Data Structures
Chapter 7: Graph

1. Introduction
   - Terminology, Representation, ADT
2. **Basic Operations**
   - **DFS,** CC, BFS, Processing
3. Digraph and Applications
4. Minimum Spanning Tree(MST)

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죄의 삯은 사망이요 하나님의 은사는 그리스도 예수 우리 주 안에 있는 영생이니라 (로마서 6:23)

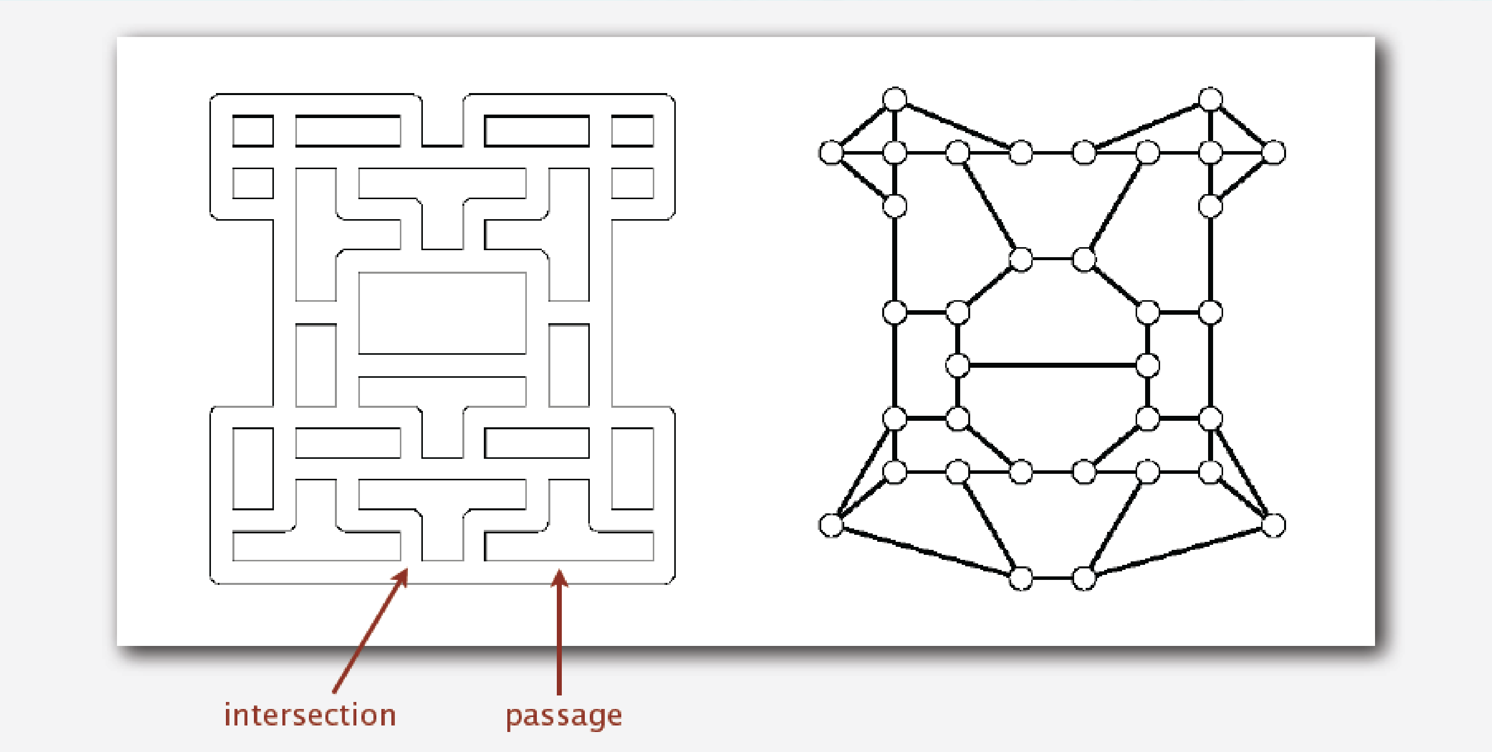
모든 사람이 죄를 범하였으매 하나님의 영광에 이르지 못하더니 그리스도 예수 안에 있는 속량으로 말미암아 하나님의 은혜로 값없이 의롭다 하심을 얻은 자 되었느니라 (로마서 3:23-24)

DFS: Depth-First Search

Algorithm:

- Vertex = intersection
- Edge = passage

pacman

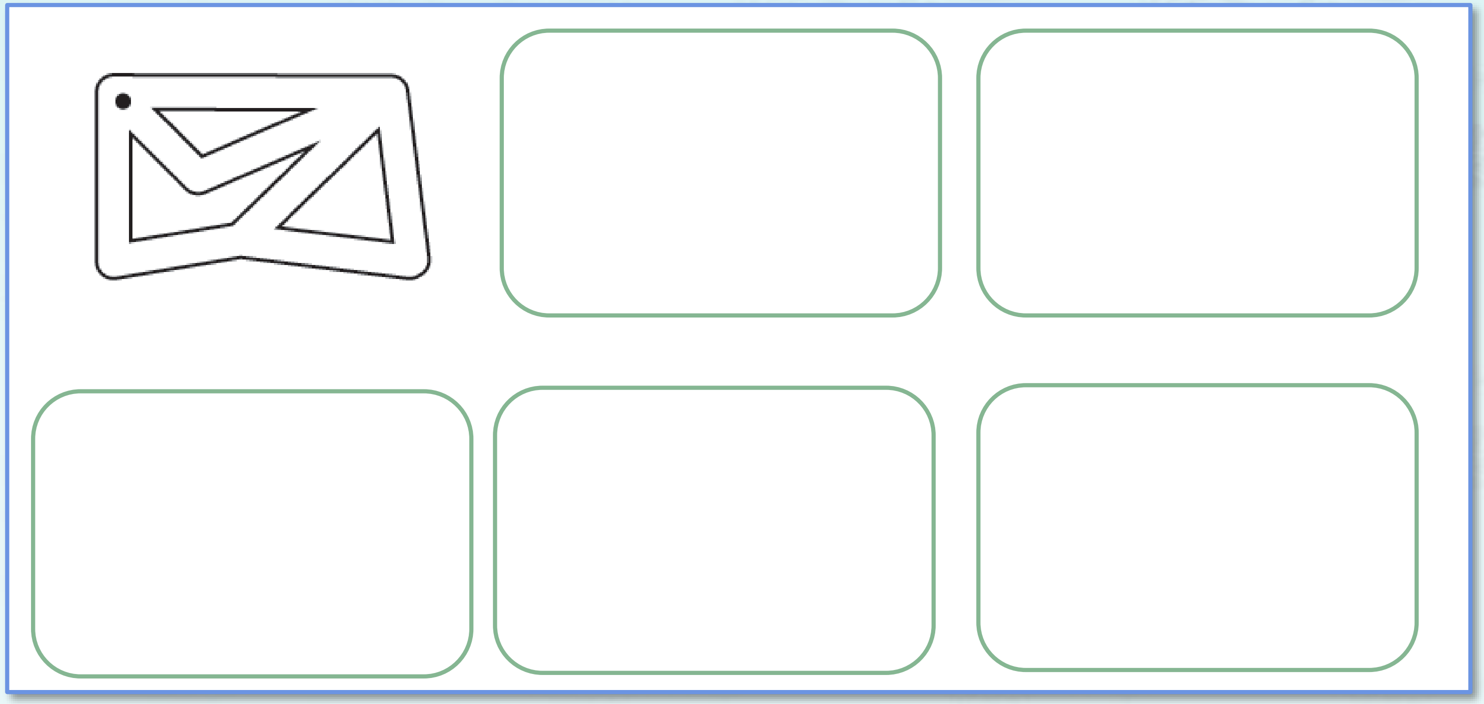


Maze Goal: Explore every intersection in the maze.

DFS: Depth-First Search

Maze graph:

- Unroll a ball of string behind you.
- Mark each visited intersection and each visited passage.
- Retrace steps when no unvisited options



Maze Goal: Explore every intersection in the maze.

Good Visualization: https://www.cs.usfca.edu/~galles/visualization/DFS.html

DFS: Depth-First Search

Maze graph:

- Unroll a ball of string behind you.
- Mark each visited intersection and each visited passage.
- Retrace steps when no unvisited options

Theseus, a hero of Greek mythology, is best known for slaying a monster called the Minotaur. When Theseus entered the Labyrinth where the Minotaur lived, he took a ball of yarn to unwind and mark his route. Once he found the Minotaur and killed it, Theseus used the string to find his way out of the maze.

Read more: http://www.mythencyclopedia.com/Sp-Tl/Theseus.html#ixzz30wFO3ofe

Maze Goal: Explore every intersection in the maze.

DFS: Depth-First Search

Maze graph:

- Unroll a ball of string behind you.
- Mark each visited intersection and each visited passage.
- Retrace steps when no unvisited options

Shannon and his famous electromechanical mouse *Theseus* (named after Theseus from Greek mythology) which he tried to have solve the maze in one of the first experiments in artificial intelligence.

The Las Vegas connection: Shannon and his wife Betty also used to go on weekends to Las Vegas with MIT mathematician Ed Thorp, and made very successful forays in blackjack using game theory.

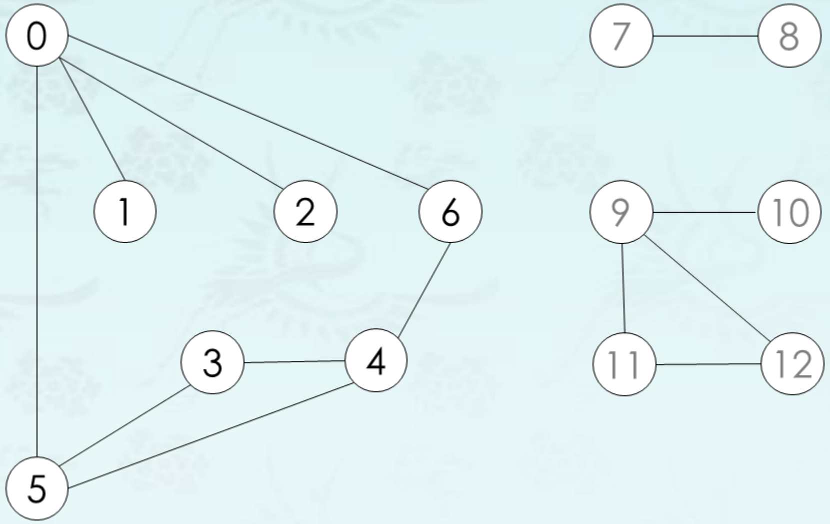
Maze Goal: Explore every intersection in the maze.

DFS: Depth-First Search

- **Design pattern:** Decouple graph data type
- **Idea:** Mimic maze exploration

DFS (to visit a vertex v)

- Mark v as visited.
- Recursively visit all unmarked vertices w adjacent to v.



Typical applications:
- Find all vertices connected to a given source vertex.
- Find a path between two vertices.

Challenge:
- How to implement?

DFS: Depth-First Search

Goal: Systematically search through a graph from graph processing

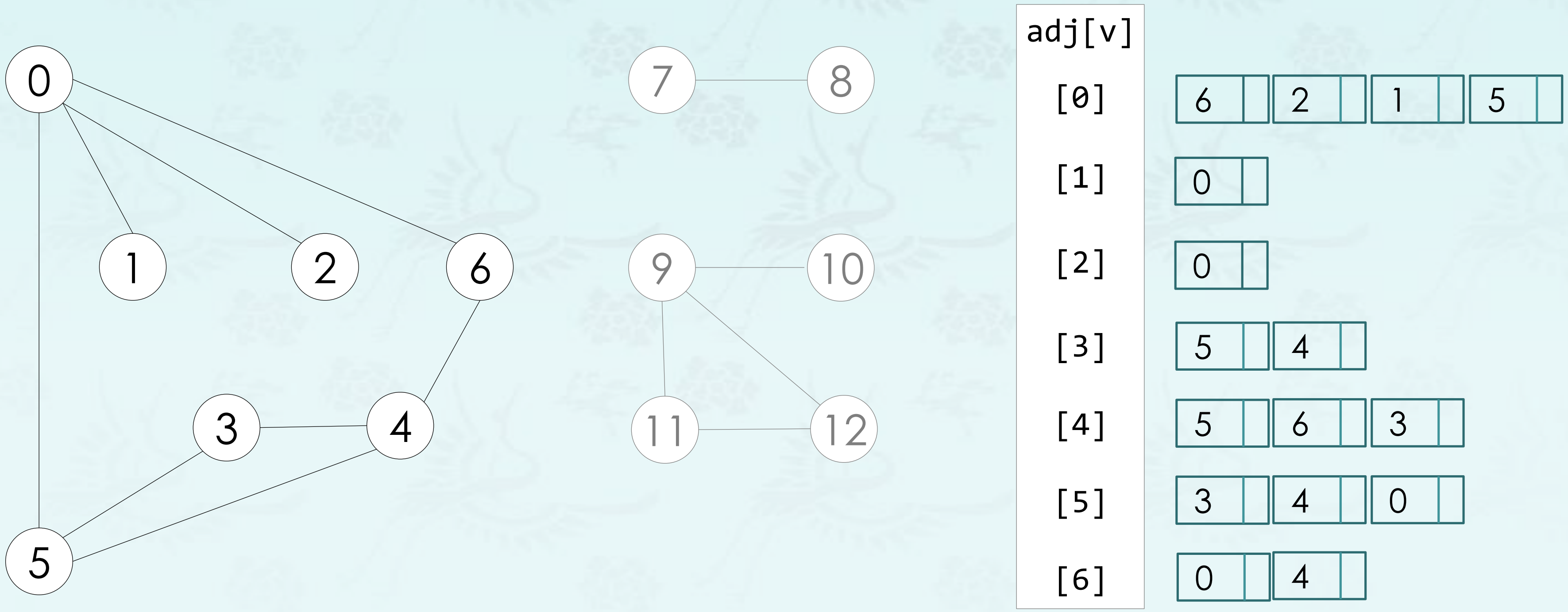
- **Create a graph object**
- **Pass the graph to a graph processing routine**
- **Query the graph-processing routine**
    - path from v to w
    - distance from v to w
    - connected
    - bipartite
    - cyclic

Graph – Build Adjacency list

- For each `edge(v, w)` in the list
- Insert front each vertex both `(adj[v], w)` and `(adj[w], v)`
  `addEdgeFromTo(g, v, w); // from v to w.`

Edge list

```
graph6.txt
13   ← V
13   ← E
0 5
4 3
0 1
9 12
6 4
5 4
0 2
11 12
9 10
0 6
7 8
9 11
5 3
```



Graph g:

Challenge: build adjacency lists?

