How do agents form inflation expectations? evidences from cross-moment estimation and the uncertainty

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Abstract

Density forecasts of macroeconomic variables provide one additional moment restriction, uncertainty, for testing and exploring the implications of theories about how people form expectations differently from full-information rationality benchmark. This paper first documents the persistent dispersion in inflation uncertainty of professionals and households, and how it conveys different information from the widely used proxies to uncertainty such as cross-sectional disagreement and forecast errors. Second, utilizing the panel data structure of both surveys, I provide additional reduced-form test results as well as structural estimates for each particular theory of "irrational expectation" by jointly accounting for its predictions for different moments. This is a natural extension of Coibion and Gorodnichenko (2012), which examines different moments separately. Also, motivated by the time-varying pattern of the uncertainty observed from surveys, I extend their work to allow for an alternative inflation process featuring stochastic volatility. These extensions allow me to match the joint dynamics of inflation and forecast moments in better goodness of fit. It also testifies how incorporating higher moments from survey data helps understand both the expectation formation mechanisms and inflation dynamics.

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1 Introduction

Theories on how agents form expectations in ways deviating from rational expectations (RE) have proliferated over the past decade. This has been facilitated by a welcoming recognition of the usefulness of surveys by macroeconomists in testing these theories empirically. On one hand, various theories built upon different micro-foundations produce somewhat similar macro patterns. For instance, the notion of expectation rigidity micro-founded by different theories mostly generates a sluggish response to new information. On the other hand, there are important and subtle differences in testable predictions from these theories for both individual forecasts and aggregate moments of the forecast. The seminal work by Coibion and Gorodnichenko (2012) examines predictions from different theories in a comparable manner using historical data on inflation expectations. The authors derive testable predictions for a variety of theory and exmains the impulse responses of the inflation expectations to different externally estimated shocks to inflation.

This paper extends the abovementioned effort by estimating each theory parametrically accounting for cross-moment restrictions. This is a natural step forward from reduced-form tests of rationality widely seen in the literature. With comparable estimates of different theories, one can evaluate to what extent each theory fits the observed expectation data and inflation dynamics. I evaluate each model of expectation in terms of its sensitivity of the parameter estimates with respect to four criteria (1) what moments are used for estimation; (2) the specification of the underlying process of the inflation; (3) if estimating the underlying process and expectations separately or jointly. The former basicaly recovers the inflation process only based on inflation data while the later let the expectations to provide information in estimating the inflation process. (4) If it accounts for households and forecasters' expectations equally well. Broadly speaking, this framework is closer to the essence of the rational expectation assumption in the sense that it focuses on the consistency of the model specification instead of the absolute rationality.

This is not the first research that estimates theory using cross-moment restrictions (For instance, Giacomini et al. (2020)). But one novel aspect of this paper lies in utilizing an additional moment of forecasts in estimation, i.e. the uncertainty. Existing work that studies expectation formation utilizing survey data mostly focuses on the individual and cross-sectional patterns of the mean forecast and forecast errors. But there are additional insights from the forecast uncertainty, which is only available if each surveyee is asked to assign their own perceived probabilities to a range of values of the variable. For instance, the dispersion of uncertainty across individual forecasters, as that of the average forecasts, are inconsistent with the benchmark full-information rational expectation as the latter assumes that agents agree on the generating process of the data and have the common knowledge of available information. Dynamically, across different vintages of the forecast, the revision in uncertainty is a measure of information gain or the degree of forecasting efficiency. Besides, the relationship between uncertainty with other moments such as forecast errors and disagreements provides a way to check if the surveyed expectations are consistent with certain theories of expectation formation. In addition, although some theories have similar qualitative predictions about the forecasting moments, the configurations of the model parameters consistent with the theory may not be empirically realistic.

With the framework of estimation presented above, this paper evaluates three workhorse theories on expectation formation. (1) Sticky expectation (SE). (2) Noisy information (NI). (3) Diagnostic expectations (DE). One major distinction between the first two and DE is that the former predict underreaction due to rigidity and the latter predicts overreaction. The distinction between SE and NI subtly depends on the quantitative realism of the data-implied parameters.

The findings from the paper are the following. Overall, sticky expectation (SE) augmented with inflation of stochastic volatility (SV) fits the joint dynamics of inflation expectation and inflations better than other theories, for both households and professionals. Three theories perform indistinguishably well in matching dynamics of forecast errors but differ in matching dynamics of disagreement and uncertainty. All of the three theories' parameter estimates are not particularly sensitive to the use of moments in estimation. Both SE and DE perform well in that the parameter estimate is not sensitive to if allowing for expectation data to be used to estimate the inflation process itself. In contrast, the NI coefficient estimate changes substantially if the process of inflation is jointly done, particularly for households. In terms of sensitivity to the specification of the inflation process, SE and DE have done a relatively good job for professionals but less consistent for households. NI estimates are very sensitive to the assumption about the inflation process. Compared to professional forecasters, households show a limited degree of consistency within many theories. This evidence of inconsistency adds to the evidence rejecting full-information-rationality based on reduced-form tests.

Because this paper adds uncertainty to the moments used for estimation, another contribution of this paper is that I derive explicitly prediction of various theories of expectation formation about the dynamics of uncertainty. For instance, different theories that entail rigidities in incorporating new information predict the inefficiency in forecast revisions. This paper shows that this inefficiency can not only be seen in the average forecast, but also in the forecast uncertainty. This imposes additional restrictions on parametric configurations that produce the degree of rigidity seen in the data.

On the empirical side, besides structural estimation, I also provide additional tests of the null of full-information rational expectation using the revision of uncertainty. I first replicate the results of tests based on forecast errors using microdata of households and professionals following the existing literature. Then, I undertake autoregression-based tests for revisions in mean forecasts and uncertainty, respectively. The new result from this paper is that the revision in uncertainty has a serial correlation that is not consistent with the level of forecast efficiency predicted by rational expectation. This provides an additional result that rejects the null of the rational expectation hypothesis.

In addition, using externally identified shocks to inflation, i.e. productivity shocks, oil price shocks, and monetary policy shocks, I find evidence that the degree of rigidity is shock-specific. In particular, professionals seem to be responsive to monetary policy shocks, while households are more responsive to oil price shocks. Unlike testing the null-hypothesis, the externally identified shocks allow us to look into the dynamic impulse responses of different moments of expectations to the particular shock. This can be directly compared with the predictions of the theories. I follow the framework by Coibion and Gorodnichenko (2012) while making two variations. First, I do not only examine the dynamics of forecast error, disagreement, but also the forecast uncertainty. Second, I include two monetary policy shocks, the shocks to current federal fund rates and future path of the federal funds rate. Related literature

1.1 Related Literature

This paper is related to four strands of literature. First, it is realted to a series of empirical work directly testing and evaluating various theories on expectations formation using survey data. For instance, Mankiw et al. (2003), Carroll (2003), Branch (2004). More recent examples include Coibion et al. (2018) on firms' managers. In addition to testing particular sets of theories, there is also a number of papers that show people's expectations are driven by idiosyncratic demographics, cognitive abilities and macroeconomic histories experienced(Malmendier and Nagel (2015), Das et al. (2017) and D'Acunto et al. (2019), etc.). In terms of the methodology, this paper is closest to Giacomini et al. (2020), which estimates theories of expectation formation using cross-moments restrictions. However, all of these studies simply rely upon point forecasts instead of density forecast or surveyed uncertainty. This is one theme on which this paper differs from the existing literature.

Second, Manski (2004), Delavande et al. (2011), Manski (2018) and many other papers have advocated long for eliciting probabilistic questions measuring subjective uncertainty in economic surveys. Although the initial suspicion concerning to people's ability in understanding, using and answering probabilistic questions is understandable, Bertrand and Mullainathan (2001) and other work have shown respondents have the consistent ability and willingness to assign a probability (or "percent chance") to future events. Armantier et al. (2017) have a thorough discussion on designing, experimenting and implementing the consumer expectation surveys to ensure the quality of the responses[1]. Broadly speaking, the advocators have argued that going beyond the revealed preference approach, availability to survey data provides economists with direct information on agents' expectations and helps avoids imposing arbitrary assumptions. This insight holds for not only point forecast but also and even more importantly, for uncertainty, because for any economic decision made by a risk-averse agent, not only the expectation but also the perceived risks matter a great deal.

Third, by approximating subjective uncertainty directly from density responses, this paper contributes to the literature that develops and uses a variety of measures of uncertainty, especially in the macroeconomic context. There is a long tradition of approximating uncertainty by measures that are more directly available in survey data or that can be estimated by econometric methods. For instance, Bachmann et al. (2013) use ex-ante disagreement and ex-post forecast errors computed from forecasters' surveys as proxies of uncertainty. Jurado et al. (2015) define the time-varying uncertainty as conditional volatility of the unforecastable component of a variable and estimate it using multiple macroeconomic series. Binder (2017) approximate uncertainty from rounding in survey data based on the insights from cognitive literature. Besides, the text-based approach such as Bloom (2009) constructs indices of policy

[1] Other literature includes Van der Klaauw et al. (2008) and Delavande (2014), etc.

uncertainty based on texts of newspaper reporting. Although these proxies are all meant to capture the notion of uncertainty, as shown in Section 3.2, cross-validation seems to suggest they are statistically uncorrelated or even negatively correlated.

Fourth, the literature that has been originally developed under the theme of forecast efficiency provides a framework analyzing the dynamics of uncertainty useful for the purpose of this paper. The focus of the forecasting efficiency literature is evaluating forecasters' performance and improving forecasting methodology, but it can be adapted to test the theories of expectation formation of different types of agents. This is especially relevant to this paper as I focus on the uncertainty.

The paper is organized as followed. Section 2 first sets up a common framework in which testable various theories can be compared. Also, I derive the testable predictions from these theories for individual and population uncertainty as well as other moments. Section 3 discusses the survey data used for this paper and presents both the stylized patterns and time-series regressions that test the implications of these theories. Section 4 includes results from estimating the impulse response of forecast moments to externally identified shocks. Section **??** concludes the paper and discusses the plan for the next step.

2 Theories of Expectation Formation

2.1 Definition of moments

An agent i is forming expectations about a stochastic variable y^i_{t+h}. The superscript i can be dropped if it is an aggregate (i.e. inflation) variable instead of individual-specific (i.e. household income or house value). This paper focuses on forecasting of aggregate variables, in particular, inflation. So we can simply denote the variable as y_{t+h}. [2]

Denote $f_{t+h|t}$ as agent i's h-period-ahead density forecast. $f_{i,t+h|t}$ is the conditional density of y_{t+h} given the information set $I_{i,t}$ available at time t.

$$f_{i,t+h|t} \equiv f_{i,t}(y_{t+h}|I_{i,t})$$

$I_{i,t}$ is the information set available for individual i at time t. The information set can be agent specific, thus it has subscript i. The specific content contained in I_t varies from different models of expectation. For instance, sticky expectation and rational inattention literature all assume that agents are not able to update new information instantaneously. So the information set may not contain the most recent realization of the variable of forecast y_t. [3]

[2]Only in the context of aggregate variable, it makes sense to study the population moments such as average expectations and disagreements. Studying expectations of idiosyncratic variables requires individual panel data, as well as the idiosyncratic realizations of the variable.

[3]Given the same information set available to agents, different theories may also differ in the underlying models each agent uses to form the conditional density of the variables by agent i. For instance, Patton and Timmermann (2010) finds that the disagreements are driven by not the only difference in information but also heterogeneity in prior and models. More theoretical work includes multi-prior or model uncertainty such as Hansen and Sargent (2001), Hansen and Sargent (2008), etc.

Accordingly, h-period-ahead mean forecast at t, denoted as $y_{i,t+h|t}$, is the conditional expectation of y_{t+h} by the agent i.

$$y_{i,t+h|t} \equiv E_{i,t}(y_{t+h}) = \int f_{i,t+h|t} dy_{t+h}$$

Similarly, individual forecasting variance $\sigma_{i,t+h|t}$, hereafter termed as individual uncertainty in this paper, is the conditional variance.

$$\sigma^2_{i,t+h|t} \equiv Var_{i,t}(y_{t+h})$$

Individual forecast error $FE_{i,t+h|t}$ is the difference of individual forecast at time t and ex post realized value of y_{t+h}. By definition. positive(negataive) forecast errors mean overpredict (underpredict) the variables.

$$FE_{i,t+h|t} = y_{i,t+h|t} - y_{t+h}$$

The population analogs of individual mean forecast, uncertainty and forecast errors are simply the average of the individual moments taken across agents. Denote them as $\bar{y}_{t+h|t}$, $\bar{\sigma}^2_{t+h|t}$, and $\overline{FE}_{t+h|t}$, respectively. Hereafter, they are termed as the population mean forecast, population uncertainty and population forecast error, respectively. In addition, disagreement is defined as the cross-sectional variance of mean forecasts of individual agents. Denote it as $\overline{Var}_{t+h|t}(y_{i,t+h|t})$. To simplify notation, let us directly call it $\overline{Disg}_{t+h|t}$.

To abuse the word moment in a way not completely complying with its statistical definition, I refer to the 3 individual indicators and 4 population indicators defined above as moments and they are listed in Table 1.

Finally, we assume the underlying true process of y_t is $AR(1)$ with persistence parameter $0 < \rho < 1$ and i.i.d. shock ω_t.

$$y_t = \rho y_{t-1} + \omega_t \tag{1}$$

$$\omega_t \sim N(0, \sigma^2_\omega)$$

Although we make the assumption of i.i.d. shock, the framework allows for the possibility of time-varying volatility, i.e. the variance of ω_t is thus $\sigma^2_{t,\omega}$ with time subscript.

2.2 Benchmark of full-information rational expectation(FIRE)

In the FIRE benchmark, it is assumed that all agents perfectly observe y_t at time t and understand the true process of y. Therefore, the individual forecast is $\rho^h y_t$, which is shared by all agents. Therefore, it is also equal to the average forecast.

Both individual and population forecast errors are simply the realized shocks between $t+1$ to $t+h$.

$$\overline{FE}^*_{i,t+h|t} = -\sum_{k=0}^{h-1} \rho^k \omega_{t+h-k} \tag{2}$$

I use superscript of $*$ to denote all the moments according to FIRE. It is easy to see that the forecast error is orthogonal to information available till time t. This provides a well known null hypothesis of FIRE.

The second implication from FIRE here is that forecast errors of non-overlapping horizon are not correlated. (Equation 3). For instance, forecast error at time t and that at time $t+h$ or further are not serially correlated. This is not the case within h periods as the realized shocks in overlapping periods enter both forecast errors. These FE-based restrictions of FIRE provide the foundations for the tests used in Section 3.3.

$$Cov(\overline{FE}^*_{t+h|t}, \overline{FE}^*_{t+s+h|t+s}) = 0 \quad \forall s \geq h \tag{3}$$

Concerning uncertainty, the first simple implication by FIRE is that all individual shares the same degree of uncertainty. The uncertainty about future y simply comes from uncertainty about unrealized shocks between t and $t+h$. With the same model in mind (Equation 1) and the same information y_t, everyone's uncertainty is equal to the weighted sum of the future volatility before its realization (Equation 4). In FIRE, there are neither disagreements about mean, nor disagreements about the uncertainty [4].

$$\bar{\sigma}^{*2}_{t+h|t} = \sum_{s=1}^{h} \rho^{2s}\sigma^2_\omega \tag{4}$$

The time series behavior of h-year-ahead uncertainty, i.e. $\sigma^2_{t+h|t}$, $\sigma^2_{t+h+1|t+1}$, etc, depends on the true process of y. Specifically, it depends if σ^2_ω is time-varying. If time-invariant, h-period-ahead uncertainty is simply as constant. In baseline case, I make such an assumption, in general it may not be true. In the later section, I make the alternative assumptions of the inflation process allowing for stochastic volatility.[5]

The testable implication of rationality lies in the revision of uncertainty. Hereafter, we refer revision (instead of change) as the difference of moments across vintages of the forecast with the fixed terminate date of realization. For instance, the difference between the uncertainty about y_{t+h} at time t and the uncertainty about y_{t+h} at time $t-1$.

More generally, denote the forecast of y_{t+h} over the horizon of $h-k$ as $y_{t+h|t+k} \quad \forall k = 0, 1...h$. Then the uncertainty at different points of the k is the following.

$$\bar{\sigma}^{*2}_{t+h|t+k} = \sum_{s=1}^{h-k} \rho^{2s}\sigma^2_\omega \tag{5}$$

As the forecaster approaches $t+h$ (k approaches to h), there is an unambiguous reduction in uncertainty (or efficiency gain in the forecasting literature) as more and

[4] This is the same to Jurado et al. (2015)'s terminology.

[5] For example, Justiniano and Primiceri (2008), Vavra (2013) on time-varying volatility of inflation.

more shocks have realized. Equivalently, revision is always negative and should be exactly equal to the variance of the realized shocks according to FIRE.

