DNK 企业部门最优化

1. 利润最大化 求最优定价（需体现垄断竞争和价格粘性）

$P\times C - TC$：消费 总成本

预期算子

书上是θ，这里用1-θ表现价格粘性 便于后续构造和计算

$$\max_{P_{i0}^*} \Pi_0^n = E_0 \sum_{t=0}^{\infty} Q_{0,t}(1-\theta)^t \left[P_{i0}^* \cdot Y_{i,t|0} - TC_{i,t|0}^n (Y_{i,t|0}) \right]$$

P_{i0}^*：体现垄断竞争，其它类型会以产量作为选择变量，i 指的是 i 厂商调价格后不变

$Q_{0,t}$：利润贴现

P_{i0}^*：（选择变量）调整后的定价

$Y_{i,t|0}$：调整后的产出，且调整后不变（粘性）（$Y_t = C_t$ 市场出清）

$TC_{i,t|0}^n(Y_{i,t|0})$：总成本是产出 $Y_{i,t|0}$ 的函数，$t|0$ 表示第0期调整定价，t 表示之后的阶段

（书上是 $t+k|t$，从第 t 期开始，没啥区别）

$$\text{s.t. 目标函数：} \quad Y_{i,t|0} = \left(\frac{P_{i0}}{P_t}\right)^{-\varepsilon} Y_t$$

（由家庭部门得到 $C_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} C_t$）

用代入法求解（将目标函数代入 max…）

得
$$\Pi_0^n = E_0 \sum_{t=0}^{\infty} Q_{0,t}(1-\theta)^t \left(P_{i0}^* \cdot \left[\left(\frac{P_{i0}^*}{P_t}\right)^{-\varepsilon} Y_t \right] - TC_{i,t|0}^n \left[\left(\frac{P_{i0}^*}{P_t}\right)^{-\varepsilon} Y_t \right] \right)$$

$P_{i0}^{*\,1-\varepsilon}$

$$\text{FOC：} \quad \frac{\partial \Pi_0^n}{\partial P_{i0}^*} = 0$$

$$\Rightarrow E_0 \sum_{t=0}^{\infty} Q_{0,t}(1-\theta)^t \left[(1-\varepsilon) \underbrace{\left(\frac{P_{i0}^*}{P_t}\right)^{-\varepsilon} Y_t}_{Y_{i,t|0}} - \overbrace{\frac{\partial TC_{i,t|0}^n}{\partial Y_{i,t|0}}}^{MC_{i,t|0}^n} \cdot \overbrace{\frac{\partial Y_{i,t|0}}{\partial P_{i0}^*}}^{-\varepsilon \cdot \frac{P_{i0}^{*\,-\varepsilon-1}}{P_t^{-\varepsilon-1}} \cdot \frac{1}{P_t} \cdot Y_t} \right] = 0$$

$$\Rightarrow E_0 \sum_{t=0}^{\infty} Q_{0,t}(1-\theta)^t \left[(1-\varepsilon) Y_{i,t|0} + MC_{i,t|0}^n \cdot \varepsilon \underbrace{\left(\frac{P_{i0}^*}{P_t}\right)^{-\varepsilon-1} \cdot \frac{Y_t}{P_t}}_{Y_{i,t|0}} \right] = 0$$

$$\Rightarrow E_0 \sum_{t=0}^{\infty} Q_{0,t}(1-\theta)^t \left[(1-\varepsilon) Y_{i,t|0} + \frac{\varepsilon}{\cancel{P_t}} MC_{i,t|0}^n \cdot Y_{i,t|0} \cdot \left(\frac{P_{i0}^*}{\cancel{P_t}}\right)^{-1} \right] = 0$$

$$\Rightarrow E_0 \sum_{t=0}^{\infty} Q_{0,t}(1-\theta)^t \cdot Y_{i,t|0} \left[(1-\varepsilon) + \varepsilon \cdot MC_{i,t|0}^n \cdot P_{i0}^{*\,-1} \right] = 0$$

接下展开移项

$$\Rightarrow (1-\varepsilon) E_0 \sum_{t=0}^{\infty} Q_{0,t} (1-\theta)^t \cdot Y_{i,t|0} = -\varepsilon \cdot P_{i0}^{*\,-1} \cdot E_0 \sum_{t=0}^{\infty} Q_{0,t} (1-\theta)^t \cdot Y_{i,t|0} \cdot MC_{i,t|0}^n$$

