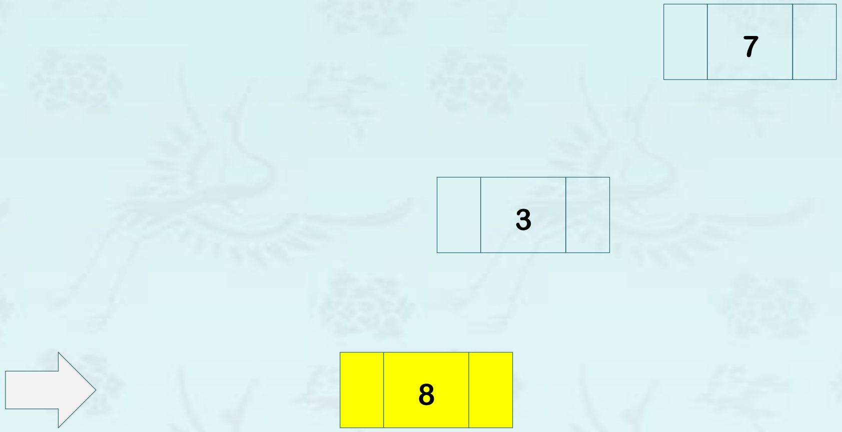
heapprint() – build a binary tree from heap/CBT using recursion



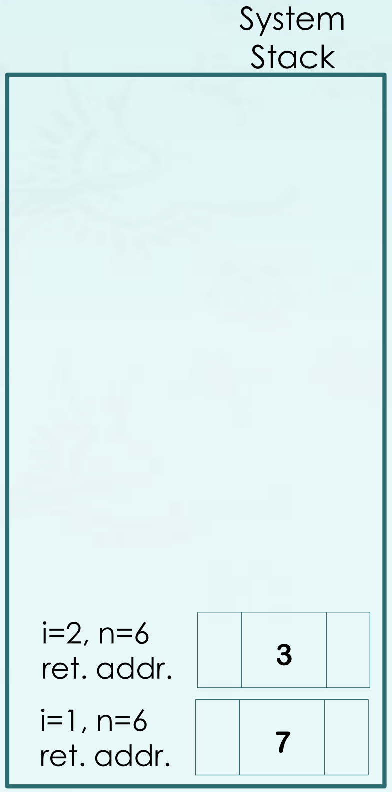
```
7
3 6
8 4 1
```

0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

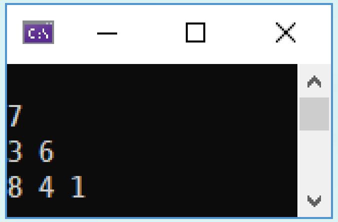
hp->nodes[] hp->N



```
tree buildBT(*nodes, i=4, n=6) { }
```



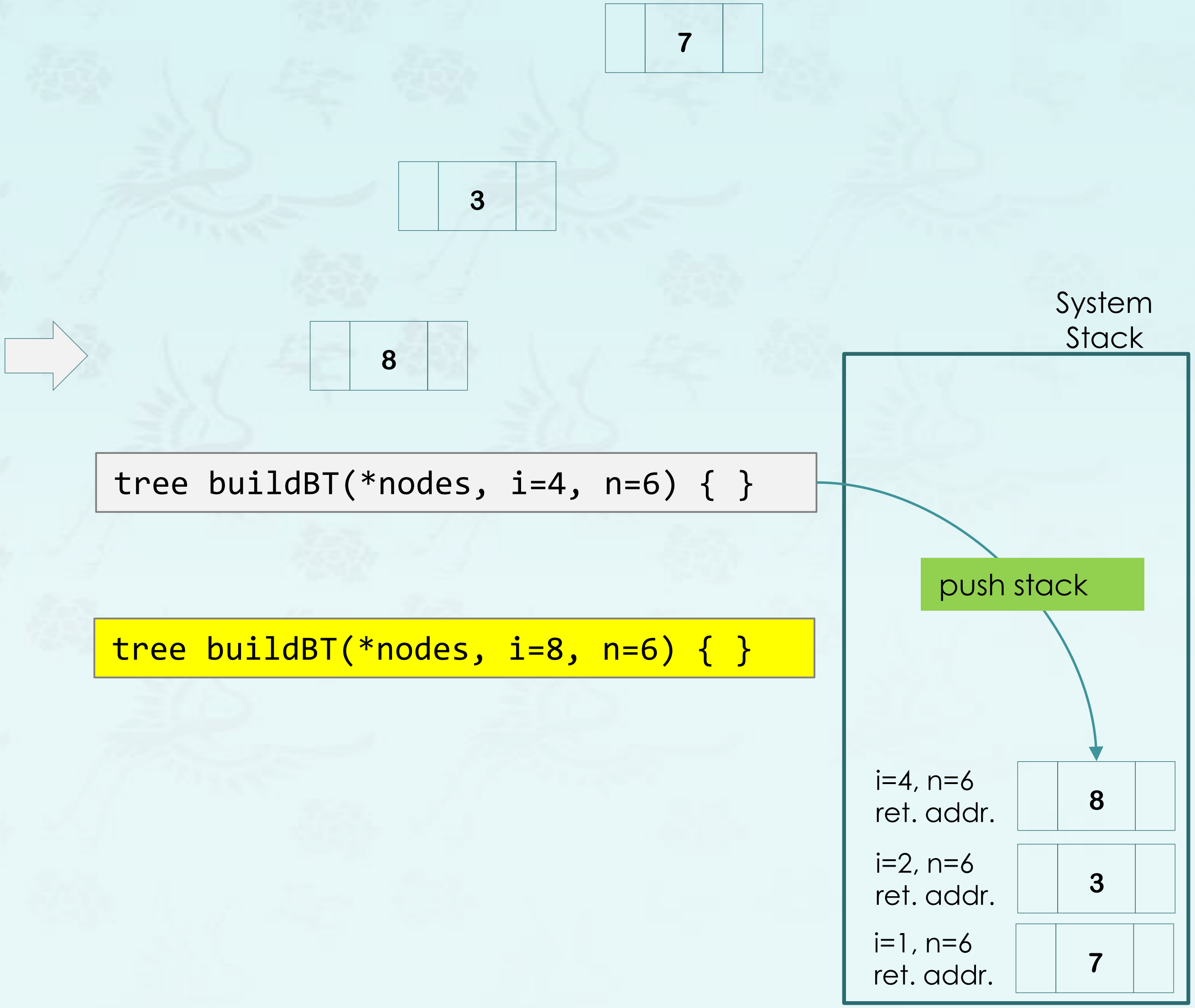
heapprint() – build a binary tree from heap/CBT using recursion



```
7
3 6
8 4 1
```

0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

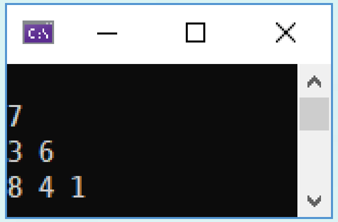
hp->nodes[] hp->N



```
tree buildBT(*nodes, i=4, n=6) { }
```

```
tree buildBT(*nodes, i=8, n=6) { }
```

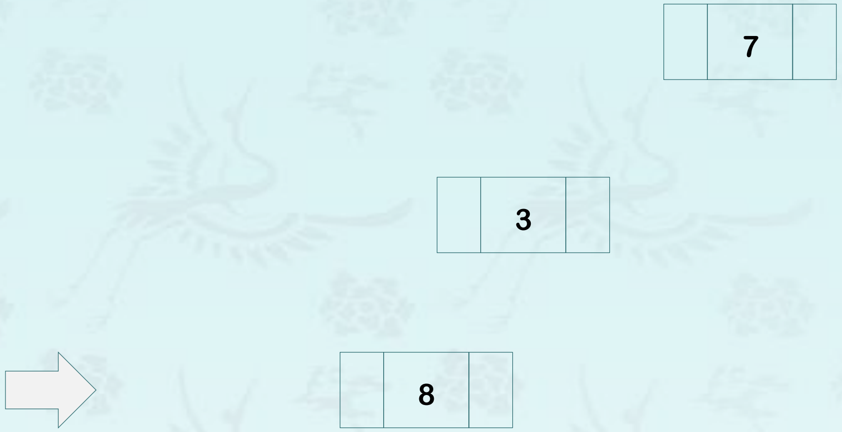
heapprint() – build a binary tree from heap/CBT using recursion



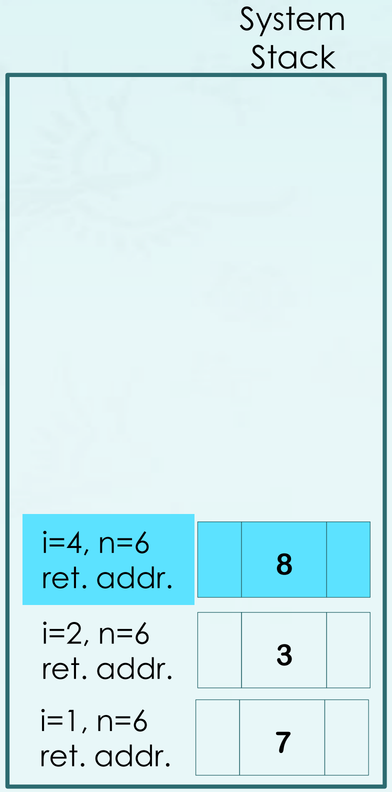
```
7
3 6
8 4 1
```

0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

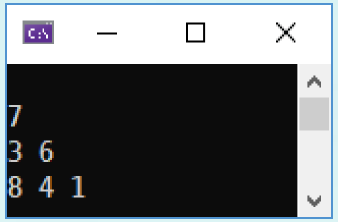
hp->nodes[] hp->N



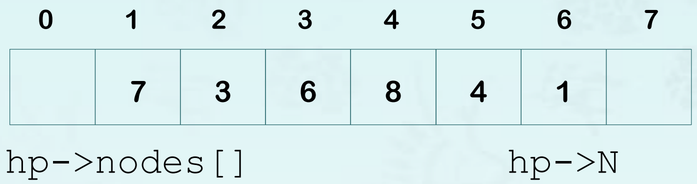
```
tree buildBT(*nodes, i=8, n=6) { }
```

