


Data Structures

Chapter 7: Graph

1. Introduction
 - Terminology, Representation, ADT
2. Basic Operations
 - DFS, **CC**, **BFS**, Processing
3. Digraph and Applications
4. Minimum Spanning Tree(MST)

A pair of black-rimmed glasses is placed on an open book. The book's pages are yellowed with age, and the text is faint and illegible. The background is a warm, golden-brown color.

네가 만일 네 입으로 예수를 주로 시인하며 또 하나님께서 그를 죽은 자 가운데서 살리신 것을 네 마음에 믿으면 구원을 받으리라 사람이 마음으로 믿어 의에 이르고 입으로 시인하여 구원에 이르느니라 (롬10:9-10)

죄의 값은 사망이요 하나님의 은사는 그리스도 예수 우리 주 안에 있는 영생이니라 (롬 6:23)

모든 사람이 죄를 범하였으매 하나님의 영광에 이르지 못하더니 그리스도 예수 안에 있는 속량으로 말미암아 하나님의 은혜로 값없이 의롭다 하심을 얻은 자 되었느니라 (롬 3:23-24)

Connectivity Queries

- **Def.:** Vertices v and w are connected if there is a path between them.
- **Goal:** Preprocess graph to answer queries of the form “*is v connected to w ?*” in constant time.

	Connected Component	
	CC(Graph g)	<i>find connected component in g</i>
bool	connected(int v , int w)	<i>are v and w connected"</i>
int	count()	<i>member of connected components</i>
int	id(int v)	<i>component identifier for v</i>

Depth-first search? Yes ...

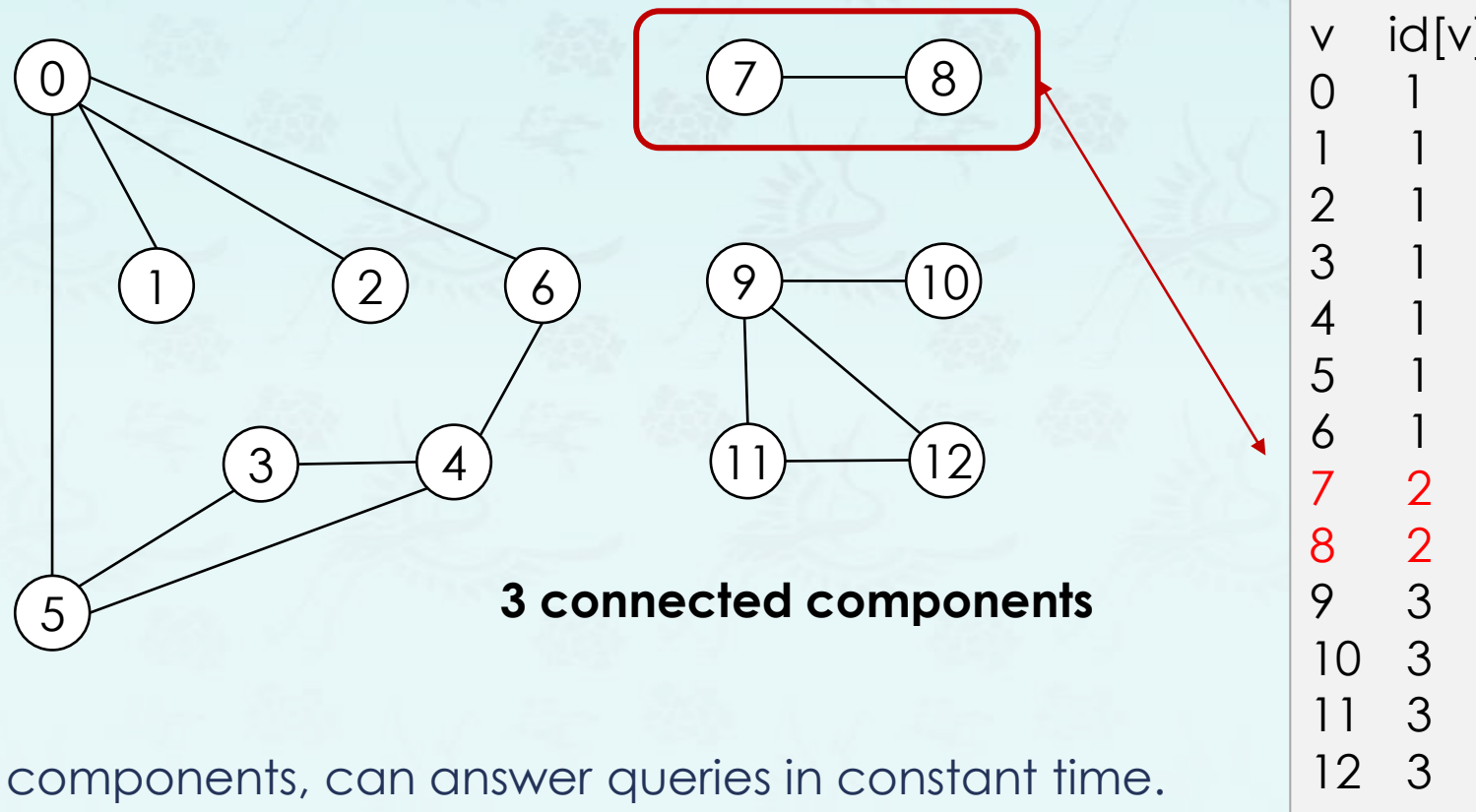
Connected Components

The relation “is connected to” is **equivalence relation**:

Reflexive: v is connected to v .

Symmetric: if v is connected to w , then w is connected to v .

Transitive: if v connected to w and w connected to x , then v connected to x



Remark:

Given connected components, can answer queries in constant time.

Connected Components

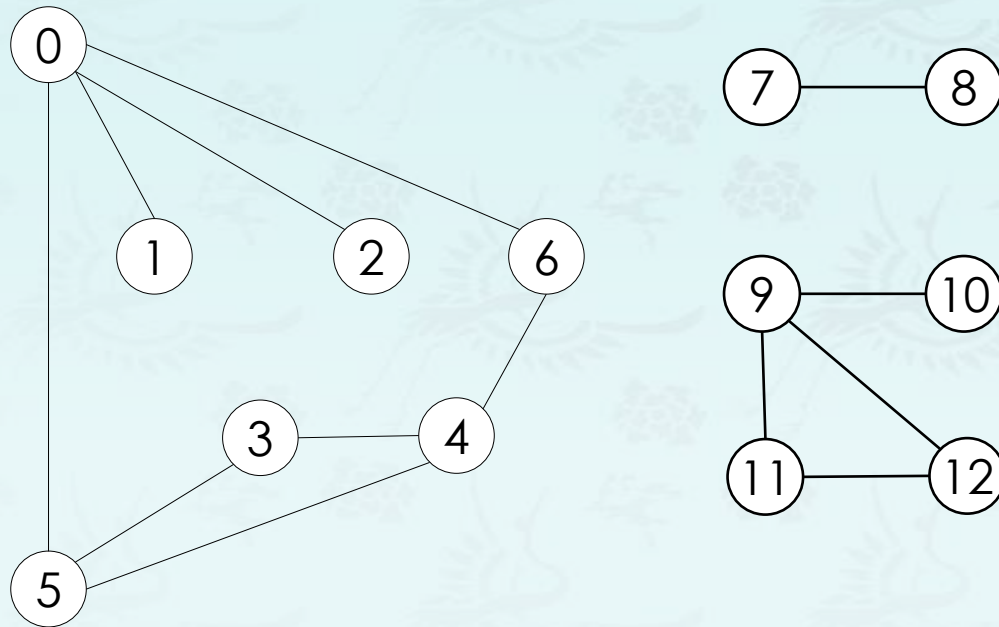
Goal: Partition vertices into connected components.

- Initialize all vertices v as unmarked.
- For each unmarked vertex v , run DFS to identify all vertices discovered as part of the same component.

Connected Components

To visit a vertex v :

- Mark vertex v as visited.
- Recursively visit all unmarked vertices adjacent to v .



Graph g :

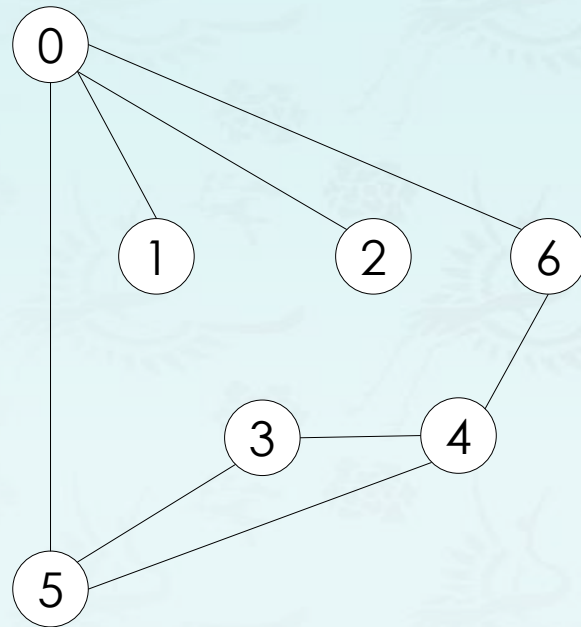
V-E lists

graph6.txt

```
13 ← V
13 ← E
0 5
4 3
0 1
9 12
6 4
5 4
0 2
11 12
9 10
0 6
7 8
9 11
5 3
```

Challenge: build adjacency lists?

Connected Components



Adjacency lists

adj[]	
0	6 2 1 5
1	0
2	0
3	5 4
4	5 6 3
5	3 4 0
6	0 4

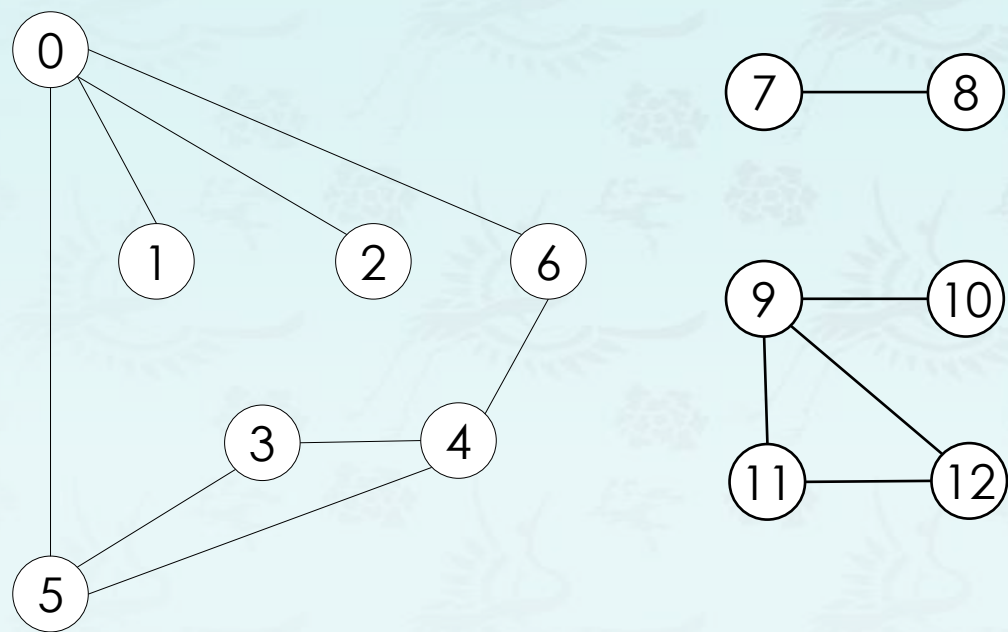
V-E lists

graph6.txt

```
13 ← V
13 ← E
0 5
4 3
0 1
9 12
6 4
5 4
0 2
11 12
9 10
0 6
7 8
9 11
5 3
```

Graph g:

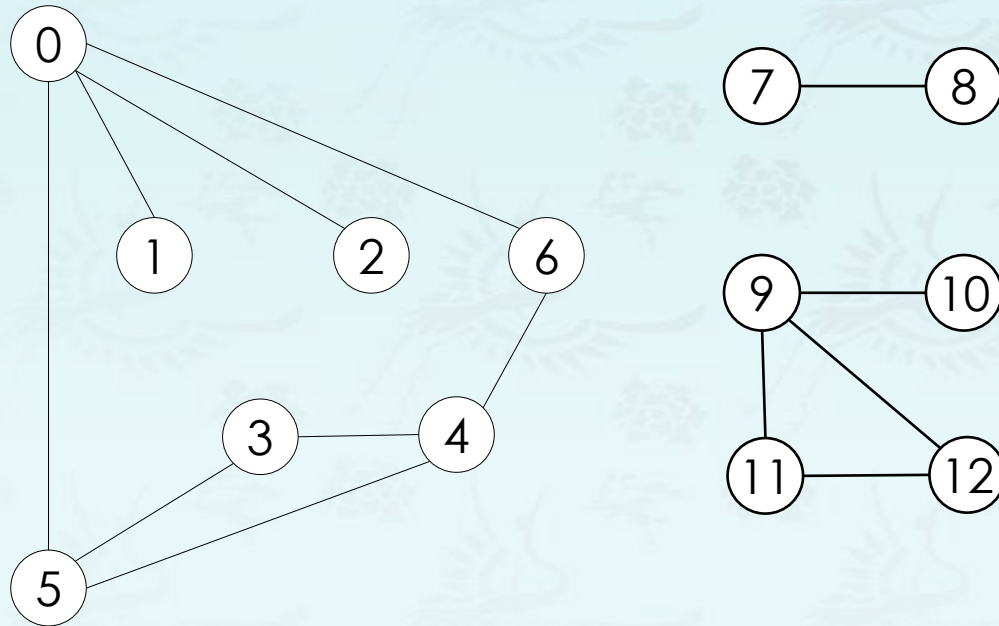
Connected Components



Graph g:

v	marked[]	id[]
0	F	-
1	F	-
2	F	-
3	F	-
4	F	-
5	F	-
6	F	-
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