Moving from t to $t+1$, for instance, revision in uncertainty can be also expressed as a function of the previous revision.

$$\begin{aligned}\bar{\sigma}^{*2}_{t+h|t+1} - \bar{\sigma}^{*2}_{t+h|t} &= -\rho^{2h}\sigma^2_\omega \\ &= \rho(\bar{\sigma}^{*2}_{t+h|t} - \bar{\sigma}^{*2}_{t+h|t-1})\end{aligned} \tag{6}$$

Lastly, FIRE has predictions about disagreements. As agents perfect update the same information, there is no disagreement at any point of the time.

$$\overline{Disg}^{*}_{t+h|t} = 0 \quad \forall t \tag{7}$$

Another simple result is that both disagreements and uncertainty do not respond to realized shocks at any point of the time. But this does not differentiate FIRE from other models.

2.3 Sticky Expectation (SE)

The theory of sticky expectation (Mankiw and Reis (2002), Carroll (2003) etc.), regardless of various micro-foundations, builds upon the assumption that agents do not update information instantaneously as they do in FIRE. One tractable assumption is that there is a homogenous Poisson rate λ of updating among the population. Specifically, at any point of time t, each agent learns about the up-to-date realization of y_t with the probability of λ; otherwise, it forms the expectation based on the most recent up-to-date realization of $y_{t-\tau}$, where τ is the time experienced since the last update.

Denote the mean forecast of a non-updater since $t-\tau$ as $y_{i,t+h|t-\tau}$ since her forecast conditions upon the information up till $t-\tau$.

$$y_{i,t+h|t-\tau} = \rho^{h+\tau} y_{t-\tau} \tag{8}$$

Now her information set is not up to date, the uncertainty to a non-updater is higher than an updater and it increases with the duration of non-updating τ.

$$\sigma^2_{i,t|t-\tau} = \sum_{s=1}^{h+\tau} \rho^{2s}\sigma^2_\omega \tag{9}$$

In FIRE, updating at each period t resolves only the uncertainty about the shocks that have just realized in t. In contrast, in SE each updating resolves the uncertainty about all the realized shocks since the last update. From $t-\tau$ to t, the revision in uncertainty is

$$\sigma^2_{i,t+1|t} - \sigma^2_{i,t+1|t-\tau} = \rho^2\sigma^2_\omega - \sum_{s=1}^{\tau+1} \rho^{2s}\sigma^2_\omega = -\sum_{s=2}^{\tau+1} \rho^{2s}\sigma^2_\omega \tag{10}$$

FIRE basically assumes $\tau = 1$ for all the agents and all the time, namely all agents' last update takes place in the previous period. So setting $\tau = 1$ in the above equation gives the reduction in uncertainty in FIRE. The reduction in uncertainty is greater in SE for any $\tau > 1$ than FIRE.

In the individual level, the key difference between FIRE and SE is that the later does not reduce uncertainty as efficiently as in the former primarily because of the rigidity incorporating new information. In the same time, note that the rigidity in updating according to SE cannot be systematically observable in the individual level, both in terms of forecasts errors and uncertainty. This is because the behaviors of each individual forecast specifically depend on if she updates or not in that period.

Relying upon the law of large numbers, one can derive testable predictions about population moments that allow us to conduct tests of sticky expectation and recover rigidity parameter λ. [6]

One well-known prediction from SE is that the average forecast is a weighted average of update-to-date rational expectation and lagged average expectation as reproduced below. [7] It can be also expressed as a weighted average to all the past realizations of y. Setting $\lambda = 1$, then the SE collapses to FIRE and the average forecast is equal to y's long-run mean of zero.

$$\begin{aligned} \bar{y}_{t+h|t} &= \lambda \underbrace{y^*_{t+h|t}}_{\text{rational expectation at t}} + (1-\lambda) \underbrace{\bar{y}_{t+h|t-1}}_{\text{average forecast at } t-1} \\ &= \lambda \sum_{\tau=0}^{\infty} (1-\lambda)^{\tau} y^*_{t+h|t-\tau} \\ &= \lambda \sum_{\tau=0}^{\infty} (1-\lambda)^{\tau} \rho^{h+\tau} y_{t-\tau} \end{aligned} \tag{11}$$

It is easy to show that the average forecast errors are serially correlated (Equation 12). For double-checking, setting $\lambda = 1$, SE collapses to FIRE, as seen in Equation 2, in which there is no serial correlation between forecast errors and it fully responds to newly realized shocks at time t.

$$\overline{FE}_{t+h|t} = (1-\lambda)\overline{FE}_{t+h|t-1} - \lambda \sum_{k=1}^{h} \rho^k \omega_{t+k} \tag{12}$$

Similarly, the inefficiency of reducing uncertainty in SE takes the following form in the aggregate level. Average uncertainty at any point of time is now a weighted average of uncertainty to agents whose last updates have taken place in different periods of past.

[6] Carroll (2003) is a good example of this for households.

[7] See Coibion and Gorodnichenko (2012) or appendix of this paper for detailed steps.

$$
\begin{aligned}
\bar{\sigma}^2_{t+h|t} &= \sum_{\tau=0}^{+\infty} \underbrace{\lambda(1-\lambda)^\tau}_{\text{fraction of non-updater until } t-\tau} \underbrace{\sigma^2_{t+h|t-\tau}}_{\text{uncertainty of most recent update at } t-\tau} \\
&= \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^\tau \sum_{s=0}^{h+\tau} \rho^{2s}\sigma^2_\omega
\end{aligned} \tag{13}
$$

Since not all agents incorporate the recently realized shocks, the revision in average uncertainty exhibits serial correlation described in Equation 14. It is a weighted average of the resolution of uncertainty from the most recent shocks and its lagged counterpart.

$$
\bar{\sigma}^2_{t+h|t+1} - \bar{\sigma}^2_{t+h|t} = (1-\lambda)(\bar{\sigma}^2_{t+h|t} - \bar{\sigma}^2_{t+h|t-1}) - \lambda\rho^{2h}\sigma^2 \tag{14}
$$

In particular, the second component is the information gain from the most recent realization of the shock underweighted by $\lambda < 1$. The first component is the inefficiency sourced from the stickiness of updating. The higher rigidity (lower λ), the smaller the efficiency gain or uncertainty reduction compared to in FIRE.

Lastly, SE also predicts non-zero disagreements and sluggish adjustment compared to FIRE. This is because of different lags in updating across populations.

$$
\overline{Disg}_{t+h|t} = \lambda \sum_{\tau=0}^{\infty} (1-\lambda)^\tau (y_{t+h|t-\tau} - \bar{y}_{t+h|t}))^2 \tag{15}
$$

From time t to $t+1$, the change in disagreements comes from two sources. One is newly realized shock at time $t+1$. The other component is from people who did not update at time t and update at time $t+1$.

Coibion and Gorodnichenko (2012) derive the impulse response of dispersion at time $t+k$ to a shock that realized at t. Disagreements increase after realization of the shock and gradually returns to its steady-state level.

$$
\rho^{2(h+k)}(1-\lambda^{k+1})\lambda^{k+1}\omega^2_t \tag{16}
$$

2.3.1 Summary of predictions of SE

- Population's mean forecast partially responds to shocks and with lags.
- Forecast errors are serially correlated.
- Population disagreements increase in response to new shocks and return to steady-state level gradually.
- Population average uncertainty revision under-reacts to the volatility of the shocks.

2.4 Noisy information(NI)

A class of models (Lucas Jr (1972), Sims (2003), Woodford (2001), etc.), noisy information(NI) hereafter, describes the expectation formation as a process of extracting or filtering true variable y_t from a sequence of realized signals. The starting assumption is that the agent cannot observe the true variable perfectly. Unlike SE, it is assumed that agents keep track of the realizations of signals instantaneously all the time.

We assume agent i observe two signals s^{pb} and s_i^{pr}, with s^{pb} being public signal common to all agents, and s_i^{pr} private signals being individual specific with subscript i. The generating process of two signals is assumed to be the following.

$$\begin{aligned} s_t^{pb} &= y_t + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\epsilon^2) \\ s_{i,t}^{pr} &= y_t + \xi_{i,t} \quad \xi_{i,t} \sim N(0, \sigma_\epsilon^2) \end{aligned} \tag{17}$$

Stacking the two signals into one vector $s_{i,t} = [s_t^{pb}, s_{i,t}^{pr}]'$ and $v_{i,t} = [\epsilon_t, \xi_{i,t}]'$, the equations above can be rewritten as

$$\begin{aligned} s_{i,t} &= Hy_t + v_{i,t} \\ &\text{where } H = [1, 1]' \end{aligned} \tag{18}$$

Now any agent trying to forecast future y has to form her expectation of the contemporaneous y. Denote it as $y_{i,t|t}$, which needs to be inferred from the signals to agent i. The agent's best h-period ahead forecast is simply iterated h periods forward based on the AR(1) process and it is equal to $\rho^h y_{i,t|t}$. This is the same as FIRE.

What is different from FIRE is that the agent makes her best guess of y_t using Kalman filtering at time t. Specifically, the mean forecast of individual i is the posterior mean based on her prior and realized signals $s_{i,t}$.

$$\begin{aligned} y_{i,t|t} &= \underbrace{y_{i,t|t-1}}_{\text{prior}} + P \underbrace{(s_{i,t|t} - s_{i,t|t-1})}_{\text{innovations to signals}} \\ &= (1 - PH)y_{t|t-1} + Ps_{i,t} \\ &= (1 - PH)y_{t|t-1} + PHy_t + Pv_{i,t} \end{aligned} \tag{19}$$

where the Kalman gain P is a vector of size of two that determines the degrees of reaction to signals.

$$P = [P_\epsilon, P_\xi] = \Sigma^y_{i,t|t-1} H (H' \Sigma^y_{i,t|t-1} H + \Sigma^v)^{-1} \tag{20}$$

$\Sigma^y_{i,t|t-1}$ is the variance of y_t based on prior belief. Here the capital case of σ is used to be consistent with vector operations.

$$\Sigma^v = \begin{bmatrix} \sigma_\epsilon^2 & 0 \\ 0 & \sigma_\xi^2 \end{bmatrix} \tag{21}$$

Individual forecast partially responds to new signals as $PH < 1$. $PH = 1$ is a special case when both signals are perfect thus $\Sigma^v = 0$, then the formula collapses to FIRE.

A comparable parameter with $1 - \lambda$ in SE that governs rigidity in NI is $1 - PH$. It is a function of previous period uncertainty about y_t and noisiness of the signals determined by Σ^v. Note P is time-variant as the variance is updated by the agent each period. The time-varying nature of rigidity will be discussed later. For now, I drop time t from P to avoid clustering.

The under-reaction to news generate serial correlation of forecast errors with the coefficient being $\rho^2(1 - PH)$. it does not only depend on PH, but also the forecast horizon h. As $\rho < 1$, the rigidity declines with a longer horizon. In this regard, the rigidities implied by NI and SE are different because the form is fixed and the later is horizon dependent.

What differentiates average forecast from individual's is the role played by private signals. On average, private signals cancel out across agents, therefore, only public signals enter the average forecast, thus, average forecast errors (Equation 22).

$$\begin{aligned}
\bar{y}_{t+h|t} &= \rho^h[(1 - PH)\underbrace{\bar{y}_{t+h|t-1}}_{\text{Average prior}} + P\underbrace{\bar{s}_t}_{\text{Average Signals}}] \\
&= (1 - PH)\bar{y}_{t+h|t-1} + P[\epsilon_t, 0]' \\
&= (1 - PH)\bar{y}_{t+h|t-1} + P\epsilon_t
\end{aligned} \tag{22}$$

Kalman filtering also updates the variance according to the rule of normal updating. The posterior variance at time t is a linear function of uncertainty in the previous period and variance of signals.

$$\Sigma^y_{i,t|t} = \Sigma^y_{i,t|t-1} - \Sigma^y_{i,t|t-1}H'(H\Sigma^y_{i,t-1}H' + \Sigma^v)^{-1}H\Sigma^y_{i,t|t-1} \tag{23}$$

This directly gives the revision in uncertainty from time $t-1$ to t. The newly arrived information, although noisy, still brings about information gains, thus leading to an unambiguously drop in uncertainty. But due to the signal is not perfect, i.e. $\Sigma^v \neq 0$, there is inefficiency in reducing uncertainty compared to FIRE.

$$\Sigma^y_{i,t|t} - \Sigma^y_{i,t|t-1} = -\Sigma^y_{i,t|t-1}H'(H\Sigma^y_{i,t-1}H' + \Sigma^v)^{-1}H\Sigma^y_{i,t|t-1} < 0 \tag{24}$$

In order to be directly comparable with the revision in uncertainty in FIRE (Equation 6) and SE (Equation 14), the nowcasting uncertainty about y_t needs to be converted to h-period-ahead forecasting uncertainty. This is simply to add the nowcasting uncertainty discounted by ρ^{2h} to the uncertainty about future unrealized shocks.

$$\Sigma_{i,t+h|t} = \rho^{2h}\Sigma_{i,t|t} + \sum_{s=1}^{h}\rho^{2s}\sigma^2_\omega \tag{25}$$

As a result, the revision in h-period-ahead uncertainty from $t-1$ to t only partially reacts to the resolution of uncertainty from newly realized shock ω_t in the past period.

Assuming the nosiness of the private signal is equal across agents, then the dynamic of population uncertainty is the same to individual counterpart $\bar{\Sigma}_{t+h|t}$, since in NI, every agent is faced with the same filtering problem.

Figure 1 illustrates the Kalman filtering problem over the 10-period horizon from to t to $t+10$ following a one-time shock of one standard deviation of ω_t. For illustration purpose, I assume that both private and public signals have an equal degree of noisiness and their standard deviation is equal to the unconditional standard deviation of y, i.e. $\sqrt{\frac{\sigma^2_\omega}{1-\rho^2}}$. A particular draw of public and private signals from their respective conditional distributions given y_t are plotted.

From the figure on the top left, the agent's nowcasting $y_{t+k|t+k}$, namely her best real-time guess based on prior and the realized signal at each period roughly tracks the y_{t+k} and responds partially to signals in each period. Correspondingly, her forecasting of y at $t+10$ also centers around the true value.

FIRE implies agent learns realized y in each period perfectly. Thus the nowcasting remains zero throughout the whole horizon. Nowcasting uncertainty picks up in the realization of shock and gradually declines as more and more signals are learned. This comes from the information gains from Kalman filtering.

In the first sight, the sharp rise in nowcasting uncertainty at the beginning of the period being only humbly reflected in the forecasting uncertainty seems counterintuitive. Actually, this is due to nowcasting uncertainty is discounted by the factor of $\rho^{2(h-k)}$. (Equation 25). It is also interesting that for the particular configurations of the noisiness of the signals, the drop in forecasting uncertainty from NI is not as inefficient as one may have expected compared to FIRE. This has important quantitative implications for NI models. In particular, the nosiness of the signals needs to be consistent with the degree of rigidity. I will return this point in the next section.

NI also predicts non-zero disagreement in the presence of private signals. The behavior of disagreement across agents come from the difference in the entire histories of realized private signals. Specifically, it is equal to the following.

$$\overline{Disg}_{t+h|t} = (1-PH)\overline{Disg}_{t+h|t-1} + \rho^{2h}P^2_\xi\sigma^2_\xi \tag{26}$$

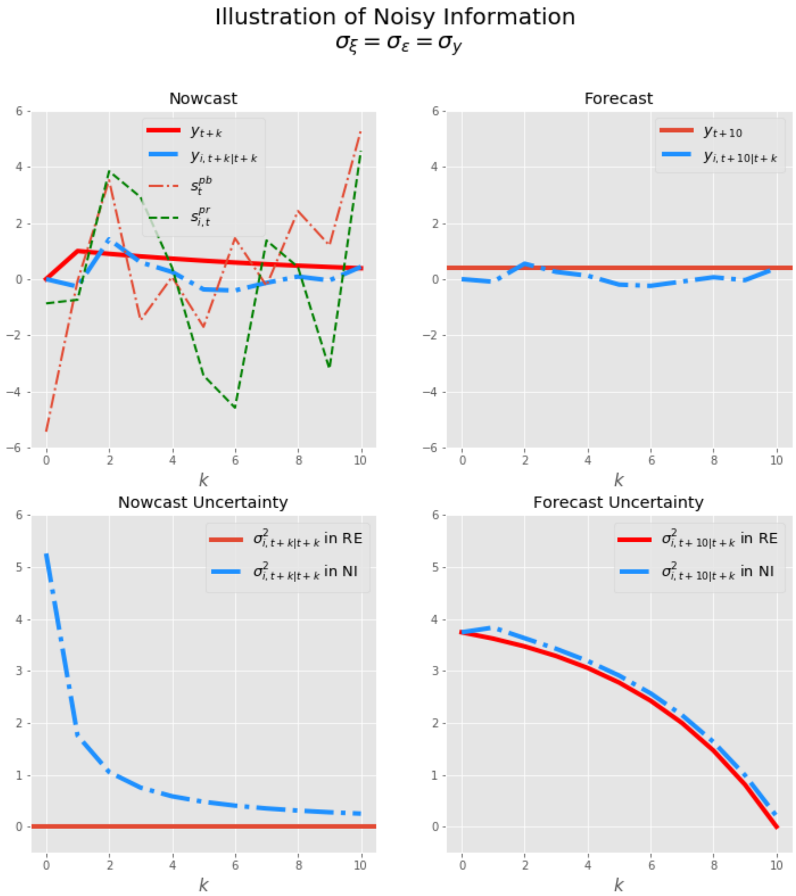
The implications for disagreement are straightforward. First, the serial correlation between disagreement is proportional to the rigidity parameter $(1-PH)$ and, similar to SE, it increases with the rigidity parameter P in this model. Second, the disagreement decrease with the forecast horizon due to discounting. Third, the disagreement depends on noisiness private signals, but not on that of public signals and the variance of the true variable y.