Graph – ADT

```
// a structure to represent an adjacency list of vertices
struct Gnode {
  int    item;
  Gnode* next;
  Gnode (int i, Gnode *p = nullptr) {
    item = i;  next = p;
  }
  ~Gnode() {}
};

using gnode = Gnode *;
```

adjacent vertices using a singly linked list

next vertex to link if any.

Graph ADT – graph.h

```
struct Graph {
  int V;                  // N vertices
  int E;                  // N edges
  gnode adj;              // array of linked lists of vertices
  Graph(int v = 0) { // constructs a graph with v vertices
    V = v;
    E = 0;
    adj = new (nothrow) Gnode[v];
    assert(adj != nullptr);

    for (int  i = 0; i < v; i++) { // initialize adj list as empty;
        adj[i].next = nullptr;
        adj[i].item = i;
    }
  }
  ~Graph() {}
};
using graph = Graph *;
```

set each adj list nullptr to begin with

unused;
but may store the degree of vertex i.

```
graph g = new Graph(v);
for (int i = 0; i < E; i++)
  addEdge(g, from[i], to[i]);
```

Graph ADT – graph.cpp

```
// add an edge from v to w to an undirected graph
// A new vertex is added at the beginning of adj list of v.
void addEdgeFromTo(graph g, int v, int w) {

  gnode node = new Gnode(w);
  g->adj[v].next = node;
  g->E++;
}
```

With a bug

```
// add an edge to an undirected graph
void addEdge(graph g, int v, int w) {
  addEdgeFromTo(g, v, w);        // edge from v to w
  addEdgeFromTo(g, w, v);        // since undirected
}
```

Graph ADT – graph.cpp

```
// add an edge from v to w to an undirected graph
// A new vertex is added at the beginning of adj list of v.
void addEdgeFromTo(graph g, int v, int w) {

  gnode node = new Gnode(w);
  g->adj[v].next = node;
  g->E++;
}
```

With a bug

instantiate a node w and make a link with vertex v. what is wrong?

```
// add an edge to an undirected graph
void addEdge(graph g, int v, int w) {
  addEdgeFromTo(g, v, w);        // edge from v to w
  addEdgeFromTo(g, w, v);        // since undirected
}
```

Graph ADT – graph.cpp

```
// add an edge from v to w to an undirected graph
// A new vertex is added at the beginning of adj list of v.
void addEdgeFromTo(graph g, int v, int w) {

  gnode node = new Gnode(w, g->adj[v].next);
  g->adj[v].next = node;
  g->E++;
}

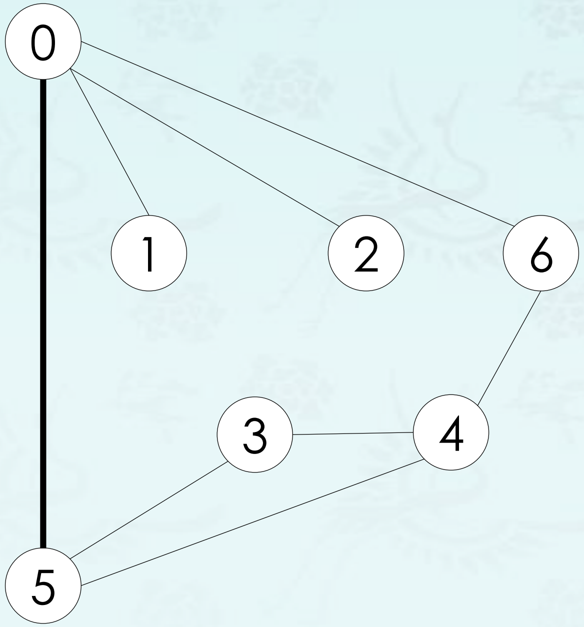
// add an edge to an undirected graph
void addEdge(graph g, int v, int w) {
  addEdgeFromTo(g, v, w);        // edge from v to w
  addEdgeFromTo(g, w, v);        // since undirected
}
```

instantiate a node w and insert it at **the front of** adj[v]

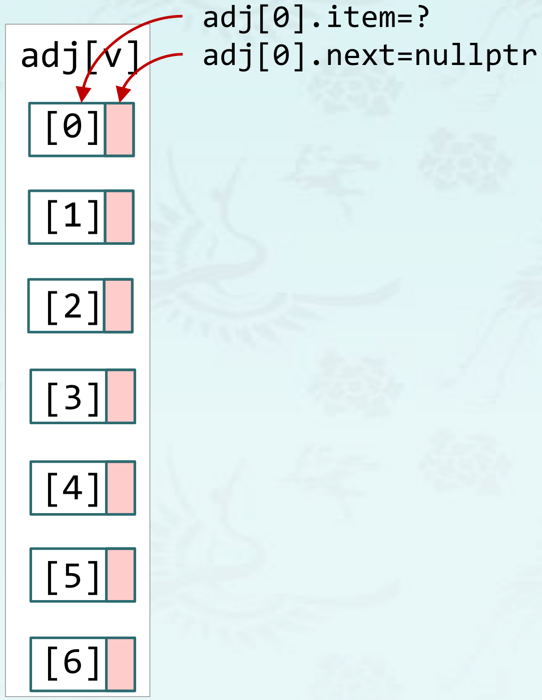
Graph – Build Adjacency list

```
void addEdgeFromTo(graph g, int v, int w) {
  gnode node = new Gnode(w, g->adj[v].next);
  g->adj[v].next = node;
  g->E++;
}
```

Graph g



Adjacency lists



Edge list

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4 3
0 1
9 12
6 4
5 4
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9 10
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7 8
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```
struct Graph {
  int V, E;
  gnode adj;
  Graph(int v = 0) {
    V = v;
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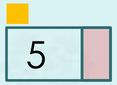
    adj = new (nothrow) Gnode[v];
    assert(adj != nullptr);

    for (int i=0; i<v; i++)
      adj[i].next = nullptr;
  }
  ~Graph() {}
};
using graph = Graph *;
```

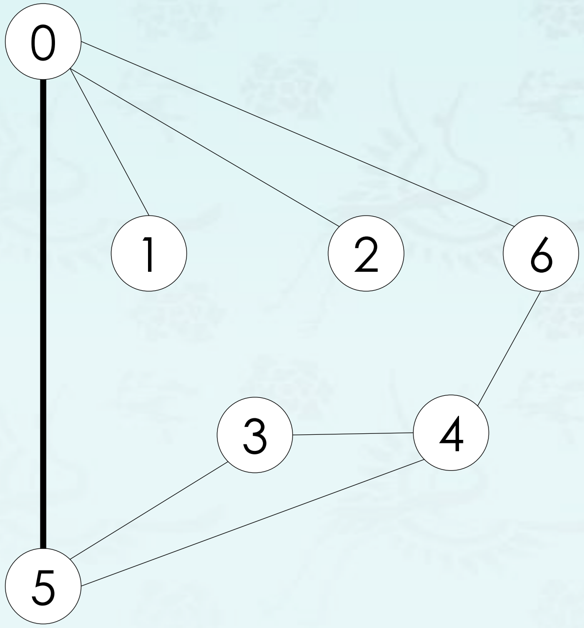
Graph – Build Adjacency list

```
void addEdgeFromTo(graph g, int v, int w) {
  gnode node = new Gnode(w, g->adj[v].next);

  g->adj[v].next = node;
  g->E++;
}
```



Graph g



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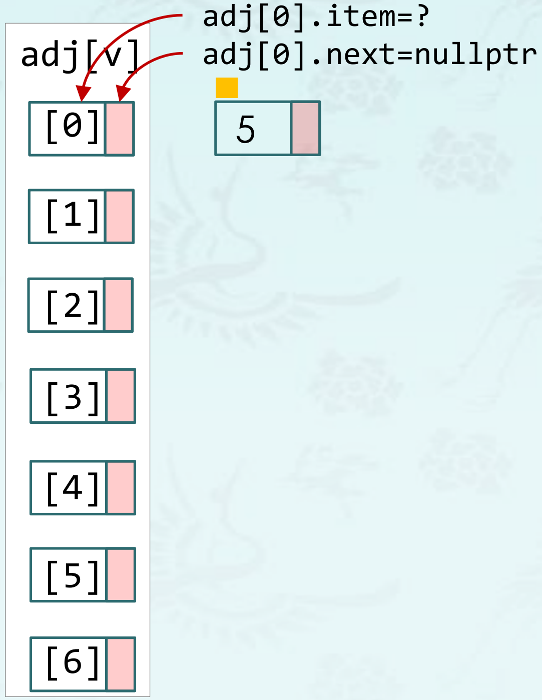

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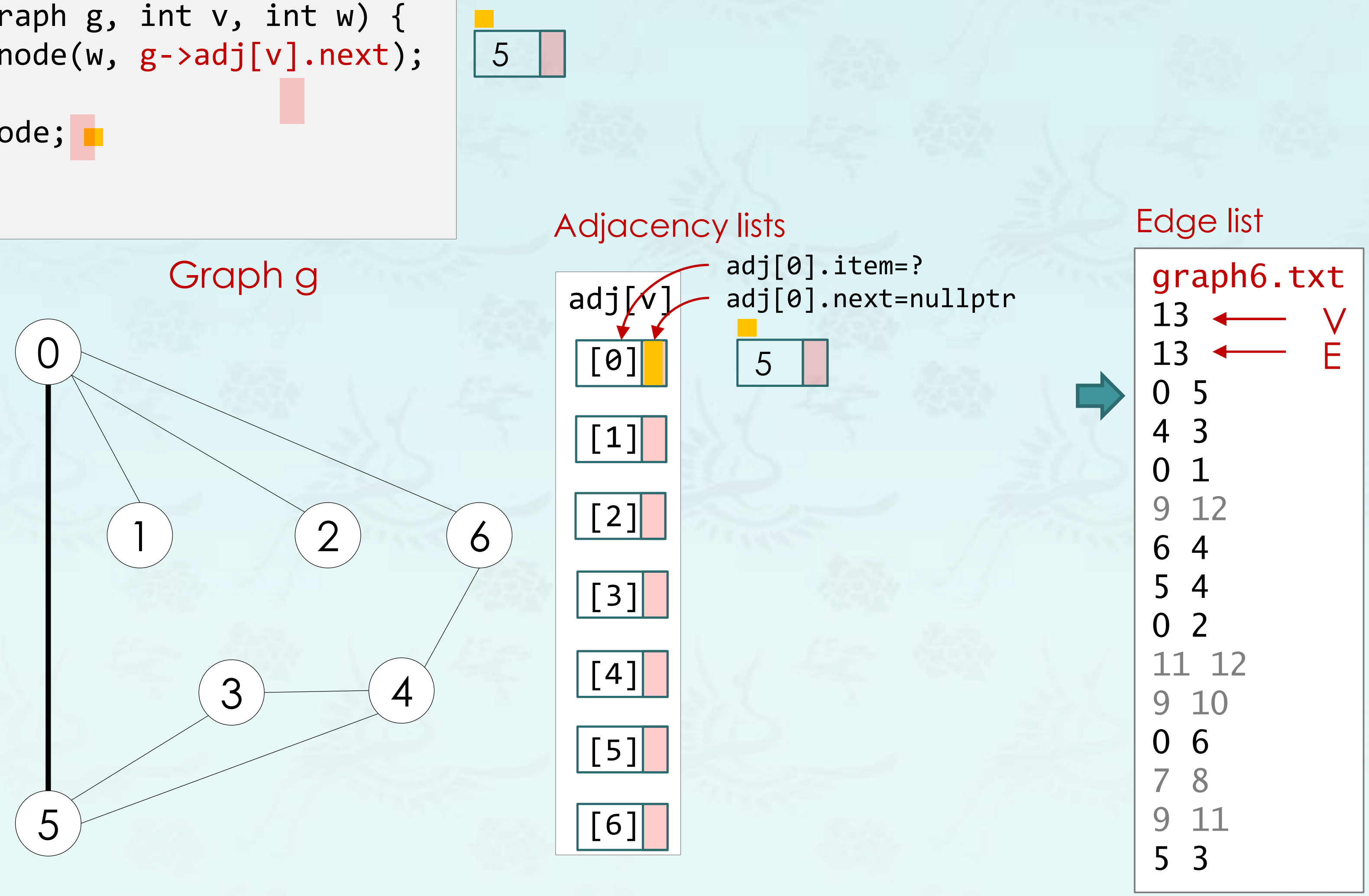
Graph – Build Adjacency list