Connected Components



Graph g:

v	marked[]	id[]
0	T	0
1	T	0
2	T	0
3	T	0
4	T	0
5	T	0
6	T	0
7	T	1
8	T	1
9	T	2
10	T	2
11	T	2
12	T	2

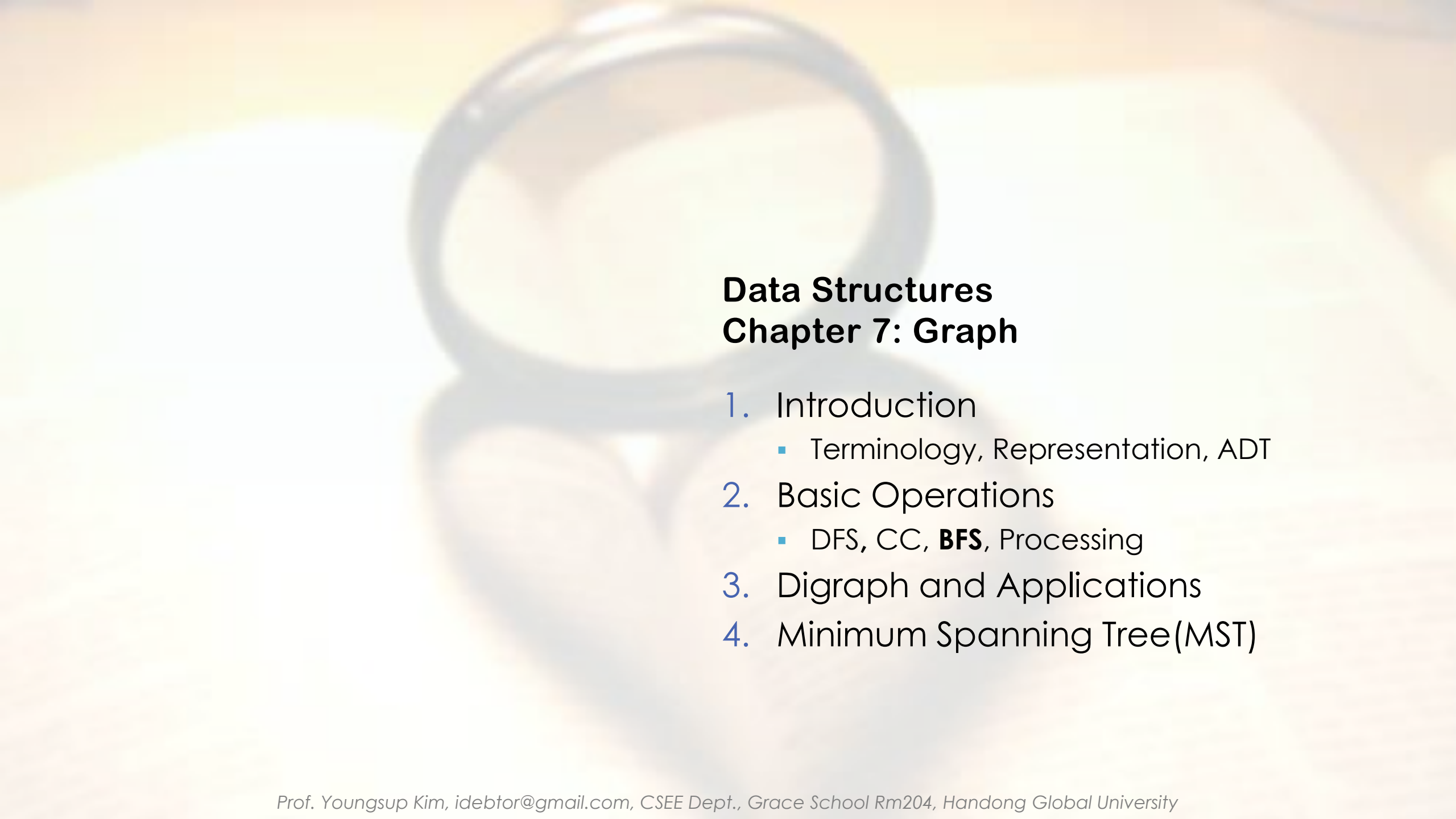
Connected Components – Coding

```
// returns true if v and w are connected.
bool connected(graph g, int v, int w) {
    if (empty(g)) return true;

    DFS_CCs(g);

    return g->CCID[v] == g->CCID[w];
}
```

```
// returns number of connected components.
int nCCs(graph g) {
    int id = g->CCID[0];
    int count = 1;
    for (int i = 0; i < V(g); i++)
        if (id != g->CCID[i]) {
            id = g->CCID[i];
            count++;
        }
    return id == 0 ? 0 : count;
}
```



Data Structures

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Design pattern for graph processing

- **Design pattern:** Decouple graph data type
- **Idea:** Mimic maze exploration

DFS (to visit a vertex v)

- **Mark v as visited.**
- **Recursively visit all unmarked vertices w adjacent to v .**

Typical applications:

- Find all vertices connected to a given source vertex.
- Find a path between two vertices.

Challenge:

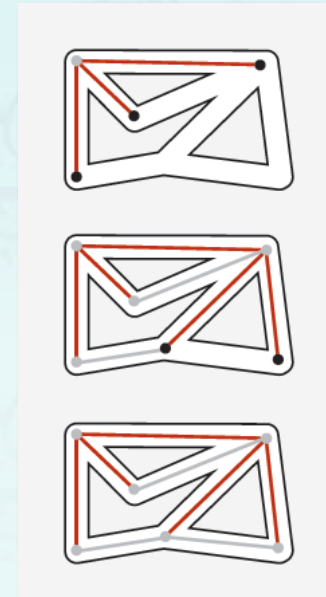
- How to implement?

Breadth-first search

- **Depth-first search:** Put unvisited vertices on a **stack**.
- **Breadth-first search:** Put unvisited vertices on a **queue**.
- **Shortest path:** Find path from s to t that uses **fewest number of edges**.

BFS: (from source vertex s)

- Put s onto a FIFO queue, and mark s as visited.
- Repeat until the queue is empty:
 - remove the least recently added vertex v
 - add each of v 's unvisited neighbors to the queue, and mark them as visited.

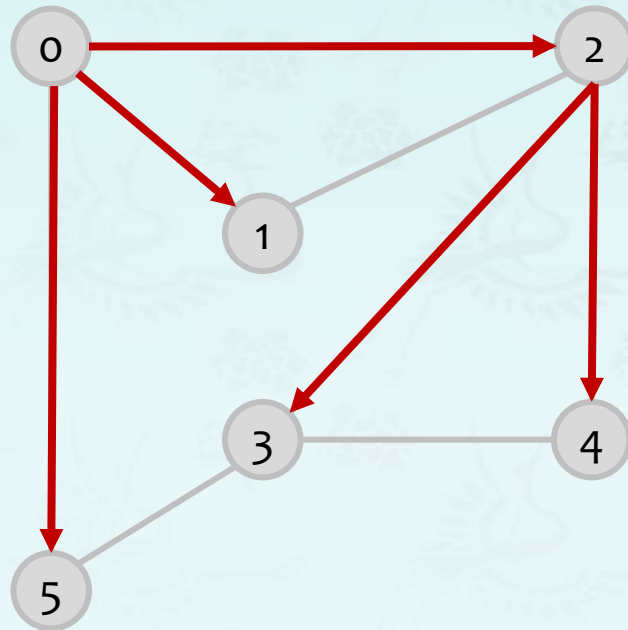


Intuition: BFS examines vertices in increasing distance from s .

Breadth-first search demo

Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.



done

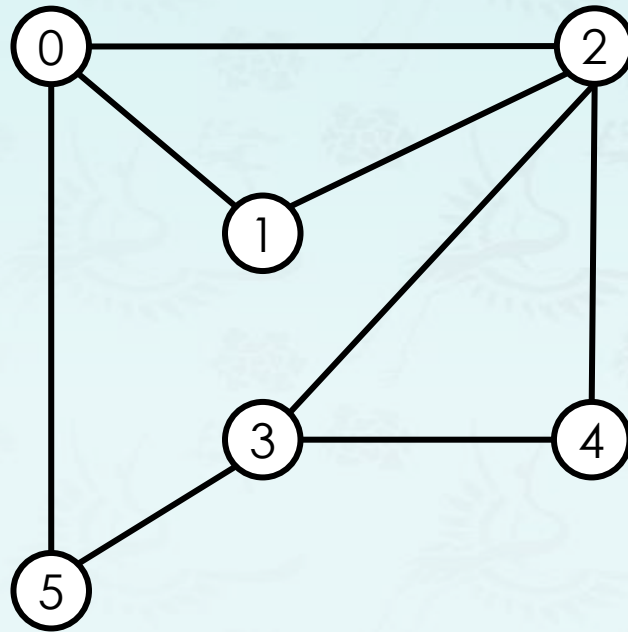
graph2.txt

v	parent[v]	distTo[]
0	-	0
1	0	1
2	0	1
3	2	2
4	2	2
5	0	1

Breadth-first search demo

Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.



Graph g :

Adjacency lists

adj[]
0
1
2
3
4
5

graph2.txt

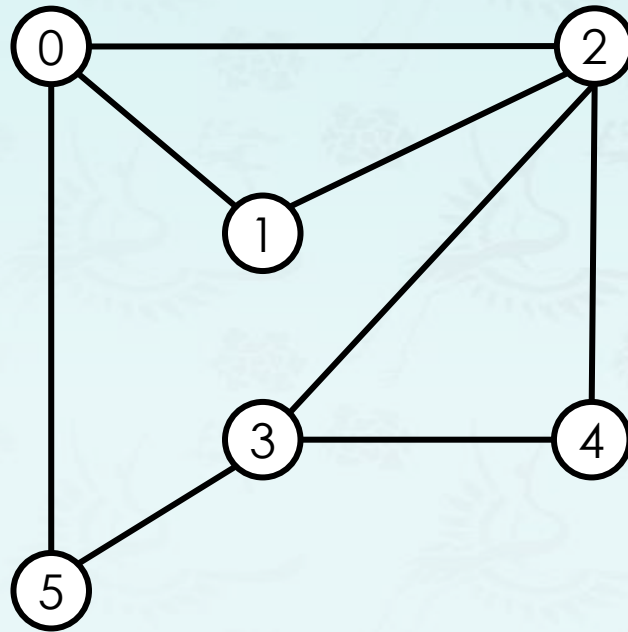
6	←	V
8	←	E
0	5	
2	4	
2	3	
1	2	
0	1	
3	4	
3	5	
0	2	

Challenge: build adjacency lists?

Breadth-first search demo

Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.



Adjacency lists

adj[]	
0	5
1	
2	4
3	
4	2
5	0

graph2.txt

6	←	V
8	←	E
0	5	
2	4	
2	3	
1	2	
0	1	
3	4	
3	5	
0	2	

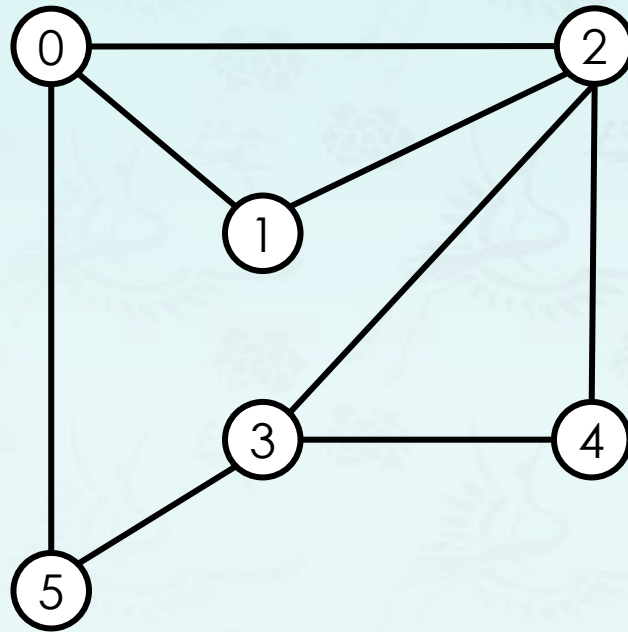
Graph g:

Challenge: build adjacency lists?

Breadth-first search demo

Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.



Adjacency lists

adj[]	
0	5
1	
2	3 4
3	2
4	2
5	0

graph2.txt	
6	← V
8	← E
0	5
2	4
2	3
1	2
0	1
3	4
3	5
0	2

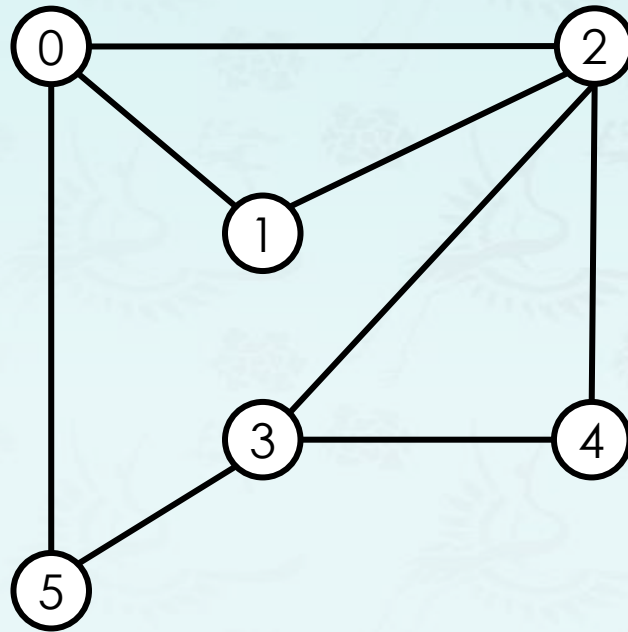
Graph g :

Challenge: build adjacency lists?

Breadth-first search demo

Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.