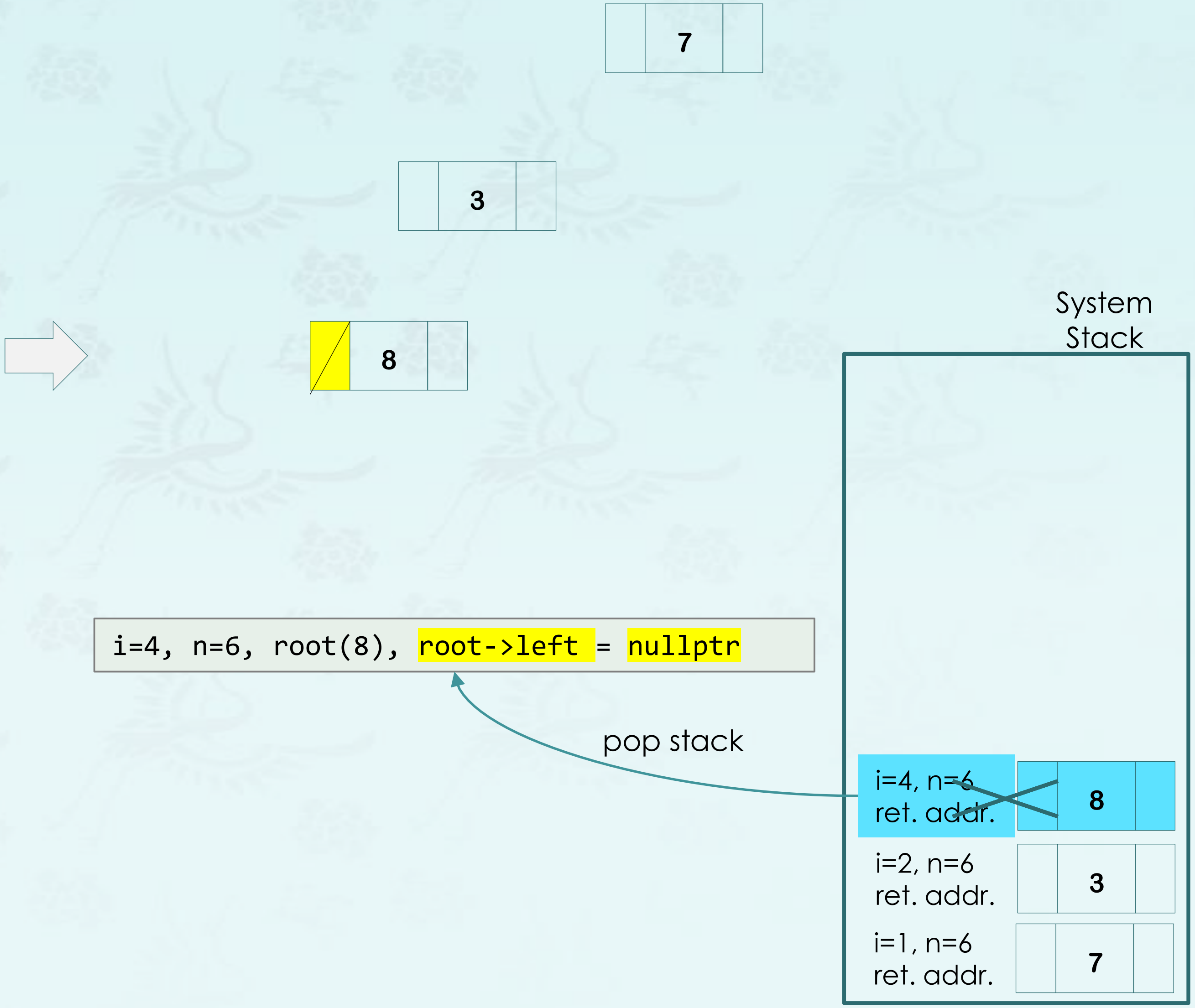


heapprint() – build a binary tree from heap/CBT using recursion



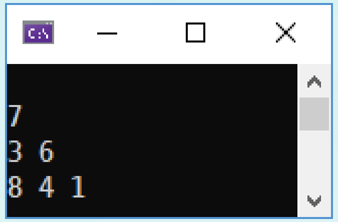
```
7
3 6
8 4 1
```





```
i=4, n=6, root(8), root->left = nullptr
```

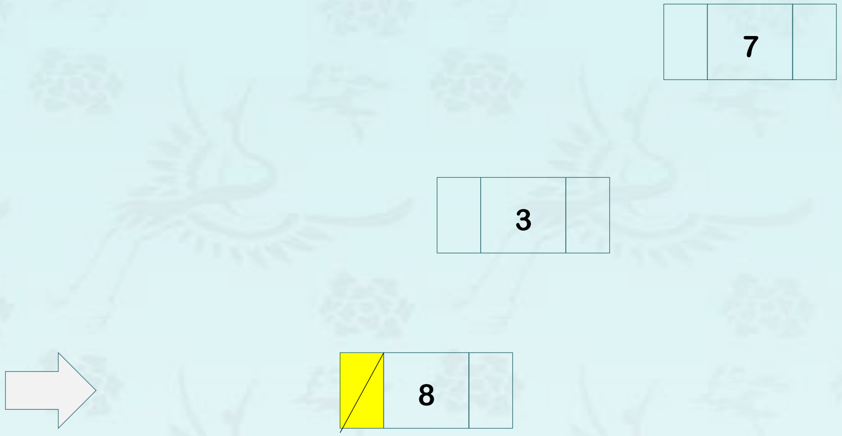
heapprint() – build a binary tree from heap/CBT using recursion



```
7
3 6
8 4 1
```

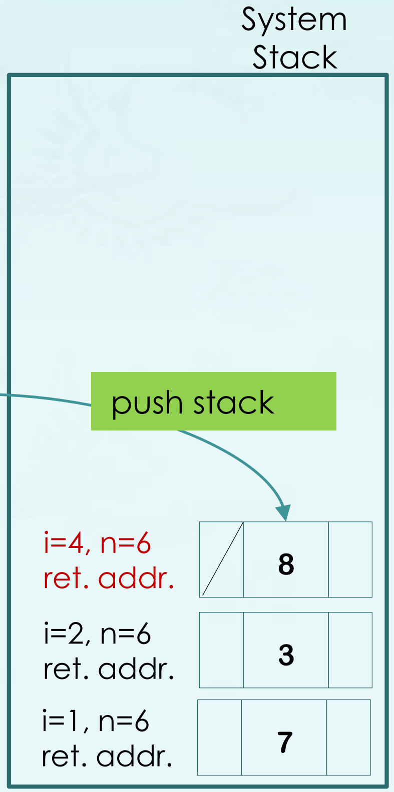
0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

hp->nodes[] hp->N

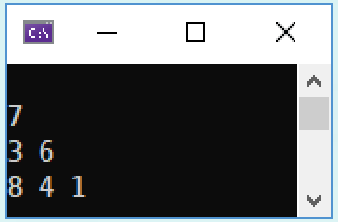


```
i=4, n=6, root(8), root->left = nullptr
```

```
tree buildBT(*nodes,i=4*2+1,n=6){}
```



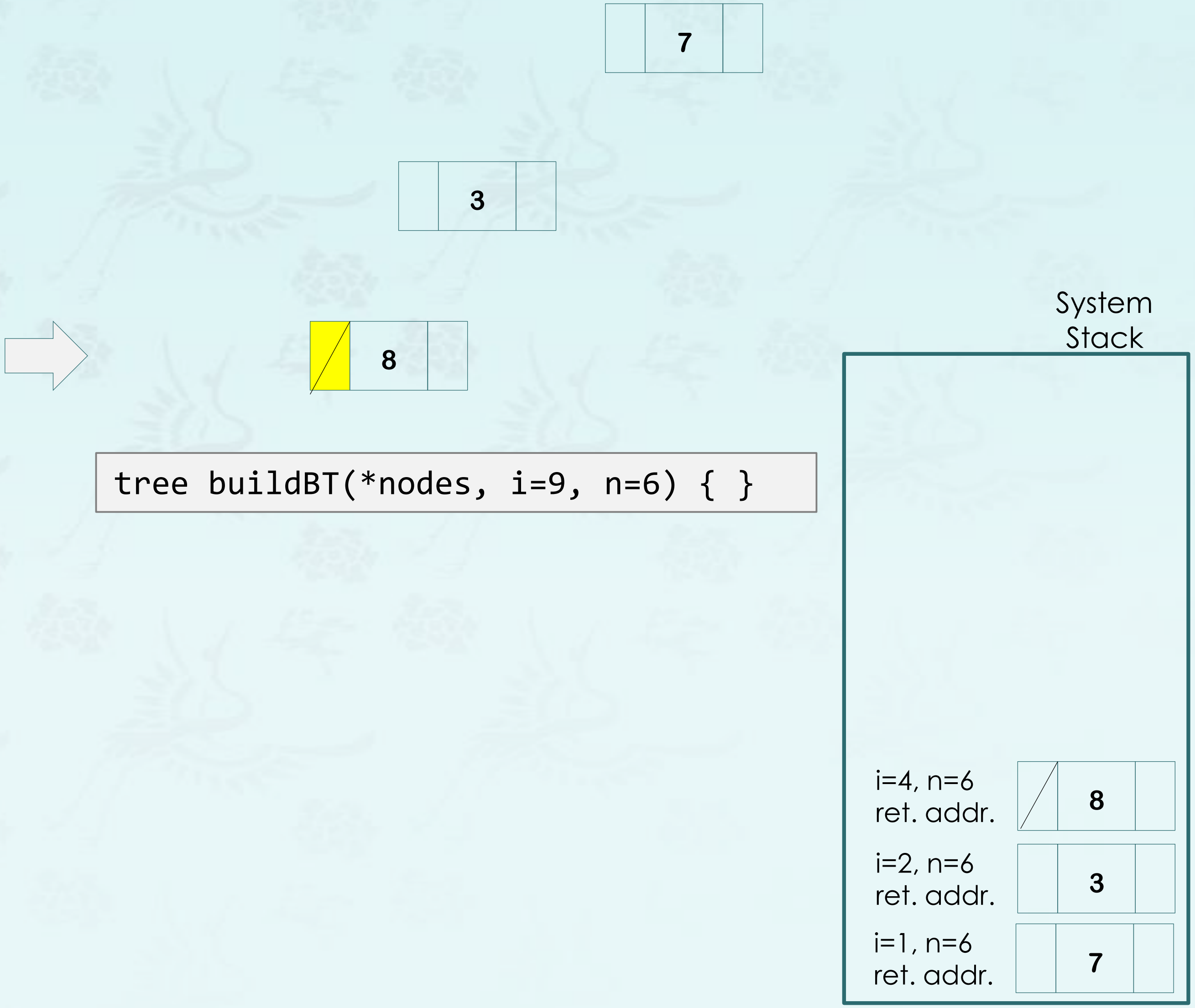
heapprint() – build a binary tree from heap/CBT using recursion



```
7
3 6
8 4 1
```