```
void addEdgeFromTo(graph g, int v, int w) {
  gnode node = new Gnode(w, g->adj[v].next);

  g->adj[v].next = node;
  g->E++;
}
```



Graph – Build Adjacency list

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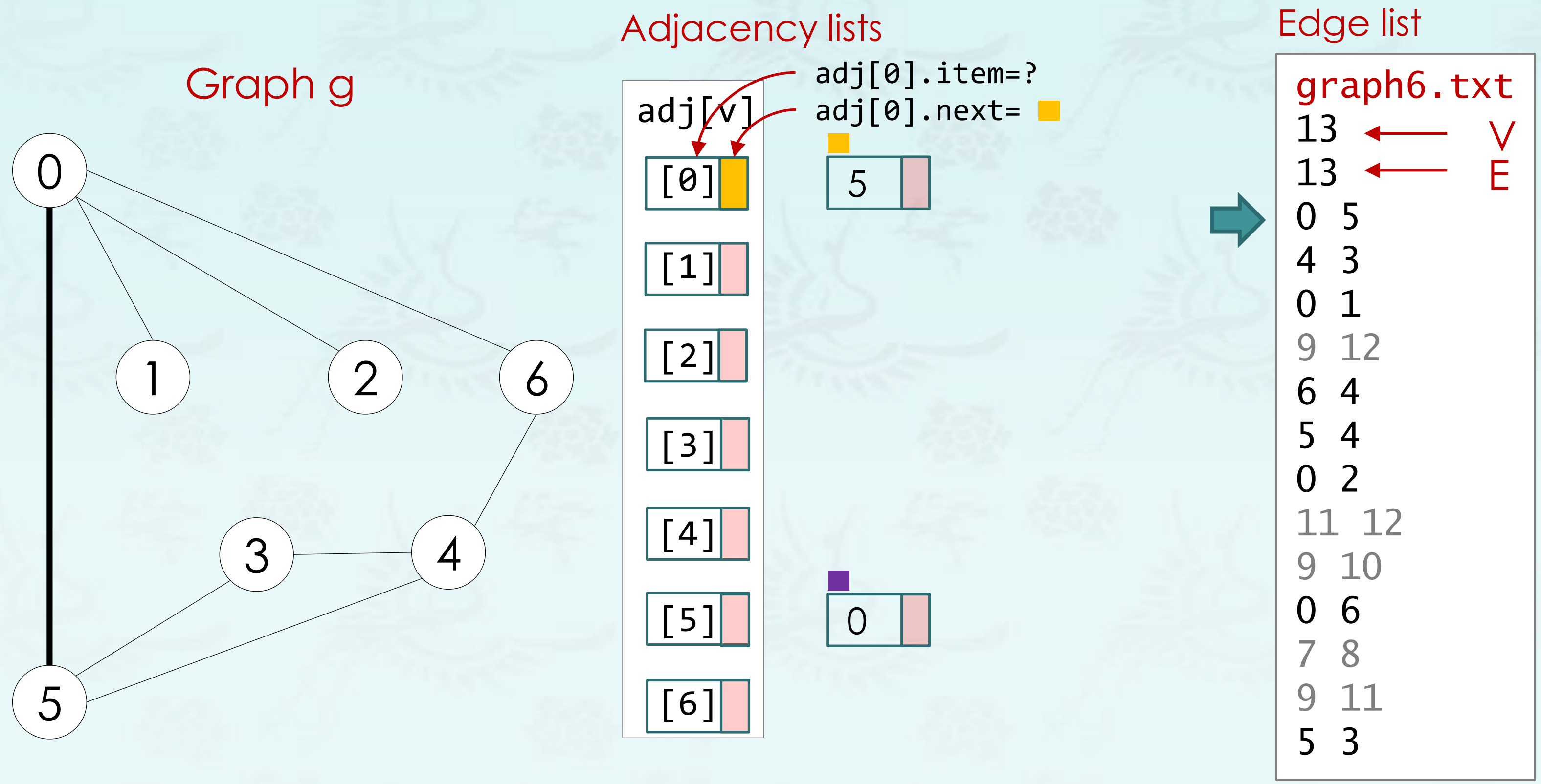

Graph – Build Adjacency list

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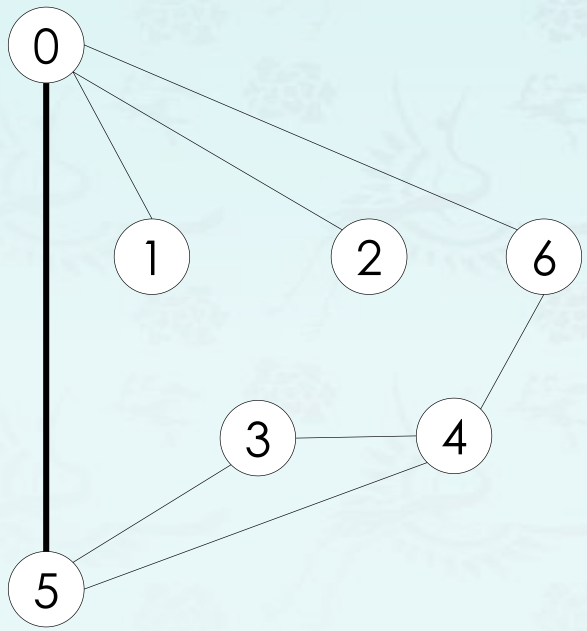

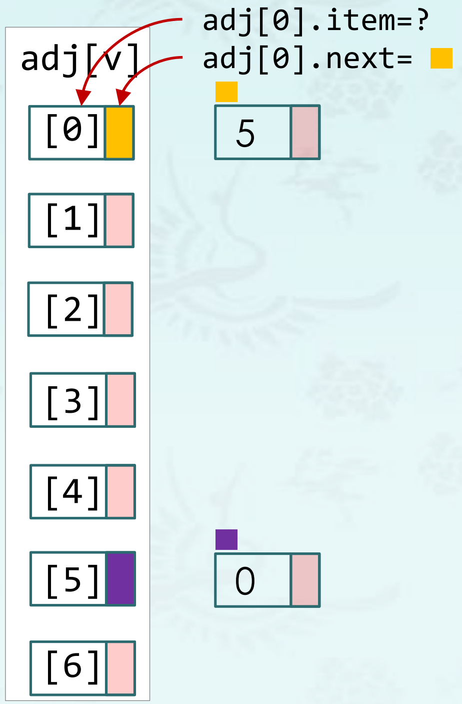

Edge list

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13	V
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0 5	
4 3	
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Graph – Build Adjacency list

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void addEdgeFromTo(graph g, int v, int w) {
  gnode node = new Gnode(w, g->adj[v].next);
  g->adj[v].next = node;
  g->E++;
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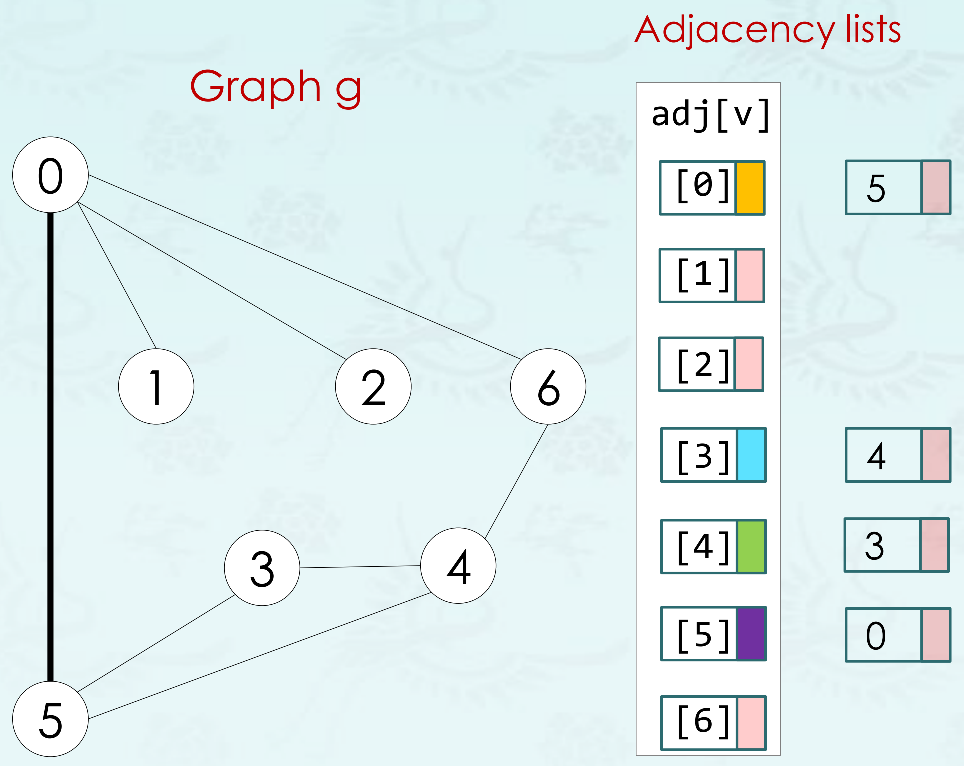
```



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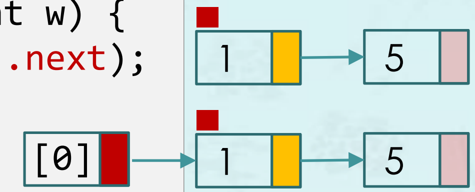
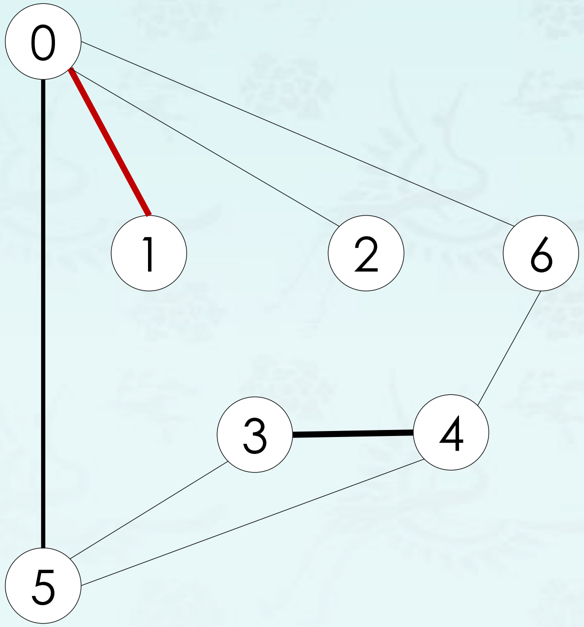
```

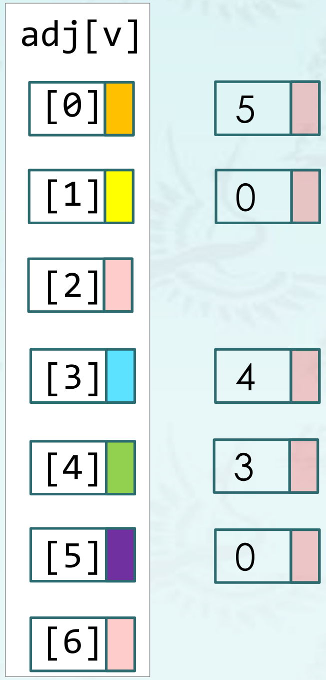
Graph – Build Adjacency list

```
void addEdgeFromTo(graph g, int v, int w) {
  gnode node = new Gnode(w, g->adj[v].next);
                             1
  g->adj[v].next = node;
  g->E++;
}
```



Graph g





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Graph – Build Adjacency list

