Data Structures
Chapter 5 Tree

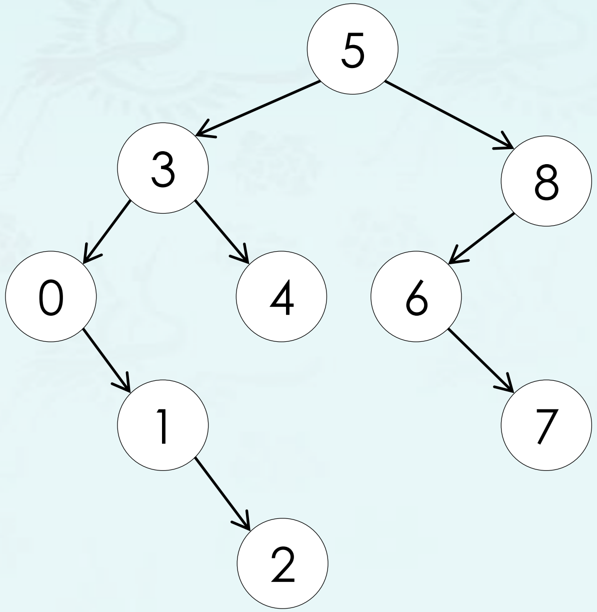
1. Introduction
2. Binary Tree
3. **Binary Search Tree**
   - Introduction
   - Operations
   - **Demo & Coding**
4. Balancing Tree

Minimum, Maximum:

- Minimum() and maximum() returns the node with min or max key.
  - Note that the entire tree does not need to be searched.
  - The minimum key is always located at the left most node, the maximum at the right most node.
  - Complexity of algorithm to find the maximum or minimum will be O(log N) in almost balanced binary tree. If tree is skewed, then we have worst case complexity of O(N).

```
tree minimum(tree node) {  // returns left-most node key


}
```

```
tree maximum(tree node) {  // returns right-most node key


}
```

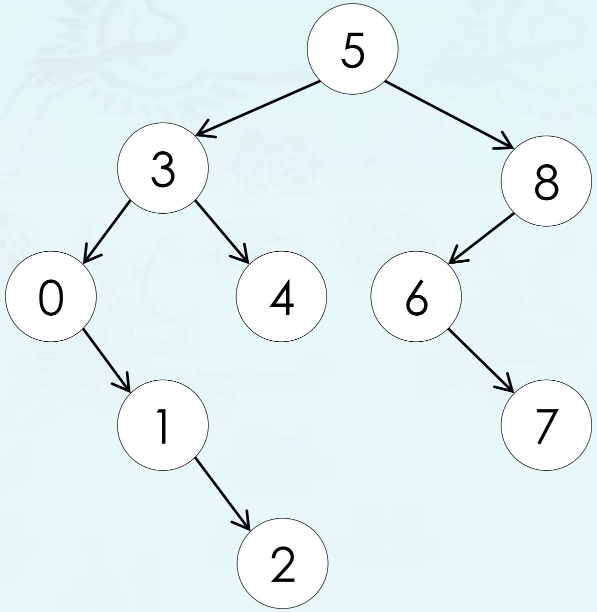


Minimum, Maximum:

- Minimum() and maximum() returns the node with min or max key.
  - Note that the entire tree does not need to be searched.
  - The minimum key is always located at the left most node, the maximum at the right most node.
  - Complexity of algorithm to find the maximum or minimum will be O(log N) in almost balanced binary tree. If tree is skewed, then we have worst case complexity of O(N).

```
tree minimum(tree node) {  // returns left-most node key
  if (node->left == nullptr) return node;
  return minimum(node->left);
}
```

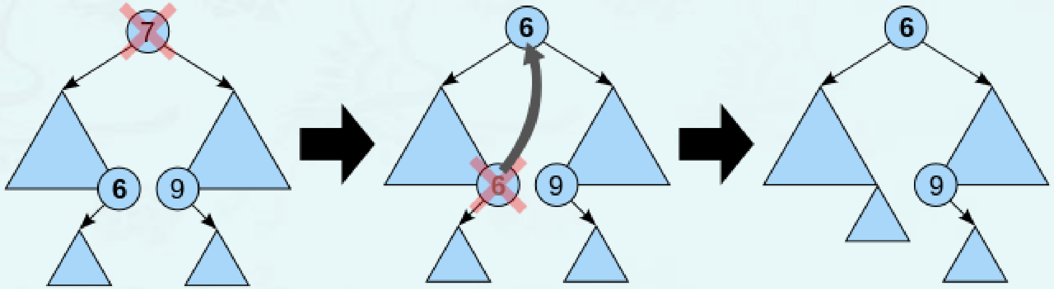
```
tree maximum(tree node) {  // returns right-most node key


}
```



pred(), succ() – predecessor, successor:

- Successor
  - If the given node has a right subtree then by the BST property the next larger key must be in the right subtree. Since all keys in a right subtree are larger than the key of the given node, the successor must be the smallest of all those keys in the right subtree.
- Predecessor
  - If the given node has a left subtree then by the BST property the next smaller key must be in the left subtree. Since all keys in a left subtree are smaller than the key of the given node, the successor must be the largest of all those keys in the left subtree.
- Complexity of algorithm
  - O(log N) in almost balanced binary tree. If tree is skewed, then we have worst case complexity of O(N).

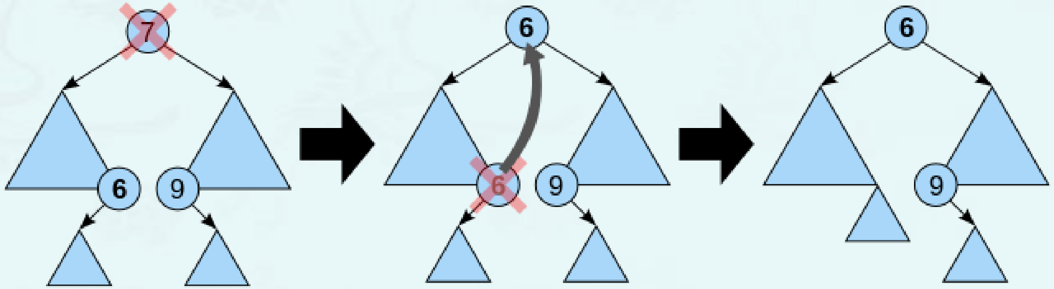
```
tree successor(tree node) {
  if (node != nullptr && node->right != nullptr)

  return nullptr;
}
```



pred(), succ() – predecessor, successor:

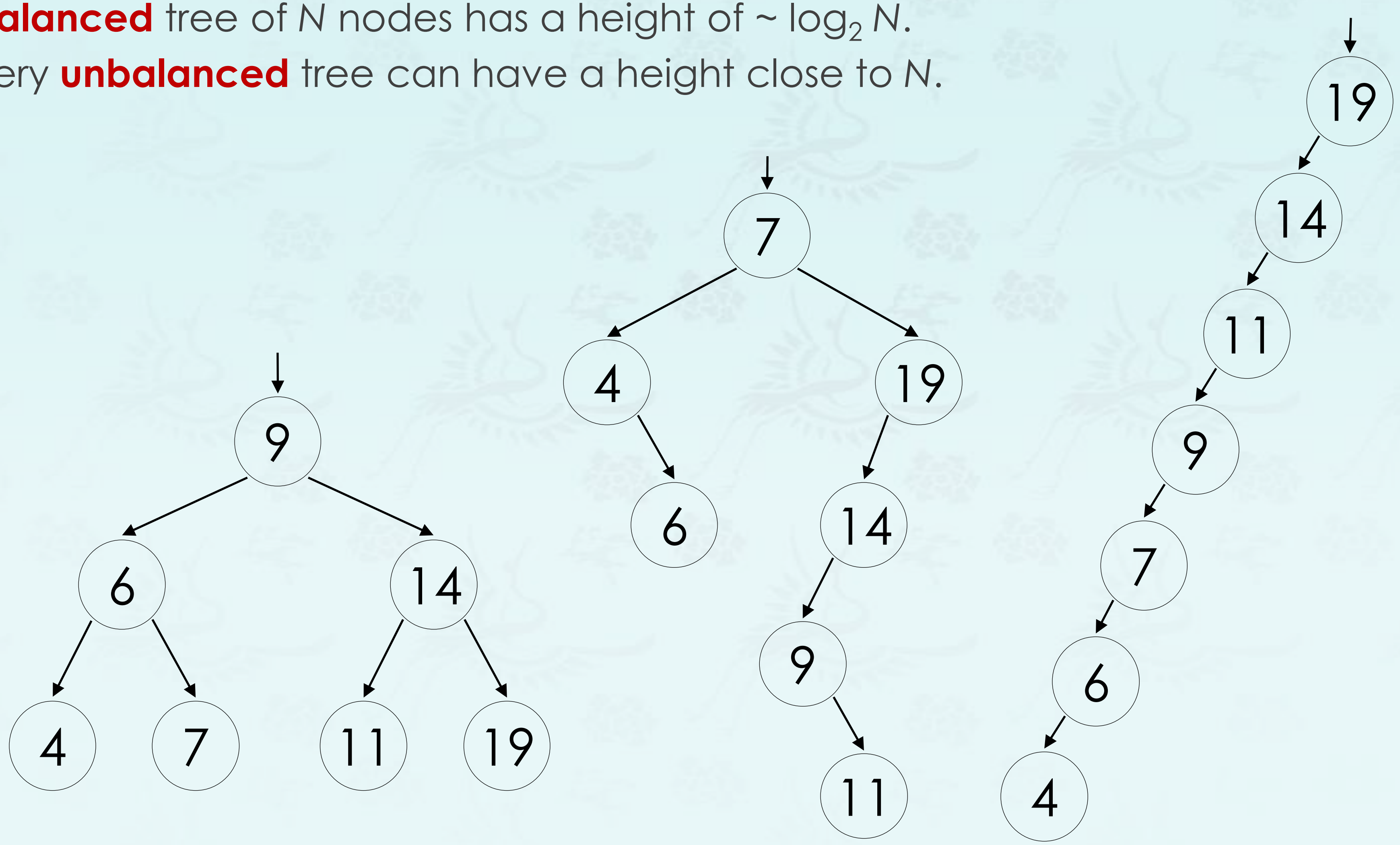
- Successor
  - If the given node has a right subtree then by the BST property the next larger key must be in the right subtree. Since all keys in a right subtree are larger than the key of the given node, the successor must be the smallest of all those keys in the right subtree.
- Predecessor
  - If the given node has a left subtree then by the BST property the next smaller key must be in the left subtree. Since all keys in a left subtree are smaller than the key of the given node, the successor must be the largest of all those keys in the left subtree.
- Complexity of algorithm
  - O(log N) in almost balanced binary tree. If tree is skewed, then we have worst case complexity of O(N).

```
tree successor(tree node) {
  if (node != nullptr && node->right != nullptr)
    return minimum(node->right);
  return nullptr;
}
```



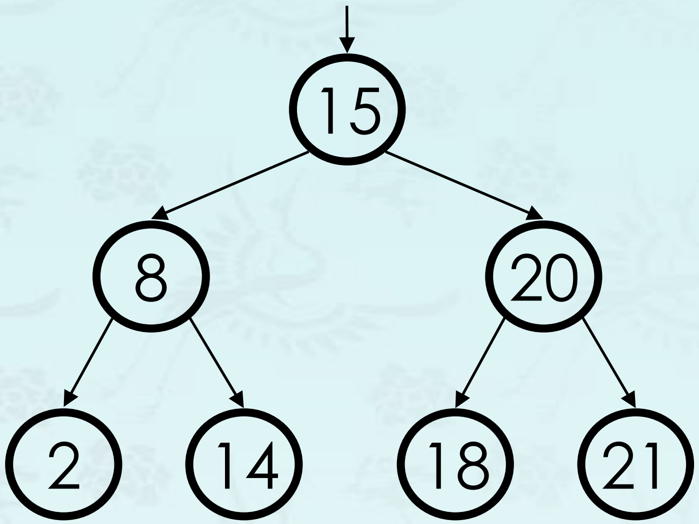
Binary Search Trees: Observations

- What do you see in the following BSTs?
  - A **balanced** tree of N nodes has a height of ~ $\log_2 N$.
  - A very **unbalanced** tree can have a height close to N.



Binary Search Trees: Observations

- For binary tree of height h:
  - max # of leaves: 2^h
  - max # of nodes: $2^{h+1} - 1$
  - min # of leaves: 1
  - min # of nodes: $h + 1$
- The shallower the BST the better.
  - Average case height is O(log *N*)
  - Worst case height is O(*N*)
  - Simple cases such as adding (1, 2, 3, ..., *N*), or the opposite order, lead to the worst case scenario: height O(*N*).



Binary Search Trees: Observations

- Q: If you have a sorted sequence, and we want to design a data structure for it. Which one are you going to use an array or BST? and why?