Adjacency lists

adj[]	
0	5
1	2
2	1 3 4
3	2
4	2
5	0

graph2.txt	
6	← V
8	← E
0	5
2	4
2	3
1	2
0	1
3	4
3	5
0	2

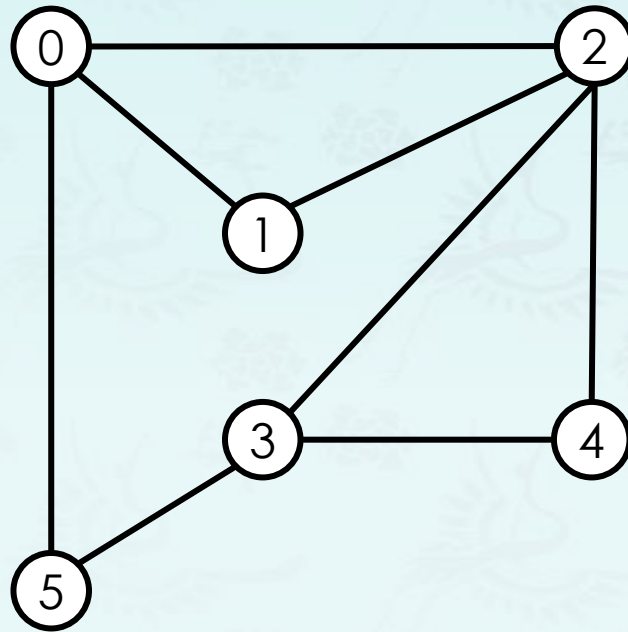
Graph g:

Challenge: build adjacency lists?

Breadth-first search demo

Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.



Adjacency lists

adj[]	
0	2 1 5
1	0 2
2	0 1 3 4
3	5 4 2
4	3 2
5	3 0

graph2.txt

```
6 ← V
8 ← E
0 5
2 4
2 3
1 2
0 1
3 4
3 5
0 2
```

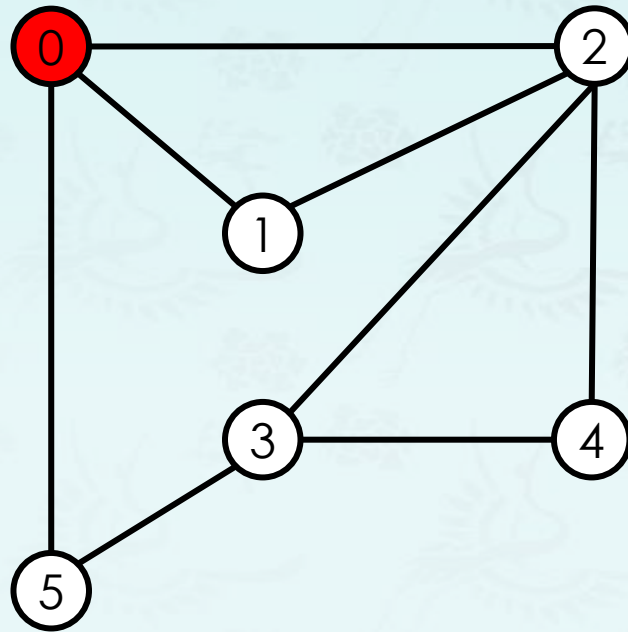
Graph g :

Challenge: build adjacency lists?
Job done

Breadth-first search demo

Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.



Adjacency lists

adj[]	
0	2 1 5
1	0 2
2	0 1 3 4
3	5 4 2
4	3 2
5	3 0

graph2.txt

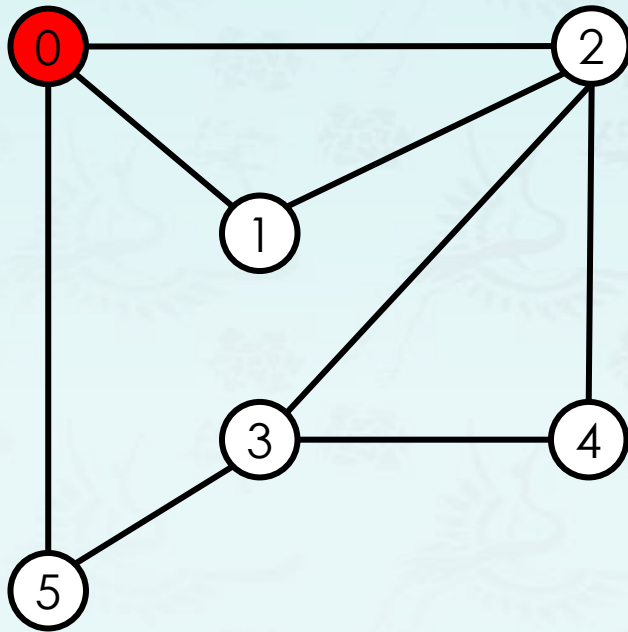
```
6 ← V
8 ← E
0 5
2 4
2 3
1 2
0 1
3 4
3 5
0 2
```

Graph g :

Breadth-first search demo

Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.



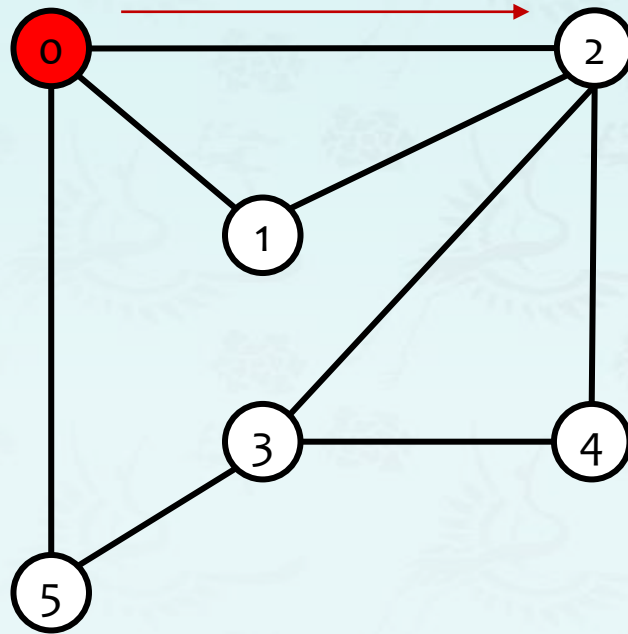
queue	v	parent[v]	distTo[]
	0	-	0
	1	-	-
	2	-	-
	3	-	-
	4	-	-
0	5	-	-

add 0 to queue:

Breadth-first search demo

Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.



queue	v	parent[v]	distTo[]
	0	-	0
	1	-	-
	2	0	1
	3	-	-
	4	-	-
0	5	-	-

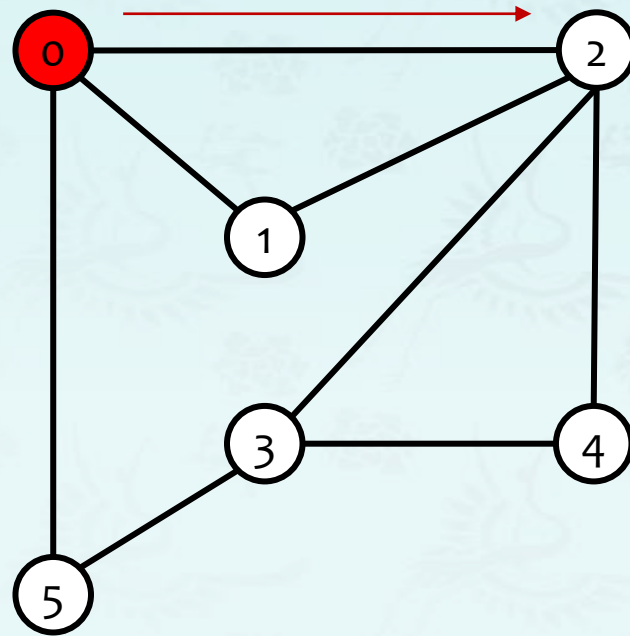
adj[0] [2] [1] [5]

dequeue 0: check 2, check 1 and check 5

Breadth-first search demo

Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.



adj[0] [2] [1] [5]

Adjacency lists

adj[]	
0	[2] [1] [5]
1	[0] [2]
2	[0] [1] [3] [4]
3	[5] [4] [2]
4	[3] [2]
5	[3] [0]

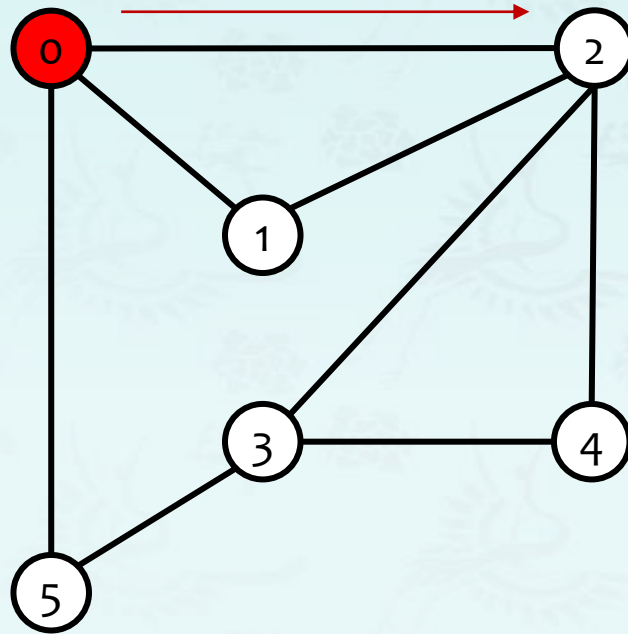
queue	v	parent[v]	distTo[]
	0	-	0
	1	-	-
	2	0	1
	3	-	-
	4	-	-
	5	-	-

dequeue 0: check 2, check 1 and check 5

Breadth-first search demo

Repeat until queue is empty:

- Remove vertex v from queue.
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queue	v	parent[v]	distTo[]
	0	-	0
	1	-	-
	2	0	1
	3	-	-
	4	-	-
2	5	-	-

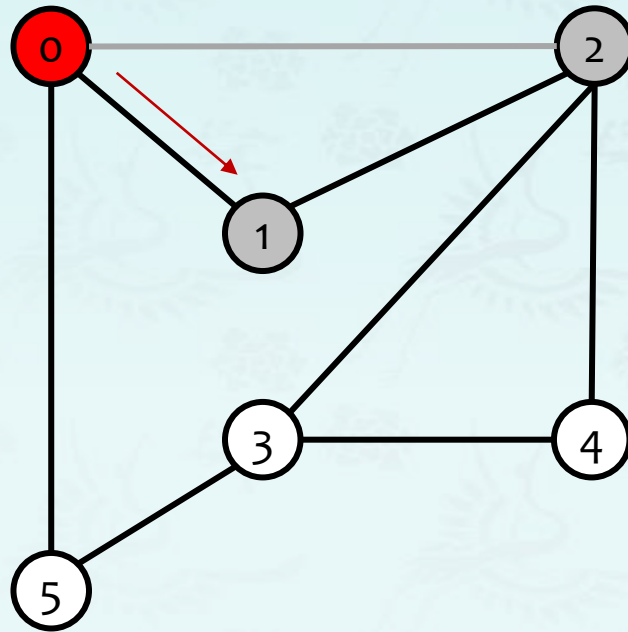
adj[0] [2] [1] [5]

dequeue 0: check 2, check 1 and check 5

Breadth-first search demo

Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.



queue	v	parent[v]	distTo[]
	0	-	0
	1	-	-
	2	0	1
	3	-	-
	4	-	-
2	5	-	-

adj[0]

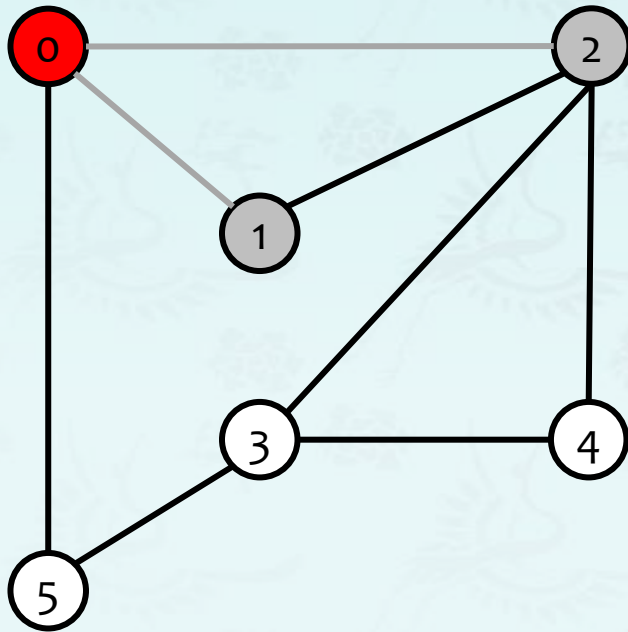
2		1		5	
---	--	---	--	---	--

dequeue 0: check 2, check 1 and check 5

Breadth-first search demo

Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.



adj[0] [2] [1] [5]

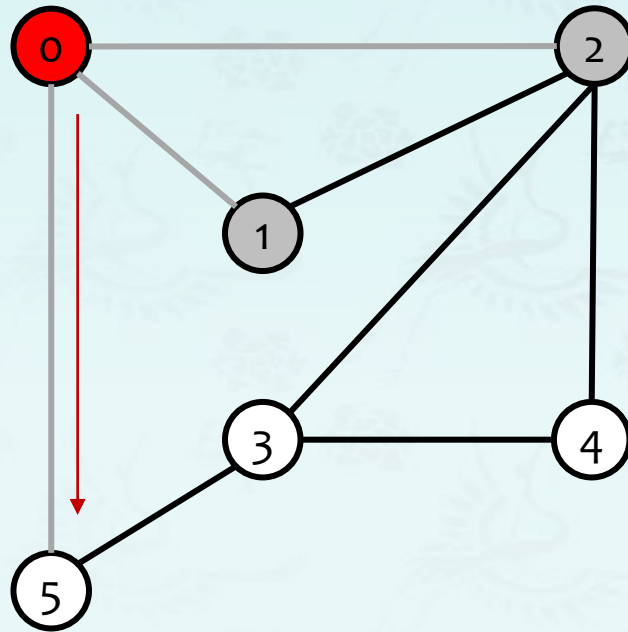
dequeue 0: check 2, check 1 and check 5

queue	v	parent[v]	distTo[]
	0	-	0
	1	0	1
	2	0	1
	3	-	-
1	4	-	-
2	5	-	-

Breadth-first search demo

Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices adjacent to v and mark them.