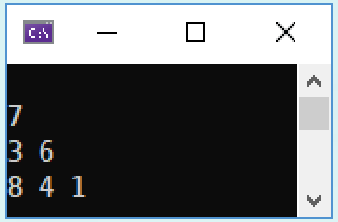
0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

hp->nodes[]   hp->N



```
tree buildBT(*nodes, i=9, n=6) { }
```

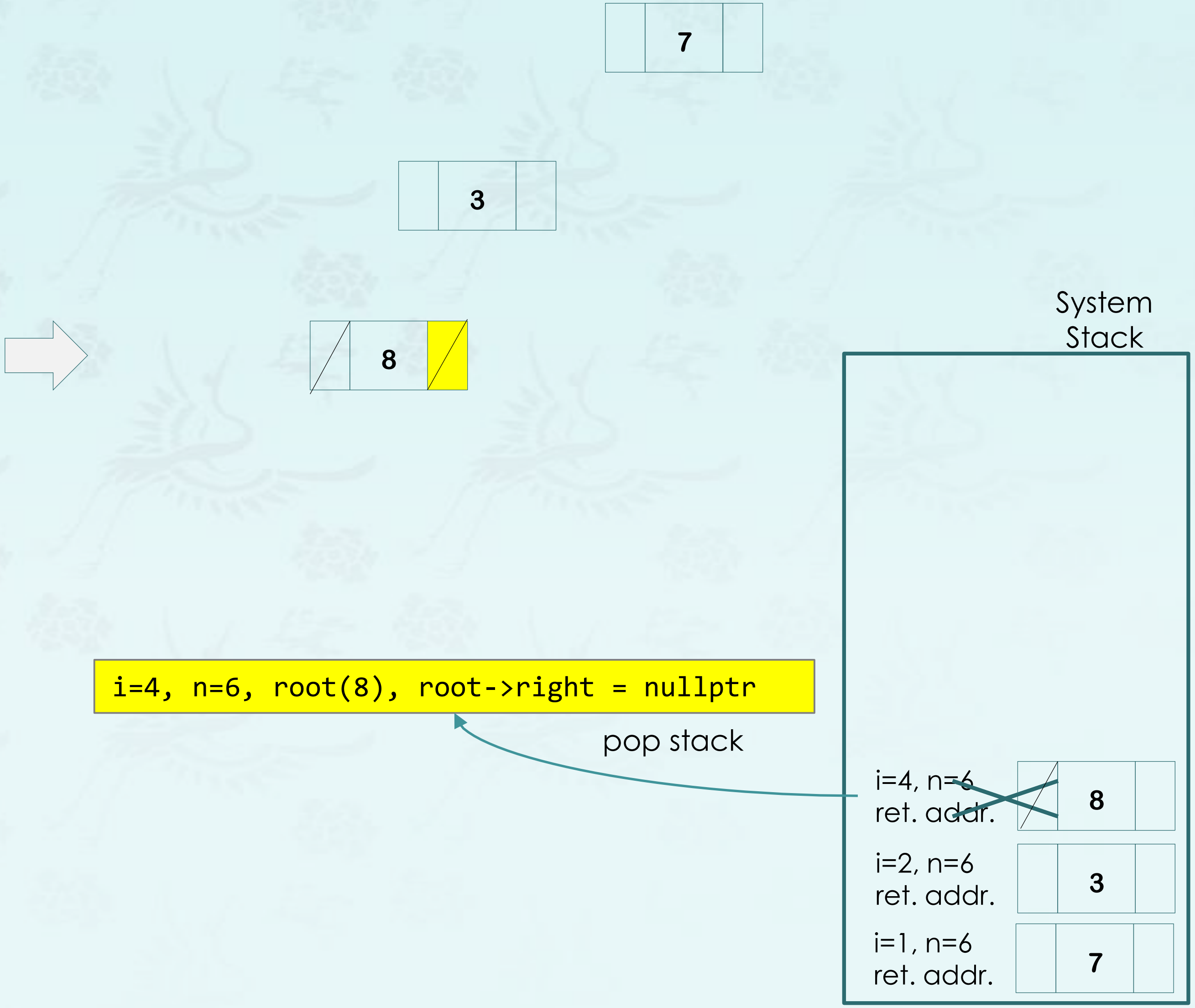
heapprint() – build a binary tree from heap/CBT using recursion



```
7
3 6
8 4 1
```

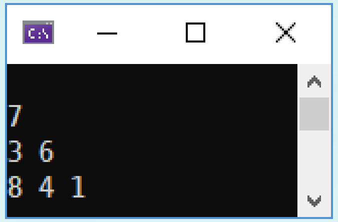
0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

hp->nodes[]   hp->N



`i=4, n=6, root(8), root->right = nullptr`

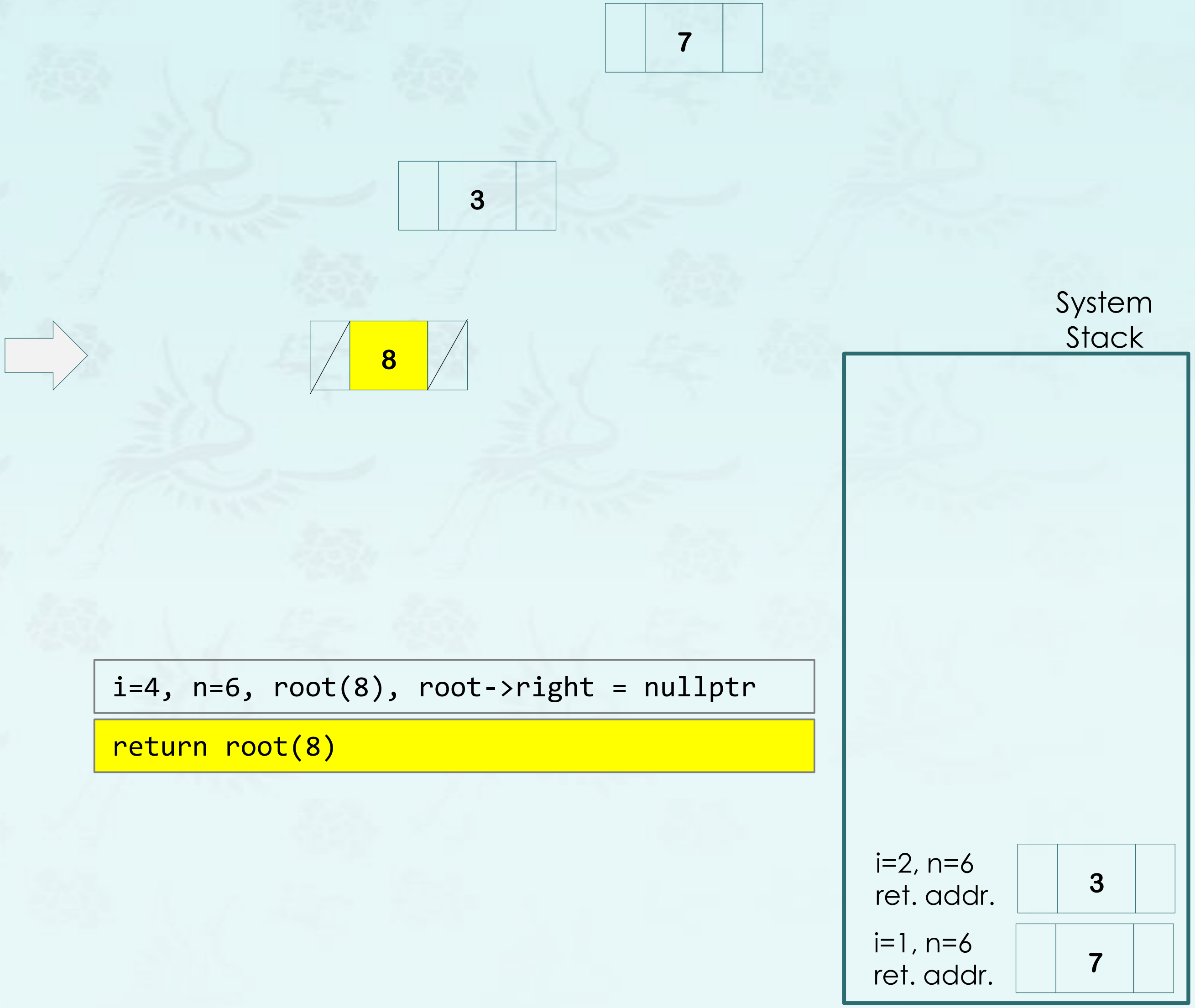
heapprint() – build a binary tree from heap/CBT using recursion



```
7
3 6
8 4 1
```

0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

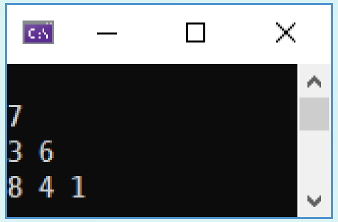
hp->nodes[] hp->N



```
i=4, n=6, root(8), root->right = nullptr
```

```
return root(8)
```

