UNIT-4: Date

2D Transformation

2D transformation refer to the manipulation of 2D geometric shapes using mathematical operations. These transformations are used to move, scale, rotate, reflect and shear 2D objects in a 2D plane.

The most common 2D transformations are

Translation: This transformation moves an object along a straight line by adding a vector to its coordinates.

Scaling: This transformation changes the size of an object by multiplying its coordinates by a scaling factor.

Rotation: This transformation rotates an object by a specified angle around a given point.

Reflection: This transformation flips an object across a line or plane, creating a mirror image.

Shearing: This transformation skews an object by moving one set of points along a fixed axis.

Each of these transformations is represented by a matrix that describes the operation to be performed on the object's coordinates. By multiplying the object's coordinate matrix by the transformation matrix, the object's new coordinates can be computed.

Translation

Translation moves an object from one position to another along a straight line in a 2D plane.

Translation is applied by adding a vector to the

Spiral

Date

object's coordinates which moves the object a specified distance in a specified direction.

Translation can be performed in any direction, including horizontal, vertical or diagonal.

To translate a point from coordinate position (x, y) to another (x_1, y_1) add translation distances T_x and T_y

$$x_1 = x + T_x$$
$$y_1 = y + T_y$$

translation pair (T_x, T_y) is called as shift vector.

Equation for performing translation operation

$$P' = T * P$$

$P \rightarrow$ matrix of original object's coordinates

$T \rightarrow$ translation matrix

$P' \rightarrow$ matrix of translated object's coordinates

translation matrix is defined as

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \qquad P = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

note $P' = P * T$

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} x & y & 1 \end{bmatrix}$$

To apply a translation to an object you would first represent its coordinates as a matrix of column vectors.

eg triangle with vertices at (0,0) (2,0) and (1,1)

translate the triangle 3 units to the right and 2 units up

$$P = \begin{bmatrix} 0 & 2 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad T = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Spiral

Date

$P' = T * P$

$$= \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 5 & 4 \\ 2 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

resulting matrix represent the translated triangle with vertices at (3,2), (5,2) and (4,3)

Scaling

Scaling changes the size of an object by multiplying its coordinates by a scaling factor.

There are two scaling factors i.e S_x in x direction
S_y in y direction

To scale a point from coordinate position (x, y) by scaling factor S_x and S_y to produced transformed coordinates (x', y')

$x' = x \cdot S_x$
$y' = y \cdot S_y$

If S_x and S_y are equal it is also called as uniform scaling if not then called as differential scaling.

If scaling factor < 1 object move closer to coordinate origin if scaling factor > 1 object move father from coordinate origin.

Date

The equation for performing a scaling operation
$P' = S * P$

$$S = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

To apply a scaling operation to an object you would first represent its coordinates as a matrix of column vectors.

eg a rectangle with vertices at (0,0), (2,0), (2,1) and (0,1)
scale the rectangle by a factor of 2 in x direction and 3 in y direction.

$$P = \begin{bmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \qquad S = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$P' = S * P$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

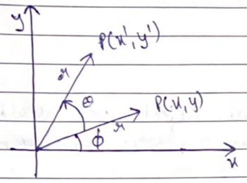
$$= \begin{bmatrix} 0 & 4 & 4 & 0 \\ 0 & 0 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

The resulting matrix represents the scaled rectangle with ~~vectors~~ vertices at (0,0), (4,0), (4,3) and (0,3)

Date

Rotation

Rotation changes the orientation of an object by rotating its coordinates around a specified point.



$x' = r\cos(\phi + \theta)$
$= r\cos\phi\cos\theta - r\sin\phi\sin\theta$ — ①

$y' = r\sin(\phi + \theta)$
$= r\cos\phi\sin\theta + r\sin\phi\cos\theta$ — ②

$x = r\cos\phi$ — ③
$y = r\sin\phi$ — ④

Substituting 3 & 4 in 1 & 2

$x' = x\cos\theta - y\sin\theta$
$y' = x\sin\theta + y\cos\theta$

The equation for performing a rotation operation on a 2D object

$P' = R(\text{theta}) * P$

P = Matrix of the original object's coordinates
R(theta) = rotation Matrix
P' = Matrix of rotated object's coordinates

$$R(\text{theta}) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where θ is the angle of rotation in radians, and cos θ and sin θ are cosine and sine of the angle

eg a rectangle with vertices at (0,0) (2,0) (2,1) and (0,1)
rotate the rectangle to 45° counterclockwise around the origin

Date

$$\begin{bmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos 45 & -\sin 45 & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1.414 & 0.707 & 2.121 & -0.707 \\ -1.414 & -0.707 & 1.414 & 0.707 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