代入具体式子

$$\Rightarrow (\varepsilon-1) E_0 \sum_{t=0}^{\infty} (1-\theta)^t \cdot \underbrace{\beta^t\left[\left(\frac{Y_0}{Y_t}\right)^{\sigma}\frac{P_0}{P_t}\right]}_{Q_{0,t}} \cdot \underbrace{\left(\frac{P_{i0}^*}{P_t}\right)^{-\varepsilon} Y_t}_{Y_{i,t|0}} = \varepsilon P_{i0}^{*-1} \cdot E_0 \sum_{t=0}^{\infty} (1-\theta)^t \cdot \underbrace{\beta^t\left[\left(\frac{Y_0}{Y_t}\right)^{\sigma}\frac{P_0}{P_t}\right]}_{Q_{0,t}} \cdot \underbrace{\left(\frac{P_{i0}^*}{P_t}\right)^{-\varepsilon} Y_t}_{Y_{i,t|0}} \cdot MC^n_{i,t|0}$$

把与t无关的常数项拎出来

$$\Rightarrow (\varepsilon-1)\cancel{Y_0^{\sigma}P_0}\,\cancel{(P_{i0}^*)^{-\varepsilon}}\, E_0 \sum_{t=0}^{\infty}[(1-\theta)\beta]^t \cdot P_t^{\varepsilon-1} \cdot Y_t^{1-\sigma} = \varepsilon \cdot P_{i0}^{*-1-\varepsilon}\,\cancel{Y_0^{\sigma}P_0}\, E_0 \sum_{t=0}^{\infty}[(1-\theta)\beta]^t \cdot P_t^{\varepsilon-1} \cdot Y_t^{1-\sigma} \cdot MC^n_{i,t|0}$$

$$\Rightarrow (\varepsilon-1) E_0 \sum_{t=0}^{\infty}[(1-\theta)\beta]^t \cdot P_t^{\varepsilon-1} \cdot Y_t^{1-\sigma} = \varepsilon \cdot (\underbrace{P_{i0}^*}_{\text{最优定价}})^{-1} E_0 \sum_{t=0}^{\infty}[(1-\theta)\beta]^t \cdot P_t^{\varepsilon-1} Y_t^{1-\sigma} \cdot MC^n_{i,t|0}$$

$$\Rightarrow P_{i0}^* = \frac{\varepsilon}{\varepsilon-1} \frac{E_0 \sum_{t=0}^{\infty}[(1-\theta)\beta]^t \cdot P_t^{\varepsilon-1} Y_t^{1-\sigma} \cdot \underbrace{MC^n_{i,t|0}}_{\text{名义边际成本}}}{E_0 \sum_{t=0}^{\infty}[(1-\theta)\beta]^t \cdot P_t^{\varepsilon-1} \cdot Y_t^{1-\sigma}}$$

求得最终的最优定价
包含了垄断竞争和价格粘性的信息

☆ 体现了垄断竞争下的成本加成

☆ 当价格弹性 $\theta=1$ 时 $1-\theta=0$ 价格完全弹性.

$0^0=1$, $P_{i0}^* = \frac{\varepsilon}{\varepsilon-1} MC^n_{i,t|0}$

可见价格弹性时 价格由MC决定

2. 求解菲利普斯曲线

失业率和通胀率的关系
与产出缺口有关

由 $P_{i0}^* = \frac{\varepsilon}{\varepsilon-1} \frac{E_0 \sum_{t=0}^{\infty}[(1-\theta)\beta]^t \cdot P_t^{\varepsilon-1} Y_t^{1-\sigma} \cdot MC^n_{i,t|0}}{E_0 \sum_{t=0}^{\infty}[(1-\theta)\beta]^t \cdot P_t^{\varepsilon-1} \cdot Y_t^{1-\sigma}}$

书上的形式： $P_t^* = \frac{\varepsilon}{\varepsilon-1} \frac{E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k \cdot P_{t+k}^{\varepsilon-1} Y_{t+k}^{1-\sigma} \cdot MC^n_{i,t+k|t}}{E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k \cdot P_{t+k}^{\varepsilon-1} \cdot Y_{t+k}^{1-\sigma}}$

从第t期开始后发展k时期，下标i即使不写也可以区别是作了价格调整的

$$\underbrace{\frac{1}{P_{t-1}} \cdot P_t^* \cdot E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k \cdot P_{t+k}^{\varepsilon-1} \cdot Y_{t+k}^{1-\sigma}}_{LHS} = \underbrace{\frac{\varepsilon}{\varepsilon-1} \cdot \frac{1}{P_{t-1}} E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k \cdot P_{t+k}^{\varepsilon-1} Y_{t+k}^{1-\sigma} MC^n_{i,t+k|t}}_{RHS}$$