```
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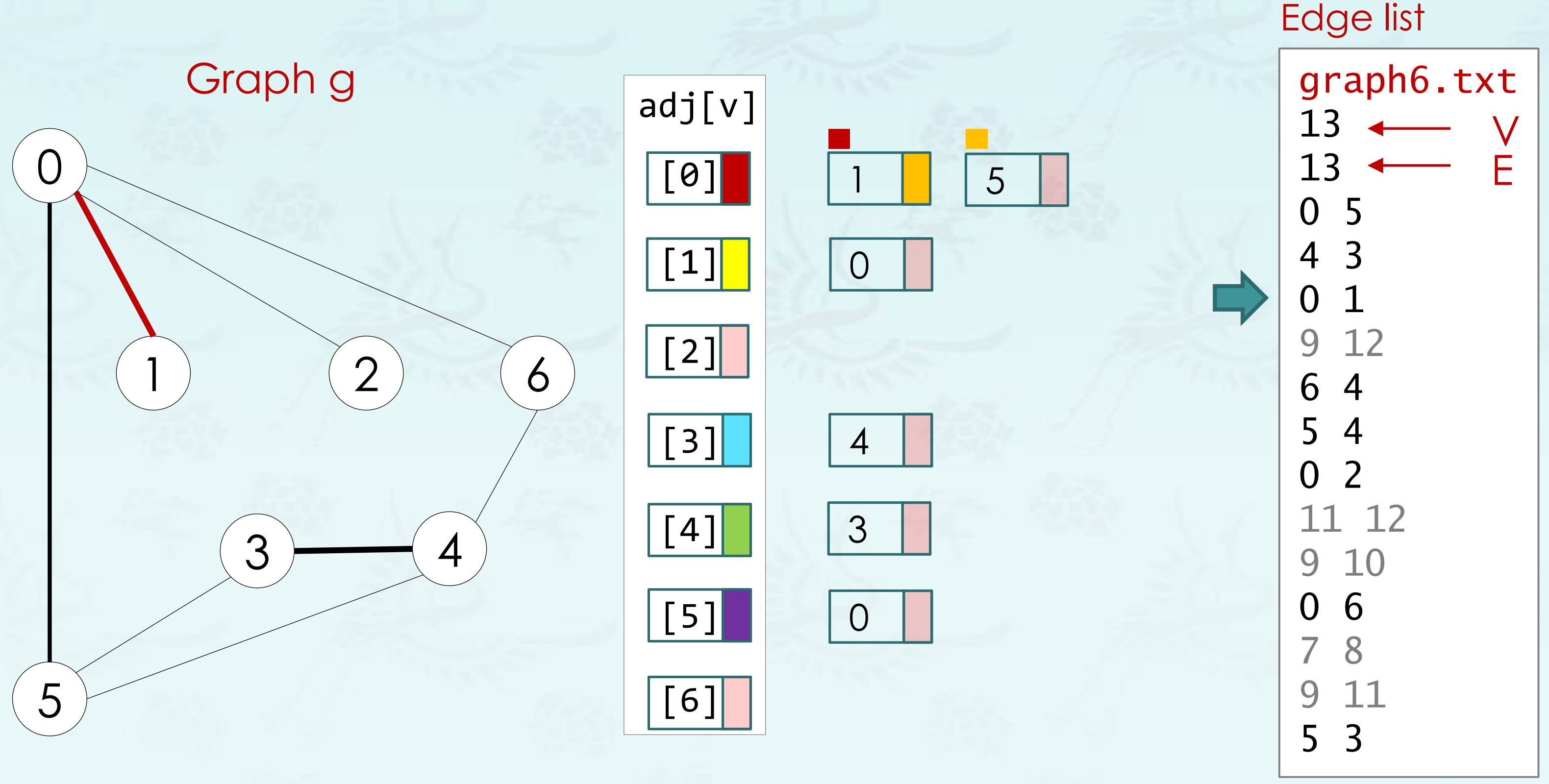

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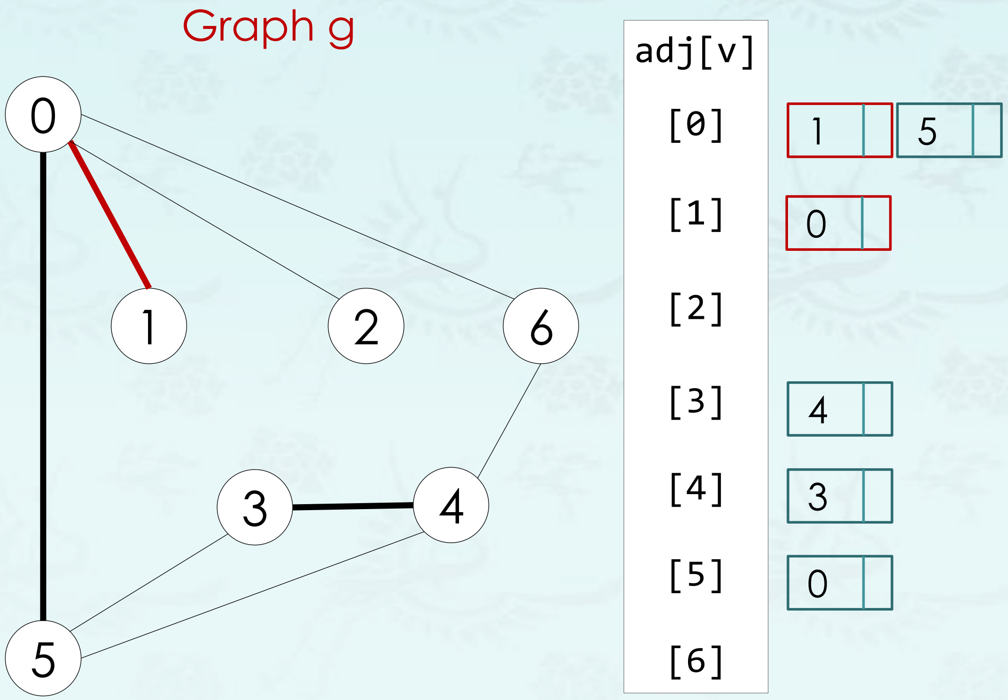

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```

Graph – Build Adjacency list

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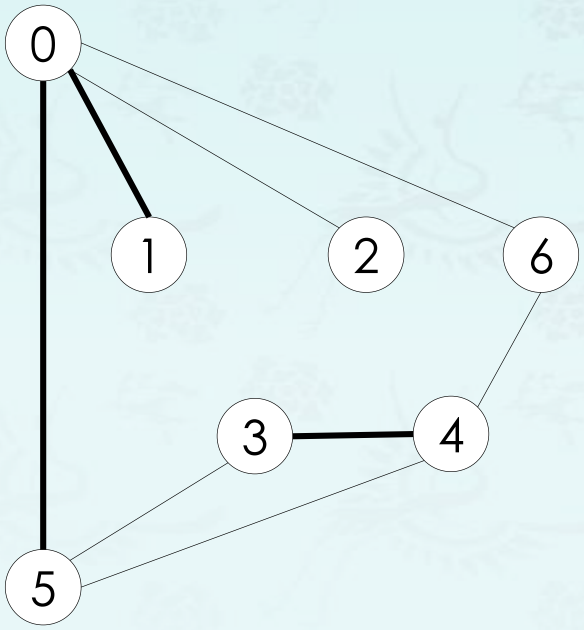
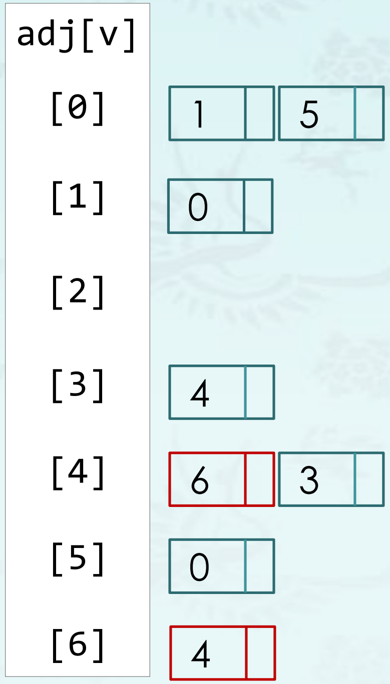
  gnode node = new Gnode(w, g->adj[v].next);
  g->adj[v].next = node;
  g->E++;
}
```

Graph g



Adjacency lists



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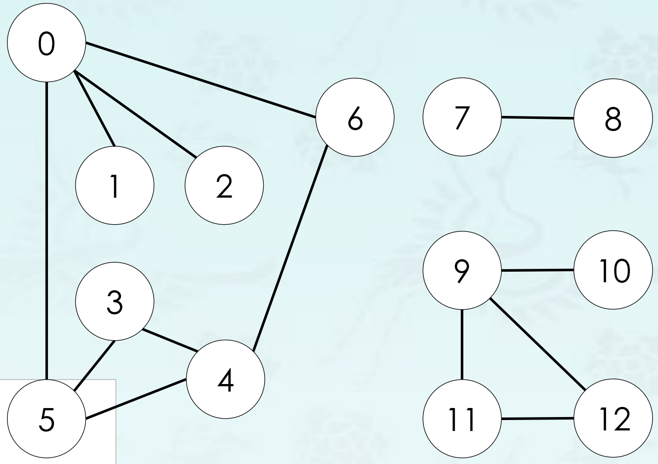
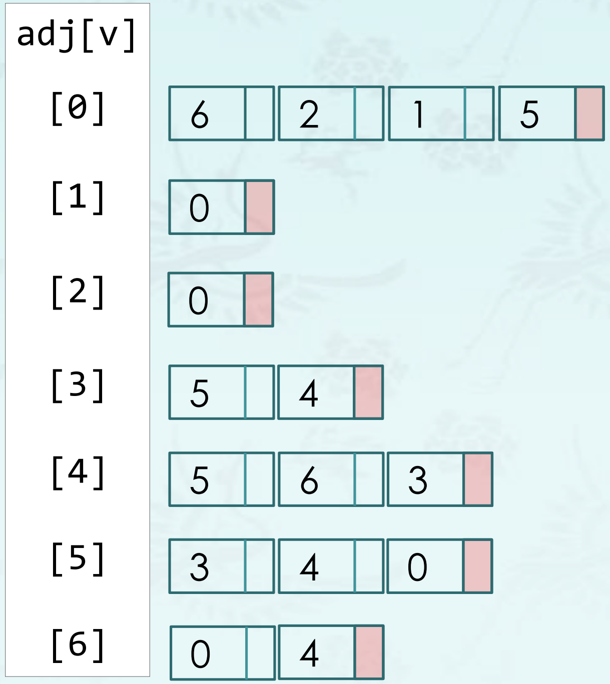
Warming-up: degree()

Adjacency lists



Graph g



Edge list

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9 12
6 4
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11 12
9 10
0 6
7 8
9 11
5 3
```

```
struct Graph {
  int V, E;
  gnode adj;
  Graph(int v = 0) {
    V = v;
    E = 0;
    adj = new (nothrow) Gnode[v];
    assert(adj != nullptr);

    for (int i=0; i<v; i++)
      adj[i].next = nullptr;
  }
  ~Graph() {}
};
using graph = Graph *;
```

Warming-up: degree()

```
int degree(graph g, int v) {
  int deg = 0;
  for (gnode w = g->adj[v].next; w != nullptr; w = w->next)
    deg++;
  return deg;
}
```

```
struct Graph {
  int V, E;
  gnode adj;
  Graph(int v = 0) {
    V = v;
    E = 0;
    adj = new (nothrow) Gnode[v];
    assert(adj != nullptr);

    for (int i=0; i<v; i++)
      adj[i].next = nullptr;
  }
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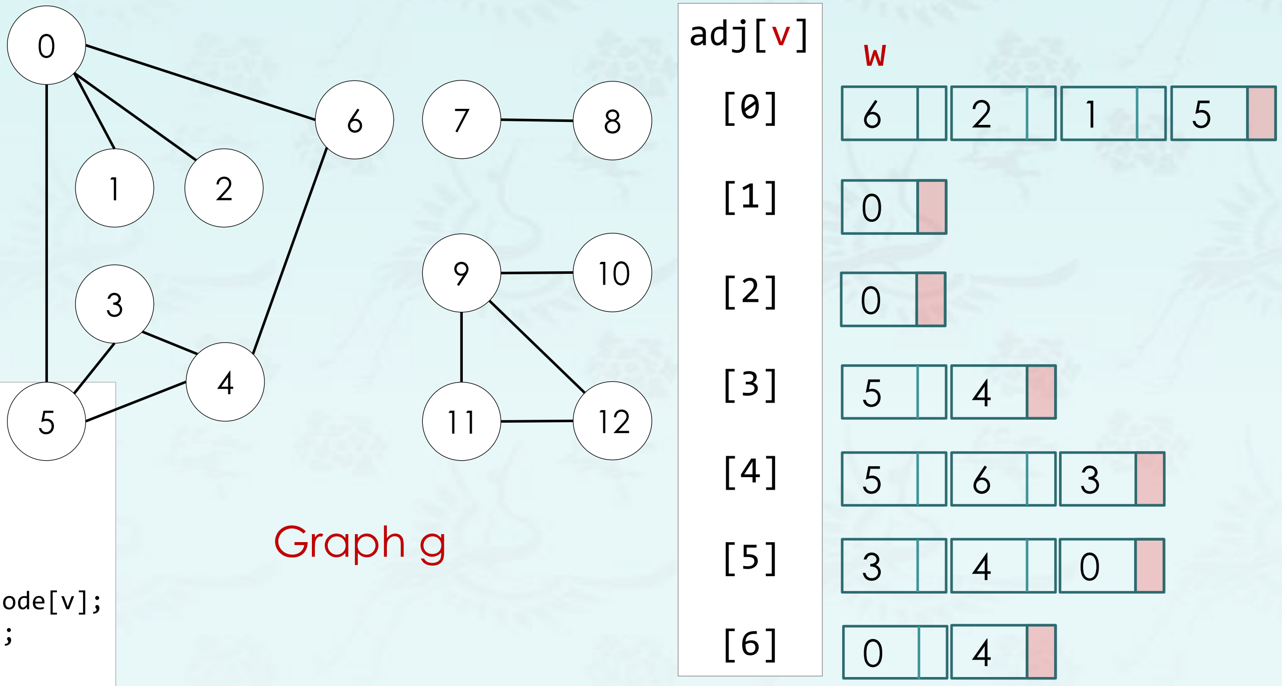
```



Edge list

```
graph6.txt
13   ← V
13   ← E
0 5
4 3
0 1
9 12
6 4
5 4
0 2
11 12
9 10
0 6
7 8
9 11
5 3
```