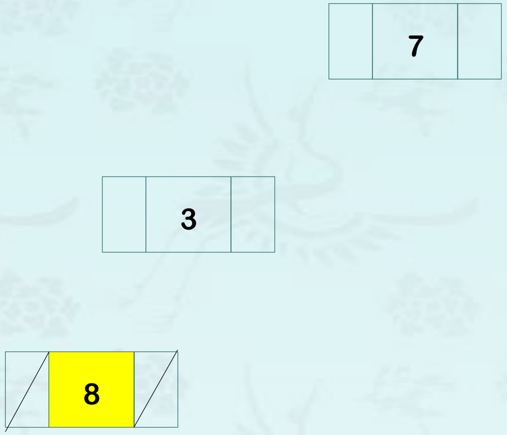
heapprint() – build a binary tree from heap/CBT using recursion



```
7
3 6
8 4 1
```

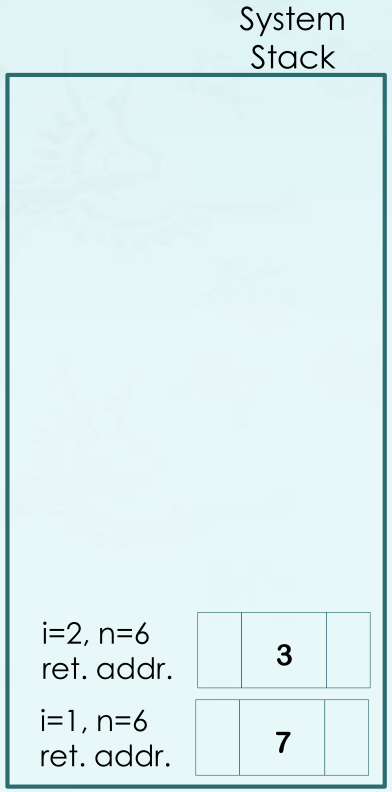
0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

hp->nodes[]　　　　hp->N

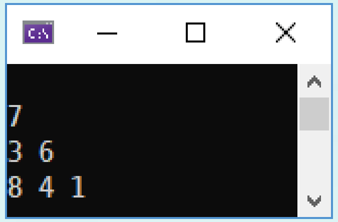


```
i=4, n=6, root(8), root->right = nullptr
```

```
return root(8)
```



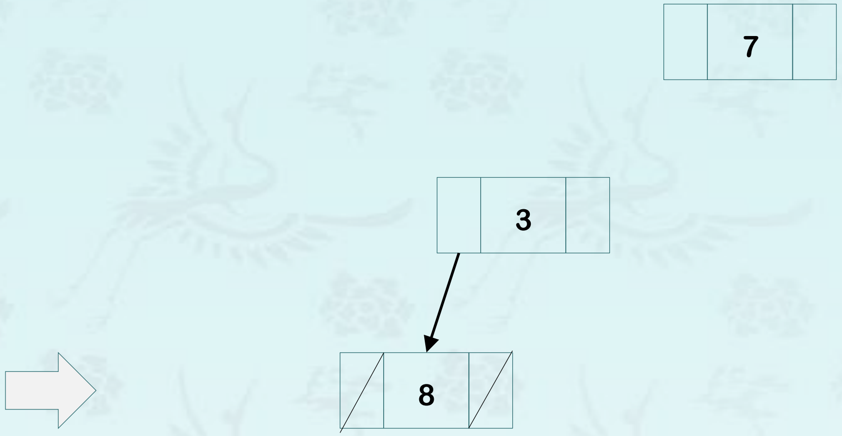
heapprint() – build a binary tree from heap/CBT using recursion



```
7
3 6
8 4 1
```

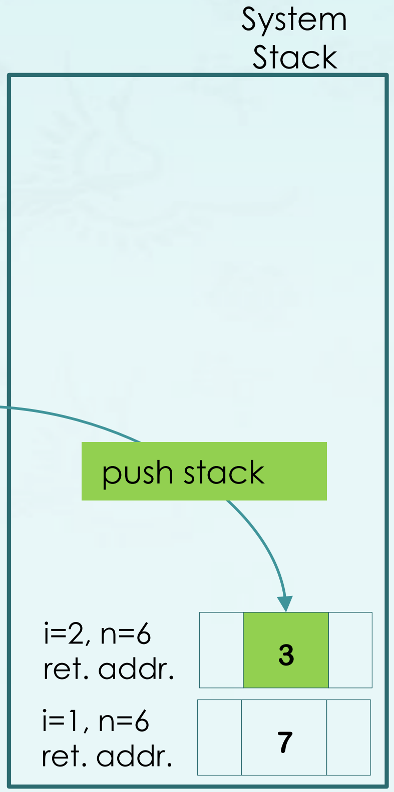
0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

`hp->nodes[]` `hp->N`

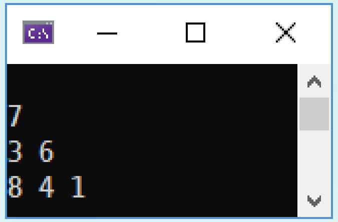


```
i=2, n=6, root(3)
```

```
tree buildBT(*nodes, i=5, n=6) { }
```



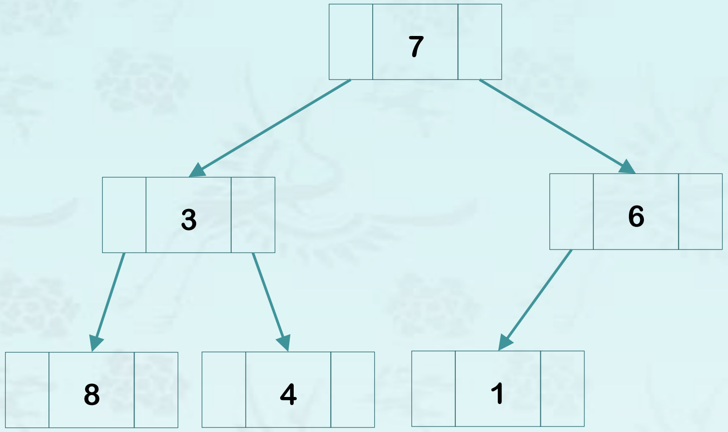
heapprint() – build a binary tree from heap/CBT using recursion

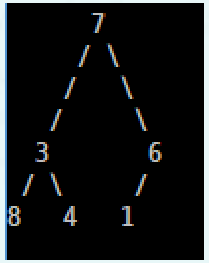


```
7
3 6
8 4 1
```

0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

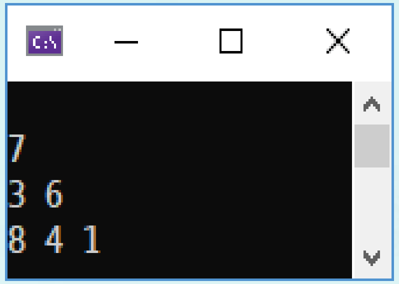
`hp->nodes[]` `hp->N`



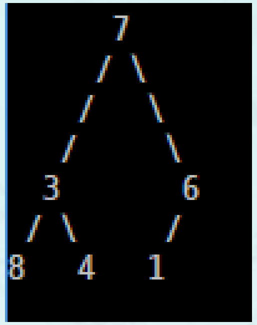


```
      7
     / \
    /   \
   /     \
  3       6
 / \     /
8   4   1
```

heapprint() – build a binary tree from heap/CBT using queue



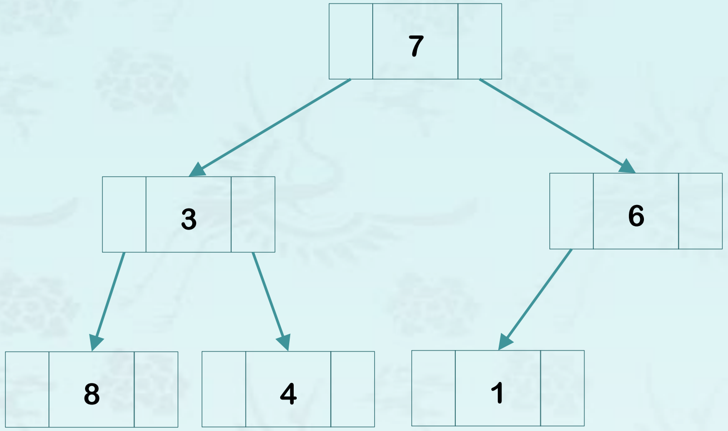
```
7
3 6
8 4 1
```



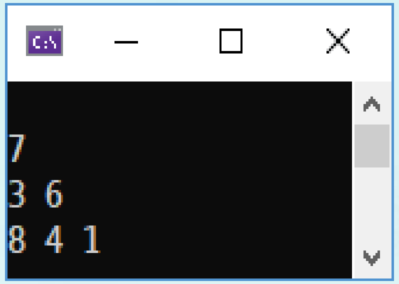
```
        7
       / \
      /   \
     /     \
    3       6
   / \     /
  8   4   1
```

0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

`hp->nodes[]` `hp->N`



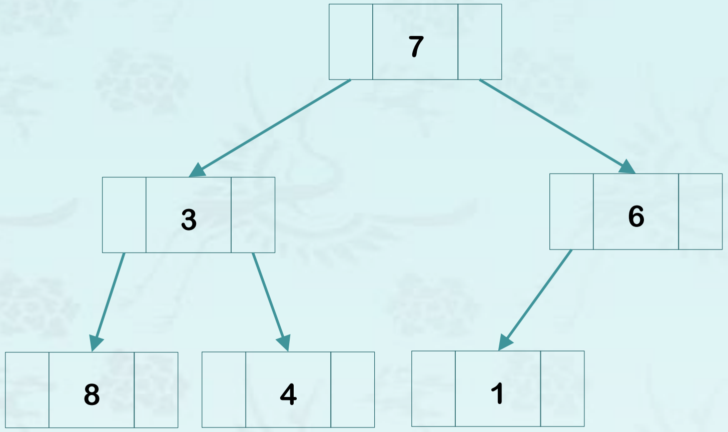
heapprint() – build a binary tree from heap/CBT using queue



```
7
3 6
8 4 1
```