左右各4个关于t的变量

进行对数线性化 → $\hat{x}_t = \ln X_t - \ln X = x_t - x$ $\quad x_t = X e^{\hat{x}_t}$

$= \frac{X_t - X}{X}$ (一阶泰勒展开)

LHS的对数线性化：(一阶泰勒展开) $f(x_0)+f'(x_0)(x-x_0)$

$$\sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-1}Y^{1-\sigma}+\frac{1}{P}E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-1}Y^{1-\sigma}\cdot(P_t^*-P)-\frac{P}{P^2}E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-1}Y^{1-\sigma}\cdot(P_{t-1}-P)$$

$$+(\varepsilon-1)\cdot\frac{P}{P}\cdot E_t\cdot\sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-2}Y^{1-\sigma}(P_{t+k}-P)+(1-\sigma)\frac{P}{P}E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-1}Y^{-\sigma}(Y_{t+k}-Y)$$

$$=\sum_{k=0}^{\infty}[(1-\theta)\beta]^k \underbrace{P^{\varepsilon-1}Y^{1-\sigma}}+E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k \underline{P^{\varepsilon-1}Y^{1-\sigma}}\cdot\frac{P_t^*-P}{P}-E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k \underline{P^{\varepsilon-1}Y^{1-\sigma}}\frac{P_{t-1}-P}{P}$$

$$+(\varepsilon-1)\cdot E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k \underbrace{P^{\varepsilon-1}Y^{1-\sigma}}\frac{P_{t+k}-P}{P}+(1-\sigma)E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k \underline{P^{\varepsilon-1}Y^{1-\sigma}}\cdot\frac{Y_{t+k}-Y}{Y}$$

$$=P^{\varepsilon-1}Y^{1-\sigma}E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k\cdot(1+\hat{p}_t^*-\hat{p}_{t-1}+(\varepsilon-1)\hat{p}_{t+k}+(1-\sigma)\hat{y}_{t+k})$$

RHS的对数线性化：

$$\frac{\varepsilon}{\varepsilon-1}\cdot\frac{1}{P}\sum_{k=0}^{\infty}[(1-\theta)\beta]^k\cdot P^{\varepsilon-1}Y^{1-\sigma}\cdot MC^n-\frac{\varepsilon}{\varepsilon-1}\frac{1}{P^2}\sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-1}Y^{1-\sigma}MC^n(P_{t-1}-P)$$

$$+\varepsilon\cdot\frac{1}{P}\sum_{k=0}^{\infty}[(1-\theta)\beta]^k\cdot P^{\varepsilon-2}Y^{1-\sigma}MC^n\cdot(P_{t+k}-P)+\frac{\varepsilon(1-\sigma)}{\varepsilon-1}\cdot\frac{1}{P}\sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-1}Y^{-\sigma}MC^n\cdot(Y_{t+k}-Y)$$

$$+\frac{\varepsilon}{\varepsilon-1}\cdot\frac{1}{P}\sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-1}Y^{1-\sigma}(MC^n_{t+k|t}-MC^n)$$

$$=\frac{\varepsilon}{\varepsilon-1}\frac{1}{P}\sum_{k=0}^{\infty}[(1-\theta)\beta]^k\cdot P^{\varepsilon-1}Y^{1-\sigma}MC^n-\frac{\varepsilon}{\varepsilon-1}\cdot\frac{1}{P}\sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-1}Y^{1-\sigma}MC^n\hat{p}_t+\varepsilon\frac{1}{P}\sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-1}Y^{1-\sigma}MC^n\cdot\hat{p}_{t+k}$$

$$+\frac{\varepsilon(1-\sigma)}{\varepsilon-1}\cdot\frac{1}{P}\sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-1}Y^{1-\sigma}MC^n\hat{y}_{t+k}+\frac{\varepsilon}{\varepsilon-1}\cdot\frac{1}{P}\cdot\sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-1}Y^{1-\sigma}MC^n\cdot\widehat{mc}^n_{t+k|t}$$

$$=\frac{\varepsilon}{\varepsilon-1}\cdot\frac{1}{P}\cdot P^{\varepsilon-1}\cdot Y^{1-\sigma}MC^n\cdot E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k\left[1-\hat{p}_{t-1}+(\varepsilon-1)\hat{p}_{t+k}+(1-\sigma)\hat{y}_{t+k}+\widehat{mc}^n_{t+k|t}\right]$$

