Chapter 2

Principles of Public-Key Cryptosystems

The concept of public-key cryptography evolved from an attempt to attack two of the most difficult problems associated with symmetric encryption. key distribution under symmetric encryption requires either (1) that two communicants already share a key, which somehow has been distributed to them; or (2) the use of a key distribution center. Whitfield Diffie, one of the discoverers of public-key encryption (along with Martin Hellman, both at Stanford University at the time), reasoned that this second requirement negated the very essence of cryptography: the ability to maintain total secrecy over your own communication.

The second problem that Diffie pondered, and one that was apparently unrelated to the first was that of "digital signatures." If the use of cryptography was to become widespread, not just in military situations but for commercial and private purposes, then electronic messages and documents would need the equivalent of signatures used in paper documents.

Public-Key Cryptosystems

Asymmetric algorithms rely on one key for encryption and a different but related key for decryption. These algorithms have the following important characteristic:

It is computationally infeasible to determine the decryption key given only knowledge of the cryptographic algorithm and the encryption key.

In addition, some algorithms, such as RSA, also exhibit the following characteristic:

Either of the two related keys can be used for encryption, with the other used for decryption.

A public-key encryption scheme has six ingredients

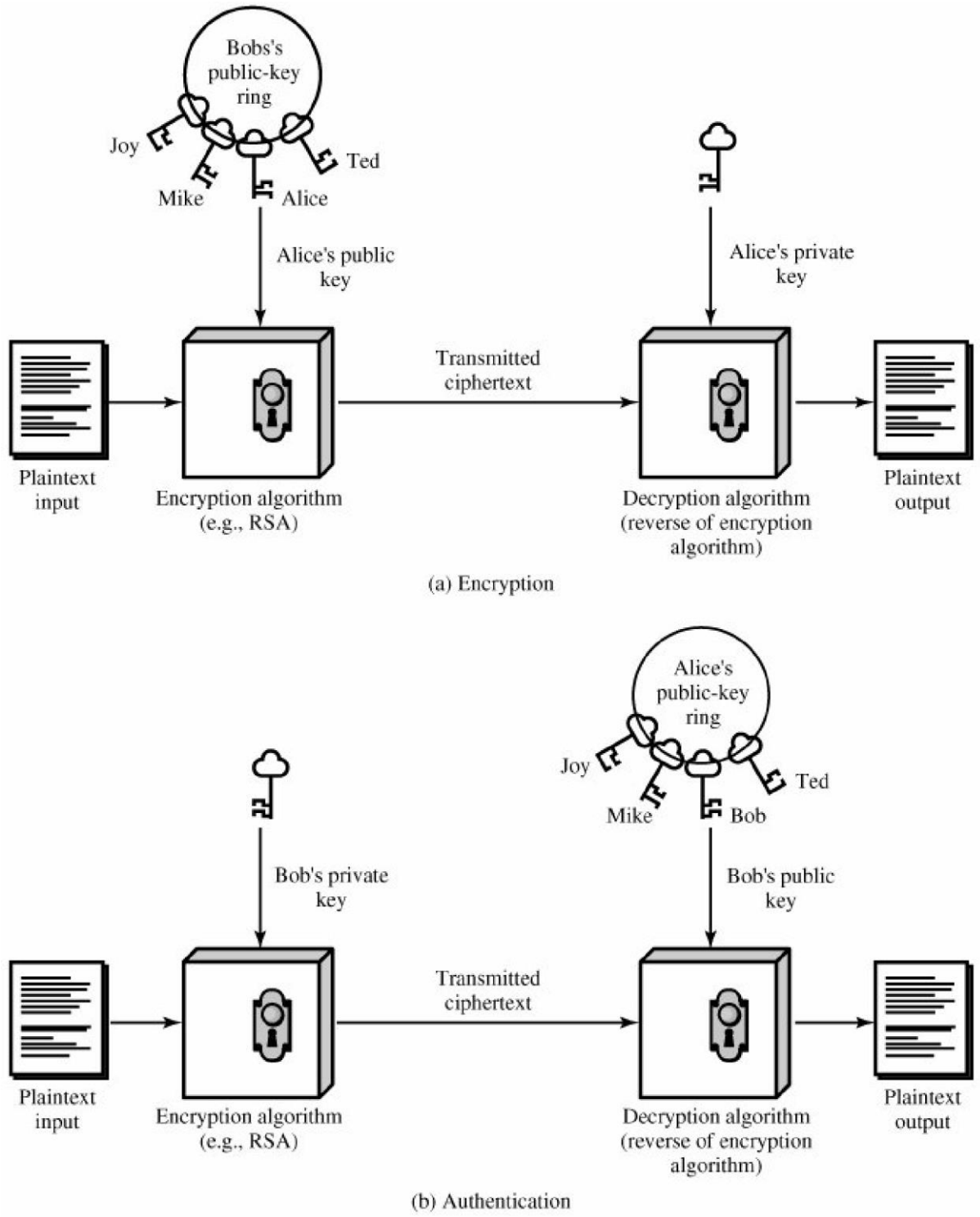
Plaintext: This is the readable message or data that is fed into the algorithm as input.

Encryption algorithm: The encryption algorithm performs various transformations on the plaintext.

Public and private keys: This is a pair of keys that have been selected so that if one is used for encryption, the other is used for decryption. The exact transformations performed by the algorithm depend on the public or private key that is provided as input.

Ciphertext: This is the scrambled message produced as output. It depends on the plaintext and the key. For a given message, two different keys will produce two different ciphertexts.

Decryption algorithm: This algorithm accepts the ciphertext and the matching key and produces the original plaintext.



The essential steps are the following:

1. Each user generates a pair of keys to be used for the encryption and decryption of messages.

2. Each user places one of the two keys in a public register or other accessible file. This is the public key. The companion key is kept private. As Figure 9.1a suggests, each user maintains a collection of public keys obtained from others.

3. If Bob wishes to send a confidential message to Alice, Bob encrypts the message using Alice's public key.

4. When Alice receives the message, she decrypts it using her private key. No other recipient can decrypt the message because only Alice knows Alice's private key.