2.4.1 Summary of predictions from noisy information

- Individual and population expectation adjust in each period, but only partially adjusts to new information.
- Individual and population uncertainty unambiguously drop each period as one approaches the period of realization.

Table 1: Definition and Notation of Moments

Individual Moments	Population Moments
Mean forecast: $y_{i,t+h\|t}$	Average forecast: $\bar{y}_{t+h\|t}$
Forecast error: $FE_{i,t+h\|t}$	Average forecast error: $\overline{FE}_{t+h\|t}$
Uncertainty: $\sigma^2_{i,t+h\|t}$	Average uncertainty: $\bar{\sigma}^2_{t+h\|t}$
	Disagreements: $\overline{Disg}_{t+h\|t}$



Note: the top two figures plot the nowcasting of y_{t+k} at time $t+k$ and forecasting of y_{t+h} at time $t+k$ through Kalman filtering and through FIRE after a one-time shock of size of one unit at time t. The nowcasting and forecasting uncertainty are correspondingly plotted in the bottom. Both private signal and public signal have the conditional variance equal to long-run variance of y, i.e. $\sigma_\epsilon = \sigma_\xi = \sigma_y$. The plotted signals are are a pair of particular draw from their respective conditional distribution.

Figure 1: Nowcasting and Forecasting in Noisy Information

- Population disagreement rises in each period as the time approaches the period of realization. Disagreements will never be zero.

2.5 Diagnostic Expectations (DE)

Different from the previous two theories, diagnostic expectation (Bordalo et al. (2018)) is developed in the literature to account for extrapolative expectations that feature overreactions to the news. Both SE and NI deviate from FIRE in terms of the information set available to the agents, while DE deviates from FIRE in terms of the processing of an otherwise fully updated information set.

Omitting the micro foundation, the following equation captures the essence of DE's mechanism. Each individual i's h-period-ahead forecast consists of two components. The first component can be considered as a rational forecast based on the fully updated y_t. The second component corresponds to the potential overreaction to the unexpected shock to inflation at time t, i.e. the forecast error. The degree of overreaction is governed by the parameter θ. The premise of DE models is that $\theta \geq 0$. The forecast collapses to the FIRE when $\theta = 0$. Any $\theta > 0$ implies cases in which the agent revises her forecast overly to the realized forecast error in the same direction.

$$\bar{y}_{i,t+h|t} = \rho^h y_t + \theta_i(\rho^h y_t - \bar{y}_{i,t+h|t-1}) \tag{27}$$

I allow θ to be different across different agents, therefore, I add the subscript i to the parameter. Since agents are equally informed about the realizations of the variable, the only room for disagreement to rise is either difference in the initial information set or the degree of overreaction differs across people. To capture this, I assume θ_i to follow a normal distribution across the population with a variance of σ_ϵ. It will be estimated along with θ_i.

Finally as to the uncertainty, since the mechanism of extrapolation in DE does not change the agent's perceived distribution of the future shocks, benchmark DE theory predicts the forecast uncertainty to remain the same as the rational expectation.

2.6 Comparing Different Theories

Taking stock, both SE and NI predict rigidity in incorporating the arrival of new information. In the former context, the rigidity comes from the simple non-updating of the most realization of the variable. In the latter case, the rigidity comes from a partial reaction to new information due to the nosiness of the signals. In addition to forecast and disagreements, the rigidity is also reflected in the dynamics of uncertainty. Uncertainty in SE does not resolve as rapidly as in FIRE because the newly realized shocks do not get updated by some of the agents. While in NI, the drop in uncertainty is dampened because there is additional uncertainty associated with the real-time realization of the variable.

Figure 2 illustrates the impulse response of population moments to a hypothetical one-time shock of size one unit at time t according to different theories. Still, I maintain the assumption in Figure 1 that the standard deviation of private and public signals are both equal to one unconditional standard deviation of y. And the λ is set to be 0.75, implying on average an update interval of four quarters.

Both FE and disagreements from SE and NI exhibit sluggish adjustment following the shock as we have anticipated. Both pick up in the time of the shock and gradually declines over the horizon as the forecaster approaches terminate date $t + 10$. The difference is that in NI, both the forecast and forecast errors move up and downs around the smooth version of the SE since signals are not perfect.

The dynamic of disagreements and uncertainty, however, illustrate one parametric difference between SE and NI. In particular, the initial increase in disagreements is higher for NI compared to SE for the particular parameter values. This implies that the noise of the private signal that drives the disagreements in NI exceeds that the extent to which the agents do not simultaneously update in SE. [8]

For the same parameters, however, the inefficiency seen in dynamics of uncertainty seems to be much more substantial for SE than in NI. Overall, the uncertainty drops almost as efficiently as in FIRE. The difference between SE and the FIRE benchmark is more notable. This is the graphical illustration of Equation 14.

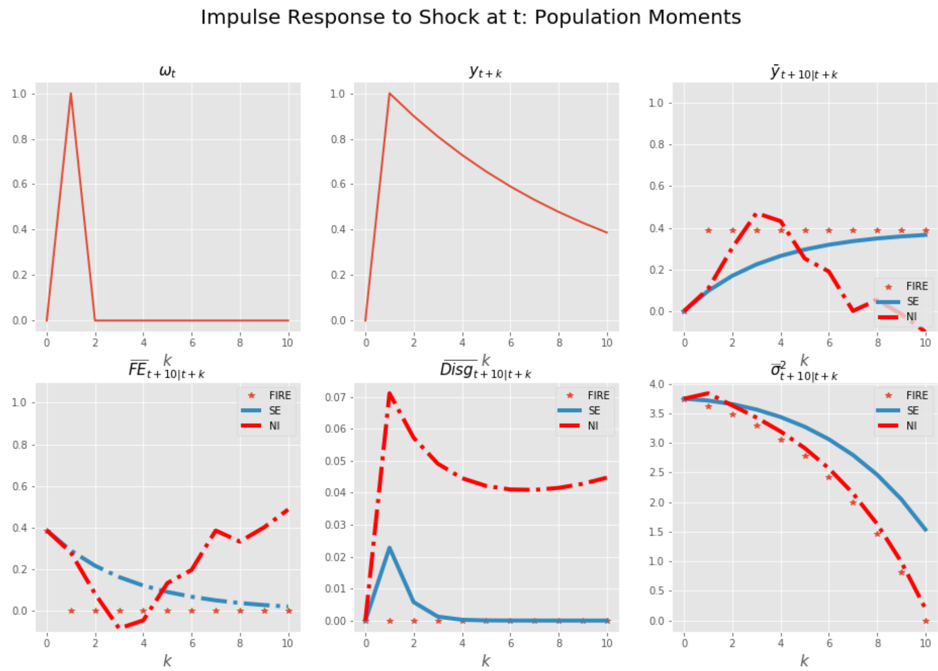
In order to understand why the rigidity seen from uncertainty is de facto much lower in NI that in SE for a sensible values of the noisiness of signals, I plot the implied rigidity from two models in Figure 3 for three different values of nosiness of signals in NI while fixing rigidity in SE to be $1 - \lambda = 0.75$. In particular, the noisiness range from 0.1, 1 and 10 times of the long-run variance of y.

As it is made clear by Figure 3, one notable difference between the two models is that SE's rigidity is exogenously fixed while NI's rigidity is endogenously determined by the horizon thus time-varying. In particular, the implied rigidity from NI in the time of the shock, given there is substantial information gain from Kalman filtering in the same period, is quite low. For a noisiness of both signals as 10 times as high of the unconditional standard deviation of y, the rigidity only reaches the fixed rigidity in SE till the 8th period. But since by that time, the initial information provided by the shock at time t has been mostly absorbed by NI agent, so in fact, the uncertainty revision in NI is quite efficient compared to SE models. This echoes our earlier observation in Figure 1.

In order to understand why the rigidity seen from uncertainty is de facto much lower in NI that in SE for a sensible values of the noisiness of signals, I plot the implied rigidity from two models in Figure 3 for three different values of nosiness of signals in NI while fixing rigidity in SE to be $1 - \lambda = 0.75$. In particular, the noisiness range from 0.1, 1 and 10 times of the long-run variance of y.

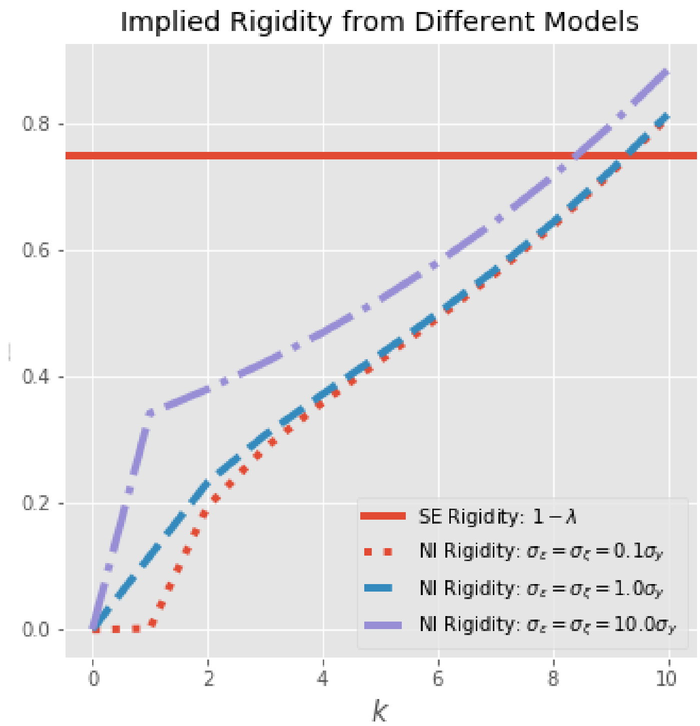
In summary, although NI and SE both predict the similar qualitative predictions of rigidity when it comes to the behavior of FE and disagreements, de facto, the efficiency of forecasting from NI is higher than SE. The only additional uncertainty in NI to that in FIRE is the nowcasting uncertainty, which is discounted in the future. While in SE, the uncertainty does come from the lagged update of information, which involve uncertainties about all the shocks that are not yet updated in agents' information set.

[8] Another subtle point is that if the disagreements jump the highest in the time of the shock turns out to depend on the value of λ. In the fixed horizon forecasting, $\lambda = 0.5$ is the maximizer of disagreements.



Note: figure 1-6 plot the dynamics of a one-time shock to inflation ω_{t+k}, the realized value of y_{t+k}, average forecast of $\bar{y}_{t+10|t+k}$, forecasting errors $\overline{FE}_{t+10|t+k}$, disagreements $\overline{Disg}_{t+10|t+k}$, and uncertainty $\bar{\sigma}^2_{t+10|t+k}$ to a one-time shock of unit one at time t, i.e. $\omega_t = 1$. For NI, public and private signals' variance is equal to long-run variance of y, i.e. $\sigma_\epsilon = \sigma_\xi = \sigma_\xi$. For SE, $\lambda = 0.25$.

Figure 2: Impulse responses of population moments to a shock to inflation



Note: implied rigidity is $1 - \lambda$ in SE and $(1 - P_{t+k}H)$ in NI. Rigidity from NI are plotted for three values of noisiness of signals, i.e. 0.1, 1 and 10 times of the unconditional variance of y.

Figure 3: Rigidity from Two Models

2.7 Stochastic volatility model of inflation

This section considers an alternative inflation process for which the volatility of shocks to the inflation are stochastic. In particular, assume that the inflation has unobserved permanent and transitory components.

$$\begin{aligned} y_t &= \tau_t + \eta_t, \quad \text{where } \eta_t = \sigma_{\eta,t}\xi_{\eta,t} \\ \tau_t &= \tau_{t-1} + \epsilon_t, \quad \text{where } \epsilon_t = \sigma_{\epsilon,t}\xi_{\epsilon,t} \\ \log\sigma^2_{\eta,t} &= \log\sigma^2_{\eta,t-1} + \mu_{\eta,t} \\ \log\sigma^2_{\epsilon,t} &= \log\sigma^2_{\epsilon,t-1} + \mu_{\epsilon,t} \end{aligned} \tag{28}$$

The distributions of shocks to levels of the components and their volatilities are, respectively, the following.

$$\begin{aligned} \xi_t &= [\xi_{\eta,t}, \xi_{\epsilon,t}] \sim N(0, I_2) \\ \mu_t &= [\mu_{\eta,t}, \mu_{\epsilon,t}]' \sim N(0, \gamma I_2) \end{aligned} \tag{29}$$

The only parameter of the model is γ, which determines the time-varying volatilities.

Consider the rational expectation as the benchmark, again. At the point of time t, the RE agent sees the realization of all stochastic variables above with subscript t, $t-1$, etc, including y_t, τ_t, η_t, $\sigma_{\eta,t}$, $\sigma_{\epsilon,t}$ and their realizations in the whose past. Again, $*$ stands for FIRE benchmark.

$$\overline{y}^*_{t+h|t} \equiv y^*_{t+h|i,t} = E^*_{i,t}(y_{t+h}|I_{i,t}) = \theta_t \tag{30}$$

Forecast error is simply the cumulated sum of unrealized shocks from t to $t+h$, which is

$$\overline{FE}^*_{t+h|t} \equiv FE^*_{t+h|i,t} = \sum_{s=1}^{h}(\eta_{t+s} + \epsilon_{t+s}) \tag{31}$$

h-step-ahead conditional variance, namely the uncertainty is

$$
\begin{aligned}
\overline{Var}^*_{t+h|t} \equiv Var^*_{t+h|i,t} &= \sum_{k=1}^{h} E_{i,t}(\sigma^2_{\eta,t+k}) + E_{i,t}(\sigma^2_{\epsilon,t+k}) \\
&= \sum_{k=1}^{h} E_{i,t}(exp^{\log\sigma^2_{\eta,t}+\sum_{k=1}^{h}\mu_{\eta,t+k}}) + E_{i,t}(exp^{\log\sigma^2_{\epsilon,t}+\sum_{f=1}^{h}\mu_{\epsilon,t+f}}) \\
&= \sum_{k=1}^{h} \sigma^2_{\eta,t} E_{i,t}(exp^{\sum_{k=1}^{h}\mu_{t+h,\eta}}) + \sigma^2_{\epsilon,t} E_{i,t}(exp^{\sum_{f=1}^{h}\mu_{\epsilon,t+f}}) \\
&= \sum_{k=1}^{h} \sigma^2_{\eta,t} exp^{E_{i,t}(\sum_{k=1}^{h}\mu_{t+k,\eta})-0.5Var_{i,t}(\sum_{k=1}^{h}\mu_{t+k,\eta})} + \sigma^2_{\epsilon,t} E_{i,t}(exp^{\sum_{f=1}^{h}\mu_{\epsilon,t+f}}) \\
&= \sigma^2_{\eta,t} \sum_{k=1}^{h} exp^{-0.5k\gamma_\eta} + \sigma^2_{\epsilon,t} exp^{-0.5h\gamma_\epsilon}
\end{aligned} \tag{32}
$$

One could immediately see that now the volatility is stochastic at any point of the time.

For instance, set $h = 1$, the conditional volatility for the 1-step-ahead inflation is

$$
Var^*_{t+1|i,t} = exp^{-0.5\gamma_\eta}\sigma^2_{\eta,t} + exp^{-0.5\gamma_\epsilon}\sigma^2_{\epsilon,t} \tag{33}
$$

Disgreement is zero across agents in RE.

$$
\overline{Disg}^*_{t+h|t} = 0 \tag{34}
$$

Under the sticky expectation (SE), an agent whose most recent up-do-date update happened at $t-\tau$, thus she sees all the realizations of stochastic variables up to $t-\tau$, including $y_{t-\tau}$, $\tau_{t-\tau}$, $\eta_{t-\tau}$, $\sigma_{\eta,t-\tau}$, $\sigma_{\epsilon,t-\tau}$.

