

# Algebra and Join Minimization

John Clara, Danyang Zhang

Winter 2016.

# Relational Algebra

## Review on Relational Algebra

# Basic Relational Algebra

- Selection:  $\sigma_{a=C}R$
- Projection:  $\Pi_a R$
- Rename:  $\delta_{a_1 \rightarrow a_2} R$
- Aggregation:  $\gamma_{a, Fn(b) \rightarrow b'} R$
- Binary operator:  $\cup, \cap, \bowtie, \div, \times, -$

# Example Schema

$S :$	sailor	sname	rating	
$B :$	boat	sname	bname	day
$R :$	reservation	bname	color	rating

# Example

List the sailors who have at least one reservation and only reserved red boats.

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$$\Pi_{sname} R - \Pi_{sname} ((\sigma_{color \neq 'red'} B) \bowtie R)$$

# Example

List the sailor name pairs who reserve the same boat.

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List the sailor name pairs who reserve the same boat.

$$\Pi_{sname1, sname2}(\sigma_{sname1 < sname2}(\Pi_{sname1, bname} \delta_{sname \rightarrow sname1} R) \bowtie (\Pi_{sname2, bname} \delta_{sname \rightarrow sname2} R))$$



# Example

List the sailor names who reserve every red boat (assuming there exists red boats). Hint: use  $\div$ .

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List the sailor names who reserve every red boat (assuming there exists red boats).

$\forall$  corresponds to  $\div$ .

$$\Pi_{sname}(R \div \sigma_{color='red'}B)$$

# Example

List the sailor names who reserve every red boat (~~assuming there exists red boats~~). Hint: use two  $-$ .

# Example

SQL using NOT IN:

```
select sname from sailor
where sname NOT IN (
  select sname from sailor, boat b
  where b.color = red and sname NOT IN (
    select sname from reservation
    where bname = b.bname))
```

Relational algebra

$$\begin{aligned} & \Pi_{sname} S - \\ & \Pi_{sname} (\Pi_{sname, boat} (\sigma_{color='red'} B \bowtie S) - \\ & \quad \Pi_{sname, boat} (\sigma_{color='red'} B \bowtie R)) \\ & ) \end{aligned}$$

# Join Minimization

## Join Minimization

# How to Optimize Queries

## Basic Rules:

- Perform different mappings to reduce rows
- Answer variables cannot map to others
- Constants cannot map to others
- Everything else is fair game!

# Example 1

What are all the books by the person who wrote “Twilight”?

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What are all the books by the person who wrote “Twilight”?

```
SELECT b1.title  
FROM Book b1, Book b2, Book b3  
WHERE b1.author = b2.author AND  
      b3.author = b2.author AND  
      b3.title = "Twilight";
```



# Example 1

What are all the books by the person who wrote "Twilight"?

```
SELECT b1.title
FROM Book b1, Book b2, Book b3
WHERE b1.author = b2.author AND
      b3.author = b2.author AND
      b3.title = "Twilight";
```

Book	title	author	answer	title
b1	d	a		
b2	-	a		d
b3	"Twilight"	a		

Can we map first row to any rows?

# Example 1

What are all the books by the person who wrote "Twilight"?

Book	title	author	answer	title
b1	d	a		
b2	-	a		d
b3	"Twilight"	a		

Map second row to some row?

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What are all the books by the person who wrote "Twilight"?

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Map second row to some row?

# Example 1

What are all the books by the person who wrote "Twilight"?

Book	title	author	answer	title
	d	a		d
	"Twilight"	a		

```
SELECT b1.title
FROM Book b1, Book b2
WHERE b1.author = b2.author AND
      b2.title = "Twilight";
```

## Example 2

```
SELECT t1.A, t2.B, t4.C
FROM R t1, R t2, R t3, R t4, R t5
WHERE t3.A=t4.A AND
      t2.B=t3.B AND
      t1.C=t2.C AND
      t3.C=t5.C AND
      t3.A=t5.A;
```

## Example 2

```
SELECT t1.A, t2.B, t4.C
FROM R t1, R t2, R t3, R t4, R t5
WHERE t3.A=t4.A AND
      t2.B=t3.B AND
      t1.C=t2.C AND
      t3.C=t5.C AND
      t3.A=t5.A;
```

R	A	B	C		answer	A	B	C
t1	<b>a</b>	-	c1					
t2	-	<b>b</b>	c1					
t3	a1	<b>b</b>	c2			<b>a</b>	<b>b</b>	<b>c</b>
t4	a1	-	<b>c</b>					
t5	a1	-	c2					

## Example 2

R	A	B	C
t1	a	-	c1
t2	-	b	c1
t3	a1	b	c2
t4	a1	-	c
t5	a1	-	c2

  

answer	A	B	C
	a	b	c

Can we reduce any rows? Cannot map t2 to t3 due to t1.