0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

`hp->nodes[]` `hp->N`



1. Create the **tree (root) node** with the first key from CBT (or **nodes[1]**).
2. Enqueue the root node.
3. Loop through from **the CBT nodes[2] to nodes[N]**
   - A. Make a **new node from nodes[].**
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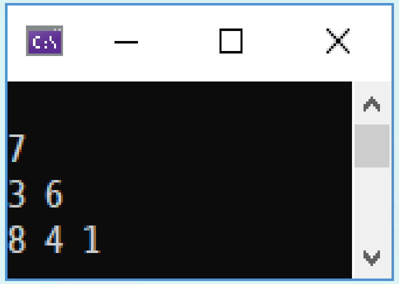
heapprint() – build a binary tree from heap/CBT using queue



```
7
3 6
8 4 1
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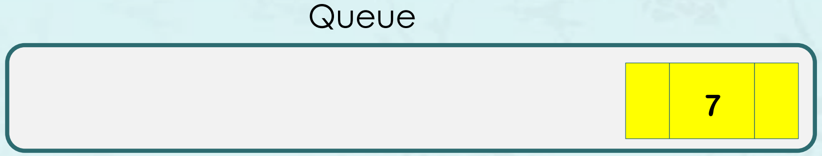

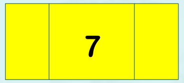

0	1	2	3	4	5	6	7
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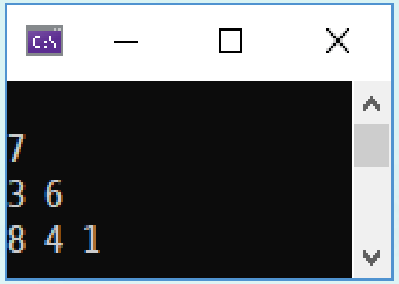

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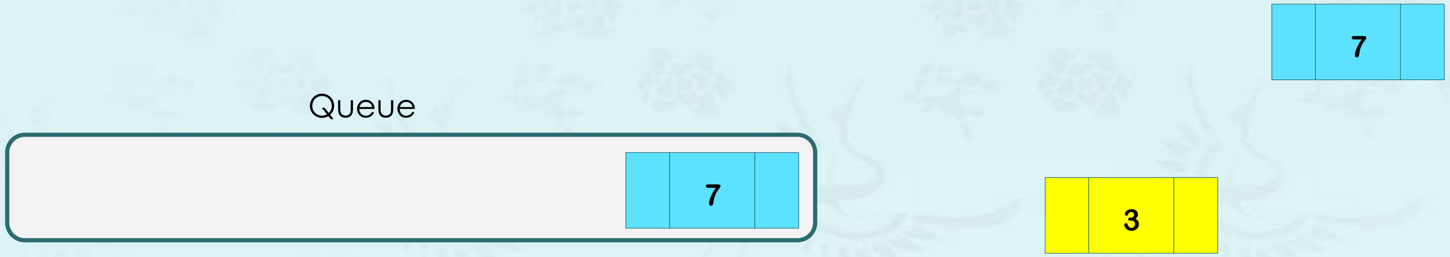

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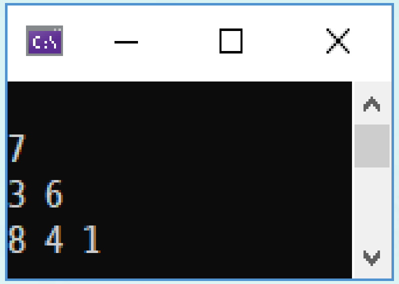

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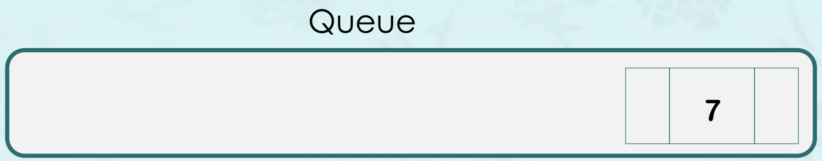

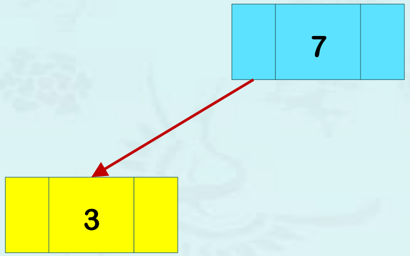

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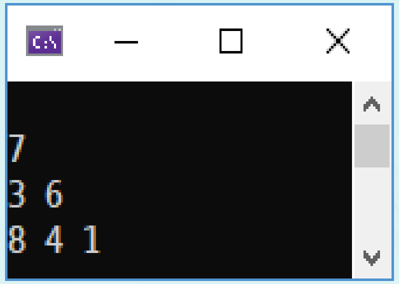

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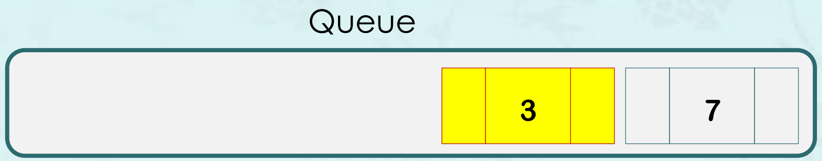

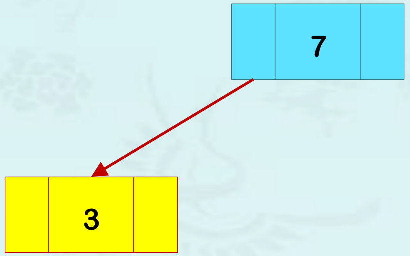

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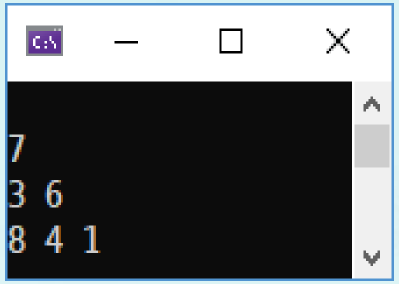

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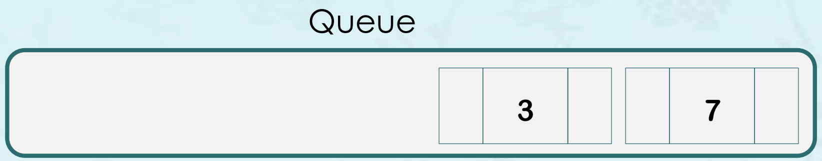

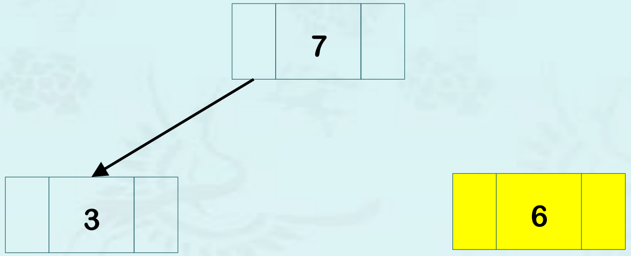

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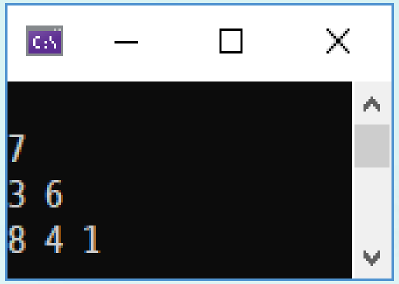

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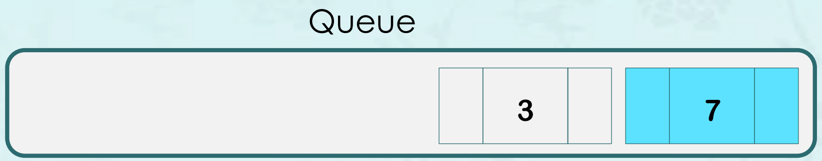

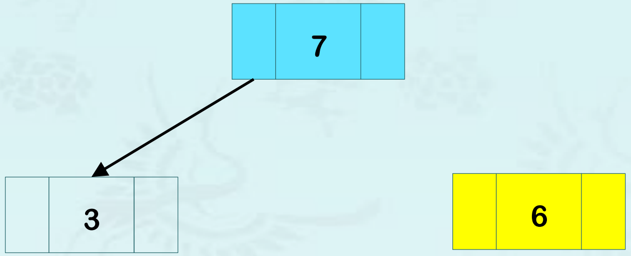

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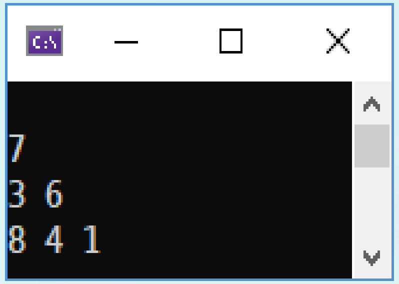

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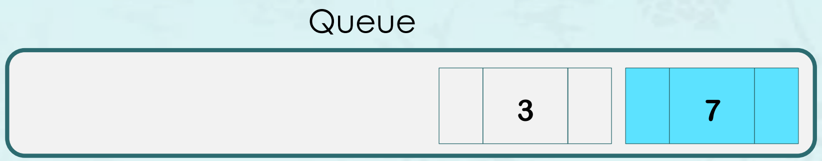

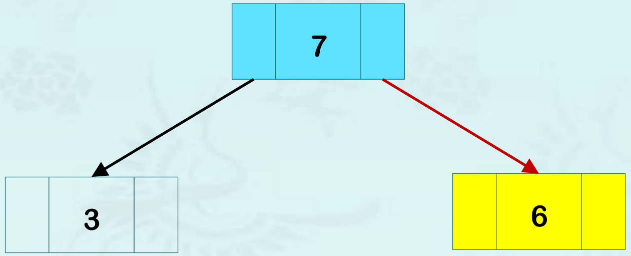