LHS = RHS：

$$\cancel{P^{\varepsilon-1}Y^{1-\sigma}}E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k\cdot(1+\hat{p}_t^*-\hat{p}_{t-1}+(\varepsilon-1)\hat{p}_{t+k}+(1-\sigma)\hat{y}_{t+k})$$

$$=\cancel{\frac{\varepsilon}{\varepsilon-1}\cdot\frac{1}{P}}\cdot \cancel{P^{\varepsilon-1}\cdot Y^{1-\sigma}}\cancel{MC^n}\cdot E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k\left[1-\hat{p}_{t-1}+(\varepsilon-1)\hat{p}_{t+k}+(1-\sigma)\hat{y}_{t+k}+\widehat{mc}^n_{t+k|t}\right]$$

$P=\frac{\varepsilon}{\varepsilon-1}MC^n$（价格粘性时）

$$\Rightarrow \quad E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k\cdot(1+\hat{p}_t^* - \hat{p}_{t-1}+(\varepsilon-1)\hat{p}_{t+k}+(1-\sigma)\hat{y}_{t+k})$$

$$= E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k\cdot(1-\hat{p}_{t-1}+(\varepsilon-1)\hat{p}_{t+k}+(1-\sigma)\hat{y}_{t+k}+\hat{mc}^n_{t+k|t})$$

因此 $\hat{p}_t^*\, E_t \boxed{\sum_{k=0}^{\infty}[(1-\theta)\beta]^k} = E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k\cdot \hat{mc}^n_{t+k|t}$

等比数列求和: $a_1=1 \quad q=(1-\theta)\beta$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\frac{a_1(1-q^n)}{1-q} = \lim_{n\to\infty}\frac{1-[(1-\theta)\beta]^n}{1-(1-\theta)\beta} = \frac{1}{1-(1-\theta)\beta}$$

$$\Rightarrow \hat{p}_t^* = [1-(1-\theta)\beta]\, E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k\cdot\hat{mc}^n_{t+k|t}$$ 最优定价方程

下面找到通胀率π与价格的关系. $\hat{\pi}_t = \theta(\hat{p}_t^* - \hat{p}_{t-1})$

$$\because \quad P_t = \left(\int_0^1 P_{it}^{1-\varepsilon}di\right)^{\frac{1}{1-\varepsilon}}$$

$$= \Big[\underbrace{\int_0^\theta P_{it}^{1-\varepsilon}di}_{\text{调价}} + \underbrace{\int_\theta^1 P_{t-1}^{1-\varepsilon}di}_{\text{未调价}}\Big]^{\frac{1}{1-\varepsilon}}$$

$$P_t = \left[\theta(P_t^*)^{1-\varepsilon} + (1-\theta)P_{t-1}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

$$P_t^{1-\varepsilon} = \theta(P_t^*)^{1-\varepsilon}+(1-\theta)P_{t-1}^{1-\varepsilon} \Rightarrow \frac{P_t^{1-\varepsilon}}{P_{t-1}^{1-\varepsilon}} = \theta\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon} + (1-\theta)$$

$$\therefore \underbrace{\pi_t^{1-\varepsilon}}_{LHS} = \underbrace{\theta\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon}}_{RHS} + (1-\theta)$$

对数线性化:(对两边一阶泰勒展开)

$\frac{\partial \frac{P_t^*}{P_{t-1}}}{\partial P_t^*}$ $\quad \frac{\partial \frac{P_t^*}{P_{t-1}}}{\partial P_{t-1}}$

$$\pi^{1-\varepsilon} + (1-\varepsilon)\pi^{1-\varepsilon}\cdot\left(\frac{\pi_t-\pi}{\pi}\right) = \theta\left(\frac{P}{P}\right)^{1-\varepsilon} + \theta(1-\varepsilon)\left(\frac{P}{P}\right)^{-\varepsilon}\cdot\left[\frac{1}{P}(P_t^*-P) - \frac{P}{P^2}(P_{t-1}-P)\right] + (1-\theta)$$

$$\Rightarrow \pi^{1-\varepsilon} + (1-\varepsilon)\pi^{1-\varepsilon}\hat{\pi}_t = \cancel{\theta} + \theta(1-\varepsilon)(\hat{p}_t^* - \hat{p}_{t-1}) + (1-\cancel{\theta})$$