Warming-up: degree()

```
int degree(graph g) {
  int deg = 0;
  for (int v = 0; v < V(g); v++)
    deg = max(degree(g, v), deg);
  return deg;
}
```

```
struct Graph {
  int V, E;
  gnode adj;
  Graph(int v = 0) {
    V = v;
    E = 0;
    adj = new (nothrow) Gnode[v];
    assert(adj != nullptr);

    for (int i=0; i<v; i++)
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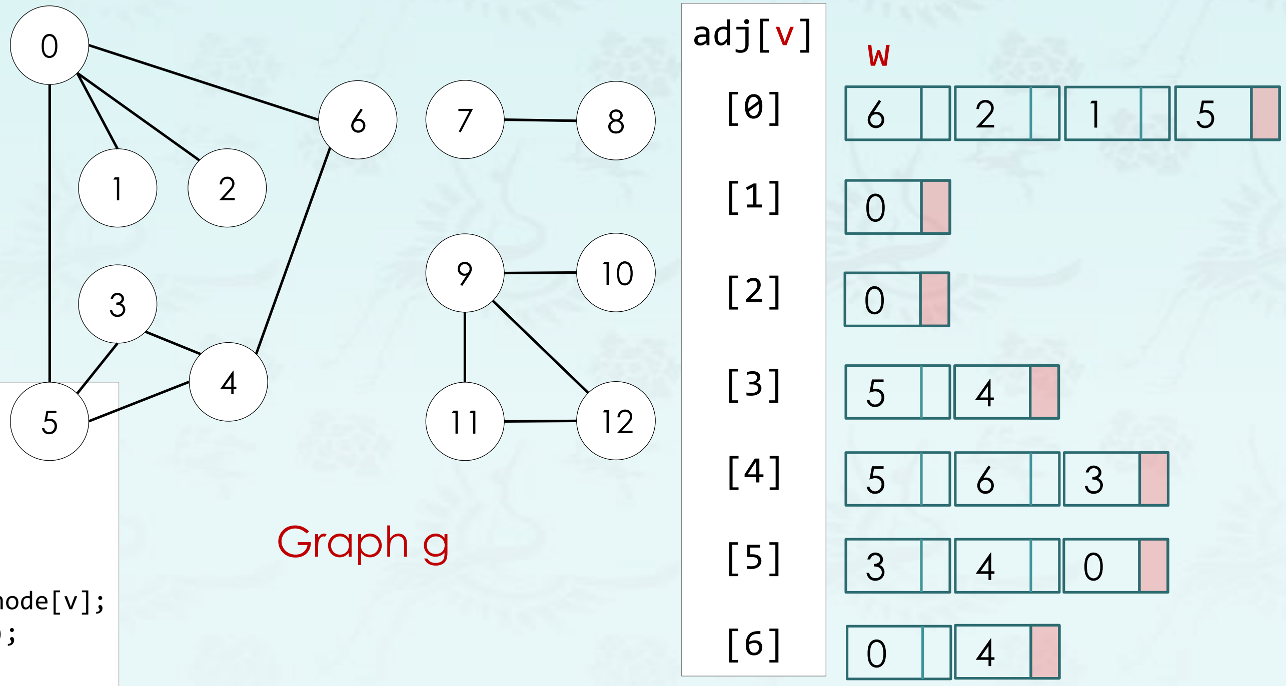

Edge list