The resulting matrix represents the rotated rectangle with vertices at approximately (-1.414, -1.414) (0.707, -0.707) (2.121, 1.414) and (-0.707, 0.707)

Composite Transformation

It involves applying multiple transformations to an object in specific order. These transformations include translation, scaling, rotation and other operations. The resulting transformation matrix is computed by multiplying the matrices of individual transformations together.

scaling → $P_s = S * P$
rotation → $P_r = R * P_s$
translation → $P_t = T * P_r$

$P_t \rightarrow T * R * S * P$

Date

eg A square with vertices at (-1,-1) (1,-1) (1,1) and (-1,1) perform following transformations
i) Scale with scaling factor of 2 in both x & y directions
ii) Rotate with 45° counterclockwise around the origin
iii) Translate (3,2) units in the x & y directions

i) $S = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$P_S = S * P$

$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 1 & 1 & -1 \\ -1 & -1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} -2 & 2 & 2 & -2 \\ -2 & -2 & 2 & 2 \end{bmatrix}$

ii) $R = \begin{bmatrix} \cos 45 & -\sin 45 \\ \sin 45 & \cos 45 \end{bmatrix}$

$P_R = R * P_S$

$= \begin{bmatrix} 0.707 & -0.707 \\ 0.707 & 0.707 \end{bmatrix} \begin{bmatrix} -2 & 2 & 2 & -2 \\ -2 & -2 & 2 & 2 \end{bmatrix}$

$= \begin{bmatrix} 0 & -2.828 & 2.828 & 0 \\ 0 & -2.828 & -2.828 & 0 \end{bmatrix}$

iii) $T = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$P_T = T * P_R$

$= \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -2.828 & 2.828 & 0 \\ 0 & -2.828 & -2.828 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3 & 0.172 & 5.828 & 3 \\ 2 & -0.828 & 0.172 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

The vertices of the transformed square are approximately
(3,2) (0.172, -0.828) (5.828, 0.172) (3,2)

Spiral

Date

Reflection

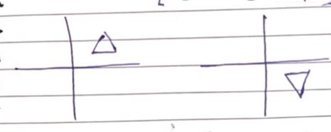
Reflection flips an object across a line or plane, creating a mirror image of the original object. It can be performed using a reflection matrix, which is defined based on the axis or line of reflection.

types of reflection

Reflection about the x-axis

The object can be reflected about x-axis with the help of the following matrix

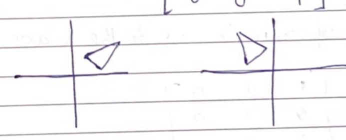
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$



Reflection about the y-axis

The object can be reflected about y-axis with the help of the following matrix

$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

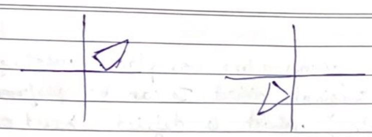


Reflection about origin

$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Spiral

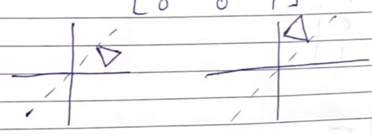
Date



Reflection about line $y = x$

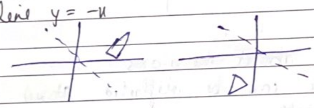
The object may be reflected about line $y = x$ with the help of following transformation matrix

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Reflection about line $y = -x$

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



eg A triangle ABC is given The coordinates of A, B, C are
A(3,4) B(6,4) C(4,8)
Find reflected position of triangle i.e to the x-axis

reflection about x axis $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$(3,4) \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= [3, -4]$ $[6, -4]$ $[4, -8]$

a(3,4) becomes a' (3,-4)
b (6,4) becomes b' (6,-4)
c (4,8) becomes c' (4,-8)

Shearing

Shearing skews the shape of an object along one or both axes. It is accomplished by applying a shearing matrix that modifies the object's coordinates. Shearing can be performed along the x-axis and y-axis or a combination of both.

In the X direction

In this horizontal shearing sliding of layers occur.

$$\begin{bmatrix} 1 & 0 & 0 \\ Sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In the Y direction

In this vertical shearing sliding of layers occur.

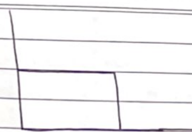
$$\begin{bmatrix} 1 & Sh_y & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In the X-Y direction

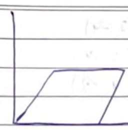
Layers will be slided in both x as well as y direction. The sliding will be in horizontal as well as vertical direction. The shape of the object will be distorted.

$$\begin{bmatrix} 1 & Sh_y & 0 \\ Sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Date



original object



Shear in x direction

Shear in Y direction

Shear in both directions

Date

3D Transformation

3D transformations are used to manipulate and position objects in 3d space. They allow for transformation such as translation, scaling, rotation, shearing, and perspective projection.