Her forecast is the permanent component that realized at time $t-\tau$.

$$
y_{t+h|i,t-\tau} = \theta_{t-\tau} \tag{35}
$$

Her forecast uncertainty is

$$
Var_{t+h|i,t-\tau} = \sigma^2_{\eta,t-\tau} \sum_{k=1}^{h+\tau} exp^{-0.5k\gamma_\eta} + \sigma^2_{\epsilon,t-\tau} exp^{-0.5(h+\tau)\gamma_\epsilon} \tag{36}
$$

The population average of the two are, respectively, a weighted average of people whose the most update was in $t, t-1...t-\tau, t-\infty$, respectively.

$$
\begin{aligned}
\overline{y}^{se}_{t+h|t} &= \sum_{\tau=0}^{\infty}(1-\lambda)^\tau \lambda y_{t+h|t-\tau} \\
&= \sum_{\tau=0}^{\infty}(1-\lambda)^\tau \lambda \theta_{t-\tau}
\end{aligned} \tag{37}
$$

$$
\begin{aligned}
\overline{Var}^{se}_{t+h|t} &= \sum_{\tau=0}^{\infty}(1-\lambda)^{\tau}\lambda Var_{t+h|t-\tau} \\
&= \sum_{\tau=0}^{\infty}(1-\lambda)^{\tau}\lambda[\sigma^2_{\eta,t-\tau}\sum_{k=1}^{h+\tau} exp^{-0.5k\gamma_{\eta}} + \sigma^2_{\epsilon,t-\tau}exp^{-0.5(h+\tau)\gamma_{\epsilon}}]
\end{aligned} \tag{38}
$$

Both forecast errors $\overline{FE}_{t+h|t}$ and disagreements takes similar form to that in AR process with time-invariant volatility.

$$
\overline{FE}^{se}_{t+h|t} = \sum_{\tau=0}^{\infty}(1-\lambda)^{\tau}\lambda FE^{*}_{t+h|t-\tau} \quad = \sum_{\tau=0}^{\infty}(1-\lambda)^{\tau}\lambda\sum_{s=1}^{\tau+h}(\theta_{t+s}+\epsilon_{t+s}) \tag{39}
$$

The disagreement is the following.

$$
\begin{aligned}
\overline{Disg}^{se}_{t+h|t} &= \sum_{\tau=0}^{\infty}(1-\lambda)^{2\tau}\lambda^2(y_{t+h|t-\tau} - \overline{y}^{se}_{t+h|t})^2 \\
&= \sum_{\tau=0}^{\infty}(1-\lambda)^{2\tau}\lambda^2(\theta_{t-\tau} - \overline{y}^{se}_{t+h|t})^2 \\
&= \sum_{\tau=0}^{\infty}(1-\lambda)^{2\tau}\lambda^2\{\theta_{t-\tau} - \sum_{\tau=0}^{\infty}(1-\lambda)^{\tau}\lambda\theta_{t-\tau}\}^2
\end{aligned} \tag{40}
$$

Under nosiy information (NI), the agent at time t needs to recover the real-time permanent component θ_t to make the best forecast for future y_{t+h} using nosiy signals.

$$
y^{ni}_{t+h|t} \equiv y^{ni}_{t+h|i,t} = \bar{\theta}_{t|t} \tag{41}
$$

where $\bar{\theta}_{t|t}$ is generated through Kalman filtering.

Assume that the nosiy signals of θ_t consists of a public signals s^{pb}_t and the private signals $s^{pr}_{i,t}$. For simplicity, let us assume the public signal is basically the y_t. A more general case would be an independently drawn public signal sequence. The two signals can be again stacked into a vector of 2×1 to $s^{\theta}_{i,t}$.

Then the filterred θ_t by agent i is

$$
\bar{\theta}_{t|t} = (1-\tilde{P}_t H)\bar{\theta}_{t|t-1} + \tilde{P}s^{\theta}_{i,t} \tag{42}
$$

where $\theta_{t|t-k}$ is the filterred forecast of θ_t using all the information up to $t-k$., and $\tilde{P}t$ is the time-specific Kalman gain that is dependent on the noisy ratios of signals.

$$
\tilde{P}_t = \Sigma^{\theta}_{i,t|t-1}H(H'\Sigma^{\theta}_{i,t|t-1}H + \Sigma^{\theta}_t)^{-1} \tag{43}
$$

Now the noisiness of signals are time varying as well.

$$\Sigma^{\theta}_{t} = \begin{bmatrix} \sigma^2_{\eta,t} & 0 \\ 0 & \sigma^2_{\xi} \end{bmatrix} \tag{44}$$

where the variance of public signal is the time-varying $\sigma^2_{\eta,t}$ and let us assume the private signals have constant nosiness σ^2_{ξ}.

The uncertainty matrix evolves in the following manner.

$$\Sigma^{\theta}_{i,t|t} = \Sigma^{\theta}_{i,t|t-1} - \Sigma^{\theta}_{i,t|t-1} H'(H\Sigma^{\theta}_{i,t-1}H' + \Sigma^{\theta}_{t})H\Sigma^{\theta}_{i,t|t-1} \tag{45}$$

Notice now that since prior of θ_t before time period t is a weighted average of previous realizations of y and past signals, current forecast depends on the past realizations even though the rational forecast is up-to-date θ_t.

Due to the time-varying volatility $\sigma^2_{\eta,t}$, the noisyness of the public signal is also time-varying, which governs the degree of rigidity. For instance, if volatility to the transitory shock is high, the Kalman gain is low, the forecast responds less to the new realizations.

It is also worth thinking about what is the one-step ahead uncertainty in the context of stochastic valotility world.

$$\begin{aligned} \Sigma^{\theta}_{i,t|t-1} &= \Sigma^{\theta}_{i,t-1|t-1} + Var^{*}_{t|t-1}(y_t) \\ &= \Sigma^{\theta}_{i,t-1|t-1} + exp^{-0.5\gamma_\eta}\sigma^2_{\eta,t-1} + exp^{-0.5\gamma_\epsilon}\sigma^2_{\epsilon,t-1} \end{aligned} \tag{46}$$

3 Empirical Results

3.1 Data

The focus of this paper naturally restricts my options of the survey data to use compared to other empirical literature of testing theories of expectation formation. The surveys that have elicited density forecasts of macroeconomic variables for a sufficiently long period are rare. Rarer, for the purpose of the paper, is the data structure that allows for comparing the revisions across vintages that have a fixed terminate date of realization, either in individual level or aggregate level.

Survey of Professional Forecasters(SPF) meet both criteria thanks to the density forecasts of a number of macroeconomic variables, including core CPI and core PCE inflation they have started eliciting since 2007, as well as a series of GDP deflator dating back to 1968. This paper focuses on the first two, for which both forecasts for current-year inflation, basically nowcast, and one-year-ahead forecast are based on densities. This makes it possible to directly test the implications of the revisions in uncertainty across different vintages of forecasts.

The New York Fed Survey of Consumer Expectation(SCE) that started in 2013 meet the first criteria and half of the second. In particular, households are asked to provide their perceived probabilities about 1-year-ahead and 3-year-ahead inflation for various ranges of values[9]. This allows for comparing 3-year-ahead forecast at time $t-3$ with 1-year-ahead forecast at $t-1$. Since the maximum duration for households to stay in the panel is 12 months (for about one-third of the households), forecast revision can be only examined in the population level. The advantage of SCE compared to SPF is its monthly frequency. This provides an invaluable chance to explore the dynamics of uncertainty.

I follow Engelberg et al. (2009) to estimate the density distribution of each individual surveyee for SPF. [10] This is the same approach adopted by the New York Fed researchers Armantier et al. (2017) for SCE and directly provided their estimate of uncertainty. I directly use them.

Unsurprisingly, both surveys need some winsorization. For SPF, I drop the outliers of mean forecast and uncertainty estimates at both top and bottom one percentile as these are typically abnormals that are due to measurement errors or other reasons. For SCE, I drop the top and bottom 5 percent of mean forecasts and uncertainty as households mean forecast are inclined to give extreme values. All the results in this paper are robust to a different threshold such as 10 and 1 percentile. [11]

A summary of the data information is in Table 2.

Throughout the paper, I use three measures of inflation: headline CPI, core CPI, and core PCE . Depending on the specific variable of forecast in the survey series, the realization of the corresponding inflation is used to compute moments such as forecast errors. Specifically, SPF has density forecasts for both core CPI and core PCE [12]. For SCE, as the households responders are asked about the overall inflation, It is the most appropriate to be interpreted as headline CPI inflation. To simplify the expression, from now on, core CPI and core PCE are simply referred to as CPI and PCE, respectively.

3.2 Stylized facts

Although the sheer magnitudes of the differences between professionals' moments and those of households are so big that a direct comparison of the two seems redundant, their respective within-agent-type correlation serves a ready checking device of some statistical consistency. Figure 4a, 4b, and 4c plot the population uncertainty against realized inflation, forecast errors, and disagreements in the first, second and third rows, respectively.

[9] Most importantly, they are kindly reminded that all the probabilities need to add up to one. NY Fed stuff has excluded those who do not meet these criteria.

[10] Answers with positive probabilities assigned to three bins is fit with a generalized beta distribution. Depending on if there is open-ended bin on either side with positive probability, 2-parameters or 4-parameters of the beta distribution are estimated. For those with only two bins with positive probabilities and adjacent, it is fit with a triangular distribution. For only one bin with probability, a uniform distribution over the positive probability bin is fit. See my online appendix for the detailed steps of estimation of python codes.

[11] For mean forecasts and uncertainty, respectively, this means dropping 6528 and 5096 observations, out of 68887 observations in total.

[12] SPF also has density forecasts for GDP deflator(for GNP prior to 1992: Q1) going back to 1968. Since the ranges of the values and the definition are not consistent over time, I do not use it in this paper.

It is widely documented in literature [13] and resonated by anecdotal narratives that high inflation is typically associated with high inflation uncertainty. However, the observable correlation of realized inflation and the directly estimated average uncertainty from both professionals and households are at most weakly positive during the period between 2007-2019. In particular, the correlation coefficients 0.14, 0.12 and -0.24 for SPF's CPI forecast, SPF's PCE forecast, and SCE's CPI expectation. This may suggest that during a period of persistently low and stable inflation, the conventional positive redux of inflation and uncertainty is not a good description of the relation of the two.

Figure 4b looks into the relationship between size of the forecast error and uncertainty. Although according to our benchmark framework in Section 2, there is no mathematical correlation between the size of the forecast errors and uncertainty as the former depends on the realized shocks and the later depends on the volatility of the shock, it is worth checking if in the data a greater ex-ante uncertainty implies bigger ex post forecast errors. The correlation coefficients of the two are -0.19, -0.18 and 0.27 for SPF CPI forecasts, SPF PCE forecasts, and SCE's forecasts, respectively. The two indicators turn out to be negatively correlated for professional forecasts. Only households forecasts exhibit such a positive correlation.

Figure 4c examines the relationship between disagreement and uncertainty. Many empirical literature in macroeconomics use disagreements and uncertainty as if they are similar concepts[14]. Such confusion in practice is partly due to the difficulty of finding appropriate measures of uncertainty in the first place. But as we have presented in 2, the two are concepts with distinct statistical definitions.[15] The empirical pattern of the two, as shown in the plots, also confirms that the two are different objects. The correlation coefficients of the two turn out to be negative, -0.31 and -0.37 for SPF forecasts. In stark contrast, households seem to bear a strong correlation of 0.43 between the two moments.

Overall, professional forecasters' moments exhibit patterns more consistency from a statistical point of view. In contrast, the positive correlation across households' ex-ante uncertainty, ex-post forecast errors, and cross-section disagreements cannot be easily reconciled by the framework we set up in Section 2.

Persistent disagreement in expectations has been used as important stylized evidence inconsistent with the assumption of identical expectation embedded in FIRE, for instance, Mankiw et al. (2003). A similar fact-checking can be done with respect to individual uncertainty. FIRE predicts individual share an equal degree of uncertainty as seen in Equation 5. In contrast, SE predicts that uncertainty of individuals differ in that agents are not equally updated at a point of the time (Equation 9). NI allows for the possibility of homogeneity in uncertainty only under the stringent conditions of an equal precision of signals and the same prior for uncertainty (Equation 25). Therefore, the presence of dispersion of uncertainty across agents is not consistent

[13] The list literature showing positive inflation and uncertainty is long. For example Ball et al. (1990).

[14] For instance, Bachmann et al. (2013) used ex-ante disagreement and ex-post forecast errors as two measures of uncertainty and find that both uncertainty indicators lead to a reduction in real economic activity.

[15] This point was made very clearly by Zarnowitz and Lambros (1987). Manski (2018) also points out many empirical work has confused the dispersion with uncertainty.

with predictions from FIRE.

Figure 5 plots the median inflation expectation along with its 25/75 percentiles in the left and its counterpart in uncertainty in the right column. Not only there is long-lasting dispersion in individual forecasts, i.e. disagreement, but also notable heterogeneity in uncertainty across agents. And not surprisingly, the dispersion of both forecasts and uncertainty of households are both of a much greater magnitude than that of the professionals. The 25/75 inter-quantile-range of households point forecasts is 4-5 percentage points compared to 1 percentage point of professionals. And the IQR of the uncertainty of households is around 150-200 times(12-14 times for standard deviation) as that of professional forecasters.

Besides, in terms of the distribution of uncertainty, there are households/professional difference and similarity. Households' uncertainty is more skewed toward the right (higher uncertainty), meaning there is a wide dispersion in the high values of uncertainty. This can be also seen in Figure 6, where I plot the kernel density estimated distribution of uncertainty by year on. What is common for both types of agents is that the dispersion in uncertainty is persistent over time and do not show much time-variation [16].

Another pattern worth discussing in Figure 5 is that there is a notable rise in the dispersion of professional forecasts in the recent 2-3 years, primarily driven by an increase of upper side of the forecast (i.e. 75 percentile forecast increases from 1% to 2%). [17] This is consistent with the observation in the top left two graphs of Figure 6 that the distribution of inflation forecasts in recent years have become flattened.

FIRE also predicts an unambiguous reduction in uncertainty as one approaches the date of realization, where the drop is exactly equal to the volatility of the realized shocks. Although quantitatively it is hard to check this, one can look if the distribution of the uncertainty revision concentrates in the negative range. Figure 7 plots the average revision in mean forecasts and uncertainty from 1-year-ahead forecast in year $t-1$ to the current-year nowcast in year t. The more negative range in which the revision lies, the more "rational" of the forecast. Looking from the histograms, uncertainty revision shows left-skewness relative to zero. This implies on average, forecasters feel more certain for her nowcasts relative to her forecast made one year before. Unfortunately, since SCE does not provide the data structure for this purpose, I cannot make a comparison between two types of agents. A formal test of revision equal to zero or being negative will be carried out in Section 3.3.

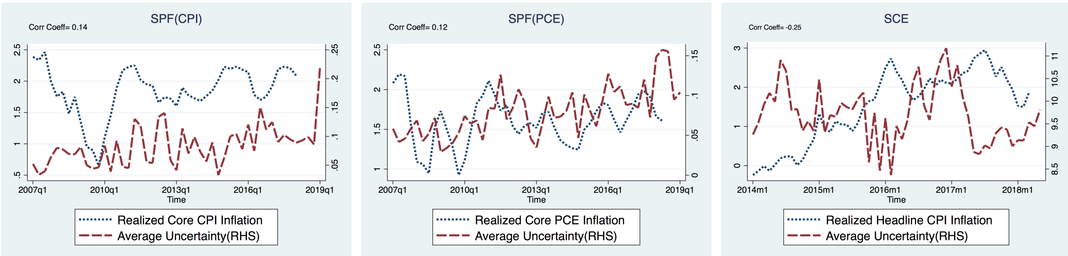
3.3 Test of null hypothesis of rational expectation

This section first reproduces a number of statistical tests of FIRE seen in existing literature in Table 3, primarily following Mankiw et al. (2003), and then extends the tests relying on uncertainty in Table 4 and 5, in the spirit of forecasting efficiency by Nordhaus (1987). It is an extension of revision tests on mean forecasts by Fuhrer (2018) to uncertainty.

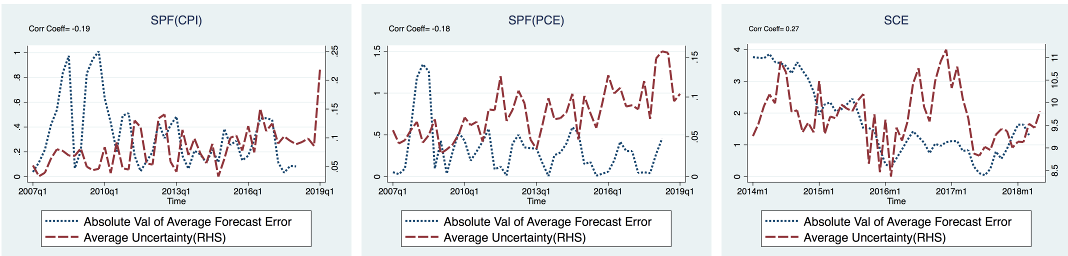
[16] Kumar et al. (2015) also presents the dispersion in uncertainty using a shorter-period of sample for SCE.

[17] This should be interpreted with caution since the disagreements of SPF forecasts shown in Figure 4c actually exhibits a gradual decline.