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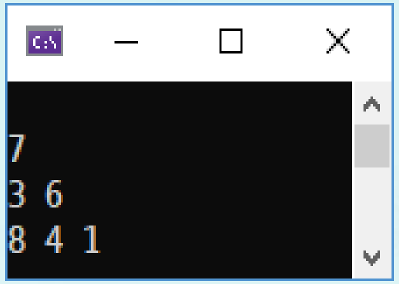

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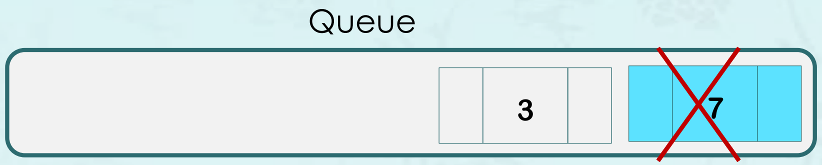

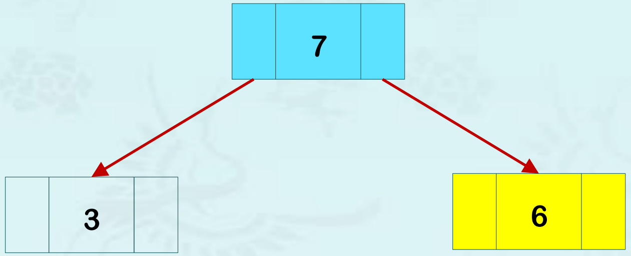

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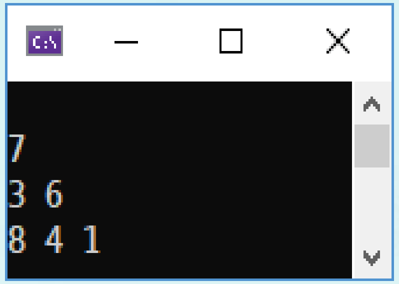

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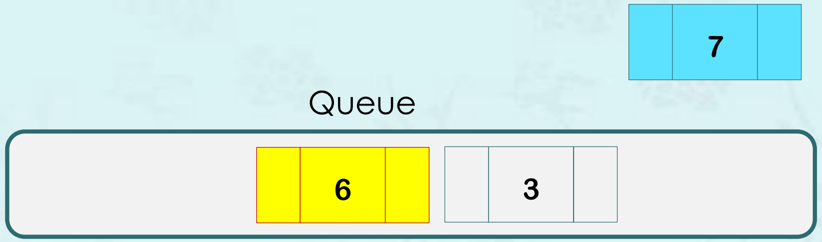

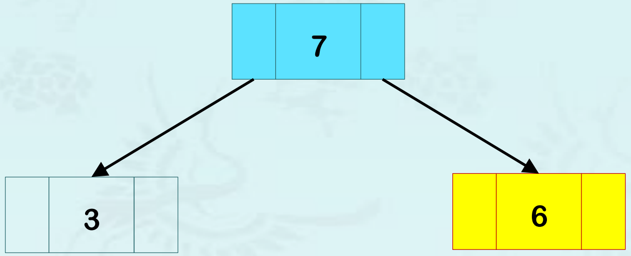

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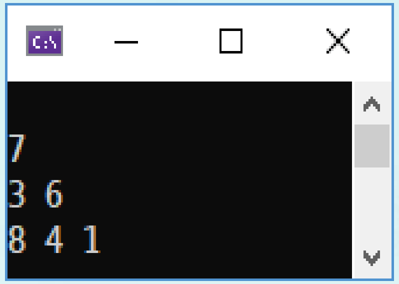

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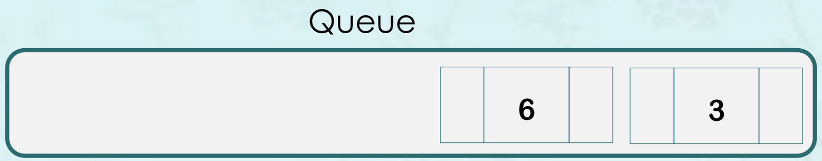

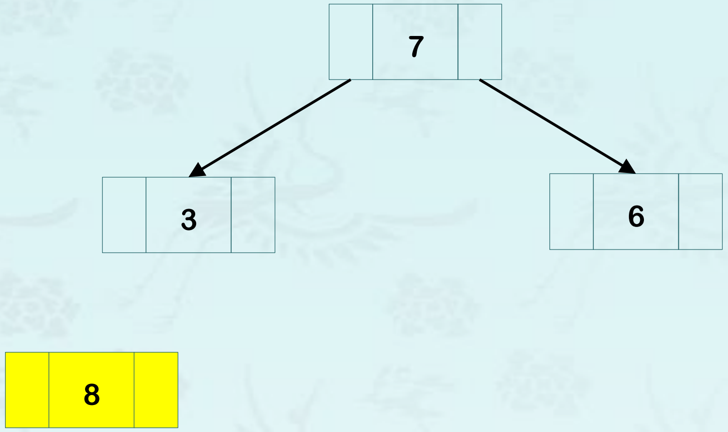

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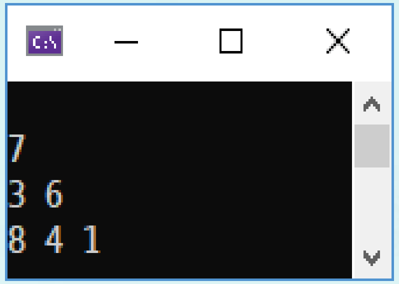

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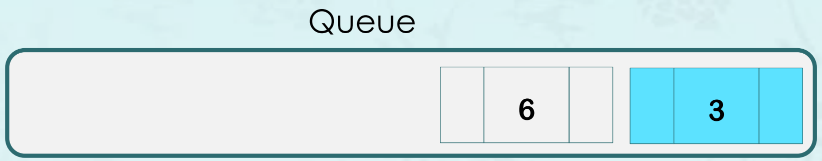

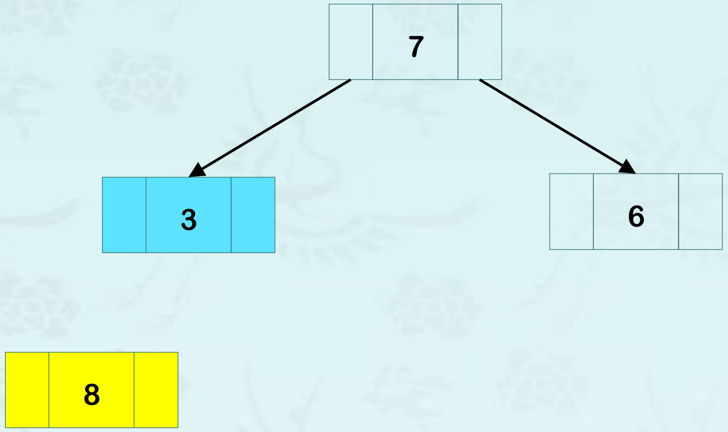

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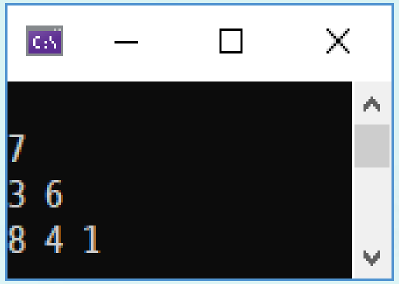

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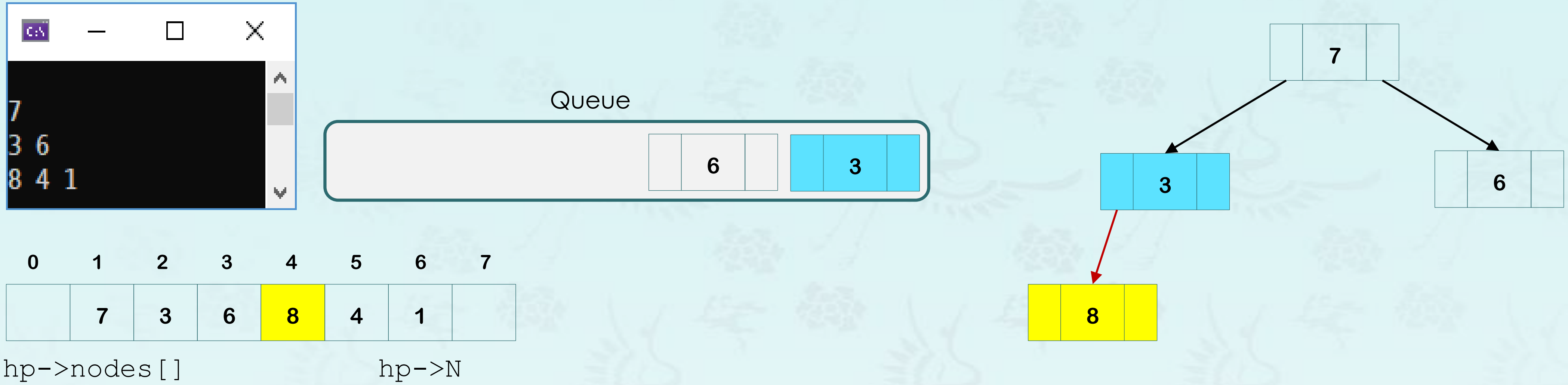

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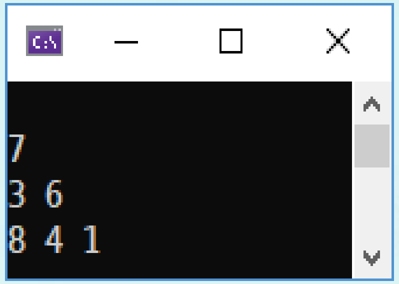

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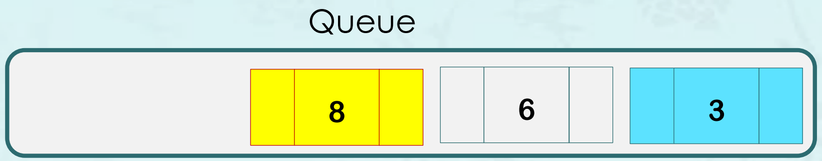

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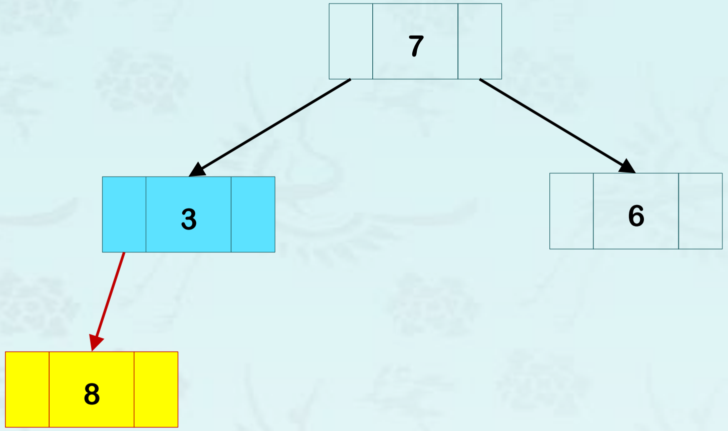

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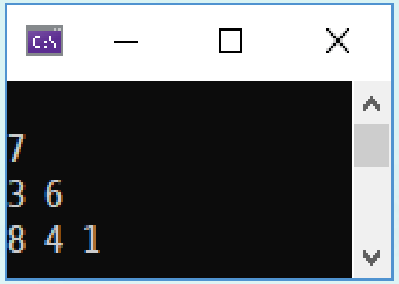

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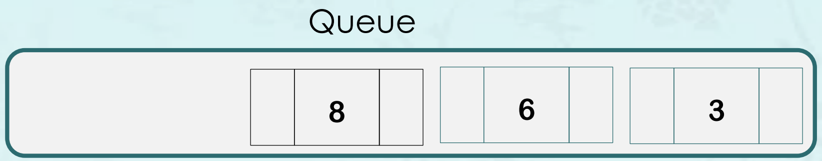

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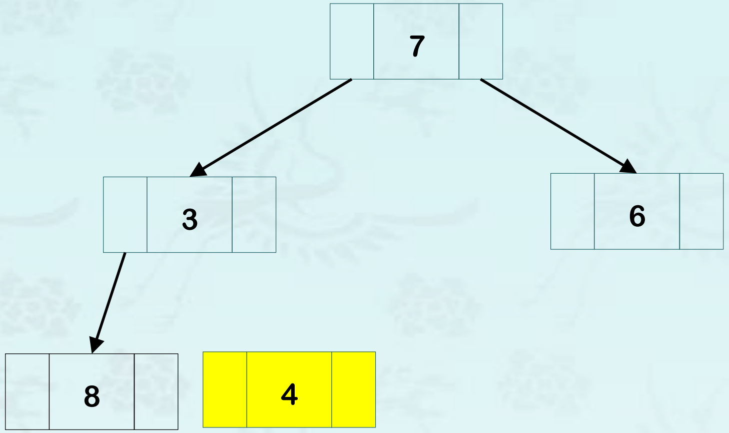

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      D.  If this tree node is full, pop (or dequeue) it.
      E.  enqueue the new node (to add children later if any).
4.  treeprint(root)
```
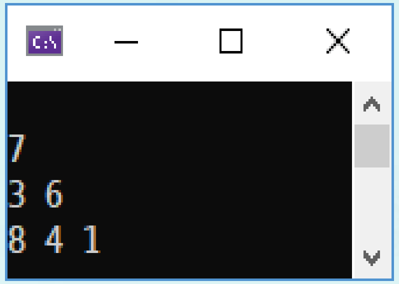
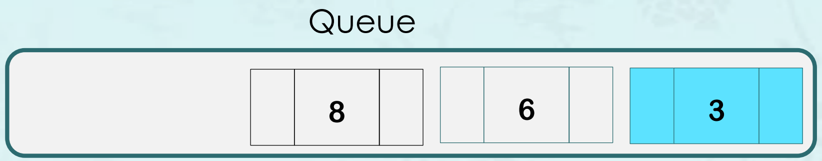