```
graph6.txt
13  <- V
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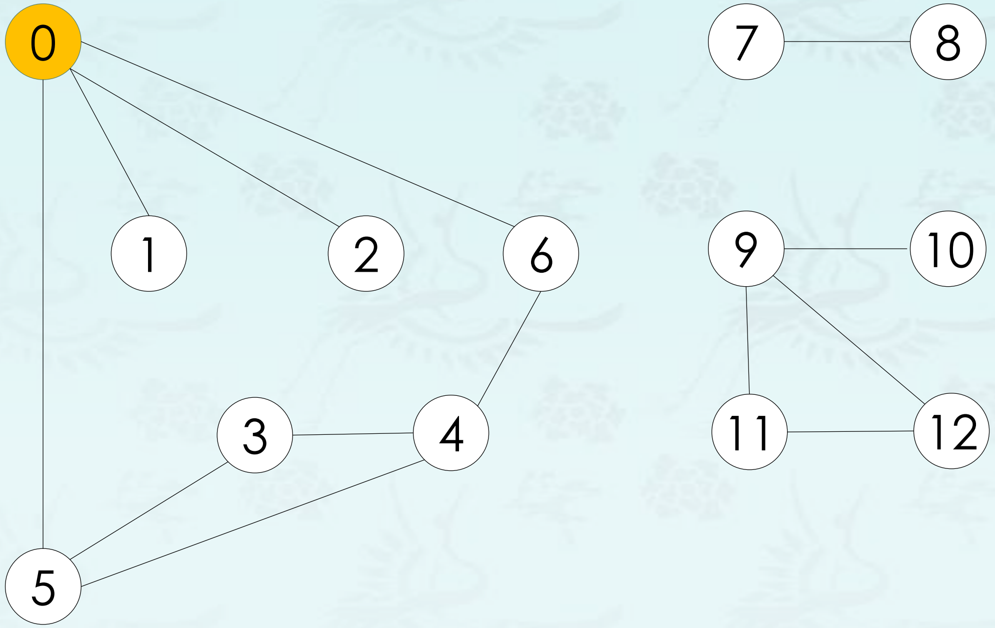
0 5
4 3
0 1
9 12
6 4
5 4
0 2
11 12
9 10
0 6
7 8
9 11
5 3
```

DFS: Depth-First Search Demo

To visit a vertex v:

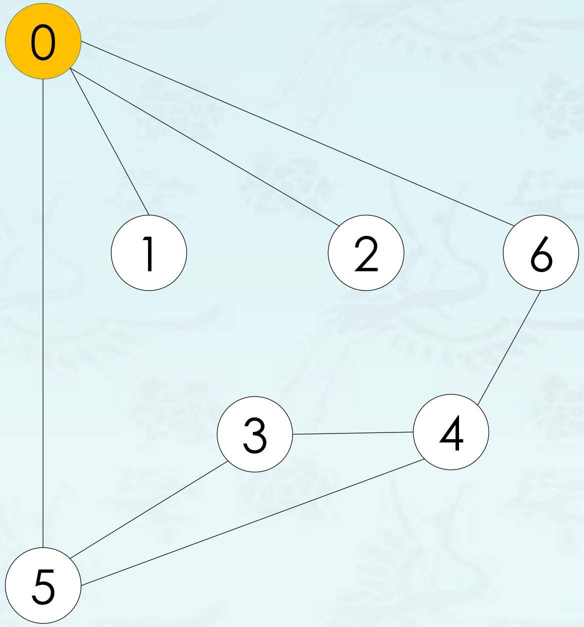
- Mark vertex v as visited.
- Recursively visit all unmarked vertices adjacent to v.



v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	F	-
4	F	-
5	F	-
6	F	-
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

Which one first? visit 0:

DFS: Depth-First Search Demo



Adjacency lists

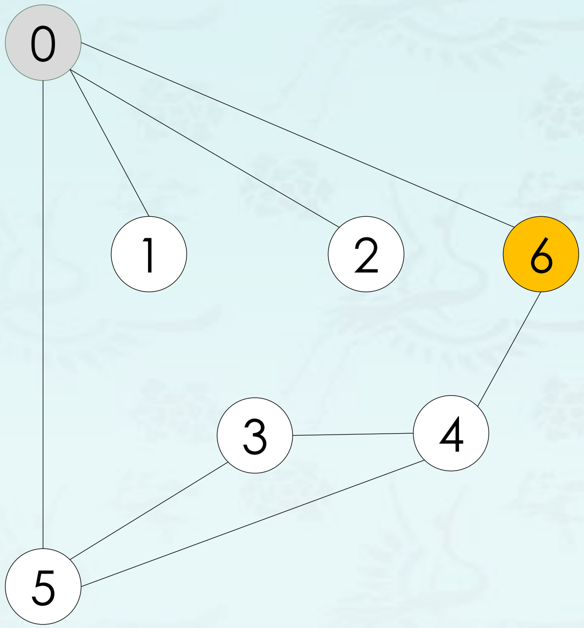
adj[v]				
[0]	6	2	1	5
[1]	0			
[2]	0			
[3]	5	4		
[4]	5	6	3	
[5]	3	4	0	
[6]	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	F	-
4	F	-
5	F	-
6	F	-
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 0: check 6, check 2, check 1, check 5

DFS 0

DFS: Depth-First Search Demo



Adjacency lists

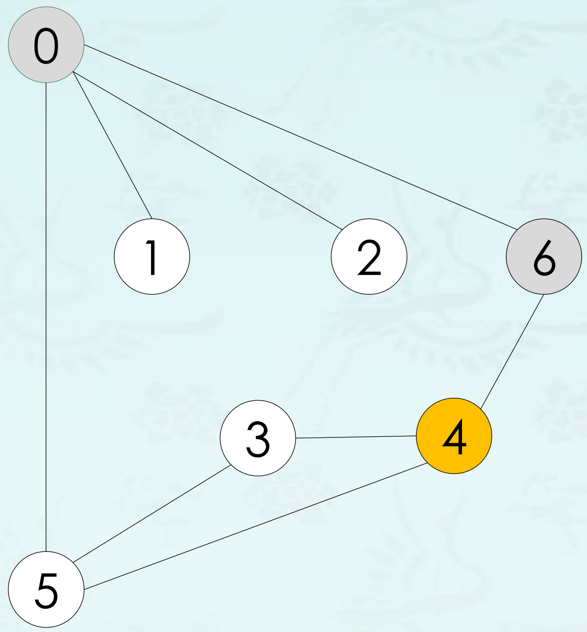
adj[v]				
[0]	6	2	1	5
[1]	0			
[2]	0			
[3]	5	4		
[4]	5	6	3	
[5]	3	4	0	
[6]	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	F	-
4	F	-
5	F	-
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 6: check 0, check 4

DFS 0 6

DFS: Depth-First Search Demo



Adjacency lists

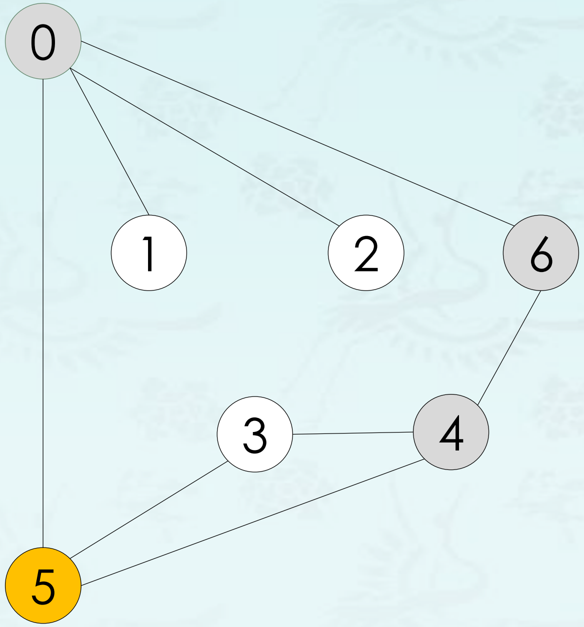
adj[v]				
[0]	6	2	1	5
[1]	0			
[2]	0			
[3]	5	4		
[4]	5	6	3	
[5]	3	4	0	
[6]	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	F	-
4	T	6
5	F	-
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 4: check 5, check 6, check 3

DFS 0 6 4

DFS: Depth-First Search Demo



Adjacency lists

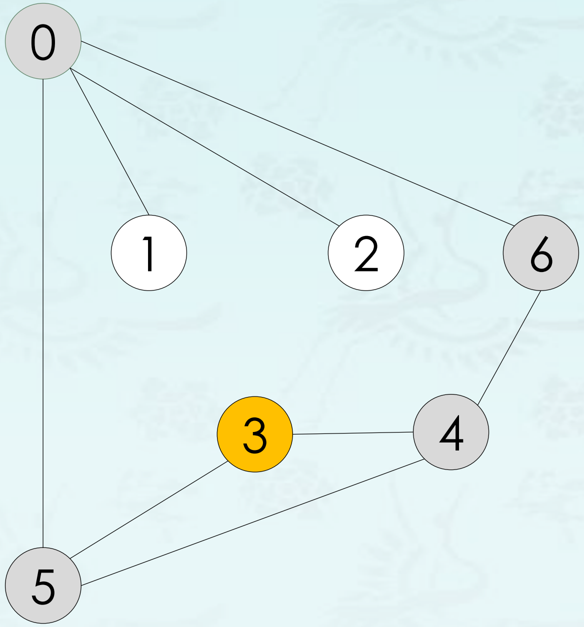
adj[v]				
[0]	6	2	1	5
[1]	0			
[2]	0			
[3]	5	4		
[4]	5	6	3	
[5]	3	4	0	
[6]	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	F	-
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 5: check 3, check 4, check 0

DFS 0 6 4 5

DFS: Depth-First Search Demo



Adjacency lists

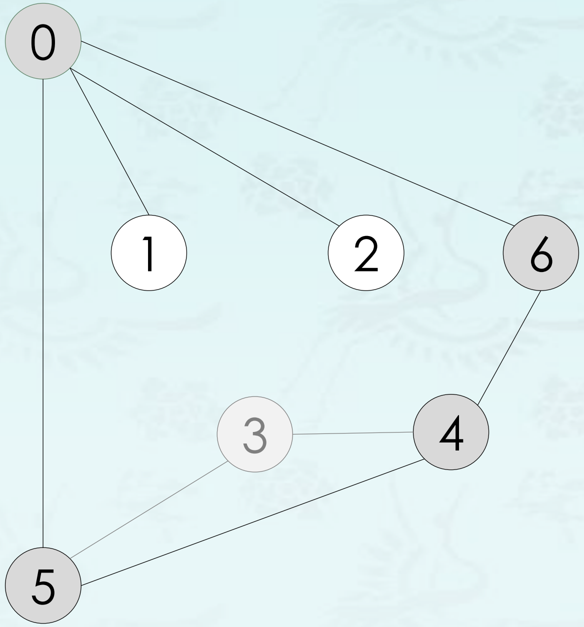
adj[v]				
[0]	6	2	1	5
[1]	0			
[2]	0			
[3]	5	4		
[4]	5	6	3	
[5]	3	4	0	
[6]	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 3: check 5, check 4

DFS 0 6 4 5 3

DFS: Depth-First Search Demo



Adjacency lists

adj[v]				
[0]	6	2	1	5
[1]	0			
[2]	0			
[3]	5	4		
[4]	5	6	3	
[5]	3	4	0	
[6]	0	4		

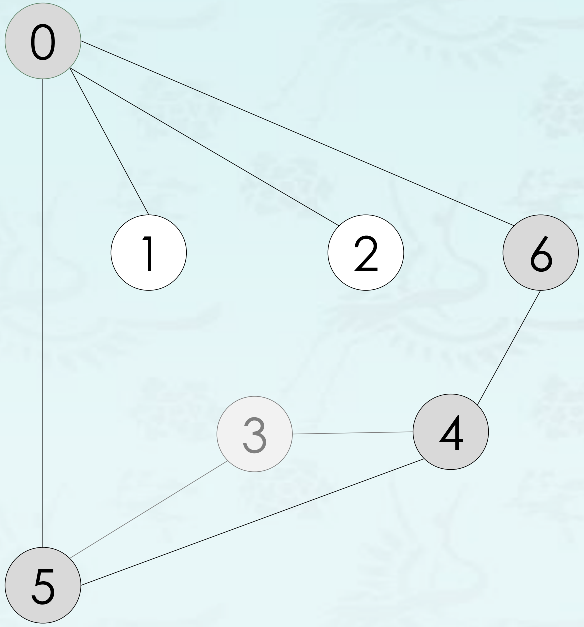
v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 3: check 5, check 4

3 done:

DFS 0 6 4 5 3

DFS: Depth-First Search Demo



Adjacency lists

adj[v]				
[0]	6	2	1	5
[1]	0			
[2]	0			
[3]	5	4		
[4]	5	6	3	
[5]	3	4	0	
[6]	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 3: check 5, check 4

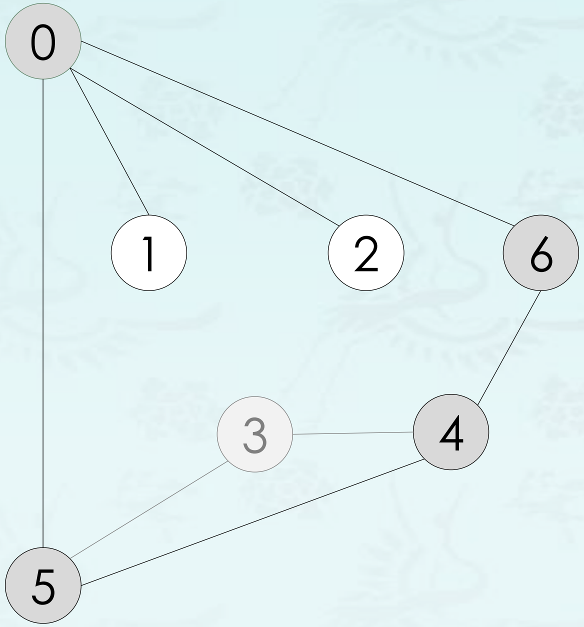
3 done:

DFS 0 6 4 5 3

What's next? **Backtrack!** How?

Use parent[v] → **parent[3] = 5**

DFS: Depth-First Search Demo



Adjacency lists

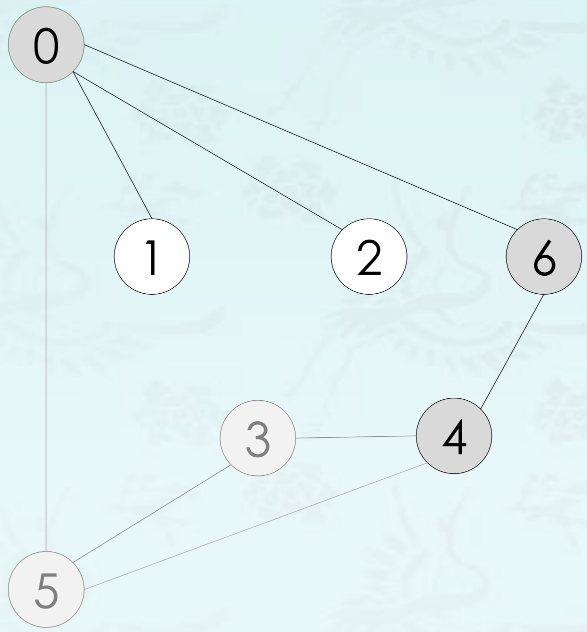
adj[v]				
[0]	6	2	1	5
[1]	0			
[2]	0			
[3]	5	4		
[4]	5	6	3	
[5]	3	4	0	
[6]	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 5: check 3, check 4, check 0

DFS 0 6 4 5 3

DFS: Depth-First Search Demo



Adjacency lists

adj[v]				
[0]	6	2	1	5
[1]	0			
[2]	0			
[3]	5	4		
[4]	5	6	3	
[5]	3	4	0	
[6]	0	4		

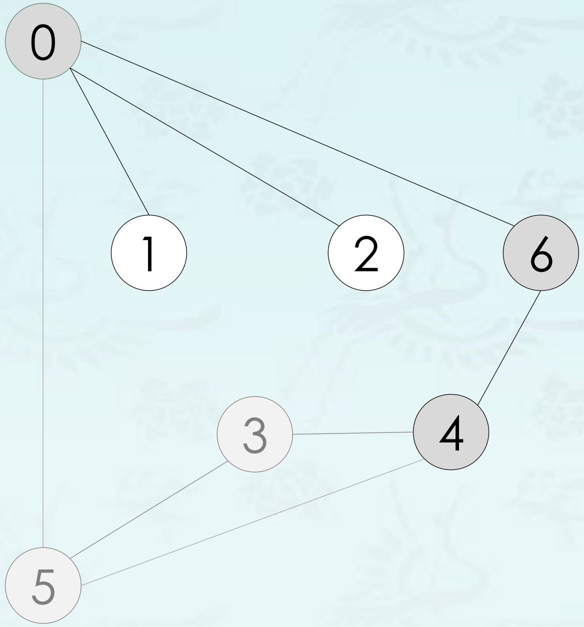
v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 5: check 3, check 4, check 0 5 done:

DFS 0 6 4 5 3

What's next? **Backtrack!** How to?
Use parent[v] → **parent[5] = 4**

DFS: Depth-First Search Demo



Adjacency lists

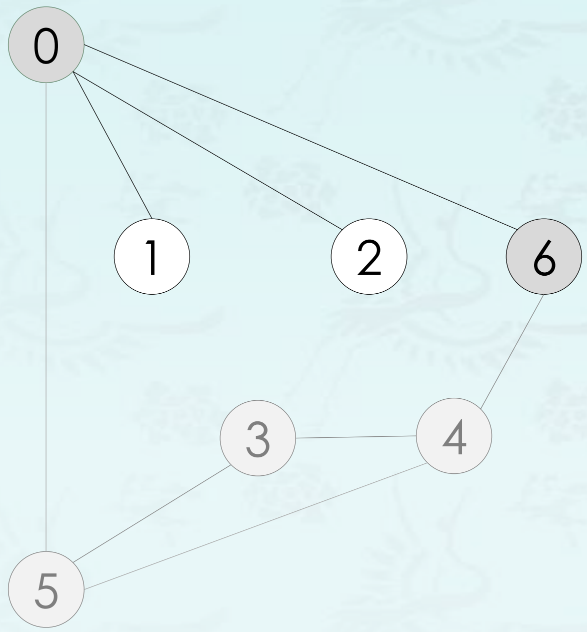
adj[v]				
[0]	6	2	1	5
[1]	0			
[2]	0			
[3]	5	4		
[4]	5	6	3	
[5]	3	4	0	
[6]	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 4: check 5, check 6, check 3

DFS 0 6 4 5 3

DFS: Depth-First Search Demo



Adjacency lists

adj[v]				
[0]	6	2	1	5
[1]	0			
[2]	0			
[3]	5	4		
[4]	5	6	3	
[5]	3	4	0	
[6]	0	4		

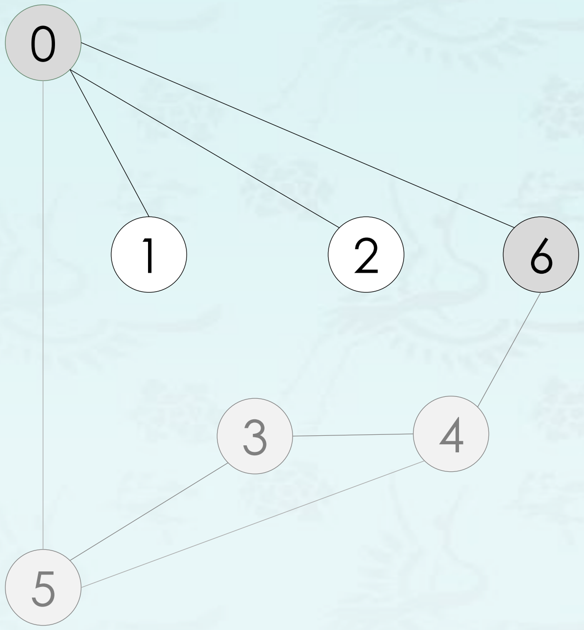
v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 4: check 5, check 6, check 3 4 done:

DFS 0 6 4 5 3

What's next? **Backtrack!** How to?
Use parent[v] → **parent[4] = 6**

DFS: Depth-First Search Demo



Adjacency lists