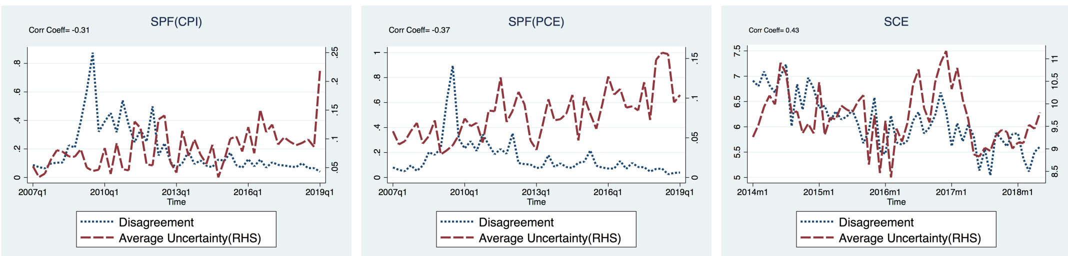
(a) Realized Inflation and Uncertainty



(b) Size of Forecast Errors and Uncertainty



(c) Disagreement and Uncertainty



Note: From left to right: SPF's forecasts of core CPI and core PCE, and SCE's forecast of headline CPI. From top to the bottom, uncertainty (red dash) versus realized inflation (blue dot) with correlation coefficient of 0.14, 0.12 and -0.25, respectively; uncertainty (red dash) versus absolute value of forecast errors (blue dot) with correlation coefficient of -0.19, -0.18, 0.27, respectively; uncertainty (red dash) versus disagreements (blue dot) with correlation coefficient of -0.31, -0.37 and 0.43,repectively. Only the Pearson tests of correlation between disagreement and uncertainty are significant for all.

Figure 4: Uncertainty and Other Moments

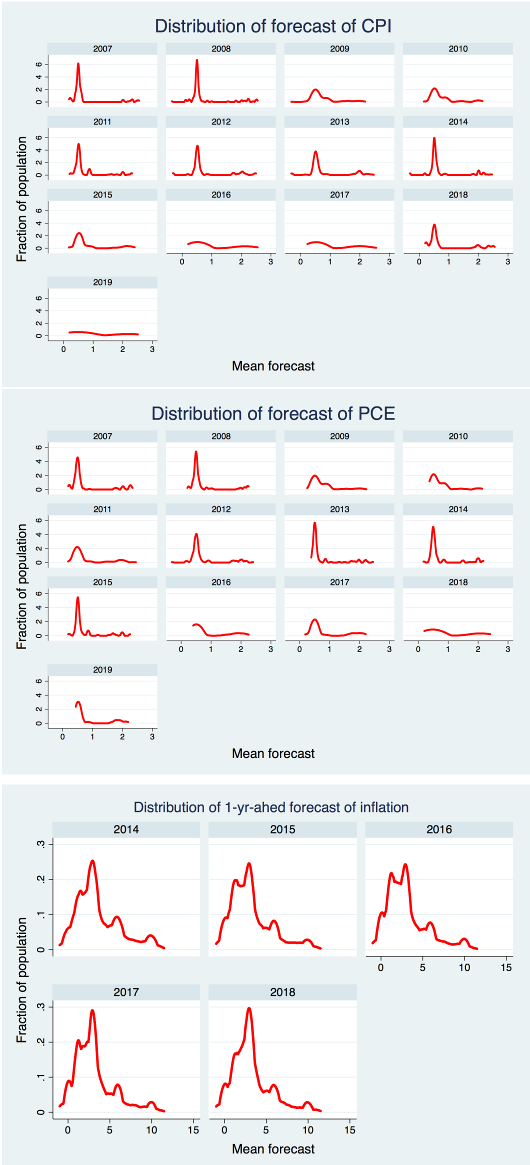
Table 2: Information of Data

	SCE	SPF
Time period	2013-2019	2007-2019
Frequency	Monthly	Quarterly
Sample Size	1,300	30-50
Var in Density	1-yr and 2-yr-ahead inflation	1-yr-ahead GDP deflator, Core CPI and Core PCE
Panel Structure	stay up to 12 months	average stay for 5 years
Individual Info	Education, Income, Age, Location	Industry

(a) Mean Forecasts (b) Uncertainty

Figure 5: Dispersion of Mean Forecasts and Unceratinty

(a) Mean Forecasts



(b) Uncertainty

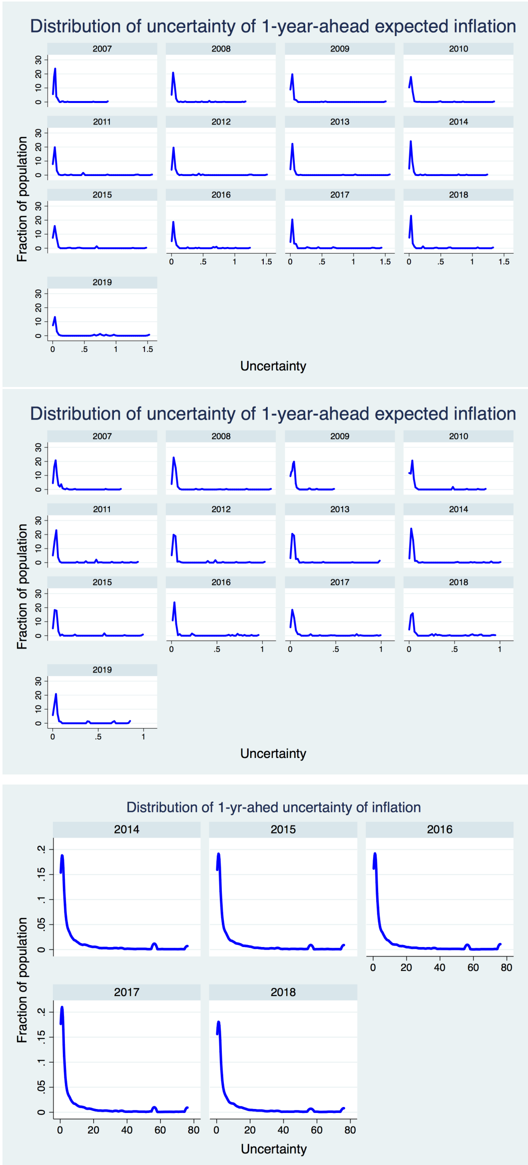


Figure 6: Distribution of Mean Forecast and Uncertainty

(a) Revision in Mean Forecasts (b) Revision in Uncertainty

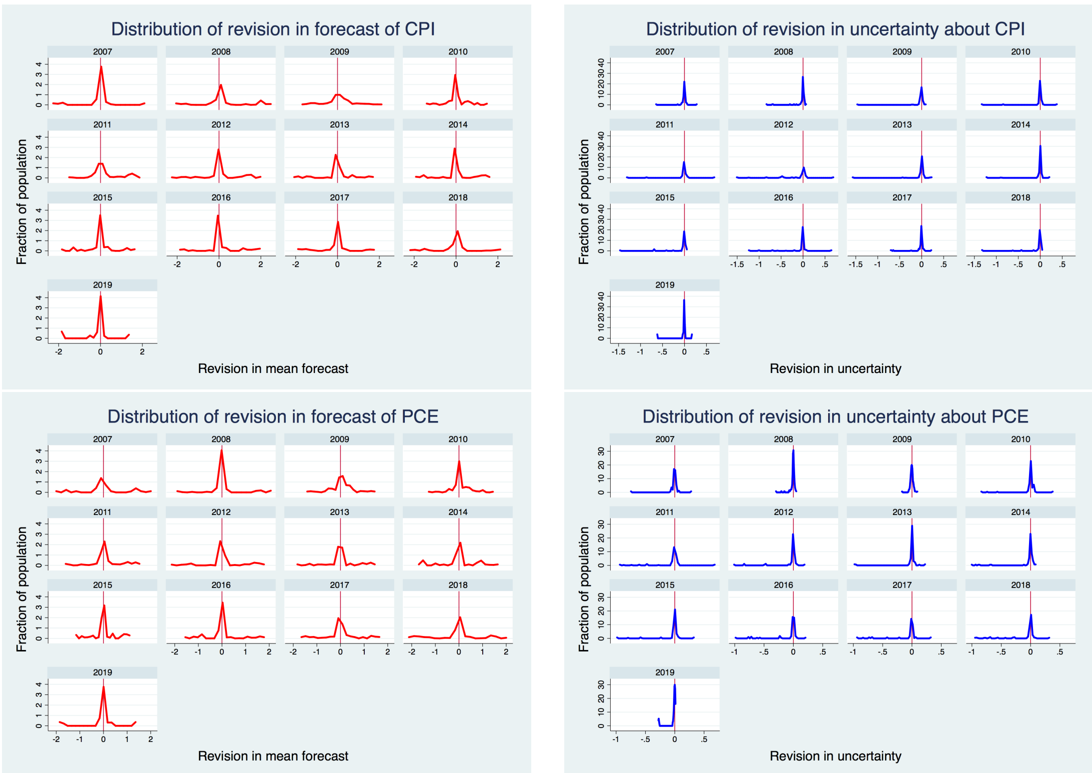


Figure 7: Distribution of Revision in Forecasts and Uncertainty

The first set of tests, hereafter, referred to as FE-based tests, utilize the moment restrictions on forecast errors. In plain words, the null hypotheses of the three tests are the following. First, since the forecasts are on average unbiased according to FIRE, forecast errors across agents should converge to zero in a large sample. Second, forecast errors of non-overlapping forecasting horizon are not serially correlated (Equation 3). Third, forecast errors cannot be predicted by any information available at the time of the forecast, including the mean forecast itself and other variables that are in the agent's information set. This follows from Equation 2. In addition, I include what is called weak version of the FE-based test which explores the serial correlation of forecast errors in overlapping periods, i.e. 1-year-ahead forecasts within one year. The forecast errors are correlated to the extent of the realized shocks in the overlapping periods. So the positive serial correlation does not directly violate FIRE. But the correlation of overlapping forecast errors still contains useful information about the size of the realized shocks.

FE-based tests results are presented in Table 3. Individual-level data are used thanks to the panel structure of both surveys. Since test 2 and 3 requires individual forecasts in vintages that are more than one year apart while SCE only surveys each household for 12 months, the two tests are done for only SPF forecasts of CPI and PCE. Also, the regressions are adjusted accordingly depending on the quarterly and monthly frequency of SPF and SCE. Since these regressions are based on 1-year inflations in overlapping periods, white standard error is computed for hypothesis testing.

First, all three forecast series easily reject the unbiasedness test at the significance level of 0.1%. There is upward bias across professional forecasters and households [18], while unsurprisingly, the bias is almost 20 times of that for professional forecasters(2.2 versus 0.12) for headline CPI.

Second, the point forecast one year ago predicts the forecast errors for professionals in the significance level of 0.1%. For headline CPI inflation, for instance, one percentage point inflation forecast corresponds to 0.3 percentage points of the forecast errors one year later. Thus test 2 in Table 3 easily rejects the second hypothesis test of FIRE that past information does not predict future forecast errors.

Third, forecast errors can be predicted by forecast errors one year ago with a significant coefficient of around 0.08 for headline CPI and 0.05 for PCE, as seen in test 3 of Table 3. Errors of non-overlapping forecasting periods are correlated, against the null of FIRE.

Lastly, test 4 in Table 3 presents a higher serial correlation of forecast errors produced within a year. For SPF forecasts, the serial correlation does not exist beyond 1 quarter, implying relative efficiency of forecasts. For the households, the forecast errors are more persistent over the entire year in that current forecast errors are correlated with all past forecast errors over the past three quarters. Again, although the persistence of 1-year forecast errors within one year does not directly violate FIRE, the fact that households' forecast errors being more persistent than professionals provide useful clues about the relative rigidity of the two types of agents.

The second batch of tests in Table 4 focus on estimating forecasting efficiency using revisions of mean forecasts and uncertainty, hereafter referred as revision-based tests. In plain words, the revision from 1-year-ahead forecast to nowcast of current-year

[18] Coibion et al. (2018) finds the same upward bias for firms' managers.

Table 3: Tests of Rationality and Efficiency Using Forecast Errors

	SPF CPI	SPF PCE	SCE
Test 1: Unbiasedness			
Constant	0.122***	0.586***	2.220***
	(0.017)	(0.061)	(0.019)
N	4697	1208	67380
Test 2: FE does not depend on past information			
Forecast 1-yr before	0.307***	0.586***	NA
	(0.020)	(0.061)	NA
Constant	-0.655***	-0.777***	NA
	(0.060)	(0.116)	NA
N	3429	1208	NA
R^2	0.0721	0.118	NA
Test 3: FEs of non-overlapping forecast horizons not serially correlated			
Forecast Error 1-year before	0.0756***	0.0503***	NA
	(0.020)	(0.035)	NA
Constant	0.145***	0.275***	NA
	(0.021)	(0.035)	NA
N	3356	1208	NA
R^2	0.00591	0.00264	NA
Test 4: Overlapping FEs are only weakly serially correlated			
Forecast Error 1-q before	0.657***	0.834***	0.297***
	(0.025)	(0.037)	(0.021)
Forecast Error 2-q before	0.0282	-0.0858	0.308***
	(0.027)	(0.048)	(0.046)
Forecast Error 3-q before	-0.0244	-0.0555	0.311***
	(0.025)	(0.038)	(0.045)
Constant	0.0626***	0.113***	0.742***
	(0.019)	(0.026)	(0.097)
N	2536	1004	2836
R^2	0.439	0.552	0.232

Note: white standard errors reported in the parentheses of estimations. *** $p<0.001$, ** $p<0.01$ and * $p<0.05$.

inflation is efficient according to two criteria (1) Forecast revision does not depend on past information, including the past revisions. (2) Drop in uncertainty is sufficiently rapid to reflect the uncertainty of all realized shocks.

The mean revision test by Fuhrer (2018) takes the following form (using 1 period as an example):

$$y_{i,t+1|t+1} - y_{i,t+1|t} = \alpha + \beta(y_{i,t+1|t} - y_{i,t+1|t-1}) + \epsilon_{i,t+1} \tag{47}$$

In the above equation $\beta = 0$ according to FIRE, because rational forecast revision only responds to newly realized shocks thus it is not predictable by past revisions.[19]. Since we have four vintages of the forecasts from SPF, the above specification can include lagged revisions up to 4 quarters.

The test with uncertainty simply replaces the revision of forecast with revision in uncertainty $\sigma^2_{i,t+1|t+1} - \sigma^2_{i.t+1|t}$, for instance. This regression follows from Equation 14 for SE and Equation 24 for NI. Although it cannot be directly used as a test against FIRE null, the autocorrelation coefficient speaks to the speed of the drop in uncertainty. Depending on the model, one can interpret it as the particular structural parameter of rigidity, as shown in Figure 3.

The top panel in Table 4 presents the results for the mean forecast. Following Fuhrer (2018), I include the median forecast available at time t and $t-1$ as an indicator of past information for the revision regression. In the first column of each panel, I report the regression on a constant.

What's surprising is the mean revision in forecast being negative and significant. Forecasts on average make downward adjustment of 1.26 percentage points of CPI and 1.1 percentage points of PCE from her previous year forecast of the same-period inflation.

The second to fourth columns of each panel in Table 4 checks autocorrelation of revisions including different lags. Revisions of forecasts are serially correlated over 4 quarters and the coefficients are all positive and significant. Also, the median forecasts as the past information always predict a negative revision with significant coefficients. This is evidence against the null hypothesis of FIRE and my estimates are comparable with those by Fuhrer (2018).

The bottom panel reports autoregression results for revision in uncertainty. Again, the first column first test the mean revision against the null being zero. The mean revisions in uncertainty are both negative (0.5-0.6 percentage points equivalence in standard deviation of uncertainty) and statistically significant, confirming our observation from Figure 7 that forecasters are more certain about current inflation compared her previous year forecast.

The second to fourth column shows a positive serial correlation of revision in uncertainty for both CPI and PCE forecasts. The revision to CPI seems more efficient as serial correlation is with only one-quarter lag. For PCE, the revisions in uncertainty are serially correlated with all past three quarters.

Table 5 presents the results with the revision replaced with change in mean forecasts and uncertainty, i.e. from $y_{t|t-1}$ to $y_{t+1|t}$. As we have discussed in Section 2, the auto-

[19] Adding $y_{t+1|t}$ to both sides of Equation 47 gives an equivalent null hypothesis used by Fuhrer (2018): coefficient of regression of $y_{t+1|t+1}$ on $y_{t+1|t}$ is $1 - \beta = 1$.