heapprint() – build a binary tree from heap/CBT using queue

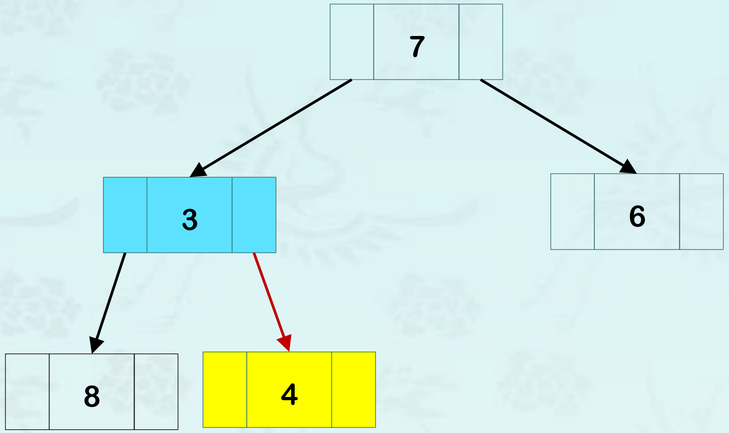


```
7
3 6
8 4 1
```



0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

`hp->nodes[]` `hp->N`



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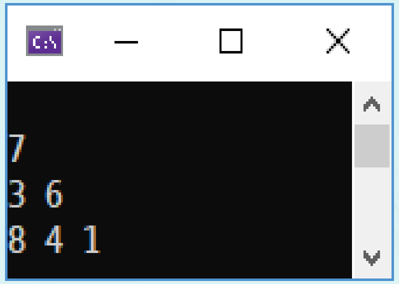

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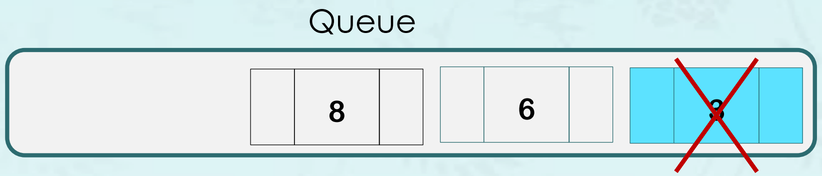

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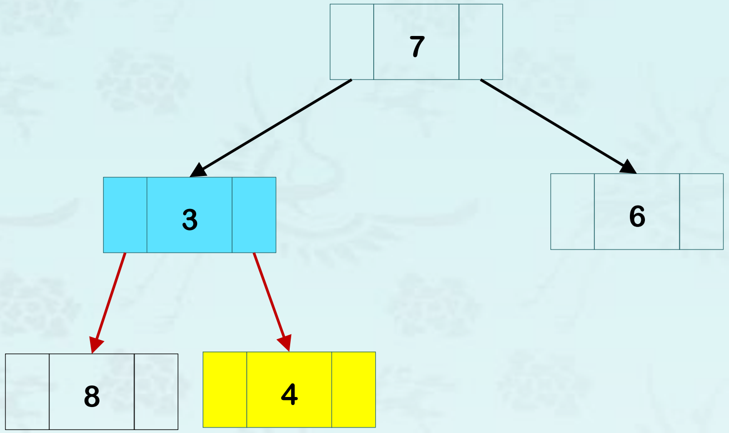
hp->nodes[]   hp->N



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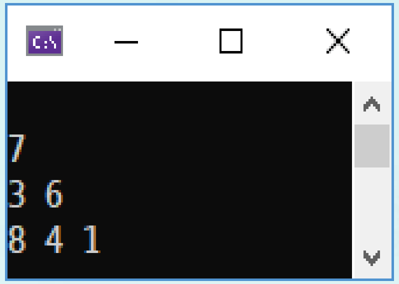

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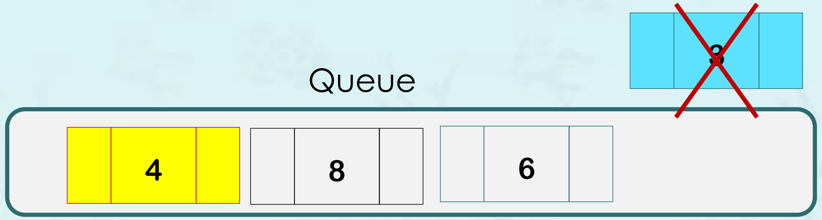

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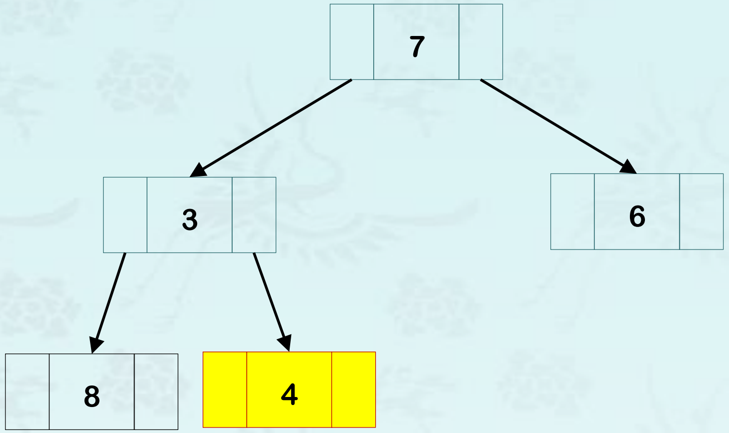

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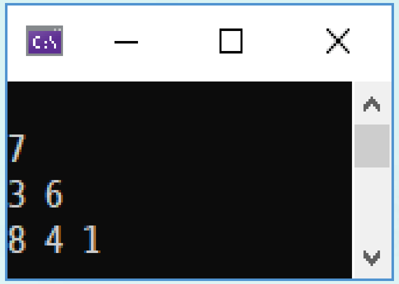