With this approach, all participants have access to public keys, and private keys are generated locally by each participant and therefore need never be distributed. As long as a user's private key remains protected and secret, incoming communication is secure. At any time, a system can change its private key and publish the companion public key to replace its old public key.

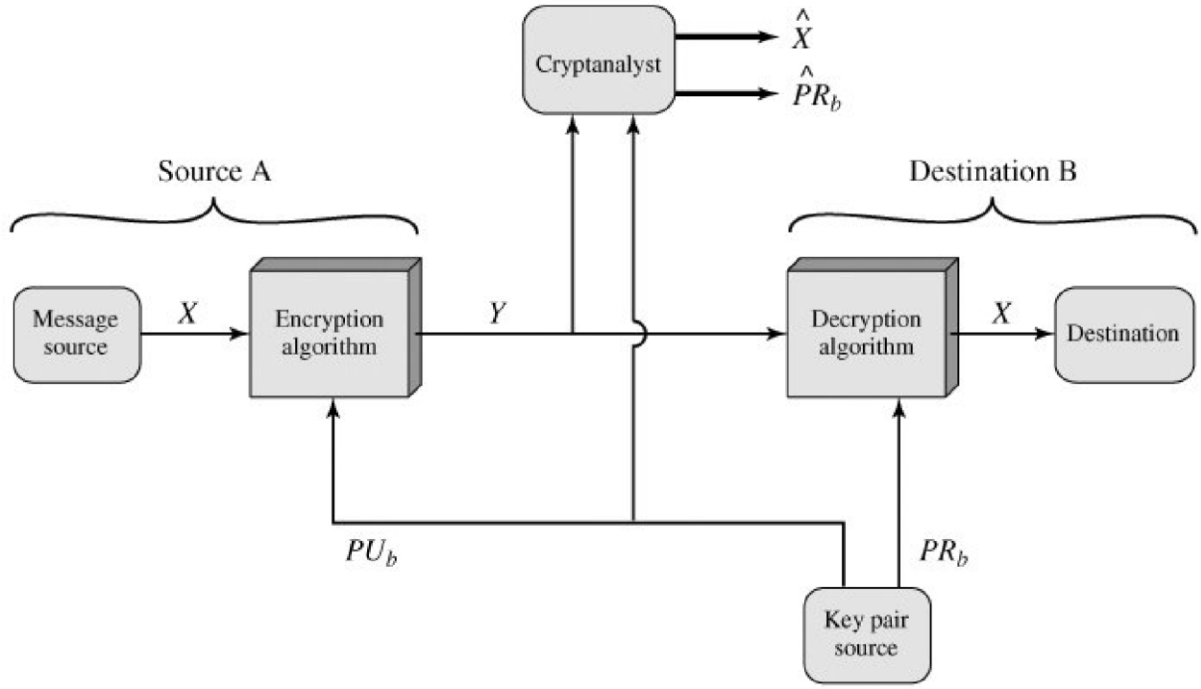
Table 9.1 summarizes some of the important aspects of symmetric and public-key encryption. To discriminate between the two, we refer to the key used in symmetric encryption as a **secret key**. The two keys used for asymmetric encryption are referred to as the **public key** and the **private key**.

[2] Invariably, the private key is kept secret, but it is referred to as a private key rather than a secret key to avoid confusion with symmetric encryption

Table 9.1. Conventional and Public-Key Encryption

Conventional Encryption	Public-Key Encryption
Needed to Work: The same algorithm with the same key is used for encryption and decryption. The sender and receiver must share the algorithm and the key.	**Needed to Work:** One algorithm is used for encryption and decryption with a pair of keys, one for encryption and one for decryption. The sender and receiver must each have one of the matched pair of keys (not the same one).
Needed for Security: Needed The key must be kept secret. It must be impossible or at least impractical to decipher a message if no other information is available. Knowledge of the algorithm plus samples of ciphertext must be insufficient to determine the key.	**Needed for Security: Needed** One of the two keys must be kept secret. It must be impossible or at least impractical to decipher a message if no other information is available. Knowledge of the algorithm plus one of the keys plus samples of ciphertext must be insufficient to determine the other key.

Let us take a closer look at the essential elements of a public-key encryption scheme, using Figure 9.2 (compare with Figure 2.2). There is some source A that produces a message in plaintext, $X = [X1, X2, ..., XM,]$. The M elements of X are letters in some finite alphabet. The message is intended for destination B. B generates a related pair of keys: a public key, PUb, and a private key, PUb. PUb is known only to B, whereas PUb is publicly available and therefore accessible by A.



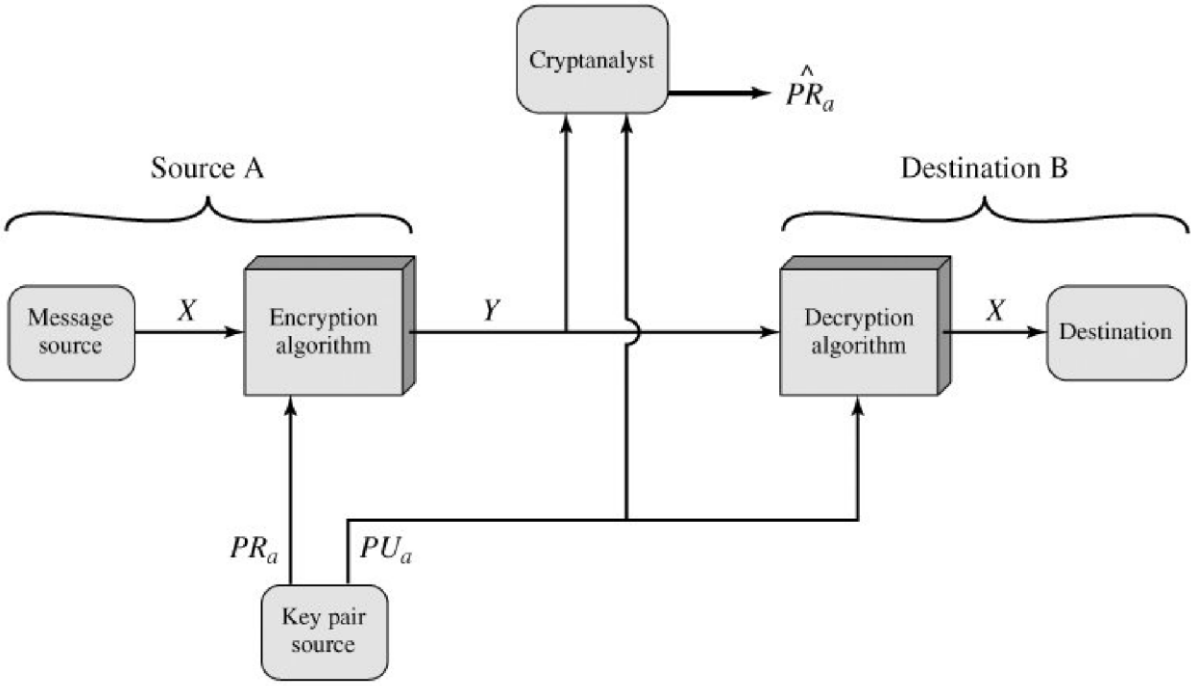
With the message X and the encryption key PUb as input, A forms the ciphertext $Y = [Y1, Y2, ..., YN]$:

$Y = E(PUb, X)$

The intended receiver, in possession of the matching private key, is able to invert the transformation:

$X = D(PRb, Y)$

An adversary, observing Y and having access to PUb but not having access to PRb or X, must attempt to recover X and/or PRb. It is assumed that the adversary does have knowledge of the encryption (E) and decryption (D) algorithms. If the adversary is interested only in this particular message, then the focus of effort is to recover X, by generating a plaintext estimate $\hat{X}$



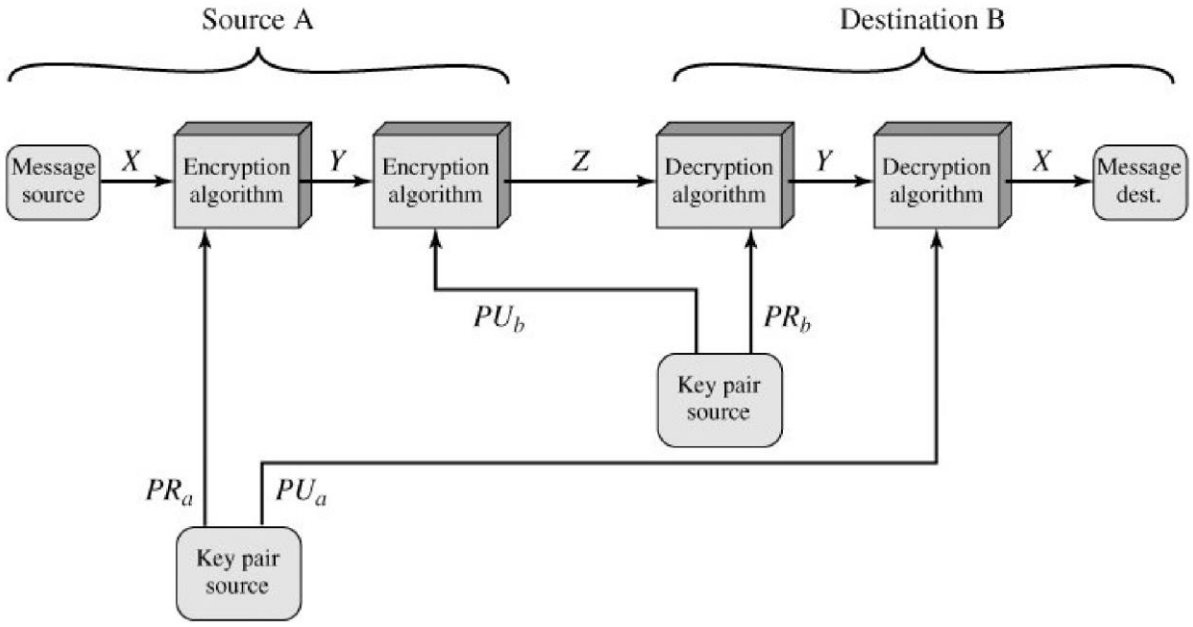
In this case, A prepares a message to B and encrypts it using A's private key before transmitting it. B can decrypt the message using A's public key. Because the message was encrypted using A's private key, only A could have prepared the message. Therefore, the entire encrypted message serves as a *digital signature*. In addition, it is impossible to alter the message without access to A's private key, so the message is authenticated both in terms of source and in terms of data integrity.

In the preceding scheme, the entire message is encrypted, which, although validating both author and contents, requires a great deal of storage. Each document must be kept in plaintext to be used for practical purposes. A copy also must be stored in ciphertext so that the origin and contents can be verified in case of a dispute. A more efficient way of achieving the same results is to encrypt a small block of bits that is a function of the document. Such a block, called an authenticator, must have the property that it is infeasible to change the document without changing the authenticator. If the authenticator is encrypted with the sender's private key, it serves as a signature that verifies origin, content, and sequencingt is important to emphasize that the encryption process depicted in Figures 9.1b and 9.3 does not provide confidentiality. That is, the message being sent is safe from alteration but not from eavesdropping. This is obvious in the case of a signature based on a portion of the message, because the rest of the message is transmitted in the clear. Even in the case of complete encryption, as shown in Figure 9.3, there is no protection of confidentiality because any observer can decrypt the message by using the sender's public key.

It is, however, possible to provide both the authentication function and confidentiality by a double use of the public-key scheme (Figure 9.4):

$Z = E(PUb, E(PRa, X))$

$X = D(PUa, E(PRb, Z))$



In this case, we begin as before by encrypting a message, using the sender's private key. This provides the digital signature. Next, we encrypt again, using the receiver's public key. The final ciphertext can be decrypted only by the intended receiver, who alone has the matching private key. Thus, confidentiality is provided. The disadvantage of this approach is that the public-key algorithm, which is complex, must be exercised four times rather than two in each communication.

Applications for Public-Key Cryptosystems

Before proceeding, we need to clarify one aspect of public-key cryptosystems that is otherwise likely to lead to confusion. Public-key systems are characterized by the use of a cryptographic algorithm with two keys, one held private and one available publicly. Depending on the application, the sender uses either the sender's private key or the receiver's public key, or both, to perform some type of cryptographic function. In broad terms, we can classify the use of public-key cryptosystems into three categories:

Encryption/decryption: The sender encrypts a message with the recipient's public key.