Table 4: Tests of Revision Efficiency Using Mean Revision and Uncertainty

	SPF CPI				SPF PCE				SCE				
Test 1. Revision efficiency of mean forecast													
	Mean revision	t-1	t-1- t-2	t-1-t-3	Mean revision	t-1	t-1- t-2	t-1-t-3		Mean revision	t-1	t-1- t-2	t-1-t-3
L.InfExp_Mean_rv		0.539***	0.418***	0.387***		0.606***	0.435***	0.369***	L.InfExp_Mean_rv		0.884***	0.750***	0.685**
		(0.031)	(0.043)	(0.052)		(0.034)	(0.042)	(0.049)			(0.084)	(0.171)	(0.211)
L2.InfExp_Mean_rv			0.218***	0.166**			0.261***	0.246***	L2.InfExp_Mean_rv			0.206	0.199
			(0.040)	(0.053)			(0.047)	(0.058)				(0.173)	(0.185)
L3.InfExp_Mean_rv				0.134**				0.116	L3.InfExp_Mean_rv			-0.073	-0.122
				(0.048)				(0.069)				(0.191)	(0.264)
SPFCPI_ct50		-0.444***	-0.391**	-0.454**					L4.InfExp_Mean_rv				0.055
		(0.105)	(0.124)	(0.138)									(0.217)
SPFPCE_ct50						-0.432***	-0.413***	-0.504***	L5.InfExp_Mean_rv				0.073
						(0.109)	(0.111)	(0.138)					(0.242)
									L6.InfExp_Mean_rv				-0.016
													(0.140)
Constant	-1.257***	0.329	0.351	0.546*	-1.095***	0.365	0.428*	0.641**	Constant	-0.263***	-0.016	-0.010	-0.005
	(0.045)	(0.191)	(0.237)	(0.269)	(0.039)	(0.188)	(0.191)	(0.228)		(0.038)	(0.029)	(0.033)	(0.042)
N	1337	1045	822	652	1111	867	683	549	N	41	40	38	35
R^2	0.000	0.335	0.355	0.372	0.000	0.409	0.444	0.452	R^2	0.000	0.717	0.732	0.690
Test 2. Revision efficiency of uncertainty													
	Mean revision	t-1	t-1- t-2	t-1-t-3	Mean revision	t-1	t-1- t-2	t-1-t-3		Mean revision	t-1	t-1- t-2	t-1-t-3
L.InfExp_Var_rv		0.290*	0.529***	0.581***		0.577***	0.477***	0.344*	L.InfExp_Var_rv		0.731***	0.723***	0.785***
		(0.122)	(0.117)	(0.145)		(0.080)	(0.130)	(0.148)			(0.108)	(0.159)	(0.161)
L2.InfExp_Var_rv			-0.059	-0.209			0.360*	0.205*	L2.InfExp_Var_rv			0.283	0.174
			(0.125)	(0.127)			(0.143)	(0.098)				(0.197)	(0.246)
L3.InfExp_Var_rv				0.353**				0.390*	L3.InfExp_Var_rv			-0.336	-0.410
				(0.121)				(0.149)				(0.186)	(0.266)
									L4.InfExp_Var_rv				0.270
													(0.219)
									L5.InfExp_Var_rv				-0.065
													(0.248)
									L6.InfExp_Var_rv				-0.082
													(0.172)
Constant	-0.034***	-0.011**	-0.008*	-0.005	-0.039***	-0.019**	-0.010**	-0.007*	Constant	-0.590**	-0.186	-0.199	-0.163
	(0.005)	(0.004)	(0.003)	(0.004)	(0.006)	(0.006)	(0.003)	(0.003)		(0.174)	(0.155)	(0.164)	(0.171)
N	1189	877	663	504	1082	801	604	458	N	41	40	38	35
R^2	0.000	0.124	0.284	0.408	0.000	0.353	0.583	0.723	R^2	0.000	0.549	0.597	0.619

Standard errors are clustered by date. *** p<0.001, ** p<0.01 and * p<0.05.

correlation of change in mean forecast and uncertainty do not bear testable predictions from FIRE (Equation 5). But if the forecasts and uncertainty are persistent in its first difference, it may imply that the agent does not react to the news and newly realized shocks sufficiently enough. In addition, the auto-correlation regressions of this kind is a useful characterization of the time series dynamics of forecasts. With the variable being the first difference, the panel structure of SCE and SPF allows for calculating changes in individual levels for greater sample size, especially for households. Besides autoregression, I also report the constant estimate of the changes in the first column of each sub-panel.

The most noticeable pattern for both professionals and households, and for both mean forecast and uncertainty is that past change predict future changes with universally negative coefficients across different horizons. Most of the negative coefficients are statistically significant in the 1% level. For instance, one percentage point of increase in SPF's CPI forecast in the previous quarter predicts around 0.29 percentage points decrease in the next quarter. This negative correlation is smaller in size but remain significant into further past, i.e. -0.24 for two quarters lag and -0.1 for three quarters lag. Data on SCE is monthly, so lags are included up to six months. The negative correlation between past and current changes are all negative and significant for households. The sizes of the correlation coefficients are comparable with professional forecasts when the monthly coefficients are converted to their quarterly average, i.e. -0.3 to -0.4.

Such an auto-correlation of change in mean forecast is very much reflected in the same regression for uncertainty, as reported in the bottom panel of Table 5. For SPF of CPI and PCE, respectively, one unit increase in uncertainty about 1-year-ahead inflation in previous quarter predicts around a 0.39 and 0.44 unit of the drop in the next quarter. The effect holds up to two quarters for professionals and 5 months for households.

These evidence suggest that both the mean forecast and uncertainty of individuals are mean-reverting. An essentially equivalent explanation is that both series are realizations of noisy signals around their respective long-run mean. This will lead to the exact negative correlation of first differences we have seen. The mean-reverting patterns might also explain what is observed from Figure 6, according to which, there are no significant changes in the distribution across different years.

The second noticeable result lies in the constant regressions reported in the first column of each sub-panel in Table 5. It implies that households constantly lower their mean forecasts as well as uncertainty from month to month, while professional forecasts do not behave in such a pattern. In particular, the constant regression of the change in the mean forecast for SCE gives an estimated coefficient of -0.05 which is significant in the 5% level. Individual households' 1-year-ahead inflation expectations keep being downward adjusted each month compared to their previous answer. What is more interesting is that their uncertainty about 1-year-ahead inflation also decreases each month. The size of the downward adjustment is -1.39 unit and statistically significant in the level of 0.1%. This negative significant and constant coefficient remains throughout all auto-regressions, implying it is not driven by time-varying changes.

The most natural explanation for this that repeatedly surveyed households have become more informative about inflation over time. Given the unconditional forecast errors of inflation by households are positive, a downward adjustment of inflation stands for a less-biased forecast. [20]

In summary, the major additional insights that arise from the empirical tests of this section is that rigidity of incorporating new information in forming expectations imply noticeable inefficiency of revisions in forecasts and drop in uncertainty.

4 Shock-based Estimates of Expectation Rigidity

Coibion and Gorodnichenko (2012) explores the implications of expectation rigidity models using externally identified shocks to inflation. Unlike the time series regressions that test null hypotheses of FIRE in the previous section, this approach allows a more straightforward comparison between the empirical evidence from surveys and the theoretical predictions illustrated in Figure 2. The difficulty of this approach lies in identifying a shock ω_t that is a pure innovation to inflation at time t.

I use three types of shocks: technology(productivity) shock from Gali (1999), oil price shock from Hamilton (1996) and monetary policy shocks from Laséen and Svensson (2011). The first two are the same in Coibion and Gorodnichenko (2012). [21]

(1) Technology shock is identified from a three-variate structural vector autoregression(SVAR) model (output per capita, hours and inflation) by imposing a widely used long-run restriction that non-technology shocks have no long-run output impacts. Based on Bayes information criterion, I use the lag of 4 quarters as in Coibion and Gorodnichenko (2012) [22]. The sample period used to identify the shock is 1949-2019, while the period where it enters my analysis is the same to the period of survey data: 2007-2019 for SPF and 2013-2019 for SCE.

(2) Oil price shock in each period is defined as the maximum of log change in crude oil prices over the past 1-year and 0. Therefore it is bounded zero.[23]

(3) Monetary policy shocks: target surprise and path surprise in the terminology of Laséen and Svensson (2011). Target surprise is defined as the unexpected change in the federal funds rate in a particular date of FOMC meetings. It is the OLS residual of federal funds rate change in the past expected change in the federal funds rate. The 2-year ahead path surprise is the regression residual of future implied federal funds

[20] The possibility that the surveys' information set is influenced by the survey itself is a double-edged sword. On one hand, this poses a methodological challenge to the survey designers of expectations as to if surveys can objectively elicit the "true" expectations held by the respondents. On the other hand, researchers can use the survey as a meaningful intervention tool to identify the effect of factors such as information provision and attention. Recent examples of this type of research includes: Coibion et al. (2018) for firms and Coibion et al. (2019) for households.

[21] Coibion and Gorodnichenko (2012) also uses the news shock by Barsky and Sims (2011). This will be included in later version of the paper.

[22] A recent alternative to long-run restriction approach is the "max-share approach" developed by Francis et al. (2014). It is shown to have better small-sample properties. In particular, the approach defines technology shock as the variation of current period labor productivity (output per hour) that explains the maximum fraction of future labor productivity over an exogenously determined horizon. The results using this shock will be included in a later version of the paper. For the replication purpose now, I stay with Gali (1999)

[23] The series of oil price is Spot Crude Oil Price: West Texas Intermediate (WTI).

Table 5: Weak Tests of Revision Efficiency Using Change in Forecasts and Uncertainty

	SPF CPI				SPF PCE					SCE			
Test 3. Weak revision efficiency of change in forecast													
	Mean change	t-1	t-1- t-2	t-1-t-3	Mean revision	t-1	t-1- t-2	t-1-t-3		Mean revision	t-1	t-1- t-2	t-1-t-3
L.InfExp_Mean_ch		-0.295***	-0.344***	-0.367***		-0.303***	-0.348***	-0.364***	L.InfExp_Mean_ch		-0.433***	-0.586***	-0.642***
		(0.034)	(0.044)	(0.045)		(0.043)	(0.059)	(0.062)			(0.01)	(0.013)	(0.025)
L2.InfExp_Mean_ch			-0.179***	-0.242***			-0.162*	-0.200**	L2.InfExp_Mean_ch			-0.336***	-0.439***
			(0.047)	(0.049)			(0.061)	(0.067)				(0.018)	(0.031)
L3.InfExp_Mean_ch				-0.097**				-0.088*	L3.InfExp_Mean_ch			-0.143***	-0.270***
				(0.032)				(0.036)				(0.012)	(0.027)
									L4.InfExp_Mean_ch				-0.183***
													(0.027)
									L5.InfExp_Mean_ch				-0.096***
													(0.021)
									L6.InfExp_Mean_ch				-0.044**
													(0.013)
Constant	-0.005	-0.004	-0.011	-0.015	0.001	0.008	-0.002	-0.007	Constant	-0.055*	-0.034	-0.001	-0.002
	(0.023)	(0.024)	(0.026)	(0.026)	(0.020)	(0.020)	(0.022)	(0.022)		-0.023	-0.023	-0.028	-0.033
N	1636	1430	1266	1141	1402	1190	1022	898	N	53016	43166	28850	14445
R^2	0.000	0.086	0.112	0.128	0.000	0.090	0.112	0.120	R^2	0.000	0.202	0.273	0.306
Test 4. Weak revision efficiency of change in uncertainty													
	Mean change	t-1	t-1- t-2	t-1-t-3	Mean change	t-1	t-1- t-2	t-1-t-3		Mean change	t-1	t-1- t-2	t-1-t-3
L.InfExp_Var_ch		-0.393**	-0.568***	-0.543**		-0.444***	-0.602***	-0.658***	L.InfExp_Var_ch		-0.382***	-0.565***	-0.652***
		(0.136)	(0.146)	(0.177)		(0.094)	(0.127)	(0.145)			-0.015	-0.022	-0.037
L2.InfExp_Var_ch			-0.322**	-0.278*			-0.289*	-0.404**	L2.InfExp_Var_ch			-0.300***	-0.406***
			(0.104)	(0.132)			(0.110)	(0.137)				-0.021	-0.031
L3.InfExp_Var_ch				0.048				-0.292	L3.InfExp_Var_ch			-0.123***	-0.265***
				(0.096)				(0.154)				-0.012	-0.027
									L4.InfExp_Var_ch				-0.130***
													-0.025
									L5.InfExp_Var_ch				-0.058**
													-0.018
									L6.InfExp_Var_ch				-0.025
													-0.012
Constant	-0.002	-0.001	0.004	0.004	0.000	0.002	0.004	0.005	Constant	-1.339***	-1.324***	-1.139***	-0.839***
	(0.005)	(0.005)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)		-0.123	-0.11	-0.104	-0.163
N	1202	950	765	625	1078	842	657	519		53016	43166	28850	14445
R^2	0.000	0.120	0.265	0.242	0.000	0.233	0.321	0.385		0	0.182	0.278	0.321

Standard errors are clustered by date. *** p<0.001, ** p<0.01 and * p<0.05.

rate on the target surprise. Depending on the frequency of the data being monthly or quarterly, I take the sum of all the changes within each period.

The reasons for including monetary policy shocks, although it turns out to provide less robust evidence, are twofold. First, the role of expectation in the working and implementation of monetary policy in the post-crisis period has been of a great relevance. Especially, around zero-lower-bound, central banks have utilized nontraditional tools such as forward guidance which primarily works through influencing expectations of market agents.

In Figure 8, all four shocks are plotted in quarterly between 1985-2019 and normalized by its respective sample standard deviations.

4.1 Forecast errors and disagreement

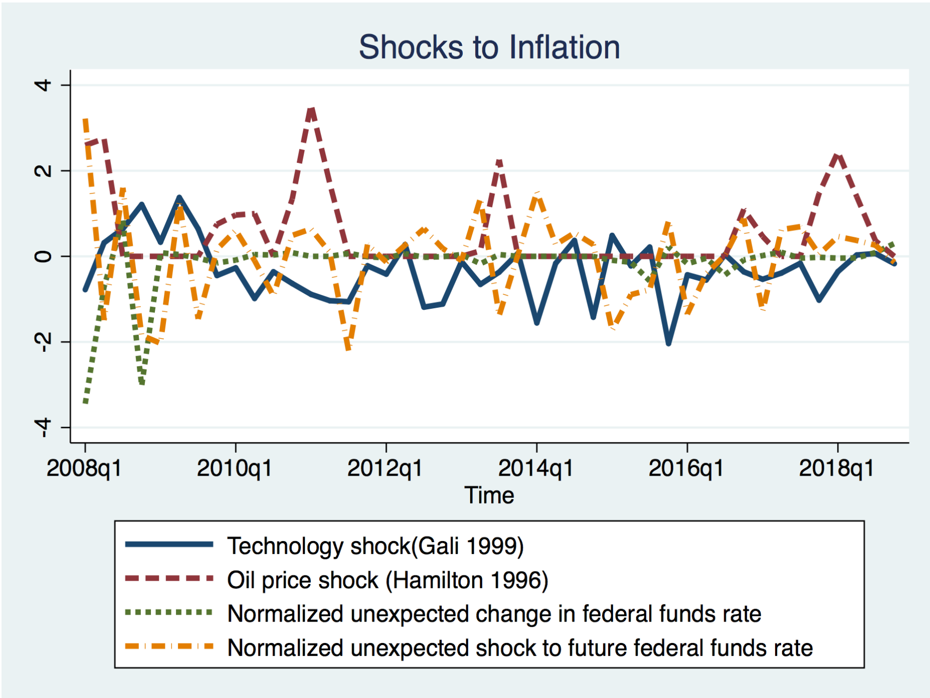
For a direct comparison of my results and Coibion and Gorodnichenko (2012), I first explore the impulse responses of CPI inflation y_{t+k}, SPF forecast errors and disagreement to one standard deviation of each shock at time t. In particular, the responses of disagreement is with respect to the absolute value of the shocks. Because the CPI inflation forecast is available only after 1984, the sample period 1984-2007 is six years shorter than that in Figure 2 from Coibion and Gorodnichenko (2012): 1976-2007. I do not include the results for PCE inflation as there is no PCE forecast during this period.

The underlying regression that produces impulse response estimates is Equation 48, according to which inflation y_t (or moments, $\overline{FE}_t$, $\overline{Disg}_t$ and $\bar{\sigma}^2_t$) in current quarter is a function of 4-period lags of itself and the realized shocks in current and previous period. The estimate of autoregression coefficients β_τ and shock-specific coefficients $\beta_{S,k}$ for different lags can be used to compute impulse responses of the variables of interest. In the impulse response graph, standard errors are computed using the parametric bootstrapping approach as in Coibion and Gorodnichenko (2012).

$$x_t = \alpha + \sum_{\tau=1}^{4} \beta_\tau y_{t-\tau} + \sum_{k=0}^{1} \sum_{S\in\{O,T\}} \beta_{S,k}\omega_{S,t-k} + \epsilon_t \tag{48}$$

FIRE predicts $\beta_{S,k} = 0 \quad \forall k = 1$ in the regressions with forecasts moments as dependent variable. i.e. forecasts react only to contemporaneous shocks instantaneously. Therefore, the average FE should not be dependent upon past shocks and only experience a one-time rise and return zero immediately. The disagreements will not respond to the shock at all.