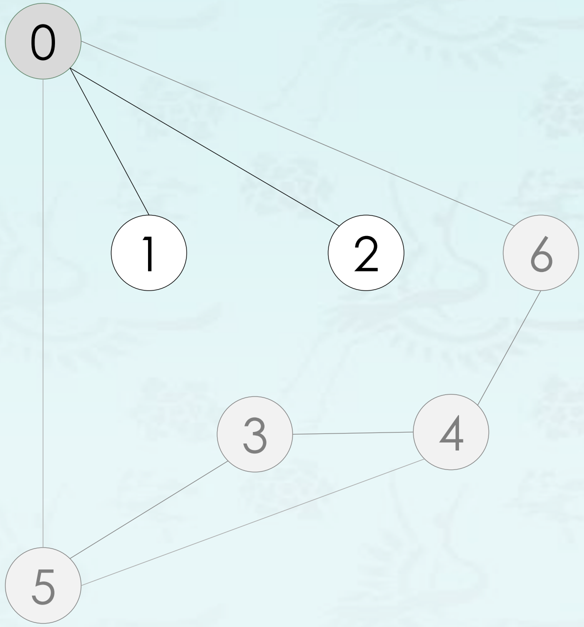
adj[v]				
[0]	6	2	1	5
[1]	0			
[2]	0			
[3]	5	4		
[4]	5	6	3	
[5]	3	4	0	
[6]	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 6: check 0, check 4

DFS 0 6 4 5 3

DFS: Depth-First Search Demo



Adjacency lists

adj[v]				
[0]	6	2	1	5
[1]	0			
[2]	0			
[3]	5	4		
[4]	5	6	3	
[5]	3	4	0	
[6]	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

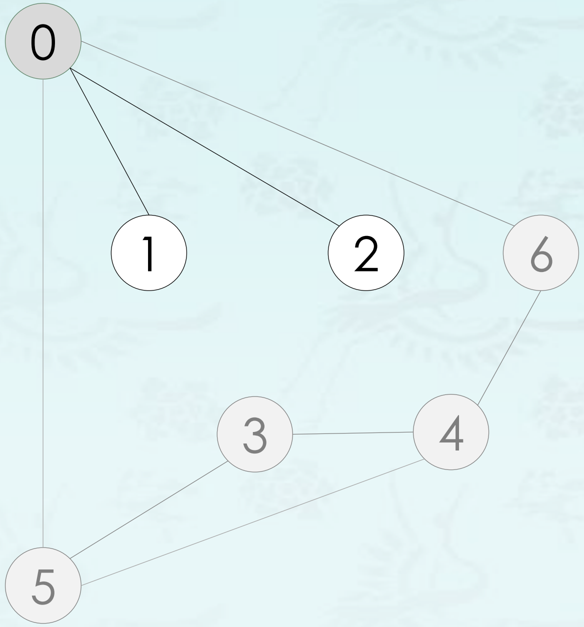
visit 6: check 0, check 4

DFS 0 6 4 5 3

6 done:

What's next? Backtrack!
parent[6] = 0

DFS: Depth-First Search Demo



Adjacency lists

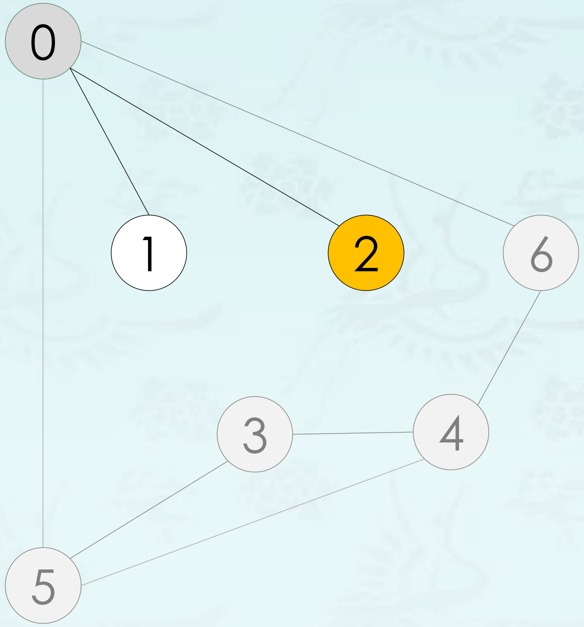
adj[v]				
[0]	6	2	1	5
[1]	0			
[2]	0			
[3]	5	4		
[4]	5	6	3	
[5]	3	4	0	
[6]	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 0: check 6, check 2, check 1, check 5

DFS 0 6 4 5 3

DFS: Depth-First Search Demo



Adjacency lists

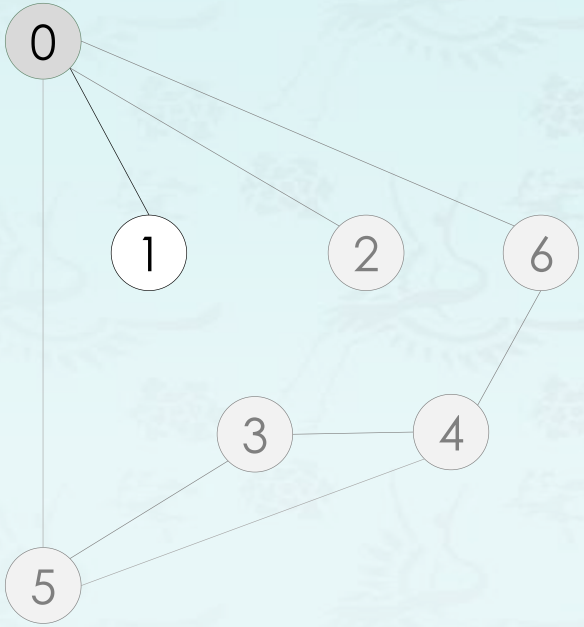
adj[v]				
[0]	6	2	1	5
[1]	0			
[2]	0			
[3]	5	4		
[4]	5	6	3	
[5]	3	4	0	
[6]	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	T	0
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 2: check 0

DFS 0 6 4 5 3 2

DFS: Depth-First Search Demo



Adjacency lists

adj[v]				
[0]	6	2	1	5
[1]	0			
[2]	0			
[3]	5	4		
[4]	5	6	3	
[5]	3	4	0	
[6]	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	T	0
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

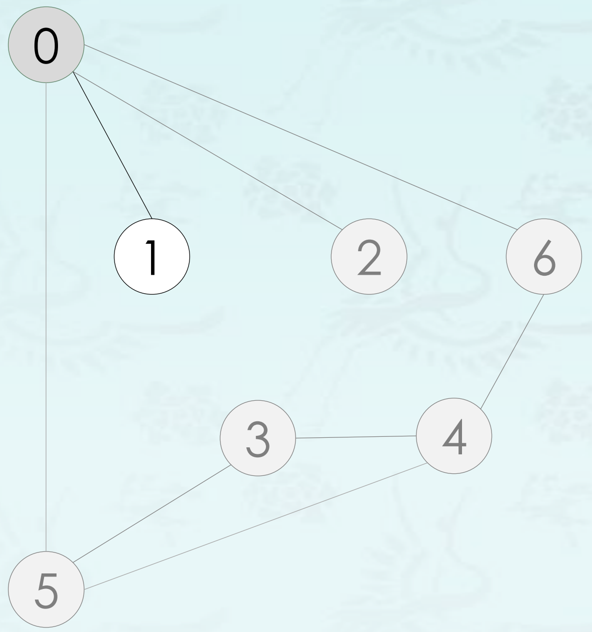
visit 2: check 0

DFS 0 6 4 5 3 2

2 done:

What's next? Backtrack!
parent[2] = 0

DFS: Depth-First Search Demo



Adjacency lists

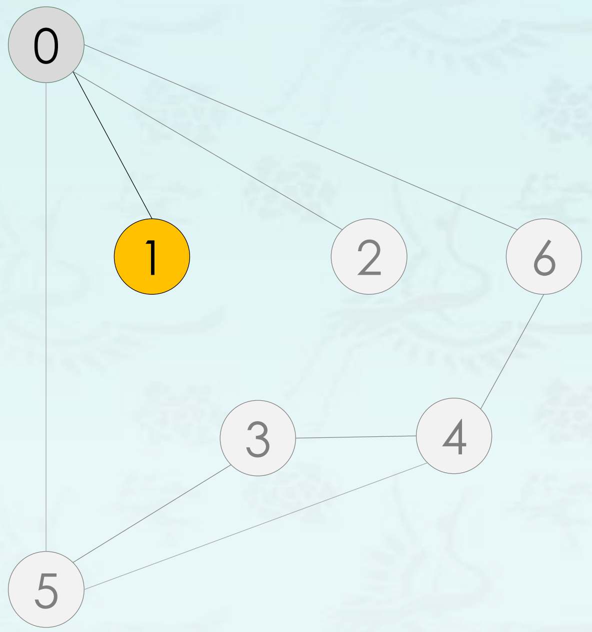
adj[v]				
[0]	6	2	1	5
[1]	0			
[2]	0			
[3]	5	4		
[4]	5	6	3	
[5]	3	4	0	
[6]	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	T	0
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 0: check 6, check 2, check 1, check 5

DFS 0 6 4 5 3 2

DFS: Depth-First Search Demo



Adjacency lists

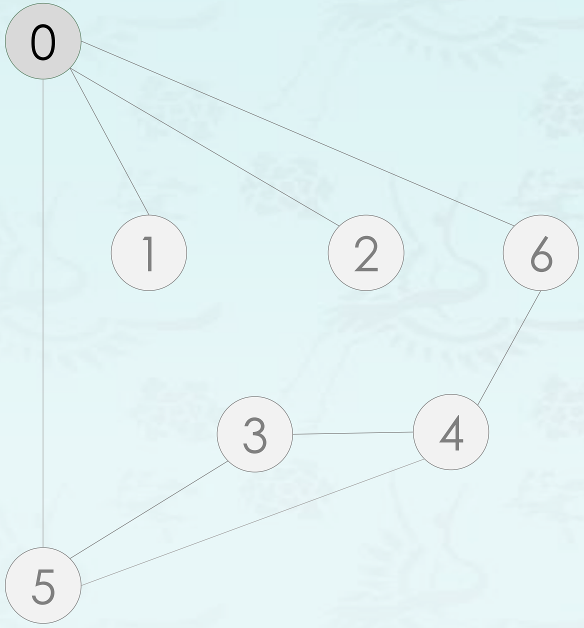
adj[v]				
[0]	6	2	1	5
[1]	0			
[2]	0			
[3]	5	4		
[4]	5	6	3	
[5]	3	4	0	
[6]	0	4		

v	marked[]	parent[v]
0	T	-
1	T	0
2	T	0
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 1: check 0

DFS 0 6 4 5 3 2 1

DFS: Depth-First Search Demo



visit 1: check 0

DFS 0 6 4 5 3 2 1

Adjacency lists

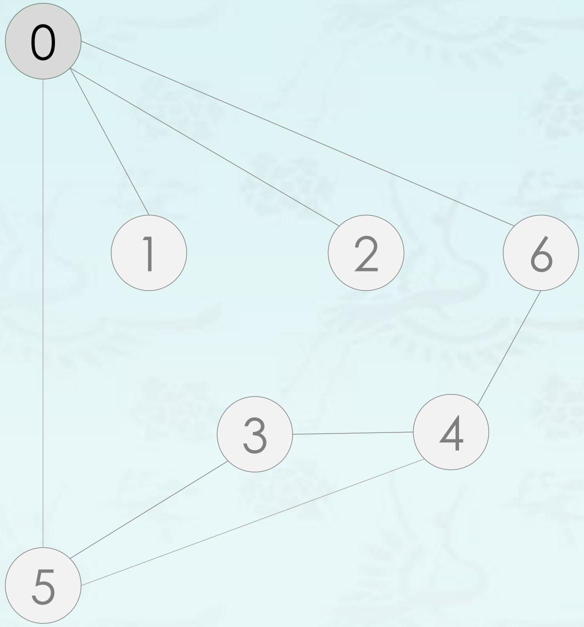
adj[v]				
[0]	6	2	1	5
[1]	0			
[2]	0			
[3]	5	4		
[4]	5	6	3	
[5]	3	4	0	
[6]	0	4		

1 done:

v	marked[]	parent[v]
0	T	-
1	T	0
2	T	0
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

What's next? Backtrack!
parent[1] = 0

DFS: Depth-First Search Demo



Adjacency lists

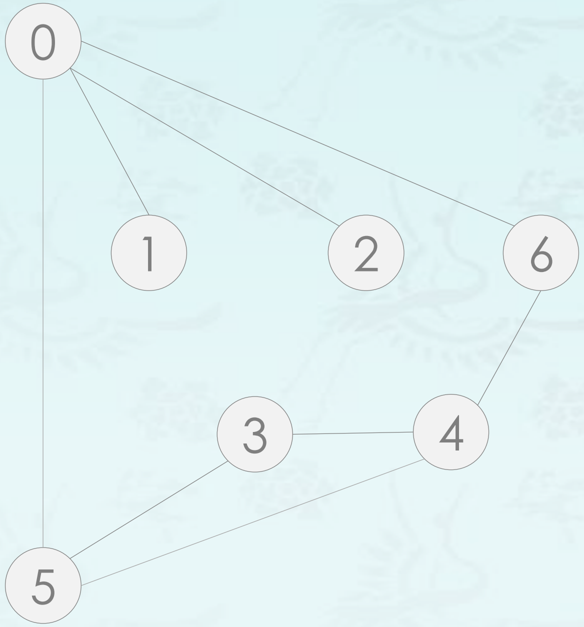
adj[v]				
[0]	6	2	1	5
[1]	0			
[2]	0			
[3]	5	4		
[4]	5	6	3	
[5]	3	4	0	
[6]	0	4		

v	marked[]	parent[v]
0	T	-
1	T	0
2	T	0
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 1: check 6, check 2, check 1, check 5

DFS 0 6 4 5 3 2 1

DFS: Depth-First Search Demo



Adjacency lists

adj[v]				
[0]	6	2	1	5
[1]	0			
[2]	0			
[3]	5	4		
[4]	5	6	3	
[5]	3	4	0	
[6]	0	4		

v	marked[]	parent[v]
0	T	-
1	T	0
2	T	0
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 1: check 6, check 2, check 1, check 5

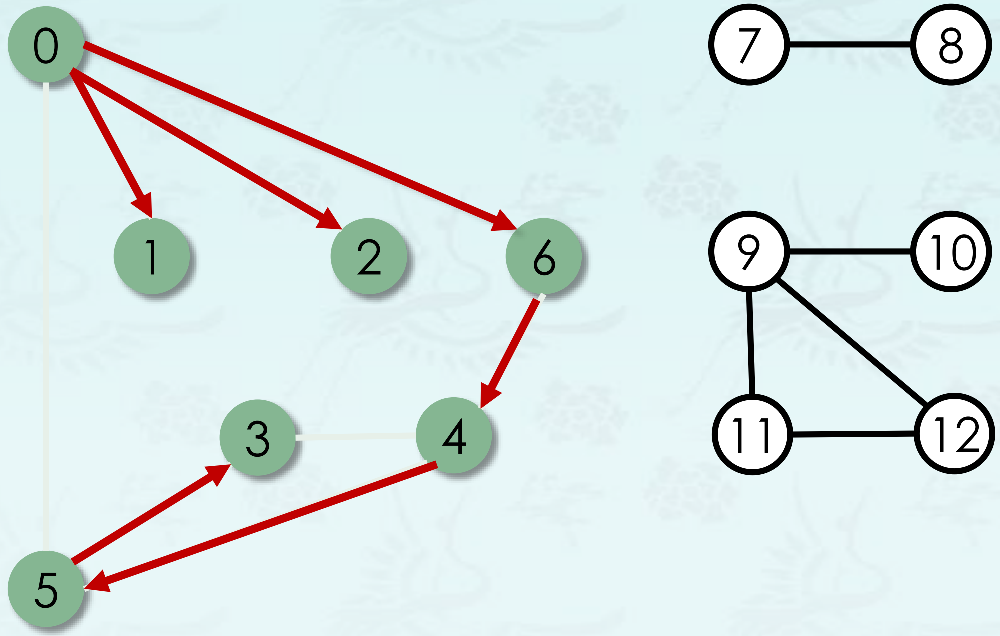
0 done:

DFS 0 6 4 5 3 2 1

DFS: Depth-First Search Demo

To visit a vertex v:

- Mark vertex v as visited.
- Recursively visit all unmarked vertices adjacent to v.



v	marked[]	parent[v]
0	T	-
1	T	0
2	T	0
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

DFS Output: DFS: 0 6 4 5 3 2 1

- found vertices reachable from 0
- build a data structure **parent[v]**

DFS: Depth-First Search Demo

Goal: Find all vertices connected to s (and a corresponding path).

Idea: Mimic maze exploration

Algorithm:

- Use recursion (ball of string).
- Mark each visited vertex (and keep track of edge taken to visit it).
- Return (retrace steps) when no unvisited options.

Data Structures:

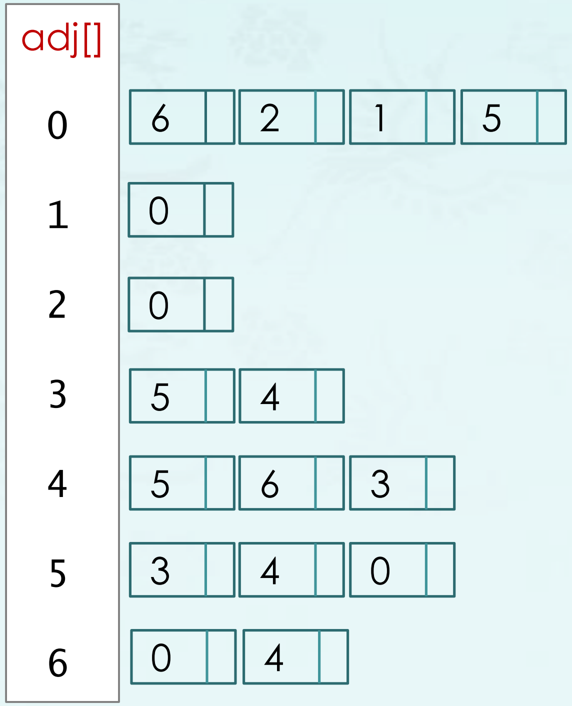
- `Boolean[] marked` to mark visited vetices.
- `int[] parent` to keep tree of paths.
  (`parent[w] == v`) means that edge v-w taken to visit w for first time

DFS: Depth-First Search Coding