Digital signature: The sender "signs" a message with its private key. Signing is achieved by a cryptographic algorithm applied to the message or to a small block of data that is a function of the message.

Key exchange: Two sides cooperate to exchange a session key. Several different approaches are possible, involving the private key(s) of one or both parties.

Table 9.2. Applications for Public-Key Cryptosystems

Algorithm	Encryption/Decryption	Digital Signature	Key Exchange
RSA	Yes	Yes	Yes
Elliptic Curve	Yes	Yes	Yes
Diffie-Hellman	No	No	Yes
DSS	No	Yes	No

The RSA Algorithm

The pioneering paper by Diffie and Hellman [DIFF76b] introduced a new approach to cryptography and, in effect, challenged cryptologists to come up with a cryptographic algorithm that met the requirements for public-key systems. One of the first of the responses to the challenge was developed in 1977 by Ron Rivest, Adi Shamir, and Len Adleman at MIT and first published in 1978. The Rivest-Shamir-Adleman (RSA) scheme has since that time reigned supreme as the most widely accepted and implemented general-purpose approach to public-key encryption.

The RSA scheme is a block cipher in which the plaintext and ciphertext are integers between 0 and n 1 for some$n$. A typical size for n is 1024 bits, or 309 decimal digits. That is, n is less than 2 1024 . We examine RSA in this section in some detail, beginning with an explanation of the algorithm. Then we examine some of the computational and cryptanalytical implications of RSA.

Description of the Algorithm

The scheme developed by Rivest, Shamir, and Adleman makes use of an expression with exponentials. Plaintext is encrypted in blocks, with each block having a binary value less than some number n. That is, the block size must be less than or equal to $\log_2(n)$; in practice, the block size is i bits, where $2^i < n < 2^{i+1}$

i+1

. Encryption and decryption are of the following form, for some plaintext block M and ciphertext block C:

$C = M^e \bmod n$

$M = C^d \bmod n = (Me)^d \bmod n = M^{ed} \bmod n$

Both sender and receiver must know the value of n. The sender knows the value of e, and only the receiver knows the value of d. Thus, this is a public-key encryption algorithm with a public key of $PU = \{e, n\}$ and a private key of $PU = \{d, n\}$. For this algorithm to be satisfactory for public-key encryption, the following requirements must be met:

1. It is possible to find values of e, d, n such that $M^{ed} \bmod n = M$ for all $M < n$.

2. It is relatively easy to calculate mod $M^e \bmod n$ and C^d for all values of $M < n$.

3. It is infeasible to determine d given e and n.

For now, we focus on the first requirement and consider the other questions later. We need to find a relationship of the form

$M^{ed} \bmod n = M$

The preceding relationship holds if e and d are multiplicative inverses modulo f(n), where f(n) is the Euler totient function. It is shown that for p, q prime, f(pq) = (p 1)(q 1) The relationship between e and d can be expressed as

$$ed \bmod \phi(n) = 1$$

This is equivalent to saying

$ed \equiv 1 \bmod$ f(n)

$d\ e \equiv 1 \bmod$ f(n)

That is, e and d are multiplicative inverses mod f(n). Note that, according to the rules of modular arithmetic, this is true only if d (and therefore e) is relatively prime to f(n). Equivalently, gcd(f(n),d) = 1.

We are now ready to state the RSA scheme. The ingredients are the following:

p,q, two prime numbers	(private, chosen)
$n = pq$	(public, calculated)
e, with gcd(f(n),e) = 1;1 < e < f(n)	(public, chosen)
$d\ e \equiv 1 \pmod{\text{f}(n)}$	(private, calculated)

The private key consists of $\{d, n\}$ and the public key consists of $\{e, n\}$. Suppose that user A has published its public key and that user B wishes to send the message M to A. Then B

calculates $C = M^e \bmod n$ and transmits C. On receipt of this ciphertext, user A decrypts by calculating $M = C^d \bmod n$.

For this example, the keys were generated as follows:

1. Select two prime numbers, $p = 17$ and $q = 11$.

2. Calculate $n = pq = 17 \times 11 = 187$.

3. Calculate $f(n) = (p\ 1)(q\ 1) = 16 \times 10 = 160$.

4. Select e such that e is relatively prime to $f(n) = 160$ and less than $f(n)$ we choose $e = 7$.

5. Determine d such that $de \equiv 1 \pmod{160}$ and $d < 160$. The correct value is $d = 23$, because $23 \times 7 = 161 = 10 \times 160 + 1$; d can be calculated using the extended Euclid's algorithm

Figure 9.5. The RSA Algorithm

Key Generation

Select p, q	p and q both prime, $p \neq q$
Calculate $n = p \times q$	
Calculate $\phi(n) = (p - 1)(q - 1)$	
Select integer e	$\gcd(\phi(n), e) = 1; 1 < e < \phi(n)$
Calculate d	$d \equiv e^{-1} \pmod{\phi(n)}$
Public key	$PU = \{e, n\}$
Private key	$PR = \{d, n\}$

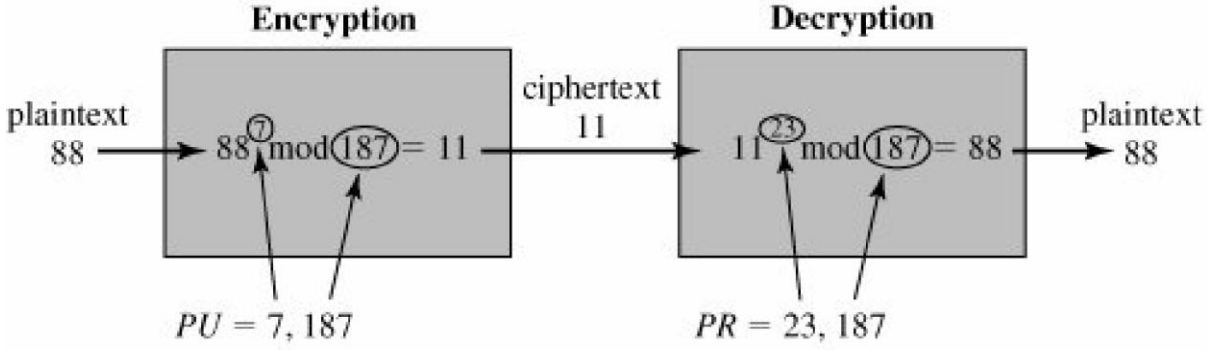
Encryption

Plaintext:	$M < n$
Ciphertext:	$C = M^e \bmod n$

Decryption

Ciphertext:	C
Plaintext:	$M = C^d \bmod n$

Figure 9.6. Example of RSA Algorithm



The resulting keys are public key $PU = \{7,187\}$ and private key $PR = \{23,187\}$. The example shows the use of these keys for a plaintext input of $M = 88$. For encryption, we need to calculate $C = 88^7 \bmod 187$. Exploiting the properties of modular arithmetic, we can do this as follows:

$88^7 \bmod 187 = [(88^4 \bmod 187) \times (88^2 \bmod 187) \times (88^1 \bmod 187)] \bmod 187$
$88^1 \bmod 187 = 88$
$88^2 \bmod 187 = 7744 \bmod 187 = 77$
$88^4 \bmod 187 = 59{,}969{,}536 \bmod 187 = 132$
$88^7 \bmod 187 = (88 \times 77 \times 132) \bmod 187 = 894{,}432 \bmod 187 = 11$

For decryption, we calculate $M = 11^{23} \bmod 187$:
$11^{23} \bmod 187 = [(11^1 \bmod 187) \times (11^2 \bmod 187) \times (11^4 \bmod 187) \times (11^8 \bmod 187) \times (11^8 \bmod 187)] \bmod 187$
$11^1 \bmod 187 = 11$
$11^2 \bmod 187 = 121$
$11^4 \bmod 187 = 14{,}641 \bmod 187 = 55$
$11^8 \bmod 187 = 214{,}358{,}881 \bmod 187 = 33$
$11^{23} \bmod 187 = (11 \times 121 \times 55 \times 33 \times 33) \bmod 187 = 79{,}720{,}245 \bmod 187 = 88$

The Security of RSA

Four possible approaches to attacking the RSA algorithm are as follows:

Brute force: This involves trying all possible private keys.

Mathematical attacks: There are several approaches, all equivalent in effort to factoring the product of two primes.

Timing attacks: These depend on the running time of the decryption algorithm.

Chosen ciphertext attacks: This type of attack exploits properties of the RSA algorithm.

The defense against the brute-force approach is the same for RSA as for other cryptosystems, namely, use a large key space. Thus, the larger the number of bits in d, the better. However, because the calculations involved, both in key generation and in encryption/decryption, are complex, the larger the size of the key, the slower the system will run.

Key Management

One of the major roles of public-key encryption has been to address the problem of key distribution. There are actually two distinct aspects to the use of public-key cryptography in this regard:
The distribution of public keys
The use of public-key encryption to distribute secret keys

Distribution of Public Keys

Several techniques have been proposed for the distribution of public keys. Virtually all these proposals can be grouped into the following general schemes:
Public announcement
Publicly available directory
Public-key authority
Public-key certificates

Public Announcement of Public Keys

On the face of it, the point of public-key encryption is that the public key is public. Thus, if there is some broadly accepted public-key algorithm, such as RSA, any participant can send his or her public key to any other participant or broadcast the key to the community at large (Figure 10.1). For example, because of the growing popularity of PGP which makes use of RSA, many PGP users have adopted the practice of appending their public key to messages that they send to public forums, such as USENET newsgroups and Internet mailing lists.

Although this approach is convenient, it has a major weakness. Anyone can forge such a public announcement. That is, some user could pretend to be user A and send a public key to another participant or broadcast such a public key. Until such time as user A discovers the forgery and alerts other participants, the forger is able to read all encrypted messages intended for A and can use the forged keys for authentication

Publicly Available Directory

A greater degree of security can be achieved by maintaining a publicly available dynamic directory of public keys. Maintenance and distribution of the public directory would have to be the responsibility of some trusted entity or organization (Figure 10.2). Such a scheme would include the following elements:

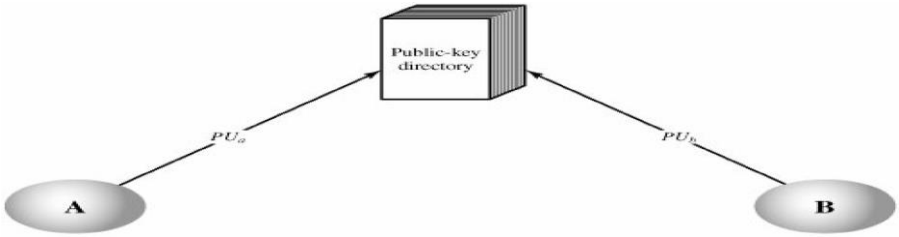
1. The authority maintains a directory with a {name, public key} entry for each participant.

2. Each participant registers a public key with the directory authority. Registration would have to be in person or by some form of secure authenticated communication.

3. A participant may replace the existing key with a new one at any time, either because of the desire to replace a public key that has already been used for a large amount of data, or because the corresponding private key has been compromised in some way.

4. Participants could also access the directory electronically. For this purpose, secure, authenticated communication from the authority to the participant is mandatory.

Figure 10.2. Public-Key Publication



This scheme is clearly more secure than individual public announcements but still has vulnerabilities. If an adversary succeeds in obtaining or computing the private key of the directory authority, the adversary could authoritatively pass out counterfeit public keys and subsequently impersonate any participant and eavesdrop on messages sent to any participant. Another way to achieve the same end is for the adversary to tamper with the records kept by the authority.