Time Complexity	
BST	$O(h)$
Array	$O(\log n)$

- Q: When searching, we're traversing a path (since we're always moving to one of the children); since the length of the longest path is the height h of the binary search tree, then finding an element takes $O(h)$.
  - Since $h = \log n$ (where n is the number of elements), then it's good! – right?
  - No, of course, it is wrong! Why?

  A: The nodes could be arranged in linear sequence in BST, so the $height\ h$ could be n. In worst case, it is $O(n)$ instead of $O(h)$.

Operations: growN() & trimN() for testing

- It performs a user specified number of insertion(or grow) or deletion(or trim) of nodes in the tree.
- The function **`growN()`** inserts a user specified number N of nodes in the tree.
  - If it is an empty tree, the value of keys to add ranges from 0 to N-1.
  - If there are some existing nodes in the tree, the value of keys to add ranges from max + 1 to max + 1 + N, where max is the maximum value of keys in the tree.
- This function growN() is provided for your reference^^.

Operations: growN() & trimN() for testing

- It performs a user specified number of insertion(or grow) or deletion(or trim) of nodes in the tree.
- The function **`growN()`** inserts a user specified number N of nodes in the tree.
  - If it is an empty tree, the value of keys to add ranges from 0 to N-1.
  - If there are some existing nodes in the tree, the value of keys to add ranges from max + 1 to max + 1 + N, where max is the maximum value of keys in the tree.
- This function growN() is provided for your reference^^.

- If the function is called with AVLtree = true, nodes are added using BST grow() function first. Then reconstruct the BST tree into an AVL tree using reconstruct() function which is much faster.

Operations: growN() & trimN() for testing

- The function **`trimN()`** deletes N number of nodes in the tree.
  - The nodes to trim are **randomly** selected from the tree.
  - If N is less than the tree size (which is not N), you just trim N nodes.
  - If the N is larger than the tree size, set it to the tree size.
  - At any case, you should trim all nodes one by one, but randomly.
  - With an AVL tree, reconstruct it **after** trimming N nodes from BST.

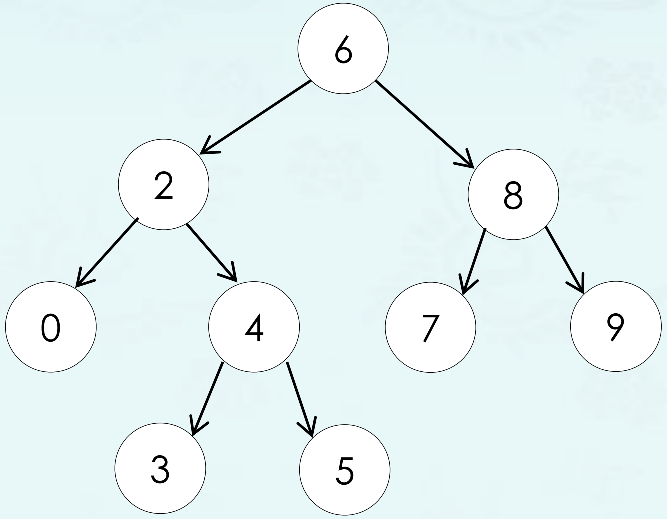
Operations: growN() & trimN() for testing

- The function **`trimN()`** deletes N number of nodes in the tree.
  - The nodes to trim are **randomly** selected from the tree.
  - If N is less than the tree size (which is not N), you just trim N nodes.
  - If the N is larger than the tree size, set it to the tree size.
  - At any case, you should trim all nodes one by one, but randomly.
  - With an AVL tree, reconstruct it **after** trimming N nodes from BST.

Step 1: Get a list (vector) of all keys from the tree first.
Get the size of the tree using the size().
Use assert to check two sizes;
Step 2: Shuffle the vector with keys. – shuffle()
Step 3: Invoke trim() N times with a key from the vector in sequence.
Inside a for loop, trim() may return a new root of the tree.
Step 4: The function is called with AVLtree = true, then reconstruct the tree.