```
// runs DFS for at vertex v recursively.
// Only que, g->marked[v] and g->parentDFS[] are updated here.
void DFS(graph g, int v, queue<int>& que) {
  g->marked[v] = true;            // visited
  que.push(v);                    // save the path

  for (gnode w = g->adj[v].next; w; w = w->next) {
    cout << "your code here (recursion) \n";
    - only for unmarked vertex, do the followings:
    - invoke DFS(g, ..., ...);
    - set g->parentDFS[...] = ...;
  }
}
```

Adjacency lists

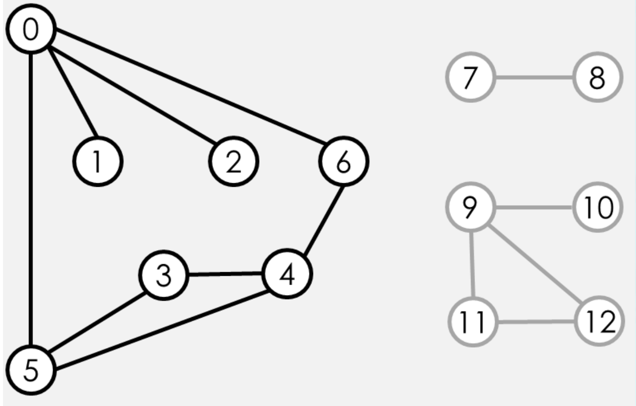


DFS: Depth-First Search Coding

```
// runs DFS for all components and produces DFS0[], CCID[] & parentDFS[].
void DFS_CCs(graph g) {
  if (empty(g)) return;
  for (int i = 0; i < V(g); i++) {
    g->marked[i] = false;
    g->parentDFS[i] = -1;
    g->CCID[i] = 0;
  }

  queue<int> que;
  DFS(g, 0, que);
  setDFS0(g, 0, que);

  g->DFSv = {};
}
```



It works only for the 1st CC at 0.
Make it run for all CC's.

```
// runs DFS for at vertex v recursively.
// Only que, g->marked[v] and g->parentDFS[] are updated here.
void DFS(graph g, int v, queue<int>& que) {
  g->marked[v] = true;                // visited
  que.push(v);                        // save the path
  for (gnode w = g->adj[v].next; w; w = w->next) {
    cout << "your code here (recursion) \n";
  }
}
```

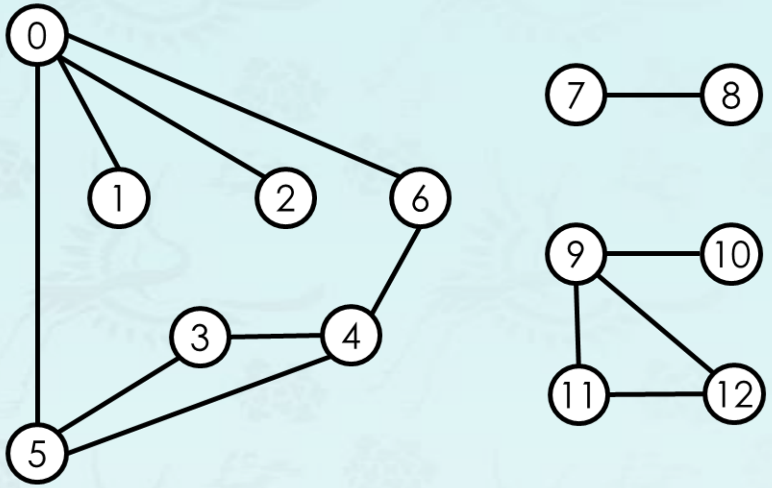
DFS: Depth-First Search Coding

```
. [0] ----------------[6]        [7]--------[8]
.  ¦ \------------[2]  ¦
.  ¦  \--------[1]     ¦         [9]--------[10]
.  ¦                  /           ¦ \
.  ¦    [3]       ---/            ¦   \
.  ¦   /   \     /                ¦     \
.  ¦  /     \   /                [11]----[12]
. [5]--------[4]

    vertex[0..12] =    0   1   2   3   4   5   6   7   8   9  10  11  12
     color[0..12] =    0   0   0   0   0   0   0   0   0   0   0   0   0

      DFS0[0..12] =    0   6   4   5   3   2   1   7   8   9  11  12  10
      CCID[0..12] =    1   1   1   1   1   1   1   2   2   3   3   3   3
DFS parent[0..12] =   -1   0   0   5   6   4   0  -1   7  -1   9   9  11
      BFS0[0..12] =    0   6   2   1   5   4   3   7   8   9  11  10  12
    DistTo[0..12] =    0   1   1   2   2   1   1   0   1   0   1   1   1
BFS parent[0..12] =   -1   0   0   5   6   0   0  -1   7  -1   9   9   9

Graph [Graph][Tablet]   file:graph3.txt V:13 E:26 CCs:3 Deg:4
n - new graph file      x - connected(v,w)
d - DFS(v=0)            e - distance(v,w)
b - BFS(v=0)            p - path(v,w)
c - cyclic(v=0)?        m - print mode[adjList/graph]
t - bigraph(v=0)?       a - bigraph using adj-list coloring
Command(q to quit):
```



DFS: Depth-First Search Properties

- **Proposition:** After DFS, can find vertices connected to s in constant time and can find a path to s (if one exists) in time proportional to its length.
- **Proof:** parent[] is parent-link representation of a tree rooted at s.

v	marked[]	parent[v]
0	T	-
1	T	0
2	T	0
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

DFS: Depth-First Search Properties

- **Proposition:** After DFS, can find vertices connected to s in constant time and can find a path to s (if one exists) in time proportional to its length.
- **Proof:** parent[] is parent-link representation of a tree rooted at s.

```
// returns a path from v to w using the DFS result or parentDFS[].
// It has to use a stack to retrace the path back to the source.
// Once the client(caller) gets a stack returned,
void DFSpath(graph g, int v, int w, stack<int>& path) {
  if (empty(g)) return;
  for (int i = 0; i < V(g); i++) {
    g->marked[i] = false;
    g->parentDFS[i] = -1;
  }
  queue<int> q;
  DFS(g, v, q);    // DFS at v, starting vertex
  g->DFSv = q;     // DFS result at v
  path = {};
  cout << "your code here\n"; // push v to w path to the stack path
}
```

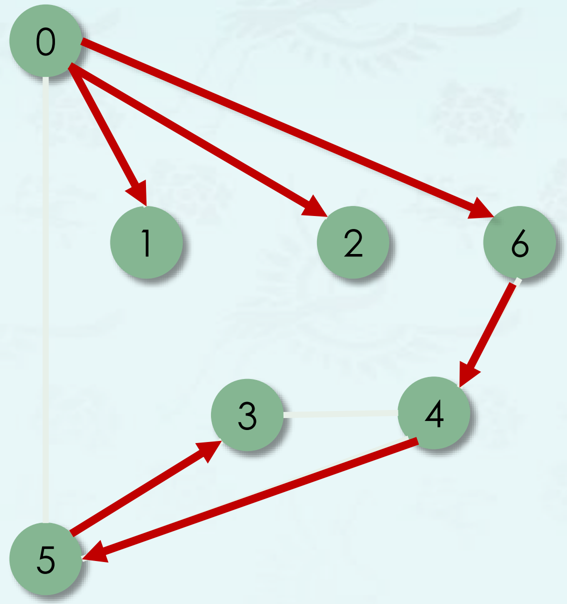
v	marked[]	parent[v]
0	T	-
1	T	0
2	T	0
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

DFS: Depth-First Search Properties

- **Proposition:** After DFS, can find vertices connected to s in constant time and can find a path to s (if one exists) in time proportional to its length.
- **Proof:** parent[] is parent-link representation of a tree rooted at s.

- What is the path from vertex 0 to vertex 3?
- In this case, what is in the stack when parent[] returns?

```
void DFSpath(graph g, int v, int w, stack<int>& path)
```

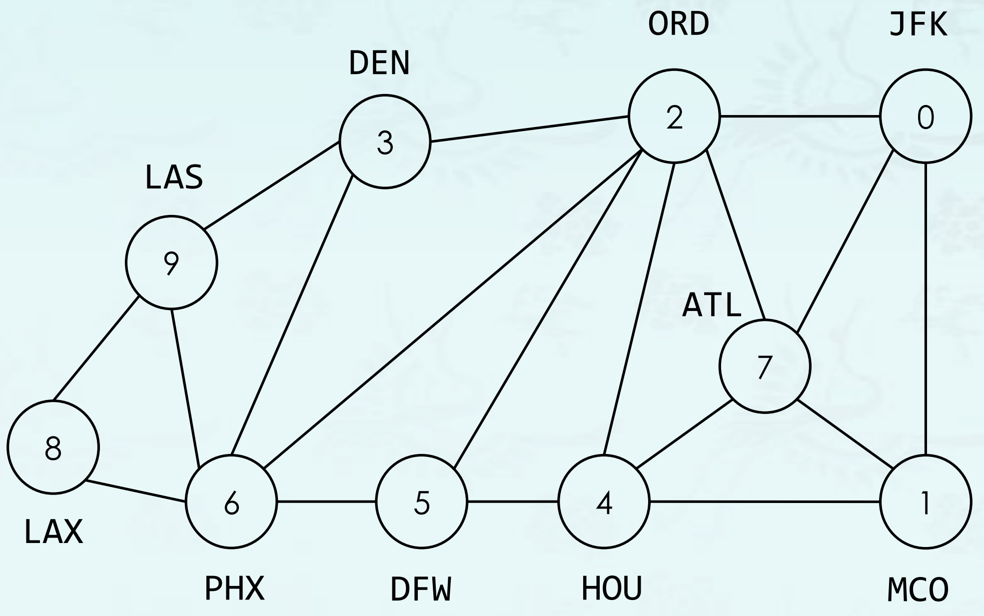


v	marked[]	parent[v]
0	T	-
1	T	0
2	T	0
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

DFS/BFS – Exercise

To visit a vertex v:

- Mark vertex v as visited.
- Recursively visit all unmarked vertices adjacent to v.



Graph g:

routes.txt
edge list

```
9
18
0 1
2 3
2 4
5 6
0 7
0 2
2 5
2 6
```

Adjacency list

```
0: 2 7 1
1: 4 7 0
2: 7 6 5 0 4 3
3: 9 6 2
4: 5 1 7 2
5: 4 2 6
6: 8 9 3 2 5
7: 1 2 4 0
8: 9 6
9: 8 6 3
```

v w

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DFS/BFS – Exercise

1. Get familiar with using graphx.exe and using ~.txt graph files provided in pset.
2. Create an **routes.txt** such that it generates the results as shown in the screen capture. Run DFS/BFS and get familiar with the results, its meaning and menu items.

```
Adjacency-list:
V[0]: 2 7 1
V[1]: 4 7 0
V[2]: 7 6 5 0 4 3
V[3]: 9 6 2
V[4]: 5 1 7 2
V[5]: 4 2 6
V[6]: 8 9 3 2 5
V[7]: 1 2 4 0
V[8]: 9 6
V[9]: 8 6 3

.    /----[3]---[2]\---------[0]
.   /      |    /|\ \        / |
.   |      |   / | \ \      /  |
.  [9]     |  /  |  \ \___[7]  |
.   | \    | /   |   \    / \  |
.   |  \  / /    |    \  /   \ |
.   |   \| /     |     \/     \|
.  [8]--[6]-----[5]----[4]----[1]

    vertex[0..9] =   0   1   2   3   4   5   6   7   8   9
     color[0..9] =   0   0   0   0   0   0   0   0   0   0
      DFS0[0..9] =   0   2   7   1   4   5   6   8   9   3
      CCID[0..9] =   1   1   1   1   1   1   1   1   1   1
DFS parent[0..9] =  -1   7   0   9   1   4   5   2   6   8
      BFS0[0..9] =   0   2   7   1   6   5   4   3   8   9
    DistTo[0..9] =   0   1   1   2   2   2   2   1   3   3
BFS parent[0..9] =  -1   0   0   2   2   2   2   0   6   6

Graph [AdjList][Graph][Tablet]   file:routes.txt V:10 E:36 CCs
n - new graph file      x - connected(v,w)
d - DFS(v=0)            e - distance(v,w)
b - BFS(v=0)            p - path(v,w)
m - print mode[adjList/graph]
Command(q to quit):
```

Data Structures
Chapter 7: Graph

1. Introduction
   - Terminology, Representation, ADT
2. Basic Operations
   - **DFS,** CC, BFS, Processing
3. Digraph and Applications
4. Minimum Spanning Tree(MST)

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