Translation

It is the movement the object from one position to another position. There are three vectors in 3D instead of two. These vectors are in x, y, and z directions. T_x, T_y, and T_y

To translate x, y, and z by $t_x, t_y,$ and t_z the new coordinates becomes $(x+t_x,\ y+t_y$ and $z+t_z)$

Matrix for translation

$$\begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} x^1 \\ y^1 \\ z^1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

eg A point coordinates in the x, y, z direction ie (5, 6, 7). The translation is done in the x-direction by 3 coordinates and y direction. Three coordinates and in the z direction by two coordinates. Shift the object. Find the coordinates of the new position.

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 7 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 9 \\ 9 \\ 1 \end{bmatrix}$$

x becomes $x^1 = 8$

y becomes $y^1 = 9$

z becomes $z^1 = 9$

Date

Scaling

Scaling is used to change the size of an object. The size can be increased or decreased. The scaling factors are S_x, S_y and S_z

S_x = Scaling factor in x-direction
S_y = Scaling factor in y-direction
S_z = Scaling factor in z-direction

Matrix for scaling

$$\begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Rotation

It is moving of an object about an angle. Movement can be anticlockwise or clockwise. 3D rotation is complex as compared to the 2D rotation. For 3D angle of rotation and axis of rotation are required. The axis can be either x, y or z.

The rotation matrix for rotating around y-axis

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The rotation matrix for rotating around x-axis

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Spiral

Date

The rotation matrix for rotating around z-axis

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Reflection

It is also called as a mirror image of an object. For this reflection of plane and reflection axis is selected.

reflection relative to XY plane

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

reflection relative YZ plane

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

reflection relative ZX plane

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

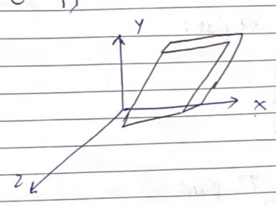
Spiral

Date

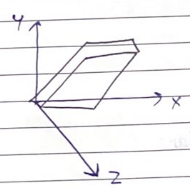
Shearing

It is change in the shape of the object. It is also called as deformation. Change can be in the x-direction or y-direction or both directions in case of 2D. If shear occurs in both directions, the object will be distorted. In 3D shear can occur in three directions.

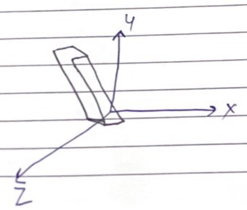
$$\begin{bmatrix} 1 & 0 & a & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Shear in X direction



Shear in X, Y direction

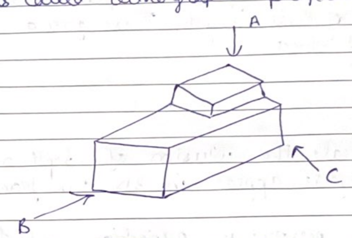


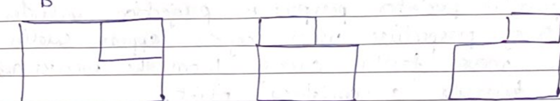
Shear in Y direction

Date

Parallel Projection

Parallel projection represent 3-d objects on a 2-d surface such as a computer screen.
It is used to display picture in its true shape and size when projectors are perpendicular to view plane then is called orthographic projection.





parallel projection from top i.e A direction	Parallel projection from top ie B direction	Parallel projection from top i.e C direction

1) Isometric Projection
All projectors make equal angles generally angle is of 30°

2) Dimetric
In these two projectors have equal angles. With respect to two principle axis.

3) Trimetric
The direction of projection makes unequal angle with their principle axis.

Date

Cavalier
All lines perpendicular to the projection plane are projected with no change in length.

Cabinet
All lines perpendicular to the projection plane are projected to one half of their length. These gives a realistic appearance of object.

Perspective Projection
Perspective projection create the illusion of depth and simulate the way objects appear in 3D scene from a specific viewpoint.
Unlike parallel projection, perspective projection includes the effects of perspective where objects appear smaller as they move farther away from the viewer and converge towards a vanishing point.

View plane
It is an area of world coordinate system which is projected into viewing plane.

Centre of Projection
It is the location of the eye on which projected light rays converge

Projectors
It is called a projection vector. These are rays start from the object scene and are used to create an image of the object on viewing or view plane

Spiral

Date

Perspective foreshortening
The size of the object will be small of its distance from the center of projection increases.

Vanishing Point
All lines appear to meet at some point in the view plane

Distortion of Lines
A range lies in front of the viewer to back of viewer is appearing to six dollars.

Spiral