## Example 2

Reduce t5

R	A	B	C
t1	a	-	c1
t2	-	b	c1
t3	a1	b	c2
t4	a1	-	c

answer	A	B	C
	a	b	c



## Example 2

Can reduce t2 to t3 or t3 to t2?

R	A	B	C									
t1	a	-	c1	<table><tr><th>answer</th><th>A</th><th>B</th><th>C</th></tr><tr><td></td><td>a</td><td>b</td><td>c</td></tr></table>	answer	A	B	C		a	b	c
answer	A	B	C									
	a	b	c									
t2	-	b	c1									
t3	a1	b	-									
t4	a1	-	c									

# Example 2

Dependencies:  $F = \{AC \rightarrow B, B \rightarrow C, C \rightarrow A\}$

R	A	B	C
	a	-	c1
	-	b	c1
	a1	b	-
	a1	-	c

  

answer	A	B	C
	a	b	c

## Example 2

Dependencies:  $F = \{AC \rightarrow B, B \rightarrow C, C \rightarrow A\}$

Use  $B \rightarrow C$

R	A	B	C		A	B	C
	a	-	c1	answer			
	-	b	c1		a	b	c
	a1	b	-				
	a1	-	c				

## Example 2

Dependencies:  $F = \{AC \rightarrow B, B \rightarrow C, C \rightarrow A\}$

Use  $C \rightarrow A$

R	A	B	C		A	B	C
	<b>a</b>	-	c1	answer			
	-	<b>b</b>	c1		<b>a</b>	<b>b</b>	<b>c</b>
	a1	<b>b</b>	c1				
	a1	-	<b>c</b>				

## Example 2

Dependencies:  $F = \{AC \rightarrow B, B \rightarrow C, C \rightarrow A\}$

Eliminate rows

R	A	B	C		answer	A	B	C
	a	-	c1					
	a	b	c1					
	a	b	c1					
	a	-	c					
						a	b	c

## Example 2

Dependencies:  $F = \{AC \rightarrow B, B \rightarrow C, C \rightarrow A\}$

Can we use any Dependencies?

R	A	B	C		answer	A	B	C
	a	b	c1			a	b	c
	a	-	c					

## Example 2

Dependencies:  $F = \{AC \rightarrow B, B \rightarrow C, C \rightarrow A\}$

R	A	B	C	answer	A	B	C
	a	b	-		a	b	c
	a	-	c				

```
SELECT r1.A, r1.B, r2.C
FROM R r1, R r2
WHERE r1.a = r2.a;
```

## Example 3

```
SELECT t1.A, t2.B, t4.C  
FROM R t1, R t2, R t3, R t4  
WHERE t2.C=5 AND t3.A=t4.A AND  
t2.B=t3.B AND t1.C=t2.C AND  
t4.A=8;
```

with functional dependencies:

$$F = \{AC \rightarrow B, B \rightarrow C, C \rightarrow A\}.$$



## Example 3

```
SELECT t1.A, t2.B, t4.C
FROM R t1, R t2, R t3, R t4
  WHERE t2.C=5 AND t3.A=t4.A AND
         t2.B=t3.B AND t1.C=t2.C AND
         t4.A=8;
```

R	A	B	C
t1	$\alpha$	-	5
t2	-	$\beta$	5
t3	8	$\beta$	-
t4	8	-	$\gamma$

  

answer	A	B	C
	$\alpha$	$\beta$	$\gamma$

## Example 3

$$F = \{AC \rightarrow B, B \rightarrow C, C \rightarrow A\}.$$

R	A	B	C		A	B	C
t1	$\alpha$	-	5	answer	$\alpha$	$\beta$	$\gamma$
t2	-	$\beta$	5				
t3	8	$\beta$	-				
t4	8	-	$\gamma$				