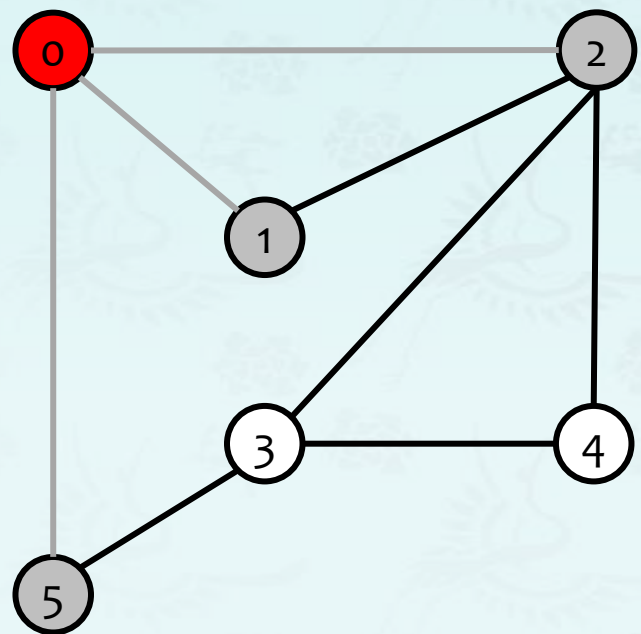


adj[0] [2] [1] [5]

dequeue 0: check 2, check 1 and check 5

queue	v	parent[v]	distTo[]
	0	-	0
	1	0	1
	2	0	1
	3	-	-
1	4	-	-
2	5	-	-

Breadth-first search demo

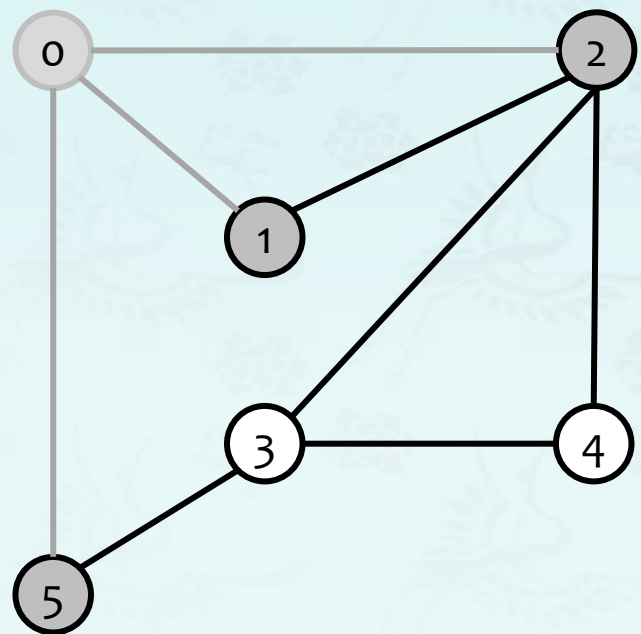


queue	v	parent[v]	distTo[]
	0	-	0
	1	0	1
	2	0	1
5	3	-	-
1	4	-	-
2	5	0	1

adj[0] [2] [1] [5]

dequeue 0: check 2, check 1 and check 5

Breadth-first search demo



queue	v	parent[v]	distTo[]
	0	-	0
	1	0	1
	2	0	1
5	3	-	-
1	4	-	-
2	5	0	1

adj[0]

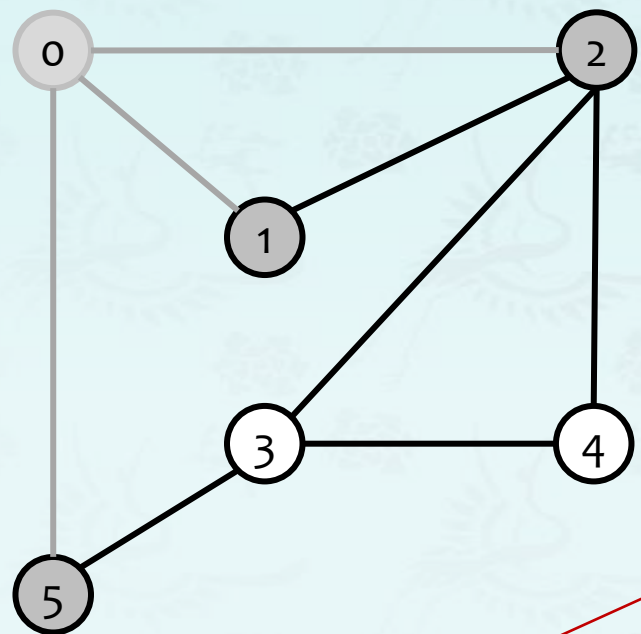
2		1		5	
---	--	---	--	---	--

0 done



BFS: 0

Breadth-first search demo

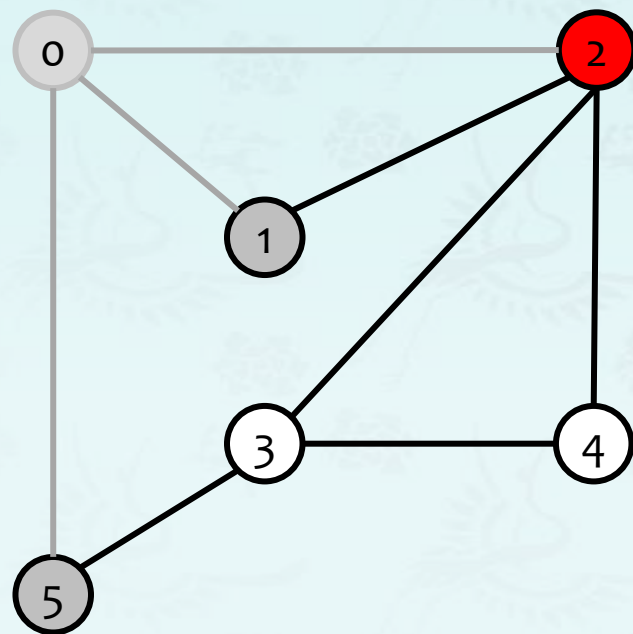


queue	v	parent[v]	distTo[]
	0	-	0
	1	0	1
	2	0	1
	3	-	-
	4	-	-
	5	0	1

dequeue 2:

BFS: 0

Breadth-first search demo



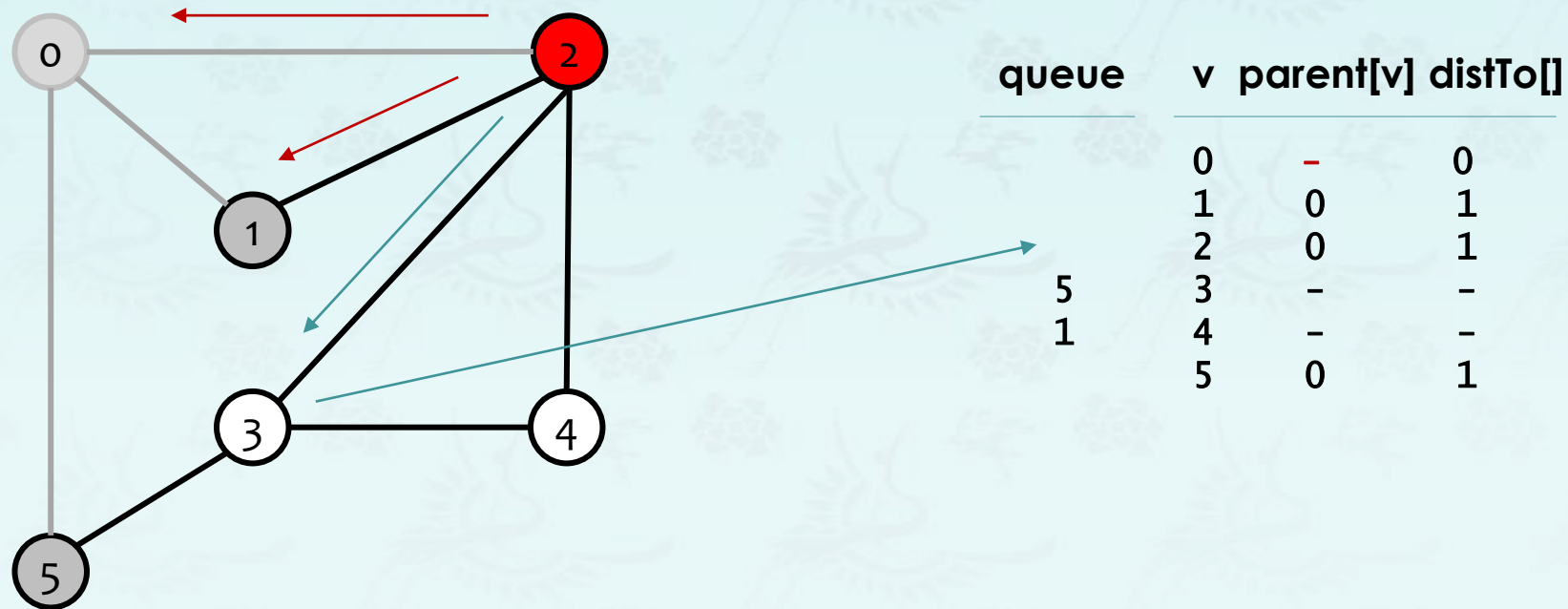
queue	v	parent[v]	distTo[]
	0	-	0
	1	0	1
	2	0	1
5	3	-	-
1	4	-	-
	5	0	1

adj[2] [0] [1] [3] [4]

dequeue 2: check 0, check 1, check 3 and check 4

BFS: 0

Breadth-first search demo

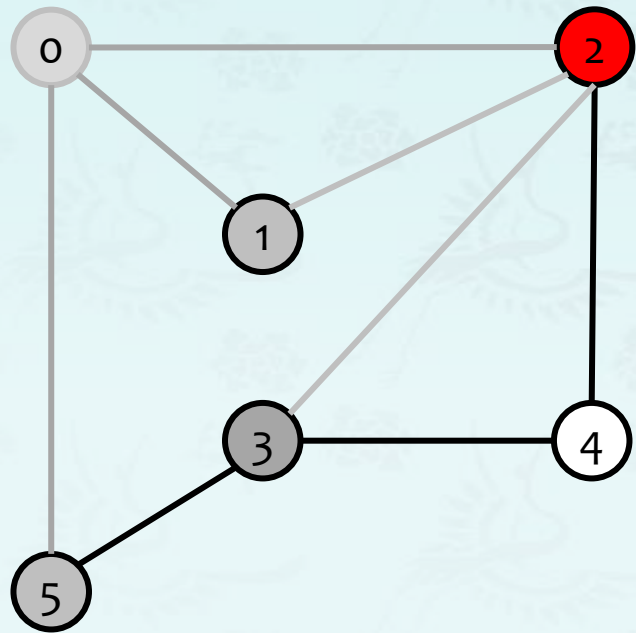


adj[2] [0] [1] [3] [4]

dequeue 2: check 0, check 1, check 3 and check 4

BFS: 0

Breadth-first search demo



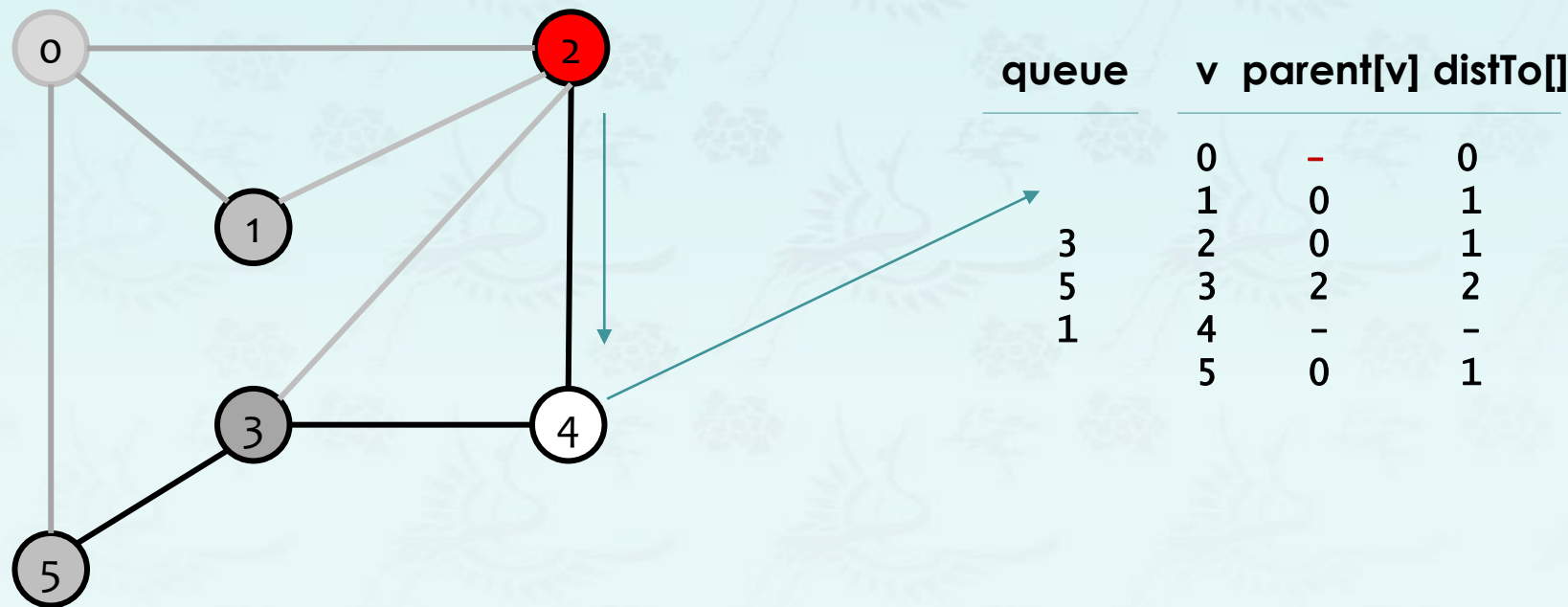
queue	v	parent[v]	distTo[]
	0	-	0
	1	0	1
3	2	0	1
5	3	2	2
1	4	-	-
	5	0	1