注：$\pi_t = \pi = \frac{P_t}{P_t} = 1$（稳态）

$\therefore \cancel{1} + (1-\cancel{\varepsilon})\hat{\pi}_t = \theta(1-\cancel{\varepsilon})(\hat{P}_t^* - \hat{P}_{t-1}) + \cancel{1}$

$\therefore \hat{\pi}_t = \theta(\hat{P}_t^* - \hat{P}_{t-1})$

$\hookrightarrow \hat{P}_t^* - \hat{P}_{t-1} = \frac{1}{\theta}\hat{\pi}_t \Rightarrow \hat{P}_{t+1}^* - \hat{P}_t = \frac{1}{\theta}\hat{\pi}_{t+1}$

最优定价方程

$$\boxed{\hat{P}_t^* = [1-(1-\theta)\beta]\, E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k\, \hat{mc}^n_{t+k|t}}$$

$$\hat{P}_t^* - \hat{P}_{t-1} = [1-(1-\theta)\beta]\, E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k\, \hat{mc}^n_{t+k|t} - \hat{P}_{t-1} = \frac{1}{\theta}\hat{\pi}_t$$

$$= [1-(1-\theta)\beta]\, E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k (\hat{mc}^n_{t+k|t} - \hat{P}_{t-1})$$

时 $\sum_{k=0}^{\infty}[(1-\theta)\beta]^k = \frac{1}{1-(1-\theta)\beta}$

$\therefore \hat{P}_{t-1} = [1-(1-\theta)\beta]\cdot E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k \cdot \hat{P}_{t-1}$

互为倒数。

$$= [1-(1-\theta)\beta]\, E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k (\boxed{\hat{mc}^r_{t+k|t} + \hat{P}_{t+k}} - \hat{P}_{t-1})$$

$$= \underbrace{[1-(1-\theta)\beta]\, E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k\, \hat{mc}^r_{t+k|t}}_{=A} + [1-(1-\theta)\beta]\, E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k (\hat{P}_{t+k} - \hat{P}_{t-1})$$

列项展开每一项 构造 π_{t+k}

$$= A + [1-(1-\theta)\beta]\, E_t \cdot \{ [(\underbrace{\hat{P}_t - \hat{P}_{t-1}}_{\hat{\pi}_t}) + [(1-\theta)\beta]\cdot(\underbrace{\hat{P}_{t+1} - \hat{P}_t}_{\hat{\pi}_{t+1}} + \underbrace{\hat{P}_t - \hat{P}_{t-1}}_{\hat{\pi}_t}) + [(1-\theta)\beta]^2(\hat{P}_{t+2} - \hat{P}_{t+1} + \hat{P}_{t+1} - \hat{P}_t + \hat{P}_t - \hat{P}_{t-1}) + \cdots \}$$

$$= A + [1-(1-\theta)\beta]\, E_t \cdot \{ \hat{\pi}_t + [(1-\theta)\beta]\cdot(\hat{\pi}_{t+1} + \hat{\pi}_t) + [(1-\theta)\beta]^2 (\hat{\pi}_{t+2} + \hat{\pi}_{t+1} + \hat{\pi}_t) + \cdots \}$$

拆开

$$= A + 1\cdot E_t \cdot \{ \hat{\pi}_t + [(1-\theta)\beta]\cdot(\hat{\pi}_{t+1} + \hat{\pi}_t) + [(1-\theta)\beta]^2 (\hat{\pi}_{t+2} + \hat{\pi}_{t+1} + \hat{\pi}_t) + \cdots \}$$
$$- (1-\theta)\beta\, E_t \cdot \{ \hat{\pi}_t + [(1-\theta)\beta]\cdot(\hat{\pi}_{t+1} + \hat{\pi}_t) + [(1-\theta)\beta]^2 (\hat{\pi}_{t+2} + \hat{\pi}_{t+1} + \hat{\pi}_t) + \cdots \}$$