After Chase

R	A	B	C		A	B	C
t1	$\alpha = 8$	-	5	answer	$\alpha$	$\beta$	$\gamma$
t2	8	$\beta$	5				
t3	8	$\beta$	5				
t4	8	-	$\gamma$				

## Example 3

After join minimization:

R	A	B	C	answer	A	B	C
t2	8	$\beta$	5		$\alpha = 8$	$\beta$	$\gamma$
t4	8	-	$\gamma$				

The final SQL query:

```
SELECT t2.A, t2.B, t4.C
FROM R t2, R t4
WHERE t2.A=8 AND t2.C=5 AND t4.A=8;
```

## Example 4

Given the following pattern, minimize the pattern.

```
SELECT t1.A, s1.E FROM R t1, R t2, R t3, R t4
S s1, S s2 WHERE
  t1.B=t2.B AND t2.C=t3.C AND
  t3.A=t4.A AND t4.B=s2.B AND
  s2.D=s1.D;
```

R	A	B	C	S	B	D	E	answer	A	E
t1	$\alpha$	b1	-	s1	-	d	$\epsilon$		$\alpha$	$\epsilon$
t2	-	b1	c	s2	b	d	-			
t3	a	-	c							
t4	a	b	-							

# Example 4

R	A	B	C	S	B	D	E	answer	A	E
t1	$\alpha$	b1	-	s1	<b>b'</b>	d	$\varepsilon$		$\alpha$	$\varepsilon$
t2	-	b1	c	s2	b	d	-			
t3	a	-	c							
t4	a	b	-							

# Example 4

R	A	B	C							
t1	$\alpha$	b1	-	S	B	D	E	answer	A	E
t2	-	b1	c	s1	b'	d	$\varepsilon$		$\alpha$	$\varepsilon$
t3	a	-	c							
t4	a	<b>b'</b>	-							

# Example 4

R	A	B	C							
t1	$\alpha$	b1	-	S	B	D	E	answer	A	E
t2	-	b1	c	s1	b'	d	$\varepsilon$		$\alpha$	$\varepsilon$
t3	a	<b>b''</b>	c							
t4	a	b'	-							

# Example 4

R	A	B	C	S	B	D	E	answer	A	E
t1	$\alpha$	b1	-	s1	<b>b''</b>	d	$\varepsilon$		$\alpha$	$\varepsilon$
t2	-	b1	c							
t3	a	b''	c							



# Example 4

R	A	B	C	S	B	D	E	answer	A	E
t1	$\alpha$	b1	-	s1	<b>b1</b>	d	$\epsilon$		$\alpha$	$\epsilon$
t2	-	b1	c							

# Example 4

R	A	B	C	S	B	D	E	answer	A	E
t1	$\alpha$	b1	-	s1	b1	-	$\varepsilon$		$\alpha$	$\varepsilon$

# Example 5

$$F = \{A \rightarrow B, BC \rightarrow A\}$$

R	A	B	C	D
	a	-	c	-
	a	b	-	-
	-	b	c	d
answer	A	B	C	D
	a	b	c	d

# Example 5

Use  $A \rightarrow B$

$$F = \{A \rightarrow B, BC \rightarrow A\}$$

R	A	B	C	D
	a	b	c	-
	a	b	-	-
	-	b	c	d
answer	A	B	C	D
	a	b	c	d

# Example 5

Reduce rows

$$F = \{A \rightarrow B, BC \rightarrow A\}$$

R	A	B	C	D
	a	b	c	-
	-	b	c	d
answer	A	B	C	D
	a	b	c	d

# Example 5

Use  $BC \rightarrow A$

$$F = \{A \rightarrow B, BC \rightarrow A\}$$

R	A	B	C	D
	a	b	c	-
	a	b	c	d
answer	A	B	C	D
	a	b	c	d

# Example 5

Reduce rows

$$F = \{A \rightarrow B, BC \rightarrow A\}$$

R	A	B	C	D
	a	b	c	d
answer	A	B	C	D
	a	b	c	d

# Reference

- ① “*Database Systems Concepts*” by Silberschatz, Korth and Sudarshan, 6th edition, McGraw-Hill.