The left panel of the Figure 9 show results are broadly consistent with Coibion and Gorodnichenko (2012) despite a wider confidence band possibly due to the change in inflation measures and the difference in the sample period. Productivity and oil price shocks are deflationary and inflationary, respectively. One standard deviation of productivity unsurprisingly brings down headline inflation by around 10 percentage points in the first quarter and the negative impacts gradually mute since then till the 7-the quarter. Oil price shock of one standard deviation increases headline inflation by around 3 percentage points in the first quarter and the inflationary impacts last for 6



Note: all the shocks are normalized by their sample standard deviation. Technology shock is idetified by long-run restriction of structural VAR with labor productivity, hours and inflation. Oil shock is defined as the max {0, the largest increase of oil price over the past 12 months.}

Figure 8: Shocks to Inflation

quarters. PCE inflation responds to both the shocks in a similar manner with slightly different magnitudes. The responses coefficients are statistically significant as the 95% confidence interval stays on one side of zero for both shocks for the same period.

Second, population forecast errors respond to shocks in a sluggish manner as predicted in rigidity models. This is illustrated using both SPF's FE of CPI in the second graph of the left panel in Figure 9. As we define the forecast error as point forecast minus realized value, forecast error rises(drops) after a positive(negative) shock and gradually return to zero around 4-5th quarters after the shock.

Lastly, SE and NI models imply disagreement pick up after shocks and gradually drop over the longer horizon. Looking into the graph, the average response of disagreement to the absolute value of technology and oil shocks exhibit patterns similar to this prediction. Note now that the initial response is no longer statistically significant(same as the original result) and the pick-up of disagreement after both shocks do not take place immediately in the same quarter but in one quarter after the shock. [24]

The responses to monetary policy shocks turn out to be noisier than those for technology and oil price shocks, as presented in the right panel of Figure 9. On the top, I first plot the impulse response of inflation to path surprises $ED8ut$ and target surprise $MP1ut$, separately. On average, inflation reduces by around 10 percentage points after one unit increase of target surprise and increases by 10 percentage points following one unit of path surprise to the federal funds rate. Both responses are statistically significant in 10% significance level. The unexpected future tightening causing a rise in inflation is consistent with the literature of the price puzzle. [25]

In the middle figure presents the responses of CPI forecast errors by SPF. Although the average response seems to go in the same direction of the respective monetary shocks, consistent with the pattern seen for technology and oil price shocks, none of the responses are statistically significant and the estimates have big standard errors of 10-20 percentage points.

Similarly, the responses of disagreements to both monetary policy shocks in the bottom panel are not statistically significant throughout the 10 quarters.

How do we interpret the finding of non-response of forecasting error and disagreement to monetary policy shocks in the context of rigidity models? Non-response of FE suggests that on average professional forecasters have incorporated the monetary policy changes in their forecasts. Non-response of disagreements could be due to that forecasters "agree" on the realization of monetary policy shocks. Although the benchmark theory of SE and NI do not differentiate shocks, the evidence here seems to suggest that the degree of rigidity depends on the nature of the shocks.

Or there are other explanations. First, overall the monetary policy shocks contribute only a small part of the variation in inflation compared to supply shocks. This was actually used by Coibion and Gorodnichenko (2012) to justify why monetary policy shock was not used in their exercise. Second, I am just running into the same difficulty in identifying real impacts of monetary policy shocks from low frequency

[24] In addition, the degree of long-run disagreements sheds light upon one subtle difference between the SE and NI. Disagreements always exist in NI with the presence of private signals, in contrast, disagreement is zero over the long run in SE.

[25] See Nakamura and Steinsson (2018) as an example of inflationary responses to a monetary tightening move of the Fed.

data as all other papers addressing this issue.

4.2 Additional evidence from uncertainty

In Figure 10, I extend the sample period to include the post-2007 years up till 2019. Importantly, post2007 is the period when SPF starts surveying density forecasts that allow us to study the dynamics of uncertainty. In addition, the post-crisis period has been characterized by a number of different features in terms of the behavior of macroeconomy(i.e. persistently low inflation) and monetary policy implementation (i.e. emphasis on monetary policy communication and zero lower bound), therefore it is worth examining if the patterns documented by Coibion and Gorodnichenko (2012) remain for the entire period. Also, since PCE inflation forecast is not available until 2007, I report results for both CPI and PCE as in previous analysis.

The left panel of Figure 10 presents impulse responses of average uncertainty of SPF in addition to that inflation, forecast error, disagreements in response to technology and oil price shocks. As shown in the top three figures, the results are consistent with that in Figure 9 for the period before 2007. Oil shock(technology shock) increases(decreases) inflation in the first quarter by 0.1 percentage points, whose impacts last till 7-8th quarters. The confidence interval of the responses is smaller compared to the previous figure probably due to the larger sample size. Average forecast errors react partially to the shock and gradually return to zero over the 4-5th quarters afterward. Disagreements picked up in the second quarter and return to zero 2 or 3 quarters later.

The primary focus of this paper is the behavior of uncertainty. Its responses to the absolute value of various shocks are presented at the bottom of Figure 10. Since average uncertainty do not directly depend on the realizations and sizes of the shocks in various models, as shown in Equation 5, Equation 13, and Equation 25, the response of uncertainty cannot be used as a screening device of FIRE from rigidity models. Instead, it checks the consistency of different theories with respect to the behavior of uncertainty.

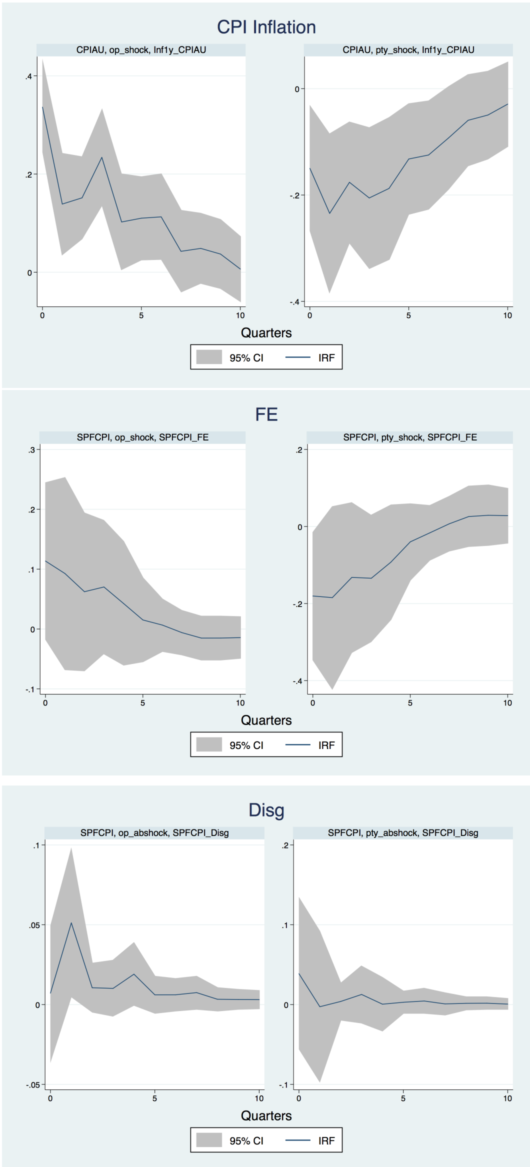
The response graph seems consistent with this simple prediction, except for the uncertainty about CPI in responses to oil price shock. The responses of average uncertainty is not significantly different from zero throughout 10 quarters. [26]

Impacts of monetary policies are shown in the right panel of Figure 10. Path surprise to federal funds rate, i.e. future monetary policy stances, again brings about the inflationary effect as before. One standard deviation of true tightening shock increases headline CPI inflation by around 0.15 percentage points in the first period. In the same time, unexpected rate change does not affect inflation significantly once we include the post-2007 period. One possible explanation for this is the small variation of target surprise after the financial crisis, as illustrated in Figure 8. Except for the dramatic monetary loosening during the financial crisis, there was a little variation in the unexpected federal fund rate change.

Similar to results for the pre-2007 period, the initial response of average forecast

[26]The impulse response that provides direct checking of rigidity models versus FIRE is the responses of revision in uncertainty, i.e. uncertainty about inflation in $t + h$ at time t relative to $t - 1$, $t - 2$, etc. The results will be presented in a revised version of the paper.

(a) Technology and Oil Shocks



(b) Monetary Policy Shocks

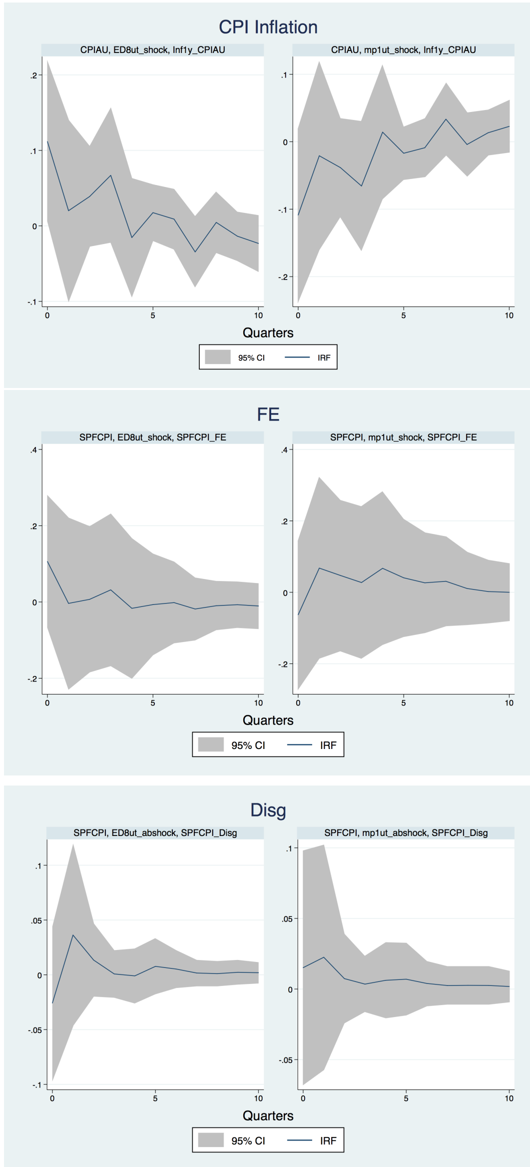


Figure 9: Responses of Inflation and Professional Forecast Moments: 1984-2007

error to future tightening shock is statistically significant in the first period but insignificant soon after. Interestingly, even though the target surprise does not affect inflation itself, forecast errors do react to it in an inflationary manner. The magnitude of the response is as large as 0.25 percentage points in the first quarter. Rigidity models predict forecast errors depend on past shocks, while there does not seem to be strong evidence for this when it comes to monetary policy shocks. Forecasters reaction to monetary policy shocks appear to more efficient than for supply shocks.

Disagreements show little response to monetary policy shocks as well, with the only exception being PCE forecast with respect to path surprise. The estimates of these responses are not significantly from zero throughout the entire horizon. Non-response of disagreements could arise if forecasters have incorporated the monetary policy shocks (thus the shock no longer qualified as a "shock"), or all forecasters simultaneously incorporating the realization of the monetary shocks. In the context of NI model, the disagreements can only arise with the presence of private signals. Therefore, the monetary policy being a more public signal can help explain the muted response of disagreements in comparison with other shocks. [27]

Lastly, average uncertainty does not react to monetary policy shocks in the extended period. None of the response is significantly different from zero. This is the same with technology and oil price shocks.

4.3 Evidence from households

It is widely documented in the literature that behaviors of households expectations are more idiosyncratic in comparison with professional forecasters. [28] This section extends the impulse responses estimation of externally identified shocks to the household forecasts moments of SCE. The monthly data of SCE allows analysis in a higher frequency but for a shorter period 2013-2019.

As technology shock can only be recovered from the quarterly time series of output, I only present results using oil price shock and monetary policy shocks in Figure 11. The responses to oil price shocks, target surprise and path surprise are plotted in column 1, 2 and 3, respectively.

Oil price shock increases CPI by 0.07-0.08 percentage points in the first period and the impacts last over the entire 10 months. This is not surprising from the previous results on quarterly data.

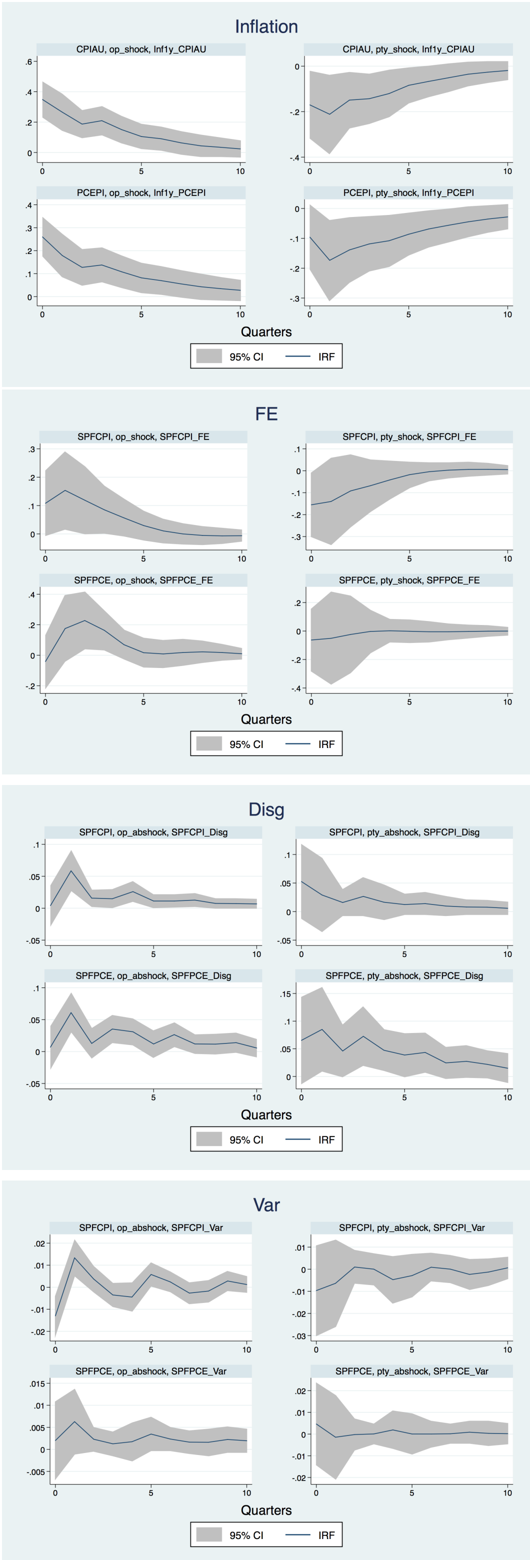
Estimated in higher frequency, the response to target surprise is on average 0.25 percentage points and statistically significant in 10% level. This confirms the wide practice of the literature that identifies monetary non-neutrality using high-frequency data. In contrast, the inflation responses to path surprise are not significant.

As to the households forecast, the starkly wider confidence band across shocks and moments seen in Figure 11 are not that surprising given the wide dispersion of the forecasts moments of a dramatically larger magnitude seen in Figure 5.

[27] Another explanation is that monetary policies actually coordinate agents' expectations, thus reduce the disagreement, according to Morris and Shin (2002). This mechanism can mute the response of the disagreement.

[28] Carroll (2003), for instance, uses median professional forecasts as an approximate of the "rational" forecasts.

(a) Technology and Oil Shocks



(b) Monetary Policy Shocks

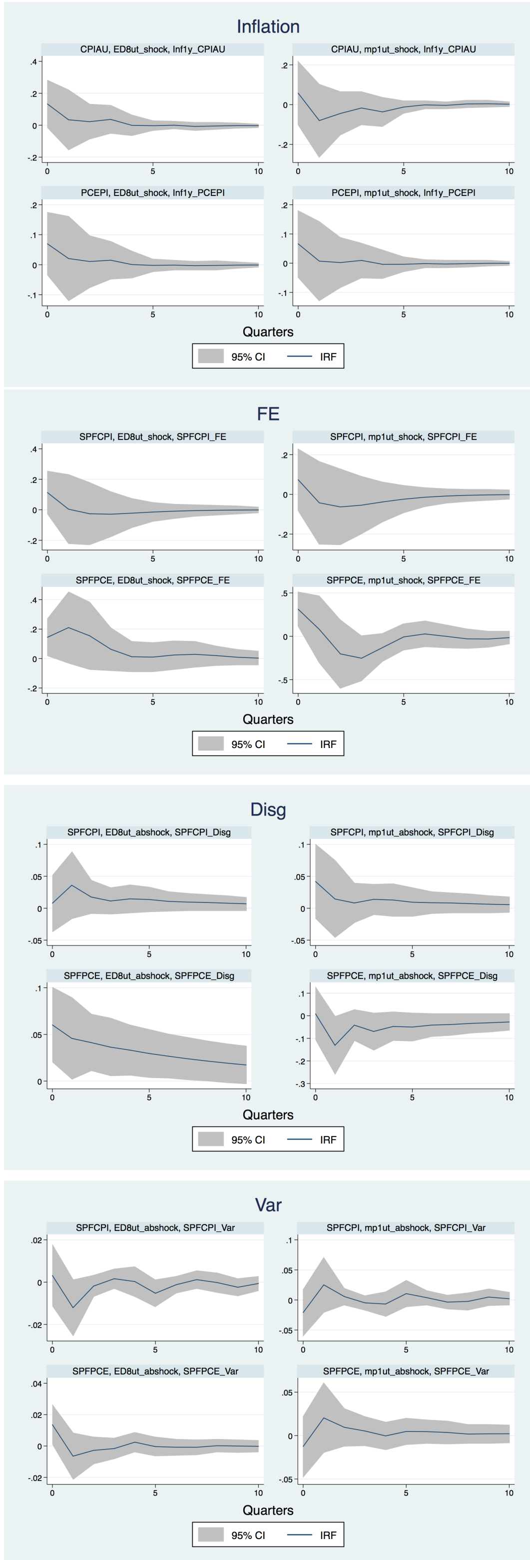


Figure 10: Responses of Inflation and Professional Forecast Moments: 1984-2019

One of the major patterns arising here is that there is no notable difference in responses to oil price shock versus monetary policy shocks, as seen in professional forecasters. Average forecast errors react to none of these shocks and neither do disagreements and uncertainty.