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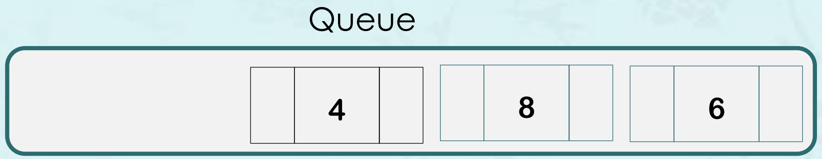

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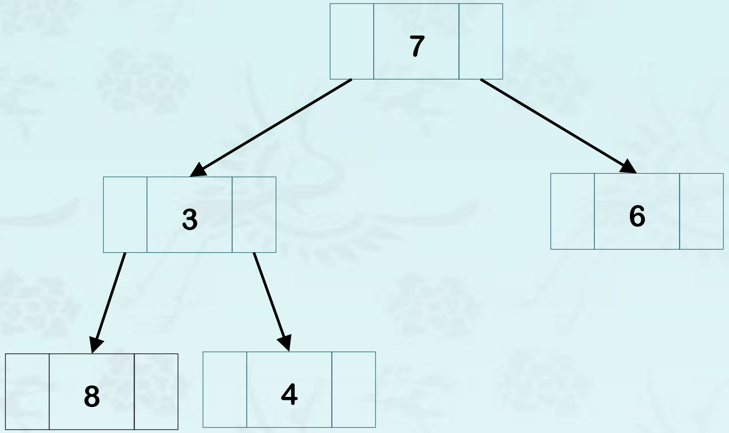

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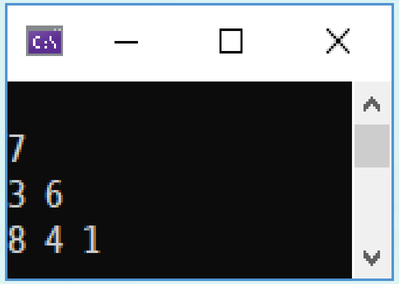

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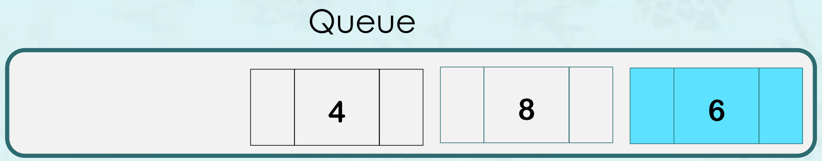

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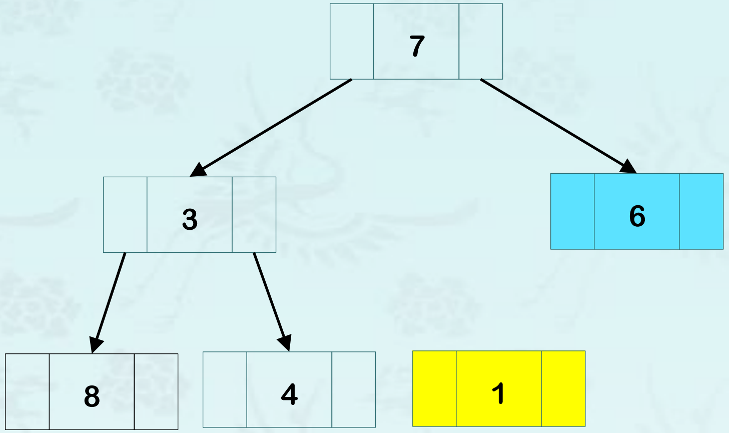

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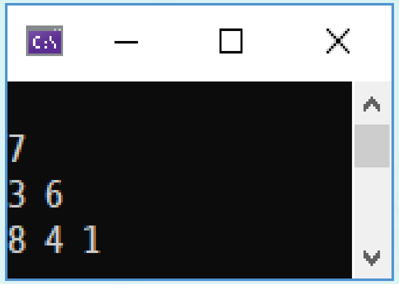

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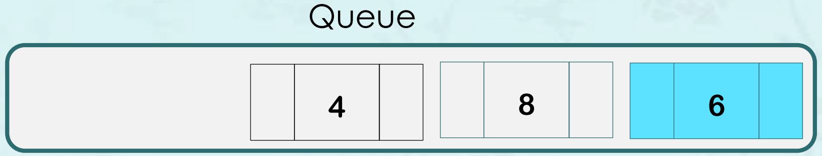

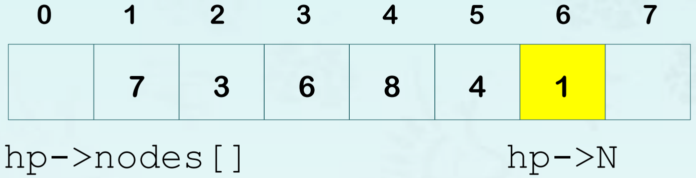

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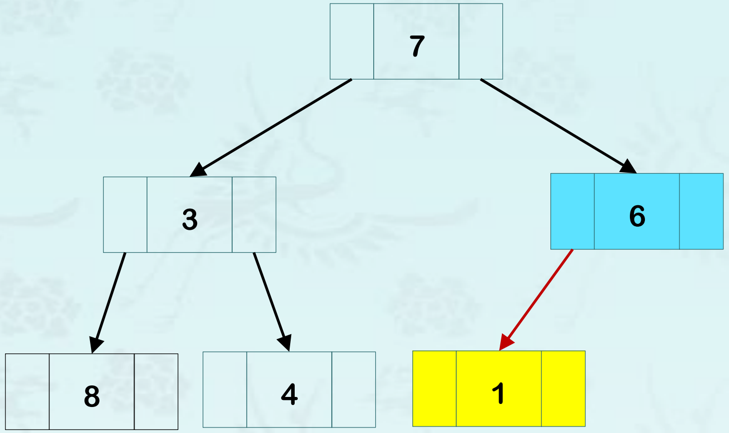

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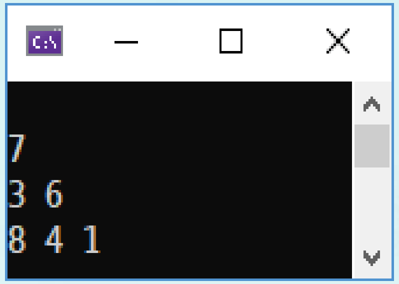

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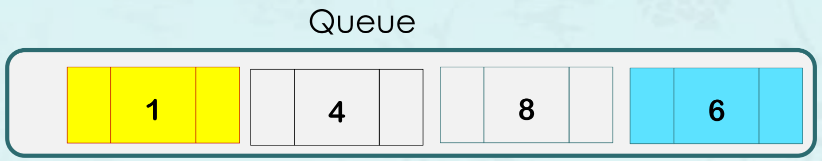

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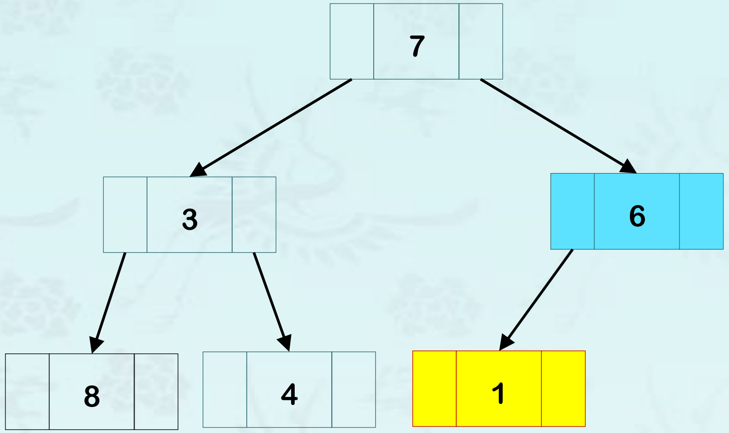
hp->nodes[]　　hp->N



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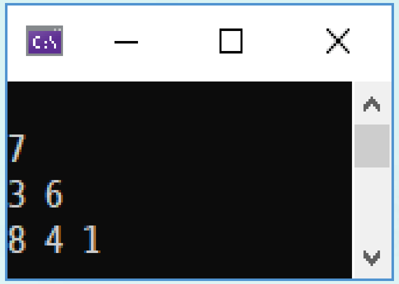

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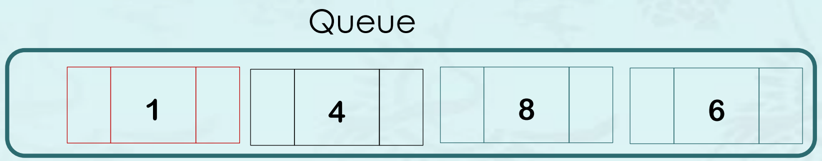

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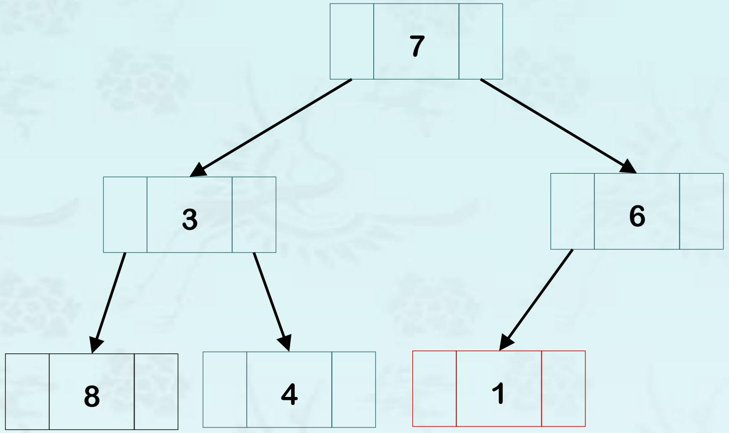
hp->nodes[] **hp->N**



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Data Structures
Chapter 5: Heap and Priority Queue

1. Heap & Priority Queue
2. Heapsort
3. **Heap & PQ Coding**