Public-Key Authority

Stronger security for public-key distribution can be achieved by providing tighter control over the distribution of public keys from the directory. A typical scenario is illustrated in Figure 10.3, which is based on a figure in [POPE79]. As before, the scenario assumes that a central authority maintains a dynamic directory of public keys of all participants. In addition, each participant reliably knows a public key for the authority, with only the authority knowing the corresponding private key. The following steps (matched by number to Figure 10.3) occur:

1. A sends a timestamped message to the public-key authority containing a request for the current public key of B.

2. The authority responds with a message that is encrypted using the authority's private key, *PRauth* Thus, A is able to decrypt the message using the authority's public key. Therefore, A is assured that the message originated with the authority. The message includes the following: B's public key, *PUb* which A can use to encrypt messages destined for B

The original request, to enable A to match this response with the corresponding earlier request and to verify that the original request was not altered before reception by the authority

The original timestamp, so A can determine that this is not an old message from the authority containing a key other than B's current public key

3. A stores B's public key and also uses it to encrypt a message to B containing an identifier of A (*IDA*) and a nonce (*N*1), which is used to identify this transaction uniquely.

4, 5. B retrieves A's public key from the authority in the same manner as A retrieved B's public key.

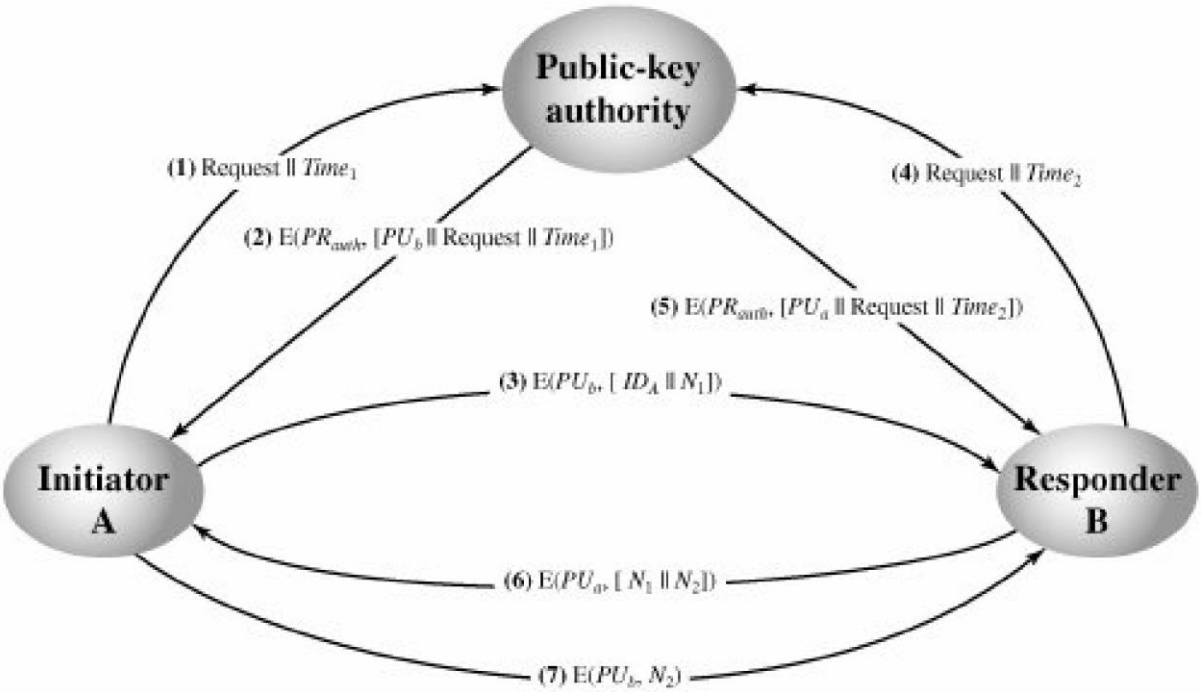
At this point, public keys have been securely delivered to A and B, and they may begin their protected exchange. However, two additional steps are desirable:

6. B sends a message to A encrypted with *PUa* and containing A's nonce (*N*1) as well as a new nonce generated by B (*N*2)

Because only B could have decrypted message (3), the presence of *N*1 in message (6) assures A that the correspondent is B.

7. A returns *N*2, encrypted using B's public key, to assure B that its correspondent is A.

Figure 10.3. Public-Key Distribution Scenario



Thus, a total of seven messages are required. However, the initial four messages need be used only infrequently because both A and B can save the other's public key for future use, a technique known as caching. Periodically, a user should request fresh copies of the public keys of its correspondents to ensure currency.

Public-Key Certificates

The scenario of Figure 10.3 is attractive, yet it has some drawbacks. The public-key authority could be somewhat of a bottleneck in the system, for a user must appeal to the authority for a public key for every other user that it wishes to contact. As before, the directory of names and public keys maintained by the authority is vulnerable to tampering.

An alternative approach, first suggested by Kohnfelder [KOHN78], is to use **certificates** that can be used by participants to exchange keys without contacting a public-key authority, in a way that is as reliable as if the keys were obtained directly from a public-key authority. In essence, a certificate consists of a public key plus an identifier of the key owner, with the whole block signed by a trusted third party.

Typically, the third party is a certificate authority, such as a government agency or a financial institution, that is trusted by the user community. A user can present his or her public key to the authority in a secure manner, and obtain a certificate. The user can then publish the certificate. Anyone needed this user's public key can obtain the certificate and verify that it is valid by way of the attached trusted signature. A participant can also convey its key information to another by transmitting its certificate. Other participants can verify that the certificate was created by the authority. We can place the following requirements on this scheme:

1. Any participant can read a certificate to determine the name and public key of the certificate's owner.

2. Any participant can verify that the certificate originated from the certificate authority and is not counterfeit.

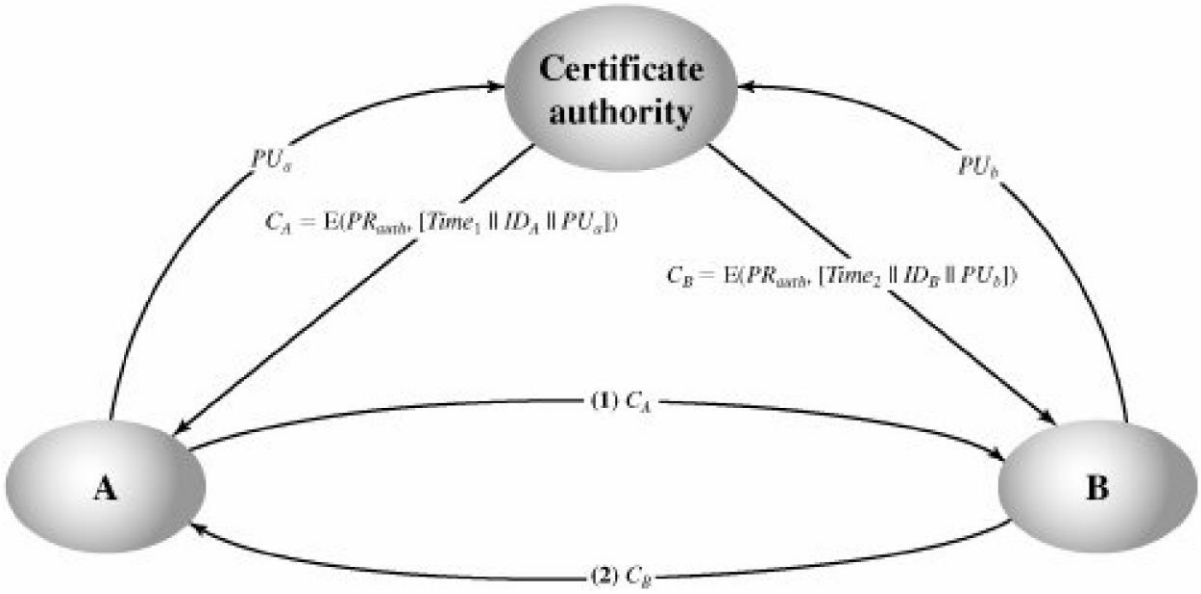
3. Only the certificate authority can create and update certificates.

These requirements are satisfied by the original proposal in [KOHN78]. Denning [DENN83] added the following additional requirement:

4. Any participant can verify the currency of the certificate.

A certificate scheme is illustrated in Figure 10.4. Each participant applies to the certificate authority, supplying a public key and requesting a certificate.

Figure 10.4. Exchange of Public-Key Certificates



Application must be in person or by some form of secure authenticated communication. For participant A, the authority provides a certificate of the form

$CA = E(PRauth, [T||IDA||PUa])$

where *PRauth* is the private key used by the authority and *T* is a timestamp. A may then pass this certificate on to any other participant, who reads and verifies the certificate as follows:

$D(PUauth, CA) = D(PUauth, E(PRauth, [T||IDA||PUa])) = (T||IDA||PUa)$

The recipient uses the authority's public key, *PUauth* to decrypt the certificate. Because the certificate is readable only using the authority's public key, this verifies that the certificate came from the certificate authority. The elements *IDA* and *PUa* provide the recipient with the name and public key of the certificate's holder. The timestamp *T* validates the currency of the certificate. The timestamp counters the following scenario. A's private key is learned by an adversary. A generates a new private/public key pair and applies to the certificate authority for a new certificate. Meanwhile, the adversary replays the old certificate to B. If B then encrypts messages using the compromised old public key, the adversary can read those messages.

In this context, the compromise of a private key is comparable to the loss of a credit card. The owner cancels the credit card number but is at risk until all possible communicants are aware

that the old credit card is obsolete. Thus, the timestamp serves as something like an expiration date. If a certificate is sufficiently old, it is assumed to be expired.

One scheme has become universally accepted for formatting public-key certificates: the X.509 standard. X.509 certificates are used in most network security applications, including IP security, secure sockets layer (SSL), secure electronic transactions (SET), and S/MIME.

Diffie-Hellman Key Exchange

The first published public-key algorithm appeared in the seminal paper by Diffie and Hellman that defined public-key cryptography and is generally referred to as Diffie-Hellman key exchange.

A number of commercial products employ this key exchange technique.

The purpose of the algorithm is to enable two users to securely exchange a key that can then be used for subsequent encryption of messages. The algorithm itself is limited to the exchange of secret values.

The Diffie-Hellman algorithm depends for its effectiveness on the difficulty of computing discrete logarithms. Briefly, we can define the discrete logarithm in the following way. First, we define a primitive root of a prime number p as one whose powers modulo p generate all the integers from 1 to p 1. That is, if a is a primitive root of the prime number p, then the numbers

a mod p, a^2 mod p,..., a^{p1} mod p are distinct and consist of the integers from 1 through p 1 in some permutation.

For any integer b and a primitive root a of prime number p, we can find a unique exponent i such that

$$b \equiv a^i (\text{mod } p) \text{ where } 0 \leq i \leq (p\ 1)$$

The exponent i is referred to as the discrete logarithm of b for the base a, mod p.

The Algorithm

Figure 10.7 summarizes the Diffie-Hellman key exchange algorithm. For this scheme, there are two publicly known numbers: a prime number q and an integer that is a primitive root of q . Suppose the users A and B wish to exchange a key. User A selects a random integer Xr <A and computes YA = a ^{XA} mod q. Similarly, user B independently selects a random integer$X$ A

< q and computes YB = a XB mod q. Each side keeps the X value private and makes the$Y$ value available publicly to the other side. User A computes the key as $K = (YB)^{XA} \bmod q$ and user B computes the key as $K = (YA)^{XB} \bmod q$. These two calculations produce identical results:

$K = (YB)^{XA} \bmod q$
$= (a^{XB} \bmod q)^{XA} \bmod q$
$= (aXB)^{XA} \bmod q$ by the rules of modular arithmetic
$= (a^{XB\,XA} \bmod q$
$= (aXA)^{XB} \bmod q$
$= (a^{XA} \bmod q)$
$= (a^{XA} \bmod q)^{XB} \bmod q$
$= (YA)^{XB} \bmod q$

Figure 10.7. The Diffie-Hellman Key Exchange Algorithm

Random Numbers

In many cases secure programs must generate "random" numbers that cannot be guessed by an adversary. Examples include session keys, public or private keys, symmetric keys, nonces and IVs used in many protocols, salts, and so on. Ideally, you should use a truly random source of data for random numbers, such as values based on radioactive decay (through precise timing of Geiger counter clicks), atmospheric noise, or thermal noise in electrical circuits. Some computers have a hardware component that functions as a real random value generator, and if it's available you should use it.

