Graphic Era Hill University, Dehradun
(Answer Sheet for Online Examination Aug. 2021)

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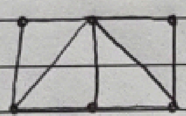
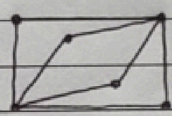
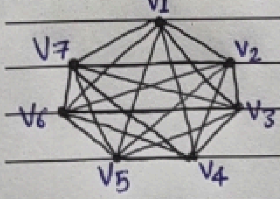
Name: Deepankar Sharma. Univ.Roll No. 2092014......... Student ID..20041299..........
Date: 12-Aug-21. Course: ..BCA.......... Branch: .... Sem.: II.... Section: ....
Subject Name: Discrete Maths & Graph Theory Subject Code: TBC 204 Page No. .01.....

Ques③ - Graph - A graph consists of set of vertices and set of edges.
G (V,E), where V → set of vertices
E → set of edges.
And each edge is associated with an unordered pair of vertices.

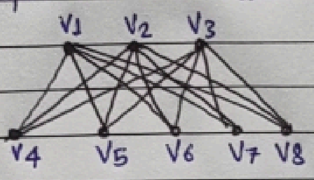
If a graph contains a Euler circuit, or a Euler Line is called an Euler graph whereas the graph which consists of a Hamiltonian circuit is called an Hamiltonian graph.
The difference between the Euler graph and the Hamiltonian graph is an Eulerian circicut traverses every edge in the graph exactly once but may repeat vertices, while a Hamiltonian circuit visits each vertex in a graph (except ending vertices) only once but may repeat the edges.


Hamiltonian graph

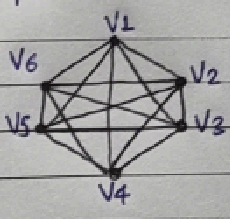

Euler graph



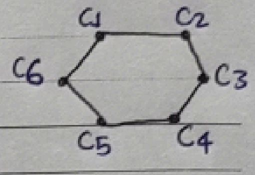
K-7 graph



K3,5 graph



5-Regular graph on 6 vertices

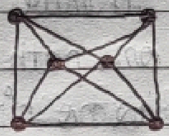


C-6 graph

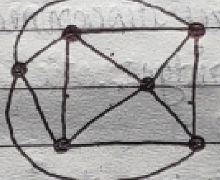
Deepankar Sharma

Name - Deepankar Sharma   Student ID - 20041299.

Ques - ④ Planar graph A graph G is said to be a planar graph if there exist some geometric representation of G which can be drawn on plane such that no two of its edges intersect.



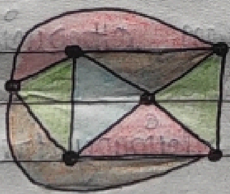
Non planar representation



planar representation

Using Euler's formula number of regions $= e - n + 2$
$= 12 - 6 + 2$
$= 8$

properly coloring the regions.



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Ques ⑤ Tree - Tree is a connected graph without any circuit. In simpler words a tree is a connected acyclic undirected graph.

Properties of tree

- In a tree of n vertices, it have n-1 edges.
- There is one and only one path between each pair of vertices
- Tree is connected and has no circuits
- Tree has no circuit but insertion of one edge can form circuit
- Removal of any edge makes tree disconnected.
- Every edge of a tree is a bridge.

Rooted Tree - A tree with one vertex designated as root and it is distinguishable from other vertices.
Any vertex can be choosen as root.
Root don't have any parent and leaf don't have child.

Binary Tree - Binary Tree is a rooted tree which can have at most two children, otherwise zero or one child.

yes the given graph is a binary tree.

F is the root, B and G are children of root, therefore siblings. I is children of B, therefore B is uncle/aunt of I.

Ques ② - the given boolean function is

$$x'y'z + xy'z' + xy'z + xyz'$$

$x \backslash yz$	$y'z'$	$y'z$	yz	yz'
$\bar{x}$		1		
x	1	1		1

from the above karnaugh map :

$$\text{pair}\,1 = xy'z' + xyz'$$
$$\text{pair}\,2 = x'y'z + xy'z$$

on solving we get $= xz' + y'z$

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Ques-① the given matrix is, $A = \begin{bmatrix} 5 & 2 \\ 1 & 4 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 5-\lambda & 2 \\ 1 & 4-\lambda \end{bmatrix}$$

characteristic polynomial, $|A - \lambda I| = 0$

putting values

$$\Rightarrow (5-\lambda)(4-\lambda) - 2 = 0$$
$$\Rightarrow \lambda^2 - 9\lambda + 18 = 0$$
$$\Rightarrow \lambda^2 - (6+3)\lambda + 18 = 0$$
$$\Rightarrow \lambda^2 - 6\lambda - 3\lambda + 18 = 0$$
$$\Rightarrow \lambda(\lambda-6) - 3(\lambda-6) = 0$$
$$\Rightarrow (\lambda-6)(\lambda-3) = 0$$
$$\lambda = 3, 6$$

for $\lambda = 3$, $(A-3I)X = 0$

$$A - 3I = \begin{bmatrix} 5-3 & 2 \\ 1 & 4-3 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$$

applying $R_2 \rightarrow R_2 - 1/2 R_1$

$$\begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix}$$

Now rank $(A-3I) = 1$ < Number of unknown variables

Equivalent system of equation

$$2x + 2y = 0$$

assuming $y = k$ as a free variable, $x = -k$

corresponding eigen vector, $X = \begin{bmatrix} -k \\ k \end{bmatrix}$, assuming $k=1$, $X = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

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for $\lambda = 6$, $(A-6I)X = 0$

$$A-6I = \begin{bmatrix} 5-6 & 2 \\ 1 & 4-6 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 1 & -2 \end{bmatrix}$$

applying $R_2 \rightarrow R_2 + R_1$

$$\begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix}$$

Rank $(A-6I) = 1 <$ Number of unknown variables

$$(A-6I)X = 0, \quad \begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

Equivalent system of equation

$$-x + 2y = 0$$

assuming $y = k$, as free variable, $x = 2k$

corresponding eigen vector $\Rightarrow X = \begin{bmatrix} 2k \\ k \end{bmatrix}$

assuming $k = 1$, $X = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$