adj[2] [0] [1] [3] [4]

dequeue 2: check 0, check 1, check 3 and check 4

BFS: 0

Breadth-first search demo

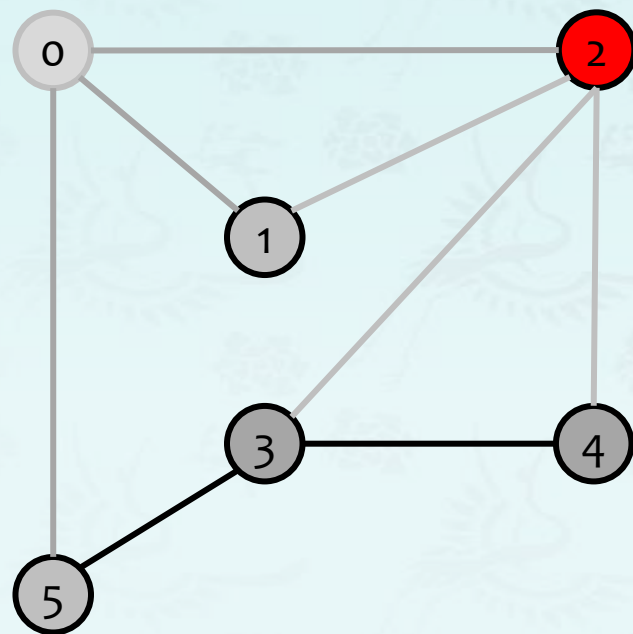


adj[2] [0] [1] [3] [4]

dequeue 2: check 0, check 1, check 3 and **check 4**

BFS: 0

Breadth-first search demo



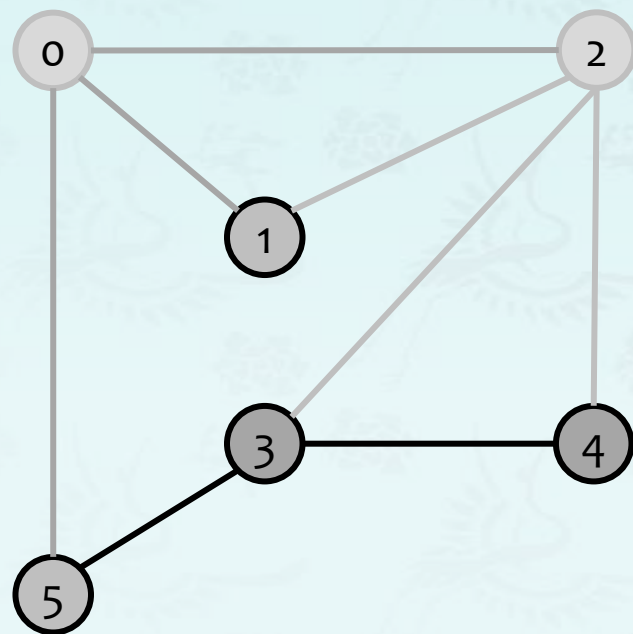
queue	v	parent[v]	distTo[]
	0	-	0
4	1	0	1
3	2	0	1
5	3	2	2
1	4	-	-
	5	0	1

adj[2] [0] [1] [3] [4]

dequeue 2: check 0, check 1, check 3 and **check 4**

BFS: 0

Breadth-first search demo



queue	v	parent[v]	distTo[]
	0	-	0
4	1	0	1
3	2	0	1
5	3	2	2
1	4	2	2
	5	0	1

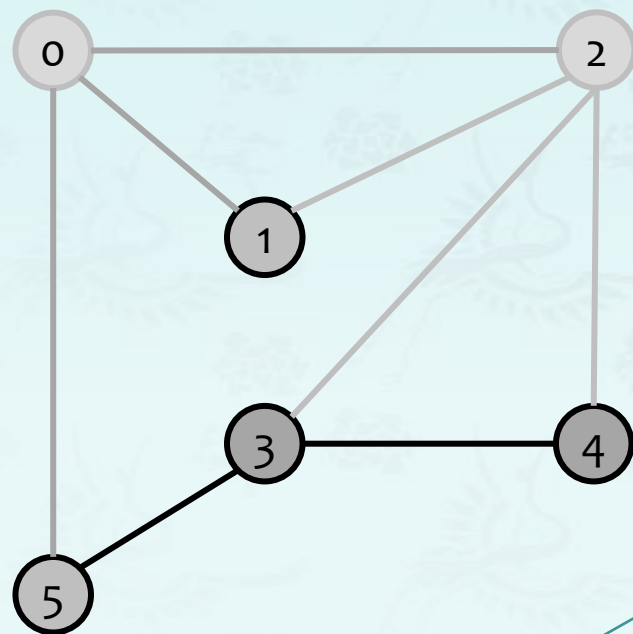
adj[2] [0] [1] [3] [4]

2 done



BFS: 0 2

Breadth-first search demo

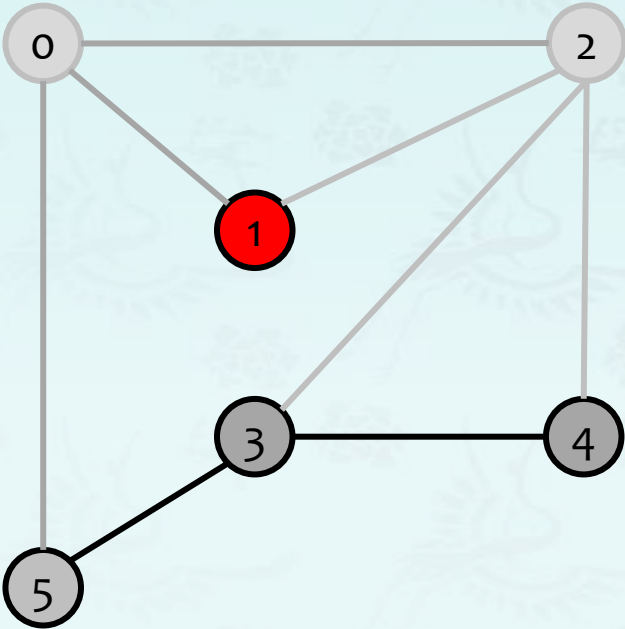


queue	v	parent[v]	distTo[]
	0	-	0
4	1	0	1
3	2	0	1
5	3	2	2
1	4	2	2
	5	0	1

dequeue 1

BFS: 0 2

Breadth-first search demo

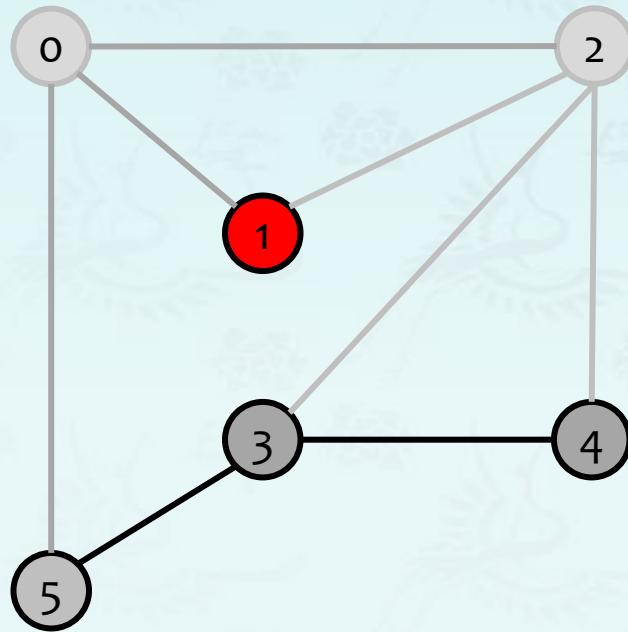


queue	v	parent[v]	distTo[]
	0	-	0
4	1	0	1
3	2	0	1
5	3	2	2
	4	2	2
	5	0	1

dequeue 1

BFS: 0 2

Breadth-first search demo



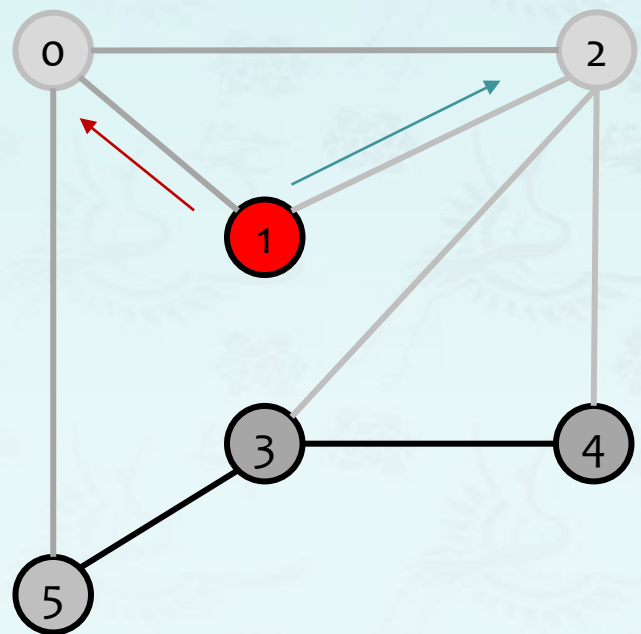
queue	v	parent[v]	distTo[]
	0	-	0
4	1	0	1
3	2	0	1
5	3	2	2
	4	2	2
	5	0	1

adj[1] [0] [2]

dequeue 1: check 0, and check 2

BFS: 0 2

Breadth-first search demo



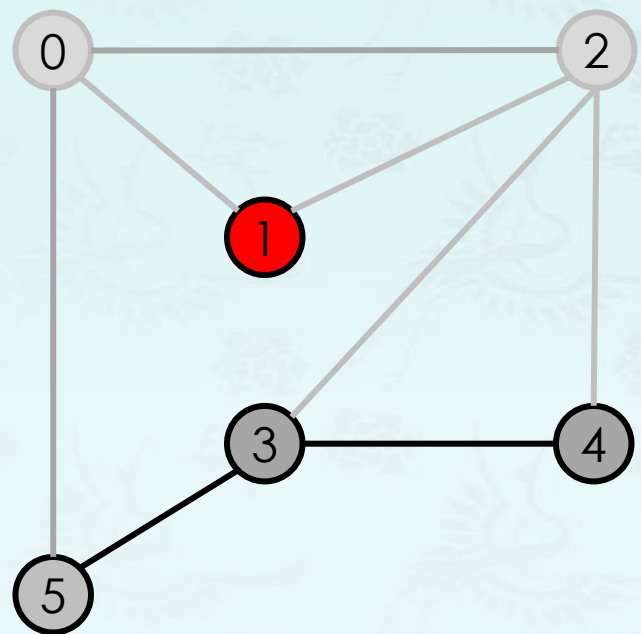
queue	v	parent[v]	distTo[]
	0	-	0
4	1	0	1
3	2	0	1
5	3	2	2
	4	2	2
	5	0	1

adj[1] [0] [2]

dequeue 1: check 0, and check 2

BFS: 0 2

Breadth-first search demo



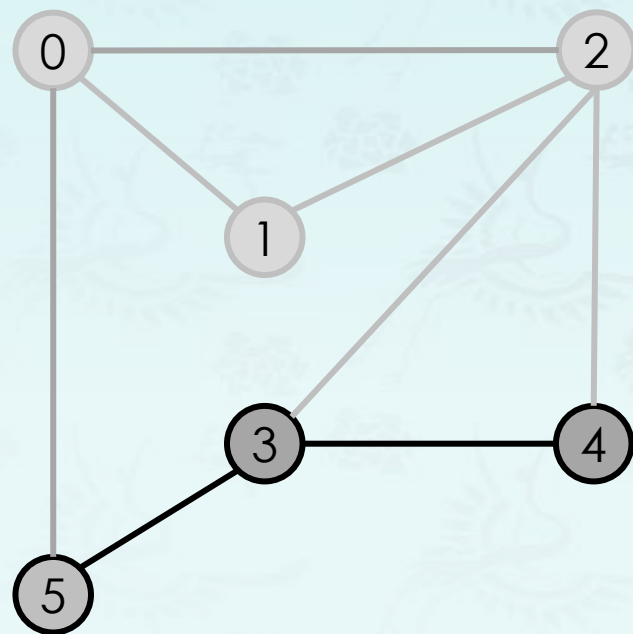
queue	v	parent[v]	distTo[]
	0	-	0
4	1	0	1
3	2	0	1
5	3	2	2
	4	2	2
	5	0	1