However, most computers don't have hardware that generates truly random values, so in most cases you need a way to generate random numbers that is sufficiently random that an adversary can't predict it. In general, this means that you'll need three things:

- An "unguessable" state; typically this is done by measuring variances in timing of low-level devices (keystrokes, disk drive arm jitter, etc.) in a way that an adversary cannot control.
- A cryptographically strong pseudo-random number generator (PRNG), which uses the state to generate "random" numbers.
- A large number of bits (in both the seed and the resulting value used). There's no point in having a strong PRNG if you only have a few possible values, because this makes it easy for an attacker to use brute force attacks. The number of bits necessary varies depending on the circumstance, however, since these are often used as cryptographic keys, the normal rules of thumb for keys apply. For a symmetric key (result), I'd use at least 112 bits (3DES), 128 bits is a little better, and 160 bits or more is even safer.

Typically the PRNG uses the state to generate some values, and then some of its values and other unguessable inputs are used to update the state. There are lots of ways to attack these systems. For example, if an attacker can control or view inputs to the state (or parts of it), the attacker may be able to determine your supposedly "random" number.

Pseudo Random Number Generator (PRNG) refers to an algorithm that uses mathematical formulas to produce sequences of random numbers. PRNGs generate a sequence of numbers

approximating the properties of random numbers. A PRNG starts from an arbitrary starting state using a seed state. Many numbers are generated in a short time and can also be reproduced later, if the starting point in the sequence is known. Hence, the numbers are deterministic and efficient.

Why do we need PRNG?

With the advent of computers, programmers recognized the need for a means of introducing randomness into a computer program. However, surprising as it may seem, it is difficult to get a computer to do something by chance as computer follows the given instructions blindly and is therefore completely predictable. It is not possible to generate truly random numbers from deterministic thing like computers so PRNG is a technique developed to generate random numbers using a computer.

Characteristics of PRNG

- **Efficient:** PRNG can produce many numbers in a short time and is advantageous for applications that need many numbers
- **Deterministic:** A given sequence of numbers can be reproduced at a later date if the starting point in the sequence is known. Determinism is handy if you need to replay the same sequence of numbers again at a later stage.
- **Periodic:** PRNGs are periodic, which means that the sequence will eventually repeat itself. While periodicity is hardly ever a desirable characteristic, modern PRNGs have a period that is so long that it can be ignored for most practical purposes

TRNGs, PRNGs, and PRFs Cryptographic applications typically make use of algorithmic techniques for random number generation. These algorithms are deterministic and therefore produce sequences of numbers that are not statistically random. However, if the algorithm is good, the resulting sequences will pass many reasonable tests of randomness. Such numbers are referred to as pseudorandom numbers. You may be somewhat uneasy about the concept of using numbers generated by a deterministic algorithm as if they were random numbers. Despite what might be called philosophical objections to such a practice, it generally works. As one expert on probability theory puts it : For practical purposes we are forced to accept the awkward concept of
“relatively random” meaning that with regard to the proposed use we can see no reason why they

will not perform as if they were random (as the theory usually requires). This is highly subjective and is not very palatable to purists, but it is what statisticians regularly appeal to when they take "a random sample" —they hope that any results they use will have approximately the same properties as a complete counting of the whole sample space that occurs in their theory. Figure 7.1 contrasts a true random number generator (TRNG) with two forms of pseudorandom number generators. A TRNG takes as input a source that is effectively random; the source is often referred to as an entropy source. . In essence, the entropy source is

drawn from the physical environment of the computer and could include things such as keystroke timing patterns, disk electrical activity, mouse movements, and instantaneous values of the system clock. The source, or combination of sources, serve as input to an algorithm that produces random binary output. The TRNG may simply involve conversion of an analog source to a binary output. The TRNG may involve additional processing to overcome any bias in the source; this is discussed in Section 7.6. In contrast, a PRNG takes as input a fixed value, called the seed, and produces a sequence of output bits using a deterministic algorithm. Typically, as shown, there is some feedback path by which some of the results of the algorithm are fed back as

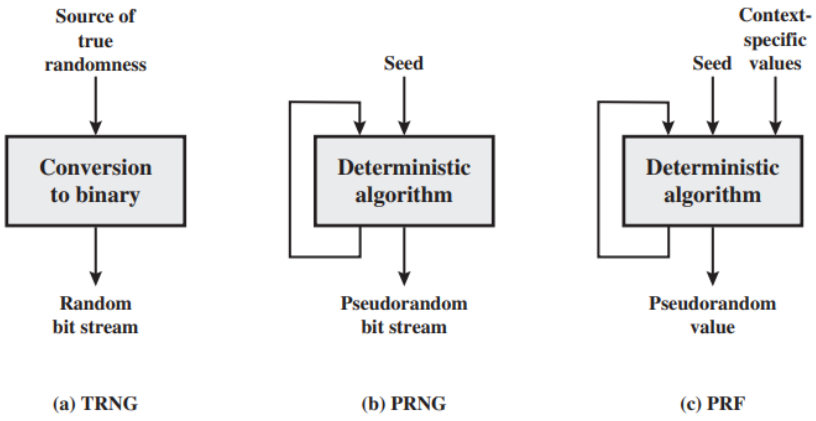


Figure 7.1 Random and Pseudorandom Number Generators

input as additional output bits are produced. The important thing to note is that the output bit stream is determined solely by the input value or values, so that an adversary who knows the algorithm and the seed can reproduce the entire bit stream. Figure 7.1 shows two different forms of PRNGs, based on application. • Pseudorandom number generator: An algorithm that is used to produce an open-ended sequence of bits is referred to as a PRNG. A common application for an open-ended sequence of bits is as input to a symmetric stream cipher, as discussed in Section 7.4. Also, see Figure 3.1a. • Pseudorandom function (PRF): A PRF is used to produced a pseudorandom string of bits of some fixed length. Examples are symmetric encryption keys and nonces. Typically, the PRF takes as input a seed plus some context specific values, such as a user ID or an application ID. A number of examples of PRFs will be seen throughout this book, notably in Chapters 16 and 17. Other than the number of bits produced, there is no difference between a PRNG and a PRF. The same algorithms can be used in both applications. Both require a seed and both must exhibit randomness and unpredictability. Further, a PRNG application may also employ context-specific input. In what follows, we make no distinction between these two applications