One interesting exception is the response of disagreements to oil price changes. Households' disagreement concerning inflation drops by one percentage point in the first period after an oil price shock and the response have a reasonably small standard error. The drop only lasts for 1 month and immediately return to zero since then. This suggests that households are digesting the shock in an efficient manner.

Researchers have also conjectured (in spite of limited evidence) that salient items such as gasoline price plays an important role in affecting household inflation expectations. The evidence presented here seems to be consistent with such a prediction.

4.4 Discussion of the results

This section presents one major additional insight in addition to Coibion and Gorodnichenko (2012): expectation rigidity is dependent on the type of shock. In particular, professional forecasters are slow in digesting oil and technology shock to resolve disagreements and reduce forecast error while reacting to monetary policy shocks relatively more efficient. This is the easiest seen in the non-response or instantaneous response if any of forecast error and disagreements to monetary policy shocks. The similar can be said for households, in contrast, for oil price shock instead of monetary policy shocks.

Shock-dependent rigidity is not a default feature in the benchmark rigidity models such as SE and NI, i.e. ω_t in Equation 1. It is true that the distinction between public and private signals within NI provides one possibility of reconciling the difference in response to shocks. If we interpret different "shocks" considered here as noisy signals instead of shocks to a fundamental variable as defined in Equation 1, then the difference across "shocks" in their degree of publicity may account for the difference in rigidity. But this implicitly implies redefining the framework presented in this paper.

Moreover, the finding that average uncertainty of fixing horizon by forecasters and households do not react to all the shocks is consistent with the current framework although it does not differentiate rigidity models from FIRE benchmark.

Efficient revision implies uncertainty drop as much as the volatility of the realized shock as one approach the terminate date. Given the volatility of shocks are different, this provides the possibility of identifying rigidity by directly comparing the observed revision in uncertainty with the externally identified volatility of various shocks. This will be the next step of the analysis.

5 Simulated Method of Moments Estimation of different models

For a given process of inflation and a particular theory of expectation formation, the SMM estimate of the vector of parameters Ω is defined as the following.

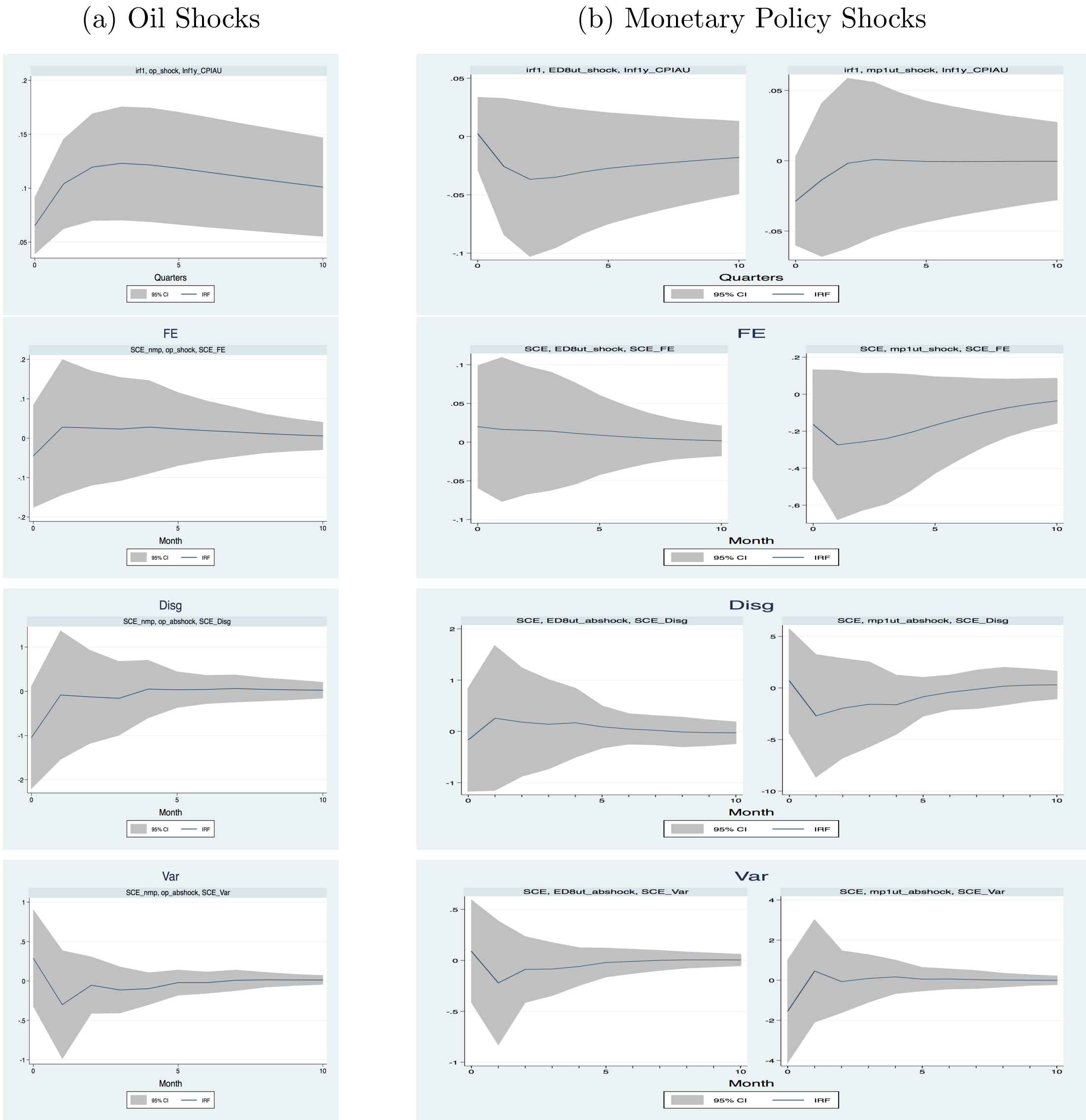


Figure 11: Responses of Inflation and Household Forecast Moments: 2013-2019

$$\widehat{\Omega} = \underset{\{\Omega \in \Gamma\}}{argmin} (M_{\text{data}} - F^o(\Omega, Y)) W (M_{\text{data}} - F^o(\Omega, Y))'$$

- Ω: parameters of the particular $o \in \{fire, se, ni, de\} \times \{ar, sv\}$
- Γ: constraints for the parameter.
- M_{data}: data moments
- F: simulated model moments according to a particular theory o, a function of parameters Ω as well as the Y, the real-time data (including history) up till each point of the time t.
  - unconditional moments, not specific to time
  - moments selected from average forecast, variance and autocovariance of forecasts, average diagreement, variance and autovariance of disagreement, average uncertainty, etc.
- W: weight matrix, identity matrix for now

The estimation procedure is as followed.

1. for each theory of expectation formation and the inflation process, start with an initial value for the parameter(s) of interest
2. simulate individual forecasts for a large enough ($N = 200$) number of forecasters
3. compute the average forecast errors, disagreement and average uncertainty across all agents
4. compute the time-series moments of the average forecast, disagreement, and uncertainty
5. compute the difference between the simulated moments and the data moments
6. keep searching the parameter value until reaching below a threshold of the loss

We evaluate a particular theory of expectation formation based on following four sensitivity criteria: if the parameter and goodness of fit is robust to

1. use of different moments in estimation
2. alternative assumption about the underlying process
3. two-step estimation or joint-estimation
4. relatively fit with professionals and households

5.1 Moments matching and parameter estimates (draft in process)

Table 6 presents the SMM estimates of different model parameters. For each theory, I estimate the theory both in two steps or jointly using expectations and inflation data. Different rows within each panel reports the estimates depending on various choices of moments used for estimation: forecast errors only, forecast error and disagreement, and the two plus uncertainty.

5.2 The scoring card of different theories (draft in process)

Table 10 reports my evaluation of the three theories based on four sensitivity criteria we discussed in the previous section. According to this evaluation, the sticky expectation seems to capture the average behavior of expectations better than the other two theories. Diagnostic expectation shows similar degree of performance according to these criteria, but the average estimation does not show overreaction in lign with the premise of the theory.

6 Conlusion

In summary, three major conclusions can be drawn from this paper. First, full information rational expectation implicitly assumes agents share the common information and agree on the model that generates the data. Therefore, agents do not only agree on the mean forecast of a macroeconomic variable but also share the same degree of uncertainty. The empirical dispersion of individual uncertainty in future inflation seems to be inconsistent with this simple prediction.

Second, the rigidity of expectation from either inattention or under-reaction due to noisiness of the signals implies agents do not revise forecasts as efficiently as they do in FIRE in the face of new information. This can be seen from both a sluggish pattern of forecast revision or uncertainty reduction across different vintages of the forecast. A weaker test of this spirit uses the changes in mean forecasts and uncertainty with fixed forecasting horizon. These tests support rigidity models in addition to other null hypotheses tests of rational expectation including unbiasedness, non-serial-correlation of forecast errors, etc.

Lastly, the directly estimated response of households and professional forecasters' expectation to shocks to inflation suggest that rigidity differs across the shocks. In particular, professional forecasters are more responsive to monetary policy shocks and households are more responsive to oil price shocks. This motivates a next-step analysis of how much resolution of uncertainty can be attributed to the volatility of various shocks. This shows the use of uncertainty.

Finally, the cross-moment estimation developed by this paper jointly accounting for the moments predictions provides a unified framework of evaluating different theories. The estimation suggest that among all combined theory of expectation formation and inflation process, sticky expectation (SE) augmented with stochastic volatility of inflation process matches data of inflation and expectations better than other theories for both professionals and households. The exercise also shows that incorporating higher

Table 6: SMM Estimates of Different Models: Professionals

SE						
Moments Used	2-Step Estimate	Joint Estimate				
	SE: $\hat{\lambda}_{SPF}$(Q)	SE: $\hat{\lambda}_{SPF}$(Q)	SE: ρ	SE: σ		
FE	0.47	0.36	1	0.08		
FE+Disg	0.47	0.38	1	0.1		
FE+Disg+Var	0.47	0.36	1	0.08		
NI						
Moments Used	2-Step Estimate		Joint Estimate			
	NI: $\hat{\sigma}_{pb,SPF}$	$\hat{\sigma}_{pr,SPF}$	NI: $\hat{\sigma}_{pb,SPF}$	$\hat{\sigma}_{pr,SPF}$	NI: ρ	NI: σ
FE	0.09	2.77	0.093	1.408	0.911	0.422
FE+Disg	0.09	2.77	0.093	1.408	0.911	0.422
FE+Disg+Var	0.14	3.85	0.133	1.359	0.911	0.422
DE						
Moments Used	2-Step Estimate		Joint Estimate			
	DE: θ_{SPF}	DE: $\sigma_{\theta,SPF}$	DE: θ_{SPF}	DE: $\sigma_{\theta,SPF}$	DE: ρ	DE: σ
FE	-0.23	0.22	NA	NA	NA	NA
FE+Disg	-0.26	1.41	-0.14	1.44	0.99	0.16
FE+Disg+Var	-0.24	1.43	-0.17	1.44	0.99	0.16

Table 7: SMM Estimates of Different Models: Households

SE						
Moments Used	2-Step Estimate	Joint Estimate				
	SE: $\hat{\lambda}_{SPF}$(Q)	SE: $\hat{\lambda}_{SPF}$(Q)	SE: ρ	SE: σ		
FE	0.2	0.5	0.84	0.25		
FE+Disg	0.21	0.54	0.92	0.18		
FE+Disg+Var	0.21	0.5	0.84	0.25		
NI						
Moments Used	2-Step Estimate		Joint Estimate			
	NI: $\hat{\sigma}_{pb,SPF}$	$\hat{\sigma}_{pr,SPF}$	NI: $\hat{\sigma}_{pb,SPF}$	$\hat{\sigma}_{pr,SPF}$	NI: ρ	NI: σ
FE	3.4	15.4	3.397	15.395	0.997	0.027
FE+Disg	3.4	15.4	3.397	15.395	0.997	0.027
FE+Disg+Var	4.9	22.4	4.860	22.367	0.997	0.027
DE						
Moments Used	2-Step Estimate		Joint Estimate			
	DE: θ_{SPF}	DE: $\sigma_{\theta,SPF}$	DE: θ_{SPF}	DE: $\sigma_{\theta,SPF}$	DE: ρ	DE: σ
FE	9.35	10.65	0.82	0.85	1	0
FE+Disg	8.2	9.52	4.79	4.59	0.58	0.55
FE+Disg+Var	4.78	3.01	NA	NA	NA	NA

Table 8: SMM Estimates of Different Models under Stocastic Volatility: Professionals

		SE			
Moments Used	2-Step Estimate	Joint Estimate			
	SE: $\hat{\lambda}_{SPF}$(Q)	SE: $\hat{\lambda}_{SPF}$(Q)	SE: γ		
FE	0.3	0.46	2.52		
FE+Disg	0.3	0.46	2.53		
FE+Disg+Var	0.3	0.46	1.26		
		NI			
Moments Used	2-Step Estimate		Joint Estimate		
	NI: $\hat{\sigma}_{pb,SPF}$	$\hat{\sigma}_{pr,SPF}$	NI: $\hat{\sigma}_{pb,SPF}$	$\hat{\sigma}_{pr,SPF}$	γ
FE	2.35	2	2.04	23.01	2.53
FE+Disg	37475663924	4.8	70269682.39	10.18	-2.91
FE+Disg+Var	3.33	1.71	2.04	22.96	2.53
		DE			
Moments Used	2-Step Estimate		Joint Estimate		
	DE: θ	σ_θ	DE: θ	σ_θ	γ
FE	-0.44	0.36	-0.43	1.03	0.13
FE+Disg	-0.44	0.27	-0.44	-0.27	0.3
FE+Disg+Var	-0.43	0.26	-0.43	0.26	0.14

Table 9: SMM Estimates of Different Models under Stocastic Volatility: Households

		SE			
Moments Used	2-Step Estimate	Joint Estimate			
	SE: $\hat{\lambda}_{SPF}$(Q)	SE: $\hat{\lambda}_{SPF}$(Q)	SE: γ		
FE	0.09	0.09	0.7		
FE+Disg	0.07	0.07	0.26		
FE+Disg+Var	0.07	0.07	0.26		
		NI			
Moments Used	2-Step Estimate		Joint Estimate		
	NI: $\hat{\sigma}_{pb,SPF}$	$\hat{\sigma}_{pr,SPF}$	NI: $\hat{\sigma}_{pb,SPF}$	$\hat{\sigma}_{pr,SPF}$	γ
FE	NA	NA	NA	NA	NA
FE+Disg	NA	NA	NA	NA	NA
FE+Disg+Var	NA	NA	NA	NA	NA
DE					
Moments Used	2-Step Estimate		Joint Estimate		
	DE: θ	σ_θ	DE: θ	σ_θ	γ
FE	7.81	4.39	7.81	2.99	0.7
FE+Disg	7.64	6.46	7.64	6.46	0.7
FE+Disg+Var	1.03	0	1.03	0	0.2

moments, i.e. uncertainty, helps "discipline" theories on expectation formation. Besides, higher moments from surveys also contain useful information about the inflation dynamics itself

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Table 10: A scoring card of different theories

Criteria	SE	NI	DE
Sensitive to moments used for estimation?	No	No	No
Sensitive to the assumed inflation process?	No	Yes	No
Sensitive to a two-step or joint estimate?	No	Yes	No
Sensitive to the type of agents?	Yes	Yes	Yes
Matching with FE	Yes	Yes	Yes
Matching with disagreement	Yes	No	Unsure
Matching with uncertainty	Unsure	No	No