adj[1] 0 2

dequeue 1: check 0, and check 2

BFS: 0 2

Breadth-first search demo



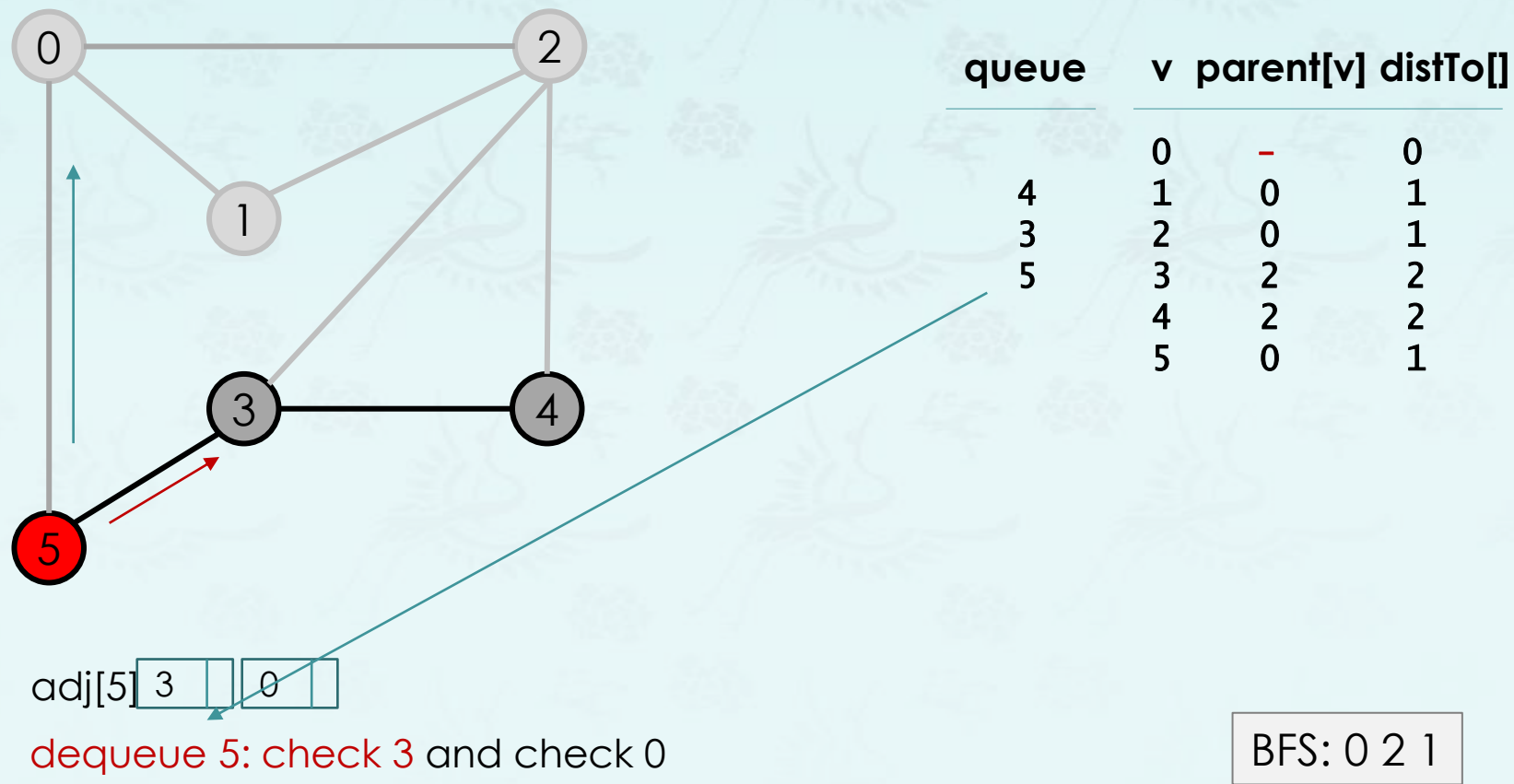
queue	v	parent[v]	distTo[]
	0	-	0
4	1	0	1
3	2	0	1
5	3	2	2
	4	2	2
	5	0	1

adj[1] 0 2

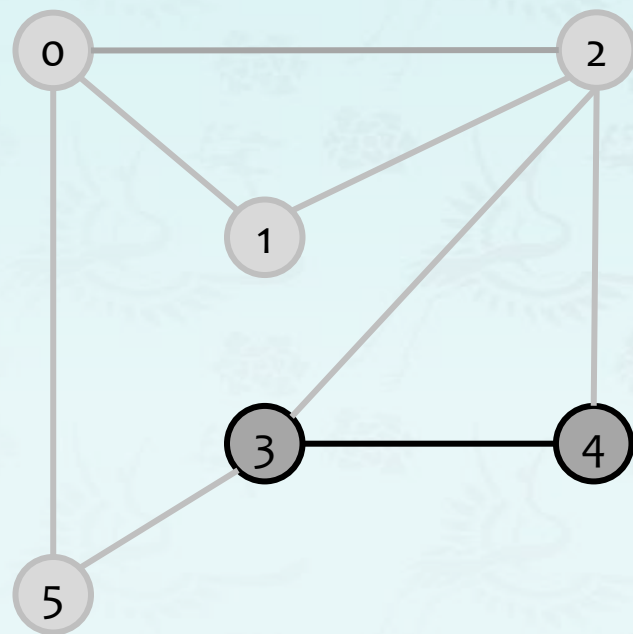
1 done →

BFS: 0 2 1

Breadth-first search demo



Breadth-first search demo



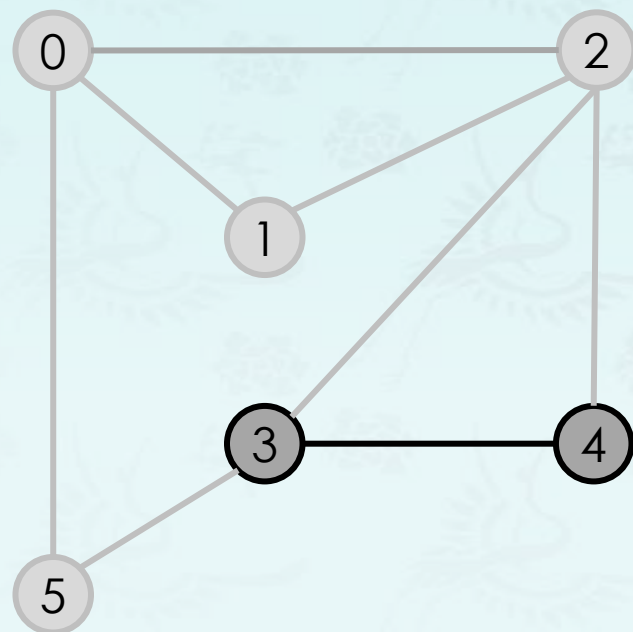
queue	v	parent[v]	distTo[]
	0	-	0
4	1	0	1
3	2	0	1
	3	2	2
	4	2	2
	5	0	1

adj[5] 3 0

5 done →

BFS: 0 2 1 5

Breadth-first search demo



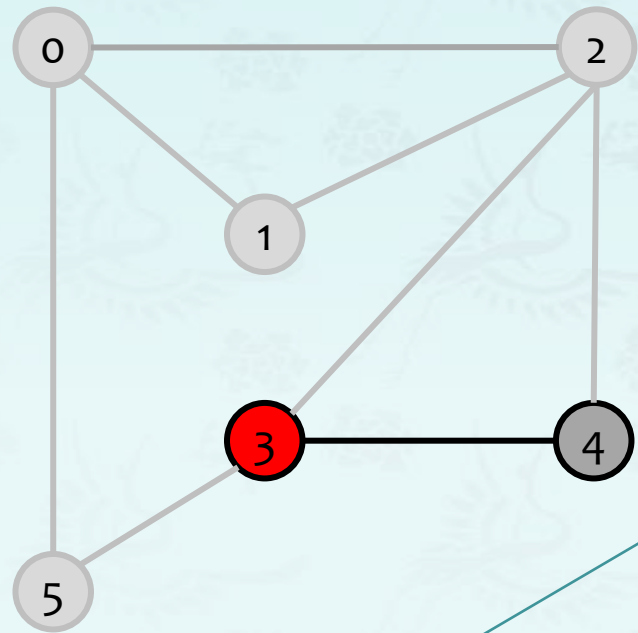
queue	v	parent[v]	distTo[]
	0	-	0
4	1	0	1
3	2	0	1
	3	2	2
	4	2	2
	5	0	1

adj[3] 5 4 2

dequeue 3: Check 5, Check 4, and Check 2

BFS: 0 2 1 5

Breadth-first search demo

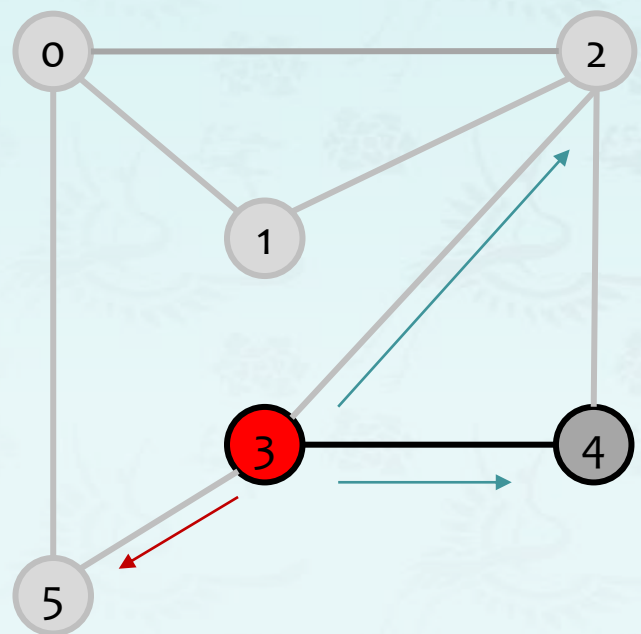


dequeue 3:

queue	v	parent[v]	distTo[]
	0	-	0
4	1	0	1
3	2	0	1
	3	2	2
	4	2	2
	5	0	1

BFS: 0 2 1 5

Breadth-first search demo



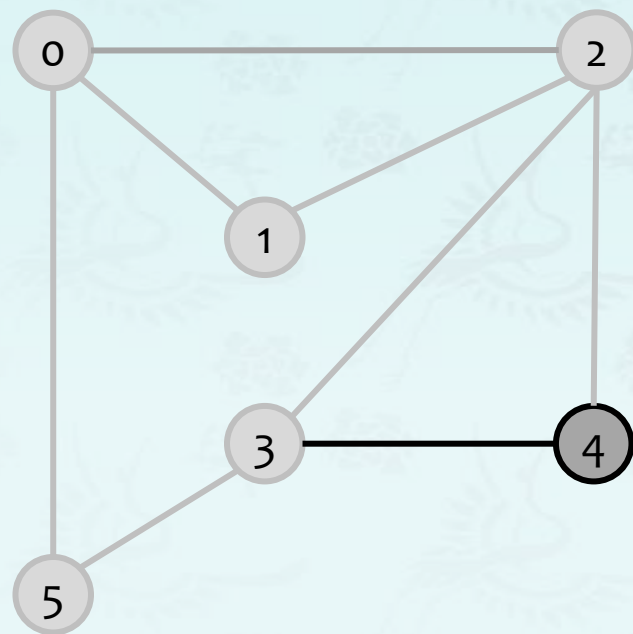
queue	v	parent[v]	distTo[]
4	0	-	0
	1	0	1
	2	0	1
	3	2	2
	4	2	2
	5	0	1

adj[3] 5 4 2

dequeue 3: Check 5, Check 4, and Check 2

BFS: 0 2 1 5

Breadth-first search demo



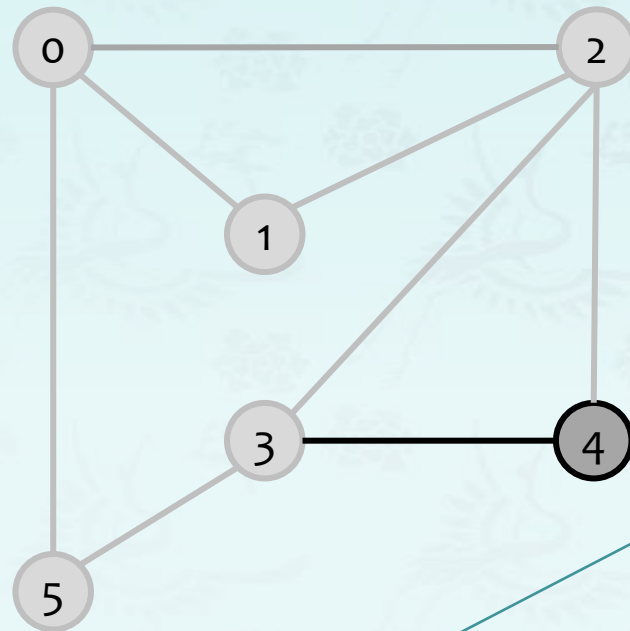
queue	v	parent[v]	distTo[]
	0	-	0
4	1	0	1
	2	0	1
	3	2	2
	4	2	2
	5	0	1

adj[3] 5 4 2

3 done →

BFS: 0 2 1 5 3

Breadth-first search demo

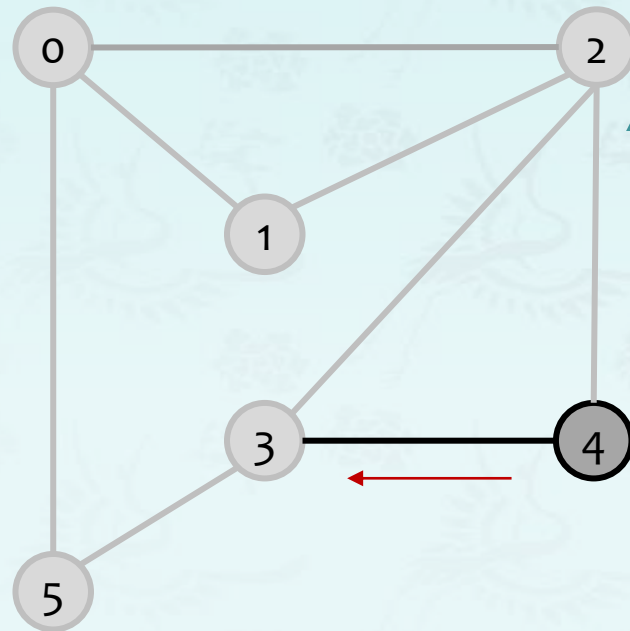


queue	v	parent[v]	distTo[]
	0	-	0
4	1	0	1
	2	0	1
	3	2	2
	4	2	2
	5	0	1

dequeue 4

BFS: 0 2 1 5 3

Breadth-first search demo



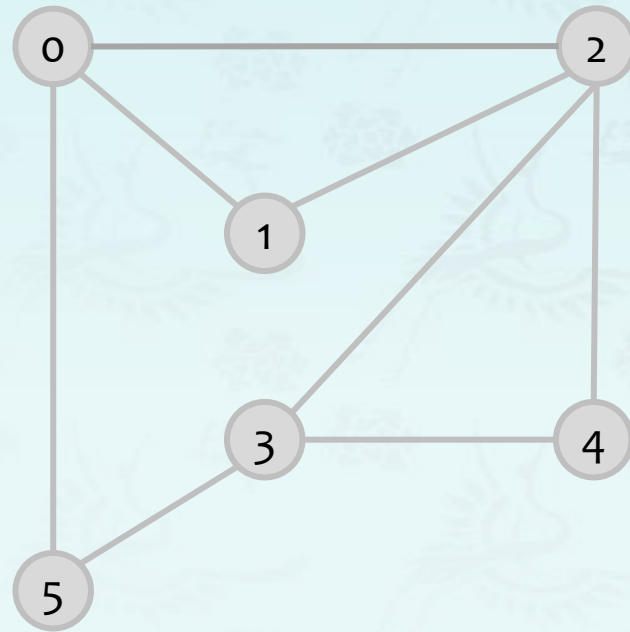
queue	v	parent[v]	distTo[]
0	-	-	0
1	0	0	1
2	0	0	1
3	2	2	2
4	2	2	2
5	0	0	1

