

1. Highway (PCH). There are n possible locations along the highway, and the distance from the start to location k is $d_k \geq 0$, where $k=1,2,\dots,n$. You may assume that $d_i < d_k$ for $i < k$. There are important constraints: each location k you can open only one charging station with the expected profit p_k . You must open at least one charging station along the whole highway. Any two stations should be at least M miles apart.

- I. Define subproblems to be solved.

Let $\text{OPT}[r]$ be the maximum profit where r is the possible locations on PCH.

- II. Write recurrence relation for subproblems.

If $d_r - d_j \geq M$ where $j = r-1$

$$\text{OPT}[r] = \text{MAX}(\text{OPT}[r-1] + p_r, p_r)$$

If $d_r - d_j \leq M$ where $j = r-1$

$$\text{OPT}[r] = \text{MAX}(\text{OPT}[r-1], p_r)$$

Base Case:

$$\text{OPT}[r] = p_r \quad \text{if } r = 1$$

$$\text{OPT}[r] = 0 \quad \text{if } r = 0$$

- III. Pseudo-code

```
PCH_Gas(int location[n]){
    for(r = 2; r ≤ n; r++){
        for(j = 1; j ≤ r - 1; j++){
            If  $d_r - d_j \geq M$ 
                 $\text{OPT}[r] = \text{MAX}(\text{OPT}[r-1] + p_r, p_r)$ 
            else if  $d_r - d_j \leq M$ 
                 $\text{OPT}[r] = \text{MAX}(\text{OPT}[r-1], p_r)$ 
            else if  $r = 1$ 
                 $\text{OPT}[r] = p_r$ 
            else if  $r = 0$ 
                 $\text{OPT}[r] = 0$ 
        }
    }
    return OPT[n]
}
```

- IV. Run-Time Complexity

$$O(n^2)$$

2. Given n balloons, indexed from 0 to $n - 1$. Each balloon is painted with a number on it represented by array `nums`. You are asked to burst all the balloons. If the you burst balloon i you will get $nums[\text{left}] \cdot nums[i] \cdot nums[\text{right}]$ coins. Here left and right are adjacent indices of i . After the burst, the left and right then becomes adjacent. You may assume $nums[-1] = nums[n] = 1$ and they are not real therefore you cannot burst them. Design a dynamic programming algorithm to find the maximum coins you can collect by bursting the balloons wisely. Analyze the running time of your algorithm.

- I. Define subproblems to be solved

Let $OPT[j, k]$ be the maximum coins collected in the range j to k .

- II. Write the recurrence relation for subproblem

$$OPT[j, k] = \text{MAX}(OPT[j, i-1] + OPT[i+1, k] + (num[j-1]*num[i]*num[k+1]), OPT[j, k])$$

Base Case:

If $j = k$ then $OPT[j, k] = num[j]$

- III. Pseudo-code

```

Ballon(int num[n]){
    num[-1] = num[n + 1] = 1
    for(int k = 1; k < n-1; k++){
        for(int j = k; j ≥ 1, j--){
            for(int x = k; x = k; x++){
                If j = k
                    OPT[j, k] = num[j]
                else
                    OPT[j, k] = MAX(OPT[j, i-1] + OPT[i+1, k]
                    + (num[j-1]*num[i]*num[k+1]), OPT[j, k])
            }
        }
    }
    return OPT[j, k]
}

```

- IV. Run-Time Complexity

Since we are testing every single element in our num array with every possible first element and last element then our complexity is as follows;

n starting points * n ending points * n last ballon to burst = $O(n^3)$