Operations: LCA in BST

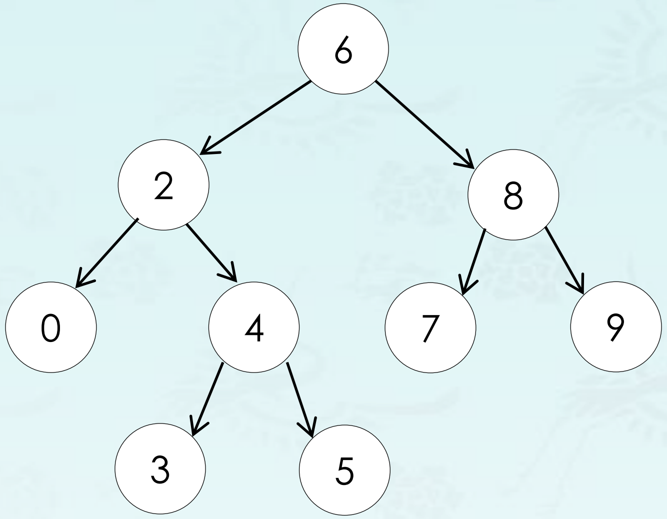
- Find the lowest common ancestor(LCA) of two given nodes, given in BST.
  - The LCA is defined between two nodes p and q as the lowest node in T that has both p and q as descendants (where we allow a node to be a descendant of itself)."
  - In BST, all of the nodes' values will be unique.
    Two nodes given, p and q, are different and both values will exist in the BST.




```
For example:
2, 8 -> 6
2, 5 -> 2
9, 5 -> 6
8, 7 -> 8
0, 5 -> 2
```

Operations: LCA(**iteration**) in BST

- **Intuition (Iteration)**: Traverse down the tree iteratively to **find the split point**. The point from where p and q won't be part of the same subtree or when one is the parent of the other.

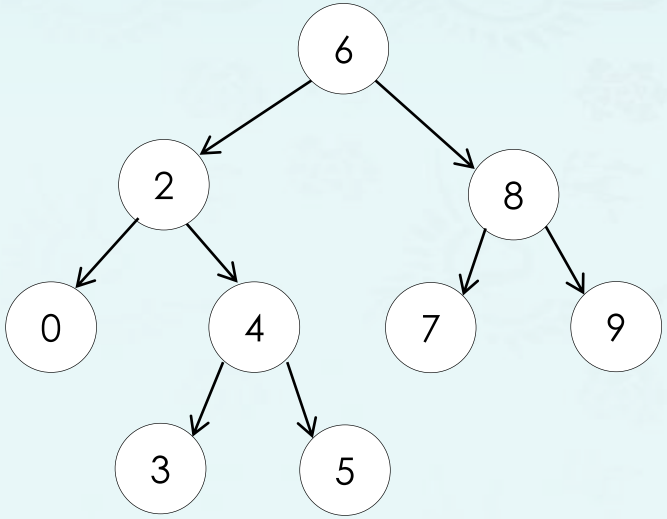



```
For example:
2, 5 -> 2
9, 7 -> 8
0, 4 -> 2
0, 5 -> 2
2, 7 -> 6
```

```
int LCAiteration(tree node, tree p, tree q) {
  while (node != nullptr) {
     if (both p & q > node)
        node move to right to search
     else if (both p & q < node)
        node moves to left to search
     else
        return node->key    // found
  }
  return 0;   // not found
} // iteration solution
```

Operations: LCA(**recursion**) in BST

- **Algorithm: (Recursion)**
  1. Start traversing the tree from the root node.
  2. If both the nodes p and q are in the right subtree, then continue the search with right subtree starting step 1.
  3. If both the nodes p and q are in the left subtree, then continue the search with left subtree starting step 1.
  4. If both step 2 and step 3 are **not true**, this means we have **found** the node which is common to node p's and q's subtrees. Hence we return this common node as the LCA.




```
tree LCA(tree root, tree p, tree q) {

  // your code here

} // recursive solution
```

Operations: LCA in BST

- Recursion Algorithm
  - Time Complexity: $O(N)$, where N is the number of nodes in the BST. In the worst case we might be visiting all the nodes of the BST.
  - Space Complexity: $O(N)$. This is because the maximum amount of space utilized by the recursion stack would be N since the height of a skewed BST could be N.
- Iteration Algorithm
  - Time Complexity : O(N), where N is the number of nodes in the BST. In the worst case we might be visiting all the nodes of the BST.
  - Space Complexity : O(1).

Data Structures
Chapter 5 Tree

1. Introduction
2. Binary Tree
3. **Binary Search Tree**
   - Introduction
   - Operations
   - Demo & Coding
4. Balancing Tree

Prof. Youngsup Kim, idebtor@gmail.com, CSEE Dept., Grace School Rm204, Handong Global University