Krister Wiklund
Dept. of Physics
Umeå University

ANSWERS TO EXERCISES

1.4* $u_r = \frac{Ur}{2(H_0 - Ut)}$ $\quad u_z = -\frac{Uz}{H_0 - Ut}$

1.5 a) $u_y = \sum_i A_i \sin(k_i x)\cos(k_i y)$ $\quad$ b) $P = -\frac{1}{4}\rho A^2[\cos(2kx) + \cos(2ky)]$

1.8 $F = \frac{1}{2}\rho g L h(2H - h)$ $\quad$ a) $T = \frac{1}{6}\rho g L h^2(3H - h)$ $\quad$ b) $T = \frac{1}{4}\rho g L^2 h(2H - h)$

1.9 $\mathbf{F}_{\text{per unit length}} = -\hat{\mathbf{n}}\frac{g}{2\sin\theta}H^2(\rho_0 + \frac{\alpha H}{3})$

1.11 $dF_{x,\text{per unit length}} = \rho g\frac{R^2}{4}$ $\quad$ $\mathrm{dF}_{z,\text{per unit length}} = \frac{5\rho g R^2}{4}$

2.2 Note, z_f is continuous at $r = R$

$$z_f = \frac{\Omega^2 r^2}{2g}, \qquad r < R$$

$$z_f = -\frac{\Omega^2 R^4}{2gr^2} + \frac{\Omega^2 R^2}{g}, \qquad r > R$$

$$\Delta h = \frac{\Omega^2 R^2}{g}$$

1.4** Note, a quasi-stationary flow ($H_0 >> Ut$), there is a typing error in the preliminary schedule where the problem is given.

$$P = P_{atm} + \frac{\rho U^2}{8H_0^2}(R^2 - r^2) \qquad P(r=0) = P_{atm}\frac{\rho U^2 R^2}{8H_0^2} \qquad F_{P,flow} = \frac{\rho U^2 \pi R^4}{16H_0^2}$$

2.3 $Q = US = \frac{\pi}{4}D^2\sqrt{2(\Theta - 1)gH}$

2.4 $\mathbf{F} = (P_1 + \rho U_1^2)S_1\mathbf{e}_y + (P_1 + \frac{1}{2}\rho U_1^2(1 + \frac{S_1^2}{S_2^2}))S_2(\cos\alpha\mathbf{e}_y - \sin\alpha\mathbf{e}_x)$

The coordinate system has been chosen as $\hat{\mathbf{n}}_1 = -\mathbf{e}_y$ and $\mathbf{e}_x$ points to the right ($\mathbf{e}_x \perp \mathbf{e}_y$).

2.6 $\mathbf{F}_{xy} = \mathbf{e}_x\sqrt{3}\rho g(h_2S_2 - h_1S_1) + \mathbf{e}_y\rho g[h_1S_1 + h_2S_2 - \frac{2}{S_3}(\sqrt{h_1}S_1 + \sqrt{h_2}S_2)^2]$

$\mathbf{F}_z = -\mathbf{e}_z 2\rho g\sqrt{h_3}(\sqrt{h_1}S_1 + \sqrt{h_2}S_2)$

The coordinate system has been chosen as x-axis to the right, y-axis backwards in the direction of the water supply pipe, and z-axis upwards.

2.7 a) $U = U_2 = \sqrt{\frac{2gL}{S_2^2 - S_3^2}} S_3$ b) $h = h_\infty + (h_0 - h_\infty)(1 - \frac{S_1}{S_0}\sqrt{\frac{gt^2}{2(h_0 - h_\infty)}})^2$,

where h_0 are h_∞ are the height of the fluid level in the reservoir at the beginning (A) and the height when the flow in the pipe stops after a long time, respectively.

2.10 a) $S_1 = \frac{S_0}{2}(1 + \cos\theta)$, $S_2 = \frac{S_0}{2}(1 - \cos\theta)$
b) $\mathbf{F}_p = \hat{\mathbf{n}}_p \rho S_0 (U_0 - U_p)^2 \sin\theta$, where the unit normal of the plate ($\hat{\mathbf{n}}_p$) has been defined that it has a component in the direction of the U_p.
c) $W_{max} = \frac{4}{27}\rho S_0 U_0^3 \sin^2\theta$ when $U_p = \frac{U_0}{3}$

2.13 $\phi = \frac{Ar^2}{4} - \frac{Az^2}{2}$ $u_r = \frac{Ar}{2}$ $u_z = -Az$, A is a constant

$zr^2 = C$, (C is a constant) for stream lines $P = P_s - \frac{\rho A^2}{2}(z^2 + \frac{r^2}{4})$

2.14 Note, in a) we calculate only the pressure difference between the pressure exerted on the plate from the left and the pressure at a location far away from the plate in the stagnation zone.

a) $\Delta P = \frac{\rho U^2}{2}\frac{S_t^2}{(S_t - S_p)^2}$ b) $F = \frac{\rho U^2}{2}\frac{S_t S_p^2}{(S_t - S_p)^2}$

2.15 F_{lift}/L_z straightforward, $F_{drag}/L_z = -\rho U \Gamma \tan\alpha$

5.1 a) $U_m = \frac{2}{3}U_0$ b) $F_0 = 0.27 F_m$

5.3 $T \propto \mu\Omega R^3$

5.4 $F \propto \sqrt{\frac{\mu^3 US}{\rho L}}$

5.7 $R \propto t^{2/5}$

5.10 $u_x = \frac{\rho g}{2\mu}\sin\alpha(R - y)y$, discharge per unit length $\frac{Q}{L_z} = \frac{\rho g}{12\mu}\sin\alpha R^3$

5.11 Note, we find u_x, $u_{x,max}$ and the viscous stress acting on the lower plate. In the selected coordinate system x axis is to the right along the lower plate.

$u_x = \frac{\rho g \sin\alpha}{2\ mu}(2R - y)y$ $u_{x,max} = \frac{\rho g \sin\alpha}{2\mu}R^2$ Viscous stress $\frac{F}{S} = \rho g R \sin\alpha$

5.12 $U = \frac{Rg\sin\alpha}{\mu S}(M + \rho RS/2)$

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5.13 $u = u_2 = \frac{\Delta P}{4\mu L}(r^2 - R_2^2) + [U + \frac{\Delta P}{4\mu L}(R_2^2 - R_1^2)]\frac{\ln(r/R_2)}{\ln(R_1/R_2)}$

5.14 $h = h_\infty + (h_0 - h_\infty)exp(-t/\tau)$, where typical time scale $\tau = \frac{8\mu L S_0}{\pi\rho g R^4}$

5.15 $u_{max} = \frac{3}{2}U\frac{x}{H}$

5.24 $u_x = U\exp(-\sqrt{\frac{\omega}{2\nu}}y)\sin(\omega t - \sqrt{\frac{\omega}{2\nu}}y)$

E1 $U = (1-\alpha)g\tau[1 - \exp(-\frac{t}{\tau})]$, where $\tau = \frac{M}{6\pi\mu R}$

6.1 $F_{drag}/L_z = 1.32\rho\nu U\sqrt{\frac{UL}{\nu}}$

7.3 $T = T_0 + \Theta\ \exp(-\alpha x)\sin(\pi y/R)\sin(\pi y/R)$

7.4 $T = T_0 + \Theta\frac{x}{R}$ $\qquad u_z = \frac{g\alpha\Theta}{6R\nu}(R^2 - x^2)x$