乘进去

$$= A + 1\cdot E_t \cdot \{ \hat{\pi}_t + [(1-\theta)\beta]\cdot(\hat{\pi}_{t+1} + \cancel{\hat{\pi}_t}) + [(1-\theta)\beta]^2 (\hat{\pi}_{t+2} + \cancel{\hat{\pi}_{t+1} + \hat{\pi}_t}) + \cdots \}$$
$$- \cancel{(1-\theta)\beta\, \hat{\pi}_t} - \cancel{[(1-\theta)\beta]^2 (\hat{\pi}_{t+1}} + \hat{\pi}_t) - [(1-\theta)\beta]^3 (\cancel{\hat{\pi}_{t+2} + \hat{\pi}_{t+1}} + \hat{\pi}_t) - \cdots \}$$

$$= A + E_t \{ \hat{\pi}_t + (1-\theta)\beta\, \hat{\pi}_{t+1} + [(1-\theta)\beta]^2 \hat{\pi}_{t+2} + \cdots \}$$

$$= A + E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k\, \hat{\pi}_{t+k}$$

$$= [1-(1-\theta)\beta]E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k \hat{mc}^r_{t+k|t} + E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k \hat{\pi}_{t+k}$$

拆成 $k=0$ 和 $k=1\to\infty$

$$= [1-(1-\theta)\beta]\,\hat{mc}^r_{t|t} + \hat{\pi}_t \qquad k=0$$

$$+ [1-(1-\theta)\beta]E_t\sum_{k=1}^{\infty}[(1-\theta)\beta]^k \hat{mc}^r_{t+k|t} + E_t\sum_{k=1}^{\infty}[(1-\theta)\beta]^k \hat{\pi}_{t+k}$$

$k=1\to\infty$

$k'=0=k-1$

$\therefore k=k'+1$

$$= [1-(1-\theta)\beta]\,\hat{mc}^r_{t|t} + \hat{\pi}_t$$

$$+ [1-(1-\theta)\beta]E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^{k+1} \hat{mc}^r_{t+k+1|t} + E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^{k+1} \hat{\pi}_{t+k+1} \qquad k=0\to\infty$$

$$= [1-(1-\theta)\beta]\,\hat{mc}^r_{t|t} + \hat{\pi}_t$$

$$+ (1-\theta)\beta\cdot\left\{[1-(1-\theta)\beta]E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k \hat{mc}^r_{t+k+1|t} + E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k \hat{\pi}_{t+k+1}\right\}$$

$$\begin{cases} \hat{p}^*_t - \hat{p}_{t-1} = [1-(1-\theta)\beta]E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k \hat{mc}^r_{t+k|t} + E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k \hat{\pi}_{t+k} \\ \hat{p}^*_{t+1} - \hat{p}_t = [1-(1-\theta)\beta]E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k \hat{mc}^r_{t+k+1|t} + E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k \hat{\pi}_{t+k+1} \end{cases}$$

$$\hookrightarrow \hat{p}^*_t - \hat{p}_{t-1} = [1-(1-\theta)\beta]\,\hat{mc}^r_{t|t} + \hat{\pi}_t + (1-\theta)\beta\cdot E_t(\hat{p}^*_{t+1} - \hat{p}_t) \qquad \frac{1}{\theta}\hat{\pi}_{t+1}$$

$$\hat{\pi}_t = \theta(\hat{p}^*_t - \hat{p}_{t-1}) \iff \hat{p}^*_t - \hat{p}_{t-1} = \frac{1}{\theta}\hat{\pi}_t$$

$$\therefore \frac{1}{\theta}\hat{\pi}_t = [1-(1-\theta)\beta]\,\hat{mc}^r_{t|t} + \hat{\pi}_t + (1-\theta)\beta\cdot E_t\cdot\frac{1}{\theta}\hat{\pi}_{t+1}$$

两边同乘 θ : $\hat{\pi}_t = \theta[1-(1-\theta)\beta]\,\hat{mc}^r_{t|t} + \theta\hat{\pi}_t + (1-\theta)\beta\cdot E_t\hat{\pi}_{t+1}$

$$\cancel{(1-\theta)}\hat{\pi}_t = \theta[1-(1-\theta)\beta]\,\hat{mc}^r_{t|t} + \cancel{(1-\theta)}\beta\cdot E_t\hat{\pi}_{t+1}$$

$$\hat{\pi}_t = \frac{\theta[1-(1-\theta)\beta]}{1-\theta}\hat{mc}^r_{t|t} + \beta E_t\cdot\hat{\pi}_{t+1}$$

基于规模报酬不变，所以 mc 相同.

$$\Longrightarrow \boxed{\hat{\pi}_t = \frac{\theta[1-(1-\theta)\beta]}{1-\theta}\hat{mc}^r_t + \beta E_t\cdot\hat{\pi}_{t+1}}$$

通胀率