adj[4] 3 2

dequeue 4: Check 3 and Check 2

BFS: 0 2 1 5 3

Breadth-first search demo



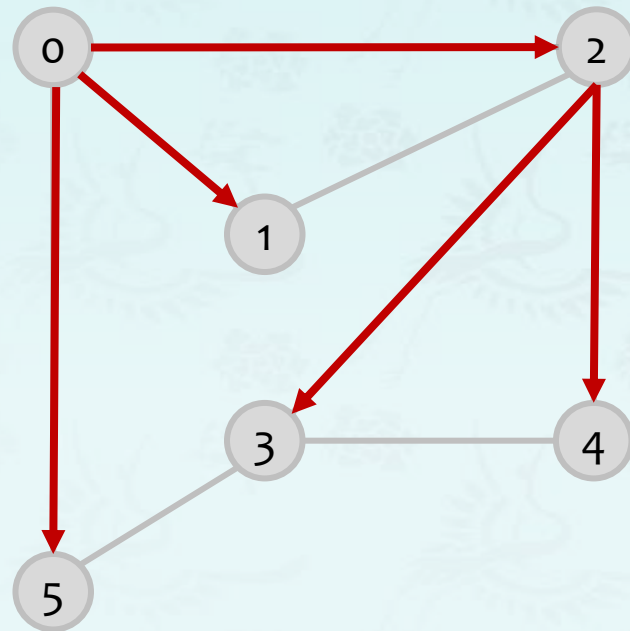
queue	v	parent[v]	distTo[]
	0	-	0
	1	0	1
	2	0	1
	3	2	2
	4	2	2
	5	0	1

4 done



BFS: 0 2 1 5 3 4

Breadth-first search demo



v	parent[v]	distTo[]
0	-	0
1	0	1
2	0	1
3	2	2
4	2	2
5	0	1

done

BFS: 0 2 1 5 3 4

Breadth-first search

- **Depth-first search:** Put unvisited vertices on a **stack**.
- **Breadth-first search:** Put unvisited vertices on a **queue**.
- **Shortest path:** Find path from s to t that uses **fewest number of edges**.

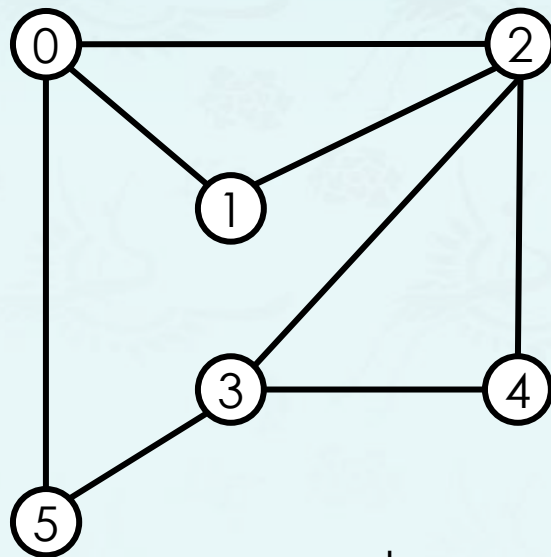
BFS: (from source vertex s)

- Put s onto a FIFO queue, and mark s as visited.
- Repeat until the queue is empty:
 - remove the least recently added vertex v
 - add each of v 's unvisited neighbors to the queue, and mark them as visited.

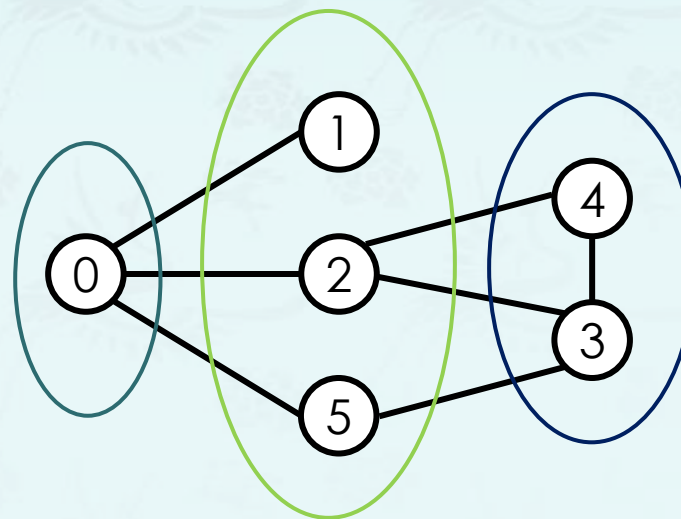
- **Intuition:** BFS examines vertices in increasing distance from s .

Breadth-first search properties

- **Proposition:** BFS computes shortest paths (fewest number of edges) from s to all other vertices in a graph in time proportional to $E + V$.
- **Proof: [correctness]** Queue always consists of zero or more vertices of distance k from s , followed by zero or more vertices of distance $k + 1$.
- **Proof: [running time]** Each vertex connected to s is visited once.



graph



dist = 0

dist = 1

dist = 2

Breadth-first search implementation

```
// runs BFS at v and produces BFS0[], distTo[] & parentBFS[]
void BFS(graph g, int v) {
    queue<int> que;           // to process each vertex
    queue<int> sav;           // BFS result saved
    for (int i = 0; i < V(g); i++) g->marked[i] = false;
    g->parentBFS[v] = -1;      g->marked[v] = true;
    g->distTo[v] = 0;          g->BFSv = {};
    que.push(v);              sav.push(v);

    while (!que.empty()) {
        int cur = que.front(); que.pop(); // remove it since processed
        for (gnode w = g->adj[cur].next; w; w = w->next) {
            if (!g->marked[w->item]) {
                g->marked[w->item] = true;
                que.push(w->item);           // queued to process next
                sav.push(w->item);           // save the result
                cout << "your code here";   // set parentBFS[] & distTo[]
            }
        }
    }
    g->BFSv = sav;                // save the result at v
    setBFS0(g, v, sav);
}
```

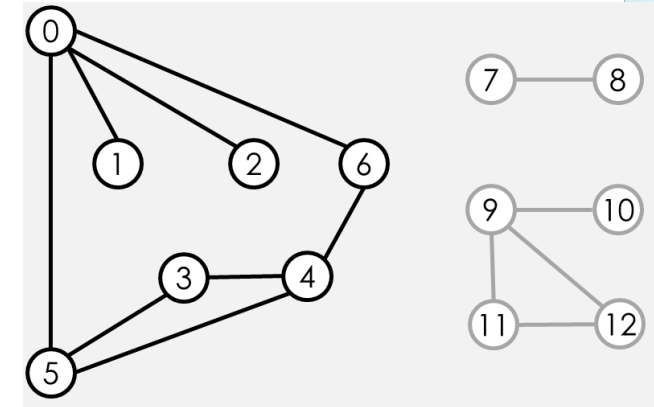
Breadth-first search implementation

```
// runs BFS for all vertices or all connected components
// It begins with the first vertex 0 at the adjacent list.
// It produces BFS0[], distTo[] & parentBFS[].
void BFS_CCs(graph g) {
    if (empty(g)) return;

    for (int i = 0; i < v(g); i++) {
        g->marked[i] = false;
        g->parentBFS[i] = -1;
        g->BFS0[i] = -1;
        g->distTo[i] = -1;
    }

    // BFS for all connected components starting from 0
    BFS(g, 0); ← make it run for all CC's

    g->BFSv = {};
}
```



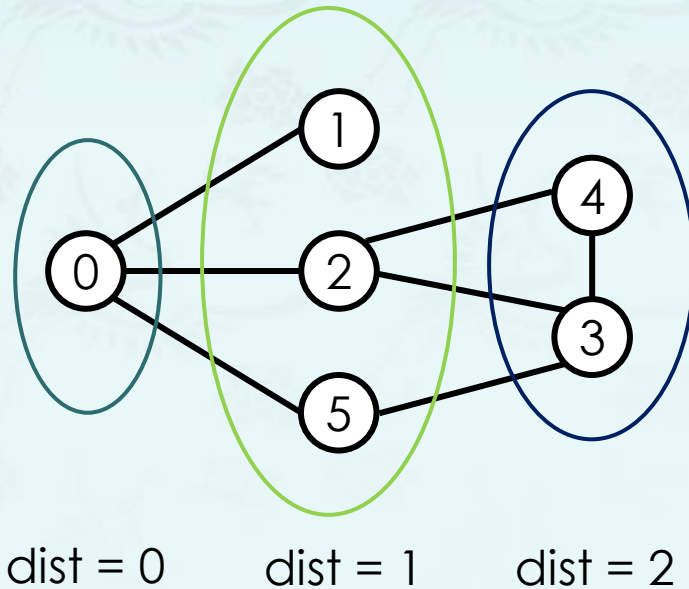
Breadth-first search implementation

```
// returns the number of edges in a shortest path between v and w
int distTo(graph g, int v, int w) {
    if (empty(g)) return 0;
    if (!connected(g, v, w)) return 0;

    BFS(g, v);

    cout << "your code here\n";

    return 0;
}
```



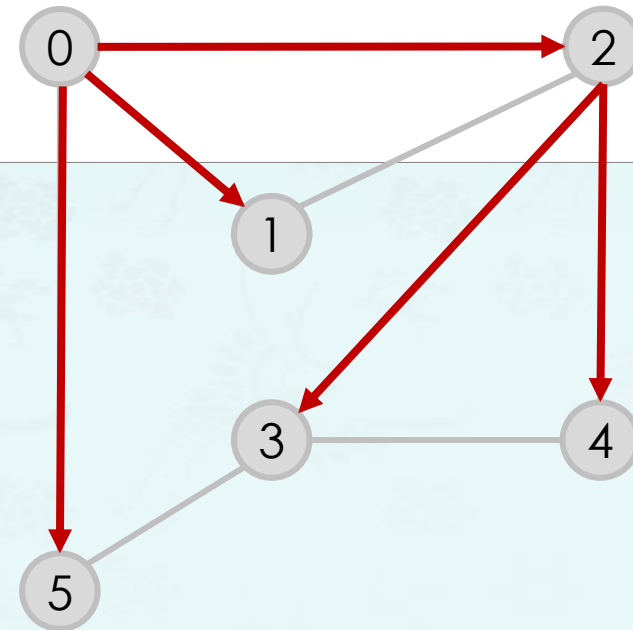
Breadth-first search implementation

```
// returns a path from v to w using the BFS result or parentBFS[].
// It has to use a stack to retrace the path back to the source.
// Once the client(caller) gets a stack returned,
void BFSpath(graph g, int v, int w, stack<int>& path) {
    if (empty(g)) return;

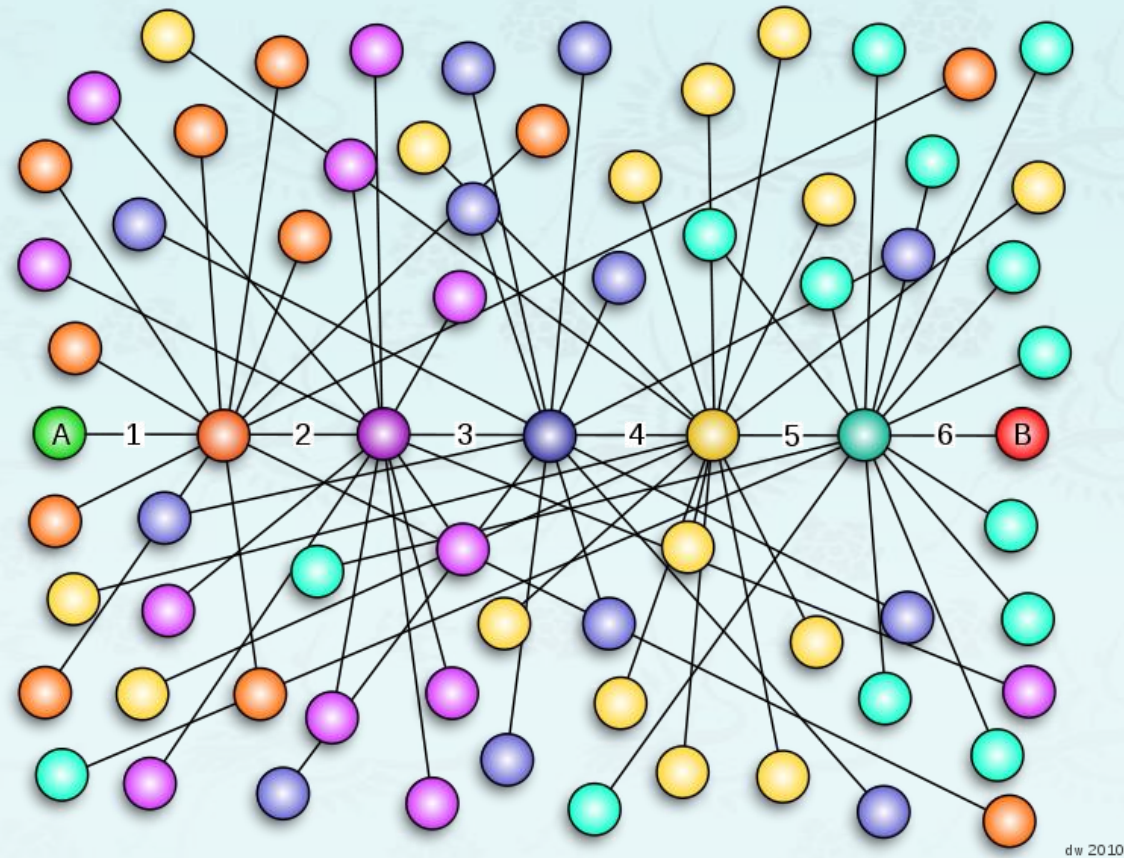
    BFS(g, v);                // g->BFSv updated already.

    path = {};                // clear path

    cout << "your code here\n";
}
```



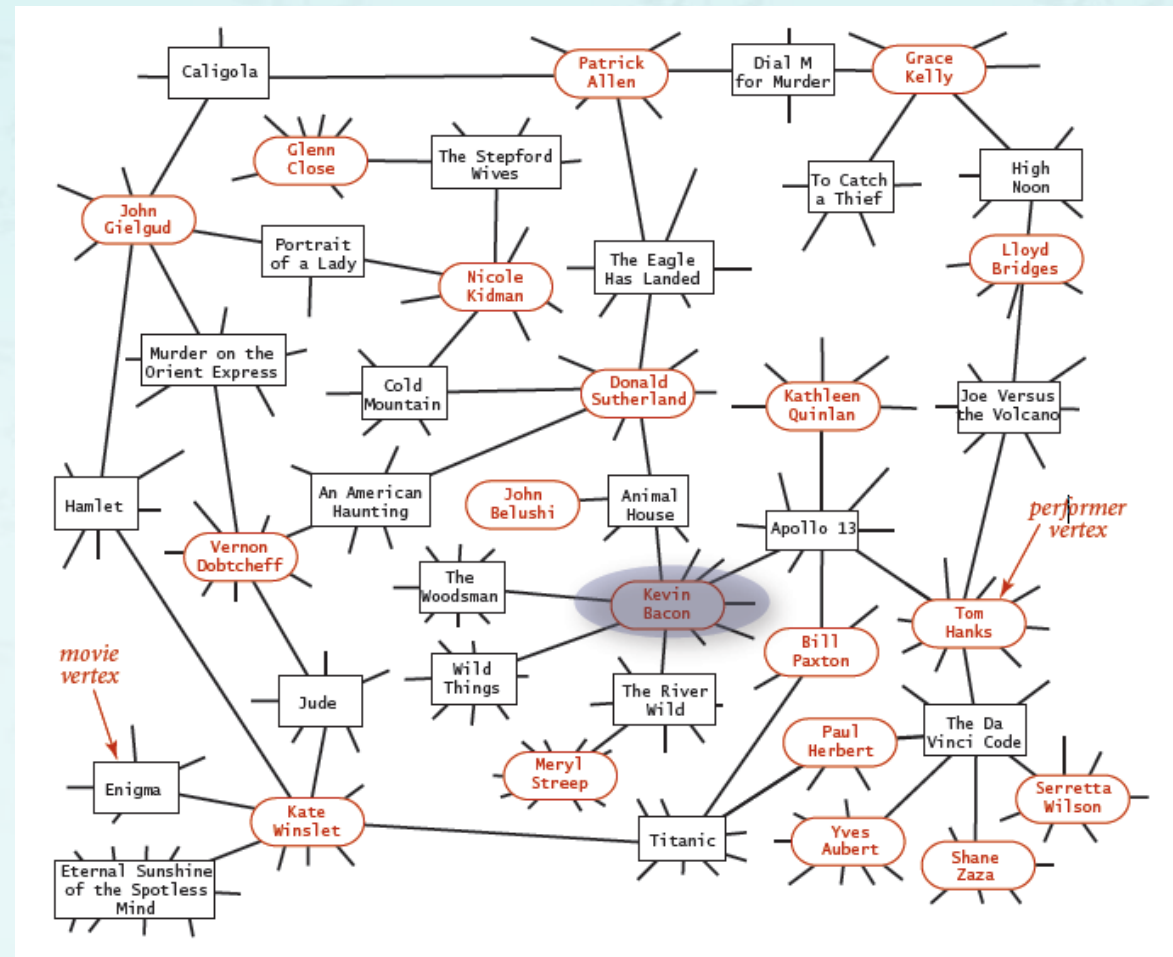
Breadth-first search application: Kevin Bacon numbers



six degrees of separation?

Breadth-first search application: Kevin Bacon numbers

- Include one vertex for each performer **and** one for each movie.
- Connect a movie to all performers that appear in that movie.
- Compute shortest path from s = Kevin Bacon.



Breadth-first search demo

SocialFacebook

4.74 — Facebook Wins By Getting Us Closer Than Six Degrees

Posted Nov 22, 2011 by [Eric Eldon \(@eldon\)](#)

35Like0Tweet327Share0

Next Story



Facebook users are getting more connected to each other as the service grows and matures, according to a [new study](#) by the company's data team and the University of Milan. Instead of the traditional "six degrees of separation" that researchers have [historically observed](#) between all people in the world (and Kevin Bacon), the number of degrees has been dropping since 2008 on the site, from 5.28 then to 4.74 now.

ADVERTISEMENT



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[Claim Basis](#)

Building You A Better Network.SM

2008: 5.28 → 2011: 4.74 → 2016.2 : 

<http://www.bbc.co.uk/newsbeat/article/35500398/how-facebook-updated-six-degrees-of-separation-its-now-357>

Breadth-first search demo

Social

Facebook

4.74 — Facebook Wins By Getting Us Closer Than Six Degrees

Posted Nov 22, 2011 by [Eric Eldon \(@eldon\)](#)

35

Like 0

Tweet 327

Share 0

Next Story



Facebook users are getting more connected to each other as the service grows and matures, according to a [new study](#) by the company's data team and the University of Milan. Instead of the traditional "six degrees of separation" that researchers have [historically observed](#) between all people in the world (and Kevin Bacon), the number of degrees has been dropping since 2008 on the site, from 5.28 then to 4.74 now.

ADVERTISEMENT



Building you a better network.SM

[Claim Basis](#)

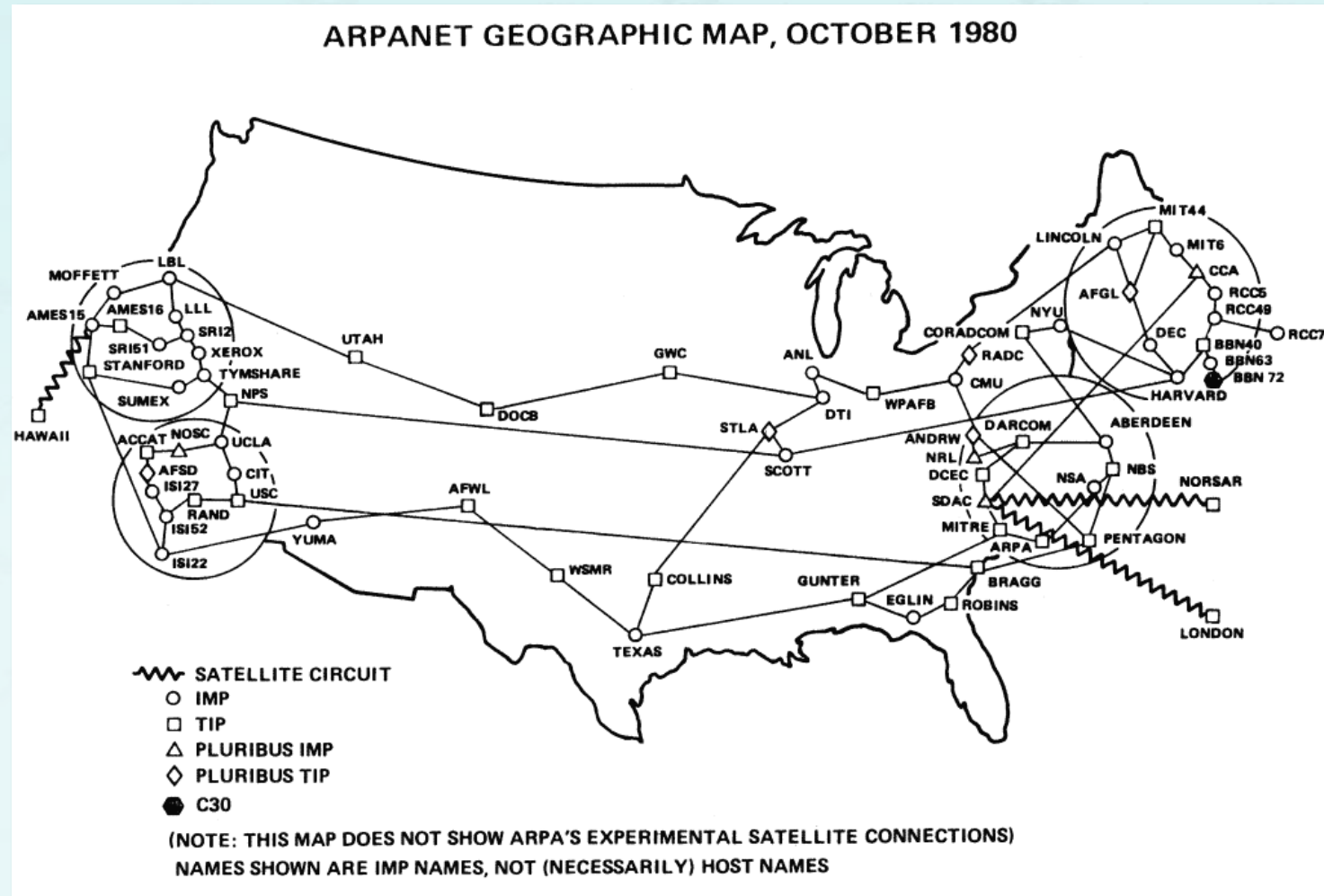
Building You A Better Network.SM

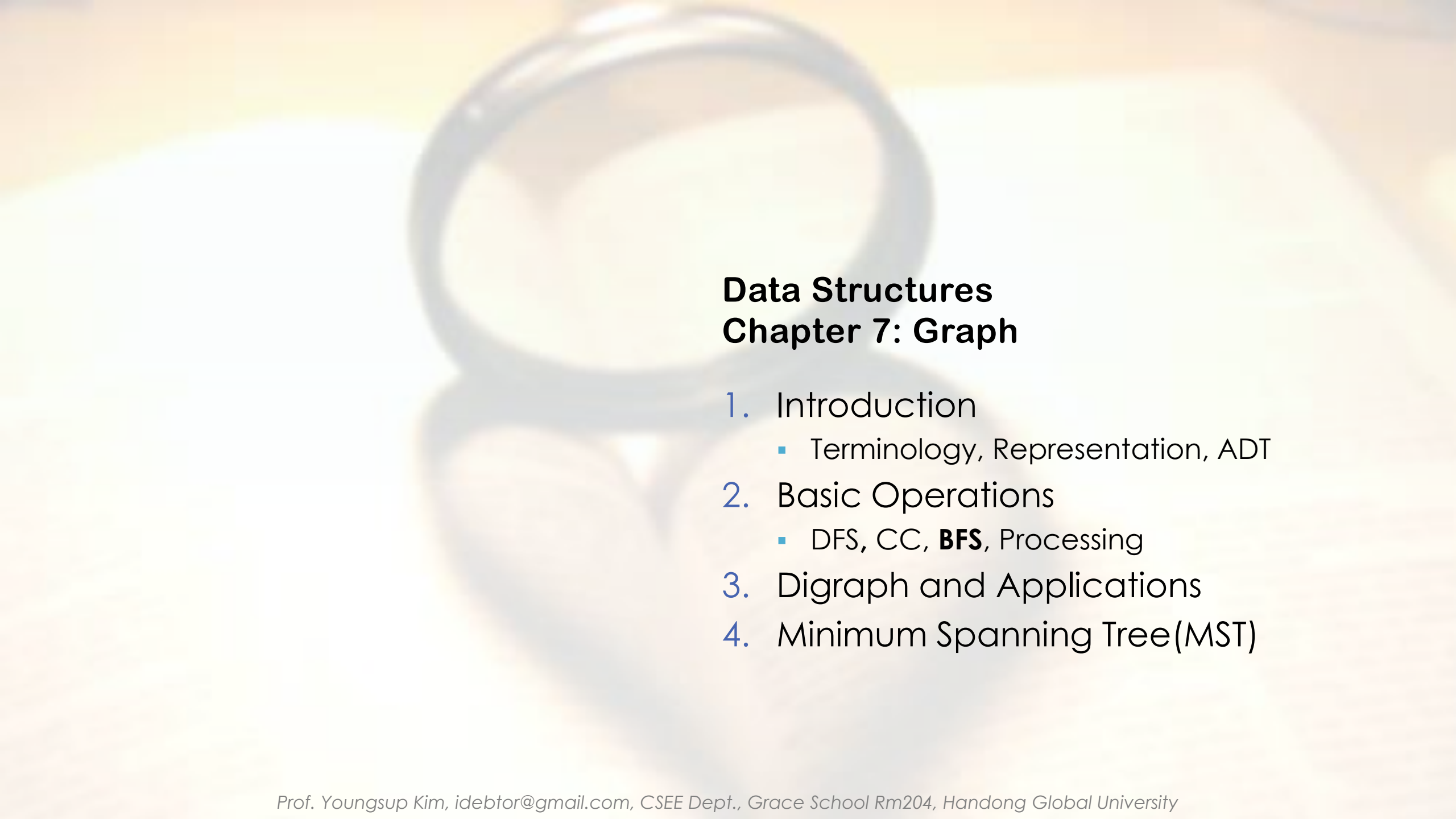
2008: 5.28 → 2011: 4.74 → 2016.2 : 3.57

<http://www.bbc.co.uk/newsbeat/article/35500398/how-facebook-updated-six-degrees-of-separation-its-now-357>

Breadth-first search application: routing

- Fewest number of hops in a communication network.



A pair of glasses with a dark frame and light-colored lenses is resting on a piece of white paper. The background is a soft, out-of-focus yellow and orange gradient.

Data Structures

Chapter 7: Graph

1. Introduction
 - Terminology, Representation, ADT
2. Basic Operations
 - DFS, CC, **BFS**, Processing
3. Digraph and Applications
4. Minimum Spanning Tree(MST)